QUADRATIC EQUATIONS [JEE ADVANCED PREVIOUS YEAR SOLVED PAPER]

JEE ADVANCED

Single Correct Answer Type

1. If l, m, n are real $l \neq m$, then the roots of the equation $(l - m)x^2 - 5(l + m)x - 2(l - m) = 0$ are

a. real and equal **b.** complex
c. real and unequal **d.** none of these

(IIT-JEE 1979)

2. If x, y, and z are real and different and $u = x^2 + 4y^2 + 9z^2 - 6yz - 3zx - 2xy$, then u is always

a. non-negative **b.** zero
c. non-positive **d.** none of these

(IIT-JEE 1979)

3. If $a > 0$, $b > 0$ and $c > 0$, then the roots of the equation $ax^2 + bx + c = 0$

a. are real and negative
b. have positive real parts
c. have negative real parts
d. none of these

(IIT-JEE 1979)

4. The entire graph of the equation $y = x^2 + kx - x + 9$ is strictly above the x-axis if and only if

a. $k < 7$ **b.** $-5 < k < 7$
c. $k > -5$ **d.** none of these

(IIT-JEE 1979)

5. Both the roots of the equation $(x - b)(x - c) + (x - a)(x - c) + (x - a)(x - b) = 0$ are always

a. positive **b.** real
c. negative **d.** none of these

(IIT-JEE 1980)

6. If $(x^2 + px + 1)$ is a factor of $(ax^3 + bx + c)$, then

a. $a^2 + c^2 = -ab$ **b.** $a^2 - c^2 = -ab$
c. $a^2 - c^2 = ab$ **d.** none of these

(IIT-JEE 1980)

7. Two towns A and B are 60 km apart. A school is to be built to serve 150 students in town A and 50 students in town B. If the total distance to be travelled by all 200 students is to be as small as possible, then the school be built at

a. town B **b.** 45 km from town A
c. town A **d.** 45 km from town B

(IIT-JEE 1982)

8. The equation $x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$ has

a. no root **b.** one root
c. two equals roots **d.** infinitely many roots

(IIT-JEE 1984)

9. If α and β are the roots of $x^2 + px + q = 0$ and α^4, β^4 are the roots of $x^2 - rx + s = 0$, then the equation $x^2 - 4qx + 2q^2 - r = 0$ has always

a. one positive and one negative root
b. two positive roots
c. two negative roots
d. cannot say anything

(IIT-JEE 1989)

10. Let a, b, c be real numbers, $a \neq 0$. If α is a root of $a^2x^2 + bx + c = 0$. β is the root of $a^2x^2 - bx - c = 0$ and $0 < \alpha < \beta$, then the equation $a^2x^2 + 2bx + 2c = 0$ has a root γ that always satisfies

a. $\gamma = \frac{\alpha+\beta}{2}$ **b.** $\gamma = \alpha + \frac{\beta}{2}$ **c.** $\gamma = \alpha$ **d.** $\alpha < \gamma < \beta$

(IIT-JEE 1989)

11. Let α, β be the roots of the equation $(x-a)(x-b)=c$, $c \neq 0$. Then the roots of the equation $(x-\alpha)(x-\beta)+c=0$ are

a. a, c **b.** b, c **c.** a, b **d.** $a+c, b+c$

(IIT-JEE 1992)

12. The number of points of intersection of two curves $y = 2\sin x$ and $y = 5x^2 + 2x + 3$ is

a. 0 **b.** 1 **c.** 2 **d.** ∞

(IIT-JEE 1994)

13. If p, q, r are positive and are in A.P., the roots of quadratic equation $px^2 + qx + r = 0$ are all real for

a. $\left|\frac{r}{p}-7\right| \geq 4\sqrt{3}$ **b.** $\left|\frac{p}{r}-7\right| \geq 4\sqrt{3}$

c. all p and r **d.** no p and r

(IIT-JEE 1994)

14. The equation $\sqrt{x+1}-\sqrt{x-1}=\sqrt{4x-1}$ has

a. no solution **b.** one solution

c. two solutions **d.** more than two solutions

(IIT-JEE 1997)

15. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3, then

a. $a < 2$ **b.** $2 \leq a \leq 3$ **c.** $3 < a \leq 4$ **d.** $a > 4$

(IIT-JEE 1999)

16. If α and β $(\alpha < \beta)$ are the roots of the equation $x^2 + bx + c = 0$, where $c < 0 < b$, then

a. $0 < \alpha < \beta$ **b.** $\alpha < 0 < \beta < |\alpha|$

c. $\alpha < \beta < 0$ **d.** $\alpha < 0 < |\alpha| < \beta$

(IIT-JEE 2000)

17. If $b > a$, then the equation $(x-a)(x-b) - 1 = 0$ has

a. both roots in (a, b)

b. both roots in $(-\infty, a)$

c. both roots in $(b, +\infty)$

d. one root in $(-\infty, a)$ and the other in $(b, +\infty)$

(IIT-JEE 2000)

18 For the equation $3x^2 + px + 3 = 0$, $p > 0$, if one of the roots is square of the other, then p is equal to

a. 1/3 **b.** 1 **c.** 3 **d.** 2/3

(IIT-JEE 2000)

19. Let $f(x) = (1 + b^2)x^2 + 2bx + 1$ and let $m(b)$ be the minimum value of $f(x)$. As b varies, the range of $m(b)$ is

a. $[0, 1]$ **b.** $\left(0, \frac{1}{2}\right]$

c. $\left[\frac{1}{2}, 1\right]$ **d.** $(0, 1]$ **(IIT-JEE 2001)**

20. Let α, β be the roots of $x^2 - x + p = 0$ and γ, δ be roots of $x^2 - 4x + q = 0$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then the integral values of p and q, respectively, are

a. $-2, -32$ **b.** $-2, 3$ **c.** $-6, 3$ **d.** $-6, -32$

(IIT-JEE 2001)

21. If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ are such that $\min f(x) > \max g(x)$, then the relation between b and c is

a. no relation **b.** $0 < c < b/2$

c. $|c| < |b|\sqrt{2}$ **d.** $|c| > |b|\sqrt{2}$ **(IIT-JEE 2003)**

22. Range of the function $f(x)=\frac{x^2+x+2}{x^2+x+1}, x \in R$ is

a. $(1, \infty)$ **b.** $(1, 11/7)$ **c.** $(1, 7/3]$ **d.** $(1, 7/5)$

(IIT-JEE 2003)

23. For all x, $x^2 + 2ax + 10 - 3a > 0$, then the interval in which a lies is

a. $a < -5$ **b.** $-5 < a < 2$

c. $a > 5$ **d.** $2 < a < 5$ **(IIT-JEE 2004)**

24. If one root is square of the other root of the equation $x^2 + px + q = 0$, then the relation between p and q is

a. $p^3 - q(3p - 1) + q^2 = 0$

b. $p^3 - q(3p + 1) + q^2 = 0$

c. $p^3 + q(3p - 1) + q^2 = 0$

d. $p^3 + q(3p + 1) + q^2 = 0$ **(IIT-JEE 2004)**

25. Let α, β be the roots of the quadratic equation $ax^2 + bx + c = 0$ and $\Delta = b^2 - 4ac$. If $\alpha + \beta$, $\alpha^2 + \beta^2$, $\alpha^3 + \beta^3$ are in G.P., then

a. $\Delta = 0$ **b.** $\Delta \neq 0$ **c.** $b\Delta = 0$ **d.** $c\Delta = 0$

(IIT-JEE 2005)

26. Let a, b, c be the sides of a triangle, where $a \neq b \neq c$ and $\lambda \in R$. If the roots of the equation $x^2 + 2(a + b + c)x + 3\lambda(ab + bc + ca) = 0$ are real. Then

a. $\lambda<\frac{4}{3}$ **b.** $\lambda>\frac{5}{3}$

c. $\lambda\in\left(\frac{1}{3},\frac{5}{3}\right)$ **d.** $\lambda\in\left(\frac{4}{3},\frac{5}{3}\right)$

(IIT-JEE 2006)

27. Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\alpha/2, 2\beta$ be the roots of the equation $x^2 - qx + r = 0$. Then the value of r is

a. $\frac{2}{9}(p-q)(2q-p)$ **b.** $\frac{2}{9}(q-p)(2p-q)$

c. $\frac{2}{9}(q-2p)(2q-p)$ **d.** $\frac{2}{9}(2p-q)(2q-p)$

(IIT-JEE 2007)

28. Let p and q be real numbers such that $p \neq 0$, $p^3 \neq q$, and $p^3 \neq -q$. If α and β are nonzero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having α/β and β/α as its roots is

a. $(p^3+q)x^2-(p^3+2q)x+(p^3+q)=0$

b. $(p^3+q)x^2-(p^3-2q)x+(p^3+q)=0$

c. $(p^3-q)x^2-(5p^3-2q)x+(p^3-q)=0$

d. $(p^3-q)x^2-(5p^3+2q)x+(p^3-q)=0$

(IIT-JEE 2010)

