Probability Distributions

EXERCISE 8.1 [PAGES 140 - 141]

Exercise 8.1 | Q 1.01 | Page 140

Let X represent the difference between number of heads and number of tails obtained when a coin is tossed 6 times. What are the possible values of X?

Solution: X represent the difference between number of heads and number of tails.

Sample space of the experiment is

S = {0 heads, 6 tails), (1 head, 5 tails), (2 heads, 4 tails),(3 heads, 3 tails), (4 heads, 2 tails), (5 heads, 1 tail), (6 heads, 0 tails)}

The value of X corresponding to these outcomes are as follow:

X (0 heads, 6 tails) = 0 – 6 = – 6

X (1 heads, 5 tails) = 1 – 5 = – 4

X (2 heads, 4 tails) = 2 – 4 = – 2

X (3 heads, 3 tails) = 3 – 3 = 0

X (4 heads, 2 tails) = 4 – 2 = 2

X (5 heads, 1 tails) = 5 – 1 = 4

X (6 heads, 0 tails) = 6 – 0 = 6

∴ Possible values of X are {–6, – 4, –2, 0, 2, 4, 6}.

Exercise 8.1 | Q 1.02 | Page 140

An urn contains 5 red and 2 black balls. Two balls are drawn at random. X denotes number of black balls drawn. What are the possible values of X?

Solution: 5 red + 2 black = 7 balls

X denote the number of black balls drawn.

Sample space of the experiment is

S = {RR, BR, RB, BB}

The value of X corresponding to these out comes are as follow:

X (RR) = 0

X(BR) = X(RB) = 1

X(BB) = 2

∴ Possible values of X are {0, 1, 2}.

Exercise 8.1 | Q 1.03 | Page 140

Determine whether each of the following is a probability distribution. Give reasons for your answer.

x	0	1	2
P(x)	0.4	0.4	0.2

Solution:

Here, pi > 0 $\forall$ i = 1, 2, 3

Now consider,

$$\sum_{i=1}^{3} P_i = 0.4 + 0.4 + 0.2 = 1$$

∴ Given distribution is a probability distribution.

Exercise 8.1 | Q 1.03 | Page 140

Determine whether each of the following is a probability distribution. Give reasons for your answer.

x	0	1	2	3	4
P(x)	0.1	0.5	0.2	–0.1	0.3

Solution: Here, P(X = 3) = –0.1 < 0

∴ Given distribution is not a probability distribution.

Exercise 8.1 | Q 1.03 | Page 140

Determine whether each of the following is a probability distribution. Give reasons for your answer.

x	0	1	2
P(x)	0.1	0.6	0.3

Solution:

Here, pi > 0, ∀ I = 1, 2, 3

Now consider,

$$\sum_{i=1}^{3} P_i = 0.1 + 0.6 + 0.3 = 1$$

∴ Given distribution is a probability distribution.

Exercise 8.1 | Q 1.03 | Page 140

Determine whether each of the following is a probability distribution. Give reasons for your answer.

z	3	2	1	0	-1
P(z)	0.3	0.2	0.4.	0.05	0.05

Solution:

Here, pi > 0, ∀∀ i = 1, 2, ...5

Now consider

$$\sum_{i=1}^{3} P_i = 0.3 + 0.2 + 0.4 + 0.05 + 0.05 = 1$$

∴ Given distribution is a probability distribution.

Exercise 8.1 | Q 1.03 | Page 141

Determine whether each of the following is a probability distribution. Give reasons for your answer.

y	–1	0	1
P(y)	0.6	0.1	0.2

Solution: Here, pi > 0, ∀ i = 1, 2, 3

Now Consider,

$$\sum_{i=1}^{3} P_i = 0.6 + 0.1 + 0.2 = 0.9 \neq 1$$

∴ Given distribution is not a probability distribution.

Exercise 8.1 | Q 1.03 | Page 141

Determine whether each of the following is a probability distribution. Give reasons for your answer.

x	0	1	2
P(x)	0.3	0.4	0.2

Solution:

Here, pi > 0, ∀ i = 1, 2, 3

Now consider,

$$\sum_{i=1}^{3} P_i = 0.3 + 0.4 + 0.2 = 0.9 \neq 1$$

∴ Given distribution is not a probability distribution.

Exercise 8.1 | Q 1.04 | Page 141

Find the probability distribution of number of number of tails in three tosses of a coin,

Solution: Let X denote the number of tails.
Sample space of the experiment is
S = {HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}
The values of X corresponding to these outcomes are as follows X(HHH) = 0
X(HHT) = X(HTH) = X(THH) = 1
X(TTH) = X(THT) = X(HTT) = 2
X(TTT) = 3
∴ X is a discrete random variable that can take values 0, 1, 2, 3.
The probability distribution of X is then obtained as follows:

X	0	1	2	3
P(X = x)	1/8	3/8	3/8	1/8

Exercise 8.1 | Q 1.04 | Page 141

Find the probability distribution of number of heads in four tosses of a coin.

Solution: Let X denote the number of heads.

Sample space of the experiment is

S = {HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTTH, TTHH, THTH, HTHT, THHT, HTTT, THTT, TTHT, TTTH, TTTT}

The values of X corresponding to these outcomes are as follows.

X(TTTT) = 0

X(HTTT) = X(THTT) = X(TTHT) = X(TTTH) = 1

X(HHTT) = X(HTTH) = X(TTHH) = X(THTH) = X(HTHT) = X(THHT) = 2

X(HHHT) = X(HHTH) = X(HTHH) = X(THHH) = 3

X(HHHH) = 4

∴ X is a discrete random variable that can take values 0, 1, 2, 3,4.

The probability distribution of X is then obtained as follows:

X	0	1	2	3	4
P(X = x)	1/16	4/16	6/16	4/16	1/16

Exercise 8.1 | Q 1.05 | Page 141

Find the probability distribution of the number of successes in two tosses of a die if success is defined as getting a number greater than 4.

Solution: Success is defined as a number greater than 4 appears on at least one die.

Let X denote the number of successes.

∴ Possible values of X and 0, 1, 2.

Let, P(getting a number greater than 4) = p = $\frac{2}{6} = \frac{1}{3}$

∴ q = 1 – p = $1 - \frac{1}{3} = \frac{2}{3}$

∴ P(X = 0) = P(no success) = qq = q^2 = $\frac{4}{9}$

9 P(X = 1) = P(one success) = qp + qp = 2pq

$= \frac{2 \times 1}{3} \times \frac{2}{3}$

$= \frac{4}{9}$

P(X = 2) = P(two successes) = pp = p^2 = $\frac{1}{9}$

∴ Probability distribution of X is as follows:

X	0	1	2
P(X = x)	4/9	4/9	1/9

Exercise 8.1 | Q 1.06 | Page 141

A sample of 4 bulbs is drawn at random with replacement from a lot of 30 bulbs which includes 6 defectives bulbs. Find the probability distribution of the number of defective bulbs.

Solution: Let X denote the number of defective bulbs.

∴ Possible values of X are 0, 1, 2, 3, 4.

Let P(getting a defective bulb) = p = $\frac{6}{30} = \frac{1}{5}$

∴ q = 1 – p = $1 - \frac{1}{5} = \frac{4}{5}$

∴ P(X = 0) = P(no defective bulb)

= qqqq = q^4 = $\left(\frac{4}{5}\right)^4$

P(X = 1) = P(one defective bulb)

= qqqp + qqpq + qpqq + pqqq

= $4pq^3$

$= 4 \times \frac{1}{5} \times \left(\frac{4}{5}\right)^3 = \left(\frac{4}{5}\right)^4$

P(X = 2) = P(two defective bulbs)

= ppqq + pqqp + qqpp + pqpq + qpqp + qppq

$= 6p^2q^2$

$= 6\left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right)^2$

P(X = 3) = P(three defective bulbs)

= pppq + ppqp + pqpp + qppp

$= 4qp^3$

$= 4\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)^3$

P(X = 4) = P(four defective bulbs)

$= pppp = p^4 = \left(\frac{1}{5}\right)^4$

∴ Probability distribution of X is as follows:

X	0	1	2	3	4
P(X = x)	$\left(\frac{4}{5}\right)^4$	$\left(\frac{4}{5}\right)^4$	$6\left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right)^2$	$4\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)$	$\left(\frac{1}{5}\right)^4$

Exercise 8.1 | Q 1.07 | Page 141

A coin is biased so that the head is 3 times as likely to occur as tail. Find the probability distribution of number of tails in two tosses.

