CHAPTER

5

Series

LEARNING OBJECTIVES

- Different types of series
- Methods to solve the questions
- Approach building

Questions from this topic are frequently asked in exams like XAT/SNAP/MAT etc., and Campus Recruitment Examinations.

'Series'questions expect students to analyse the pattern, and extrapolate on the basis of the rule or pattern identified. There are different types of rules in a series. Some of these are listed below:

The Prime Number Series

Prime numbers have only two distinct factors like 2, 3, 5, 7, 11, 13, 17…. Occurrence of these numbers does not follow any particular rule.

Examples

(a) 3, 5, 7, 11, 13, 17, _____

Rule – Series contains a prime number series will have the next number as 19.

(b) 5, 7, 10, 15, 22, _____

Rule – Series contains the difference between two successive numbers is 2, 3, 5, 7 and so on, which are prime numbers. Hence the next number of the series will be 22 (the last given number) + 11 (the next prime number) = 33.

(c) 4, 9, 25, 49, 121, _____

Rule – Series is the squares of the prime numbers. $(2)^2 = 4$, $(3)^2 = 9$ and so on. Hence the next number will be 169 which is $(13)^2$.

(d) 1, 5, 14, 39, 88, _____

Rule – Difference between two consecutive numbers of the given series is the squares of prime numbers. Hence the next number will be 88 + 121 = 209.

The Difference Series

In these types of series, pattern would be obtained by finding the difference between terms. Differences are categorized as 1st order difference, 2nd order difference, 3rd order difference and so on.

What is the "Order of Difference":

Let us understand this with the help of some examples:

1st order difference is the difference between two consecutive terms of the series. If 1st order difference is constant, then general term of the series will be a linear equation.

1st order difference →

2		5		10		17		26		37
	3		5		7		9		11	

2nd order difference is the difference between the consecutive terms obtained from the 1st order difference. If 2nd order difference is constant, then general term of the series will be a quadratic equation.

1st order difference →
2nd order difference →

2		5		10		17		26		37
	3		5		7		9		11	
		2		2		2		2		

And so on, 3rd order difference will be difference between the consecutive terms obtained from the 2nd order difference. If 3rd order difference is constant, then general term of the series will be a cubic equation.

1st order difference →
2nd order difference →
3rd order difference →

2		5		10		17		26		37
	3		5		7		9		11	
		2		2		2		2		
			0		0		0			

Since in the series 2, 5, 10, 17, 26, 37 – 2nd order difference is constant, hence the general term of this series is a quadratic equation. We can also conclude that if 2nd order difference is constant, then general term of the series cannot be linear equation.

Rule followed here = $(n^2 + 1)$. Hence next term = $7^2 + 1 = 50$. In general, we can find out any term of this series by putting the value of n. For example, 10th term will be $10^2 + 1 = 101$.

How to Identify the General Rule

For this series, 2nd order difference is constant. Hence general term will be a quadratic equation.

Let us assume that the general term for n^{th} term = $an^2 + bn + c$.

$T_1 = a + b + c$ (we obtain this by putting n = 1 in $an^2 + bn + c$) = 2 (as given in the question) ...(1)

$T_2 = 4a + 2b + c$ (we obtain this by putting n = 2 in $an^2 + bn + c$) = 5 ...(2)

$T_3 = 9a + 3b + c$ (we obtain this by putting n = 3 in $an^2 + bn + c$) = 10 ...(3)

Now, we have three variables and three equations. To solve further, we would 1st eliminate one variable keeping the equations in two variable. Then, we would have two simultaneous equations in two variables

Subtract equation (1) from equation (2):

$$\begin{array}{r} 4a+2b+c=5 \\ -\quad a+b+c=2 \\ \hline 3a+b=3 \end{array}$$

Subtract equation (1) from equation (3):

$$\begin{array}{r} 9a+3b+c=10 \\ -\quad a+\ \ b+c=2 \\ \hline 8a+2b=8 \end{array}$$

Now we obtain two new equations in two variables:

$3a + b = 3$ (4)

$8a + 2b = 8$ OR $4a + b = 4$ (5)

Solving (4) and (5) gives:

$a = 1, b = 0, c = 1$

Hence general term n^{th} term = $an^2 + bn + c = n^2 + 1$

Examples:

(a) 8, 16, 24, 32, 40, _____

In this series, the difference between two successive terms is same i.e., 8. Hence the next number in the series will be 40 + 8 = 48.

(b) 6, 13, 27, 55, 111, _____

In this series, the difference between two consecutive numbers is 2, 4, 6, 8... Hence the next term will be 33.