29. A value of b for which the equations $x^2 + bx - 1 = 0$, $x^2 + x + b = 0$ have one root in common is

a. $-\sqrt{2}$ **b.** $-i\sqrt{3}$ **c.** $\sqrt{2}$ **d.** $\sqrt{3}$

(IIT-JEE 2011)

30. Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\dfrac{a_{10} - 2a_8}{2a_9}$ is

a. 1 **b.** 2 **c.** 3 **d.** 4

(IIT-JEE 2011)

31. The quadratic equation $p(x) = 0$ with real coefficients has purely imaginary roots. Then the equation $p(p(x)) = 0$ has

a. only purely imaginary roots
b. all real roots
c. two real and two purely imaginary roots
d. neither real nor purely imaginary roots

(JEE Advanced 2014)

Multiple Correct Answers Type

1. For real x, the function $\dfrac{(x-a)(x-b)}{x-c}$ will assume all real values provided

a. $a > b > c$ **b.** $a < b < c$
c. $a > c > b$ **d.** $a < c < b$ **(IIT-JEE 1984)**

2. Let S be the set of all non-zero real numbers such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is(are) a subset(s) of S?

a. $\left(-\dfrac{1}{2}, -\dfrac{1}{\sqrt{5}}\right)$ **b.** $\left(-\dfrac{1}{\sqrt{5}}, 0\right)$

c. $\left(0, \dfrac{1}{\sqrt{5}}\right)$ **d.** $\left(\dfrac{1}{\sqrt{5}}, \dfrac{1}{2}\right)$

(JEE Advanced 2015)

Matching Column Type

1. Match the statements/expressions in Column I with the statements/expressions in Column II.

Column I	Column II
(a) The minimum value of $\dfrac{x^2+2x+4}{x+2}$ is	(p) 0
(b) Let A and B be 3×3 matrices of real numbers, where A is symmetric, B is skew symmetric, and $(A + B)(A - B) = (A - B)(A + B)$. If $(AB)^t = (-1)^k AB$, where $(AB)^t$ is the transpose of the matrix AB, then the possible values of k are	(q) 1
(c) Let $a = \log_3 \log_3 2$. An integer k satisfying $1 < 2^{(-k+3^{-a})} < 2$, must be less than	(r) 2
(d) If $\sin \theta = \cos \phi$, then the possible values of $\dfrac{1}{\pi}\left(\theta \pm \phi - \dfrac{\pi}{2}\right)$ are	(s) 3

(IIT-JEE 2008)

Integer Answer Type

1. The smallest value of k, for which both the roots of the equation $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real, distinct and have values at least 4, is **(IIT-JEE 2009)**

2. The number of distinct real roots of $x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is **(IIT-JEE 2011)**

Assertion-Reasoning Type

1. Let a, b, c, p, q be real numbers. Suppose α, β are the roots of the equation $x^2 + 2px + q = 0$, α and $1/\beta$ are the roots of the equation $ax^2 + 2bx + c = 0$, where $\beta^2 \notin \{-1, 0, 1\}$.

Statement 1: $(p^2 - q)(b^2 - ac) \geq 0$

Statement 2: $b \neq pa$ or $c \neq qa$

a. Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1.
b. Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1.
c. Statement 1 is true, statement 2 is false.
d. Statement 1 is false, statement 2 is true.

(IIT-JEE 2008)

Fill in the Blanks Type

1. The coefficient of x^{99} in the polynomial $(x-1)(x-2)\cdots(x-100)$ is ________. **(IIT-JEE 1982)**

2. If $2 + i\sqrt{3}$ is a root of the equation $x^2 + px + q = 0$, where p and q are real, then $(p, q) =$ (________, ________).

(IIT-JEE 1982)

3. If $x<0, y<0, x+y+(x/y)=(1/2)$ and $(x+y)(x/y)=-(1/2)$, then $x =$ ________ and $y =$ ________.

(IIT-JEE 1982)

4. If the product of the roots of the equation $x^2 - 3kx + 2e^{2 \ln k} - 1 = 0$ is 7, then the roots are real for ________.

(IIT-JEE 1984)

5. If the quadratic equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ $(a \neq b)$ have a common root, then the numerical value of $a + b$ is ________. **(IIT-JEE 1986)**

True/False Type

1. The equation $2x^2 + 3x + 1 = 0$ has an irrational root.

(IIT-JEE 1983)

2. If $a < b < c < d$, then the roots of the equation $(x-a)(x-c) + 2(x-b)(x-d) = 0$ are real and distinct.

(IIT-JEE 1984)

3. If $n_1, n_2, \ldots, n_p$ are p positive integers, whose sum is an even number, then the number of odd integers among them is odd. **(IIT-JEE 1985)**

4. If $P(x) = ax^2 + bx + c$ and $Q(x) = -ax^2 + dx + c$, where $ac \neq 0$, then $P(x)\,Q(x) = 0$ has at least two real roots.

(IIT-JEE 1985)

Subjective Type

1. Solve for x: $4^x - 3^{x-1/2} = 3^{x+1/2} - 2^{2x-1}$. **(IIT-JEE 1978)**
2. Solve for x: $\sqrt{x+1} - \sqrt{x-1} = 1$. **(IIT-JEE 1978)**
3. Show that the square of $\left(\sqrt{26-15\sqrt{3}}\right)/\left(5\sqrt{2}-\sqrt{38+5\sqrt{3}}\right)$ is a rational number. **(IIT-JEE 1978)**
4. If α, β are the roots of $x^2 + px + q = 0$ and γ, δ are the roots of $x^2 + rx + s = 0$, evaluate $(\alpha - \gamma)(\alpha - \delta)(\beta - \gamma)(\beta - \delta)$ in terms of p, q, r, and s. Deduce the condition that the equation has a common root. **(IIT-JEE 1979)**
5. Prove that the minimum value of $\dfrac{(a+x)(b+x)}{(c+x)}, a, b > c$, $x > -c$ is $(\sqrt{a-c} + \sqrt{b-c})^2$. **(IIT-JEE 1979)**
6. If one root of the quadratic equation $ax^2 + bx + c = 0$ is equal to the nth power of the other, then show that $(ac^n)^{\frac{1}{n+1}} + (a^n c)^{\frac{1}{n+1}} + b = 0$ **(IIT-JEE 1983)**
7. Solve for $x: (5+2\sqrt{6})^{x^2-3} + (5-2\sqrt{6})^{x^2-3} = 10$. **(IIT-JEE 1985)**
8. For $a \le 0$, determine all real roots of the equation $x^2 - 2a|x-a| - 3a^2 = 0$. **(IIT-JEE 1985)**
9. Let a, b, c be real. If $ax^2 + bx + c = 0$ has two real roots α and β, where $\alpha < -1$ and $\beta > 1$, then show that $1 + \dfrac{c}{a} + \left|\dfrac{b}{a}\right| < 0$ **(IIT-JEE 1995)**
10. The real numbers x_1, x_2, x_3 satisfying the equation $x^3 - x^2 + bx + \gamma = 0$ are in A.P. Find the intervals in which β and γ lie. **(IIT-JEE 1996)**
11. Let S be a square of unit area. Consider any quadrilateral, which has one vertex on each side of S. If a, b, c, and d denote the lengths of the sides of the quadrilateral, prove that $2 \le a^2 + b^2 + c^2 + d^2 \le 4$. **(IIT-JEE 1997)**
12. Let $f(x) = Ax^2 + Bx + C$, where A, B, C are real numbers. Prove that if $f(x)$ is an integer whenever x is an integer, then the numbers $2A, A + B$, and C are all integers. Conversely, prove that if the number $2A, A + B$, and C are all integers, then $f(x)$ is an integer whenever x is an integer. **(IIT-JEE 1998)**
13. If α, β are the roots of $ax^2 + bx + c = 0$ $(a \ne 0)$ and $\alpha + \delta$, $\beta + \delta$ are the roots of $Ax^2 + Bx + C = 0$ $(A \ne 0)$ for some constant δ, then prove that $(b^2 - 4ac)/a^2 = (B^2 - 4AC)/A^2$. **(IIT-JEE 2000)**
14. Let a, b, c be real numbers with $a \ne 0$ and let α, β be the roots of the equation $ax^2 + bx + c = 0$. Express the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of α, β. **(IIT-JEE 2001)**
15. If $x^2 + (a-b)x + (1-a-b) = 0$, where $a, b \in R$, then find the values of a for which equation has unequal real roots for all values of b. **(IIT-JEE 2003)**
16. Let a and b be the roots of the equation $x^2 - 10cx - 11d = 0$ and those of $x^2 - 10ax - 11b = 0$ are c, d. Then find the value of $a + b + c + d$, when $a \ne b \ne c \ne d$. **(IIT-JEE 2006)**

Answer Key

JEE Advanced

Single Correct Answer Type

1. c. **2.** a. **3.** c. **4.** b. **5.** b.
6. c. **7.** c. **8.** a. **9.** a. **10.** d.
11. c. **12.** a. **13.** b. **14.** a. **15.** a.
16. b. **17.** d. **18.** c. **19.** d. **20.** a.
21. d. **22.** c. **23.** b. **24.** a. **25.** d.
26. a. **27.** d. **28.** b. **29.** b. **30.** c.
31. d.

