## ACTIVITY 4

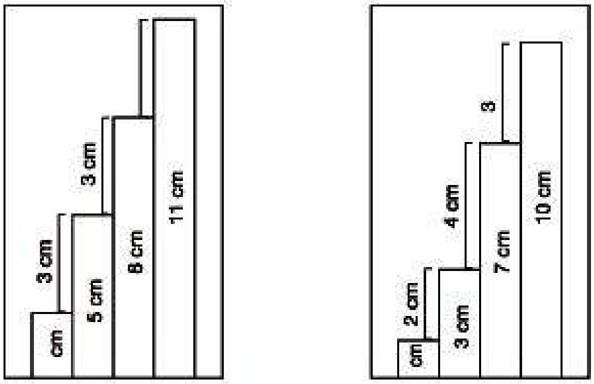
### HANDS ON ACTIVITY

***Concept***

Verification of an A.P.

***Objective:***

To check whether the given sequence is an A.P. or not by paper cutting and pasting.




Paste the paper strip according to given terms and observe the difference for consideration of an A.P. This activity will help to visualise the sequences representing an A.P., able to visualise an A.P using a paper model.

## WORKSHEET 1

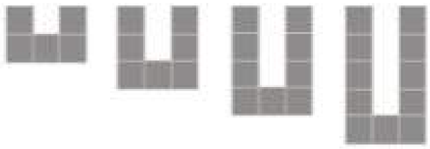
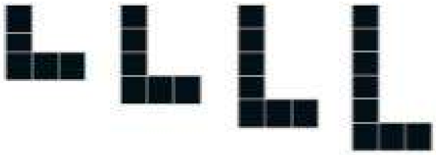
In which of the following situations, the list of numbers generated will form an A.P.? Why?

1. The taxi fare after each kilometre when the fare is ` 30 for the first kilometre and ` 15 for the each next kilometre.
2. Sudhir had ` 500 on 1st January, 2015. He decided to save ` 200 more every month.
3. Reena planted 6 plants every day.
4. The price of a bouquet when it's cost is ` 100 with 10 flowers and increases by Rs. 10 with an additional flower.
5. Honey had ` 200 on 1st January, 2015. He decided to save ` 100 more in February, ` 200 more in March and so on.

# WORKSHEET 2

## DESIGNS AND NUMBERS

Observe the following diagrams and write your observations. Do you observe any pattern? Draw the next shape for each of the following. Express the given designs in terms of numbers.

| Diagrams | Observations |
|---|---|
|  | |
|  | |

# WORKSHEET 3

## GENERATING NUMBER PATTERNS

Generate number patterns with the given information:

1. First term is 5 and the succeeding term is 2 more than the previous term.
2. First term is 0 and the succeeding term is obtained by subtracting 2 from the preceding term.
3. Choose any first term. Each succeeding term is obtained by adding 2.5 to the preceding term.
4. First term is x and each succeeding term is obtained by adding 4 to the preceding term.

## WORKSHEET 4

### $n^{th}$ TERM OF AN A.P.

Let *a* be the first term, *d* be the common difference of a given sequence. Observe the following diagram and write the successive terms in terms of *a* and *d*.

| | | | | | |
|---|---|---|---|---|---|
| a | | | | | |
| a | d | | | | |
| a | d | d | | | |
| a | d | d | d | | |
| a | d | d | d | d | |
| a | d | d | d | d | d |

First term, $a_1$ = ………………

Second term, $a_2$ = …………..

Third term, $a_3$ = ………..

Third term, $a_4$ = ………..

Third term, $a_5$ = ………..

What do you think would be the 28th term?

Write the formula for nth term.

If a is the first term of an A.P. and d is the common difference then the general sequence will be written as

## WORKSHEET 5

**Multiple Choice Quesitons:**

1. The value of $2x + 3$ where, 4/5, $x$ and 34/5 are in A.P is
A. 68/5 B. 14/5 C. 53/5 D. 58/5
2. 7th term of an A.P. is 77 and 77th term is 7. The term which is 0 is
A. 83rd B. 84th C. 85th D. 78th 
3. The heights $h_1$, $h_2$ and $h_3$ of three right triangles are in A.P.. Which of the following is true?
A. $h_1 - h_2 = h_1 - h_3$ B. $2h_3 = h_1 + h_2$
C. $h_2 : h_1 = h_3 : h_2$ D. $2h_2 = h_1 + h_3$
4. Which term of A.P. 7, 22/3, 23/3,… is 11?
A. $11^{th}$ term B. $12^{th}$ term
C. $13^{th}$ term D. $14^{th}$ term
5. The number of multiples of 9 between 9 and 999 is
A. 19 B. 109 C. 108 D. 110
6. If the $10^{th}$ term of an A.P. is 0 then the ratio of its $100^{th}$ and $1000^{th}$ terms will be
A. 1:10 B. 10:1 C. 11:1 D. 1:11
7. The number of multiples of 7 from 3 to 777 is
A. 99 B. 100 C. 111 D. 112

## WORKSHEET 6

### MATCHING ACTIVITY

Match each sequence with the expression that describes the relationship between the nth terms of the sequence. Some choices will not be used.

| Column A | | Column B | |
|---|---|---|---|
| 1. | 2, 4, 6, 8, 10… | a. | n+10 |
| 2. | 1, 3, 9, 27, 81… | b. | 0.3n |
| 3. | 45, 41, 37, 33, 29… | c. | 5n |
| 4. | 11, 21, 31, 41, 51… | d. | n + 5 |
| 5. | 10, 5, 2.5, 1.25, 0.625… | e. | n + 2 |
| 6. | 11, 3, –5, –13, –21… | f. | 0.5n |
| 7. | 100, 20, 4, 0.8, 0.16… | g. | n + (–2) |
| 8. | 7, 10.5, 15.75, 23.625, 35.4375… | h. | n + (–4) |
| | | i. | 1.5n |
| | | j. | 3n |
| | | k. | n + (–8) |
| | | 1. | 2n |
| | | m. | 0.2n |

## SUGGESTED SELF EXPLORATORY ACTIVITIES

**Activity 1**

- Take any A.P..
- Find Sum of its n terms.
- Add a constant P to each term.
- Find the new sum
- Explore the relation between the old sum and the new sum.

**Activity 2**

- Take any A.P.
- Find Sum of its n terms.
- Subtract a constant P from each term.
- Find the new sum
- Explore the relation between the old sum and the new sum.

**Activity 3**

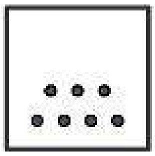
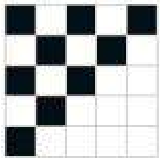
- Take any A.P.
- Find Sum of its n terms.
- Multiply each term by a constant P.
- Find the new sum
- Explore the relation between the old sum and the new sum.

**Activity 4**

- Take any A.P.
- Find Sum of its n terms.
- Divide each term by a constant P.
- Find the new sum
- Explore the relation between the old sum and the new sum.

## WORKSHEET 7

Using the given picture, frame a question based on the concept of A.P..

| | |
|---|---|
|  | |
|  | |
| | |

## PROJECT WORK

Tell students that an arithmetic sequence is a list of numbers with a common difference. Challenge them to create a way of testing the following hypotheses:

- The sum of two arithmetic sequences is another arithmetic sequence. (For the first sequence, assign n for the first number and n + d for the next number. For the second sequence, assign n + 1 for the first number and n + 1 + d for the next number. Then, determine the sum. Note that some students may notice that this is not a true generalized test of the hypothesis.)
- The product of two arithmetic sequences is another arithmetic sequence. (For the first sequence, assign n for the first number and n + d for the next number. For the second sequence, assign n + 1 for the first number and n + 1 + d for the next number. Then, determine the product.)

Suggest that students test each hypotheses using arithmetic sequences. Then, have they tested with another pair of sequences. If the hypothesis appears to be true, ask them to use a general case to summarize their findings.

**Misconceptions/Common Errors**

| *Misconceptions/Common Errors* | *Enrichment Task* |
|---|---|
| Some students may have difficulty understanding that arithmetic sequences have a common difference between terms. | Students should get in the habit of using at least three sets of consecutive terms to determine the common difference |
| Some students may not be able to see that any term in the sequence can be determined by adding the first term of one less than the term number and the common difference. | Lead students to see the relationship by using one on-one discussions about the connection between the sum of the first term and one less than the term number. For example, you might ask<br>- If you use n to represent the term number, how would you write one less than the term number? |
| Some students may not understand how to use subscripts to define specific terms in the sequence. | Give students practice by, for example, asking them to represent the fifth term of a sequence using t to represent a term and a subscript to represent the term number. |
| Some students may find it challenging to identify the variables for the equation. | Encourage students to develop a habit of writing down the known information by listing ti, d, and/or n, and then writing the equation for tn. Suggest that they always identify the target or goal of each question by indicating which variable is unknown, for example, $n$ = ? They can then substitute the known data into the equation and solve for the unknown value. |
| Some students may not recognize when they need to use a system of equations to solve a problem. | Help students to understand that whenever two variables are unknown, they should use a system of equations to solve a problem. |
| Some students may not recognize a sequence as arithmetic. | Students should identify the difference for at least three pairs of consecutive terms. This practice will help them to see the pattern. |
| Some students may confuse arithmetic sequences and arithmetic series. | Discuss with students the different targets of sequences and series. Ask:<br>- Are you trying to determine a specific term?<br>- Are you trying to determine the sum of all the terms? |

| | |
|---|---|
| Some students may be unable to decide which sum formula to use. | Have students write down the known values for $t_1$, d, n, and $t_n$, and then use the appropriate formula based on the known information. |
| Some students may choose an inappropriate formula for determining the sum of a series. | By writing down the values of $t_1$, d, n, and $t_n$, students will recognize which formula they can use based on the data that are known. They should form a habit of writing down the formula before doing any substitution. |
| Some students may not know when to use a system of equations. | By writing down the known values of $t_1$, d, n, and $t_n$, they will know when two variables are unknown. They will then be aware that they may need to use a system of equations to solve for the missing terms. |

# 4
# GEOMETRY

## LINES AND ANGLES

## INTRODUCTION

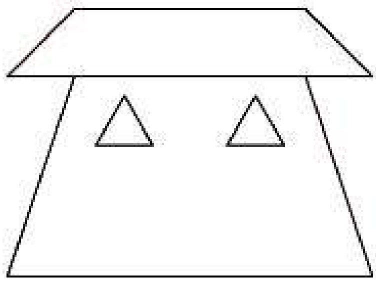
Why we are teaching lines and angles?

