5.3 DIFFRACTION OF LIGHT

5.97 The radius of the periphery of the N^{th} Fresnel zone is

$$r_N = \sqrt{N b \lambda}$$

Then by conservation of energy

$$I_0 \pi (\sqrt{N b \lambda})^2 = \int_0^\infty 2 \pi r \, dr \, I(r)$$

Here r is the distance from the point P.

Thus $$I_0 = \frac{2}{N b \lambda} \int_0^\infty r \, dr \, I(r).$$

5.98 By definition

$$r_k^2 = k \frac{a b \lambda}{a + b}$$

for the periphery of the k[th] zone. Then

$$a r_k^2 + b r_k^2 = k a b \lambda$$

So $$b = \frac{a r_k^2}{k a \lambda - r_k^2} = \frac{a r^2}{k a \lambda - r^2} = 2 \text{ metre}.$$

on putting the values. (It is given that $r = r_k$) for $k = 3$).

5.99 Suppose maximum intensity is obtained when the aperture contains k zones. Then a minimum will be obtained for $k+1$ zones. Another maximum will be obtained for $k+2$ zones. Hence

$$r_1^2 = k \lambda \frac{a b}{a + b}$$

$$r_2^2 = (k+2) \lambda \frac{a b}{a + b}$$

Thus $$\lambda = \frac{a + b}{2 a b} (r_2^2 - r_1^2) = 0{\cdot}598 \, \mu\text{m}$$

On putting the values.

5.100 (a) When the aperture is equal to the first Fresnel Zone :-

The amplitude is A_1 and should be compared with the amplitude $\frac{A}{2}$ when the aperture is very wide. If I_0 is the intensity in the second case the intensity in the first case will be $4 I_0$.

When the aperture is equal to the internal half of the first zone :- Suppose A_{in} and A_{out} are the amplitudes due to the two halves of the first Fresnel zone. Clearly A_{in} and A_{out} differ in phase by $\frac{\pi}{2}$ because only half the Fresnel zone in involved. Also in magnitude $|A_{in}| = |A_{out}|$. Then

$$A_1^2 = 2\,|A_{in}|^2 \quad \text{so} \quad |A_{in}|^2 = \frac{A_1^2}{2}$$

Hence following the argument of the first case. $I_{in} = 2\,I_0$

(b) The aperture was made equal to the first Fresnel zone and then half of it was closed along a diameter. In this case the amplitude of vibration is $\frac{A_1}{2}$. Thus

$$I = I_0 .$$

5.101 (a) Suppose the disc does not obstruct light at all. Then

$$A_{disc} + A_{remainder} = \frac{1}{2} A_{disc}$$

(because the disc covers the first Fresnel zone only).

So$A_{remainder} = -\frac{1}{2} A_{disc}$

Hence the amplitude when half of the disc is removed along a diameter

$= \frac{1}{2} A_{disc} + A_{remainder} = \frac{1}{2} A_{disc} - \frac{1}{2} A_{disc} = 0$

Hence $I = 0$.

(b) In this case

$$A = \frac{1}{2} A_{external} + A_{remainder}$$
$$= \frac{1}{2} A_{external} - \frac{1}{2} A_{disc}$$

We write $$A_{disc} = A_{in} + i A_{out}$$

where A_{in} (A_{out}) stands for $A_{internal}$ ($A_{external}$). The factor i takes account of the $\frac{\pi}{2}$ phase difference between two halves of the first Fresnel zone. Thus

$$A = -\frac{1}{2} A_{in} \quad \text{and} \quad I = \frac{1}{4} A_{in}^2$$

On the other hand $$I_0 = \frac{1}{4}(A_{in}^2 + A_{out}^2) = \frac{1}{2} A_{in}^2$$

so $$I = \frac{1}{2} I_0 .$$

5.102 When the screen is fully transparent, the amplitude of vibrations is $\frac{1}{2} A_1$ (with intensity $I_0 = \frac{1}{4} A_1^2$).

(a) (1) In this case $A = \frac{3}{4}\left(\frac{1}{2} A_1\right)$ so squaring $I = \frac{9}{16} I_0$

(2) In this case $\frac{1}{2}$ of the plane is blacked out so

$$A = \frac{1}{2}\left(\frac{1}{2}A_1\right) \text{ and } I = \frac{1}{4}I_0$$

(3) In this case $A = \frac{1}{4}(A_1/2)$ and $I = \frac{1}{16}I_0$.

(4) *In this case* $A = \frac{1}{2}\left(\frac{1}{2}A_1\right)$ *again and* $I = \frac{1}{4}I_0$ so $I_4 = \frac{I}{2}$

In general we get $I(\varphi) = I_0\left(1 - \left(\frac{\varphi}{2\pi}\right)\right)^2$

where φ is the total angle blocked out by the screen.

(b) (5) Here $A = \frac{3}{4}\left(\frac{1}{2}A_1\right) + \frac{1}{4}A_1$

A_1 being the contribution of the first Fresnel zone.

Thus $$A = \frac{5}{8}A_1 \text{ and } I = \frac{25}{16}I_0$$

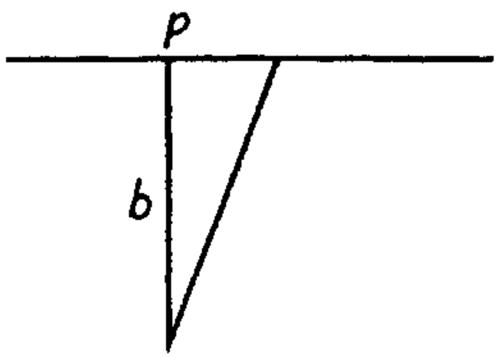
(6) $A = \frac{1}{2}\left(\frac{1}{2}A_1\right) + \frac{1}{2}A_1 = \frac{3}{4}A_1$ and $I = \frac{9}{4}I_0$

(7) $A = \frac{1}{4}\left(\frac{1}{2}A_1\right) + \frac{3}{4}A_1 = \frac{7}{8}A_1$ and $I = \frac{49}{16}I_0$

(8) $A = \frac{1}{2}\left(\frac{1}{2}A_1\right) + \frac{1}{2}A_1 = \frac{3}{4}A_1$ and $I = \frac{9}{4}I_0$ $(I_8 = I_6)$

In 5 to 8 the first term in the expression for the amplitude is the contribution of the plane part and the second term gives the expression for the Fresnel zone part. In general in (5) to (8) $I = I_0\left(1 + \left(\frac{\varphi}{2\pi}\right)\right)^2$ when φ is the angle covered by the screen.

5.103 We would require the contribution to the amplitude of a wave at a point from half a Fresnel zone. For this we proceed directly from the Fresnel Huyghens principle. The complex amplitude is written as



$$E = \int K(\varphi)\,\frac{a_0}{r}\,e^{-ikr}\,dS$$

Here $K(\varphi)$ is a factor which depends on the angle φ between a normal $\vec{n}$ to the area dS and the direction from dS to the point P and r is the distance from the element dS to P.

We see that for the first Fresnal zone

$$\left(\text{using } r = b + \frac{\rho^2}{2b} \left(\text{for } \sqrt{\rho^2 + b^2}\right)\right)$$

$$E = \frac{a_0}{b} \int_0^{\sqrt{b\lambda}} e^{-ikb - ik\rho^2/2b} 2\pi\rho\, d\rho \quad (K(\varphi) = 1)$$

For the first Fresnel zone $r = b + \lambda/2$ so $r^2 = b^2 + b\lambda$ and $\rho^2 = b\lambda$.

Thus $$E = \frac{a_0}{b} e^{-ikb}\, 2\pi \int_0^{\frac{b\lambda}{2}} e^{-i\frac{kx}{b}}\, dx$$

$$= \frac{a_0}{b} 2\pi e^{-ikb} \frac{e^{-ik\lambda/2} - 1}{-ik/b}$$

$$= \frac{a_0}{k} 2\pi i e^{-ikb} (-2) = -\frac{4\pi}{k} i a_0 e^{-ikb} = A_1$$

For the next half zone

$$E = \frac{a_0}{b} e^{-ikb} 2\pi \int_{\frac{b\lambda}{2}}^{\frac{3b\lambda}{4}} e^{-ikx/b}\, dx x$$

$$= \frac{a_0}{k} 2\pi i e^{-ikb} \left(e^{-i\frac{3k\lambda}{4}} - e^{-ik\lambda/2}\right)$$

$$= \frac{a_0}{k} 2\pi i\, e^{-ikb}\, (+1 + i) = -\frac{A_1(1+i)}{2}$$

If we calculate the contribution of the full 2nd Fresnel zone we will get $-A_1$. If we take account of the factors $K(\varphi)$ and $\frac{1}{r}$ which decrease monotonically we expect the contribution to change to $-A_2$. Thus we write for the contribution of the half zones in the 2nd Fresnel zone as

$$-\frac{A_2(1+i)}{2} \quad \text{and} \quad -\frac{A_2(1-i)}{2}$$

The part lying in the recess has an extra phase difference equal to $-\delta = -\frac{2\pi}{\lambda}(n-1)h$. Thus the full amplitude is (note that the correct form is e^{-ikr})

$$\left(A_1 - \frac{A_2}{2}(1+i)\right) e^{+i\delta} - \frac{A_2}{2}(1-i) + A_3 - A_4 + \ldots$$

$$= \left(\frac{A_1}{2}(1-i)\right) e^{+i\delta} - \frac{A_2}{2}(1-i) + \frac{A_3}{2}$$

$$= \left(\frac{A_1}{2}(1-i)\right)e^{+i\delta} + i\frac{A_1}{2} \text{ (as } A_2 = A_3 = A_1) \text{ and } A_3 - A_4 + A_5 \ldots = \frac{A_3}{2}.$$

The corresponding intensity is

$$I = \frac{A_1^2}{4}\left[(1-i)e^{+i\delta} + \frac{i}{e}\right][(1+i)e^{-i\delta} - i]$$

$$= I_0[3 - 2\cos\delta + 2\sin\delta] = I_0\left[3 + 2\sqrt{2}\sin\left(\delta - \frac{\pi}{4}\right)\right]$$

(a) For maximum intensity $\sin\left(\delta - \frac{\pi}{4}\right) = +1$

or
$$\delta - \frac{\pi}{4} = 2k\pi + \frac{\pi}{2}, \quad k = 0, 1, 2, \ldots$$

$$\delta = 2k\pi + \frac{3\pi}{4} = \frac{2\pi}{\lambda}(n-1)h$$

so
$$h = \frac{\lambda}{n-1}\left(k + \frac{3}{8}\right)$$

(b) For minimum intensity

$$\sin\left(\delta - \frac{\pi}{4}\right) = 1$$

$$\delta - \frac{\pi}{4} = 2k\pi + \frac{3\pi}{2} \quad \text{or} \quad \delta = 2k\pi + \frac{7\pi}{4}$$

so
$$h = \frac{\lambda}{n-1}\left(k + \frac{7\pi}{8}\right)$$

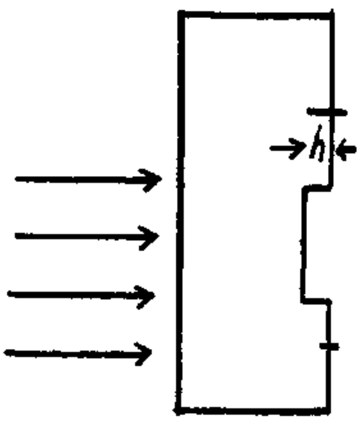
(c) For $I = I_0$, $\left.\begin{matrix}\cos\delta = 0 \\ \sin\delta = -1\end{matrix}\right\}$ or $\left\{\begin{matrix}\sin\delta = 0 \\ \cos\delta = +1\end{matrix}\right.$

Thus
$$\delta = 2k\pi \quad h = \frac{k\lambda}{n-1}$$

or
$$\delta = 2k\pi + \frac{3\pi}{2}, \; h = \frac{\lambda}{n-1}\left(k + \frac{3\pi}{4}\right)$$

5.104 The contribution to the wave amplitude of the inner half-zone is

$$\frac{2\pi a_0 e^{-ikb}}{b}\int_0^{\sqrt{b\lambda/2}} e^{-ik\rho^2/2b}\rho\, d\rho$$



$$= \frac{2\pi a_0 e^{-ikb}}{b}\int_0^{b\lambda/4} e^{-ikx/b}dx$$

$$= \frac{2\pi a_0 e^{-ikb}}{b}\left(e^{-ik\lambda/4} - 1\right) \times \frac{1}{\frac{-ik}{b}}$$

$$= \frac{2\pi i\, a_0\, e^{-ikb}}{k}(-i-1) = +\frac{A_1}{2}(1+i)$$

With phase factor this becomes $\frac{A_1}{2}(1+i)e^{i\delta}$ where $\delta = \frac{2\pi}{\lambda}(n-1)h$. The contribution of the remaining aperture is $\frac{A_1}{2}(1-i)$

