14. CHAIN RULE

IMPORTANT FACTS AND FORMULAE

1. **Direct Proportion** : *Two quantities are said to be directly proportional, if on the increase (or decrease) of the one, the other increases (or decreases) to the same extent.*

   Ex. 1. ***Cost is directly proportional to the number of articles.***
   (More Articles, More Cost)

   Ex. 2. ***Work done is directly proportional to the number of men working on it.***
   (More Men, More Work)

2. **Indirect Proportion** : *Two quantities are said to be indirectly proportional, if on the increase of the one, the other decreases to the same extent and vice-versa.*

   Ex. 1. ***The time taken by a car in covering a certain distance is inversely proportional to the speed of the car.***
   (More speed, Less is the time taken to cover a distance)

   Ex. 2. ***Time taken to finish a work is inversely proportional to the number of persons working at it.***
   (More persons, Less is the time taken to finish a job)

   Remark : In solving questions by chain rule, we compare every item with the term to be found out.

SOLVED EXAMPLES

Ex. 1. ***If 15 toys cost Rs. 234, what do 35 toys cost ?***

Sol. Let the required cost be Rs. x. Then,

More toys, More cost *(Direct Proportion)*

$\therefore \quad 15 : 35 :: 234 : x \quad \Leftrightarrow \quad (15 \times x) = (35 \times 234) \quad \Leftrightarrow \quad x = \left(\frac{35 \times 234}{15}\right) = 546.$

Hence, the cost of 35 toys is Rs. 546.

Ex. 2. ***If 36 men can do a piece of work in 25 hours, in how many hours will 15 men do it ?***

Sol. Let the required number of hours be x. Then,

Less men, More hours *(Indirect Proportion)*

$\therefore \quad 15 : 36 :: 25 : x \quad \Leftrightarrow \quad (15 \times x) = (36 \times 25) \quad \Leftrightarrow \quad x = \frac{36 \times 25}{15} = 60.$

Hence, 15 men can do it in 60 hours.

Ex. 3. ***If the wages of 6 men for 15 days be Rs. 2100, then find the wages of 9 men for 12 days.***

Sol. Let the required wages be Rs. x.

More men, More wages *(Direct Proportion)*

Less days, Less wages *(Direct Proportion)*

$$\left.\begin{matrix}\text{Men} & 6 : 9 \\ \text{Days} & 15 : 12\end{matrix}\right\} :: 2100 : x$$

$\therefore \quad (6 \times 15 \times x) = (9 \times 12 \times 2100) \quad \Leftrightarrow \quad x = \left(\frac{9 \times 12 \times 2100}{6 \times 15}\right) = 2520.$

Hence, the required wages are Rs. 2520.

Ex. 4. ***If 20 men can build a wall 56 metres long in 6 days, what length of a similar wall can be built by 35 men in 3 days ?***

Sol. Let the required length be x metres.

More men, More length built (*Direct Proportion*)

Less days, Less length built (*Direct Proportion*)

$$\left.\begin{matrix}\text{Men} & 20 : 35 \\ \text{Days} & 6 : 3\end{matrix}\right\} :: 56 : x$$

$\therefore \quad (20 \times 6 \times x) = (35 \times 3 \times 56) \quad \Leftrightarrow \quad x = \frac{(35 \times 3 \times 56)}{120} = 49.$

Hence, the required length is 49 m.

Ex. 5. ***If 15 men, working 9 hours a day, can reap a field in 16 days, in how many days will 18 men reap the field, working 8 hours a day ?***

Sol. Let the required number of days be x.

More men, Less days (*Indirect Proportion*)

Less hours per day, More days (*Indirect Proportion*)

$$\left.\begin{matrix}\text{Men} & 18 : 15 \\ \text{Hours per day} & 8 : 9\end{matrix}\right\} :: 16 : x$$

$\therefore \quad (18 \times 8 \times x) = (15 \times 9 \times 16) \quad \Leftrightarrow \quad x = \left(\frac{15 \times 144}{144}\right) = 15.$

Hence, required number of days = 15.

Ex. 6. ***If 9 engines consume 24 metric tonnes of coal, when each is working 8 hours a day, how much coal will be required for 8 engines, each running 13 hours a day, it being given that 3 engines of former type consume as much as 4 engines of latter type?***

Sol. Let 3 engines of former type consume 1 unit in 1 hour.

Then, 4 engines of latter type consume 1 unit in 1 hour.

$\therefore$ 1 engine of former type consumes $\frac{1}{3}$ unit in 1 hour.

1 engine of latter type consumes $\frac{1}{4}$ unit in 1 hour.

Let the required consumption of coal be x units.

Less engines, Less coal consumed (*Direct Proportion*)

More working hours, More coal consumed (*Direct Proportion*)

Less rate of consumption, Less coal consumed (*Direct Proportion*)

$$\left.\begin{matrix}\text{Number of engines} & 9 : 8 \\ \text{Working hours} & 8 : 13 \\ \text{Rate of consumption} & \frac{1}{3} : \frac{1}{4}\end{matrix}\right\} :: 24 : x$$

$\therefore \quad \left(9 \times 8 \times \frac{1}{3} \times x\right) = \left(8 \times 13 \times \frac{1}{4} \times 24\right) \quad \Leftrightarrow \quad 24x = 624 \quad \Leftrightarrow \quad x = 26.$

Hence, the required consumption of coal = 26 metric tonnes.

Ex. 7. ***A contract is to be completed in 46 days and 117 men were set to work, each working 8 hours a day. After 33 days, $\frac{4}{7}$ of the work is completed. How many additional men may be employed so that the work may be completed in time, each man now working 9 hours a day ?***

Sol. Remaining work $= \left(1-\frac{4}{7}\right) = \frac{3}{7}$. Remaining period = (46 – 33) days = 13 days.

Let the total men working at it be x.