Solution: Let X denote the number of tails.

∴ Possible values of X are 0, 1, 2.

Let P(getting tail) = p

According to the given condition,

P(getting head) = q = 3p

As p + q = 1,

p + 3p = 1

$\therefore p = \frac{1}{4}$ and $q = \frac{3}{4}$

$\therefore P(X = 0) = P(\text{no tails}) = qq = q^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$

$P(X = 1) = P(\text{one tail}) = pq + qp = 2pq = 2\left(\frac{1}{4}\right)\left(\frac{3}{4}\right) = \frac{6}{16}$

$P(X = 2) = P(\text{two tails}) = pp = p^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{6}$

∴ Probability distribution of X is as follows:

X	0	1	2
P(X = x)	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

Exercise 8.1 | Q 1.08 | Page 141

A random variable X has the following probability distribution:

x	1	2	3	4	5	6	7
P(x)	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Determine k

Solution:

The table gives a probability distribution and therefore $\sum_{i=1}^{3} P_i = 1$

$\therefore$ k + 2k + 2k + 3k + k^2 + $2k^2$ + $7k^2$ + k = 1

$\therefore$ $10k^2 + 9k - 1 = 0$

$\therefore$ $10k^2 + 10k - k - 1 = 0$

$\therefore$ $10k(k + 1) - 1(k + 1) = 0$

$\therefore$ $(10k - 1)(k + 1) = 0$

$\therefore$ $k = \frac{1}{10}$ or $k = -1$

But k cannot be negative

$\therefore$ $k = \frac{1}{10}$.

Exercise 8.1 | Q 1.08 | Page 141

A random variable X has the following probability distribution:

x	1	2	3	4	5	6	7
P(x)	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Determine P(X < 3)

Solution: P(X < 3)

= P(X = 1 or X = 2)

= P(X = 1) + P(X = 2)

= k + 2k

= 3k = 3/10.

Exercise 8.1 | Q 1.08 | Page 141

A random variable X has the following probability distribution:

x	1	2	3	4	5	6	7
P(x)	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Determine $P(0 < X < 3)$

Solution: $P(0 < X < 3)$

$= P(X = 1 \text{ or } X = 2)$

$= P(X = 1) + P(X = 2)$

$= k + 2k$

$= 3k$

$= 3/10.$

Exercise 8.1 | Q 1.08 | Page 141

A random variable X has the following probability distribution:

x	1	2	3	4	5	6	7
P(x)	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Determine $P(X > 4)$

Solution: $P(X > 4)$

$= P(X = 5 \text{ or } X = 6 \text{ or } X = 7)$

$= P(X = 5) + P(X = 6) + P(X = 7)$

$= k^2 + 2k^2 + 7k^2 + k$

$= 10k^2 + k$

$$= 10\left(\frac{1}{10}\right)^2 + \frac{1}{10}$$

$$= \frac{1}{10} + \frac{1}{10}$$

$$= \frac{1}{5}$$

$= 0.2$

Exercise 8.1 | Q 1.09 | Page 141

Find expected value and variance of X using the following p.m.f.

X	–2	–1	0	1	2

P(x)	0.2	0.3	0.1	0.15	0.25

Solution: Expected value of X 5

$$= E(X) = \sum_{i=1}^{5} x_i . P_i(x_i)$$

= (–2) x (0.2) + (–1) x (0.3) + 0 x (0.1) + 1 x (0.15) + 2 x (0.25)

= – 0.4 – 0.3 + 0 + 0.15 + 0.5

= – 0.05

$$E(X^2) = \sum_{i=1}^{5} x_i . P_i(x_i)$$

= $(–2)^2$ x (0.2) + (–1)2 x (0.3) + 02 x (0.1) + 12 x (0.15) + 22 x (0.25)

= 0.8 + 0.3 + 0 + 0.15 + 1

= 2.25

∴ Variance of X

= Var(X)

= $E(X^2) – [E(X)]^2$

= $2.25 – (– 0.05)^2$

= 2.2475.

Exercise 8.1 | Q 1.1 | Page 141

Find expected value and variance of X, the number on the uppermost face of a fair die.

Solution: Let X denote the number on uppermost face.
∴ Possible values of X are 1, 2, 3, 4, 5, 6.
Each outcome is equiprobable.
∴ P(X = 1) = P(X = 2) = P(X = 3) = P(X = 4) = P(X = 5) = P(X = 6) = 1/6

∴ Expected value of X
= E(X)

$$= \sum_{i=1}^{6} x_i . \mathrm{P}(x_i)$$

$$= 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6}$$

$$= \frac{1}{6}(1 + 2 + 3 + 4 + 5 + 6)$$

$$= \frac{21}{6}$$

$$= \frac{7}{2}$$

$$E(X^2) = \sum_{i=1}^{6} x_i^2 . \mathrm{P}(x_i)$$

$$= 1^2 \times \frac{1}{6} + 2^2 \times \frac{1}{6} + 3^2 \times \frac{1}{6} + 4^2 \times \frac{1}{6} + 5^2 \times \frac{1}{6} + 6^2 \times \frac{1}{6}$$

$$= \frac{1}{6}\left(1^2 + 2^+ 3^2 + 4^2 + 5^2 + 6^2\right)$$

$$= \frac{(6 \times 7 \times 13)}{6 \times 6}$$

$$= \frac{91}{6}$$

∴ Variance of X

= Var(X)

$= E(X^2) - [E(X)]^2$

$$= \frac{91}{6} - \left(\frac{7}{2}\right)^2$$

$$= \frac{35}{12}.$$

Exercise 8.1 | Q 1.11 | Page 141

Find the mean of number of heads in three tosses of a fair coin.

Solution: Let X denote the number of heads.

∴ Possible values of X are 0,1, 2, 3.

Let P(getting head) = p = $\frac{1}{2}$

∴ q = 1 – p = $1 - \frac{1}{2} = \frac{1}{2}$

∴ P(X = 0) = P(no head) = qqq = q^3 = $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$

P(X = 1) = P(one head) = pqq + qpq + qqp = $3pq^2$

= $3\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^2$

= $\frac{3}{8}$

P(X = 2) = P(two heads) = ppq + pqp + qpp = $3p^2q$

= $3\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)$

$= \frac{3}{8}$

P(X = 2) = P(two heads) = ppq + pqp + qpp = $3p^2q$

$= 3\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)$

$= \frac{3}{8}$

P(X = 3) = P(three heads) = ppp

$= p^3$

$= \left(\frac{1}{2}\right)^3$

$= \frac{1}{8}$

∴ Mean number of heads

= E(X)

$= \sum_{i=1}^{4} x_i . \mathrm{P}(x_i)$

$= 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$

$= \frac{3}{2}$

= 1.5

Exercise 8.1 | Q 1.12 | Page 141

Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X.

Solution: Let X denote the number of sixes.

∴ Possible values of X are 0, 1, 2.

Let P(getting six when a die is thrown) = p = $\frac{1}{6}$

∴ q = 1 – p = $1 - \frac{1}{6} = \frac{5}{6}$

∴ P(X = 0) = P(no six) = qq = q^2 = $\left(\frac{5}{6}\right)^2 = \frac{25}{36}$

P(X = 1) = P(one six) = pq + qp = 2pq = $2\left(\frac{1}{6}\right)\left(\frac{5}{6}\right) = \frac{10}{36}$

P(X = 2) = P(two sixes) = pp = p^2 = $\left(\frac{1}{6}\right)^2 = \frac{1}{6}$

Expectation of X

= E(X)

= $\sum_{i=1}^{3} x_i . \mathrm{P}(x_i)$

= $0 \times \frac{25}{36} + 1 \times \frac{10}{36} + 2 \times \frac{1}{36}$

= $\frac{1}{36}(0 + 10 + 2)$

= $\frac{1}{3}$.

Exercise 8.1 | Q 1.13 | Page 141

Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers. Find E(X).

Solution: Two number are selected without replacement from {1, 2, 3, 4, 5, 6}.

Let S = sample space

$\therefore$ n(S) = ${}^6C_2$ = $\dfrac{6!}{2! \times 4!} = \dfrac{6 \times 5 \times 4!}{2 \times 1 \times 4!}$ = 15

Let X denote the larger of the two numbers obtained.

$\therefore$ Possible values of X are 2, 3, 4, 5, 6.