(so that the sum of the two parts when $\delta = 0$ is A_1)

Thus the complete amplitude is

$$\frac{A_1}{2}(1+i)e^{i\delta} + \frac{A_1}{2}(1-i)$$

and the intensity is

$$I = I_0[(1+i)e^{i\delta} + (1-i)][(1-i)e^{-i\delta} + (1+i)]$$
$$= I_0[2+2+(1-i)^2 e^{-i\delta} + (1+i)^2 e^{i\delta}]$$
$$= I_0[4 - 2ie^{-i\delta} + 2ie^{i\delta}] = I_0(4 - 4\sin\delta)$$

Here $I_0 = \frac{A_1^2}{4}$ is the intensity of the incident light which is the same as the intensity due to an aperture of infinite extent (and no recess). Now

I is maximum when $\sin\delta = -1$

or $$\delta = 2k\pi + \frac{3\pi}{2}$$

so $$h = \frac{\lambda}{n-1}\left(k + \frac{3}{4}\right) \quad \text{and (b)} \quad I_{max} = 8I_0.$$

5.105 We follow the argument of 5.103. we find that the contribution of the first Fresnel zone is

$$A_1 = -\frac{4\pi i}{k} a_0 e^{-ikb}$$

For the next half zone it is $-\frac{A_2}{2}(1+i)$

(The contribution of the remaining part of the 2nd Fresnel zone will be $-\frac{A_2}{2}(1-i)$)

If the disc has a thickness h, the extra phase difference suffered by the light wave in passing through the disc will be

$$\delta = \frac{2\pi}{\lambda}(n-1)h.$$

Thus the amplitude at P will be

$$E_P = \left(A_1 - \frac{A_2}{2}(1+i)\right)e^{-i\delta} - \frac{A_2}{2}(1-i) + A_3 - A_4 - A_5 + \dots$$
$$= \left(\frac{A_1(1-i)}{2}\right)e^{-i\delta} + \frac{iA_1}{2} = \frac{A_1}{2}[(1-i)e^{-i\delta} + i]$$

The corresponding intensity will be

$$I = I_0(3 - 2\cos\delta - 2\sin\delta) = I_0\left(3 - 2\sqrt{2}\sin\left(\delta + \frac{\pi}{4}\right)\right)$$

The intensity will be a maximum when

$$\sin\left(\delta + \frac{\pi}{4}\right) = -1$$

or $$\delta + \frac{\pi}{4} = 2k\pi + \frac{3\pi}{2}$$

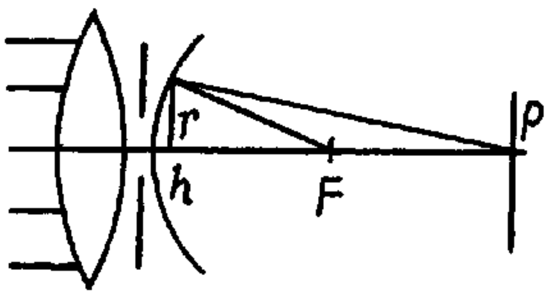
i.e. $$\delta = \left(k + \frac{5}{8}\right).2\pi$$

so $$h = \frac{\lambda}{n-1}\left(k + \frac{5}{8}\right),\ k = 0, 1, 2, \ldots$$

Note :- It is not clear why $k = 2$ for h_{min}. The normal choice will be $k = 0$. If we take $k = 0$ we get $h_{min} = 0{\cdot}59\ \mu$ m.

5.106 Here the focal point acts as a virtual source of light. This means that we can take spherical waves converging towards F. Let us divide these waves into Fresnel zones just after they emerge from the stop. We write

$r^2 = f^2 - (f-h)^2 = (b - m\lambda/2)^2 - (b-h)^2$



Here r is the radius of the m^{th} fresnel zone and h is the distance to the left of the foot of the perpendicular. Thus

$$r^2 = 2fh = -bm\lambda + 2bh$$

So $$h = bm\lambda/2(b-f)$$

and $$r^2 = fbm\lambda/(b-f).$$

The intensity maxima are observed when an odd number of Fresnel zones are exposed by the stop. Thus

$$r_k = \sqrt{\frac{kbf\lambda}{b-f}} \quad \text{where} \quad k = 1, 3, 5, \ldots$$

5.107 For the radius of the periphery of the kth zone we have

$$r_k = \sqrt{k\lambda\frac{ab}{a+b}} = \sqrt{k\lambda b} \quad \text{if } a = \infty.$$

If the aperture diameter is reduced η times it will produce a similar deffraction pattern (reduced η times) if the radii of the Fresnel zones are also η times less. Thus

$r'_k = r_k/\eta$

This requires $b' = b/\eta^2$.

5.108 (a) If a point source is placecd before an opaque ball, the diffraction pattern consists of a bright spot inside a dark disc followed by fringes. The bright spot is on the line joining the point source and the centre of the ball. When the object is a finite source of transverse diamension y, every point of the source has its corresponding image on the line joining that point and the centre of the ball. Thus the transverse dimension of the image is given by

$$y' = \frac{b}{a} y = 9 \text{ mm.}$$

(b) The minimum height of the irregularities covering the surface of the ball at random, at which the ball obstructs light is, according to the note at the end of the problem, comparable with the width of the Fresnel zone along which the edge of opaque screen passes. So

$$h_{min} \approx \Delta r$$

To find Δr we note that

$$r^2 = \frac{k \lambda a b}{a+b}$$

or

$$2 r \Delta r = D \Delta r = \frac{\lambda a b}{a+b} \Delta k$$

Where D = diameter of the disc (= diameter of the last Fresnel zone) and $\Delta k = 1$

Thus $h_{min} = \dfrac{\lambda a b}{D(a+b)} = 0{\cdot}099$ mm.

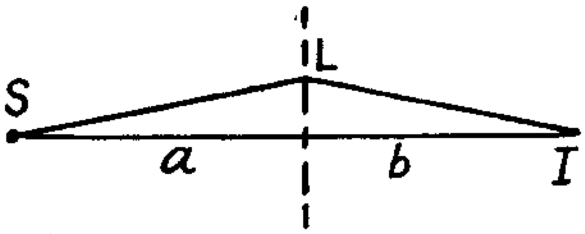
5.109 In a zone plate an undarkened circular disc is followed by a number of alternately undarkened and darkened rings. For the proper case, these correspond to 1^{st}, 2^{nd}, 3^{rd}..... Fresnel zones.

Let r_1 = radius of the central undarkened circle. Then for this to be first Fresnel zone in the present case, we must have

$$SL + LI - SI = \lambda/2$$

Thus if r_1 is the radius of the periphery of the first zone

$$\sqrt{a^2 + r_1^2} + \sqrt{b^2 + r_1^2} - (a+b) = \frac{\lambda}{2}$$



or

$$\frac{r_1^2}{2}\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{\lambda}{2} \quad \text{or} \quad \frac{1}{a} + \frac{1}{b} = \frac{1}{r_1^2/\lambda}$$

It is clear that the plate is acting as a lens of focal length

$$f_1 = \frac{r_1^2}{\lambda} = \frac{a b}{a+b} = {\cdot}6 \text{ metre.}$$

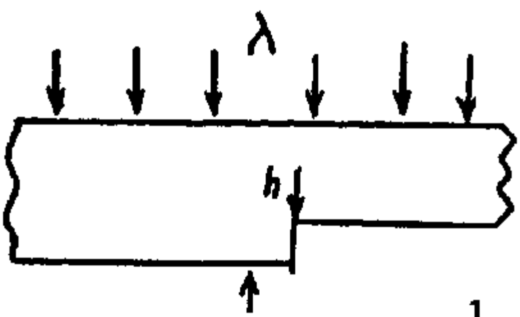
This is the principle focal length.

Other maxima are obtained when

$$SL + LI - SI = 3\frac{\lambda}{2}, 5\frac{\lambda}{2}, \ldots$$

These focal lengths are also $\quad \dfrac{r_1^2}{3\lambda}, \dfrac{r_1^2}{5\lambda}, \ldots$

5.110 Just below the edge the amplitude of the wave is given by



$$A = \frac{1}{2}(A_1 - A_2 + A_3 - A_4 + \dots)\,e^{-i\delta}$$

$$+\frac{1}{2}(A_1 - A_2 + A_3 - A_4 + \dots)$$

Here the quantity in the brackets is the contribution of various Fresnel zones; the factor $\frac{1}{2}$ is to take account of the division of the plate into two parts by the ledge; the phase factor δ is given by

$$\delta = \frac{2\pi}{\lambda}h(n-1)$$

and takes into account the extra length traversed by the waves on the left.

Using $$A_1 - A_2 + A_3 - A_4 + \dots = \frac{A_1}{2}$$

we get $$A = \frac{A_1}{4}(1 + e^{i\delta})$$

and the corresponding intensity is

$$I = I_0\frac{1+\cos\delta}{2}, \quad \text{where } I_0 \propto \left(\frac{A_1}{2}\right)^2$$

(a) This is minimum when

$$\cos\delta = -1$$

So $$\delta = (2k+1)\pi$$

and $$h = (2k+1)\frac{\lambda}{2(n-1)}, \quad k = 0, 1, 2, \dots$$

using $n = 1{\cdot}5$, $\lambda = 0{\cdot}60\,\mu m$

$$h = 0{\cdot}60\,(2k+1)\,\mu\text{m}\,.$$

(b) $I = I_0/2$ when $\cos\delta = 0$

or $$\delta = k\pi + \frac{\pi}{2} = (2k+1)\frac{\pi}{2}$$

Thus in this case $$h = 0{\cdot}30\,(2k+1)\,\mu\text{m}\,.$$

5.111 (a) From the Cornu's spiral, the intensity of the first maximum is given as

$$I_{max,1} = 1{\cdot}37\,I_0$$

and the intensity of the first minimum is given by

$$I_{min} = 0{\cdot}78\,I_0$$

so the required ratio is

$$\frac{I_{max}}{I_{min}} = 1{\cdot}76$$

(b) The value of the distance x is related to the parameter ν in Fresnel's integral by

$$\nu = x\sqrt{\frac{2}{b\lambda}}.$$

For the first two maxima the distances x_1, x_2 are related to the parameters ν_1, ν_2 by

$$x_1 = \sqrt{\frac{b\lambda}{2}}\,\nu_1,\ x_2 = \sqrt{\frac{b\lambda}{2}}\,\nu_2$$

Thus
$$(\nu_2 - \nu_1)\sqrt{\frac{b\lambda}{2}} = x_2 - x_1 = \Delta x$$

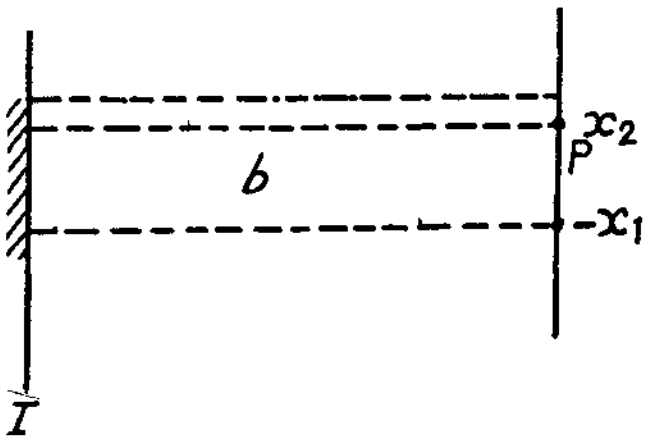
or
$$\lambda = \frac{2}{b}\left(\frac{\Delta x}{\nu_2 - \nu_1}\right)^2$$

From the Cornu's spiral the positions of the maxima are

$$\nu_1 = 1{\cdot}22,\ \nu_2 = 2{\cdot}34,\ \nu_3 = 3{\cdot}08 \text{ etc}$$

Thus
$$\lambda = \frac{2}{b}\left(\frac{\Delta x}{1{\cdot}12}\right)^2 = 0{\cdot}63\ \mu\text{m}.$$

5.112 We shall use the equation written down in 5.103, the Fresnel-Huyghens formula.



Suppose we want to find the intensity at P which is such that the coordinates of the edges (x-coordinates) with respect to P are x_2 and $-x_1$. Then, the amplitude at P is

$$E = \int K(\varphi)\,\frac{a_0}{r}\,e^{-ikr}\,dS$$

We write $dS = dx\,dy$, y is to integrated from $-\infty + 0 + \infty$. We write

$$r = b + \frac{x^2 + y^2}{2b} \tag{1}$$

(r is the distance of the element of surface on I from P. It is $\sqrt{b^2 + x^2 + y^2}$ and hence approximately (1)). We then get

$$E = A_0(b)\left[\int_{x_2}^{\infty} e^{-ikx^2/2b}\,dx + \int_{-\infty}^{-x_1} e^{-ikx^2/2b}\,dx\right]$$

$$= A'_0(b)\left[\int_{v_2}^{\infty} e^{-i\frac{\pi u^2}{2}}\,du + \int_{-\infty}^{-v_1} e^{-i\pi u^2/2}\,du\right]$$

where $$v_2 = \sqrt{\frac{2}{b\lambda}}\,x_2,\ v_1 = \sqrt{\frac{2}{b\lambda}}\,x_1$$