From for practical purposes namely

Construction of different types of 'Vedies' for performing religious rituals and also in astronomy and astrology.

Measurement for constructing 'Vedies'

In the present context any building cannot be constructed without knowledge of cube, cuboid, surface area, area of rectangle, square measurement of sides of various plane figures, which are there in any building For example constructing a house of the following type we need to know the concept of line, line segment, various types of angles etc.



## KEY CONCEPTS

1. Point, line, plane, angle
2. Intersecting and non-intersecting lines

3. Pair of angles
4. Parallel lines and transversal
5. Parallel lines to a given line
6. Angle sum property of a triangle

## LEARNING TEACHING STRATEGY

### ACTIVITY I

By paper cutting and pasting verity if two lines intersect each other then vertically opposite angles are equal.

***Material Required***

| | |
|---|---|
| Coloured paper | Carbon paper |
| Ruler | Pencil |
| Pair of scissors | Glue |

***Procedure***

**Step-1:** On a sheet of paper draw two intersecting lines AB and CD. Let the two lines intersect at O.

**Step-2:** Lable two pairs of vertically opposite angles as angle 1 opposite to angle 2 and angles 3 opposite to angle 4.

**Step-3:** Make the replica of angle 2 and angle 3 and cut it.

**Step-4:** Place the cut out of angle 2 on angle 1. Are they equal?

**Step-5:** Place the cut out of angle 3 on angle 4. Are they equal?

Write the observation and result.

**Basic Geometrical Concepts**

Point, Line and plane are the building blocks of geometry. Teacher can explain these ideas through some physical examples and experiences.

**Activity:** Each student of the class may be asked to write the possible definition of a 'line'.

It will be observed that different students will use different words to define a line and these words themselves need to be defined first. You may now ask the student concerned to define these words. On getting his/her answer, it will be seen that for defining these words also some more 'new words' have been used, which are required to define themselves again. In this way the process may never end and hence need of taking the line as an undefined term. Some



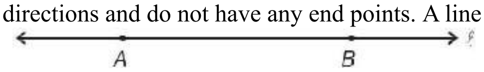
process can be followed for explaining the need of taking point and plane as the undefined terms.

**Activity:** Each students is asked to draw the figure of park where all types of figures are there e.g. parallel line (flower bed), non-parallel lines, rectangular stairs, sea saws, swings etc.

**Discussion Points**

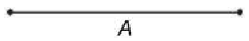
***Basic Concepts***

A line is extended in both the directions and do not have any end points. A line do not have specific length.




$AB$ or $\ell$ is a line.

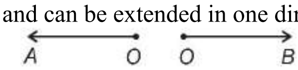
Part of line having two end points and cannot be extended in both the directions is called a line segment. Line segment has fixed length




B

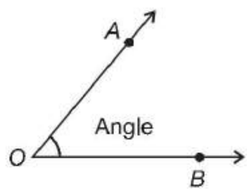
$AB$ is a line.

Ray is a part of line having one fixed point and can be extended in one direction and do not have a specific length



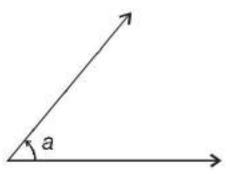

$AO$ and $OB$ are two rays

If three points lie on a line they are called collinear point else non collinear points




Two rays having common end points make an angle.

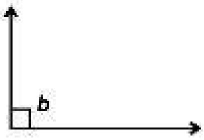
$\angle AOB$ is an angle



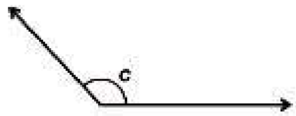
***Types of angles***

(i) Acute angle: $0° < a < 90°$

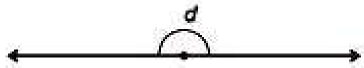
(ii) Right angle: $b = 90°$



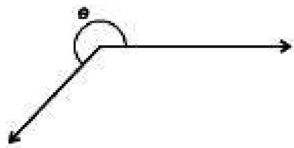

(iii) Obtuse angle: $90° < c < 180°$




(iv) Straight angle: $d = 180°$




(v) Reflex angle: $180° < e < 360°$

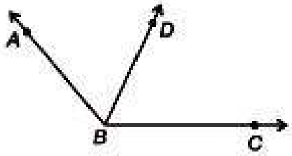



Complementary Angles: If sum of two angles is 90°, they are called complementary angles.

Supplementary Angles: If sum of two angles 180°, they are called supplementary angles.

In general a pair of supplementary angles = $x°$, $(180 - x)°$

Adjacent Angles:

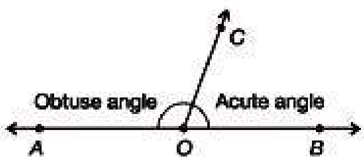



B ⊠ Common vertex

BD = Common ray

∠ABD and ∠DBC is a pair of adjacent angles.

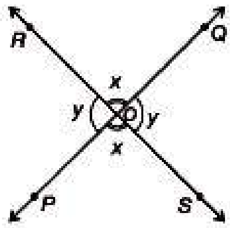
Linear Pair: A pair of adjacent angles which are supplementary i.e. their sum is 180°.



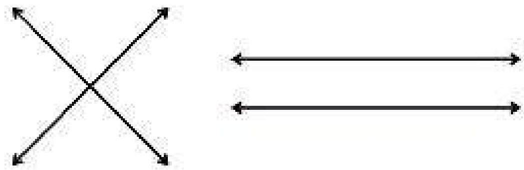

∠AOC, ∠BOC is a linear pair ∠AOC + ∠BOC = 180

⊠

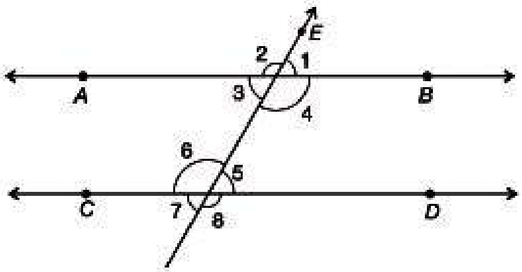
**Vertically Opposite Angles:** ⊠ROQ ,⊠POS and ⊠POR ,⊠QOS are two pairs of Vertically opposite angles.




**Intersecting and Non-intersecting Lines:** In a plane two lines either intersecting each other or do not intersect. It they do not intersect each other, they are parallel.



**Parallel Lines and Transversal line**




If a transversal intersect two parallel lines then

(i) Pair of corresponding angles are equal.

(a) $\angle 1 = \angle 5$ (b) $\angle 2 = \angle 6$ (c) $\angle 3 = \angle 7$ (d) $\angle 4 = \angle 8$

(ii) Pair of interior alternate angles are equal.

(a) $\angle 3 = \angle 5$ (b) $\angle 4 = \angle 6$

(iii) Pair of exterior alternate angles are equal.

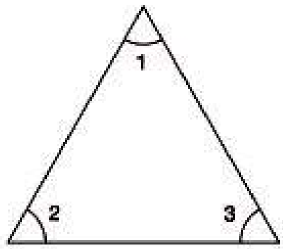
(a) $\angle 2 = \angle 8$ (b) $\angle 1 = \angle 7$

(iv) Interior angles on the same side of transversal are supplementary.

(a) $\angle 3 + \angle 6 = 180°$ (b) $\angle 4 + \angle 5 = 180°$

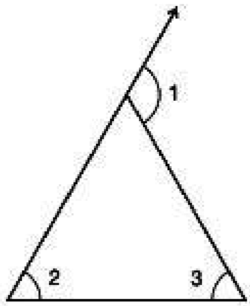
- Sum of the interior angles of a triangle is 180°.

  $\angle 1 + \angle 2 + \angle 3 = 180$



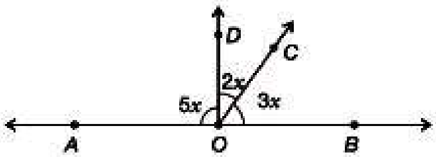

- If one side of a triangle is extended then exterior angle so formed is equal to the sum of interior opposite angles.

  $\angle 1 = \angle 2 + \angle 3$

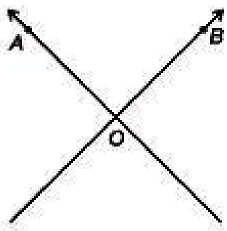



## THOUGHT PROVOKING PROBLEMS

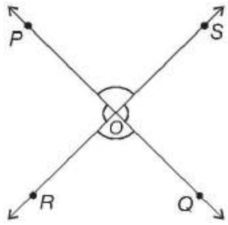
1. What is the measure of $\angle AOC$



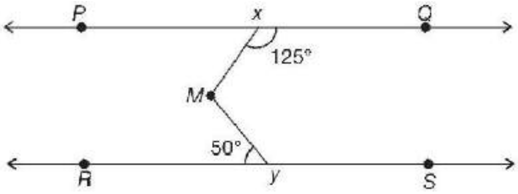

2. If AC and BD intersect each other at O and if $\angle AOD : \angle DOC = 1:4$ then what is measure of $\angle COB$



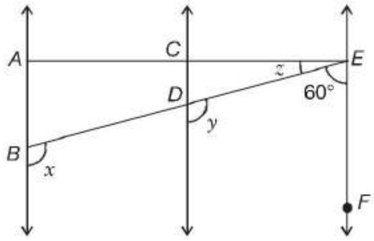
3. If PQ and RS intersect each other at O and if $\angle POR : \angle ROQ = 2 : 3$ then find all the angles