Less work, Less men *(Direct Proportion)*

Less days, More men *(Indirect Proportion)*

More Hrs/Day, Less men *(Indirect Proportion)*

$$\left.\begin{matrix}\text{Work} & \frac{4}{7} : \frac{3}{7} \\ \text{Days} & 13 : 33 \\ \text{Hrs/Day} & 9 : 8\end{matrix}\right\} :: 117 : x$$

$\therefore \quad \frac{4}{7}\times 13\times 9\times x = \frac{3}{7}\times 33\times 8\times 117$ or $x = \left(\frac{3\times 33\times 8\times 117}{4\times 13\times 9}\right) = 198.$

$\therefore$ Additional men to be employed = (198 – 117) = 81.

Ex. 8. ***A garrison of 3300 men had provisions for 32 days, when given at the rate of 850 gms per head. At the end of 7 days, a reinforcement arrives and it was found that the provisions will last 17 days more, when given at the rate of 825 gms per head. What is the strength of the reinforcement ?***

Sol. ***The problem becomes :***

3300 men taking 850 gms per head have provisions for (32 – 7) or 25 days. How many men taking 825 gms each have provisions for 17 days ?

Less ration per head, more men *(Indirect Proportion)*

Less days, More men *(Indirect Proportion)*

$$\left.\begin{matrix}\text{Ration} & 825 : 850 \\ \text{Days} & 17 : 25\end{matrix}\right\} :: 3300 : x$$

$\therefore \quad 825\times 17\times x = 850\times 25\times 3300$ or $x = \frac{850\times 25\times 3300}{825\times 17} = 5000.$

$\therefore$ Strength of reinforcement = (5500 – 3300) = 1700.

EXERCISE 14

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. If the cost of x metres of wire is d rupees, then what is the cost of y metres of wire at the same rate ? **(M.B.A. 2002)**

   (a) Rs. $\left(\frac{xy}{d}\right)$ (b) Rs. (xd) (c) Rs. (yd) (d) Rs. $\left(\frac{yd}{x}\right)$

2. If the price of 6 toys is Rs. 264.37, what will be the approximate price of 5 toys ?

   (a) Rs. 140 (b) Rs. 100 (c) Rs. 200 (d) Rs. 220 (e) Rs. 240

   (Bank P.O. 2000)

3. The price of 357 mangoes is Rs. 1517.25. What will be the approximate price of 9 dozens of such mangoes ?
(a) Rs. 3000 (b) Rs. 3500 (c) Rs. 4000 (d) Rs. 2500

4. If a quarter kg of potato costs 60 paise, how many paise will 200 gm cost ?
(a) 48 paise (b) 54 paise (c) 56 paise (d) 72 paise
(C.B.I. 2001)

5. If 11.25 m of a uniform iron rod weighs 42.75 kg, what will be the weight of 6 m of the same rod ?
(a) 22.8 kg (b) 25.6 kg (c) 28 kg (d) 26.5 kg

6. On a scale of map, 0.6 cm represents 6.6 km. If the distance between the points on the map is 80.5 cm, the actual distance between these points is :
(a) 9 km (b) 72.5 km (c) 190.75 km (d) 885.5 km

7. An industrial loom weaves 0.128 metres of cloth every second. Approximately, how many seconds will it take for the loom to weave 25 metres of cloth ?
(a) 178 (b) 195 (c) 204 (d) 488
(M.B.A. 2003)

8. A flagstaff 17.5 m high casts a shadow of length 40.25 m. The height of the building, which casts a shadow of length 28.75 m under similar conditions will be : **(M.B.A. 2002)**
(a) 10 m (b) 12.5 m (c) 17.5 m (d) 21.25 m

9. A man completes $\frac{5}{8}$ of a job in 10 days. At this rate, how many more days will it take him to finish the job ? **(M.B.A. 2003)**
(a) 5 (b) 6 (c) 7 (d) $7\frac{1}{2}$

10. 36 men can complete a piece of work in 18 days. In how many days will 27 men complete the same work ? **(Bank P.O. 1998)**
(a) 12 (b) 18 (c) 22 (d) 24 (e) None of these

11. A fort had provision of food for 150 men for 45 days. After 10 days, 25 men left the fort. The number of days for which the remaining food will last, is : **(S.S.C. 2001)**
(a) $29\frac{1}{5}$ (b) $37\frac{1}{4}$ (c) 42 (d) 54

12. A wheel that has 6 cogs is meshed with a larger wheel of 14 cogs. When the smaller wheel has made 21 revolutions, then the number of revolutions made by the larger wheel is : **(M.A.T. 2000)**
(a) 4 (b) 9 (c) 12 (d) 49

13. In a camp, there is a meal for 120 men or 200 children. If 150 children have taken the meal, how many men will be catered to with the remaining meal ?
(a) 20 (b) 30 (c) 40 (d) 50
(Railways, 2003)

14. The cost of 16 packets of salt, each weighing 900 grams is Rs. 28. What will be the cost of 27 packets, if each packet weighs 1 kg ?
(a) Rs. 52.50 (b) Rs. 56 (c) Rs. 58.50 (d) Rs. 64.75

15. 4 mat-weavers can weave 4 mats in 4 days. At the same rate, how many mats would be woven by 8 mat-weavers in 8 days ? **(S.S.C. 2004)**
(a) 4 (b) 8 (c) 12 (d) 16

16. Running at the same constant rate, 6 identical machines can produce a total of 270 bottles per minute. At this rate, how many bottles could 10 such machines produce in 4 minutes ? **(M.A.T. 2004)**
(a) 648 (b) 1800 (c) 2700 (d) 10800

17. In a dairy farm, 40 cows eat 40 bags of husk in 40 days. In how many days one cow will eat one bag of husk ? **(Railways, 2003)**

(a) 1 (b) $\frac{1}{40}$ (c) 40 (d) 80

18. 12 men working 8 hours per day complete a piece of work in 10 days. To complete the same work in 8 days, working 15 hours a day, the number of men required, is :

(a) 4 (b) 5 (c) 6 (d) 8

19. 10 men, working 6 hours a day can complete a work in 18 days. How many hours a day must 15 men work to complete the same work in 12 days ? **(S.S.C. 2004)**