When X = 2,

one of the two numbers is 2 and remaining one is smaller than 2, i.e., 1.

$\therefore$ Remaining number can be selected in 1 way only.

$\therefore$ n(X = 2) = 1

$\therefore$ P(X = 2) = $\dfrac{1}{15}$

When X = 3,

one of the two numbers is 3 and remaining one is smaller than 3, i.e., 1 or 2.

$\therefore$ n(X = 3) = 2

$\therefore$ P(X = 3) = $\dfrac{2}{15}$

Similarly, P(X = 4) = $\dfrac{3}{15}$, P(X = 5) = $\dfrac{4}{15}$, P(X = 6) = $\dfrac{5}{15}$

$\therefore$ E(X) = $\sum_{i=1}^{5} x_i . \mathrm{P}(x_i)$

= $2 \times \dfrac{1}{15} + 3 \times \dfrac{2}{15} + 4 \times \dfrac{3}{15} + 5 \times \dfrac{4}{15} + 6 \times \dfrac{5}{15}$

$$= \frac{1}{15}(2 + 6 + 12 + 20 + 30)$$

$$= \frac{14}{3}$$

= 4.667

Exercise 8.1 | Q 1.14 | Page 141

Let X denote the sum of the numbers obtained when two fair dice are rolled. Find the variance of X.

Solution: The sample space of the experiment consists of 36 elementary events in the form of ordered pairs (x_i, y_i), where x_i = 1, 2, 3, 4, 5, 6 and y_i = 1, 2, 3, 4, 5, 6.
The random variable X, i.e., the sum of the numbers on the two dice takes the values 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 or 12.

X = x_i	**P(x_i)**	**x_iP(x_i)**	**x_i^2P(x_i)**
2	1/36	2/36	4/36
3	2/36	6/36	18/36
4	3/36	12/36	48/36
5	4/36	20/36	100/36
6	5/36	30/36	180/36
7	6/36	42/36	294/36
8	5/36	40/36	320/36
9	4/36	36/36	324/36
10	3/36	30/36	300/36
11	2/36	22/36	242/36
12	1/36	12/36	144/36
		$\sum_{i=1}^{n} x_i \mathrm{P}(x_i) = \frac{252}{36} = 7$	$\sum_{i=1}^{n} x_i^2 \mathrm{P}(x_i) = \frac{1974}{36}$

E(X²) = $\sum_{i=1}^{11} x_i^2 \mathrm{P}(x_i)$ = $\frac{1974}{36}$

Var(X) = E(X²) – [E(X)]²

= $\frac{1974}{36} - (7)^2$

= $\frac{1974}{36} - 49$

= $\frac{35}{6}$

= 5.8333

Exercise 8.1 | Q 1.15 | Page 141

A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. If X denotes the age of a randomly selected student, find the probability distribution of X. Find the mean and variance of X.

Solution: Let X denote the age of the selected student.
∴ Possible values of X are 14, 15, 16, 17, 18, 19, 20, 21.
There are 2 students of age 14, 1 student of age 15, 2 students of age 16, 3 students of age 17, 1 student of age 18, 2 students of age 19, 3 students of age 20, 1 student of age 21.

∴ P(X = 14)
= 2/15,P(X=15)=1/15,P(X=16)=2/15,P(X=17)=3/15,P(X=18)=1/15,P(X=19)=2/15,P(X=20)=3/15,P(X=21)=1/15,

∴ Mean of X

= E(X)

= $\sum_{i=1}^{8} x_i . \mathrm{P}(x_i)$

$= 14 \times \frac{2}{15} + 15 \times \frac{1}{15} + 16 \times \frac{2}{15} + 17 \times \frac{3}{15} + 18 \times \frac{1}{15} + 19 \times \frac{2}{15} + 20 \times \frac{3}{15} + 21 \times \frac{1}{15}$

$= \frac{1}{15}(28 + 15 + 32 + 51 + 18 + 38 + 60 + 21)$

$= \frac{263}{15}$

= 17.53

E(X^2) = $\sum_{i=1}^{8} x_i^2 \mathrm{P}(x_i)$

$= \frac{1}{15}\left(14^2 \times 2 + 15^2 + 16^2 \times 2 + 17^2 \times 3 + 18^2 + 19^2 \times 2 + 20^2 \times + 21^2\right)$

$= \frac{4683}{15}$

$= \frac{1561}{5}$

Variance of X = Var(X)

= E(X^2) – [E(X)]^2

$= \frac{1561}{5} - \left(\frac{263}{15}\right)^2$

$= \frac{70245 - 69169}{225}$

$= \frac{1076}{225}$

= 4.8

Exercise 8.1 | Q 1.16 | Page 141

70% of the members favour and 30% oppose a proposal in a meeting. The random variable X takes the value 0 if a member opposes the proposal and the value 1 if a member is in favour. Find E(X) and Var(X).

Solution: According to the given conditions, P(X = 0) = $\frac{30}{100} = \frac{3}{10}$ and $P(X = 1) = \frac{70}{100} = \frac{7}{10}$

∴ E(X) = $\sum_{i=1}^{2} x_i P(x_i)$ = $0 \times \frac{3}{10} + 1 \times \frac{7}{10} = \frac{7}{10}$ = 0.7

∴ E(X^2) = $\sum_{i=1}^{2} x_i^2 P(x_i)$ = $0^2 \times \frac{3}{10} + 1^2 \times \frac{7}{10} = \frac{7}{10}$ = 0.7

∴ Var(X) = E(X^2) – $[E(X)]^2$ = $\frac{7}{10} - \left(\frac{7}{10}\right)^2$

= $\frac{70 - 49}{100}$

= $\frac{21}{100}$

= 0.21

EXERCISE 8.2 [PAGES 144 - 145]

Exercise 8.2 | Q 1.01 | Page 144

Check whether the following is a p.d.f.

f(x) = $\begin{cases} x & \text{for } 0 \leq x \leq 1 \\ 2 - x & \text{for } 1 < x \leq 2. \end{cases}$

Solution: Here, f(x) ≥ 0 ∀ x ∈[0, 2]

Also, f(x) is continuous.

Now consider,

$$\int_0^2 f(x) \cdot dx = \int_0^1 f(x) \cdot dx + \int_1^2 f(x) \cdot dx$$

= $\int_0^1 x \cdot dx + \int_1^2 (2 - x) \cdot dx$

$= \frac{1}{2}\left[x^2\right]_0^1 + 2[x]_1^2 - \frac{1}{2}\left[x^2\right]_1^2$

$= \frac{1}{2}[1 - 0] + 2[2 - 1] - \frac{1}{2}[4 - 1]$

$= \frac{1}{2} + 2 - \frac{3}{2}$

= 1

∴ f(x) is p.d.f. of r.v.X

Exercise 8.2 | Q 1.01 | Page 144

Check whether the following is a p.d.f.

f(x) = 2 for 0 < x < q.

Solution: Here, f(x) > 0 ∀ x ∈[0, 1]

Now consider,

$\int_0^1 f(x) \cdot dx$

$= \int_0^1 2 \cdot dx$

$= 2[x]_0^1$

= 2[1 – 0]

= 2 ≠ 1

f(x) is not p.d.f. of r.v.X.

Exercise 8.2 | Q 1.02 | Page 144

The following is the p.d.f. of a r.v. X.

f(x) = $\begin{cases} \frac{x}{8} & \text{for } 0 < x < 4 \\ 0 & \text{otherwise.} \end{cases}$

Find P(X < 1.5),

Solution:

P(X < 1.5) = $\int_0^{1.5} f(x) \cdot dx$

= $\int_0^{1.5} \frac{x}{8} \cdot dx$

= $\frac{1}{8} \int_0^{1.5} \cdot dx$

= $\frac{1}{16} \left[x^2\right]_0^{1.5}$

= $\frac{1}{16}[2.25 - 0]$

= $\frac{2.25}{16}$.

Exercise 8.2 | Q 1.02 | Page 144

The following is the p.d.f. of a r.v. X.

f(x) = $\begin{cases} \frac{x}{8} & \text{for } 0 < x < 4 \\ 0 & \text{otherwise.} \end{cases}$

Find P(1 < X < 2),

Solution:

P(1 < X < 2) = $\int_1^2 f(x) \cdot dx$

= $\int_1^2 \frac{x}{8} \cdot dx$

= $\frac{1}{8} \int_1^2 x \cdot dx$

= $\frac{1}{16} \left[x^2\right]_1^2$

= $\frac{1}{16} [4 - 1]$

= $\frac{3}{16}$.