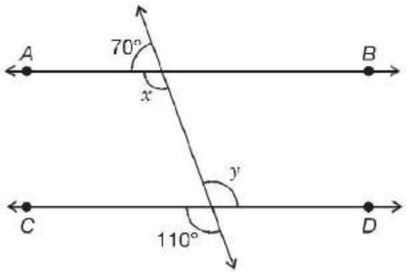

4. In given figure if $PQ \parallel RS$, $\angle MXQ = 125°$, $\angle MYR = 50°$ then find $\angle XMY$



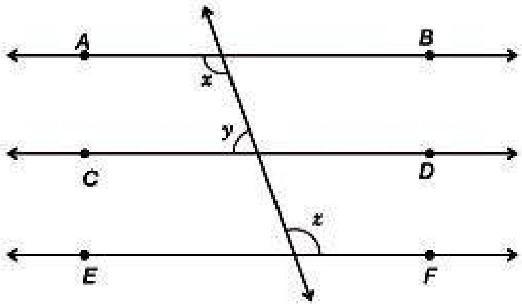

5. In given figure if $AB \parallel CD$ and $CD \parallel EF$, $EA \perp AB$, if $\angle BEF = 60°$ then find $x$, $y$, $z$



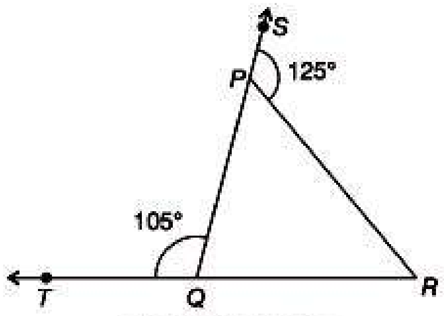

6. Find the values of $x$ and $y$. Also show that $AB \parallel CD$



7. In adjacent figure if $AB \parallel CD$ and $CD \parallel EF$, $y : z = 2 : 3$ then find x



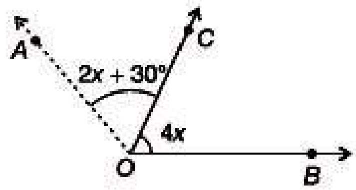

8. In $\Delta PQR$ sides QP and RQ are extended to S and T resp. If $\angle SPR = 125°$ and $\angle PQT = 105°$ then find $\angle PRQ$




## WORKSHEET 1

Multiple Choice Question:

1. In the given figure the value of x which makes AOB a straight line is

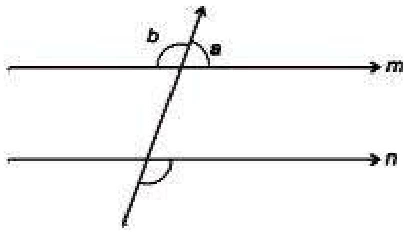



(a) 35° (b) 30° (c) 25° (d) 40°

2. The angle which is equal to 8 times its complement is

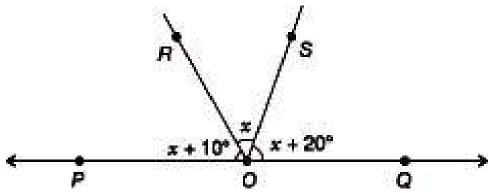
(a) 80° (b) 72° (c) 90° (d) 88°

3. In the figure if $m \parallel n$ and $\angle a : \angle b = 2 : 3$ then measure of $\angle c$ is




(a) 72° (b) 108° (c) 120° (d) 150°

4. In the figure, value of $x$ is




(a) 50° (b) 40° (c) 60° (d) 70°

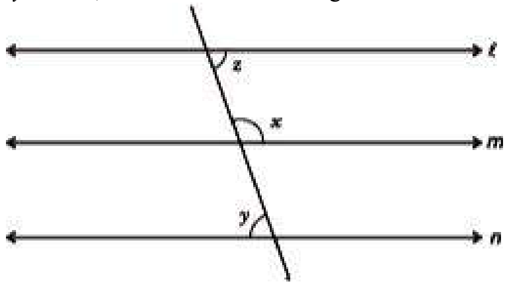
5. An exterior angle of a triangle is 80° and the interior opposite angles are in a ratio 1 : 3, then measure of each interior opposite angle is

(a) 30°, 90° (b) 40°, 120° (c) 20°, 60° (d) 30°, 60°

6. The complementary angles are in the ratio 1 : 5. Find the measure of the angles:

(a) 15°, 75° (b) 75°, 15° (c) 12°, 60° (d) 60°, 12°

7. If the given figure $l \parallel m \parallel n$, if $x : y = 5 : 4$, then the measure of angle $z$ is:




(a) 40° (b) 50° (c) 90° (d) 80°

8. The complement of an angle $m$ is

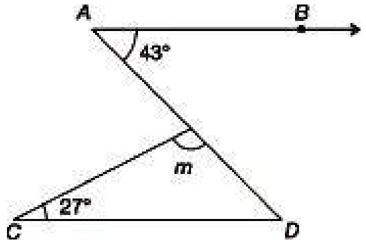
(a) $m$ (b) $90° + m$ (c) $90° – m$ (d) $m \times 90°$

9. If the measure of an angle is twice the measure of its supplementary angle, then the measure of the angle is
(a) 60° (b) 90° (c) 120° (d) 130°

10. If two interior angles on the same side of a transversal intersecting two parallel lines are in the ratio 2 : 3 then the larger of two angles is:

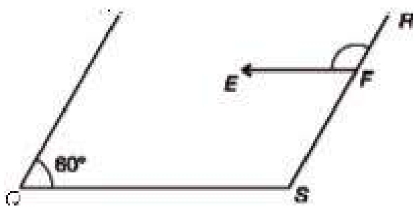
(a) 72° (b) 108° (c) 54° (d) 36°

11. If the given figure $AB \parallel CD$ then the measure of $m$ is



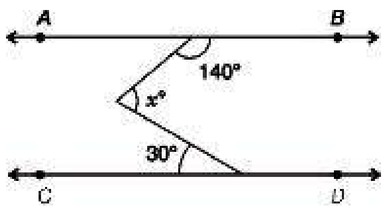

(a) 110° (b) 100° (c) 90° (d) 137°

12. If the given figure $PQ \parallel RS$ and $EF \parallel QS$, if $\angle PQS = 60°$, then the measure of $\angle RFE$ is



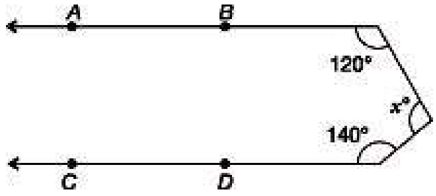

(a) 115° (b) 120° (c) 60° (d) 180°

13. If the given figure $AB \parallel CD$ the value of $x$ is




(a) 35° (b) 40° (c) 60° (d) 75°

14. If the given figure $AB$ and $CD$ are parallel to each other the value of $x$ is




(a) 90° (b) 100° (c) 45° (d) 180°

15. Find the measure of the angle which is complement of itself

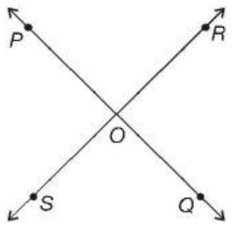
(a) 30° (b) 90° (c) 45° (d) 180°

16. Two lines are respectively perpendicular to two perpendicular lines then these two lines to each other are

(a) parallel (b) perpendicular

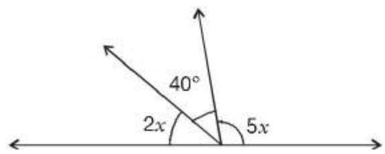
(c) inclined at same $O$ cute angle (d) intersecting at 110°

17. In the figure $PQ$ and $RS$ intersect at $O$. If $\angle POS : \angle SOQ = 4 : 5$ $\angle ROQ$ is




(a) 80° (b) 100° (c) 90° (d) 70°

18. In the figure the value of $x$ is

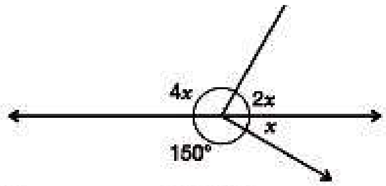



(a) 30° (b) 10° (c) 20° (d) 40°

19. Measure of an angle which is supplement to itself

(a) 45° (b) 30° (c) 90° (d) 180°

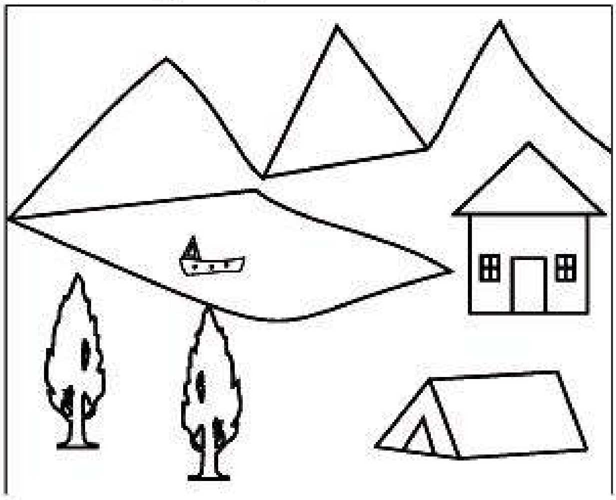
20. In the figure, value of x is




(a) 45° (b) 30° (c) 60° (d) 20°

## COUNT AND WRITE

How many triangles are there in given figure?