(a) 6 (b) 10 (c) 12 (d) 15

20. 39 persons can repair a road in 12 days, working 5 hours a day. In how many days will 30 persons, working 6 hours a day, complete the work ? **(C.B.I. 2003)**

(a) 10 (b) 13 (c) 14 (d) 15

21. 3 pumps, working 8 hours a day, can empty a tank in 2 days. How many hours a day must 4 pumps work to empty the tank in 1 day ? **(M.B.A. 2002)**

(a) 9 (b) 10 (c) 11 (d) 12

22. If 8 men can reap 80 hectares in 24 days, then how many hectares can 36 men reap in 30 days ? **(C.B.I. 2001)**

(a) 350 (b) 400 (c) 425 (d) 450

23. A certain number of persons can dig a trench 100 m long, 50 m broad and 10 m deep in 10 days. The same number of persons can dig another trench 20 m broad and 15 m deep in 30 days. The length of the second trench is :

(a) 400 m (b) 500 m (c) 800 m (d) 900 m

24. If 5 men or 9 women can do a piece of work in 19 days, then in how many days will 3 men and 6 women do the same work ?

(a) 12 (b) 15 (c) 18 (d) 21

25. 49 pumps can empty a reservoir in $6\frac{1}{2}$ days, working 8 hours a day. If 196 pumps are used for 5 hours each day, then the same work will be completed in :

(a) 2 days (b) $2\frac{1}{2}$ days (c) $2\frac{3}{5}$ days (d) 3 days

26. 30 labourers, working 7 hours a day can finish a piece of work in 18 days. If the labourers work 6 hours a day, then the number of labourers to finish the same piece of work in 30 days, will be :

(a) 15 (b) 21 (c) 22 (d) 25

27. If 7 spiders make 7 webs in 7 days, then 1 spider will make 1 web in how many days ?

(a) 1 (b) $\frac{7}{2}$ (c) 7 (d) 49

(Railways, 2003)

28. If 18 pumps can raise 2170 tonnes of water in 10 days, working 7 hours a day; in how many days will 16 pumps raise 1736 tonnes of water, working 9 hours a day ?

(a) 6 (b) 7 (c) 8 (d) 9

29. If 80 lamps can be lighted, 5 hours per day for 10 days for Rs. 21.25, then the number of lamps, which can be lighted 4 hours daily for 30 days, for Rs. 76.50, is :

(a) 100 (b) 120 (c) 150 (d) 160

30. If 12 carpenters, working 6 hours a day, can make 460 chairs in 24 days, how many chairs will 18 carpenters make in 36 days, each working 8 hours a day ?

(a) 1260 (b) 1320 (c) 920 (d) 1380

31. 400 persons, working 9 hours per day complete $\frac{1}{4}$th of the work in 10 days. The number of additional persons, working 8 hours per day, required to complete the remaining work in 20 days, is :

(*a*) 675 (*b*) 275 (*c*) 250 (*d*) 225

32. If 9 examiners can examine a certain number of answer books in 12 days, working 5 hours a day; for how many hours a day would 4 examiners have to work in order to examine twice the number of answer books in 30 days ?

(*a*) 6 (*b*) 8 (*c*) 9 (*d*) 10

33. If 17 labourers can dig a ditch 20 m long in 18 days, working 8 hours a day; how many more labourers should be engaged to dig a similar ditch 39 m long in 6 days, each labourer working 9 hours a day ?

(*a*) 34 (*b*) 51 (*c*) 68 (*d*) 85

34. 20 men complete one-third of a piece of work in 20 days. How many more men should be employed to finish the rest of the work in 25 more days ?

(*a*) 10 (*b*) 12 (*c*) 15 (*d*) 20

35. If 18 binders bind 900 books in 10 days, how many binders will be required to bind 660 books in 12 days ?

(*a*) 22 (*b*) 14 (*c*) 13 (*d*) 11

36. If $\frac{3}{5}$ of a cistern is filled in 1 minute, how much more time will be required to fill the rest of it ?

(*a*) 30 sec (*b*) 40 sec (*c*) 36 sec (*d*) 24 sec

37. If x men, working x hours per day, can do x units of work in x days, then y men, working y hours per day would be able to complete how many units of work in y days ?

(*a*) $\frac{x^2}{y^3}$ (*b*) $\frac{x^3}{y^2}$ (*c*) $\frac{y^2}{x^3}$ (*d*) $\frac{y^3}{x^2}$

38. A rope makes 70 rounds of the circumference of a cylinder whose radius of the base is 14 cm. How many times can it go round a cylinder with radius 20 cm ?

(*a*) 40 (*b*) 49 (*c*) 100 (*d*) None of these

39. If 5 engines consume 6 metric tonnes of coal when each is running 9 hours a day, how many metric tonnes of coal will be needed for 8 engines, each running 10 hours a day, it being given that 3 engines of the former type consume as much as 4 engines of the latter type ?

(*a*) $3\frac{1}{8}$ (*b*) 8 (*c*) $8\frac{8}{9}$ (*d*) $6\frac{12}{25}$

40. If a certain number of workmen can do a piece of work in 25 hours, in how many hours will another set of an equal number of men, do a piece of work, twice as great, supposing that 2 men of the first set can do as much work in an hour, as 3 men of the second set do in an hour ?

(*a*) 60 (*b*) 75 (*c*) 90 (*d*) 105

41. Some persons can do a piece of work in 12 days. Two times the number of such persons will do half of that work in :

(*a*) 6 days (*b*) 4 days (*c*) 3 days (*d*) 12 days

42. A certain number of men can finish a piece of work in 100 days. If, there were 10 men less, it would take 10 days more for the work to be finished. How many men were there originally ?