The intensity is the square of the amplitude. In our case, at the centre

$$v_1 = v_2 = \sqrt{\frac{2}{b\lambda}}\cdot\frac{a}{2} = \sqrt{\frac{a^2}{2b\lambda}} = 0{\cdot}64$$

(a = width of the strip = 0·7 mm, b = 100 cm, λ = 0·60 μm)

At, say, the lower edge $v_1 = 0,\ v_2 = 1{\cdot}28$

Thus

$$\frac{I_{\text{centre}}}{I_{\text{edge}}} = \left|\frac{\int_{0\cdot64}^{\infty} e^{-i\pi u^2/2}\,du + \int_{-\infty}^{-0\cdot64} e^{-i\pi u^2/2}\,du}{\int_{1\cdot28}^{\infty} e^{-i\pi u^2/2}\,du + \int_{-\infty}^{0} e^{-i\pi u^2/2}\,du}\right|^2 = 4\,\frac{\left(\frac{1}{2}-C(0{\cdot}64)\right)^2+\left(\frac{1}{2}-S(0{\cdot}64)\right)^2}{(1-C(1{\cdot}28))^2+(1-S(1{\cdot}28))^2}$$

where $$C(v) = \int_0^v \cos\frac{\pi u^2}{2}\,du$$

$$S(v) = \int_0^v \sin\frac{\pi u^2}{2}\,du$$

Rough evaluation of the integrals using cornu's spiral gives

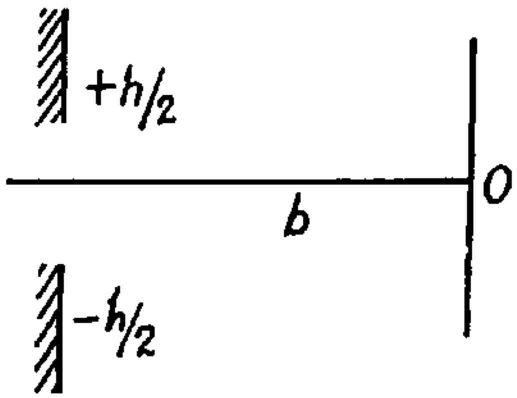
$$\frac{I_{\text{centre}}}{I_{\text{edge}}} = 2{\cdot}4$$

(We have used $\int_0^{\infty} \cos\frac{\pi u^2}{2}\,du = \int_0^{\infty} \sin\frac{\pi u^2}{2}\,du = \frac{1}{2}$

$C(0{\cdot}64) = 0{\cdot}62,\ S(0{\cdot}64) = 0{\cdot}15$

$C(1{\cdot}28) = 0{\cdot}65\ \ S(1{\cdot}28) = 0{\cdot}67$

5.113 If the aperture has width h then the parameters $(\nu, -\nu)$ associated with $\left(h/2, -\frac{h}{2}\right)$ are given by



$$\nu = \frac{h}{2}\sqrt{\frac{2}{b\lambda}} = h/\sqrt{2b\lambda}$$

The intensity of light at O on the screen is obtained as the square of the amplitude A of the wave at O which is

$$A \sim \text{const} \int_{-\nu}^{\nu} e^{-i\pi u^2/2}\, du$$

Thus $$I = 2I_0((C(\nu))^2 + (S(\nu))^2)$$

where $C(\nu)$ and $S(\nu)$ have been defined above and I_0 is the intensity at O due to an infinitety wide ($\nu = \infty$) aperture for then

$$I = 2I_0\left(\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right) = 2I_0 \times \frac{1}{2} = I_0.$$

By definition ν corresponds to the first minimum of the intensity. This means

$$\nu = \nu_1 \approx \cdot 90$$

when we increase h to $h + \Delta h$, the corresponding $\nu_2 = \frac{h + \Delta h}{\sqrt{2b\lambda}}$ relates to the second minimum of intensity. From the cornu's spiral $\nu_2 \approx 2{\cdot}75$

Thus $$\Delta h = \sqrt{2b\lambda}\,(\nu_2 - \nu_1) = 0{\cdot}85\sqrt{2b\lambda}$$

or $$\lambda = \left(\frac{\Delta h}{0{\cdot}85}\right)^2 \frac{1}{2b} = \left(\frac{0{\cdot}70}{0{\cdot}85}\right)^2 \frac{1}{2 \times 0{\cdot}6}\,\mu\text{m} = 0{\cdot}565\,\mu\text{m}$$

5.114 Let a = width of the recess and

$$\nu = \frac{a}{2}\sqrt{\frac{2}{b\lambda}} = \frac{a}{\sqrt{2b\lambda}} = \frac{0{\cdot}6}{\sqrt{2 \times 0{\cdot}77 \times 0{\cdot}65}} \approx 0{\cdot}60$$

be the parameter along Cornu's spiral corresponding to the half-width of the recess.

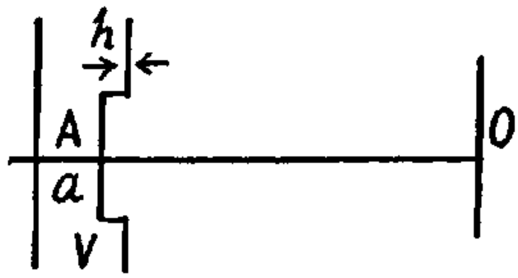
The amplitude of the diffracted wave is given by

$$\sim \text{const}\left[e^{i\delta}\int_{-\nu}^{\nu} e^{-i\pi u^2/2}\, du + \int_{\nu}^{\infty} e^{-i\pi u^2/2}\, du + \int_{-\infty}^{\nu} e^{-i\pi u^2/2}\, du\right]$$

where $$\delta = \frac{2\pi}{\lambda}(n-1)h$$

is the extra phase due to the recess. (Actually an extra phase $e^{-i\delta}$ appears outside the recess. When we take it out and absorb it in the constant we get the expression written).

Thus the amplitude is



$$\sim \text{const}\left[(C(v)-iS(v))e^{i\delta}+\left(\frac{1}{2}-C(v)\right)-i\left(\frac{1}{2}-S(v)\right)\right]$$

From the Cornu's spiral, the coordinates corresponding to the parameter $v = 0{\cdot}60$ are

$$C(v) = 0{\cdot}57,\ S(v) = 0{\cdot}13$$

so the intensity at O is proportional to

$$\left|\left[(0{\cdot}57-0{\cdot}13\,i)e^{i\delta}-0{\cdot}07-i\,0{\cdot}37\right]\right|^2$$
$$= (0{\cdot}57^2+0{\cdot}13^2)+0{\cdot}07^2+0{\cdot}37^2$$
$$+\ (0{\cdot}57-0{\cdot}13\,i)(-0{\cdot}07+0{\cdot}37\,i)e^{i\delta}$$
$$+\ (0{\cdot}57+0{\cdot}13\,i)(-0{\cdot}07-i\,0{\cdot}37\,i)e^{-i\delta}$$

We write

$$0{\cdot}57\mp 0{\cdot}13\,i = 0{\cdot}585\ e^{\mp i\alpha}\quad \alpha = 12{\cdot}8^\circ$$
$$-0{\cdot}07\pm 0{\cdot}37\,i = 0{\cdot}377\ e^{\pm i\beta}\quad \beta = 100{\cdot}7^\circ$$

Thus the cross term is

$$2\times 0{\cdot}585\times 0{\cdot}377\ \cos(\delta+88^\circ)$$
$$\approx 2\times 0{\cdot}585\times 0{\cdot}377\ \cos\left(\delta+\frac{\pi}{2}\right)$$

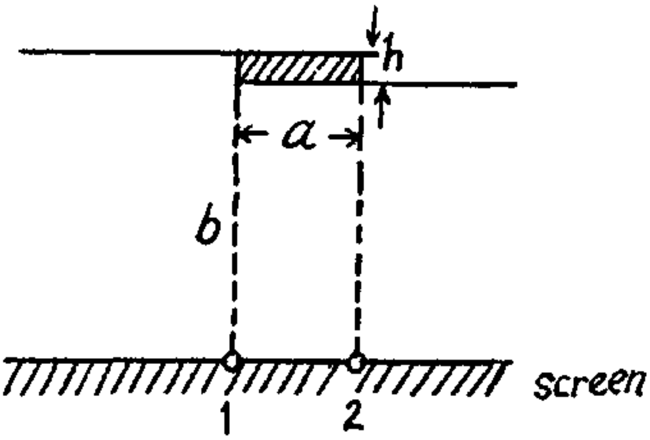
For maximum intensity

$$\delta+\frac{\pi}{2} = 2k'\pi,\quad k' = 1,2,3,4,\ldots$$
$$= 2(k+1)\pi,\ k = 0,1,2,3,\ldots$$

or
$$\delta = 2k\pi+\frac{3\pi}{2}$$

so
$$h = \frac{\lambda}{n-1}\left(k+\frac{3}{4}\right)$$

5.115



Using the method of problem 5.103 we can immediately write down the amplitudes at 1 and 2. We get :

At 1 $\quad$ amplitude $A_1 \sim \text{const}\left[\int_{-\infty}^{0} e^{-i\pi u^2/2}du + e^{-i\delta}\int_{v}^{\infty} e^{-i\pi u^2/2}du\right]$

At 2 amplitude $A_2 \sim$ const $\left[\int_{-\infty}^{-v} e^{-i\pi u^2/2} du + e^{-i\delta} \int_0^{\infty} e^{-i\pi u^2/2} du \right]$

where $$v = a\sqrt{\frac{2}{b\lambda}}$$

is the parameter of Cornu's spiral and constant factor is common to 1 and 2.

With the usual notation

$$C = C(v) = \int_0^v \cos\frac{\pi u^2}{2} du$$

$$S = S(v) = \int_0^v \sin\frac{\pi u^2}{2} du$$

and the result $$\int_0^\infty \cos\frac{\pi u^2}{2} du = \int_0^\infty \sin\frac{\pi u^2}{2} du = \frac{1}{2}$$

We find the ratio of intensities as

$$\frac{I_2}{I_1} = \left| \frac{\left(\frac{1}{2} - C\right) - i\left(\frac{1}{2} - S\right) + e^{-i\delta}\frac{(1-i)}{2}}{\frac{1-i}{2} + e^{-i\delta}\left\{\left(\frac{1}{2} - C\right) - i\left(\frac{1}{2} - S\right)\right\}} \right|^2$$

(The constants in A_1 and A_2 must be the same by symmetry)

In our case, $a = 0{\cdot}30$ mm, $\lambda = 0{\cdot}65\ \mu$m, $b = 1{\cdot}1$ m

$$v = 0{\cdot}30 \times \sqrt{\frac{2}{1{\cdot}1 \times 0{\cdot}65}} = 0{\cdot}50$$

$$C(0{\cdot}50) = 0{\cdot}48 \qquad S(0{\cdot}50) = 0{\cdot}06$$

$$\frac{I_2}{I_1} = \left| \frac{0{\cdot}02 - 0{\cdot}44\,i + e^{-i\delta}\frac{(1-i)}{2}}{\frac{1-i}{2}e^{i\delta} + 0{\cdot}02 - 0{\cdot}44\,i} \right|^2 = \left| \frac{1 + (0{\cdot}02 - 0{\cdot}44\,i)\sqrt{2}\,e^{i\delta + \frac{i\pi}{4}}}{1 + (0{\cdot}02 - 0{\cdot}44\,i)\sqrt{2}\,e^{-i\delta + \frac{i\pi}{4}}} \right|^2$$

But $0{\cdot}02 - 0{\cdot}44\,i = 0{\cdot}44\,e^{i\alpha}$, $\alpha = 1{\cdot}525$ rad ($= 87{\cdot}4°$)

So $$\frac{I_2}{I_1} = \left| \frac{1 + 0{\cdot}44 \times \sqrt{2} \times e^{i(\delta - 0{\cdot}740)}}{1 + 0{\cdot}44 \times \sqrt{2} \times e^{-i(\delta + 0{\cdot}740)}} \right|^2 = \frac{1 + 2(0{\cdot}44)^2 + 2\sqrt{2} \times 0{\cdot}44 \cos(\delta - 0{\cdot}740)}{1 + 2(0{\cdot}44)^2 + 2\sqrt{2} \times 0{\cdot}44 \cos(\delta + 0{\cdot}740)}$$