Multiple Correct Answers Type

1. c., d. **2.** a., d.

Matching Column Type

1. (a)–(r)

Integer Answer Type

1. (2) **2.** (2)

Assertion–Reasoning Type

1. b.

Fill in the Blanks Type

1. –5050 **2.** $p = -4, q = 7$
3. $x = -1/4$ and $y = -1/4$ **4.** 2 **5.** –1

True/False Type

1. False **2.** True **3.** False **4.** True

Subjective Type

1. $\dfrac{3}{2}$ **2.** $\dfrac{5}{2}$
4. $q(r-p)^2 - p(r-p)(s-q) + (s-q)^2$; $(q-s)^2 = (r-p)(ps-qr)$
7. $\pm 2, \pm\sqrt{2}$
8. $\{a - a\sqrt{2}, -a + a\sqrt{6}\}$
10. $\beta \in (-\infty, 1/3]$, $\gamma \in [-1/27, \infty)$
14. $\alpha^2\beta, \alpha\beta^2$ **15.** $a > 1$ **16.** 1210

Hints and Solutions

JEE Advanced

Single Correct Answer Type

1. c. l, m, n are real and $l \neq m$. Given equation is

$$(l-m)x^2 - 5(l+m)x - 2(l-m) = 0$$

$$D = 25(l+m)^2 + 8(l-m)^2 > 0, \forall l, m \in R$$

Therefore, the roots are real and unequal.

2. a. $u = x^2 + 4y^2 + 9z^2 - 6yz - 3zx - 2xy$

$$= \frac{1}{2}\left[2x^2 + 8y^2 + 18z^2 - 12yz - 6zx - 4xy\right]$$

$$= \frac{1}{2}\left[(x^2 - 4xy + 4y^2) + (4y^2 + 9z^2 - 12yz) + (x^2 + 9z^2 - 6zx)\right]$$

$$= \frac{1}{2}\left[(x-2y)^2 + (2y-3z)^2 + (3z-x)^2\right] \geq 0$$

Hence, u is always non-negative.

3. c. As $a, b, c > 0$, so a, b, c should be real (note that other relation is not defined in the set of complex numbers). Therefore, the roots of equation are either real or complex conjugate.

Let α, β be the roots of $ax^2 + bx + c = 0$. Then,

$$\alpha + \beta = -\frac{b}{a} = -\text{ve and } \alpha\beta = \frac{c}{a} = +\text{ve}$$

Hence, either both α, β are –ve (if roots are real) or both α, β have –ve real part (if roots are complex conjugate).

4. b. Given equation is $y = x^2 + (k-1)x + 9$.

Since coefficient of x^2 is positive, graph is concave upward.

If graph strictly lies above the x-axis, then we can say that equation $x^2 + (k-1)x + 9 = 0$ has imaginary roots.

$\therefore \quad D = (k-1)^2 - 36 < 0$

or $\quad -6 < k - 1 < 6$

or $\quad -5 < k < 7$

5. b. The given equation is

$$(x-b)(x-c) + (x-a)(x-c) + (x-a)(x-b) = 0$$

$$\Rightarrow \quad 3x^2 - 2(a+b+c)x + (ab+bc+ca) = 0$$

$$D = 4(a+b+c)^2 - 12(ab+bc+ca)$$

$$= 4[a^2 + b^2 + c^2 - ab - bc - ca]$$

$$= 2[(a-b)^2 + (b-c)^2 + (c-a)^2] \geq 0, \forall a, b, c$$

Therefore, the roots of the given equation are always real.

6. c.

$$\begin{array}{r} ax - ap \\ x^2 + px + 1\overline{)\,ax^3 + bx + c} \\ ax^3 + apx^2 + ax \\ \hline -apx^2 + (b-a)x + c \\ -apx^2 - ap^2x - ap \\ \hline (b - a + ap^2)x + c + ap \end{array}$$

Now, remainder must be zero.

or $\quad (b - a + ap^2)x + c + ap = 0, \forall x \in R$

Hence, $b - a + ap^2 = 0$ and $c + ap = 0$

$$\Rightarrow \quad p = -\frac{c}{a} \text{ and } p^2 = \frac{a-b}{a}$$

or $\quad \left(\frac{-c}{a}\right)^2 = \frac{a-b}{a}$

or $\quad c^2 = a^2 - ab$

or $\quad a^2 - c^2 = ab$

7. c. Let the distance of the school from A be x. Therefore, the distance of the school from B is $60 - x$. The total distance covered by 200 students is

$$[150x + 50(60 - x)] = [100x + 3000]$$

This is minimum when $x = 0$. Hence, the school should be at town A.

8. a. Given equation is

$$x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$$

Clearly, the given equation is defined if $x - 1 \neq 0$. We can cancel the common term $-2/(x-1)$ on both sides to get $x = 1$, but it is not possible. So, given equation has no roots.

9. a. α, β are roots of $x^2 + px + q = 0$. Hence,

$$\alpha + \beta = -p \text{ and } \alpha\beta = q$$

α^4, β^4 are roots of $x^2 - rx + s = 0$. Hence,

$$\alpha^4 + \beta^4 = r, \alpha^4\beta^4 = q$$

Now for equation $x^2 - 4qx + 2q^2 - r = 0$, the product of roots is

$$2q^2 - r = 2(\alpha\beta)^2 - (\alpha^4 + \beta^4)$$

$$= -(\alpha^2 - \beta^2)^2$$

$$< 0$$

Therefore, the product of roots is negative. So, the roots must be real and of opposite signs.

10. d. We know that if $f(\alpha)$ and $f(\beta)$ are of opposite signs, then there must be a value γ between α and β such that $f(\gamma) = 0$. Here, a, b, c are real numbers and $a \neq 0$. As α is a root of $a^2x^2 + bx + c = 0$, so

$$a^2\alpha^2 + b\alpha + c = 0 \qquad (1)$$

Also, β is a root of $a^2x^2 - bx - c = 0$, so

$$a^2\beta^2 - b\beta - c = 0 \qquad (2)$$

Now, let $f(x) = a^2x^2 + 2bx + 2c$. Then,

$$f(\alpha) = a^2\alpha^2 + 2b\alpha + 2c$$

$$= a^2\alpha^2 + 2(b\alpha + c)$$

$$= a^2\alpha^2 + 2(-a^2\alpha^2) \qquad \text{[Using (1)]}$$

$$= -a^2\alpha^2 < 0$$

and

$$f(\beta) = a^2\beta^2 + 2b\beta + 2c$$

$= a^2\beta^2 + 2(b\beta + c)$

$= a^2\beta^2 + 2(a^2\beta^2)$ [Using (2)]

$= 3a^2\beta^2 > 0$

Since $f(\alpha)$ and $f(\beta)$ are of opposite signs and γ is a root of equation $f(x) = 0$, γ must lie between α and β. Thus, $\alpha < \gamma < \beta$.

11. c. α, β are roots of the equation $(x - a)(x - b) = c, c \neq 0$.

$\therefore \quad (x - a)(x - b) - c = (x - \alpha)(x - \beta)$

or $\quad (x - \alpha)(x - \beta) + c = (x - a)(x - b)$

Hence, the roots of $(x - \alpha)(x - \beta) + c = 0$ are a and b.

12. a. Minimum value of $5x^2 + 2x + 3$ is

$$-\frac{D}{4a} = -\frac{(2)^2 - 4(5)(3)}{4(5)} > 2$$

where maximum value of 2 sin x is 2. Hence, graph of $y = 5x^2 + 2x + 3$ lies above the graph of $y = 2\sin x$ without touching or intersecting. Therefore, the two curves do not meet at all.