(*a*) 75 (*b*) 82 (*c*) 100 (*d*) 110

43. In a camp, 95 men had provisions for 200 days. After 5 days, 30 men left the camp. For how many days will the remaining food last now ?
(a) 180 (b) 285 (c) $139\frac{16}{19}$ (d) None of these

44. A garrison of 500 men had provisions for 27 days. After 3 days a reinforcement of 300 men arrived. For how many more days will the remaining food last now ?
(a) 15 (b) 16 (c) $17\frac{1}{2}$ (d) 18

45. A garrison had provisions for a certain number of days. After 10 days, $\frac{1}{5}$ of the men desert and it is found that the provisions will now last just as long as before. How long was that ?
(a) 15 days (b) 25 days (c) 35 days (d) 50 days

46. 15 men take 21 days of 8 hours each to do a piece of work. How many days of 6 hours each would 21 women take, if 3 women do as much work as 2 men ?
(a) 18 (b) 20 (c) 25 (d) 30

47. A contractor undertook to do a certain piece of work in 9 days. He employed certain number of men, but 6 of them being absent from the very first day, the rest could finish the work in 15 days. The number of men originally employed were :
(a) 12 (b) 15 (c) 18 (d) 24

48. A contractor undertakes to do a piece of work in 40 days. He engages 100 men at the beginning and 100 more after 35 days and completes the work in stipulated time. If he had not engaged the additional men, how many days behind schedule would it be finished ?
(a) 3 (b) 5 (c) 6 (d) 9

49. A contractor employed 30 men to do a piece of work in 38 days. After 25 days, he employed 5 men more and the work was finished one day earlier. How many days he would have been behind, if he had not employed additional men ?
(a) 1 (b) $1\frac{1}{4}$ (c) $1\frac{3}{4}$ (d) $1\frac{1}{2}$

50. 12 men and 18 boys, working $7\frac{1}{2}$ hours a day, can do a piece of work in 60 days. If a man works equal to 2 boys, then how many boys will be required to help 21 men to do twice the work in 50 days, working 9 hours a day ?
(a) 30 (b) 42 (c) 48 (d) 90

51. If 3 men or 6 boys can do a piece of work in 10 days, working 7 hours a day; how many days will it take to compete a piece of work twice as large with 6 men and 2 boys working together for 8 hours a day ?
(a) 6 (b) $7\frac{1}{2}$ (c) $8\frac{1}{2}$ (d) 9

52. 2 men and 7 boys can do a piece of work in 14 days; 3 men and 8 boys can do the same in 11 days. Then, 8 men and 6 boys can do three times the amount of this work in :
(a) 18 days (b) 21 days (c) 24 days (d) 30 days

ANSWERS

1. (d)	2. (d)	3. (d)	4. (a)	5. (a)	6. (d)	7. (b)	8. (b)	9. (b)
10. (d)	11. (c)	12. (b)	13. (b)	14. (a)	15. (d)	16. (b)	17. (c)	18. (d)
19. (a)	20. (b)	21. (d)	22. (d)	23. (b)	24. (b)	25. (c)	26. (b)	27. (c)

28. (*b*)	**29.** (*b*)	**30.** (*d*)	**31.** (*b*)	**32.** (*c*)	**33.** (*b*)	**34.** (*b*)	**35.** (*d*)	**36.** (*b*)
37. (*d*)	**38.** (*b*)	**39.** (*b*)	**40.** (*b*)	**41.** (*c*)	**42.** (*d*)	**43.** (*b*)	**44.** (*a*)	**45.** (*d*)
46. (*d*)	**47.** (*b*)	**48.** (*b*)	**49.** (*a*)	**50.** (*b*)	**51.** (*b*)	**52.** (*b*)		

SOLUTIONS

1. Cost of x metres = Rs. d. Cost of 1 metre = Rs. $\left(\frac{d}{x}\right)$.

   Cost of y metres = Rs. $\left(\frac{d}{x} \times y\right)$ = Rs. $\left(\frac{yd}{x}\right)$.

2. Let the required price be Rs. x. Then, ***Less toys, Less cost (Direct Proportion)***

   $\therefore \quad 6 : 5 :: 264.37 : x \quad \Leftrightarrow \quad 6x = (5 \times 264.37) \quad \Leftrightarrow \quad x = \frac{(5 \times 264.37)}{6} \quad \Leftrightarrow \quad x = 220.308.$

   $\therefore$ Approximate price of 5 toys = Rs. 220.

3. Let the required price be Rs. x. Then, ***More mangoes, More price (Direct Proportion)***

   $\therefore \quad 357 : (49 \times 12) :: 1517.25 : x$

   $\Leftrightarrow \quad 357x = (49 \times 12 \times 1517.25) \quad \Leftrightarrow \quad x = \frac{(49 \times 12 \times 1517.25)}{357} \quad \Leftrightarrow \quad x = 2499.$

   Hence, the approximate price is Rs. 2500.

4. Let the required cost be x paise. ***Less weight, Less cost (Direct Proportion)***

   $\therefore \quad 250 : 200 :: 60 : x \quad \Leftrightarrow \quad 250 \times x = (200 \times 60) \quad \Leftrightarrow \quad x = \frac{(200 \times 60)}{250} \quad \Leftrightarrow \quad x = 48.$

5. Let the required weight be x kg. Then, ***Less length, Less weight (Direct Proportion)***

   $\therefore \quad 11.25 : 6 :: 42.75 : x \quad \Leftrightarrow \quad 11.25 \times x = 6 \times 42.75 \quad \Leftrightarrow \quad x = \frac{(6 \times 42.75)}{11.25} \quad \Leftrightarrow \quad x = 22.8.$

6. Let the actual distance be x km. Then,

   More distance on the map, More is the actual distance (Direct Proportion)

   $\therefore \quad 0.6 : 80.5 :: 6.6 : x \quad \Leftrightarrow \quad 0.6x = 80.5 \times 6.6 \quad \Leftrightarrow \quad x = \frac{80.5 \times 6.6}{0.6} \quad \Leftrightarrow \quad x = 885.5.$

7. Let the required time be x seconds. Then, ***More metres, more time (Direct Proportion)***

   $\therefore \quad 0.128 : 25 :: 1 : x$

   $\Leftrightarrow \quad 0.128 \times x = 25 \times 1 \quad \Leftrightarrow \quad x = \frac{25}{0.128} = \frac{25 \times 1000}{128} \quad \Leftrightarrow \quad x = 195.31.$

   $\therefore$ Required time = 195 sec (approximately).