I_2 is maximum when $\delta - 0{\cdot}740 = 0$ (modulo 2π)

Thus in that case $$\frac{I_2}{I_1} = \frac{1{\cdot}387 + 1{\cdot}245}{1{\cdot}387 + 1{\cdot}245\cos(1{\cdot}48)} = \frac{2{\cdot}632}{1{\cdot}5} = 1{\cdot}75$$

5.116 We apply the formula of problem 5.103 and calculate

$$\int_{aperture} \frac{a_0}{r} e^{-ikr} dS = \int_{Semicircle} + \int_{Slit}$$

The contribution of the full 1st Fresnel zone has been evaluated in 5.103. The contribution of the semi-circle is one half of it and is

$$-\frac{2\pi}{k} i a_0 e^{-ikb} = -i a_0 \lambda e^{-ikb}$$

The contribution of the slit is

$$\frac{a_0}{b} \int_0^{0{\cdot}90\sqrt{b\lambda}} e^{-ikb} e^{-ik\frac{x^2}{2b}} dx \int_{-\infty}^{\infty} e^{-iky^2/2b} dy$$

Now

$$\int_{-\infty}^{\infty} e^{-iky^2/2b} dy = \int_{-\infty}^{\infty} e^{-i\frac{\pi y^2}{b\lambda}} dy$$

$$\sqrt{\frac{b\lambda}{2}} \int_{-\infty}^{\infty} e^{-i\pi u^2/2} du = \sqrt{b\lambda}\; e^{-i\pi/4}$$

Thus the contribution of the slit is

$$\frac{a_0}{b}\sqrt{b\lambda}\; e^{-ikb-i\pi/4} \int_0^{0{\cdot}9\times\sqrt{2}} e^{-i\pi u^2/2} du \sqrt{\frac{b\lambda}{2}}$$

$$= a_0 \lambda e^{-ikb-i\pi/4} \frac{1}{\sqrt{2}} \int_0^{1{\cdot}27} e^{-i\pi u^2/2} du$$

Thus the intensity at the observation point P on the screen is

$$a_0^2 \lambda^2 \left| -i + \frac{1-i}{2}(C(1{\cdot}27) - iS(1{\cdot}27)) \right|^2 = a_0^2 \lambda^2 \left| -i + \frac{(1-i)(0{\cdot}67 - 0{\cdot}65\, i)}{2} \right|^2$$

(on using $C(1{\cdot}27) = 0{\cdot}67$ and $S(1{\cdot}27) = 0{\cdot}65$)

$$= a_0^2 \lambda^2 \,|-i + 0{\cdot}01 - 0{\cdot}66\, i\,|^2$$

$$= a_0^2 \lambda^2 \,|\,0{\cdot}01 - 1{\cdot}66\, i\,|^2$$

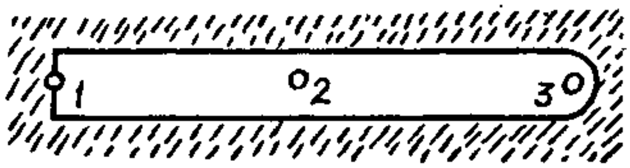
$$= 2{\cdot}76\; a_0^2 \lambda^2$$

Now $a_0^2 \lambda^2$ is the intensity due to half of 1st Fresnel zone and is therefore equal to I_0. (It can also be obtained by doing the x-integral over $-\infty$ to $+\infty$).

Thus

$$I = 2{\cdot}76\, I_0 .$$

5.117 From the statement of the problem we know that the width of the slit = diameter of the first Fresnel zone = $2\sqrt{b\lambda}$ where b is the distance of the observation point from the slit.



We calculate the amplitudes by evaluating the integral of problem 5.103 We get

$$A_1 = \frac{a_0}{b}\int_{-\sqrt{b\lambda}}^{\sqrt{b\lambda}} e^{-ikb}\, e^{-ik\frac{x^2}{2b}}\,dx\int_0^{\infty} e^{-ik\frac{y^2}{2b}}\,dy$$

$$= \frac{a_0}{b}\,e^{-ikb}\,\frac{b\lambda}{2}\int_{-\sqrt{2}}^{\sqrt{2}} e^{-i\pi u^2/2}\,du\times\int_0^{\infty} e^{-i\pi u^2/2}\,du$$

$$= \frac{a_0\lambda}{2}(1-i)\,e^{-ikb}\left(C\left(\sqrt{2}\right)-iS\left(\sqrt{2}\right)\right)$$

$$A_2 = \frac{a_0}{b}\int_{-\sqrt{b\lambda}}^{\sqrt{b\lambda}} e^{-ikb}\, e^{-ik\frac{x^2}{2b}}\,dx\int_{-\infty}^{\infty} e^{-iky^2/2b}\,dy$$

$$= 2A_1$$

$$A_3 = -i\,a_0\lambda\,e^{-ikb}+\frac{a_0\lambda(1-i)}{2}\left(C\left(\sqrt{2}\right)-iS\left(\sqrt{2}\right)\right)e^{-ikb}$$

where the contribution of the 1st half Fresnel zone (in A_3, first term) has been obtained from the last problem.

Thus
$$I_1 = a_0^2\lambda^2\left|\frac{(1-i)(0{\cdot}53-0{\cdot}72\,i)}{2}\right|^2$$

$\left(\text{on using } C\left(\sqrt{2}\right) = 0{\cdot}53,\ S\left(\sqrt{2}\right) = 0{\cdot}72\right)$

$$= a_0^2\lambda^2\,|-0{\cdot}095-0{\cdot}625\,i\,|^2 = 0{\cdot}3996\,a_0^2\lambda^2$$

$$I_2 = 4I_1$$

$$I_3 = a_0^2\lambda^2\,|-0{\cdot}095-0{\cdot}625\,i-i\,|^2$$

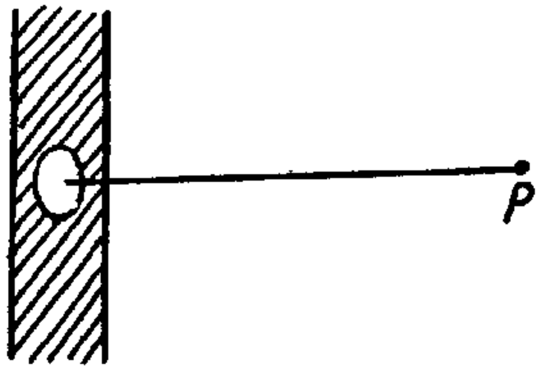
$$= a_0^2\lambda^2\,|-0{\cdot}095-1{\cdot}625\,i\,|^2$$

$$= 2{\cdot}6496\,a_0^2\lambda^2$$

So
$$I_3 = 6{\cdot}6\,I_1$$

Thus
$$I_1 : I_2 : I_3 = 1 : 4 : 7$$

5.118 The radius of the first half Fresnel zone is $\sqrt{b\lambda/2}$ and the amplitude at P is obtained using problem 5.103.



$$A = \frac{a_0}{b}\left[\int_{-\infty}^{-\eta\sqrt{b\lambda/2}} + \int_{\eta\sqrt{b\lambda/2}}^{\infty}\right] e^{-ikb-\frac{kx^2}{2b}}\,dx$$

$$\int_{-\infty}^{\infty} e^{-iky^2/2b}\,dy + \frac{a_0}{b}e^{-ikb}\int_0^{\sqrt{b\lambda/2}} e^{-ik\rho^2/2b}\,2\pi\rho\,d\rho\,.$$

We use
$$\int_{-\infty}^{-\eta\sqrt{b\lambda/2}} e^{-ikx^2/2b}\,dx$$

$$= \int_{\eta\sqrt{b\lambda/2}}^{\infty} e^{-ikx^2/2b}\,dx = \int_{\eta\sqrt{b\lambda/2}}^{\infty} e^{-i\frac{\pi x^2}{b\lambda}}\,dx$$

$$= \int_{\eta}^{\infty} e^{-i\pi u^2/2}\sqrt{\frac{b\lambda}{2}}\,du = \sqrt{\frac{b\lambda}{2}}\left(\int_0^{\infty} - \int_0^{\eta}\right) e^{-i\pi u^2/2}\,du.$$

$$= \sqrt{\frac{b\lambda}{2}}\left(\left(\frac{1}{2}-C(\eta)\right) - i\left(\frac{1}{2}-S(\eta)\right)\right)$$

Thus
$$A = a_0\frac{\lambda}{2}\times 2\times(1-i)e^{-ikb}\left[\left(\frac{1}{2}-C(\eta)\right)\right.$$
$$\left.-i\left(\frac{1}{2}-S(\eta)\right)\right] + a_0\lambda(1-i)\,e^{-ikb}$$

where we have used
$$\int_0^{\sqrt{b\lambda/2}} e^{-ik\rho^2/2b}\,2\pi\rho\,d\rho = \frac{2\pi i b}{k}(-1-i) = \frac{2\pi b}{k}(1-i) = \lambda b(1-i)$$

Thus the intensity is
$$I = |A|^2 = a_0^2\lambda^2\times 2\left[(3/2-C(\eta))^2 + \left(\frac{1}{2}-S(\eta)\right)^2\right]$$

From Cornu's Spiral,

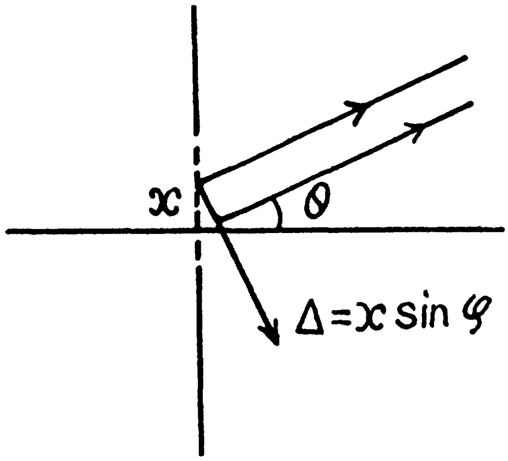
$$C(\eta) = C(1{\cdot}07) = 0{\cdot}76$$

$$S(\eta) = S(1{\cdot}07) = 0{\cdot}50$$

$$I = a_0^2 \lambda^2 \times 2 \times (0{\cdot}74)^2 = 1{\cdot}09\, a_0^2 \lambda^2$$

As before $\quad I_0 = a_0^2 \lambda^2$ so $I \approx I_0$.

5.119 If a plane wave is incident normally from the left on a slit of width b and the diffracted wave is observed at a large distance, the resulting pattern is called Fraunhofer diffraction. The condition for this is $b^2 \ll l\lambda$ where l is the distance between the slit and the screen. In practice light may be focussed on the screen with the help of a lens (or a telescope).



Consider an element of the slit which is an infinite strip of width dx. We use the formula of problem 5.103 with the following modifications.

The factor $\frac{1}{r}$ characteristic of spherical waves will be omitted. The factor $K(\varphi)$ will also be dropped if we confine overselves to not too large φ. In the direction defined by the angle φ the extra path difference of the wave emitted from the element at x relative to the wave emitted from the centre is

$$\Delta = -x \sin\varphi$$

Thus the amplitude of the wave is given by

$$\alpha \int_{-b/2}^{+b/2} e^{ik\sin\varphi}\, dx = \left(e^{i\frac{1}{2}kb\sin\varphi} - e^{-i\frac{1}{2}kb\sin\varphi} \right) \Big/ ik\sin\varphi$$

$$= \frac{\sin\left(\frac{\pi b}{\lambda}\sin\varphi\right)}{\frac{\pi b}{\lambda}\sin\varphi}$$

Thus $\quad I = I_0 \dfrac{\sin^2\alpha}{\alpha^2}$

where $\quad \alpha = \dfrac{\pi b}{\lambda}\sin\varphi$ and

I_0 is a constant

Minima are observed for $\sin\alpha = 0$ but $\alpha \neq 0$

Thus we find minima at angles given by

$$b\sin\varphi = k\lambda,\; k = \pm 1, \pm 2, \pm 3, \ldots$$

5.120 Since $I(\alpha)$ is +ve and vanishes for $b \sin\varphi = k\lambda$ i.e for $\alpha = k\pi$, we expect maxima of $I(\alpha)$ between $\alpha = +\pi$ & $\alpha = +2\pi$, etc. We can get these values by.

$$\frac{d}{d\alpha}(I(\alpha)) = I_0\, 2\frac{\sin\alpha}{\alpha}\frac{d}{d\alpha}\frac{\sin\alpha}{\alpha} = 0$$

$$\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0 \quad \text{or} \quad \tan\alpha = \alpha$$

Solutions of this transcendental equation can be obtained graphically.