13. b. For real roots,

$$q^2 - 4pr \geq 0$$

$\Rightarrow \quad \left(\frac{p+r}{2}\right)^2 - 4pr \geq 0$ ($\because p, q, r$ are in A.P.)

or $\quad p^2 + r^2 - 14pr \geq 0$

or $\quad \frac{p^2}{r^2} - 14\frac{p}{r} + 1 \geq 0$

or $\quad \left(\frac{p}{r} - 7\right)^2 - 48 \geq 0$

or $\quad \left|\frac{p}{r} - 7\right| \geq 4\sqrt{3}$

14. a. The given equation i

$$\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$$

Squaring both sides, we get

$$x + 1 + x - 1 - 2\sqrt{x^2 - 1} = 4x - 1$$

$\Rightarrow \quad -2\sqrt{x^2 - 1} = 2x - 1$

Again squaring both sides, we get

$$4(x^2 - 1) = 4x^2 - 4x + 1$$

or $\quad -4x = -5$

or $\quad x = 5/4$

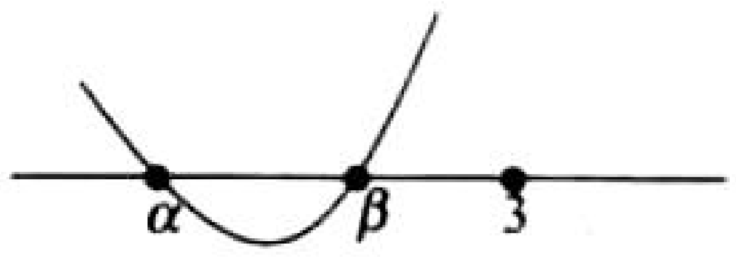
Substituting this value of x in given equation, we get

$$\sqrt{\frac{5}{4} + 1} - \sqrt{\frac{5}{4} - 1} = \sqrt{4 \times \frac{5}{4} - 1}$$

$\Rightarrow \quad \frac{3}{2} - \frac{1}{2} = 2$ (not satisfied)

Therefore, 5/4 is not a solution of given equation. Hence, the given equation has no solution.

15. a. If both the roots of a quadratic equation $Ax^2 + Bx + C = 0$ are less than k, then $Af(k) > 0$, $-B/2A < k$ and $D \geq 0$. Now,



$$f(x) = x^2 - 2ax + a^2 + a - 3$$

$\Rightarrow \quad f(3) > 0, a < 3, -4a + 12 \geq 0$

$\Rightarrow \quad a^2 - 5a + 6 > 0, a < 3, -4a + 12 \geq 0$

$\Rightarrow \quad a < 2$ or $a > 3, a < 3, a \leq 3$

$\Rightarrow \quad a < 2$

16. b. Here $D = b^2 - 4c > 0$ because $c < 0 < b$. So, roots are real and unequal. Now,

$$\alpha + \beta = -b < 0 \text{ and } \alpha\beta = c < 0$$

Therefore, one root is positive and the other root is negative, the negative root being numerically bigger. As $\alpha < \beta$, so α is the negative root while β is the positive root. So, $|\alpha| > \beta$ and $\alpha < 0 < \beta < |\alpha|$.

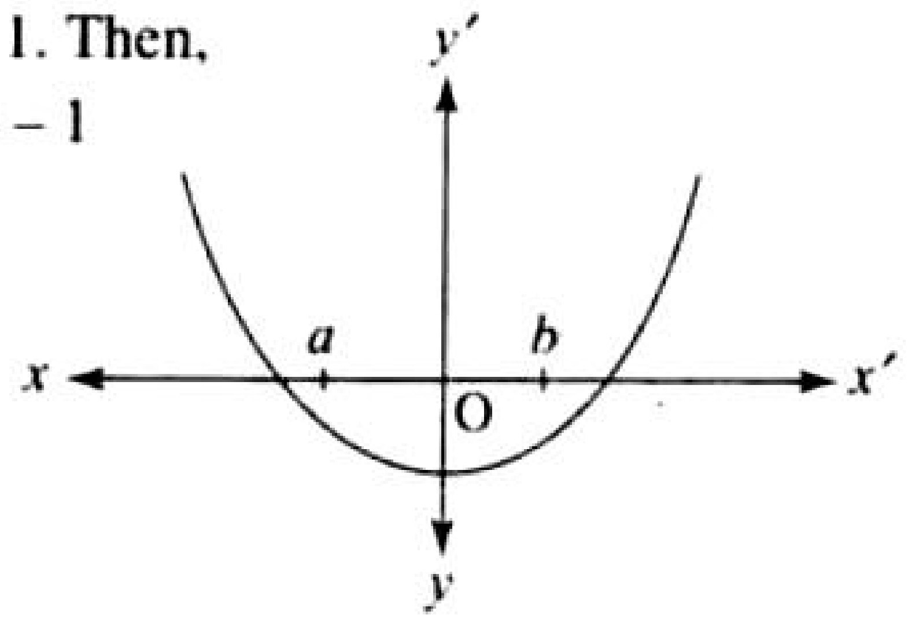
17. d. Given equation is

$$(x - a)(x - b) - 1 = 0$$

Let $f(x) = (x - a)(x - b) - 1$. Then,

$$f(a) = -1 \text{ and } f(b) = -1$$

Also, graph of $f(x)$ is concave upward; hence, a and b lie between the roots. Also, if $b > a$, then one root lies in $(-\infty, a)$ and the other root lies in $(b, +\infty)$.



18. c. Let α, α^2 be the roots of $3x^2 + px + 3 = 0$. Now,

$$S = \alpha + \alpha^2 = -p/3, p = \alpha^3 = 1$$

$\Rightarrow \quad \alpha = 1, \omega, \omega^2 \quad \left(\text{where } \omega = \frac{-1 + \sqrt{3}i}{2}\right)$

$\alpha + \alpha^2 = -p/3 \Rightarrow \omega + \omega^2 = -p/3$

$\Rightarrow \quad -1 = -p/3 \Rightarrow p = 3$

19. d. Minimum value of $f(x) = (1 + b^2)^{2} + 2bx + 1$ is

$$m(b) = -\frac{(2b)^2 - 4(1 + b^2)}{4(1 + b^2)} = \frac{1}{1 + b^2}$$

Clearly, $m(b)$ has range (0, 1].

20. a. Clearly, $\alpha + \beta = 1, \alpha\beta = p, \gamma + \delta = 4, \gamma\delta = q\ (p, q \in I)$.

Since $\alpha, \beta, \gamma, \delta$ are in G.P. (with common ratio r), so

$$\alpha + \alpha r = 1, \alpha(r^2 + r^3) = 4$$

$\Rightarrow \quad \alpha(1 + r) = 1, \alpha r^2(1 + r) = 4$

$\Rightarrow \quad r^2 \times 1 = 4 \Rightarrow r^2 = 4 \Rightarrow r = 2, -2$

If $\quad r = 2$,

$$\alpha + 2\alpha = 1 \Rightarrow \alpha = \frac{1}{3}$$

If $\quad r = -2$,

$$\alpha - 2\alpha = 1 \Rightarrow \alpha = -1$$

But $\quad p = \alpha\beta \in I$

$\therefore \quad r = -2$ and $\alpha = -1$

$\Rightarrow \quad p = -2$,

$$q = \alpha^2 r^5 = 1(-2)^5 = -32$$

21. d. $f(x) = x^2 + 2bx + 2c^2$

$= (x + b)^2 + 2c^2 - b^2$

$g(x) = -x^2 - 2cx + b^2$

$= -(x + c)^2 + b^2 + c^2$

Given that

$$\min f(x) > \max g(x)$$

$\Rightarrow \quad 2c^2 - b^2 > b^2 + c^2$

or $\quad c^2 > 2b^2$

$\Rightarrow \quad |c| > |b|\sqrt{2}$

22. c. We have $y = \frac{x^2 + x + 2}{x^2 + x + 1}$

or $\quad yx^2 + yx + y = x^2 + x + 2$

or $(y-1)x^2 + (y-1)x + y - 2 = 0$

Clearly $y \neq 1$

Since x is real,

$$D \geq 0$$

$\Rightarrow \quad (y-1)^2 - 4(y-1)(y-2) \geq 0$

$\Rightarrow \quad (y-1)[y - 1 - 4y + 8] \geq 0$

$\Rightarrow \quad (y-1)(3y-7) \leq 0$

$\therefore \quad y \in (1, 7/3]$

23. b. $x^2 + 2ax + 10 - 3a > 0, \forall\, x \in R$

$\Rightarrow \quad D < 0$

$\Rightarrow \quad 4a^2 - 4(10 - 3a) < 0$

or $\quad a^2 + 3a - 10 < 0$

or $\quad (a + 5)\,(a - 2) < 0$

or $\quad a \in (-5, 2)$

24. a. α and α^2 are the roots of the equation $x^2 + px + q = 0$. Hence,

$$\alpha + \alpha^2 = -p \qquad (1)$$

and

$$\alpha\alpha^2 = q \text{ or } \alpha^3 = q \qquad (2)$$

Cubing (1),

$$\alpha^3 + \alpha^6 + 3\alpha\,\alpha^2(\alpha + \alpha^2) = -p^3$$

or $\quad q + q^2 + 3q(-p) = -p^3$

or $\quad p^3 + q^2 - q\,(3p - 1) = 0$

25. d. $\alpha + \beta$, $\alpha^2 + \beta^2$, $\alpha^3 + \beta^3$ are in G.P. Hence,

$$(\alpha^2 + \beta^2)^2 = (\alpha + \beta)\,(\alpha^3 + \beta^3)$$

$\Rightarrow \quad \alpha\beta(\alpha - \beta)^2 = 0$

$\Rightarrow \quad c\Delta = 0$

26. a. a, b, c are sides of a triangle and $a \neq b \neq c$.