## PROJECT WORK 1

Puzzle : Lines and angles

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| I | N | T | E | R | I | O | R | E | Z | C | C | C | T | R |
| T | C | E | S | R | E | T | N | I | N | S | I | D | E | N |
| N | R | D | M | E | L | N | L | E | L | L | A | R | A | P |
| E | E | A | T | I | A | A | E | P | V | E | R | T | E | X |
| U | L | G | N | I | D | N | O | P | S | E | R | R | O | C |

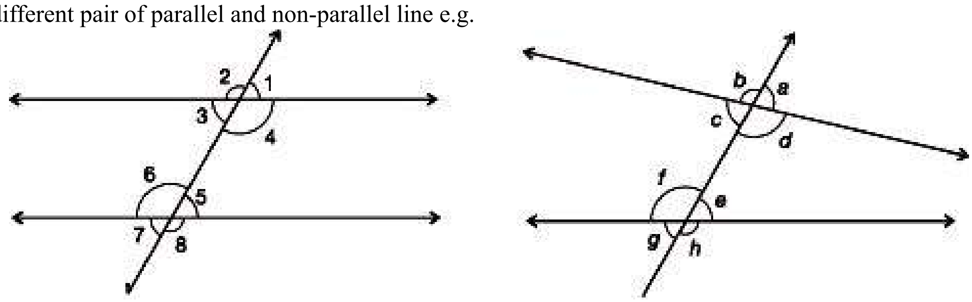
| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| R | A | E | I | S | A | G | L | T | C | Y | P | A | O | T |
| G | C | T | O | L | V | L | N | O | V | E | E | P | E | E |
| N | S | U | P | P | L | E | M | E | N | T | E | R | Y | T |
| O | H | CY | | E | C | P | R | D | I | G | U | O | P | A |
| C | O | A | F | A | A | T | I | S | T | S | C | T | R | N |
| W | R | T | J | S | I | C | O | R | A | D | O | R | O | R |
| E | T | D | S | C | U | P | I | E | E | L | O | A | U | E |
| K | A | RA | | L | P | G | M | G | E | W | A | C | G | T |
| S | E | L | A | O | H | O | R | I | Z | O | N | T | A | L |
| L | Y | RA | | T | N | E | M | E | L | P | M | O | C | A |
| O | B | T | U | S | E | N | O | I | T | C | E | R | I | D |

| | | |
|---|---|---|
| ACUTE | ADJACENT | ALTERNATE |
| ANGLE | ARROW | COMPASS |
| COMPLEMENTARY | CONGRUENT | CORRESPONDING |
| DEGREE | DIRECTION | HORIZONTAL |
| INSIDE | INTERIOR | INTERSECT |
| LEFT | LINE | LONG |
| MEASURE | OBTUSE | OPPOSITE |
| PARALLEL | PERPENDICULAR | PLANE |
| POINT | PROTRACTOR | RAY |
| RIGHT | SCALE | SHORT |
| SKEW | SUPPLEMENTARY | TRANSVERSAL |
| VERTEX | VERTICAL | ZERO |

## PROJECT WORK 2

1. Draw a Badminton Court
2. Draw a Football Court

3. Draw different pair of parallel and non-parallel line e.g.




**Measure angles and Draw Conclusions from following Table Different figures**

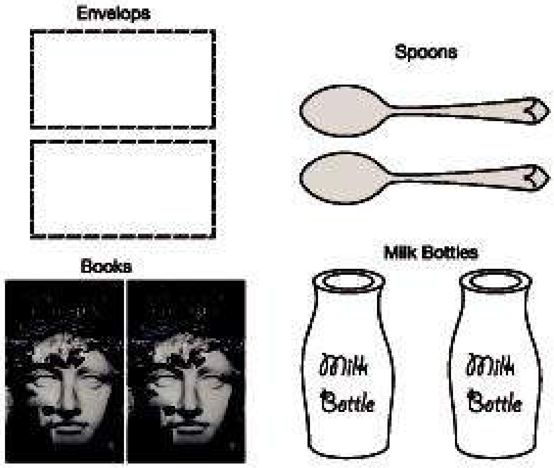
| of parallel lines | ∠1 | ∠2 | ∠3 | ∠4 | ∠5 | ∠6 | ∠7 | ∠8 | ∠1 = ∠5 | ∠2 = 6 | ∠3 = ∠7 | ∠4 = 8 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | |
| 2 | | | | | | | | | | | | |
| 3 | | | | | | | | | | | | |
| 4 | | | | | | | | | | | | |
| 5 | | | | | | | | | | | | |
| 6 | | | | | | | | | | | | |

| | ∠a | ∠b | ∠c | ∠d | ∠e | ∠f | ∠g | ∠h | ∠a ≠ ∠e | ∠b ≠ f | ∠c ≠ ∠g | ∠d ≠ h |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |

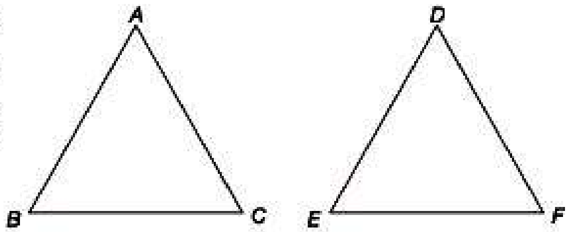
# CONGRUENT TRIANGLES

## Introduction

Two figures are said to be congruent to each other if they have exactly same shape and size.




A triangle is a closed figure having three sides, three angels and three vertices. Thus two triangles are congruent if the sides and angles of one triangle are equal to the corresponding sides and angles of the other triangle.




Thus $\Delta ABC \cong \Delta DEF$

If

A ↔ D

B ↔ E

C ↔ F

$\overline{AB} = \overline{DE}$

$\overline{BC} = \overline{EF}$

$\overline{AC} = \overline{DF}$

$\angle A = \angle D$

$\angle B = \angle E$

$\angle C = \angle F$

$\angle C = \angle F$

**Key Concepts**

1. **SAS (Congruence rule):** Two triangles are congruent if two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle.
2. **(ASA Congruence rule):** Two triangles are congruent if two angles and the included side of one triangle are equal to the two angles and the included side of the other triangle.
3. **(AAS Corrolary rule):** Two triangles are congruent if any two pairs of angles and one pair of corresponding sides are equal.
4. **(SSS Congruence rule):** If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.
5. **(RHS Congruence rule):** If in two right triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then two triangle are congruent.
6. Angles opposite to equal sides of a triangle are equal.
7. The sides opposite to equal angles of a triangle are equal.

## LEARNING TEACHING STRATEGY

### ACTIVITY 1

The explore criteria of congruent triangles, using a set of triangles cut outs.

1. Cut out a triangle $ABC$ (1) on red coloured glazed paper. Now cut another $\Delta PQR$ on green coloured glazed paper such that $PQ = AB$, $QR = BC$ and $PR = AC$.

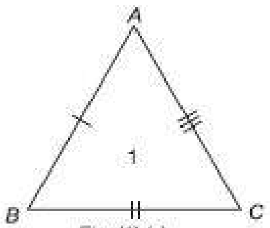


Fig. (1) (a)

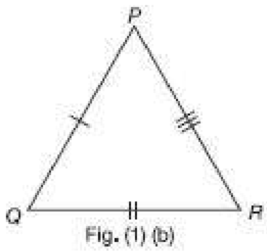


Fig. (1) (b)

2. Construct a $\Delta ABC$ (2) with known base $BC$ and base angles $\angle B$ and $\angle C$ on yellow coloured glazed paper such that $EF = BC$, $\angle E = \angle B$ and $\angle F = \angle C$.

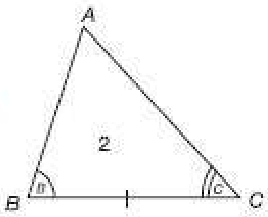


Fig. (2) (a)

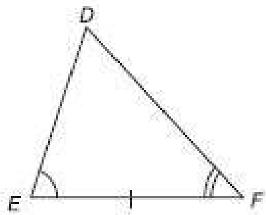


Fig. (2) (b)

LEARNING TEACHING STRATEGY

3. Draw a $\Delta ABC$ (3) with two known sides $AB$ and $BC$ and $\angle B$ on voilet coloured glazed paper. Take its cut out. Now cut another $\Delta LMN$ on pink coloured glazed paper such that $LM = AB$, $MN = BC$, $\angle M = \angle B$.

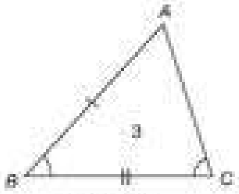


Fig. (3) (a)

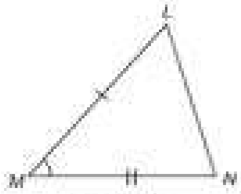


Fig. (3) (b)

4. Draw a right angled $\Delta ABC$ (4) on green coloured glazed paper. Take its cut out. Now cut another $XYZ$ such that side $YZ = BC$ and hypotenuse $ZX = CA$ on blue coloured glazed paper.

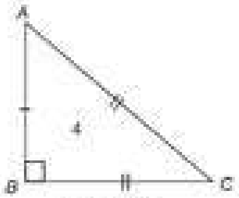


Fig. (4) (a)

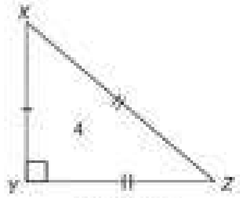


Fig. (4) (b)

***Observation***

(i) We observe that $ABC$ (1) superimposes $DEF$ exactly and vice-versa. Therefore $ABC$ (1) is congruent to $PQR$. This type of congruency is called $SSS$ congruency.

(ii) We observe that $ABC$ (2) superimposes $DEF$ exactly vice-versa. This means that $ABC$ (2) is congruent to $DEF$. This is called $ASA$ congruency of triangles.

(iii) We see that $ABC$ (3) superimposes $LMN$ exactly and vice-versa which implies that $ABC$ (3) is congruent to $LMN$ (3). This type of congruency is $SAS$ congruency.

(iv) We see that $ABC$ (4) superimposes $XYZ$ exactly and vice-versa..Therefore $ABC$ (4) is congruent to $XYZ$. This is RHS type of congruency.

## THOUGHT PROVOKING PROBLEMS 2

1. Can equiangular triangles be always congruent? Explain.
2. Is there any $SSA$ congruency criterion for the triangles? Explain.