Exercise 8.2 | Q 1.02 | Page 144

The following is the p.d.f. of a r.v. X.

f(x) = $\begin{cases} \frac{x}{8} & \text{for } 0 < x < 4 \\ 0 & \text{otherwise.} \end{cases}$

Find P(X > 2)

Solution:

P(X > 2) = $\int_2^4 \frac{x}{8}$

= $\frac{1}{8} \int_2^4 x \cdot dx$

= $\frac{1}{16} \left[x^2\right]_2^4$

= $\frac{1}{16} [16 - 4]$

$$= \frac{12}{16}$$

$$= \frac{3}{4}.$$

Exercise 8.2 | Q 1.03 | Page 144

It is felt that error in measurement of reaction temperature (in celsius) in an experiment is a continuous r. v. with p. d. f.

$$f(x) = \begin{cases} \frac{x^3}{64} & \text{for } 0 \leq x \leq 4 \\ 0 & \text{otherwise.} \end{cases}$$

Verify whether f(x) is a p.d.f.

Solution: Here, f(x) ≥ 0, x ∈ [0, 4]

Now consider,

$$\int_0^4 f(x) \cdot dx$$

$$= \int_0^4 \frac{x^3}{64} \cdot dx$$

$$= \frac{1}{64} \int_0^4 x^3 \cdot dx$$

$$= \frac{1}{256} \left[x^4\right]_0^4$$

$$= \frac{1}{256} [256 - 0]$$

= 1

∴ f(x) is p.d.f of r.v.X.

Exercise 8.2 | Q 1.03 | Page 144

It is felt that error in measurement of reaction temperature (in celsius) in an experiment is a continuous r. v. with p. d. f.

$$f(x) = \begin{cases} \frac{x^3}{64} & \text{for } 0 \le x \le 4 \\ 0 & \text{otherwise.} \end{cases}$$

Find P(0 < X ≤ 1).

Solution:

$$P(0 < X \le 1) = \int_0^1 f(x) \cdot dx$$

$$= \int_0^1 \frac{x^3}{64} \cdot dx$$

$$= \frac{1}{64} \int_0^1 x^3 \cdot dx$$

$$= \frac{1}{256} \left[x^4\right]_0^1$$

$$= \frac{1}{256}.$$

Exercise 8.2 | Q 1.03 | Page 144

It is felt that error in measurement of reaction temperature (in celsius) in an experiment is a continuous r. v. with p. d. f.

$$f(x) = \begin{cases} \frac{x^3}{64} & \text{for } 0 \le x \le 4 \\ 0 & \text{otherwise.} \end{cases}$$

Find probability that X is between 1 and 3..

Solution: P(X is between 1 and 3) = P(1 < X < 3)

$$= \int_1^3 f(x) \cdot dx$$

$= \int_1^3 f(x) \cdot dx$

$= \frac{1}{64} \int_1^3 x^3 \cdot dx$

$= \frac{1}{256} \left[x^4\right]_1^3$

$= \frac{1}{256} [81 - 1]$

$= \frac{5}{16}.$

Exercise 8.2 | Q 1.04 | Page 144

Find k if the following function represents the p. d. f. of a r. v. X.

f(x) = $\begin{cases} kx & \text{for } 0 < x < 2 \\ 0 & \text{otherwise.} \end{cases}$

Also find $\mathrm{P}\left[\frac{1}{4} < \mathrm{X} < \frac{1}{2}\right]$

Solution: Given that f(x) represents p.d.f of r.v. X.

$\therefore \int_0^2 f(x) \cdot dx = 1$

$\therefore \int_0^2 \mathrm{k}x \cdot dx = 1$

$\therefore \mathrm{k} \int_0^2 x \cdot dx = 1$

$\therefore \frac{\mathrm{k}}{2} \left[x^2\right]_0^2 = 1$

$\therefore \frac{\mathrm{k}}{2} [4 - 0] = 1$

$\therefore \frac{k}{2}[4] = 1$

$\therefore k = \frac{1}{2}$

$$P\left[\frac{1}{4} < X < \frac{1}{2}\right] = \int_{\frac{1}{4}}^{\frac{1}{2}} f(x) \cdot dx$$

$$= \int_{\frac{1}{4}}^{\frac{1}{2}} \frac{x}{2} \cdot dx$$

$$= \frac{1}{2} \int_{\frac{1}{4}}^{\frac{1}{2}} x \cdot dx$$

$$= \frac{1}{4} \left[x^2\right]_{\frac{1}{4}}^{\frac{1}{2}}$$

$$= \frac{1}{4} \left[\frac{1}{4} - \frac{1}{16}\right]$$

$$= \frac{1}{4} \left[\frac{4-1}{16}\right]$$

$$= \frac{3}{64}.$$

Exercise 8.2 | Q 1.04 | Page 145

Find k if the following function represents the p. d. f. of a r. v. X.

$$f(x) = \begin{cases} kx(1-x) & \text{for } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Also Find

a. $P\left[\frac{1}{4} X < \frac{1}{2}\right]$

b. $P\left[X < \frac{1}{2}\right]$

Solution: Given that f(x) represents p.d.f. of r.v. X.

$$\therefore \int_0^1 f(x) \cdot dx = 1$$

$$\therefore \int_0^1 \mathrm{k}x(1-x) \cdot dx = 1$$

$$\therefore \mathrm{k}\left[\int_0^1 x \cdot dx - \int_0^1 x^2 \cdot dx\right] = 1$$

$$\therefore \mathrm{k}\left\{\frac{1}{2}\left[x^2\right]_0^1 - \frac{1}{3}\left[x^3\right]_0^1\right\} = 1$$

$$\therefore \mathrm{k}\left[\frac{1}{2} - \frac{1}{3}\right] = 1$$

$$\therefore \mathrm{k} = 6$$

a. $$\mathrm{P}\left[\frac{1}{4}\,\mathrm{X} < \frac{1}{2}\right] = \int_{\frac{1}{4}}^{\frac{1}{2}} f(x) \cdot dx$$

$$=6\int_{\frac{1}{4}}^{\frac{1}{2}} \left(x - x^2\right) \cdot dx$$

$$= 6\left[\int_{\frac{1}{4}}^{\frac{1}{2}} x \cdot dx - \int_{\frac{1}{4}}^{\frac{1}{2}} x^2 \cdot dx\right]$$

$$= 6\left[\frac{1}{2}\left[x^2\right]_{\frac{1}{\square}}^{\frac{1}{2}} - \frac{1}{3}\left[x^3\right]_{\frac{1}{4}}^{\frac{1}{2}}\right]$$

$$= 6\left\{\frac{1}{2}\left[\frac{1}{4} - \frac{1}{16}\right] - \frac{1}{3}\left[\frac{1}{8} - \frac{1}{64}\right]\right\}$$

$$= 6\left[\frac{1}{2} \times \frac{3}{16} - \frac{1}{3} \times \frac{7}{64}\right]$$

$= 6 \times \frac{11}{192} = \frac{11}{32}$

b. $\text{P}\left[\text{X} < \frac{1}{2}\right] = \int_0^{\frac{1}{2}} f(x) \cdot dx$

$= 6 \int_0^{\frac{1}{2}} (x - x^2) \cdot dx$

$= 6\left[\int_0^{\frac{1}{2}} x \cdot dx - \int_0^{\frac{1}{2}} x^2 \cdot dx\right]$

$= 6\left[\frac{1}{2}\left[x^2\right]_0^{\frac{1}{2}} - \frac{1}{3}\left[x^3\right]_0^{\frac{1}{2}}\right]$

$= 6\left\{\frac{1}{2}\left[\frac{1}{4} - 0\right] - \frac{1}{3}\left[\frac{1}{8} - 0\right]\right\}$

$= 6\left[\frac{1}{2} \times \frac{1}{4} - \frac{1}{3} \times \frac{1}{8}\right]$

$= 6 \times \frac{1}{12} = \frac{1}{2}$.

