Algebraic Expressions and Linear Equation

NOTES

Algebraic Expression

Algebraic expression is the combination of constants and variables along with the fundamental operations $(+,-,\times,\div)$. The part of an algebraic expression which is separated by the sign of addition and subtraction are called terms.

Types of Algebraic Expression

T-re following are the few types of algebraic expression:

- Monomials
- Binomials
- Trinomials

Finding the Value of an Algebraic Expression

To find the value of an algebraic expression, first simplify the given algebraic expression if possible and replace the variable with given numerical value.

- **Example:**

List the following algebraic expression into monomial, binomial or trinomial:

$3z^2$, $3a + 4b + 5c$, $a^3 + b^3 - 3ab$, $5a$, $6x + 4y$, $35x + 5y - 35(x - y)$

Solution: Monomial: $3z^2$, $5a$, $35x + 5y - 35(x - y)$, as these have only one term.

Binomial: $6x + 4y$, because it has only two terms.

Trinomial: $3a + 4b + 5c$, $a^3 + b^3 - 3ab$, because they have only 3 terms.

Algebraic Identities

- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$
- $a^2 - b^2 = (a-b)(a+b)$
- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
- $(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$ or $(a+b)^3 = a^3 + b^3 ab(a+b)$
- $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
- $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$ or $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a-b)^3 + 3ab(a-b)$ or $a^3 + b^3 = (a-b)(a^2 + ab + b^2)$
- $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 + c^2 - ab - bc - ca)$

If $(a + b + c) = 0$, *then* $a^3 + b^3 + c^3 = 3abc$

- **Example:**

Find the value of $\frac{\mathbf{a^3+b^3+c^3-3abc}}{\mathbf{a+b+c}}$ **If a = 1, b = 2 and c = - 1.**

(a) 4 (b) 1

(c) 7 (d) 2

(e) None of these

Ans. (c)

Explanation: $\frac{a^3+b^3+c^3-3abc}{a+b+c}$

$$= \frac{(a+b+c)\left(a^2+b^2+c^2-ab-bc-ca\right)}{a+b+c} = \left(a^2+b^2+c^2-ab-bc-ca\right)$$

Now putting the values of a, b and c, we get

$$\left(a^2+b^2+c^2-ab-bc-ca\right) = 1+4+1-2+2+1 = 7$$

Polynomials

Polynomials are algebraic expressions in which the variables have only non-negative integral power. In other words, a polynomial is an algebraic expression involving a sum of powers in one or more variables. Degree of Polynomial is the highest power of a variable in it.

Note: The degree of a polynomial is the largest exponent of that variable. The Polynomials of degree 1 and degree 2 are called linear polynomials and quadratic polynomials respectively.

Factorization of Polynomials

An algebraic expression is expressed as the product of two or more expressions and each of which is a factor of the given algebraic expression. The process of writing a given algebraic expression as the product of two or more factors is called factorization.

- **Example:**

Factorize, $\mathbf{a^3+8b^3+6a^2b+12ab^2}$**.**

(a) $(a+2b)^3$ (b) $(-a+2b)^3$

(c) $(a-2b)^3$ (d) $-(a+2b)^3$

(e) None of these

Ans. (a)

Explanation: $a^3+8b^3+6a^2b+12ab^2 = a^3+8b^3+6ab(a+2b) = (a+2b)^3$

Linear Equation

We know that a mathematical statement of equality which involves one or more than one variables is called an equation. An equation in which variables are of degree one is called linear equation.

For example, $4x + 5 = 9$ is a linear equation in one variable.

Solution of the Linear Equation

The real number which satisfies the given linear equation is called the solution of the equation.

The following methods are used to solve a linear equation:

- Trial and error method
- Systematic method
- Transposition Method

- **Example:**

Find the solution of the equation $\mathbf{x + 4 = 5x - 8}$.

(a) $x = 3$ (b) $x = -5$

(c) $x = -2$ (d) $x = -9$

(e) None of these

Ans. (a)

Explanation: $x + 4 = 5x - 8 \Rightarrow 5x - x = 4 + 8 \Rightarrow 4x = 12 \Rightarrow x = 3$

- **Example:**

Solve for $\mathbf{x}$: $\mathbf{19x + 2 = 40}$

(a) 0 (b) 1

(c) 2 (d) 3

(e) None of these

Ans. (c)

Explanation: Using trial and error method: for x = 1, $19x + 2 = 40$

$\Rightarrow 19x + 2 = 19 \times 1 + 2 = 21 \neq R.H.S$

for $x = 2$, $19x + 2 = 19 \times 2 + 2 = 40 = R.H.S$

So, $x = 2$ is the solution of the given equation

Systematic method: $19x + 2 = 40$

$\Rightarrow 19x + 2 - 2 = 40 - 2$ (subtracting 2 from both sides)

$\Rightarrow 19x + 0 = 38 \Rightarrow 19x = 38$

$\Rightarrow \frac{19x}{19} = \frac{38}{19}$ (dividing both sides by 19) $\Rightarrow x = 2$

Therefore, $x = 2$ is the solution of given equation.

- **Example:**

Amit is 46 years old. He is 4 years older than thrice his son's age. Find the age of his son.

(a) 16 years (b) 14 years

(c) 22 years (d) 24 years

(e) None of these

Ans. (b)

Explanation: let his son's age be x years.

Then, $3x + 4 = 46 \Rightarrow 3x = 46 - 4 \Rightarrow 3x = 42 \Rightarrow x = 14$

So, his son's age is 14 years.