8. Let the height of the building be x metres.

   Less lengthy shadow, Less is the height (Direct Proportion)

   $\therefore \quad 40.25 : 28.75 :: 17.5 : x \quad \Leftrightarrow \quad 40.25 \times x = 28.75 \times 17.5$

   $\Leftrightarrow \quad x = \frac{(28.75 \times 17.5)}{40.25} \quad \Leftrightarrow \quad x = 12.5.$

9. Work done = $\frac{5}{8}$. Balance work = $\left(1 - \frac{5}{8}\right) = \frac{3}{8}$.

   Less work, Less days (Direct Proportion)

   Let the required number of days be x.

   Then, $\frac{5}{8} : \frac{3}{8} :: 10 : x \quad \Leftrightarrow \quad \frac{5}{8} \times x = \frac{3}{8} \times 10 \quad \Leftrightarrow \quad x = \left(\frac{3}{8} \times 10 \times \frac{8}{5}\right) = 6.$

10. Let the required number of days be x.
Then, ***Less men, More days*** (***Indirect Proportion***)

$$\therefore \quad 27 : 36 :: 18 : x \quad \Leftrightarrow \quad 27 \times x = 36 \times 18 \quad \Leftrightarrow \quad x = \frac{36 \times 18}{27} \quad \Leftrightarrow \quad x = 24.$$

11. **After 10 days :** 150 men had food for 35 days.
Suppose 125 men had food for x days. Now, ***Less men, More days*** (***Indirect Proportion***)

$$\therefore \quad 125 : 150 :: 35 : x \quad \Leftrightarrow \quad 125 \times x = 150 \times 35 \quad \Leftrightarrow \quad x = \frac{150 \times 35}{125} \quad \Leftrightarrow \quad x = 42.$$

Hence, the remaining food will last for 42 days.

12. Let the required number of revolutions made by larger wheel be x.
Then, ***More cogs, Less revolutions*** (***Indirect Proportion***)

$$\therefore \quad 14 : 6 :: 21 : x \quad \Leftrightarrow \quad 14 \times x = 6 \times 21 \quad \Leftrightarrow \quad x = \left(\frac{6 \times 21}{14}\right) = 9.$$

13. There is a meal for 200 children. 150 children have taken the meal.
Remaining meal is to be catered to 50 children.
Now, 200 children = 120 men

$$50 \text{ children} = \left(\frac{120}{200} \times 50\right) \text{ men} = 30 \text{ men.}$$

14. Let the required cost be Rs. x. Then,
More packets, More cost (***Direct Proportion***)
More weight, More cost (***Direct Proportion***)

$$\left.\begin{array}{ll} \text{Packets} & 16 : 27 \\ \text{Weight} & 900 : 1000 \end{array}\right\} :: 28 : x$$

$$\therefore \quad (16 \times 900 \times x) = (27 \times 1000 \times 28) \quad \Leftrightarrow \quad x = \frac{(27 \times 1000 \times 28)}{16 \times 900} = \frac{105}{2} = 52.50.$$

15. Let the required number of mats be x.
More weavers, More mats (***Direct Proportion***)
More days, More mats (***Direct Proportion***)

$$\left.\begin{array}{ll} \text{Weavers} & 4 : 8 \\ \text{Days} & 4 : 8 \end{array}\right\} :: 4 : x$$

$$\therefore \quad 4 \times 4 \times x = 8 \times 8 \times 4 \quad \Leftrightarrow \quad x = \frac{(8 \times 8 \times 4)}{(4 \times 4)} = 16.$$

16. Let the required number of bottles be x.
More machines, More bottles (***Direct Proportion***)
More minutes, More bottles (***Direct Proportion***)

$$\left.\begin{array}{ll} \text{Machines} & 6 : 10 \\ \text{Time (in Minutes)} & 1 : 4 \end{array}\right\} :: 270 : x$$

$$\therefore \quad 6 \times 1 \times x = 10 \times 4 \times 270 \quad \Leftrightarrow \quad x = \frac{10 \times 4 \times 270}{6} \quad \Leftrightarrow \quad x = 1800.$$

17. Let the required number of days be x.
Less cows, More days (***Indirect Proportion***)
Less bags, Less days (***Direct Proportion***)

$$\left.\begin{array}{ll} \text{Cows} & 1 : 40 \\ \text{Bags} & 40 : 1 \end{array}\right\} :: 40 : x$$

$$\therefore \quad 1 \times 40 \times x = 40 \times 1 \times 40 \quad \Leftrightarrow \quad x = 40.$$

18. Let the required number of men be x.
Less days, More men **(*Indirect Proportion*)**
More working hrs per day, Less men **(*Indirect Proportion*)**

$$\left.\begin{array}{ll}\text{Days} & 8 : 10\\ \text{Working Hrs} & 15 : 8\end{array}\right\} :: 12 : x$$

$\therefore \quad 8 \times 15 \times x = 10 \times 8 \times 12 \Leftrightarrow x = \dfrac{10 \times 8 \times 12}{8 \times 15} \Leftrightarrow x = 8.$

19. Let the required number of hours per day be x.
More men, Less hours per day **(*Indirect Proportion*)**
Less days, More hours per day **(*Indirect Proportion*)**

$$\left.\begin{array}{ll}\text{Men} & 15 : 10\\ \text{Days} & 12 : 18\end{array}\right\} :: 6 : x$$

$\therefore \quad 15 \times 12 \times x = 10 \times 18 \times 6 \Leftrightarrow x = \dfrac{10 \times 18 \times 6}{15 \times 12} \Leftrightarrow x = 6.$