$\therefore \quad |a - b| < |c| \Rightarrow c^2 + b^2 - 2ab < c^2$

Similarly, we have

$$b^2 + c^2 - 2bc < a^2$$

and

$$c^2 + a^2 - 2ca < b^2$$

On adding, we get

$$a^2 + b^2 + c^2 < 2(ab + bc + ca)$$

or $\quad \dfrac{a^2 + b^2 + c^2}{ab + bc + ca} < 2 \qquad (1)$

Since the roots of the given equation are real, therefore

$$D \geq 0$$

$\therefore \quad (a + b + c)^2 - 3\lambda(ab + bc + ca) \geq 0$

$\Rightarrow \quad \dfrac{a^2 + b^2 + c^2}{ab + bc + ca} > 3\lambda - 2 \qquad (2)$

From (1) and (2), we get

$$3\lambda - 2 < 2 \text{ or } \lambda < \frac{4}{3}$$

27. d. α, β are the roots of $x^2 - px + r = 0$. Hence,

$$\alpha + \beta = p \qquad (1)$$

and

$$\alpha\beta = r \qquad (2)$$

Also, $\alpha/2$, 2β are the roots of $x^2 - qx + r = 0$. Hence,

$$\frac{\alpha}{2} + 2\beta = q \qquad (3)$$

or

$$\alpha + 4\beta = 2q$$

Solving (1) and (3) for α and β, we get

$$\beta = \frac{1}{3}(2q - p) \text{ and } \alpha = \frac{2}{3}(2p - q)$$

Substituting values of α and β, in Eq. (2), we get

$$\frac{2}{9}(2p - q)(2q - p) = r$$

28. b. $\alpha^3 + \beta^3 = q$

$\Rightarrow \quad (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = q$

$\Rightarrow \quad -p^3 + 3p\alpha\beta = q \Rightarrow \alpha\beta = \dfrac{q + p^3}{3p}$

Required equation is

$$x^2 - \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)x + \frac{\alpha}{\beta}\cdot\frac{\beta}{\alpha} = 0$$

$$x^2 - \frac{(\alpha^2 + \beta^2)}{\alpha\beta}x + 1 = 0$$

$\Rightarrow \quad x^2 - \left(\dfrac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}\right)x + 1 = 0$

$\Rightarrow \quad x^2 - \dfrac{p^2 - 2\left(\dfrac{p^3 + q}{3p}\right)}{\dfrac{p^3 + q}{3p}}x + 1 = 0$

$\Rightarrow \quad (p^3 + q)x^2 - (3p^3 - 2p^3 - 2q)x + (p^3 + q) = 0$

$\Rightarrow \quad (p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0.$

29. b. $x^2 + bx - 1 = 0 \qquad (1)$

$x^2 + x + b = 0 \qquad (2)$

Common root is $(b - 1)\,x - 1 - b = 0$

$\Rightarrow \quad x = \dfrac{b + 1}{b - 1}$

This value of x satisfies equation (2)

$\Rightarrow \quad \dfrac{(b + 1)^2}{(b - 1)^2} + \dfrac{b + 1}{b - 1} + b = 0$

$\Rightarrow \quad b = \sqrt{3}\,i, -\sqrt{3}\,i, 0$

30. c. $a_n = \alpha^n - \beta^n$

Also $\alpha^2 - 6\alpha - 2 = 0$

Multiply with α^8 on both sides

$\Rightarrow \quad \alpha^{10} - 6\alpha^9 - 2\alpha^8 = 0 \qquad (1)$

similarly $\beta^{10} - 6\beta^9 - 2\beta^8 = 0 \qquad (2)$

Subtracting (2) from (1) we have

$$\alpha^{10} - \beta^{10} - 6\,(\alpha^9 - \beta^9) = 2(\alpha^8 - \beta^8)$$

$\Rightarrow \quad a_{10} - 6a_9 = 2a_8 \Rightarrow \dfrac{a_{10} - 2a_8}{2a_9} = 3.$

31. d. Since $p(x) = 0$ has purely imaginary roots,

$p(x) = ax^2 + c$, where a and c have same sign.

Also, $p(p(x)) = 0$

$\Rightarrow \quad p(x)$ is purely imaginary

$\Rightarrow \quad ax^2 + c$ is purely imaginary

Hence, x cannot be either purely real or purely imaginary.

Multiple Correct Answers Type

1. c., d. Let

$$y = \frac{(x - a)(x - b)}{(x - c)}$$

or $(x - c)\, y = x^2 - (a + b)x + ab$

or $x^2 - (a + b + y)x + ab + cy = 0$

Since x is real, so

$D \geq 0$

$\Rightarrow$ $(a + b + y)^2 - 4(ab + cy) \geq 0,\ \forall\ y \in R$

or $y^2 + 2y\,(a + b - 2c) + (a - b)^2 \geq 0,\ \forall\ y \in R$

or $4(a + b - 2c)^2 - 4(a - b)^2 < 0$

or $(a + b - 2c + a - b)\,(a + b - 2c - a + b) < 0$

or $4\,(a - c)\,(b - c) < 0$

$\Rightarrow$ $a - c < 0$ and $b - c > 0$ or $a - c > 0$ and $b - c < 0$

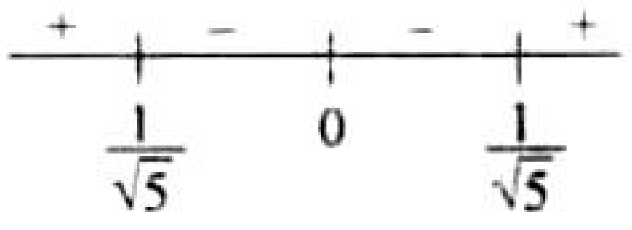
$\Rightarrow$ $a < c < b$ or $a > c > b$

2. a., d. $|x_1 - x_2| < 1$

$\therefore$ $(x_1 + x_2)^2 - 4x_1x_2 < 1$

$\Rightarrow$ $\dfrac{1}{\alpha^2} - 4 < 1$

$\Rightarrow$ $5 - \dfrac{1}{\alpha^2} > 0$



$\Rightarrow$ $\dfrac{5\alpha^2 - 1}{\alpha^2} > 0$

$\Rightarrow$ $\alpha \in \left(-\infty, -\dfrac{1}{\sqrt{5}}\right) \cup \left(\dfrac{1}{\sqrt{5}}, \infty\right)$ (1)

Also $D > 0$

$\therefore$ $1 - 4\alpha^2 > 0$

$\therefore$ $\alpha \in \left(-\dfrac{1}{2}, \dfrac{1}{2}\right)$ (2)

From (1) and (2)

$$\alpha \in \left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{2}}\right)$$

Matching Column Type

1. (a) – (r)

We have $y = \dfrac{x^2 + 2x + 4}{x + 2}$

$\Rightarrow$ $x^2 + (2 - y)\,x + 4 - 2y = 0$

Since x is real,

$D \geq 0$

$\Rightarrow$ $y^2 + 4y - 12 \geq 0$

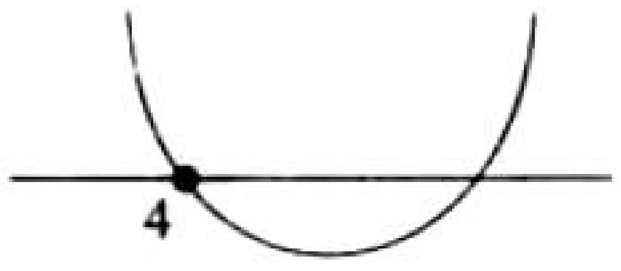
$\Rightarrow$ $y \leq -6$ or $y \geq 2$

minimum value is 2

Note: Solutions of the remaining parts are given in their respective chapters.

Integer Answer Type

1. (2) $x^2 - 8kx + 16\,(k^2 - k + 1) = 0$



Since roots are real and distinct,

$D > 0$

$\Rightarrow$ $64k^2 - 64(k^2 - k + 1) > 0$

$\Rightarrow$ $k > 1$ (1)

Also, from the figure, we have

$\dfrac{-b}{2a} > 4 \Rightarrow \dfrac{8k}{2} > 4$

$\Rightarrow$ $k > 1$ (2)

Further

$f(4) \geq 0$

$\Rightarrow$ $16 - 32k + 16\,(k^2 - k + 1) \geq 0$

$\Rightarrow$ $k^2 - 3k + 2 \geq 0$

$\Rightarrow$ $k \leq 1$ or $k \geq 2$ (3)

From (1), (2) and (3)

$k_{\min} = 2.$

2. b. Let $f(x) = x^4 - 4x^3 + 12x^2 + x - 1 = 0$

$f'(x) = 4x^3 - 12x^2 + 24x + 1$

$f''(x) = 12x^2 - 24x + 24 = 12(x^2 - 2x + 2) > 0$

$\Rightarrow$ $f''(x) = 0$ has imaginary roots

$\Rightarrow$ $f'(x) = 0$ has only one real root

$\Rightarrow$ $f(x) = 0$ has maximum 2 distinct real roots.