Exercise 8.2 | Q 1.05 | Page 145

Let X be the amount of time for which a book is taken out of library by a randomly selected student and suppose that X has p.d.f.

$$\text{f(x)} = \begin{cases} 0.5x & \text{for} 0 \leq x \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$

Calculate : P(X ≤ 1)

Solution:

$\text{P(X} \leq 1) = \int_0^1 f(x) \cdot dx$

$= \int_0^1 0.5x \cdot dx$

$= 0.5 \int_0^2 \cdot dx$

$= \frac{0.5}{2} \left[x^2\right]_0^1$

$= \frac{1}{4}[1 - 0]$

$= \frac{1}{4}.$

Exercise 8.2 | Q 1.05 | Page 145

Let X be the amount of time for which a book is taken out of library by a randomly selected student and suppose that X has p.d.f.

f(x) = $\begin{cases} 0.5x & \text{for} 0 \le x \le 2 \\ 0 & \text{otherwise.} \end{cases}$

Calculate : P(0.5 ≤ X ≤ 1.5)

Solution:

P(0.5 ≤ X ≤ 1.5) = $\int_{0.5}^{1.5} f(x) \cdot dx$

$= 0.5 \int_{0.5}^{1.5} x \cdot dx$

$= \frac{0.5}{2} \left[x^2\right]_{0.5}^{1.5}$

$= \frac{1}{4}[2.25 - 0.25]$

$= \frac{1}{4} \times 2$

$= \frac{1}{2}.$

Exercise 8.2 | Q 1.05 | Page 145

Let X be the amount of time for which a book is taken out of library by a randomly selected student and suppose that X has p.d.f.

f(x) = $\begin{cases} 0.5x & \text{for} 0 \leq x \leq 2 \\ 0 & \text{otherwise.} \end{cases}$

Calculate : P(X ≥ 1.5)

Solution:

$$P(X \geq 1.5) = \int_{1.5}^{2} f(x) \cdot dx$$

$$= 0.5 \int_{1.5}^{2} x \cdot dx$$

$$= \frac{0.5}{2} \left[x^2\right]_{1.5}^{2}$$

$$= \frac{1}{4} [4 - 2.25]$$

$$= \frac{1}{4} \times \frac{7}{4}$$

$$= \frac{7}{16}.$$

Exercise 8.2 | Q 1.06 | Page 145

Suppose X is the waiting time (in minutes) for a bus and its p. d. f. is given by

f(x) = $\begin{cases} \frac{1}{5} & \text{for } 0 \leq x \leq 5 \\ 0 & \text{otherwise.} \end{cases}$

Find the probability that waiting time is between 1 and 3 minutes

Solution: waiting time is between 1 and 3 minutes,

$$= P(1 < X < 3) = \int_1^3 f(x) \cdot dx$$

$$= \int_1^3 \frac{1}{5} \cdot dx$$

$$= \frac{1}{5} \int_1^3 1 \cdot dx$$

$$= \frac{1}{5} \left[x^2\right]_1^3$$

$$= \frac{1}{5} [3 - 1]$$

$$= \frac{2}{5}.$$

Exercise 8.2 | Q 1.06 | Page 145

Suppose X is the waiting time (in minutes) for a bus and its p. d. f. is given by

$$f(x) = \begin{cases} \frac{1}{5} & \text{for } 0 \leq x \leq 5 \\ 0 & \text{otherwise.} \end{cases}$$

Find the probability that waiting time is more than 4 minutes.

Solution: P(waiting time is more than 4 minutes)

$$= P(X > 4) = \int_4^5 f(x) \cdot dx$$

$$= \int_4^5 \frac{1}{5} \cdot dx$$

$$= \frac{1}{5} \int_4^5 1 \cdot dx$$

$$= \frac{1}{5} [x]_4^5$$

$= \frac{1}{5}[5-4]$

$= \frac{1}{5}.$

Exercise 8.2 | Q 1.07 | Page 145

Suppose error involved in making a certain measurement is a continuous r. v. X with p.d.f.

$$f(x) = \begin{cases} k(4-x^2) & \text{for } -2 \le x \le 2 \\ 0 & \text{otherwise.} \end{cases}$$

compute P(X > 0)

Solution: Given that f(x) represents a p.d.f. of r.v. X.

$$\therefore \int_{-2}^{2} f(x) \cdot dx = 1$$

$$\therefore \int_{-2}^{2} k(4-x^2) \cdot dx = 1$$

$$\therefore k\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = 1$$

$$\therefore k\left[\left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)\right] = 1$$

$$\therefore k\left(\frac{16}{3} + \frac{16}{3}\right) = 1$$

$$\therefore k\left(\frac{32}{3}\right) = 1$$

$$\therefore k = \frac{3}{32}$$

F(x) = $\int_{-2}^{2} f(x) \cdot dx$

= $\int_{-2}^{2} \mathrm{k}\left(4 - x^2\right) \cdot dx$

= $\frac{3}{32}\left[4x - \frac{x^3}{3}\right]_{-2}^{2}$

= $\frac{3}{32}\left[4x - \frac{x^3}{3} + 8 - \frac{8}{3}\right]$

∴ F(x) = $\frac{3}{32}\left[4x - \frac{x^3}{3} + \frac{16}{3}\right]$

P(X > 0) = 1 – P(X ≤ 0)

= 1 – F(0)

= $1 - \frac{3}{32}\left(0 - 0 + \frac{16}{3}\right)$

= $1 - \frac{1}{2}$

= $\frac{1}{2}$.

Exercise 8.2 | Q 1.07 | Page 145

Suppose error involved in making a certain measurement is a continuous r. v. X with p.d.f.

f(x) = $\begin{cases} \mathrm{k}\left(4 - x^2\right) & \text{for} - 2 \leq x \leq 2 \\ 0 & \text{otherwise.} \end{cases}$

compute P(–1 < X < 1)

Solution: Given that f(x) represents a p.d.f. of r.v. X.

$\therefore \int_{-2}^{2} f(x) \cdot dx = 1$

$\therefore \int_{-2}^{2} \mathrm{k}(4 - x^2) \cdot dx = 1$

$\therefore \mathrm{k}\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = 1$

$\therefore \mathrm{k}\left[\left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)\right] = 1$

$\therefore \mathrm{k}\left(\frac{16}{3} + \frac{16}{3}\right) = 1$

$\therefore \mathrm{k}\left(\frac{32}{3}\right) = 1$

$\therefore \mathrm{k} = \frac{3}{32}$

$\mathrm{F}(x) = \int_{-2}^{2} f(x) \cdot dx$

$= \int_{-2}^{2} \mathrm{k}(4 - x^2) \cdot dx$

$= \frac{3}{32}\left[4x - \frac{x^3}{3}\right]_{-2}^{2}$

$= \frac{3}{32}\left[4x - \frac{x^3}{3} + 8 - \frac{8}{3}\right]$

$\therefore \mathrm{F}(x) = \frac{3}{32}\left[4x - \frac{x^3}{3} + \frac{16}{3}\right]$

$P(-1 < X < 1) = F(1) - F(-1)$

$$= \frac{3}{32}\left(4 - \frac{1}{3} + \frac{16}{3}\right) - \frac{3}{32}\left(-4 + \frac{1}{3} + \frac{16}{3}\right)$$

$$= \frac{3}{32}\left(9 - \frac{5}{3}\right)$$

$$= \frac{3}{32}\left(\frac{22}{3}\right)$$

$$= \frac{11}{16}.$$

Exercise 8.2 | Q 1.07 | Page 145

Suppose error involved in making a certain measurement is a continuous r. v. X with p.d.f.

$$f(x) = \begin{cases} \mathrm{k}\left(4 - x^2\right) & \text{for } -2 \leq x \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$

compute P(X < – 0.5 or X > 0.5)

Solution: Given that f(x) represents a p.d.f. of r.v. X.

$$\therefore \int_{-2}^{2} f(x) \cdot dx = 1$$

$$\therefore \int_{-2}^{2} \mathrm{k}\left(4 - x^2\right) \cdot dx = 1$$

$$\therefore \mathrm{k}\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = 1$$

$$\therefore \mathrm{k}\left[\left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)\right] = 1$$

$$\therefore \mathrm{k}\left(\frac{16}{3} + \frac{16}{3}\right) = 1$$

$\therefore \mathrm{k}\left(\frac{32}{3}\right) = 1$

$\therefore \mathrm{k} = \frac{3}{32}$

$F(x) = \int_{-2}^{2} f(x) \cdot dx$

$= \int_{-2}^{2} \mathrm{k}\left(4 - x^2\right) \cdot dx$

$= \frac{3}{32}\left[4x - \frac{x^3}{3}\right]_{-2}^{2}$

$= \frac{3}{32}\left[4x - \frac{x^3}{3} + 8 - \frac{8}{3}\right]$

$\therefore F(x) = \frac{3}{32}\left[4x - \frac{x^3}{3} + \frac{16}{3}\right]$

P(X < – 0.5 or X > 0.5)

= 1 – P(– 0.5 ≤ X ≤ 0.5)

= 1 – [F(0.5) – F(– 0.5)]

$= 1 - \left\{\frac{3}{32}\left[4(0.5) - \frac{(0.5)^3}{3} + \frac{16}{3}\right] - \frac{3}{32}\left[4(-0.5) - \frac{(0.5)^3}{3} + \frac{16}{3}\right]\right.$

$= 1 - \frac{3}{32}\left(2 - \frac{1}{24} + \frac{16}{3} + 2 - \frac{1}{24} - \frac{16}{3}\right)$

$= 1 - \frac{3}{32}\left(4 - \frac{1}{12}\right)$

$= 1 - \frac{3}{32} \times \frac{47}{12}$

$= 1 - \frac{47}{128}$

$= \frac{81}{128}$.