20. Let the required number of days be x.
Less persons, More days **(*Indirect Proportion*)**
More working hrs per day, Less days **(*Indirect Proportion*)**

$$\left.\begin{array}{ll}\text{Persons} & 30 : 39\\ \text{Working hrs/day} & 6 : 5\end{array}\right\} :: 12 : x$$

$\therefore \quad 30 \times 6 \times x = 39 \times 5 \times 12 \Leftrightarrow x = \dfrac{39 \times 5 \times 12}{30 \times 6} \Leftrightarrow x = 13.$

21. Let the required number of working hours per day be x.
More pumps, Less working hours per day **(*Indirect Proportion*)**
Less days, More working hours per day **(*Indirect Proportion*)**

$$\left.\begin{array}{ll}\text{Pumps} & 4 : 3\\ \text{Days} & 1 : 2\end{array}\right\} :: 8 : x$$

$\therefore \quad 4 \times 1 \times x = 3 \times 2 \times 8 \Leftrightarrow x = \dfrac{3 \times 2 \times 8}{4} \Leftrightarrow x = 12.$

22. Let the required number of hectares be x. Then,
More men, More hectares **(*Direct Proportion*)**
More days, More hectares **(*Direct Proportion*)**

$$\left.\begin{array}{ll}\text{Men} & 8 : 36\\ \text{Days} & 24 : 30\end{array}\right\} :: 80 : x$$

$\therefore \quad 8 \times 24 \times x = 36 \times 30 \times 80 \Leftrightarrow x = \dfrac{(36 \times 30 \times 80)}{(8 \times 24)} \Leftrightarrow x = 450.$

23. Let the required length be x metres.
More breadth, Less length **(*Indirect Proportion*)**
More depth, Less length **(*Indirect Proportion*)**
More days, More length **(*Direct Proportion*)**

$$\left.\begin{array}{ll}\text{Breadth} & 20 : 50\\ \text{Depth} & 15 : 10\\ \text{Days} & 10 : 30\end{array}\right\} :: 100 : x$$

$\therefore \quad 20 \times 15 \times 10 \times x = 50 \times 10 \times 30 \times 100 \Leftrightarrow x = \dfrac{(50 \times 10 \times 30 \times 100)}{(20 \times 15 \times 10)} \Leftrightarrow x = 500.$

24. Let the required number of days be x.

5 men = 9 women $\Leftrightarrow$ 3 men = $\left(\frac{9}{5}\times 3\right)$ women = $\frac{27}{5}$ women.

$\therefore$ (3 men and 6 women) = $\left(\frac{27}{5}+6\right)$ women = $\frac{57}{5}$ women.

Now, ***More women, Less days (Indirect Proportion)***

$\therefore \quad \frac{57}{5} : 9 :: 19 : x \quad \Leftrightarrow \quad \frac{57}{5}\times x = 9\times 19 \quad \Leftrightarrow \quad x = \left(9\times 19\times \frac{5}{57}\right) = 15.$

25. Let the required number of days be x. Then,

More pumps, Less days ***(Indirect Proportion)***

Less working hrs/day, More days ***(Indirect Proportion)***

Pumps	196 : 49	$:: \frac{13}{2} : x$
Working Hrs/Day	5 : 8	

$\therefore \quad 196\times 5\times x = 49\times 8\times \frac{13}{2} \quad \Leftrightarrow \quad x = \left(49\times 8\times \frac{13}{2}\times \frac{1}{196\times 5}\right) \quad \Leftrightarrow \quad x = \frac{13}{5} = 2\frac{3}{5}.$

26. Let the required number of labourers be x. Then,

Less working hrs/day, More labourers ***(Indirect Proportion)***

More days, Less labourers ***(Indirect Proportion)***

Working Hrs/Day	6 : 7	$:: 30 : x$
Days	30 : 18	

$\therefore \quad 6\times 30\times x = 7\times 18\times 30 \quad \Leftrightarrow \quad 6x = 126 \quad \Leftrightarrow \quad x = 21.$

27. Let the required number of days be x. Then,

Less spiders, More days ***(Indirect Proportion)***

Less webs, Less days ***(Direct Proportion)***

$\therefore$

Spiders	1 : 7	$:: 7 : x$
Webs	7 : 1	

$\therefore \quad 1\times 7\times x = 7\times 1\times 7 \quad \Leftrightarrow \quad x = 7.$

28. Let the required number of days be x. Then,

Less pumps, More days ***(Indirect Proportion)***

Less weight, Less days ***(Direct Proportion)***

More hours/day, Less days ***(Indirect Proportion)***

Pumps	16 : 18	
Weight	2170 : 1736	$:: 10 : x$
Hours/Day	9 : 7	

$\therefore \quad (16\times 2170\times 9\times x) = (18\times 1736\times 7\times 10) \quad \Leftrightarrow \quad x = \frac{18\times 1736\times 7\times 10}{16\times 2170\times 9} = 7.$

29. Let the required number of lamps be x.

Less hours per day, More lamps ***(Indirect Proportion)***

More money, More lamps ***(Direct Proportion)***

More days, Less lamps ***(Indirect Proportion)***

Hours per day	4 : 5	
Money	21.25 : 76.50	$:: 80 : x$
Number of days	30 : 10	

$\therefore \quad 4 \times 21.25 \times 30 \times x = 5 \times 76.50 \times 10 \times 80 \quad \Leftrightarrow$

$$x = \frac{5 \times 76.50 \times 10 \times 80}{4 \times 21.25 \times 30} \Leftrightarrow x = 120.$$

30. Let the required number of chairs be x. Then,

More carpenters, More chairs *(Direct Proportion)*
More hours per day, More chairs *(Direct Proportion)*
More days, More chairs *(Direct Proportion)*

Carpenters	12 : 18	
Hours per day	6 : 8	:: 460 : x
Days	24 : 36	

$\therefore$ $(12 \times 6 \times 24 \times x) = (18 \times 8 \times 36 \times 460) \Leftrightarrow x = \frac{(18 \times 8 \times 36 \times 460)}{(12 \times 6 \times 24)} = 1380.$

$\therefore$ Required number of chairs = 1380.