The first three solutions are

$$\alpha_1 = 1{\cdot}43\,\pi,\ \alpha_2 = 2{\cdot}46\,\pi,\ \alpha_3 = 3{\cdot}47\,\pi$$

on the +ve side. (On the negative side the solution are $-\alpha_1, -\alpha_2, -\alpha_3, \ldots$)

Thus

$$b\sin\varphi_1 = 1{\cdot}43\,\lambda$$

$$b\sin\varphi_2 = 2{\cdot}46\,\lambda$$

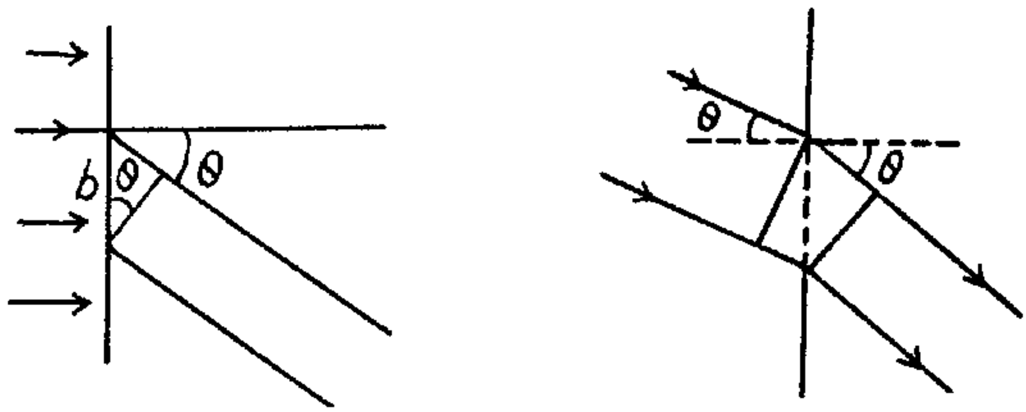
$$b\sin\varphi_3 = 3{\cdot}47\,\lambda$$

Asymptotically the solutions are

$$b\sin\varphi_m = \left(M + \frac{1}{2}\right)\lambda$$

5.121 The relation $b\sin\theta = k\lambda$

for minima (when light is incident normally on the slit) has a simple interpretation : $b\sin\theta$ is the path difference between extreme wave normals emitted at angle θ



When light is incident at an angle θ_0 the path difference is

$$b(\sin\theta - \sin\theta_0)$$

and the condition of minima is

$$b(\sin\theta - \sin\theta_0) = k\lambda$$

For the first minima

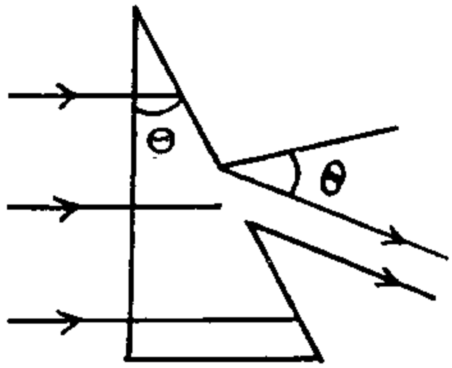
$$b(\sin\theta - \sin\theta_0) = \pm\lambda \quad \text{or} \quad \sin\theta = \sin\theta_0 \pm \frac{\lambda}{b}$$

Putting in numbers $\theta_0 = 30°$, $\lambda = 0{\cdot}50\,\mu\text{m}$ $b = 10\,\mu\text{m}$

$$\sin\theta = \frac{1}{2} \pm \frac{1}{20} = 0{\cdot}55 \text{ or } 0{\cdot}45$$

$$\theta_{+1} = 33° - 20' \quad \text{and} \quad \theta_{-1} = 26°\,44'$$

5.122 (a) This case is analogous to the previous one except that the incident wave moves in glass of RI n. Thus the expression for the path difference for light diffracted at angle θ from the normal to the hypotenuse of the wedge is



$$b(\sin\theta - n\sin\Theta)$$

we write $$\theta = \Theta + \Delta\theta$$

Then for the direction of principal Fraunhofer maximum

$$b(\sin(\Theta + \Delta\theta) - n\sin\Theta) = 0$$

or $$\Delta\theta = \sin^{-1}(n\sin\Theta) - \Theta$$

Using $\Theta = 15°$, $n = 1{\cdot}5$ we get

$$\Delta\theta = 7{\cdot}84°$$

(b) The width of the central maximum is obtained from ($\lambda = 0{\cdot}60\,\mu\text{m}$, $b = 10\,\mu m$)

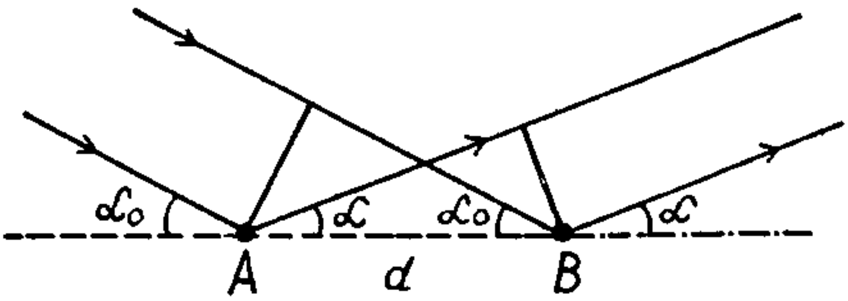
$$b(\sin\theta_1 - n\sin\Theta) = \pm\lambda$$

Thus $$\theta_{+1} = \sin^{-1}\left(n\sin\Theta + \frac{\lambda}{b}\right) = 26{\cdot}63°$$

$$\theta_{-1} = \sin^{-1}\left(n\sin\Theta\frac{\lambda}{b}\right) = 19{\cdot}16°$$

$$\therefore\quad \delta\theta = \theta_{+1} - \theta_{-1} = 7{\cdot}47°$$

5.123



The path difference between waves reflected at A and B is

$$d(\cos\alpha_0 - \cos\alpha)$$

and for maxima

$$d(\cos\alpha_0 - \cos\alpha) = k\lambda,\quad k = 0, \pm 1, \pm 2, \ldots$$

In our case, $k = 2$ and α_0, α are small in radiaus. Then

$$2\lambda = d\left(\frac{\alpha^2 - \alpha_0^2}{2}\right)$$

Thus $$\lambda = \frac{(\alpha^2 - \alpha_0^2)d}{4} = 0{\cdot}61\,\mu\text{m}$$

for $$\alpha = \frac{3\pi}{180},\ \alpha_0 = \frac{\pi}{180},\ d = 10^{-3}\,\text{m}$$

5.124 The general formula for diffraction from N slits is

$$I = I_0 \frac{\sin^2 \alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2 \beta}$$

where
$$\alpha = \frac{\pi a \sin\theta}{\lambda}$$

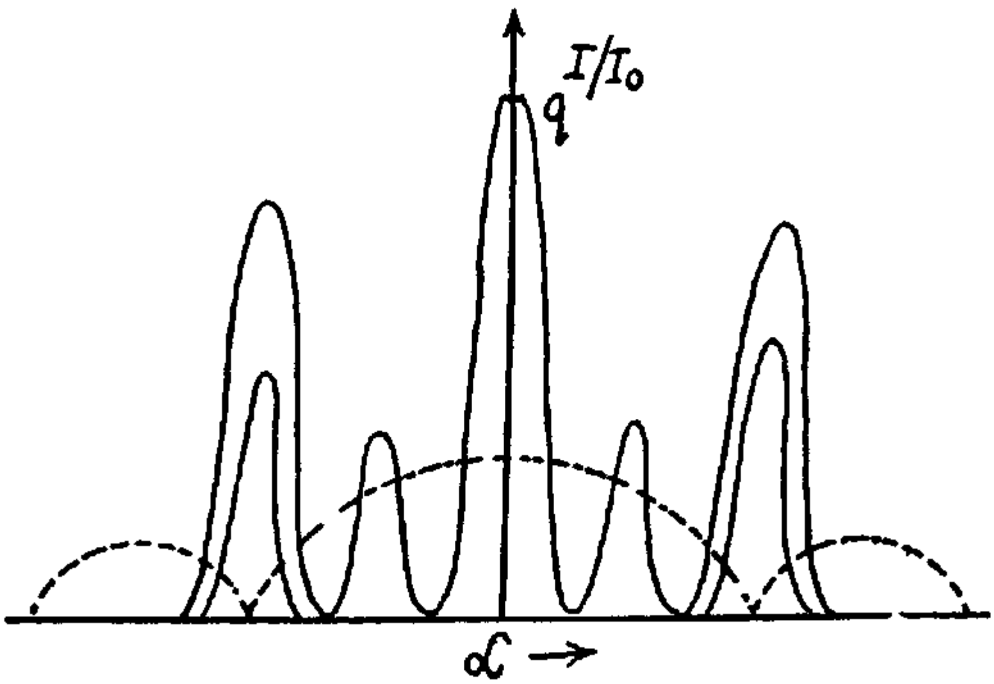
$$\beta = \frac{\pi (a+b)\sin\theta}{\lambda}$$

and $N = 3$ in the cases here.

(a) In this case $a + b = 2a$

so
$$\beta = 2\alpha \quad \text{and} \quad I = I_0 \frac{\sin^2 \alpha}{\alpha^2} (3 - 4\sin^2 2\alpha)^2$$

On plotting we get a curve that qualitatively looks like the one below



(b) In this case $a + b = 3a$

so
$$\beta = 3\alpha$$

and
$$I = I_0 \frac{\sin^2 \alpha}{\alpha^2} (2 - 4\sin^2 3\alpha)^2$$

This has 3 minima between the principal maxima

5.125 From the formula $d \sin\theta = m\lambda$

we have $d \sin 45° = 2\lambda_1 = 2 \times 0{\cdot}65\ \mu\text{m}$

or
$$d = 2\sqrt{2} \times 0{\cdot}65\ \mu\text{m}$$

Then for $\lambda_2 = 0{\cdot}50$ in the third order

$$2\sqrt{2} \times 0{\cdot}65 \sin\theta = 3 \times 0{\cdot}50$$

$$\sin\theta = \frac{1{\cdot}5}{1{\cdot}3 \times \sqrt{2}} = 0{\cdot}81602$$

This gives $\theta = 54{\cdot}68° \approx 55°$

5.126 The diffraction formula is

$$d \sin \theta_0 = n_0 \lambda$$

where $\theta_0 = 35°$ is the angle of diffraction corresponding to order n_0 (which is not yet known).

Thus
$$d = \frac{n_0 \lambda}{\sin \theta_0} = n_0 \times 0{\cdot}9327 \, \mu\text{m}$$

on using $\lambda = 0{\cdot}535 \, \mu\text{m}$

For the n^{th} order we get

$$\sin \theta = \frac{n}{n_0} \sin \theta_0 = \frac{n}{n_0} (0{\cdot}573576)$$

If $n_0 = 1$, then $n > n_0$ is at least 2 and $\sin \theta > 1$ so $n = 1$ is the highest order of diffraction.

If $n_0 = 2$ then $n = 3, 4$, but $\sin \theta > 1$ for $n = 4$ thus the highest order of diffraction is 3.