## WORKSHEET 2

***Choose the correct option in the following questions***

1. In $\Delta ABC$, $\angle C = \angle A$ and $BC = 6$ cm and $AC = 5$ cm then the length of $AB$ is

   (a) 6 cm (b) 5 cm (c) 3 cm (d) 3.5 cm

2. $\Delta ABC \cong \Delta DEF$ by SSS congruence rule then

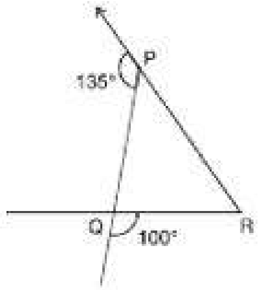
   (a) $AB = DE, BC = EF, \angle C = \angle E$ (b) $AB = FD, BC = DE, CA = EF$

   (c) $AB = DE, BC = EF, CA = FD$ (d) $AB = EF, BC = FD, CA = DE$

3. In $\Delta XYZ$ of $XY = YZ$, then

   (a) $\angle Y > \angle Z$ (b) $\angle X = \angle Z$ (c) $\angle X = \angle Y$ (d) $\angle X < \angle Y$

4. For the $\Delta PQR$, which of the following are true?




   (a) $PQ > QR$

   (b) $PQ < QR$

   (c) $PQ = QR$

   (d) $\angle P = \angle Q$

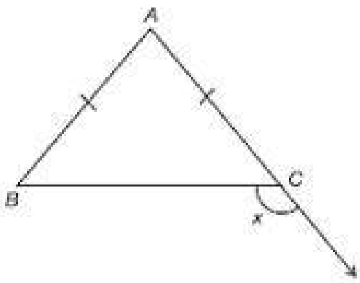
5. $\Delta ABC \cong \Delta PQR$ if $AB = 5$ cm, $\angle B = 40°$ and $\angle A = 80°$, then which is ture?

   (a) $QR = 5$ cm (b) $\angle R = 40°$

   (c) $PQ = 5$ cm and $\angle Q = 40°$ (d) $\angle R = 80°$, $BC = 5$ cm

6. If $\Delta ABC$ and $\Delta DEF$, AB = $DE$, $BC = EF$ and $\angle A = \angle D$ then the triangles are ______ and congruency rule used is __________.

7. In figure $\Delta ABC$ is such that $AB = AC$ then $x$ is




   (a) 80°

   (b) 100°

   (c) 130°

   (d) 120°



A BC

8. In and D E F, $AB = EF$, $AC = ED$, then A BC ≅ ______

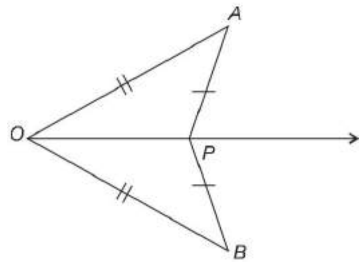
   (a) D EF (b) ED F (c) EFD (d) FED

9. If A B C ≅ PQ R , then which of the following is ture?

   (a) $AB = QR$ (b) $AC = QR$ (c) $AB = PQ$ (d) $AC = QR$

O A

10. Given P ≅ O BP in the figure the criteria by which S are congruent is




(a) *SAS*
(b) *SSS*
(c) *RHS*
(d) *ASA*

## WORKSHEET 3

***Choose the correct option in the following question***

1. Two equilateral triangles are congruent when
   (a) their angles are equal
   (b) their sides are equal
   (c) their sides are proportional
   (d) their areas are equal.
2. One of the angles of a triangle is 75°. If difference of the other two angles is 35°, then the largest angle of the triangle is
   (a) 80° (b) 100° (c) 75° (d) 135°
3. In $ABC$ if $AB = BC$, then
   (a) $\angle B > \angle C$ (b) $\angle A = \angle B$ (c) $\angle A = \angle C$ (d) $\angle A < \angle B$
4. In $AOC$ and $XYZ$, $\angle A = \angle X$, $AO = XY$, $AC = XZ$ then by which congruence rule, $AOC \cong XYZ$?
   (a) *ASA* (b) *SAS* (c) *SSS* (d) *RHS*

5. In $ABC \cong PQR$ then which of the following is true?

(a) $AB = YZ$ (b) $AB = XY$ (c) $AB = XZ$ (d) $BC = XY$

6. If $ABC \cong PQR$, $\angle B = 40°$ and $\angle C = 95°$ then $\angle P$ is

(a) 40° (b) 45° (c) 35° (d) 48°

7. If $ABC$, $AB$ = BC = 5 cm and $\angle A = 55°$ then $\angle B$ is

(a) 55° (b) 70° (c) 45° (d) 60°

8. Complete the following statement

The sides _______ equal angles of a triangle are equal.

9. In $ABC$, $\angle A > \angle B$, then which is true?

(a) $BC > AC$ (b) $BC < AC$ (c) $AB > BC$ (d) $BC > AB$

10. Each angle of an equilateral triangle is

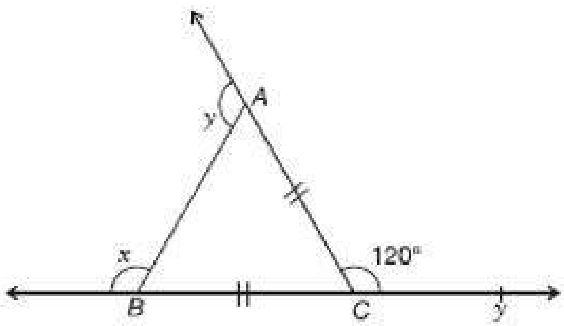
(a) 55° (b) 60° (c) 80° (d) 45°

## PROJECT WORK 3

Take a rectangular Geo-Board. Find out the number of triangles with area 20 sq. units.

## ASSIGNMENT

1. In fig $AC = BC$ and $\angle ACY = 120°$. Find $\angle x$ and $\angle y$

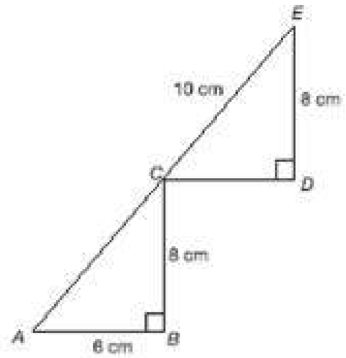


2. Prove that $\Delta ABC \cong \Delta CDE$

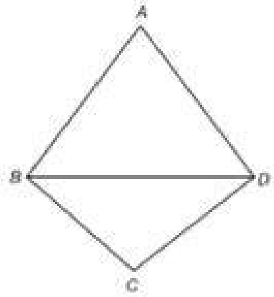
Given that, $CE = 10$ cm

$DE = 8$ cm

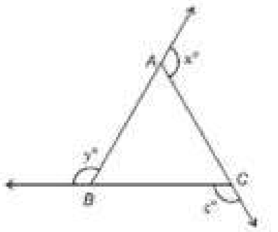
$AB = 6$ cm




3. In figure $AB = AD = 6$ cm and $BC = CD = 3.5$ cm. Prove that $\angle ABC = \angle ADC$




4. In figure $AB = 8.5$ cm, $BC = 6$ cm and $CA = 7.2$ cm. Write $x$, $y$ and $z$ is descending order.




## Misconceptions/Common Errors

(a) **The naming of the triangle on the basis of**

Ordering of the congruency is very important i.e. the corresponding angles and sides should be in same order. If $\Delta ABC \cong \Delta DEF$ then

$A \longleftrightarrow D,\ B \longleftrightarrow E,\ C \longleftrightarrow F$

(b) **Congruent parts are generally misunderstood by student.**

Problem in the understanding of questions and application of criteria to be applied.

**Remedial Strategies**

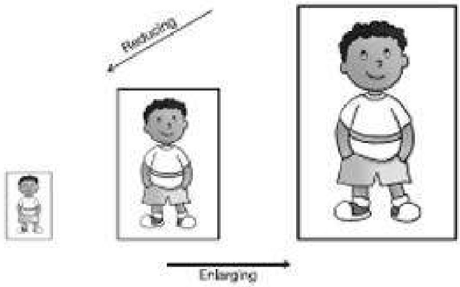
(a) Congruent triangles should be taught to write correctly by matching the corresponding vertices for this "match the following" activities should be suggested.

(b) Making student to comprehend the question.

## SIMILAR TRIANGLES

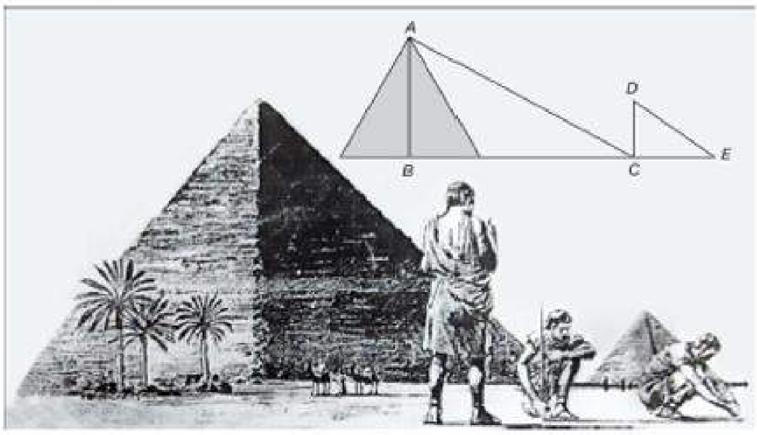
# Introduction

People often enlarge a photograph after developing a film. The enlarged picture and the original scene have the same shape but not the same size. Similarly a reduced copy of the diagram has the same shape as the original but is smaller is size.




hey are called similar shapes. Two shapes are called similar when they have the same shape.

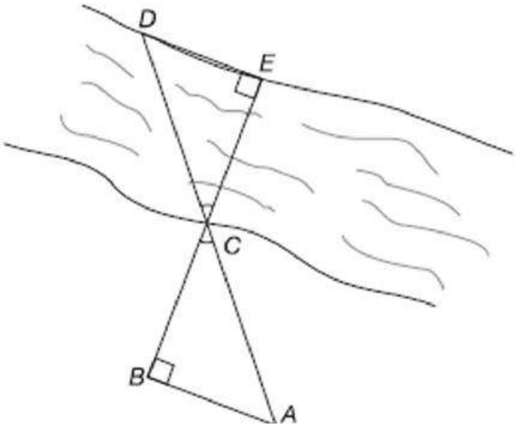
bout 2500 years ago, a Greek Mathematician Thales surprised people by calculating the height of a yramid from the length of the shadow of a stick.



Thales used the fact that the big triangle ABC formed by the pyramid and its shadow and the small triangle DCE formed by the stick and its shadow are similar. So we have $\frac{AB}{BC} = \frac{DC}{CE}$

Thales could measure the length BC, DC and CE. So he used this equation to calculate the height AB of the pyramid.