31. Let the number of persons completing the work in 20 days be x.

Work done = $\frac{1}{4}$, Remaining work = $\left(1 - \frac{1}{4}\right) = \frac{3}{4}$.

Less hours per day, More men required *(Indirect Proportion)*
More work, More men required *(Direct Proportion)*
More days, Less men required *(Indirect Proportion)*

Hours per day	8 : 9	
Work	$\frac{1}{4} : \frac{3}{4}$	:: 400 : x
Days	20 : 10	

$\therefore$ $8 \times \frac{1}{4} \times 20 \times x = 9 \times \frac{3}{4} \times 10 \times 400 \Leftrightarrow 40x = 27000 \Leftrightarrow x = 675.$

$\therefore$ Additional men = (675 − 400) = 275.

32. Let the required number of working hours per day be x.

Less examiners, More working hours per day *(Indirect Proportion)*
More days, Less working hours per day *(Indirect Proportion)*
More answer books, More working hours per day *(Direct Proportion)*

Examiners	4 : 9	
Days	30 : 12	:: 5 : x
Answer books	1 : 2	

$\therefore$ $(4 \times 30 \times 1 \times x) = (9 \times 12 \times 2 \times 5) \Leftrightarrow 120x = 1080 \Leftrightarrow x = 9.$

33. Let the total number of men to be engaged be x.

More length, More labourers *(Direct Proportion)*
Less days, More labourers *(Indirect Proportion)*
More hours per day, Less labourers *(Indirect Proportion)*

Length	26 : 39	
Days	6 : 18	:: 17 : x
Hours per day	9 : 8	

$\therefore$ $(26 \times 6 \times 9 \times x) = (39 \times 18 \times 8 \times 17) \Leftrightarrow x = \frac{(39 \times 18 \times 8 \times 17)}{(26 \times 6 \times 9)} = 68.$

$\therefore$ Number of more labourers = (68 − 17) = 51.

34. Let the total number of men be x. Work done = $\frac{1}{3}$, Remaining work = $\left(1 - \frac{1}{3}\right) = \frac{2}{3}$.

More work, More men *(Direct Proportion)*
More days, Less men *(Indirect Proportion)*

$$\left.\begin{array}{ll}\text{Work} & \frac{1}{3} : \frac{2}{3} \\ \text{Days} & 25 : 20\end{array}\right\} :: 20 : x$$

$$\therefore \quad \left(\frac{1}{3} \times 25 \times x\right) = \left(\frac{2}{3} \times 20 \times 20\right) \Leftrightarrow x = \frac{800}{25} = 32.$$

$\therefore$ More men to be employed = (32 − 20) = 12.

35. Let the required number of binders be x.

Less books, Less binders *(Direct Proportion)*
More days, Less binders *(Indirect Proportion)*

$$\left.\begin{array}{ll}\text{Books} & 900 : 600 \\ \text{Days} & 12 : 10\end{array}\right\} :: 18 : x$$

$$\therefore \quad (900 \times 12 \times x) = (600 \times 10 \times 18) \Leftrightarrow x = \frac{600 \times 10 \times 18}{900 \times 12} = 11.$$

36. Let the required time be x seconds.

Part filled = $\frac{3}{5}$, Remaining part = $\left(1 - \frac{3}{5}\right) = \frac{2}{5}$.

Less part, Less time *(Direct Proportion)*

$$\therefore \quad \frac{3}{5} : \frac{2}{5} :: 60 : x \Leftrightarrow \left(\frac{3}{5} \times x\right) = \left(\frac{2}{5} \times 60\right) \Leftrightarrow x = 40.$$

37. Let the required number of units of work be z.

More men, More work *(Direct Proportion)*
More working hours, More work *(Direct Proportion)*
More days, More work *(Direct Proportion)*

$$\left.\begin{array}{ll}\text{Men} & x : y \\ \text{Hours per day} & x : y \\ \text{Days} & x : y\end{array}\right\} :: x : z$$

$$\therefore \quad (x \times x \times x \times z) = (y \times y \times y \times x) \Leftrightarrow z = \frac{y^3}{x^2}.$$

38. Let the required number of rounds be x.

More radius, Less rounds *(Indirect Proportion)*

$$\therefore \quad 20 : 14 :: 70 : x \Leftrightarrow (20 \times x) = (14 \times 70) \Leftrightarrow x = \frac{14 \times 70}{20} \Leftrightarrow x = 49.$$

Hence, the required number of rounds = 49.

39. Let the required quantity of coal be x metric tonnes.

More engines, More coal *(Direct Proportion)*
More hours per day, More coal *(Direct Proportion)*
More rate, More coal *(Direct Proportion)*

$$\left.\begin{array}{ll}\text{Engines} & 5 : 8 \\ \text{Hours per day} & 9 : 10 \\ \text{Rate} & \frac{1}{3} : \frac{1}{4}\end{array}\right\} :: 6 : x$$

$\therefore \quad \left(5 \times 9 \times \frac{1}{3} \times x\right) = \left(8 \times 10 \times \frac{1}{4} \times 6\right) \Leftrightarrow 15x = 120 \Leftrightarrow x = 8.$

40. Let the required number of hours be x.

Speeds of working of first and second type of men are $\frac{1}{2}$ and $\frac{1}{3}$.