If $n_0 = 3$,

then
$$n = 4, 5, 6 .$$

For $n = 6$, $\sin \theta = 2 \times 0{\cdot}57 > 1$, so not allowed while for

$$n = 5, \ \sin \theta = \frac{5}{3} \times 0{\cdot}573576 < 1$$

is allowed. Thus in this case the highest order of diffraction is five as given. Hence

$$n_0 = 3$$

and
$$d = 3 \times 0{\cdot}9327 = 2{\cdot}7981 \approx 2{\cdot}8 \, \mu\text{m} .$$

5.127 Given that

$$d \sin \theta_1 = \lambda$$

$$d \sin \theta_2 = d \sin (\theta_1 + \Delta \theta) = 2 \lambda$$

Thus
$$\sin \theta_1 \cos \Delta \theta + \cos \theta_1 \sin \Delta \theta = 2 \sin \theta_1$$

or
$$\sin \theta_1 (2 - \cos \Delta \theta) = \cos \theta_1 \sin \Delta \theta$$

or
$$\tan \theta_1 = \frac{\sin \Delta \theta}{2 - \cos \Delta \theta}$$

or
$$\sin \theta_1 = \frac{\sin \Delta \theta}{\sqrt{\sin^2 \Delta \theta + (2 - \cos \Delta \theta)^2}}$$

$$= \frac{\sin \Delta \theta}{\sqrt{5 - 4 \cos \Delta \theta}}$$

Finally
$$\lambda = \frac{d \sin \Delta \theta}{\sqrt{5 - 4 \cos \Delta \theta}} .$$

Substitution gives $\lambda \approx 0{\cdot}534 \, \mu\text{m}$

5.128 (a) Here the simple formula

$$d \sin \theta = m_1 \lambda \text{ holds.}$$

Thus
$$1{\cdot}5 \sin \theta = m \times 0{\cdot}530 \quad \sin \theta = \frac{m \times 0{\cdot}530}{1{\cdot}5}$$

Highest permissible m is $m = 2$ because $\sin\theta > 1$ if $m = 3$. Thus

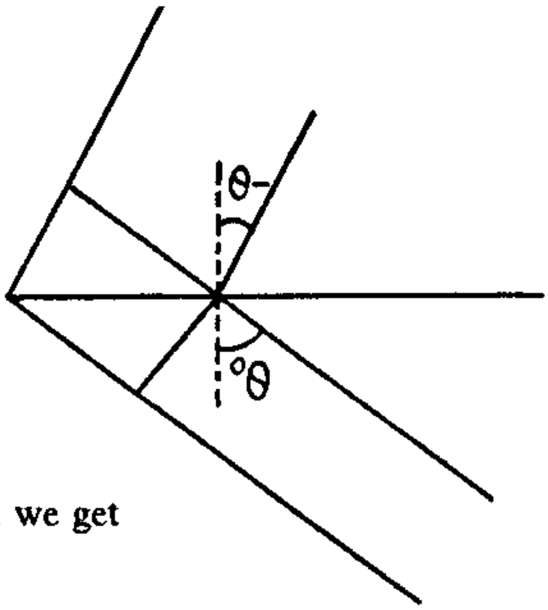
$$\sin\theta = \frac{1{\cdot}06}{1{\cdot}50} \text{ for } m = 2, \text{ This gives } \theta = 45^\circ \text{ nearby.}$$

(b) Here $d(\sin\theta_0 - \sin\theta) = n\lambda$

Thus $\sin\theta = \sin\theta_0 - \dfrac{n\lambda}{d}$

$$= \sin 60^\circ - n \times \frac{0{\cdot}53}{1{\cdot}5}$$

$$= 0{\cdot}86602 - n \times 0{\cdot}353333 .$$



For $n = 5$, $\sin\theta = -0{\cdot}900645$

for $n = 6$, $\sin\theta < -1$.

Thus the highest order is $n = 5$ and we get

$\theta = \sin^{-1}(-0{\cdot}900645) \cong -64^\circ$

5.129 For the lens

$$\frac{1}{f} = (n-1)\left(\frac{1}{R} - \frac{1}{\infty}\right) \quad \text{or} \quad f = \frac{R}{n-1}$$

For the grating

$$d\sin\theta_1 = \lambda \quad \text{or} \quad \sin\theta_1 = \frac{\lambda}{d}$$

$$\operatorname{cosec}\theta_1 = \frac{d}{\lambda}, \ \cot\theta_1 = \sqrt{\left(\frac{d}{\lambda}\right)^2 - 1}$$

$$\tan\theta_1 = \frac{1}{\sqrt{\left(\frac{d}{\lambda}\right)^2 - 1}}$$

Hence the distance between the two symmetrically placed first order maxima

$$= 2f\tan\theta_1 = \frac{2R}{(n-1)\sqrt{\left(\frac{d}{\lambda}\right)^2 - 1}}$$

On putting $R = 20$, $n = 1{\cdot}5$, $d = 6{\cdot}0\,\mu\text{m}$

$\lambda = 0{\cdot}60\,\mu\text{m}$ we get 8·04 cm.

5.130 The diffraction formula is easily obtained on taking account of the fact that the optical path in the glass wedge acquires a factor n (refractive index). We get

$$d(n\sin\Theta - \sin(\Theta\,\theta_k)) = k\lambda$$

Since $n > 0$, $\Theta - \theta_0 > \Theta$ and so θ_0 must be negative. We get, using $\Theta = 30^\circ$

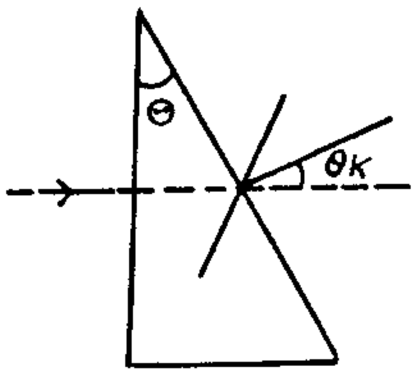
$$\frac{3}{2} \times \frac{1}{2} = \sin(30^\circ - \theta_0) = \sin 48{\cdot}6^\circ$$

Thus $$\theta_0 = -18{\cdot}6°$$

Also for $k = 1$

$$\frac{3}{4} - \sin(30° - \theta_{+1}) = \frac{\lambda}{d} = \frac{0{\cdot}5}{2{\cdot}0} = \frac{1}{4}$$

Thus $$\theta_{+1} = 0°$$



We calculate θ_k for various k by the above formula. For $k = 6$.

$$\sin(\theta_k - 30°) = \frac{3}{4} \Rightarrow \theta_k = 78{\cdot}6°$$

For $k = 7$

$$\sin(\theta_k - 30°) = +1 \Rightarrow \theta_k = 120°$$

This is in admissible. Thus the highest order that can be observed is

$$k = 6$$

corresponding to $$\theta_k = 78{\cdot}6°$$

(for $k = 7$ the diffracted ray will be grazing the wedge).

5.131 The intensity of the central Fraunhofer maximum will be zero if the waves from successive grooves (not in the same plane) differ in phase by an odd multiple of π. Then since the phase difference is

$$\delta = \frac{2\pi}{\lambda}(n-1)h$$

for the central ray we have

$$\frac{2\pi}{\lambda}(n-1)h = \left(k - \frac{1}{2}\right)2\pi, \; k = 1, 2, 3, \ldots$$

or $$h = \frac{\lambda}{n-1}\left(k - \frac{1}{2}\right).$$

The path difference between the rays 1 & 2 is approximately (neglecting terms of order θ^2)

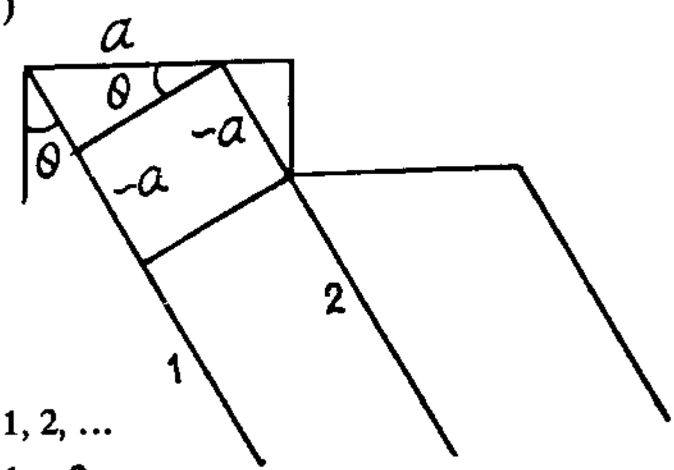
$a\sin\theta + a - na$

$= a\sin\theta - (n-1)a$

Thus for a maximum

$$a\sin\theta - \left(k' + \frac{1}{2}\right)\lambda = m\lambda$$

or $$a\sin\theta = \left(m + k' + \frac{1}{2}\right)\lambda,$$



$$k' = 0, 1, 2, \ldots$$

$$m = 0, \pm 1, \pm 2, \ldots$$

The first maximum after the central minimum is obtained when $m + k' = 0$

We get $$a\sin\theta_1 = \frac{1}{2}\lambda$$

5.132 When standing ultra sonic waves are sustained in the tank it behaves like a grating whose grating element is

$$d = \frac{v}{\nu} = \text{wavelength of the ultrasonic}$$

v = velocity of ultrasonic. Thus for maxima

$$\frac{v}{\nu} \sin \theta_m = m \lambda$$

On the other hand

$$f \tan \theta_m = m \Delta x$$

Assuming θ_m to be small $\left(\text{because } \lambda << \frac{v}{\nu}\right)$

we get $$\Delta x = \frac{f \tan \theta_m}{m} = \frac{f \tan \theta_m}{\frac{v}{\nu \lambda} \sin \theta_m} = \frac{\lambda \nu f}{v}$$

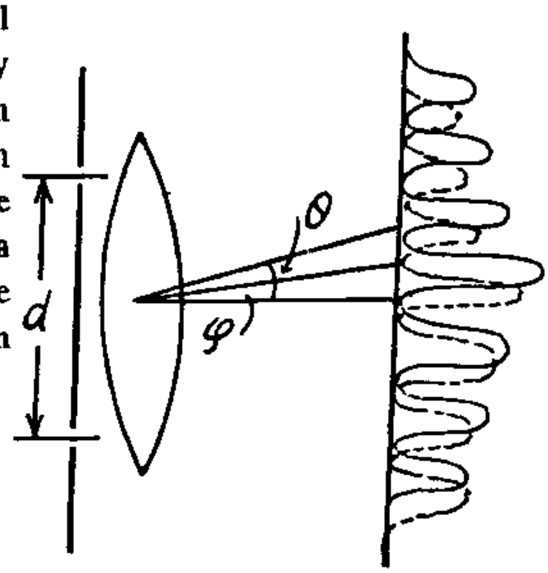
or $$v = \frac{\lambda \nu f}{\Delta x}$$

Putting the values $\lambda = 0{\cdot}55\ \mu\text{m}$, $\nu = 4{\cdot}7\ \text{MHz}$

$f = 0{\cdot}35\ \text{m}$ and $\Delta x = 0{\cdot}60 \times 10^{-3}\ \text{m}$ we easily get

$$v = 1{\cdot}51\ \text{km/sec}.$$

5.133 Each star produces its own diffraction pattern in the focal plane of the objective and these patterns are separated by angle ψ. As the distance d decreases the angle θ between the neighbouring maxima in either diffraction pattern increases ($\sin \theta = \lambda/d$). When θ becomes equal to 2ψ the first deterioration of visibility occurs because the maxima of one system of fringes coincide with the minima of the other system. Thus from the condition $\theta = 2\psi$ and $\sin \theta = \frac{\lambda}{d}$ we get



$$\psi = \frac{1}{2}\theta = \frac{\lambda}{2d} \text{ (radians)}$$

Putting the values we get $\psi = 0{\cdot}06''$

5.134 (a) For normal incidence, the maxima are given by

$$d \sin \theta = n \lambda$$

so $$\sin \theta = n \frac{\lambda}{d} = n \times \frac{0{\cdot}530}{1{\cdot}500}$$

Clearly $n \leq 2$ as $\sin \theta > 1$ for $n = 3$.

Thus the highest order is $n = 2$. Then

$$D = \frac{d\theta}{d\lambda} = \frac{k}{d\cos\theta} = \frac{k}{d}\frac{1}{\sqrt{1-\left(\frac{k\lambda}{d}\right)^2}}$$

Putting $k = 2$, $\lambda = 0{\cdot}53\ \mu\text{m}$, $d = 1{\cdot}5\ \mu\text{m} = 1500\ \text{nm}$

we get $D = \frac{2}{1500}\frac{1}{\sqrt{1-\left(\frac{1{\cdot}06}{1{\cdot}5}\right)^2}} \times \frac{180}{\pi} \times 60 = 6{\cdot}47$ ang. min/nm.

(b) We write the diffraction formula as

$$d(\sin\theta_0 + \sin\theta) = k\lambda$$

so
$$\sin\theta_0 + \sin\theta = k\frac{\lambda}{d}$$

Here
$$\theta_0 = 45^\circ \text{ and } \sin\theta_0 = 0{\cdot}707$$

so
$$\sin\theta_0 + \sin\theta \le 1{\cdot}707 \text{. Since}$$

$$\frac{\lambda}{d} = \frac{0{\cdot}53}{1{\cdot}5} = 0{\cdot}353333, \text{ we see that}$$

$$k \le 4$$

Thus highest order corresponds to $k = 4$.

Now as before $D = \frac{d\theta}{d\lambda}$ so

$$D = \frac{k}{d\cos\theta} = \frac{k/d}{\sqrt{1-\left(\frac{k\lambda}{d} - \sin\theta_0\right)^2}}$$

$$= 12{\cdot}948 \text{ ang. min/nm},$$

5.135 We have
$$d\sin\theta = k\lambda$$

so
$$\frac{d\theta}{d\lambda} = D = \frac{k}{d\cos\theta} = \frac{\tan\theta}{\lambda}$$

5.136 For the second order principal maximum

$$d\sin\theta_2 = 2\lambda = k\lambda$$

or
$$\frac{N\pi}{\lambda} d\sin\theta_2 = 2N\pi$$

minima adjacent to this maximum occur at

$$\frac{N\pi}{\lambda} d\sin(\theta_2 \pm \Delta\theta) = (2N \pm 1)\pi$$

or
$$d\cos\theta_2\,\Delta\theta = \frac{\lambda}{N}$$

Finally angular width of the 2^{nd} principal maximum is

$$2\,\Delta\theta = \frac{2\lambda}{N d\cos\theta_2} = \frac{2\lambda}{N d\sqrt{1-(k\lambda/d)^2}} = \frac{\tan\theta_2}{N}$$

On putting the values we get $11{\cdot}019''$ of arc

5.137 Using

$$R = \frac{\lambda}{\delta\lambda} = kN = \frac{N d\sin\theta}{\lambda}$$

$$= \frac{l\sin\theta}{\lambda} \le \frac{l}{\lambda}.$$

5.138 For the just resolved waves the frequency difference

$$\delta\nu = \frac{c\,\delta\lambda}{\lambda} = \frac{c}{\lambda R} = \frac{c}{\lambda kN}$$

$$= \frac{c}{N d\sin\theta} = \frac{1}{\delta t}$$

since $N d\sin\theta$ is the path difference between waves emitted by the extremities of the grating.