The same concept of similarly can be used to find the width of the river also.




$ABC$ and $DEC$ are similar. So $\frac{AB}{DE} = \frac{BC}{EC}$

We can measure the distance *AB, BC* and *DE* and using the equation we can find the width of the river.

The same concept can be used to find the height of the mountain, tree and so on.

**Key Concepts**

1. Two figures having the same shape but not necessarily the same size are called similar figures.
2. All the congruent figures are similar but the converse is not true.
3. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.
4. If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

5. If in two triangles, two angles of one triangle are respectively equal to the two angles of the other triangle, then the two triangles are similar (*AA* similarity criterion).
6. If in two triangles, corresponding sides are is the same ratio, then their corresponding angles are equal and hence the triangles are similar (*SSS* similarity criterion).
7. If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are in the same ratio then the triangles are similar (*SAS* similarity criterion).
8. The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.
9. In two similar triangles the ratio of their corresponding sides is equal to the ratio of their respective perimeters.
10. If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then the triangles on both sides of the perpendicular are similar to the whole triangle and also to each other.
11. In a right triangle, the square of the hypotenuse is equal to the sum of the square of the other two sides (Pythagoras theorem).
12. If in a triangle, square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

## LEARNING TEACHING STRATEGIES

### ACTIVITY 1

***Choose the correct option***

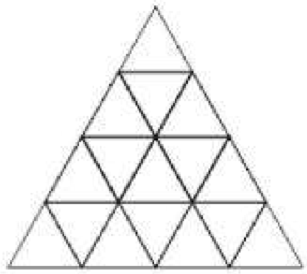
How many triangles similar to this equilateral triangle Δ are there in the following diagram:

(a) 10

(b) 16

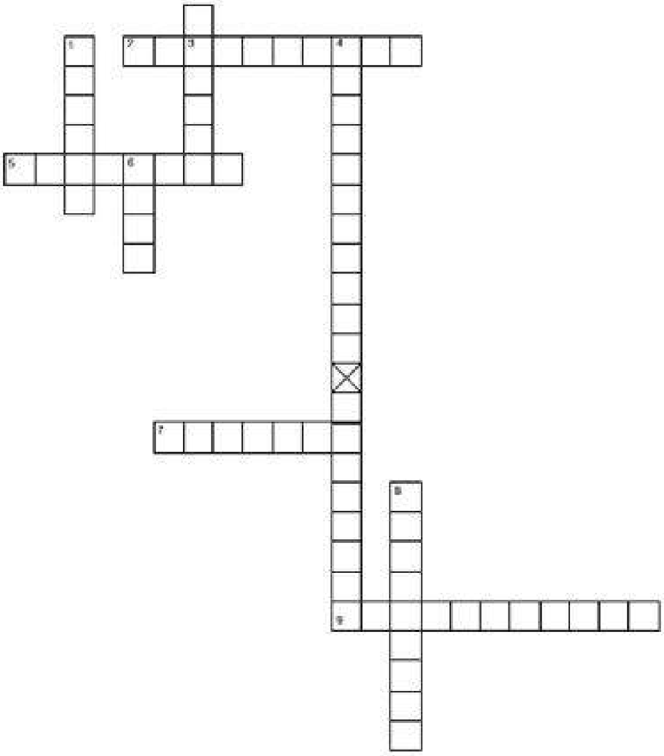
(c) 26

(d) 27



# ACTIVITY 2

## CROSS WORD PUZZLE



***Down***

1. The ratio of the areas of two similar triangles is equal to the _______ of the ratio of their corresponding sides.
3. Mathematician with whose name basic proportionality theorem is known.
4. Triangles in which Pythagoras theorem is applicable.
6. A side of triangle is a ______ segment.
8. Two figures with same shape and size.

***Across***

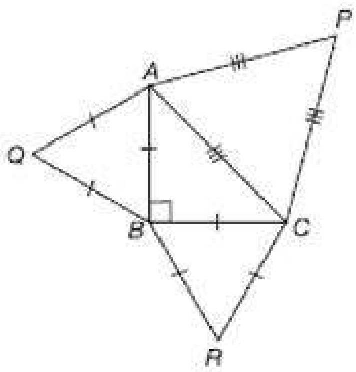
2. Mathematician who proved that in right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.
5. If a line divides any two sides of a triangle in the same ratio, then the line is _____ to the third side.
7. Two figures with same shape.
9. Triangles which are always similar.

## THOUGHT PROVOKING QUESTION

1. Rajesh has a land in the form as given in adjacent figure, in which APC, AQB and BRC are equilateral triangles. He donated the triangular land ABC to Old Age Home. He gave triangular land APC to his daughter and the land AQB and BRC to his son.

   (a) Was he fair in dividing his land to his children. Explain

   (b) Which value is depicted in the question.



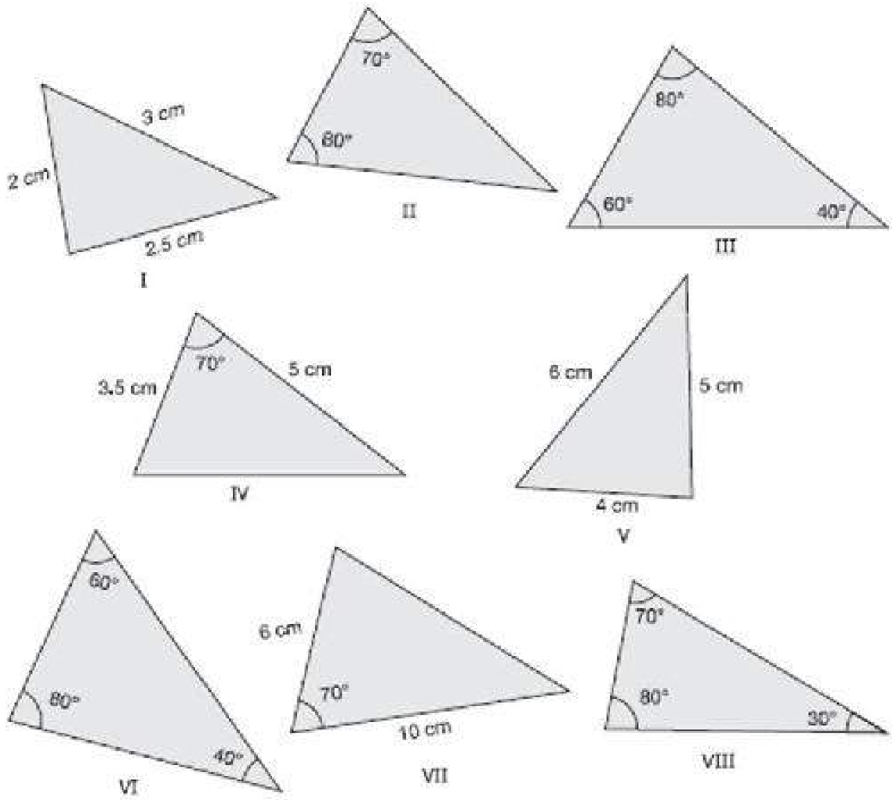

2. CE and DE are equal chords of a circle with centre O. If angle AOB = 90°.

   Find $ar(\Delta CED) : ar(\Delta AOB)$

E

## ACTIVITY 1

Take the following cutouts of the triangle and complete the following table:

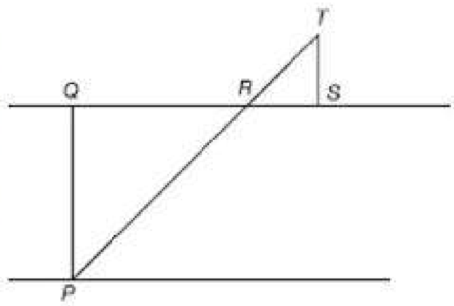



| *Triangle No.* | *Similar Triangle No.* ~ Δ | *Similarity Criterion Applied* |
|---|---|---|
| I | | |
| IV | | |
| II | | |
| VII | | |

## PROJECT WORK

**AIM:** To find the width of a pathway.

Fix a pole at $Q$ directly opposite to tree $P$ on the other side of pathway. Walk along the pathway, fix another pole at $R$ at known distance. Walk another known distance to $S$. From here, walk at right angle, to the pathway till the points $T$ is reached such that $T$ is directly in line with $R$ and $P$. Measure the distance $ST$.



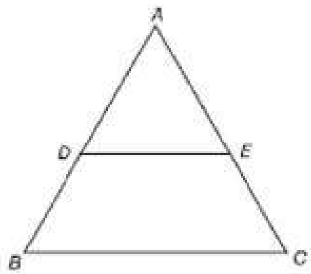

Using the property of similarity of triangles, the width of the pathway is determined.

## WORKSHEET 1

### ASSESSMENT

***Choose the correct option in the following questions:***

1.