(c) 13, 14, 18, 27, 43, _____

Difference between the two successive terms are 1, 4, 9, 16, 25, etc., which are the squares of 1, 2, 3, 4, 5… Hence the next term of the series will be 43 + 25 = 68

The Product Series

A series is called a product series when the term is obtained by multiplying by a constant or any other number.

Examples

(a) 3, 12, 48, 192, 768, _____

In this series, multiplying the previous term by four gets each next term. Hence the next term will be obtained by multiplying 768 by 4. The term will be 3072

(b) 3, 6, 18, 90, 630, _____

Here the numbers are being multiplied by consecutive prime numbers i.e., 2, 3, 5, 7, 11 and so on. Hence the next number in the series will be 630 × 11 = 6930.

The Mixed Series

The mixed series has three types of forms involved, which can be classified as follows:

(a) A mixed series may be formed by mixing two different series.

Example

3, 6, 24, 30, 63, 72, _____, 132

In this series, the difference between the terms of each pair formed by two consecutive terms is 3, 6, 9, 12 ……. Hence the missing number is 132 – 12 = 120.

(b) A mixed series may be formed by the mixture of two series, which are placed alternately.

Example

2, 6, 10, 3, 9, 13, 4, 12, _____

In this series, the first term is multiplied by 3 to get the second term and 4 is added to the product to get the 3rd term. Hence the answer will be 12 + 4 = 16. here a new series starts after every three terms.

The Square And Cube Series

Example

1, 4, 27, 16, 125, _____

Here the numbers in the series are formed by alternately placing the cubes of odd numbers and squares of even numbers. Hence the next term will be 36.

PRACTICE EXERCISES

Q.1 8, 12, 10, 16, 12, _____
(a) 10 (b) 20
(c) 30 (d) 40

Q.2 7, 15, 32, _____, 138, 281
(a) 57 (b) 67
(c) 77 (d) 87

Q.3 4, 6, 9, 14, _____
(a) 16 (b) 18
(c) 20 (d) 23

Q.4 36, 28, 24, 22, _____
(a) 18 (b) 20
(c) 21 (d) 22

Q.5 260, 216, 128, 108, 62, 54, _____, 27
(a) 39 (b) 49
(c) 29 (d) 19

Q.6 2, 6, 12, 20, 30, _____
(a) 32 (b) 42
(c) 52 (d) 62

Q.7 1, 5, 11, 19, 29, _____, 55
(a) 45 (b) 39
(c) 41 (d) 47

Q.8 0, 3, 8, 15, _____, 35, 48
(a) 22 (b) 11
(c) 24 (d) 26

Q.9 36, 30, 24, 18, _____
(a) 22 (b) 12
(c) 21 (d) 11

Q.10 2, 5, 9, 19, 37, _____
(a) 47 (b) 87
(c) 75 (d) 85

Q.11 6, 8, 9, 12, 14, 18, _____
(a) 19 (b) 21
(c) 23 (d) 25

Q.12 9, 15, 23, 33, 45, _____
(a) 55 (b) 57
(c) 59 (d) 61

Q.13 1, 0, 3, 2, 5, 6, _____, 12, 9, 20
(a) 10 (b) 7
(c) 8 (d) 12

Q.14 2, 0, 5, 3, _____, 8, 17
(a) 9 (b) 10
(c) 6 (d) 8

Q.15 3, 7, 13, 21, 31, _____
(a) 43 (b) 34
(c) 56 (d) 45

Q.16 6, 15, 33, 69, _____, 285
(a) 137 (b) 141
(c) 143 (d) 147

Q.17 0, 5, 22, 57, 116, _____
(a) 216 (b) 205
(c) 207 (d) 192

Q.18 1, 2, 4, 8, 16, _____
(a) 20 (b) 24
(c) 28 (d) 32

Q.19 0, 7, 26, 63, 124, _____
(a) 195 (b) 208
(c) 215 (d) 240

Q.20 1, 5, 11, 19, 29, 41, 55, _____
(a) 68 (b) 70
(c) 82 (d) 71

Q.21 123, 129, 141, _____, 159, 165
(a) 147 (b) 148
(c) 149 (d) 151

Q.22 27, 64, 125, 216, 343, _____
(a) 416 (b) 512
(c) 686 (d) 559

Directions for questions 23 to 27: ***In each of the following question a number series is given. After the series, a number is given followed by (a), (b), (c), (d) and (e). You have to complete the series starting with the number given, following the sequence of the original series and answer the questions that follow the series.***

Q.23

12	30	120	460	1368	2730
16	(A)	(B)	(C)	(D)	(E)

What will come in place of (D)?
(a) 1384 (b) 2642
(c) 2808 (d) 1988

Q.24

154	462	231	693	346.5	1039.5
276	(a)	(b)	(c)	(d)	(e)

What will come in place of (e)?
(a) 1746 (b) 621
(c) 1242 (d) 1863

Q.25

582	574	601	537	662	446
204	(A)	(B)	(C)	(D)	(E)

What will come in place of (D)?