Assertion-Reasoning Type

1. b. Suppose the roots are imaginary. Then

$$\beta = \bar{\alpha} \text{ and } \frac{1}{\beta} = \bar{\alpha} \Rightarrow \beta = \frac{1}{\beta}$$

which is not possible. The roots are real, so

$(p^2 - q)\,(b^2 - ac) \geq 0$

Hence, statement 1 is correct.

Also, $-2b/a = \alpha + \beta$ and $\alpha/\beta = c/a$, $\alpha + \beta = -2p$, $\alpha\beta = q$.

If $\beta = 1$, then

$\alpha = q \Rightarrow c = qa$ (which is not possible)

Also,

$$s + 1 = \frac{-2b}{a} \Rightarrow -2p = \frac{-2b}{a} \Rightarrow b = ap$$

(which is not possible)

Hence, statement 2 is correct, but it is not correct explanation of statement 1.

Fill in the Blanks Type

1. Given polynomial is

$(x - 1)\,(x - 2)\,(x - 3) \cdots (x - 100)$

$= x^{100} - (1 + 2 + 3 + \cdots + 100)\,x^{99} + (\cdots)x^{98} \cdots$

Hence, coefficient of x^{99} is

$$-(1 + 2 + 3 + \cdots + 100) = \frac{-100 \times 101}{2}$$

$$= -5050$$

2. As p and q are real and one root is $2 + i\sqrt{3}$, so the other root must be $2 - i\sqrt{3}$. Then,

$p = -(\text{sum of roots}) = -4$

$q = \text{product of roots} = (2 + i\sqrt{3})(2 - i\sqrt{3}) = 4 + 3 = 7$

3. Given $x < 0$, $y < 0$.

$$x + y + \frac{x}{y} = \frac{1}{2} \text{ and } (x + y)\frac{x}{y} = -\frac{1}{2}$$

Let

$x + y = a$ and $\dfrac{x}{y} = b$ (1)

Therefore, we get

$a + b = \dfrac{1}{2}$, $ab = -\dfrac{1}{2}$

Solving these two, we get

$a + \left(-\dfrac{1}{2a}\right) = \dfrac{1}{2}$

$\Rightarrow$ $2a^2 - a - 1 = 0$

$\Rightarrow$ $a = 1, -1/2$

$\Rightarrow$ $b = -1/2, 1$

$\therefore$ from (1) $x + y = 1$ and $\frac{x}{y} = -\frac{1}{2}$

or

$$x + y = -\frac{1}{2} \text{ and } \frac{x}{y} = 1$$

But $x, y < 0$

$$\therefore \quad x + y < 0 \Rightarrow x + y = \frac{-1}{2} \text{ and } \frac{x}{y} = 1$$

On solving, we get $x = -1/4$ and $y = -1/4$.

4. Given equation is

$$x^2 - 3kx + 2e^{2 \ln k} - 1 = 0$$

$$\Rightarrow \quad x^2 - 3kx + (2k^2 - 1) = 0$$

Here, product of roots is $2k^2 - 1$.

$$\therefore \quad 2k^2 - 1 = 7 \text{ or } k^2 = 4 \text{ or } k = 2, -2$$

Now for real roots, we must have

$$D \geq 0$$

$$\Rightarrow \quad 9k^2 - 4(2k^2 - 1) \geq 0$$

$$\Rightarrow \quad k^2 + 4 \geq 0$$

which is true for all k. Thus, $k = 2, -2$. But for $k = -2$, $\ln k$ is not defined. Therefore, rejecting $k = -2$, we get $k = 2$.

5. By observation, one root is $x = 1$,

$$\Rightarrow \quad a + b = -1$$

True/False Type

1. False.

$$2x^2 + 3x + 1 = 0$$

$$\Rightarrow \quad (2x + 1)(x + 1) = 0$$

$$\Rightarrow \quad x = -1, -1/2, \text{ both are rational}$$

2. True. Given equation is

$$(x - a)(x - c) + 2(x - b)(x - d) = 0$$

Let

$$f(x) = (x - a)(x - c) + 2(x - b)(x - d)$$

$$f(b) = (b - a)(b - c) < 0$$

$$f(d) = (d - a)(d - c) > 0$$

Thus,

$$f(b)\, f(d) < 0$$

Therefore, one root lies between b and d; hence, the roots are real.

3. False. Consider $N = n_1 + n_2 + n_3 + \cdots + n_p$, where N is an even number. Let k numbers among these p numbers be odd, then $p - k$ are even numbers.

Now, sum of $p - k$ even numbers is even and for N to be an even number, sum of k odd numbers must be even, which is possible only when k is even.

4. True. We have $P(x) = ax^2 + bx + c$, for which

$$D_1 = b^2 - 4ac \qquad (1)$$

and $Q(x) = -ax^2 + dx + c$, for which

$$D_2 = d^2 + 4ac \qquad (2)$$

Given that $ac \neq 0$. Following two cases are possible.

If $ac > 0$, then from Eq. (2), D_2 is +ve $\Rightarrow Q(x)$ has real roots.

If $ac < 0$, then from Eq. (1), D_1 is +ve $\Rightarrow P(x)$ has real roots.

Thus, $P(x)\,Q(x) = 0$ has at least two real roots.

Subjective Type

1. $4^x - 3^{x - \frac{1}{2}} = 3^{x + \frac{1}{2}} - 2^{2x - 1}$

$$\text{or} \quad 4^x - \frac{3^x}{\sqrt{3}} = 3^x\sqrt{3} - \frac{4^x}{2}$$

$$\text{or} \quad \frac{3}{2}4^x = 3^x\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)$$

$$\text{or} \quad \frac{3}{2}4^x = 3^x\frac{4}{\sqrt{3}}$$

$$\text{or} \quad \frac{4^{x-1}}{4^{1/2}} = \frac{3^{x-1}}{\sqrt{3}}$$

$$\text{or} \quad 4^{x - 3/2} = 3^{x - 3/2}$$

$$\text{or} \quad \left(\frac{4}{3}\right)^{x - 3/2} = 1$$

$$\text{or} \quad x - \frac{3}{2} = 0$$

$$\text{or} \quad x = 3/2$$

2. We have,

$$\sqrt{x + 1} = 1 + \sqrt{x - 1}$$

Squaring both sides, we get

$$x + 1 = 1 + x - 1 + 2\sqrt{x - 1}$$

$$\text{or} \quad 1 = 2\sqrt{x - 1}$$

$$\text{or} \quad 1 = 4(x - 1)$$

$$\text{or} \quad x = 5/4$$

3. Let

$$x = \frac{\sqrt{26 - 15\sqrt{3}}}{5\sqrt{2} - \sqrt{38 + 5\sqrt{3}}}$$

$$\text{or} \quad x^2 = \frac{26 - 15\sqrt{3}}{50 + 38 + 5\sqrt{3} - 10\sqrt{76 + 10\sqrt{3}}}$$

$$= \frac{26 - 15\sqrt{3}}{88 + 5\sqrt{3} - 10\sqrt{75 + 1 + 10\sqrt{3}}}$$

$$= \frac{26 - 15\sqrt{3}}{88 + 5\sqrt{3} - 10\sqrt{(5\sqrt{3})^2 + (1)^2 + 2 \times 5\sqrt{3} \times 1}}$$

$$= \frac{26 - 15\sqrt{3}}{88 + 5\sqrt{3} - 10\sqrt{(5\sqrt{3} + 1)^2}}$$

$$= \frac{26 - 15\sqrt{3}}{88 + 5\sqrt{3} - 10(5\sqrt{3} + 1)}$$

$$= \frac{26 - 15\sqrt{3}}{78 - 45\sqrt{3}}$$

$$= \frac{26 - 15\sqrt{3}}{3(26 - 15\sqrt{3})} = \frac{1}{3}$$

which is a rational number

4. α, β are the roots of $x^2 + px + q = 0$.

$$\therefore \quad \alpha + \beta = -p, \alpha\beta = q$$

γ, δ are the roots of $x^2 + rx + s = 0$.

$$\therefore \quad \gamma + \delta = -r, \gamma\delta = s$$

Now,

$$E = (\alpha - \gamma)(\alpha - \delta)(\beta - \gamma)(\beta - \delta)$$

$$= [\alpha^2 - (\gamma + \delta)\alpha + \gamma\delta]\,[\beta^2 - (\gamma + \delta)\beta + \gamma\delta]$$

$$= [\alpha^2 + r\alpha + s]\,[\beta^2 + r\beta + s]$$

Also $\alpha^2 + p\alpha + q = 0$ and $\beta^2 + p\beta + q = 0$

$\Rightarrow \quad E = [(r-p)\alpha + (s-q)][(r-p)\beta + (s-q)]$

$= (r-p)^2 \alpha\beta + (r-p)(s-q)(\alpha+\beta) + (s-q)^2$

$= q(r-p)^2 - p(r-p)(s-q) + (s-q)^2$

Now if the equations $x^2 + px + q = 0$ and $x^2 + rx + s = 0$ have a common root say α, then

$$\alpha^2 + p\alpha + q = 0$$

and

$$\alpha^2 + r\alpha + s = 0$$

$\Rightarrow \quad (q-s)^2 = (r-p)(ps-qr),$

which is the required condition.