In the figure $DE \parallel BC$ and $\frac{AD}{DB} = \frac{3}{5}$, if $AC = 4.8$ cm then $AE$ is

(a) 1.6 cm (b) 1.8 cm (c) 3 cm (d) 3.5 cm

2. $\Delta ABC \sim \Delta DEF$ and $AB = 1.2$ cm, $DE - 1.4$ cm then $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} =$

(a) 49 : 36 (b) 49 : 30 (c) 7 : 6 (d) 36 : 49

3. $\Delta PQR$ is a right angled triangle, right angled at $P$, $PS \perp QR$ then $\frac{QS}{PQ} =$

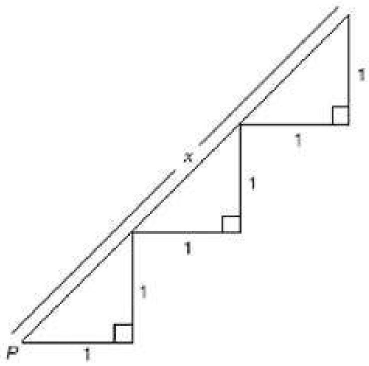
(a) $\frac{PQ}{PR}$ (b) $\frac{(PQ)^2}{PS}$ (c) $\frac{PQ}{QR}$ (d) $\left(\frac{PQ}{PR}\right)^2$

4. $\Delta ABC$ is an isosceles right triangle right angled at C then $AB^2 =$

(a) $AC^2$ (b) $3\,AC^2$ (c) $2\,AC^2$ (d) $BC^2$

5. ABC is an equilateral triangle of side $2a$. Find each of its altitudes.

6. In figure, find $x$




7. If $D$, $E$, $F$ are the mid points of the sides of $BC$, $CA$, $AB$ respectively of $ABC$, then $ar\,(DEF) = ?$

$ar(ABC)$

8. The perimeter of an isosceles right triangle, the length of the hypotenuse is 10 cm, is

(a) $(10\sqrt{2}+9)$ cm (b) $10(\sqrt{2}+1)$ cm (c) 20 cm (d) $20\sqrt{2}$ cm

9. If in two triangles $ABC$ and $PQR$, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$ then

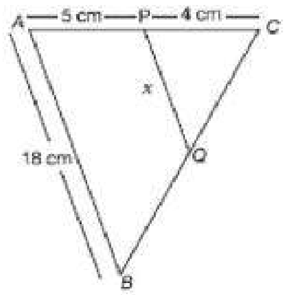
(a) $PQR \sim CAB$ (b) $PQR \sim ABC$

(c) $CBA \sim PQR$ (d) $BCA \sim PQR$

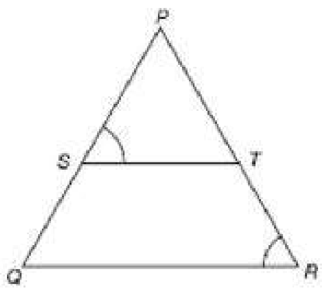
10. If a ladder is kept is such a way that its foot is at a distance of 12 m from the wall and its top reaches a window 9 m above the ground, what is the length of the ladder.

## WORKSHEET 2

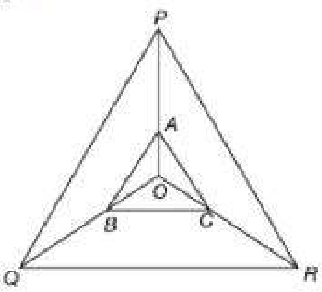
1. In the given figure if $PQ \parallel AB$. Find $x$



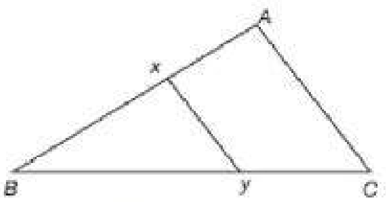

2. In figure $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$ . Prove that $PQR$ is an isosceles triangle




3. In given figure $A$, $B$, $C$ are the mid points of $OP$, OQ and OR respectively. IF $PQ$ = PR. Prove that $ABC$ is an isosceles triangle.

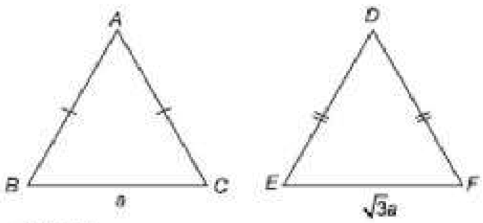


4. In figure $XY \parallel AC$ and $ar(\Delta BXY) = ar(\text{trap } AXYC)$. Prove that: $\frac{AX}{AB} = \frac{2-\sqrt{2}}{2}$




5. In $\Delta ABC$, $BD \perp AC$; prove that $\Delta BDC \sim \Delta ABC$.

6.



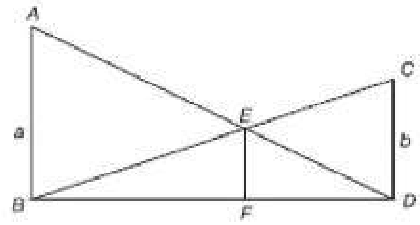

$AB = AC$

$DE = DF$

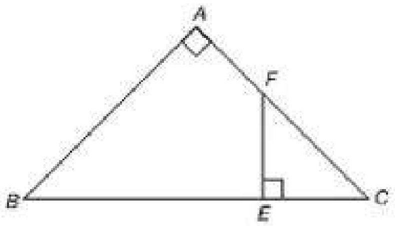
$BC = a, \quad EF = \sqrt{3}a$

Find $\frac{ar(\Delta ABC)}{ar(DEF)}$

7. Two poles of height '$a$' meters and '$b$' metres are '$q$' metres apart. Prove that the height of $EF = \left(\frac{ab}{a+b}\right)$ metres.



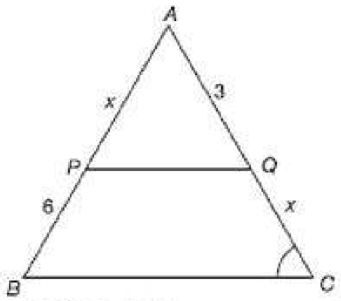

8.



In figure, prove that $\frac{AB}{AC} = \frac{EF}{EC}$

9. The perpendicular from $A$ on side $BC$ of a $\Delta ABC$ intersect $BC$ at D such that $BD$ : $3$ CD. Prove that 2 $AB^2$ = 2 $AC^2$ + $BC^2$.

10.




In figure $PQ \parallel BC$. Find $x$.

$PQ \parallel BC$. Find $x$.

**Misconceptions/Common Errors**

1. In case of SAS criterion, children sometimes do not check if the angle is included between the proportionate sides. Which is to be emphasised.
2. Many-a-times students do not write the similar triangles in the right order i.e. the correspondence of the sides and angles are not right.

**Remedial Strategies**

1. Dividing the questions in simple steps and making the student to practice it again.
2. Making student to comprehend the question by reading the questions again and again.

## QUADRILATERALS

From the moment we wake up till we get into the bed, most of the objects we come across are composed of surfaces in the shape of quadrilaterals. The bed on which we sleep, the floor at which we put our steps, the plots of land on which our houses are constructed, surfaces of beams used in the constructions of building, the newspaper we read in the morning, the blackboard on which a teacher writes, etc. —all involve quadrilaterals. Study of quadrilaterals is the therefore indispensable.

**Key Concepts**

Basics of Quadrilaterals have been discussed at upper primary level. So briefly mentioning the basic concepts, we shall move on to the following topics: Angle sum property of a quadrilateral

- Types of quadrilaterals
- Properties of various types of quadrilaterals
- Mid-point theorem and its converse.

## ACTIVITY

***Angle sum property of a quadrilateral***

Students should be divides into groups and each group should be given a cut–out of a quadrilateral and the should be directed to measure all angles and find their sum. Results obtained by different groups should be tabulated on the blackboard:

| Group No. | Measure of four angles | Sum of the four angles |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| . | | |
| . | | |
| . | | |

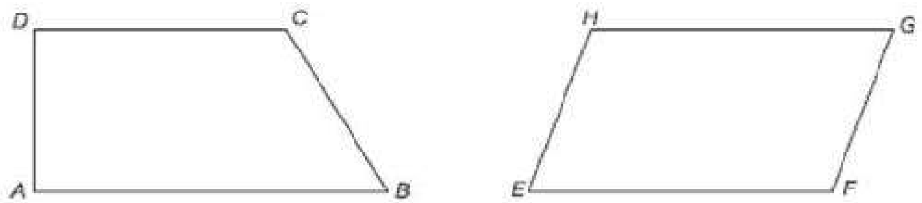
Students will observe that though the measures of the individual angles is quite different, the sum of the four angles in always 360°.

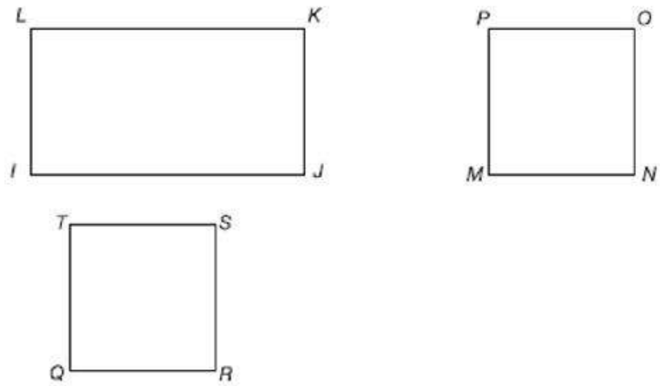
***Teachers Statement***

Sum of the four angles of a quadrilateral is always 360°.

## Types of quadrilaterals

**Activity:** Different groups of students should be provided with sticks of different sizes and should be asked to join them to form quadrilaterals. They may get figures as shown below:






Students will observes that in quadrilateral *ABCD* one pair of opposite sides is parallel, namely, $AB \parallel DC$ . A quadrilateral with one pair of opposite sides parallel in called a trapezium. So *ABCD* is a trapezium.

In *EFGH*, both pairs of opposite sides are parallel. It is a parallelogram.

In *IJKL*, both pairs of opposite sides are parallel and each angle is 90°. It is a rectangle.

*MNOP* is a parallelogram whose all sides are equal. It is a rhombus.

*QRST* is a rhombus whose each angle is 90°. It is a square.

The following properties of different types of quadrilaterals will be discussed through measurement of different parts by students.

| ***Types of Quadrilateral*** | ***Properties*** |
|---|---|
| Parallelogram | (i) Opposite sides are equal.<br>(ii) Opposite angles are equal.<br>(iii) Diagonals bisect each other. |
| Rectangle | (i) All properties of parallelogram.<br>(ii) Diagonals are equal. |
| Rhombus | (i) All properties of parallelogram.<br>(ii) Diagonals are perpendicular. |
| Square | (i) All properties of a parallelogram.<br>(ii) Diagonals are equal.<br>(iii) Diagonals are perpendicular. |

## Midpoint Theorem:

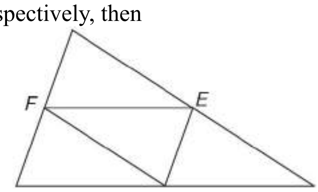
**Activity:** Student will be given cutouts of triangles, preferably, scalene triangle. They will be directed to locate midpoints of the sides of triangles, and join them pairwise. Students will have to measure various line segments as well as angles.