Exercise 8.2 | Q 1.08 | Page 145

Following is the p. d. f. of a continuous r.v. X.

f(x) = $\begin{cases} \frac{x}{8} & \text{for } 0 < x < 4 \\ 0 & \text{otherwise.} \end{cases}$

Find expression for the c.d.f. of X.

Solution: c.d.f. of X

= F(X)

$= \int_0^x \frac{x}{8} \cdot dx$

$= \frac{1}{8} \int_0^x x \cdot dx$

$= \frac{1}{16} \left[x^2\right]_0^x$

$= \frac{1}{16} \left[x^2 - 0\right]$

$= \frac{x^2}{16}$.

Exercise 8.2 | Q 1.08 | Page 145

Following is the p. d. f. of a continuous r.v. X.

f(x) = $\begin{cases} \frac{x}{8} & \text{for } 0 < x < 4 \\ 0 & \text{otherwise.} \end{cases}$

Find F(x) at x = 0.5, 1.7 and 5.

Solution: To find

a. F(x) at x = 0.5

$\therefore$ F(0.5) = $\dfrac{(0.5)^2}{16} = \dfrac{0.25}{16} = \dfrac{1}{64}$

b. F(x) at x = 1.7

$\therefore$ F(1.7) = $\dfrac{(1.7)^2}{16} = \dfrac{2.89}{16}$

c. F(x) at x = 5

$\therefore$ F(5) = 1 ... $\begin{bmatrix} \text{f}(x) = 0 \text{ if } x \notin (0.4) \\ \therefore \text{F}(x) = 1 \text{ for } x \geq 4 \end{bmatrix}$

Exercise 8.2 | Q 1.09 | Page 145

The p.d.f. of a continuous r.v. X is

f(x) = $\begin{cases} \frac{3x^2}{8} & \text{for } 0 < x < 2 \\ 0 & \text{otherwise.} \end{cases}$

Determine the c.d.f. of X and hence find P(X < 1)

Solution:

F(x) = $\displaystyle\int_0^x (x) \cdot dx$

= $\displaystyle\int_0^x \frac{3x^2}{8} \cdot dx$

= $\displaystyle\frac{3}{8} \int_0^x x^2 \cdot dx$

$= \frac{1}{8}\left[x^3\right]_0^x$

$= \frac{x^3}{8}$

P(X < 1) = F(1)

$= \frac{(1)^3}{8}$

$= \frac{1}{8}.$

Exercise 8.2 | Q 1.09 | Page 145

The p.d.f. of a continuous r.v. X is

f(x) = $\begin{cases} \frac{3x^2}{8} & \text{for} 0 < x < 2 \\ 0 & \text{otherwise.} \end{cases}$

Determine the c.d.f. of X and hence find P(X < –2)

Solution:

F(x) = $\int_0^x (x) \cdot dx$

$= \int_0^x \frac{3x^2}{8} \cdot dx$

$= \frac{3}{8}\int_0^x x^2 \cdot dx$

$= \frac{1}{8}\left[x^3\right]_0^x$

$= \frac{x^3}{8}$

P(X < –2)

= F(–2)

= 0 $\quad \ldots \begin{bmatrix} \mathrm{f}(x) = 0 \text{ if } x \notin (0.2) \\ \therefore \mathrm{F}(x) = 1 \text{ for } x \leq 0 \end{bmatrix}$

Exercise 8.2 | Q 1.09 | Page 145

The p.d.f. of a continuous r.v. X is

f(x) = $\begin{cases} \frac{3x^2}{8} & \text{for} 0 < x < 2 \\ 0 & \text{otherwise.} \end{cases}$

Determine the c.d.f. of X and hence find P(X > 0)

Solution:

F(x) = $\int_0^x (x) \cdot dx$

= $\int_0^x \frac{3x^2}{8} \cdot dx$

= $\frac{3}{8} \int_0^x x^2 \cdot dx$

= $\frac{1}{8} \left[x^3\right]_0^x$

= $\frac{x^3}{8}$

P(X > 0) = 1 – P(X ≤ 0)

= 1 – F(0)

= 1 – 0

= 1.

Exercise 8.2 | Q 1.09 | Page 145

The p.d.f. of a continuous r.v. X is

f(x) = $\begin{cases} \frac{3x^2}{8} & \text{for} 0 < x < 2 \\ 0 & \text{otherwise.} \end{cases}$

Determine the c.d.f. of X and hence find P(1 < X < 2)

Solution:

F(x) = $\int_0^x (x) \cdot dx$

= $\int_0^x \frac{3x^2}{8} \cdot dx$

= $\frac{3}{8} \int_0^x x^2 \cdot dx$

= $\frac{1}{8} \left[x^3\right]_0^x$

= $\frac{x^3}{8}$

P(1 < X < 2) = F(2) – F(1)

= $\frac{(2)^3}{8} - \frac{(1)^3}{8}$

= $1 - \frac{1}{8}$

= $\frac{7}{8}$.

Exercise 8.2 | Q 1.1 | Page 145

If a r.v. X has p.d.f

f(x) = $\begin{cases} \frac{c}{x} & \text{for} 1 < x < 3 \\ 0 & \text{otherwise.} \end{cases}$, c $> 0.$.

Find c, E(X), and Var(X).Also Find F(x).

Solution: i. Given that f(x) represents p.d.f. of r.v. X

$$\therefore \int_1^3 f(x) \cdot dx = 1$$

$$\therefore \int_1^3 \frac{c}{x} \cdot dx = 1$$

$$\therefore c \int_1^3 \frac{1}{x} \cdot dx = 1$$

$$\therefore c[\log x]_1^3 = 1$$

$\therefore$ c [log 3 – log 1] = 1

$\therefore$ c [log 3 – 0] = 1

$$\therefore c = \frac{1}{\log 3}$$

ii. $E(X) = \int_{-\infty}^{\infty} x f(x)$

$$= \int_1^3 x f(x) \cdot dx$$

$$= \int_1^3 x \frac{c}{x} \cdot dx$$

$$= c \int_1^3 1 \cdot dx$$

$$= \frac{1}{\log 3} [x]_1^3$$

$$= \frac{1}{\log 3} [3 - 1]$$

$= \dfrac{2}{\log 3}$.

iii. $E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)$

$= \int_1^3 x^2 f(x) \cdot dx$

$= -\int_1^3 x^2 . \dfrac{c}{x} \cdot dx$

$= c \int_1^3 x \cdot dx$

$= \dfrac{1}{2\log 3} \left[x^2\right]_1^3$

$= \dfrac{1}{2\log 3}$

$= \dfrac{4}{\log 3}$

$\therefore$ Var(X) = $E(X^2) - [E(x)]^2$

$= \dfrac{4}{\log 3} - \left(\dfrac{2}{\log 3}\right)^2$

$= \dfrac{4}{(\log 3)} - \dfrac{4}{(\log 3)^2}$

$= \dfrac{4\log 3 - 4}{(\log 3)^2}$

$= \dfrac{4(\log 3 - 1)}{(\log 3)^2}$

F(x) = $\int_1^x f(x) \cdot dx$

= $\int_1^x \frac{c}{x} \cdot dx$

= c $\int_1^x \frac{1}{x} \cdot dx$

= c$[\log x]_1^x$

= c[logx – log 1]

= c log x.

EXERCISE 8.3 [PAGES 150 - 151]

Exercise 8.3 | Q 1.01 | Page 150

A die is thrown 4 times. If ‘getting an odd number’ is a success, find the probability of 2 success

Solution: Let X denote the number of odd numbers.