(a) 284 (b) 68
(c) 174 (d) 331

Q.26

7 91 1001 7007 35035

14.5 (A) (B) (C) (D)

What will come in place of (C)?

(a) 21132.5 (b) 14514.5
(c) 20020.5 (d) 13864.5

Q.27

85 43 44 67.5 137 345

125 (A) (B) (C) (D) (E)

What will come in place of (C)?

(a) 86 (b) 107.5
(c) 112.5 (d) 97.5

Directions for questions 28 to 30: ***Find what should come in place of question mark (?) in the following number series:***

Q.28

12 12 18 45 180 1170 ?

(a) 12285 (b) 10530
(c) 11700 (d) 12870

Q.29

444 467 513 582 674 789 ?

(a) 950 (b) 904
(c) 927 (d) 881

Q.30

23 25 53 163 657 ?

(a) 4096 (b) 2401
(c) 1764 (d) 3291

ANSWER KEYS

1. (b)	**2.** (b)	**3.** (d)	**4.** (c)	**5.** (c)	**6.** (b)	**7.** (c)	**8.** (c)	**9.** (b)	**10.** (c)
11. (b)	**12.** (c)	**13.** (b)	**14.** (b)	**15.** (a)	**16.** (b)	**17.** (b)	**18.** (d)	**19.** (c)	**20.** (d)
21. (a)	**22.** (b)	**23.** (c)	**24.** (d)	**25.** (a)	**26.** (b)	**27.** (d)	**28.** (a)	**29.** (c)	**30.** (d)

HINTS AND SOLUTIONS

1. In this series the 1st, 3rd, 5th terms are multiplied by 2 and 4 is subtracted from the product whereas the 2nd, 4th terms are multiplied by 2 and 4 is added to the product.

Hence, option (b) is the correct answer.

2. Each term is doubled and consecutive numbers starting from 1, 2, 3 are added to the sum to get the next term. i.e., $2 \times 32 = 64 + 3 = 67$

Hence, option (b) is the correct answer.

3. Here the preceding term is multiplied by 2 and then 2, 3, 4 etc. are subtracted from the product.

Hence, option (d) is the correct answer.

4. Each number is half the preceding number plus 10.

Hence, option (c) is the correct answer.

5. The alternate numbers form two different series. The missing number belongs to the one beginning with 260. in this subtract 4 and divide the difference by 2 to get the next number. So the missing number is $62 - 4 = 58/2 = 29$

Hence, option (c) is the correct answer.

6. There is a gap of 4, 6, 8, 10, 12,…Therefore $30 + 12 = 42$

Hence, option (b) is the correct answer.

7. The terms are got by adding even numbers starting from 4 to the preceding term.

Hence, option (c) is the correct answer.

8. The terms are obtained by adding odd numbers from 3, 5, … to the preceding term.

Hence, option (c) is the correct answer.

9. The number goes down by 6 each time.

Hence, option (b) is the correct answer.

10. Multiply each preceding number by 2, add 1 and subtract 1 alternately.

Hence, option (c) is the correct answer.

11. Alternate terms of the given series form two series …. 1st 6, 9, 14, 21, and the 2nd 8, 12, 18

Hence, option (b) is the correct answer.

12. The terms of the given series are 3^2, $4^2 - 1$, $5^2 - 2$, $6^2 - 3$, $7^2 - 4$, $8^2 - 5$

There is a gap of 6, 8, 10, 12,14, …Therefore $45 + 14 = 59$

Hence, option (c) is the correct answer.

13. Alternate terms of the given series form tow different series. 1st 1, 3, 5, 7, 9 and the 2nd is 0, 2, 6, 12, 20.

Hence, option (b) is the correct answer.

14. Alternate terms of the given series form two different series. 1st 2, 5, 10, 17 ($n^2 + 1$) and the 2nd is 0, 3, 8 ($n^2 - 1$).

Hence, option (b) is the correct answer.

15. Series is: +2, +4, +6, +8, +10. Hence next term will be + 12 = 31 + 12 = 43.

Hence, option (a) is the correct answer.

16. The terms are 6, 15, 33, 69, x, 285. These terms are 3×2, 3×5, 3×11, 3×23, x, 3×95

It can be observed all terms are $3 \times a_n$. Where $a_1 = 2$.

There is a gap of (3×3), (3×6), (3×12), (3×24), (3×48),….Therefore, $x = 69 + (3 \times 24) = 141$

Hence, option (b) is the correct answer.