More work, More time **(*Direct Proportion*)**
Less speed, More time **(*Indirect Proportion*)**

$$\left.\begin{array}{l}\text{Work} \quad 1 : 2 \\ \text{Speed} \quad \frac{1}{3} : \frac{1}{2}\end{array}\right\} :: 25 : x$$

$\therefore \quad \left(1 \times \frac{1}{3} \times x\right) = \left(2 \times \frac{1}{2} \times 25\right) \Leftrightarrow x = 75.$

41. Let x men can do the work in 12 days and the required number of days be z.

More men, Less days **(*Indirect Proportion*)**
Less work, Less days **(*Direct Proportion*)**

$$\left.\begin{array}{l}\text{Men} \quad 2x : x \\ \text{Work} \quad 1 : \frac{1}{2}\end{array}\right\} :: 12 : z$$

$\therefore \quad (2x \times 1 \times z) = \left(x \times \frac{1}{2} \times 12\right) \Leftrightarrow 2xz = 6x \Leftrightarrow z = 3.$

42. Originally, let there be x men.

Less men, More days **(*Indirect Proportion*)**

$\therefore \quad (x - 10) : x :: 100 : 110 \Leftrightarrow (x - 10) \times 110 = x \times 100 \Leftrightarrow 10x = 1100 \Leftrightarrow x = 110.$

43. Let the remaining food will last for x days.

95 men had provisions for 195 days. 65 men had provisions for x days.

Less men, More days **(*Indirect Proportion*)**

$\therefore \quad 65 : 95 :: 195 : x \Leftrightarrow (65 \times x) = (95 \times 195) \Leftrightarrow x = \frac{95 \times 195}{65} = 285.$

44. Let the remaining food will last for x days.

500 men had provisions for $(27 - 3) = 24$ days.

$(500 + 300)$ men had provisions for x days.

More men, Less days **(*Indirect Proportion*)**

$\therefore \quad 800 : 500 :: 24 : x \Leftrightarrow (800 \times x) = (500 \times 24) \Leftrightarrow x = \left(\frac{500 \times 24}{800}\right) = 15.$

45. Initially, let there be x men having food for y days.

After 10 days, x men had food for $(y - 10)$ days. Also, $\left(x - \frac{x}{5}\right)$ men had food for y days.

$\therefore \quad x(y - 10) = \frac{4x}{5} \times y \Leftrightarrow 5xy - 50x = 4xy \Leftrightarrow xy - 50x = 0$

$\Leftrightarrow x(y - 50) = 0 \Leftrightarrow y - 50 = 0 \Leftrightarrow y = 50.$

46. 3 women = 2 men. So, 21 women = 14 men.

Less men, More days **(*Indirect Proportion*)**
Less hours per day, More days **(*Indirect Proportion*)**

$$\left.\begin{array}{ll}\text{Men} & 14 : 15 \\ \text{Hours per day} & 6 : 8\end{array}\right\} :: 21 : x$$

$\therefore \quad (14 \times 6 \times x) = (15 \times 8 \times 21) \quad \Leftrightarrow \quad x = \dfrac{(15 \times 8 \times 21)}{(14 \times 6)} = 30.$

$\therefore$ Required number of days = 30.

47. Let there be x men at the beginning.

Less men, More days ***(Indirect Proportion)***

$\therefore \quad 15 : 9 :: x : (x - 6) \quad \Leftrightarrow \quad 15\,(x - 6) = 9x \quad \Leftrightarrow \quad 6x = 90 \quad \Leftrightarrow \quad x = 15.$

48. $[(100 \times 35) + (200 \times 5)]$ men can finish the work in 1 day.

$\therefore$ 4500 men can finish the work in 1 day. 100 men can finish it in $\dfrac{4500}{100} = 45$ days.

This is 5 days behind schedule.

49. After 25 days, 35 men complete the work in 12 days.

Thus, 35 men can finish the remaining work in 12 days.

$\therefore \quad$ 30 men can do it in $\dfrac{(12 \times 35)}{30} = 14$ days, which is **1 day behind.**

50. 1 man = 2 boys $\Leftrightarrow$ (12 men + 18 boys) = $(12 \times 2 + 18)$ boys = 42 boys.

Let required number of boys = x. 21 men + x boys = $(21 \times 2 + x)$ boys = $(42 + x)$ boys.

Less days, More boys ***(Indirect Proportion)***

More hrs per day, Less boys ***(Indirect Proportion)***

$$\left.\begin{array}{ll} \text{Days} & 50 : 60 \\ \text{Hours per day} & 9 : \dfrac{15}{2} \\ \text{Work} & 1 : 2 \end{array}\right\} :: 42 : (42 + x)$$

$\therefore \quad [50 \times 9 \times 1 \times (42 + x)] = \left(60 \times \dfrac{15}{2} \times 2 \times 42\right)$

$\Leftrightarrow \quad (42 + x) = \dfrac{37800}{450} \quad \Leftrightarrow \quad 42 + x = 84 \quad \Leftrightarrow \quad x = 42.$

51. 3 men = 6 boys $\Leftrightarrow$ (6 men + 2 boys) = 14 boys.

More work, More days ***(Direct Proportion)***

More boys, Less days ***(Indirect Proportion)***

More hours per day, Less days ***(Indirect Proportion)***

$$\left.\begin{array}{ll} \text{Work} & 1 : 2 \\ \text{Boys} & 14 : 6 \\ \text{Hours per day} & 8 : 7 \end{array}\right\} :: 10 : x$$

$\therefore \quad (1 \times 14 \times 8 \times x) = (2 \times 6 \times 7 \times 10) \quad \Leftrightarrow \quad x = \dfrac{840}{112} = 7\dfrac{1}{2}.$

52. (2×14) men + (7×14) boys = (3×11) men + (8×11) boys.

$\Leftrightarrow$ 5 men = 10 boys $\Leftrightarrow$ 1 man = 2 boys.

$\therefore$ (2 men + 7 boys) = $(2 \times 2 + 7)$ boys = 11 boys.

(8 men + 6 boys) = $(8 \times 2 + 6)$ boys = 22 boys.

Let the required number of days be x.

Now, ***More boys, Less days*** ***(Indirect Proportion)***

More work, More days ***(Direct Proportion)***

$$\left.\begin{array}{ll} \text{Boys} & 22 : 11 \\ \text{Work} & 1 : 3 \end{array}\right\} :: 14 : x$$

$\therefore \quad (22 \times 1 \times x) = (11 \times 3 \times 14) \quad \therefore \quad x = \dfrac{462}{22} = 21.$

Hence, the required number of days = 21.