Students will note that line segment joining the midpoints of two sides are:

(i) Parallel to the third side; and

(ii) half of it.

If *D, E, F* are midpoints of sides *BC, CA* and *AB* respectively, then




(i) $DE \parallel AB$ and $DE = \frac{1}{2}AB$

(ii) $EF \parallel BC$ and $EF = \frac{1}{2}BC$

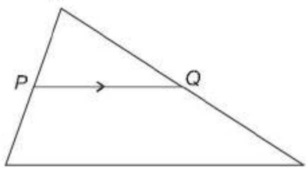
(iii) $FD \parallel AC$ and $FD = \frac{1}{2}AC$

### *Teachers Statements*

The fact which we have observed in case of a few triangles is true for every triangle and is known as midpoint theorem. Line segment joining the midpoint of any two sides of triangle is parallel to the third side and half of it.

### *Converse of Midpoint theorem*

Line segment drawn through midpoint of one side of a triangle parallel to the other side bisects the third side.




Students will verify the above for a few case.

*P* is midpoint of *AB* and $PQ \parallel BC$

⇒ *Q* is midpoint of *AC*.

## WORKSHEET 1

### *F.A.*

1. State true or False

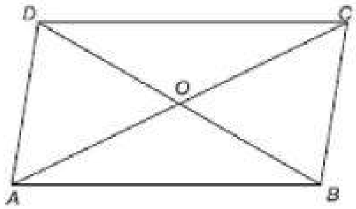
   (a) A parallelogram is a trapezium.

(b) A quadrilateral whose both pairs of opposite sides are equal is a rectangle.

(c) Diagonals of a rhombus are equal.

2. *ABCD* is a parallelogram

Diagonals *AC* and *BD* meet at *O*, *AO* = 4.5 cm, OD = 4 cm, DC = 6 cm, then



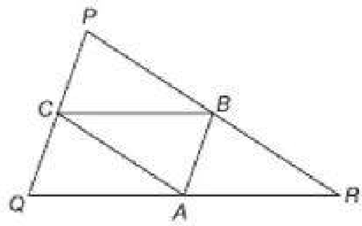

(a) *OC* = ............

(b) *BD* = ............

(c) *AB* = ............

3. A, B, C are the mid-points of the sides *QR*, *PR* and *PQ* respectively. *AB* = 2.5 cm, *PR* = 6 cm, *QR* = 8 cm

Then

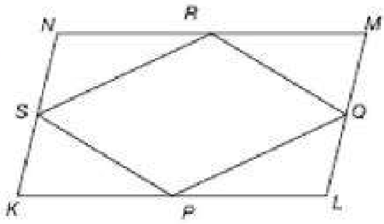



*PQ* = ............

*BC* = ............

4. If *KLMN* is a parallelogram, and

*P, Q, R, S* are mid-points of *KL, LM, MN* and *NK* respectively *PQRS* is a




(a) Square

(b) Rhombus

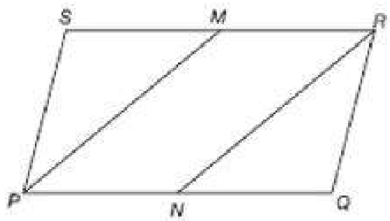
(c) Parallelogram

(d) None of these

Tick the correct option

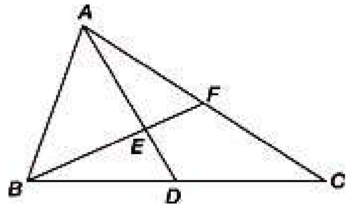
**S.A.**

1. *PQRS* is a parallelogram. *PM* and *RN* bisect $\angle SPQ$ and $\angle SRQ$ respectively. Prove that *PNRM* is a parallelogram.



2. Prove that perpendiculars drawn from two opposite vertices of a parallelogram on the diagonal joining other two vertices are equal.
3. *ABCD* is a trapezium in which $AB \| DC$ . *AB* =14 cm, *DC* = 18 cm, *E* and *F* are midpoints of non-parallel sides *AD* and *BC*. Find *EF*. Justify your answers.
4. *AD* is a median of *ABC*. *E* is midpoint of *AD*. *BE* is joined and produced to meet *AC* in *F*.

If *AC = 7.8* cm, find *FC*.




**Recreational Methematics**

Ramesh rejects a plot of land in the shape of a quadrilateral saying that it is not possible to walk straight from one of the corner to all the other corners, within plot. Guess the shape of the plot.

**Challenging Questions**

1. Prove that the line segment joining the midpoints of non-parallel sides of a trapezium is parallel to each of the parallel sides and is equal to half the sum of the parallel sides
2. Prove that the line segment joining the midpoints of the diagonals of a trapezium is parallel to each of the parallel sides and is equal to half the difference of these sides.

## PROJECT WORK

Develop a crossword puzzle based on terms and properties related to quadrilaterals.

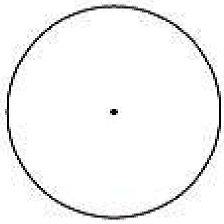
**Art of Questioning**

1. Given the lengths of two adjacent sides say *AB* and *AD* of a parallelogram *ABCD* what measures can we find?
2. Given measure of one angle of a parallelogram, what measures can we find, about other angles.
3. How many parallelograms can be formed given the lengths of the two diagonals?

## CIRCLE

# Introduction

Circle is a set of points in a plane which are equidistant from the given fixed points in the same plane.



In daily life, you see several things which are round (in circular shapes) like wheel, bangles, watches, coins, plates etc. In this chapter student will study the circle and related terms and some properties of the circle.

Teaching of circles enhances the learning and listening skills of children as examples can be taken from real life. Learning circle promote theme based project work integrated with class room curriculum. It also help students develop interpersonal skills. It also encourages interaction among teachers providing a very different model of professional development.

**Key Concepts**

1. Equal chords of a circle subtend equal angles at the centre of the circle.
2. If the angles subtended by chords of a circle at the centre are equals, then the chords are equal.
3. The perpendicular drawn from the centre of a circle to a chord of the circle bisects the chord.
4. The line drawn through the centre of a circle to bisect the chord is perpendicular to the chord.
5. Equal chords of a circle or (of Congruent Circles) are equidistant from the centre.
6. One and only one circle can be drawn to pass through three non-collinear points.
7. The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
8. Angle subtended in a semicircle is right angle.
9. Angle in the same segment of a circle are equal.

## ACTIVITY 1

***Topic***
Properties of the chords of a circle.
***Objective***
To verify the statement that the perpendicular drawn form the centre of a circle to a chord of the circle bisects the chord.
***Pre-requisite Knowledge***
To draw a circle with given centre.
To draw perpendicular from a point to a straight line.
***Material required***
Pair of scissors Tracing paper Fevistic White sheets of paper Black ball point pen Geometry Box Green and red sketch pen
Teacher will perform the activity in the class slowly and carefully so that every student may understand it.

## ACTIVITY 2

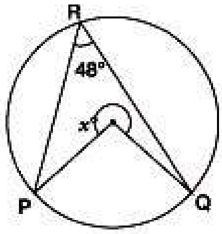
***Objective***
To verify there all the equal chords of a circle are equidistant from the centre.
***Material Required***
As above
Through the above two activities, teacher will transact the teaching points in the class room. Other activities may be performed too.

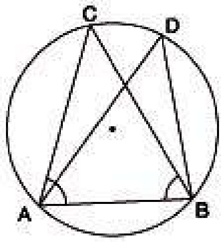
## WORKSHEET 1

1. Largest chord of the circle is called…………….

2. In the following figure find $x$.

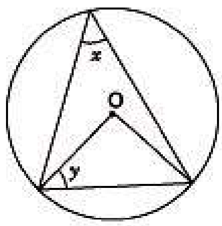
3. $\angle CAB = 70°$, $\angle CBA = 50°$ , find value of $\angle ADB$ .



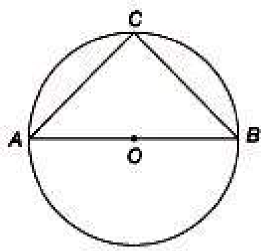

4. If $AB = 12$ cm, $BC = 16$ cm and $\angle ABC = 90°$ then find the radius of the circle passing through $A, B, C$.

5. In the figure

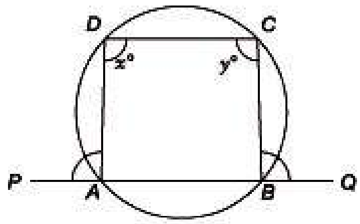



$x + y$ is (choose the correct option) (a) less than 90° (b) greater than 90° (c) equal to 90° (d) None

6. In figure $AOB$ is the diameter and $AC = BC$ then angle $\angle CBA$ is equal to (choose the correct option)
   (a) 30°
   (b) 60°
   (c) 45°
   (d) 15°

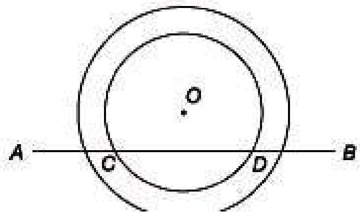


7. In given figure $\angle CBQ = 70°$,




$\angle DAP = 75°$ . Find $x + y$

8. In figure two concentric circles are given. If $AB = b$, $CD = a$. Then $AC$ is (Choose the correct option)




(a)
$a - b$ 2

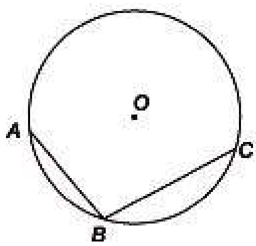
(b)
$b - a$
(c)

2
$a + b$ 2

(d) $b - a$

## WORKSHEET 2

### S.A.




1. In figure

Chord $AB$ = Chord $BC$, then justify that $OB$ is bisector of angle $\angle AOC$ .

2. Prove that the line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.

3. $AB$ and $CD$ are parallel chords of a circle whose centre is O and radius = 20 cm. If $CD$ = 32 cm, $AB$ = 24 cm, find the distance between the chords when it is given that they lie on the opposite sides of the centre O.