P(getting and odd number) = p = $\frac{3}{6} = \frac{1}{2}$

∴ q = 1 – p = $1 - \frac{1}{2} = \frac{1}{2}$

Given, n = 4

∴ X ~ B$\left(4, \frac{1}{2}\right)$

The p.m.f. of X is given by

$$P(X = x) = {}^4C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{4-x}$$

$$= {}^4C_x \left(\frac{1}{2}\right)^4, x = 0, 1,...,4$$

P(2 successes) = P(X = 2)

$$= {}^4C_2 \left(\frac{1}{2}\right)^4$$

$$= \frac{4!}{2! \times 2!} \left(\frac{1}{2}\right)^4$$

$$= \frac{4 \times 3 \times 2!}{2 \times 1 \times 2!} \left(\frac{1}{16}\right)$$

$$= \frac{3}{8}$$

= 0.375.

Exercise 8.3 | Q 1.01 | Page 150

A die is thrown 4 times. If 'getting an odd number' is a success, find the probability of at least 3 successes

Solution: Let X denote the number of odd numbers.

$$P(\text{getting and odd number}) = p = \frac{3}{6} = \frac{1}{2}$$

$$\therefore q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$$

Given, n = 4

$\therefore X \sim B\left(4, \frac{1}{2}\right)$

The p.m.f. of X is given by

$$P(X = x) = {}^4C_x\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{4-x}$$

$$= {}^4C_x\left(\frac{1}{2}\right)^4, x = 0, 1,...,4$$

P(at least 3 successes)

= P(X ≥ 3)

= 1 – P(X < 3)

= 1 – P(X = 0 or X = 1 or X = 2)

= 1 – [P(X = 0) + P(X = 1) + P(X = 2)]

$$= 1 - \left[{}^4C_0\left(\frac{1}{2}\right)^4 + {}^4C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^3 + {}^4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2\right]$$

$$= 1 - \left[\frac{1}{16} + \frac{4}{16} + \frac{61}{16}\right]$$

$$= 1 - \frac{11}{16}$$

$$= \frac{5}{16}$$

= 0.3215.

Exercise 8.3 | Q 1.01 | Page 150

A die is thrown 4 times. If 'getting an odd number' is a success, find the probability of at most 2 successes.

Solution: Let X denote the number of odd numbers.

P(getting and odd number) = p = $\frac{3}{6} = \frac{1}{2}$

$\therefore$ q = 1 – p = $1 - \frac{1}{2} = \frac{1}{2}$

Given, n = 4

$\therefore$ X ~ B$\left(4, \frac{1}{2}\right)$

The p.m.f. of X is given by

P(X = x) = ${}^4C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{4-x}$

= ${}^4C_x \left(\frac{1}{2}\right)^4$, x = 0, 1,...,4

P(at most 2 successes)

= P(X ≤ 2)

= P(X = 0 or X = 1 or X = 2)

= P(X = 0 + P(X = 1) + P(X = 2)

= 11/16

= 0.6875 ...[From (ii)]

Exercise 8.3 | Q 1.02 | Page 150

A pair of dice is thrown 3 times. If getting a doublet is considered a success, find the probability of two successes.

Solution: Let X denote the number of times of getting doublet.

If a pair of dice is thrown, then there are total 36 possible outcomes, out of which 6 [i.e. (1,1), (2, 2), …,(6,6)] are doublets.

$\therefore$ P(getting a doublet) = p = $\frac{6}{36} = \frac{1}{6}$

$\therefore$ q = 1 – p = $1 - \frac{1}{6} = \frac{5}{6}$

Given, n = 3

$\therefore$ X ~ B $\left(3, \frac{1}{6}\right)$

The p.m.f. of X is given by

$$P(X = x) = {}^3C_x \left(\frac{1}{6}\right)^x \left(\frac{5}{6}\right)^{3-x}, x = 0, 1, 2, 3$$

$\therefore$ P(to successes) = P(X = 2)

$$= {}^3C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)$$

$$= 3 \times \frac{1}{36} \times \frac{5}{6}$$

$$= \frac{5}{72}.$$

Exercise 8.3 | Q 1.03 | Page 150

There are 10% defective items in a large bulk of items. What is the probability that a sample of 4 items will include not more than one defective item?

Solution: Let X denote the number of defective items.

$$P(\text{getting defective item}) = p = \frac{10}{100} = 0.1 \quad \text{...[Given]}$$

$\therefore$ q = 1 – p = 1 – 0.1 = 0.9

Given, n = 4

$\therefore$ X ~ B(4, 0.1)

The p.m.f. of X is given by

P(X = x) = $^4C_x(0.1)^x(0.9)^{4-x}$, x = 0, 1,...,4

P(sample will include not more than one defective item)

= P(X ≤ 1) = P(X = 0 or X = 1)

= P(X = 0) + P(X = 1)

= $^4C_0(0.1)^0(0.9)^4 + {}^4C_1(0.1)^1(0.9)^3$

= $(0.9)^4$ + 4 x 0.1 x $(0.9)^3$

= $(0.9)^3$ (0.9 + 0.4)

= 1.3 x $(0.9)^3$.

Exercise 8.3 | Q 1.04 | Page 150

Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards, find the probability that all the five cards are spades.

Solution 1: Let X = number of spade cards.

p = probability of drawing a spade card from pack of 52 cards.

Since, there are 13 spade cards in the pack of 52 cards,

∴ p = $\frac{13}{52} = \frac{1}{4}$ and $q = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$

Given: n = 5

∴ X ~ B $\left(5, \frac{1}{4}\right)$

The p.m.f. of X is given by

P(X = x) = ${}^{n}C_{x}\ p^{x}\ q^{n-x}$

i.e. p(x) = ${}^{5}C_{x}\ \left(\frac{1}{4}\right)^{x}\ \left(\frac{3}{5}\right)^{5-x}$, x = 0, 1, 2,...,5

P(all five cards are spade)

= P(X = 5) = p(5) = ${}^{5}C_{5}\left(\frac{1}{4}\right)^{5}\left(\frac{3}{4}\right)^{5-5}$

$= 1\left(\frac{1}{4}\right)^{5}\left(\frac{3}{4}\right)^{0}$

$= 1 \times \frac{1}{1024} \times 1 = \frac{1}{1024}$

Hence, the probability of all the five cards are spades = $\frac{1}{1024}$

Solution 2: Let X denote the number of spades.

P(getting spade) = p = $\frac{13}{52} = \frac{1}{4}$

∴ q= 1 – p = $1 - \frac{1}{4} = \frac{3}{4}$

Given, n = 5

∴ X ~ B$\left(5, \frac{1}{4}\right)$

The p.m.f. of X is given by

P(X = x) = ${}^{5}\mathrm{C}_{x}\left(\frac{1}{4}\right)^{x}\left(\frac{3}{4}\right)^{5-x}$, x = 0, 1, ...,5

P(all five cards are spades)

= P(X = 5)

$= {}^5C_5 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^0$

$= \frac{1}{4^5}.$

Exercise 8.3 | Q 1.04 | Page 150

Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards, find the probability that only 3 cards are spades

Solution 1: Let X = number of spade cards.

p = probability of drawing a spade card from pack of 52 cards.

Since, there are 13 spade cards in the pack of 52 cards,

$\therefore$ p = $\frac{13}{52} = \frac{1}{4}$ and $\mathrm{q} = 1 - \mathrm{p} = 1 - \frac{1}{4} = \frac{3}{4}$

Given: n = 5

$\therefore$ X ~ B$\left(5, \frac{1}{4}\right)$

The p.m.f. of X is given by

P(X = x) = ${}^nC_x\, p^x\, q^{n-x}$

i.e. p(x) = ${}^5C_x \left(\frac{1}{4}\right)^x \left(\frac{3}{5}\right)^{5-x}$, x = 0, 1, 2,...,5

P(only 3 cards are spades) = P(X = 3)

= p(3) = ${}^5C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^{5-3}$

$$= \frac{5!}{3! \cdot 2!}\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2$$

$$= \frac{5 \cdot 4 \cdot 3!}{3! \cdot 2 \cdot 1} \times \frac{1}{64} \times \frac{9}{16} = \frac{45}{512}$$

Hence, the probability of only 3 cards are spades = $\frac{45}{512}$

Solution 2: Let X denote the number of spades.