5. $y = \dfrac{(a+x)(b+x)}{(c+x)}$

Let $x + c = t$

$$\Rightarrow \quad y = \frac{(a-c+t)(b-c+t)}{t}$$

$$= \frac{t^2 + [(a-c)+(b-c)]t + (a-c)(b-c)}{t}$$

$$= t + \frac{(a-c)(b-c)}{t} + (a-c) + (b-c)$$

$$= \left(t - \sqrt{\frac{(a-c)(b-c)}{t}}\right)^2 + (\sqrt{(a-c)} + \sqrt{(b-c)})^2$$

Hence maximum value of y is $\left(\sqrt{a-c} + \sqrt{b-c}\right)^2$

when $t = \sqrt{\dfrac{(a-c)(b-c)}{t}}$.

6. Let α, β be the roots of equation $ax^2 + bx + c = 0$. Given that $\beta = \alpha^n$. Also, $\alpha + \beta = -b/a$, $\alpha\beta = c/a$. Now,

$$\alpha\beta = \frac{c}{a} \Rightarrow \alpha\,\alpha^n = \frac{c}{a} \Rightarrow \alpha = \left(\frac{c}{a}\right)^{\frac{1}{n+1}}$$

$$\alpha + \beta = -b/a \Rightarrow \alpha + \alpha^n = \frac{-b}{a}$$

or

$$\left(\frac{c}{a}\right)^{\frac{1}{n+1}} + \left(\frac{c}{a}\right)^{\frac{n}{n+1}} = \frac{-b}{a}$$

or $\quad a\left(\dfrac{c}{a}\right)^{\frac{1}{n+1}} + a\left(\dfrac{c}{a}\right)^{\frac{n}{n+1}} + b = 0$

or $\quad a^{\frac{n}{n+1}} c^{\frac{1}{n+1}} + a^{\frac{1}{n+1}} c^{\frac{n}{n+1}} + b = 0$

or $\quad (a^n c)^{\frac{1}{n+1}} + (a\,c^n)^{\frac{1}{n+1}} + b = 0$

7. $(5+2\sqrt{6})(5-2\sqrt{6}) = 25 - 24 = 1$

$\Rightarrow \quad 5 - 2\sqrt{6} = \dfrac{1}{5+2\sqrt{6}}$

Hence, the given equation is

$$(5+2\sqrt{6})^{x^2-3} + \frac{1}{(5+2\sqrt{6})^{x^2-3}} = 10$$

$\Rightarrow \quad y + \dfrac{1}{y} = 10$, where $y = (5+2\sqrt{6})^{x^2-3}$

$\Rightarrow \quad y^2 - 10y + 1 = 0$

or $\quad y = \dfrac{10 \pm \sqrt{100-4}}{2}$

or $\quad y = 5 \pm 2\sqrt{6}$

$\Rightarrow \quad (5+2\sqrt{6})^{x^2-3} = 5 \pm 2\sqrt{6}$

$\Rightarrow \quad (5+2\sqrt{6})^{x^2-3} = (5+2\sqrt{6})$

or

$$(5+2\sqrt{6})^{x^2-3} = \frac{1}{5+2\sqrt{6}}$$

$\Rightarrow \quad (5+2\sqrt{6})^{x^2-3} = (5+2\sqrt{6})^1$ or $(5+2\sqrt{6})^{x^2-3} = (5+2\sqrt{6})^{-1}$

$\Rightarrow \quad x^2 - 3 = 1$ or $x^2 - 3 = -1$

$\Rightarrow \quad x^2 = 4$ or $x^2 = 2$

$\Rightarrow \quad x = \pm 2$ or $\pm \sqrt{2}$

8. The given equation is

$$x^2 - 2a|x-a| - 3a^2 = 0$$

Case I: If $x - a \geq 0$, then $|x-a| = x - a$. Hence, the equation becomes

$$x^2 - 2a(x-a) - 3a^2 = 0$$

or $\quad x^2 - 2ax - a^2 = 0$

or $\quad x = \dfrac{2a \pm \sqrt{4a^2 + 4a^2}}{2} \Rightarrow a - a\sqrt{2} \qquad (\because x \geq a)$

Case II: If $x - a < 0$, then $|x-a| = -(x-a)$. Hence, the equation becomes

$$x^2 + 2a(x-a) - 3a^2 = 0$$

or $\quad x^2 + 2ax - 5a^2 = 0$

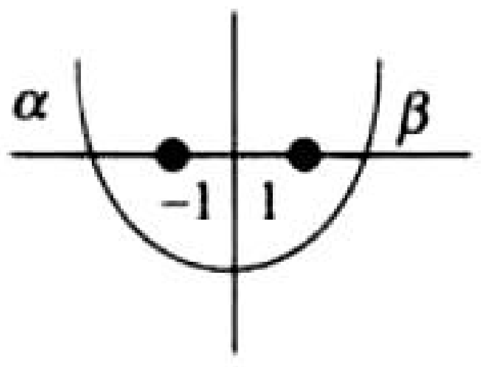
or $\quad x = \dfrac{-2a \pm \sqrt{4a^2 + 20a^2}}{2}$

or $\quad = \dfrac{-2a \pm 2a\sqrt{6}}{2}$

$= -a + a\sqrt{6} \qquad (\because x < a)$

Thus, the solution set is $\{a - a\sqrt{2}, -a + a\sqrt{6}\}$.

9. Let $f(x) = x^2 + (b/a)x + (c/a)$. According to the question, we have the following graph.



From graph, $f(-1) < 0$ and $f(1) < 0$. So,

$$1 + \frac{c}{a} - \frac{b}{a} < 0 \text{ and } 1 + \frac{c}{a} + \frac{b}{a} < 0$$

$\Rightarrow \quad 1 + \dfrac{c}{a} + \left|\dfrac{b}{a}\right| < 0$

10. Since x_1, x_2, x_3 are in A.P.

Let $x_1 = a - d$, $x_2 = a$ and $x_3 = a + d$

Also, x_1, x_2, x_3 are the roots of $x^3 - x^2 + \beta x + \gamma = 0$.