5.139 $\delta\lambda = {\cdot}050$ nm

$$R = \frac{\lambda}{\delta\lambda} = \frac{600}{{\cdot}05} = 12000 \text{ (nearly)}$$

$$= kN$$

On the other hand

$$d\sin\theta = k\lambda$$

Thus $$\frac{l}{kN}\sin\theta = \lambda$$

where $l = 10^{-2}$ metre is the width of the grating

Hence $$\sin\theta = 12000\times\frac{\lambda}{l}$$

$$= 12000\times 600\times 10^{-7} = 0{\cdot}72$$

or $$\theta = 46^\circ.$$

5.140 (a) We see that

$$N = 6{\cdot}5\times 10\times 200 = 13000$$

Now to resolve lines with $\delta\lambda = 0{\cdot}015$ nm and $\lambda = 670{\cdot}8$ nm we must have

$$R = \frac{670{\cdot}8}{0{\cdot}015} = 44720$$

Since $3N < R < 4N$ one must go to the fourth order to resolve the said components.

(b) we have $d = \frac{1}{200}\text{mm} = 5\,\mu\text{m}$

so $$\sin\theta = \frac{k\lambda}{d} = \frac{k\times 0{\cdot}670}{5}$$

since $|\sin\theta| \le 1$ we must have $k \le 7{\cdot}46$

so $$k_{max} = 7 \approx \frac{d}{\lambda}$$

Thus $$R_{max} = k_{max}\,N = 91000 \approx \frac{N d}{\lambda} = \frac{l}{\lambda}$$

where $l = 6{\cdot}5$ cm is the grating width.

Finally $$\delta\lambda_{min} = \frac{\lambda}{R_{max}} = \frac{670}{91000} = {\cdot}007\,\text{nm} = 7\,\text{pm} \approx \frac{\lambda^2}{l}.$$

5.141 Here

$$R = \frac{\lambda}{\delta\lambda} = \frac{589{\cdot}3}{0{\cdot}6} = \text{kN} = 5\,\text{N}$$

so $$N = \frac{589{\cdot}3}{3} = \frac{10^{-2}}{d}$$

$$d = \frac{3\times 10^{-2}}{589{\cdot}3}\,\text{m} = {\cdot}0509\,\text{mm}$$

(b) To resolve a doublet with $\lambda = 460{\cdot}0$ nm and $\delta\lambda = 0{\cdot}13$ nm in the third order we must have

$$N = \frac{R}{3} = \frac{460}{3\times 0{\cdot}13} = 1179$$

This means that the grating is
$$N d = 1179 \times 0{\cdot}0509 = 60{\cdot}03\,\text{mm}$$
wide = 6 cm wide.

5.142 (a) From $d\sin\theta = k\lambda$

we get $$\delta\theta = \frac{k\,\delta\lambda}{d\cos\theta}$$

On the other hand $$x = f\sin\theta$$

so $$\delta x = f\cos\theta\,\delta\theta = \frac{k f}{d}\,\delta\lambda$$

For $f = 0{\cdot}80$ m, $\delta\lambda = 0{\cdot}03$ nm, and

$$d = \frac{1}{250}\,\text{mm}$$

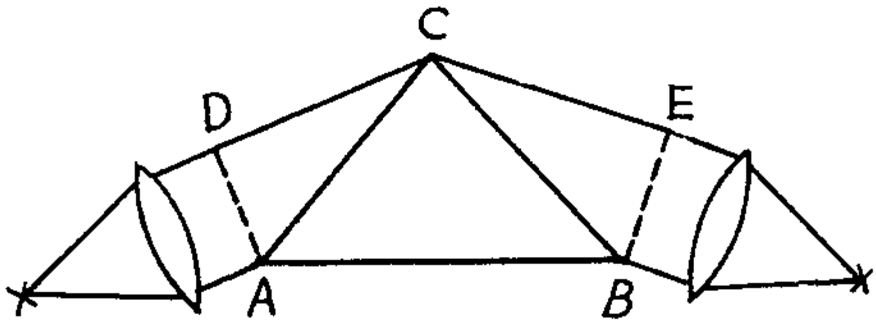
we get $$\delta x = \begin{cases} 6\,\mu\text{m} & \text{if } k = 1 \\ 12\,\mu\text{m} & \text{if } k = 2 \end{cases}$$

(b) Here $N = 25\times 250 = 6250$

and $$\frac{\lambda}{\delta\lambda} = \frac{310{\cdot}169}{0{\cdot}03} = 10339\cdots > N$$

and so to resolve we need $k = 2$ For $k = 1$ gives an R.P. of only 6250.

5.143



Suppose the incident light consists of two wavelengths λ and $\lambda + \delta\lambda$ which are just resolved by the prism. Then by Rayleigh's criterion, the maximum of the line of wavelength λ must coincide with the first minimum of the line of wavelength $\lambda + \delta\lambda$. Let us write both conditions in terms of the optical path differences for the extreme rays :

For the light of wavelength λ

$$bn - (DC + CE) = 0$$

For the light of wavelength $\lambda + \delta\lambda$

$$b(n + \delta n) - (DC + CE) = \lambda + \delta\lambda$$

because the path difference between extreme rays equals λ for the first minimum in a single slit diffraction (from the formula $a \sin\theta = \lambda$).

Hence $$b\,\delta n \approx \lambda$$

and $$R = \frac{\lambda}{\delta\lambda} = b\left|\frac{\delta n}{\delta\lambda}\right| = b\left|\frac{dn}{d\lambda}\right|$$

5.144 (a) $\dfrac{\lambda}{\delta\lambda} = R = b\left|\dfrac{dn}{d\lambda}\right| = 2Bb/\lambda^3$

For $b = 5$ cm, $B = 0{\cdot}01\ \mu m^2$ $\quad \lambda_1 = 0{\cdot}434\ \mu m = 5 \times 10^4\ \mu m$

$$R_1 = 1{\cdot}223 \times 10^4$$

for $$\lambda_2 = 0{\cdot}656\ \mu m$$

$$R_2 = 0{\cdot}3542 \times 10^4$$

(b) To resolve the *D*-lines we require

$$R = \frac{5893}{6} = 982$$

Thus $$982 = \frac{0{\cdot}02 \times b}{(0{\cdot}5893)^3}$$

$$b = \frac{982 \times (0{\cdot}5893)^3}{0{\cdot}02}\ \mu m = 1{\cdot}005 \times 10^4\ \mu m = 1{\cdot}005 \text{ cm}$$

5.145 $b\left|\dfrac{dn}{d\lambda}\right| = kN = 2 \times 10{,}000$

$$b \times 0{\cdot}10\ \mu m^{-1} = 2 \times 10^4$$

$$b = 2 \times 10^5\ \mu m = 0{\cdot}2\, m = 20 \text{ cm}.$$

5.146 Resolving power of the objective

$$= \frac{D}{1{\cdot}22\,\lambda} = \frac{5 \times 10^{-2}}{1{\cdot}22 \times 0{\cdot}55 \times 10^{-6}} = 7{\cdot}45 \times 10^4$$

Let $(\Delta y)_{min}$ be the minimum distance between two points at a distance of 3·0 km which the telescope can resolve. Then

$$\frac{(\Delta y)_{min}}{3 \times 10^3} = \frac{1{\cdot}22\,\lambda}{D} = \frac{1}{7{\cdot}45 \times 10^4}$$

or $$(\Delta y)_{min} = \frac{3 \times 10^3}{7{\cdot}45 \times 10^4} = 0{\cdot}04026 \text{ m} = 4{\cdot}03 \text{ cm}.$$

5.147 The limit of resolution of a reflecting telescope is determined by diffraction from the mirror and obeys a formula similar to that from a refracting telescope. The limit of resolution is

$$\frac{1}{R} = \frac{1{\cdot}22\,\lambda}{D} = \frac{(\Delta y)_{min}}{L}$$

where L = distance between the earth and the moon = 384000 km

Then putting the values $\lambda = 0{\cdot}55\ \mu$m, $D = 5$ m

we get $$(\Delta y)_{min} = 51{\cdot}6 \text{ metre}$$

5.148 By definition, the magnification

$$\Gamma = \frac{\text{angle subtended by the image at the eye}}{\text{angle subtended by the object at the eye}} = \frac{\psi'}{\psi}$$

At the limit of resolution $$\psi = \frac{1{\cdot}22\,\lambda}{D}$$

where D = diameter of the objective

On the other hand to be visible to the eye $\psi' \geq \dfrac{1{\cdot}22\,\lambda}{d_0}$

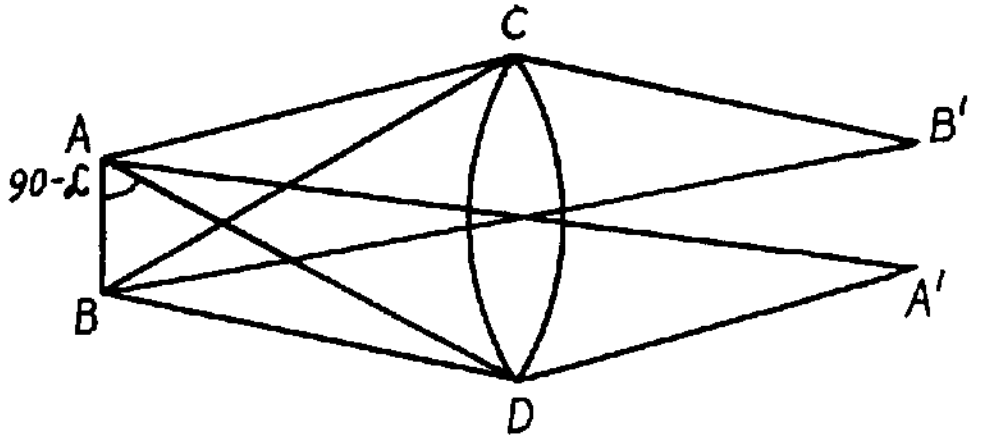
where d_0 = diameter of the pupil

Thus to avail of the resolution offered by the telescope we must have

$$\Gamma \geq \frac{1{\cdot}22\,\lambda}{d_0} \Big/ \frac{1{\cdot}22\,\lambda}{D} = \frac{D}{d_0}$$

Hence $$\Gamma_{min} = \frac{D}{d_0} = \frac{50 \text{ mm}}{4 \text{ mm}} = 12{\cdot}5$$

5.149



Let A and B be two points in the field of a microscope which is represented by the lens C D. Let A', B' be their image points which are at equal distances from the axis of the lens CD. Then all paths from A to A' are equal and the extreme difference of paths from A to B' is equal to

$$ADB' - ACB'$$
$$= AD + DB' - (AC + CB')$$
$$= AD + DB' - BD - DB'$$
$$+ BC + CB' - AC - CB'$$
$$(\text{as } BD + DB' = BC + CB')$$
$$= AD - BD + BC - AC$$
$$= 2AB\cos(90° - \alpha) = 2AB\sin\alpha$$

From the theory of diffraction by circular apertures this distance must be equal to $1{\cdot}22\,\lambda$

when B' coincides with the minimum of the diffraction due to A and A' with the minimum of the diffraction due to B. Thus

$$AB = \frac{1{\cdot}22\,\lambda}{2\sin\alpha} = 0{\cdot}61\,\frac{\lambda}{\sin\alpha}$$

Here 2α is the angle subtended by the objective of the microscope at the object.

Substiturting the values

$$AB = \frac{0{\cdot}61 \times 0{\cdot}55}{0{\cdot}24}\,\mu\text{m} = 1{\cdot}40\,\mu\text{m}.$$

5.150 Suppose d_{min} = minimum separation resolved by the microscope

ψ = angle subtended at the eye by this object when the object is at the least distance of distinct vision l_0 (= 25 cm).