17. The 1st to 5th terms are 0, 5, 22, 57, 116. These terms are 1–1, 8–3, 27–5, 64–7, 125–9. So, these are 1^3–1, 2^3–3, 3^3–5, 4^3–7, 5^3–9. So, nth term of series is $n^3 - (2n - 1)$. So, 6th term of series is $6^3 - (2 \times 6 - 1) = 216 - 11 = 205$

Hence, option (b) is the correct answer.

18. In this series each term is twice of its last term. So, the term after 16 is twice of 16 i.e. 32.

Hence, option (d) is the correct answer.

19. The nth term of series is given as $n^3 - 1$. So, the missing term (i.e. 6th term) = $6^3 - 1 = 216 - 1 = 215$.

Hence, option (c) is the correct answer.

20. The common difference between consecutive terms is increasing by 2. There is difference of 14 between 41 & 55. So, the difference between 55 & next term must be 16. So, next term is 55 + 16 = 71

Hence, option (d) is the correct answer.

21. The common difference between consecutive terms is 6, 12, 6, 12 & so on. So, the term next to 141 is 141 + 6 = 147

Hence, option (a) is the correct answer.

22. The general term is $T_n = (n + 2)^3$. So, the 6th term is $T_6 = (6 + 2)^3 = 8^3 = 512$

Hence, option (b) is the correct answer.

23. The given series is based on the following pattern:

$30 = 12 \times 6 - 7 \times 6$

$120 = 30 \times 5 - 6 \times 5$

$460 = 120 \times 4 - 5 \times 4$

$1368 = 460 \times 3 - 4 \times 3$

$2730 = 1368 \times 2 - 3 \times 2$

Similarly,

(a) $= 16 \times 6 - 7 \times 6 = 96 - 42 = 54$

(b) $= 54 \times 5 - 6 \times 5 = 240$

(c) $= 240 \times 4 - 5 \times 4 = 940$

(d) $= 940 \times 3 - 4 \times 3 = 2808$

Hence, 2808 will come in place of d.

Hence, option (c) is the correct answer.

24. The given series is based on the following pattern:

154 462 231 693 346.5 1039.5

×3 ÷ 2 ×3 ÷ 2 ×3

Similarly,

a b c d e

276 828 414 1242 621 1863

×3 ÷ 2 ×3 ÷ 2 ×3

Hence, 1863 will come in place of e.

Hence, option (d) is the correct answer.

25. The given series is based on the following pattern:

582 574 601 537 662 446

-2^3 $+3^3$ -4^3 $+5^3$ -6^3

Similarly,

a b c d

204 196 223 159 284

-2^3 $+3^3$ -4^3 $+5^3$

Hence, 284 will come in place of d.

Hence, option (a) is the correct answer.

26. The given series is based on the following pattern:

7 91 1001 7007 35035

×13 × 11 ×7 ×5

Similarly,

a b c

14.5 188.5 2073.5 14514.5

×13 ×11 ×7

Hence, 14514.5 will come in place of c.

Hence, option (b) is the correct answer.

27. The given series is based on the following pattern:

85 43 44 67.5 137 345

× 0.5 + 0.5 ×1 ÷ 1 ×1.5 + 1.5 ×2 ÷ 2 ×2.5 ÷ 2.5

Similarly,

a b c

125 63 64 97.5

×0.5 + 0.5 ×1 ÷ 1 ×1.5 ÷ 1.5

Hence, 97.5 will come in place of c.

Hence, option (d) is the correct answer.

28. The given number series is based on the following pattern:

$12 \times 1 = 12$

$12 \times 1.5 = 18$

$18 \times (1 + 1.5) = 18 \times 2.5 = 45$

$45 \times (1.5 + 2.5) = 45 \times 4 = 180$

$180 \times (4 + 2.5) = 180 \times 6.5 = 1170$

$\therefore \quad ? = 1170 \times (4 + 6.5) = 12285$

Hence, 12285 will replace the question mark.

Hence, option (a) is the correct answer.

29. The given number series is based on the following pattern:

$467 - 444 = 23 = 23 \times 1$

$513 - 467 = 46 = 23 \times 2$

$582 - 513 = 69 = 23 \times 3$

$674 - 582 = 92 = 23 \times 4$

$789 - 674 = 115 = 23 \times 4$

$\therefore \quad ? = 789 + 23 \times 6 = 789 + 138 = 927$

Hence, 927 will replace the question mark.

Hence, option (c) is the correct answer.

30. The given number series is based on the following pattern:

$23 \times 1 + 2 = 25$

$25 \times 2 + 3 = 53$

$53 \times 3 + 4 = 163$

$163 \times 4 + 5 = 657$

$657 \times 5 + 6 = 3291$

Hence, 3291 will replace the question mark.

Hence, option (d) is the correct answer.