We have

Sum of roots $= \Sigma\alpha = a - d + a + a + d = 1 \qquad (1)$

Sum of product of roots taken two at a time $= \Sigma\alpha\beta$

$\Rightarrow \quad (a-d)a + a(a+d) + (a-d)(a+d) = \beta \qquad (2)$

Product of roots $= \alpha\beta\gamma$

$\Rightarrow \quad (a-d)a(a+d) = -\gamma \qquad (3)$

From (1), we get, $3a = 1 \Rightarrow a = 1/3$

From (2), we get, $3a^2 - d^2 = \beta$

$\Rightarrow \quad 3(1/3)^2 - d^2 = \beta \Rightarrow 1/3 - \beta = d^2$

$$\Rightarrow \quad \frac{1}{3} - \beta \ge 0 \qquad (\because d^2 \ge 0)$$

$$\Rightarrow \quad \beta \le \frac{1}{3}$$

$$\Rightarrow \quad \beta \in (-\infty, 1/3]$$

From (3), $a(a^2 - d^2) = -\gamma$

$$\Rightarrow \quad \frac{1}{3}\left(\frac{1}{9} - d^2\right) = -\gamma$$

$$\Rightarrow \quad \frac{1}{27} - \frac{1}{3}d^2 = -\gamma$$

$$\Rightarrow \quad \gamma + \frac{1}{27} = \frac{1}{3}d^2$$

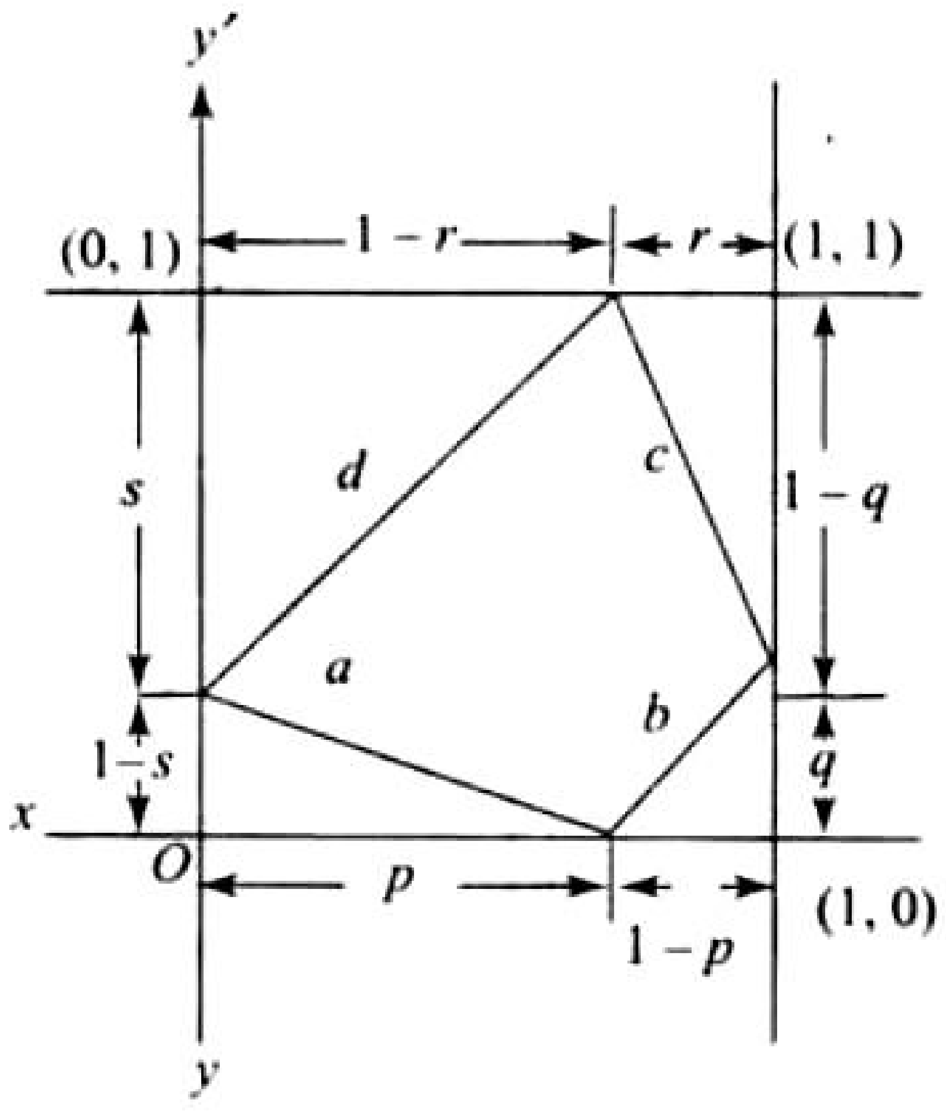
$$\Rightarrow \quad \gamma + \frac{1}{27} \ge 0$$

$$\Rightarrow \quad \gamma \ge -\frac{1}{27}$$

$$\Rightarrow \quad \gamma \in \left[-\frac{1}{27}, \infty\right)$$

Hence, $\beta \in (-\infty, 1/3]$ and $\gamma \in [-1/27, \infty)$.

11.



$$a^2 = p^2 + (1 - s)^2$$
$$b^2 = (1 - p)^2 + q^2$$
$$c^2 = (1 - q)^2 + r^2$$
$$d^2 = (1 - r)^2 + s^2$$

$\therefore \quad a^2 + b^2 + c^2 + d^2 = [p^2 + (1 - p)^2] + [q^2 + (1 - q)^2] + [r^2 + (1 - r)^2] + [s^2 + (1 - s)^2]$, where $p, q, r, s \in [0, 1]$

Now consider the function

$$y = x^2 + (1 - x)^2, 0 \le x \le 1$$

$$\Rightarrow \quad y = 2x^2 - 2x + 1$$

which has vertex (1/2, 1/2).

Hence, minimum value is 1/2 when $x = 1/2$ and maximum value is at $x = 1$, which is 1. Therefore, minimum value of $a^2 + b^2 + c^2 + d^2$ is 1/2 + 1/2 + 1/2 + 1/2 = 2 and maximum value is 1 + 1 + 1 + 1 = 4.

12. Let us consider the integral values of x as 0, 1, −1. Then $f(0)$, $f(1)$ and $f(-1)$ are all integers. Therefore, C, $A + B + C$ and $A - B + C$ are all integers.

Therefore, C is integer and hence, $A + B$ is an integer and also $A - B$ is an integer,

$$2A = (A + B) + (A - B)$$

Therefore, $2A$, $A + B$ and C are all integers. Conversely, let $n \in I$. Then,

$$f(n) = An^2 + Bn + C = 2A\left[\frac{n(n-1)}{2}\right] + (A + B)n + C$$

Now, A, $A + B$, and C are all integers and

$$\frac{n(n-1)}{2} = \frac{\text{Even number}}{2} = \text{Integer}$$

Therefore, $f(n)$ is also an integer.

13. We know that

$$(\alpha - \beta)^2 = [(\alpha + \delta) - (\beta + \delta)]^2$$

$$\Rightarrow \quad (\alpha + \beta)^2 - 4\alpha\beta = (\alpha + \delta + \beta + \delta)^2 - 4(\alpha + \delta)(\beta + \delta) \quad (1)$$

Now

$$\alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$$

and

$$(\alpha + \delta) + (\beta + \delta) = -\frac{B}{A}, (\alpha + \delta)(\beta + \delta) = \frac{C}{A}$$

$$\Rightarrow \quad \frac{b^2 - 4ac}{a^2} = \frac{B^2 - 4AC}{A^2} \qquad \text{[From (1)]}$$

14. $\alpha + \beta = \frac{-b}{a}, \alpha\beta = \frac{c}{a}$

Roots of the equation $a^3x^2 + abcx + c^3 = 0$ are

$$x = \frac{-abc \pm \sqrt{(abc)^2 - 4a^3c^2}}{2a^3}$$

$$= \frac{1}{2}\left(-\frac{b}{a}\right)\left(\frac{c}{a}\right) \pm \frac{\sqrt{\left(\frac{b}{a}\right)^2\left(\frac{c}{a}\right)^2 - 4\left(\frac{c}{a}\right)^3}}{2}$$

$$= \frac{(\alpha + \beta)(\alpha\beta) \pm \sqrt{(\alpha + \beta)^2(\alpha\beta)^2 - 4(\alpha\beta)^3}}{2}$$

$$= (\alpha\beta)\frac{[(\alpha + \beta) \pm \sqrt{(\alpha - \beta)^2}]}{2}$$

$$= \alpha\beta\frac{[(\alpha + \beta) \pm (\alpha - \beta)]}{2}$$

$$= \alpha^2\beta, \alpha\beta^2$$

15. The given equation is

$$x^2 + (a - b)x + (1 - a - b) = 0, a, b \in R$$

For this equation to have unequal real roots ∀ b,

$$D > 0$$

$$\Rightarrow \quad (a - b)^2 - 4(1 - a - b) > 0$$

$$\Rightarrow \quad a^2 + b^2 - 2ab - 4 + 4a + 4b > 0$$

$$\Rightarrow \quad b^2 + b(4 - 2a) + a^2 + 4a - 4 > 0 \qquad (1)$$

which is a quadratic expression in b, and it will be true ∀ $b \in R$. Then its discriminant will be less than 0. Hence,

$$(4 - 2a)^2 - 4(a^2 + 4a - 4) < 0$$

or $\quad (2 - a)^2 - (a^2 + 4a - 4) < 0$

or $\quad 4 - 4a + a^2 - a^2 - 4a + 4 < 0$

or $\quad -8a + 8 < 0$

or $\quad a > 1$

16. Roots of $x^2 - 10cx - 11d = 0$ are a and b. Hence,

$$a + b = 10c \text{ and } ab = -11d$$

c and d are the roots of $x^2 - 10ax - 11b = 0$. Hence,

$$c + d = 10a \text{ and } cd = -11b$$

$\Rightarrow \quad a + b + c + d = 10\,(a + c)$ and $abcd = 121\,bd$

$\Rightarrow \quad b + d = 9\,(a + c)$ and $ac = 121$

Also, we have

$$a^2 - 10ac - 11d = 0 \text{ and } c^2 - 10ac - 11b = 0$$

$\Rightarrow \quad a^2 + c^2 - 20ac - 11\,(b + d) = 0$

$\Rightarrow \quad (a + c)^2 - 22 \times 121 - 99\,(a + c) = 0$

$\Rightarrow \quad a + c = 121$ or -22

For $a + c = -22$, we get $a = c$. Rejecting these values, we have $a + c = 121$. Therefore,

$$a + b + c + d = 10\,(a + c) = 1210$$