ψ' = minimum angular separation resolved by the eye = $\dfrac{1{\cdot}22\,\lambda}{d_0}$

From the previous problem

$$d_{min} = \frac{0{\cdot}61\,\lambda}{\sin\alpha}$$

and

$$\psi = \frac{d_{min}}{l_0} = \frac{0{\cdot}61\,\lambda}{l_0\sin\alpha}$$

Now

$$\Gamma = \text{magnifying power} = \frac{\text{angle subtended at the eye by the image}}{\text{angle subtended at the eye by the object}}$$

when the object is at the least distance of distinct vision

$$\geq \frac{\psi'}{\psi} = 2\left(\frac{l_0}{d_0}\right)\sin\alpha$$

Thus

$$\Gamma_{min} = 2\left(\frac{l_0}{d_0}\right)\sin\alpha = 2 \times \frac{25}{0{\cdot}4} \times 0{\cdot}24 = 30$$

5.151 Path difference

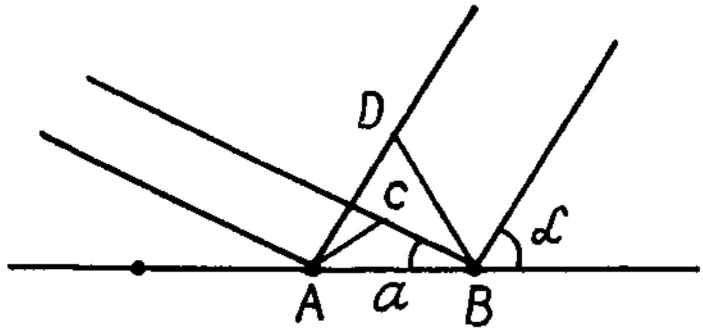
$= BC - AD$

$= a(\cos 60° - \cos\alpha)$

For diffraction maxima

$a(\cos 60° - \cos\alpha) = k\lambda$,

since $\lambda = \frac{2}{5}a$, we get



$$\cos\alpha = \frac{1}{2} - \frac{2}{5}k$$

and we get
$$k = -1, \cos\alpha = \frac{1}{2} + \frac{2}{5} = 0{\cdot}9, \ \alpha = 26°$$
$$k = 0, \cos\alpha = \frac{1}{2} = 0{\cdot}5, \ \alpha = 60°$$
$$k = 1, \cos\alpha = \frac{1}{2} - \frac{2}{5} = 0{\cdot}1, \ \alpha = 84°$$
$$k = 2, \cos\alpha = \frac{1}{2} - \frac{4}{5} = -0{\cdot}3, \ \alpha = 107{\cdot}5°$$
$$k = 3, \cos\alpha = \frac{1}{2} - \frac{6}{5} = -0{\cdot}7, \ \alpha = 134{\cdot}4°$$

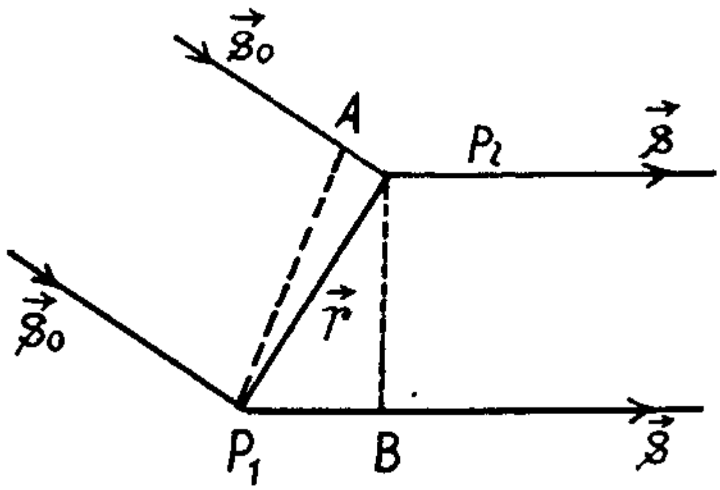
Other values of k are not allowed as they lead to $|\cos\alpha| > 1$.

5.152 We give here a simple derivation of the condition for diffraction maxima, known as Laue equations. It is easy to see form the above figure that the path difference between waves scattered by nearby scattering centres P_1 and P_2 is

$P_2A - P_1B = \vec{r}\cdot\vec{s_0} - \vec{r}\cdot\vec{s}$

$= \vec{r}\cdot(\vec{s_0} - \vec{s}) = \vec{r}\cdot\vec{S}$.

Here $\vec{r}$ is the radius vector $\overrightarrow{P_1P_2}$. For maxima this path difference must be an integer multiple of λ for any two neighbouring atoms. In the present case of two dimensional lattice with $X-$ rays incident normally $\vec{r}\cdot\vec{s} = 0$. Taking successively · nearest neighbours in the $x-$ & $y-$ directions



We get the equations

$$a\cos\alpha = h\lambda$$
$$b\cos\beta = k\lambda$$

Here $\cos\alpha$ and $\cos\beta$ are the direction cosines of the ray with respect to the x & y axes of the two dimensional crystal.

$$\cos\alpha = \frac{\Delta x}{\sqrt{(\Delta x)^2 + 4l^2}} = \sin\left(\tan^{-1}\frac{\Delta x}{2l}\right) = 0{\cdot}28735$$

so using $h = k = 2$ we get

$$a = \frac{40 \times 2}{\cdot 28735} \text{pm} = 0{\cdot}278 \text{ nm}$$

Similarly $$\cos\beta = \frac{\Delta y}{\sqrt{(\Delta y)^2 + 4l^2}} = \sin\left(\tan^{-1}\frac{\Delta y}{2l}\right) = 0{\cdot}19612$$

$$b = \frac{80}{\cos\beta} \text{p m} = 0{\cdot}408 \text{ nm}$$

5.153 Suppose α, β, and γ are the angles between the direction to the diffraction maximum and the directions of the array along the periods a, b, and c respectively (call them x, y, & z axes). Then the value of these angles can be found from the following familiar conditions

$$a(1 - \cos\alpha) = k_1 \lambda$$

$$b\cos\beta = k_2 \lambda \quad \text{and} \quad c\cos\gamma = k_3\lambda$$

where k_1, k_2, k_3 are whole numbers (+, −, or 0)

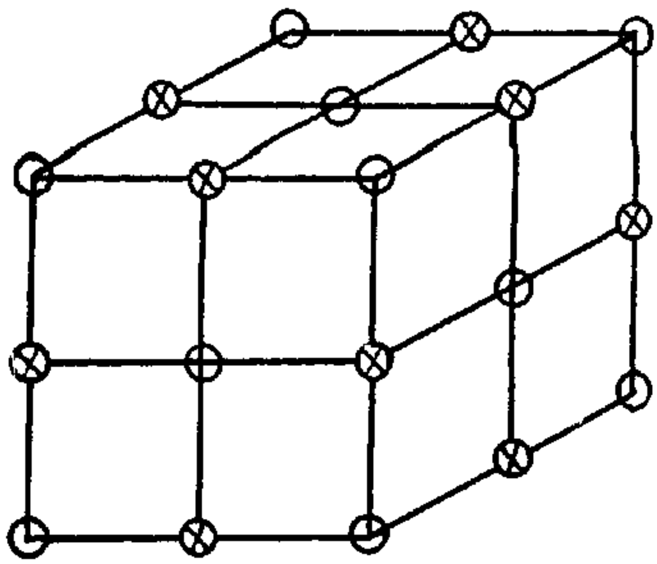
(These formulas are, in effect, Laue equations, see any text book on modern physics). Squaring and adding we get on using $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$2 - 2\cos\alpha = \left[\left(\frac{k_1}{a}\right)^2 + \left(\frac{k_2}{b}\right)^2 + \left(\frac{k_3}{c}\right)^2\right]\lambda^2 = \frac{2k_1\lambda}{a}$$

Thus $$\lambda = \frac{2k_1/a}{\left[(k_1/a)^2 + (k_2/a)^2 + (k_3/a)^2\right]}.$$

Knowing a, b, c and the integer k_1, k_2, k_3 we can find α, β, γ as well as λ.

5.154 The unit cell of $NaCl$ is shown below. In an infinite crystal, there are four Na^+ and four Cl^- ions per unit cell. (Each ion on the middle of the edge is shared by four unit cells; each ion on the face centre by two unit cells, the ion in the middle of the cell by one cell only and finally each ion on the corner by eight unit cells.) Thus



$$4\frac{M}{N_A} = \rho \cdot a^3$$

where M = molecular weight of NaCl in gms = 58·5 gms

N_A = Avogadro number = $6{\cdot}023 \times 10^{23}$

Thus $$\frac{1}{2}a = \sqrt[3]{\frac{M}{2N_A\rho}} = 2{\cdot}822 \text{ Å}$$

The natural facet of the crystal is one of the faces of the unit cell. The interplanar distance

$$d = \frac{1}{2}a = 2{\cdot}822 \text{ Å}$$

Thus $$2d\sin\alpha = 2\lambda$$

So $$\lambda = d\sin\alpha = 2{\cdot}822\,\text{Å}\times\frac{\sqrt{3}}{2} = 244 \text{ pm.}$$

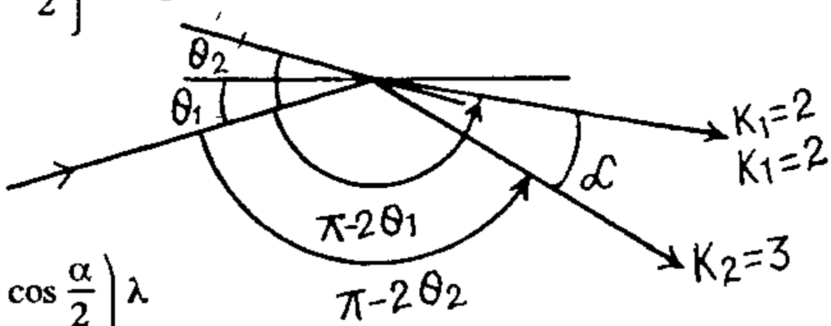
5.155 When the crystal is rotated, the incident monochromatic beam is diffracted from a given crystal plane of interplanar spacing d whenever in the course of rotation the value of θ satisfies the Bragg equation.

We have the equations $2d\sin\theta_1 = k_1\lambda$ and $2d\sin\theta_2 = k_2\lambda$

But $\pi - 2\theta_1 = \pi - 2\theta_2 + \alpha$ or $2\theta_1 = 2\theta_2 - \alpha$

so $$\theta_2 = \theta_1 + \frac{\alpha}{2}.$$

Thus $$2d\left\{\sin\theta_1\cos\frac{\alpha}{2} + \cos\theta_1\sin\frac{\alpha}{2}\right\} = k_2\lambda$$



Hence $$2d\sin\frac{\alpha}{2}\cos\theta_1 = \left(k_2 - k_1\cos\frac{\alpha}{2}\right)\lambda$$

also $$2d\sin\frac{\alpha}{2}\sin\theta_1 = k_1\lambda\sin\frac{\alpha}{2}$$

Squaring and adding $$2d\sin\frac{\alpha}{2} = \left(k_1^2 + k_2^2 - 2k_1k_2\cos\frac{\alpha}{2}\right)^{1/2}\lambda$$

Hence $$d = \frac{\lambda}{2\sin\frac{\alpha}{2}}\left[k_1^2 + k_2^2 - 2k_1k_2\cos\frac{\alpha}{2}\right]^{1/2}$$

Substituting $\alpha = 60°$, $k_1 = 2$, $k_2 = 3$, $\lambda = 174\text{ pm}$

we get $$d = 281\text{ pm} = 2{\cdot}81\text{ Å}$$

(and not 0·281 p m as given in the book.)

(Lattice parameters are typically in Å' s and not in fractions of a pm.)

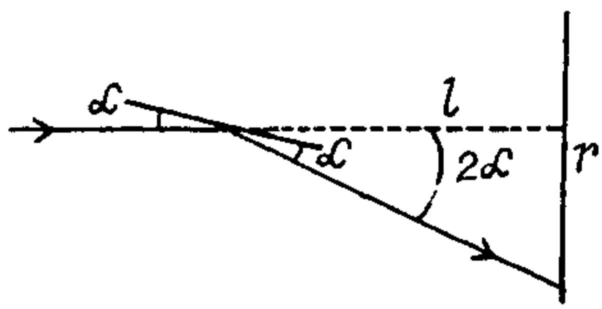
5.156 In a polycrystalline specimen, microcrystals are oriented at various angles with respect to one another. The microcrystals which are oriented at certain special angles with respect to the incident beam produce diffraction maxima that appear as rings.

The radial of these rings are given by

$$r = l\tan 2\alpha$$

where the Bragg's law gives

$$2d\sin\alpha = k\lambda$$



In our case $k = 2$, $d = 155\text{ pm}$, $\lambda = 17{\cdot}8\text{ pm}$

so $\alpha = \sin^{-1}\frac{17{\cdot}8}{155} = 6{\cdot}6°$ and $r = 3{\cdot}52$ cm.