P(getting spade) = p = $\frac{13}{52} = \frac{1}{4}$

∴ q= 1 – p = $1 - \frac{1}{4} = \frac{3}{4}$

Given, n = 5

∴ X ~ B$\left(5, \frac{1}{4}\right)$

The p.m.f. of X is given by

P(X = x) = ${}^5C_x\left(\frac{1}{4}\right)^x\left(\frac{3}{4}\right)^{5-x}, x = 0, 1, ..., 5$

P(only 3 cards are spades)

= P(X = 3)

$= {}^5C_3\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2$

$= \frac{5!}{3! \times 2!} \times \frac{3^2}{4^3 \times 4^2}$

$= \dfrac{5 \times 4 \times 3!}{3! \times 2 \times 1} \times \dfrac{9}{4^5}$

$= \dfrac{90}{4^5}$.

Exercise 8.3 | Q 1.04 | Page 150

Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards, find the probability that none is a spade.

Solution 1: Let X = number of spade cards.

p = probability of drawing a spade card from pack of 52 cards.

Since, there are 13 spade cards in the pack of 52 cards,

$\therefore$ p = $\dfrac{13}{52} = \dfrac{1}{4}$ and $q = 1 - p = 1 - \dfrac{1}{4} = \dfrac{3}{4}$

Given: n = 5

$\therefore$ X ~ B$\left(5, \dfrac{1}{4}\right)$

The p.m.f. of X is given by

P(X = x) = ${}^nC_x\, p^x\, q^{n-x}$

i.e. p(x) = ${}^5C_x \left(\dfrac{1}{4}\right)^x \left(\dfrac{3}{5}\right)^{5-x}$, x = 0, 1, 2,...,5

P(none of cards is spade) = P(X = 0)

= p(0) = ${}^5C_0 \left(\dfrac{1}{4}\right)^0 \left(\dfrac{3}{4}\right)^{5-0}$

$= 1 \times 1 \times \left(\dfrac{3}{4}\right)^5 = \dfrac{243}{1024}$

Hence, the probability of only 3 cards are spades = $\frac{243}{1024}$

Solution 2:

Let X = number of spade cards.

p = probability of drawing a spade card from pack of 52 cards.

Since, there are 13 spade cards in the pack of 52 cards,

$\therefore$ p = $\frac{13}{52} = \frac{1}{4}$ and $q = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$

Given: n = 5

$\therefore X \sim B\left(5, \frac{1}{4}\right)$

The p.m.f. of X is given by

P(X = x) = ${}^{n}C_{x}\, p^{x}\, q^{n-x}$

i.e. p(x) = ${}^{5}C_{x}\left(\frac{1}{4}\right)^{x}\left(\frac{3}{5}\right)^{5-x}$, x = 0, 1, 2,...,5

P(none is a spade)

= P(X = 0)

= ${}^{5}C_{0}\left(\frac{1}{4}\right)^{0}\left(\frac{3}{4}\right)^{5}$

= $\left(\frac{3}{4}\right)^{5}$.

Exercise 8.3 | Q 1.05 | Page 150

The probability that a bulb produced by a factory will fuse after 200 days of use is 0.2. Let X denote the number of bulbs (out of 5) that fuse after 200 days of use. Find the probability of X = 0

Solution: Let X denote the number of bulbs that will fuse after 200 days.

P(bulb will fuse after 200 days) = p = 0.2

∴ q = 1 – p = 1 – 0.2 = 0.8

Given, n = 5

∴ X ~ B(5, 0.2)

The p.m.f. of X is given by

$$P(X = x) = {}^5C_x(0.2)^x(0.8)^{5-x}, x = 0.1,...,5$$

$$P(X = 0) = {}^5C_0(0.2)^0(0.8)^5 = (0.8)^5$$

Exercise 8.3 | Q 1.05 | Page 150

The probability that a bulb produced by a factory will fuse after 200 days of use is 0.2. Let X denote the number of bulbs (out of 5) that fuse after 200 days of use. Find the probability of X ≤ 1

Solution: Let X denote the number of bulbs that will fuse after 200 days.

P(bulb will fuse after 200 days) = p = 0.2

∴ q = 1 – p = 1 – 0.2 = 0.8

Given, n = 5

∴ X ~ B(5, 0.2)

The p.m.f. of X is given by

$$P(X = x) = {}^5C_x(0.2)^x(0.8)^{5-x}, x = 0.1,...,5$$

$$P(X \le 1) = P(X = 0 \text{ or } X = 1)$$

$$= P(X = 0) + P(X = 1)$$

$$= (0.8)^5 + {}^5C_1(0.2)(0.8)^4 \qquad \text{...[From (i)]}$$

= (0.8)4 [0.8 + 5 x 0.2]

= $(1.8)\ (0.8)^4$.

Exercise 8.3 | Q 1.05 | Page 150

The probability that a bulb produced by a factory will fuse after 200 days of use is 0.2. Let X denote the number of bulbs (out of 5) that fuse after 200 days of use. Find the probability of X > 1

Solution: Let X denote the number of bulbs that will fuse after 200 days.

P(bulb will fuse after 200 days) = p = 0.2

∴ q = 1 – p = 1 – 0.2 = 0.8

Given, n = 5

∴ X ~ B(5, 0.2)

The p.m.f. of X is given by

$$P(X = x) = {}^5C_x(0.2)^x(0.8)^{5-x}, x = 0.1,...,5$$

P(X > 1) = 1 – p(X ≤ 1)

= 1 – (1.8) $(0.8)^4$ ...[From (ii)]

Exercise 8.3 | Q 1.05 | Page 150

The probability that a bulb produced by a factory will fuse after 200 days of use is 0.2. Let X denote the number of bulbs (out of 5) that fuse after 200 days of use. Find the probability of X ≥ 1

Solution: Let X denote the number of bulbs that will fuse after 200 days.

P(bulb will fuse after 200 days) = p = 0.2

∴ q = 1 – p = 1 – 0.2 = 0.8

Given, n = 5

∴ X ~ B(5, 0.2)

The p.m.f. of X is given by

$$P(X = x) = {}^5C_x(0.2)^x(0.8)^{5-x}, x = 0.1,...,5$$

P(X ≥ 1) = 1 – P(X < 1)

= 1 – P(X = 0)

= 1 – $(0.8)^5$.

Exercise 8.3 | Q 1.06 | Page 150

10 balls are marked with digits 0 to 9. If four balls are selected with replacement. What is the probability that none is marked 0?

Solution: Let X denote the number of times of getting a ball marked with the digit 0.

P(getting a ball marked with the digit 0) = p = 1/10

$\therefore$ q = 1 – p = $1 - \frac{1}{10} = \frac{9}{10}$

Given, n = 4

$\therefore X \sim B\left(4, \frac{1}{10}\right)$

The p.m.f. of X is given by

$P(X = x) = {}^4C_x \left(\frac{1}{10}\right)^x \left(\frac{9}{10}\right)^{4-x}, x = 0, 1, \ldots, 4$

P(none is marked with the digit 0) = P(X = 0)

$= {}^4C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^4$

$= \left(\frac{9}{10}\right)^4.$

Exercise 8.3 | Q 1.07 | Page 151

In a multiple choice test with three possible answers for each of the five questions, what is the probability of a candidate getting four or more correct answers by random choice?

Solution: Let X denote the number of correct answers.

Since only one of the three options is correct, P(getting correct answer by guessing) = p = $\frac{1}{3}$

$\therefore$ q = 1 – p = $1-\frac{1}{3}=\frac{2}{3}$

Given, n = 5

$\therefore X \sim B\left(5, \frac{1}{3}\right)$

The p.m.f. of X is given by

$$P(X = x) = {}^5C_x\left(\frac{1}{3}\right)^x\left(\frac{2}{3}\right)^{5-x}, x = 0, 1, \ldots, 5$$

P(getting 4 or more correct answers by guessing) = P(X ≥ 4) = P(X = 4 or X = 5)

= P(X = 4) + (X = 5)

$$= {}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)+{}^5C_5\left(\frac{1}{3}\right)^5$$

$$= 5\times\frac{1}{3^4}\times\frac{2}{3}+\frac{1}{3^5}$$

$$= \frac{10+1}{3^5}$$

$$= \frac{11}{3^5}$$

$$= \frac{11}{243}.$$

Exercise 8.3 | Q 1.08 | Page 151

Find the probability of throwing at most 2 sixes in 6 throws of a single die.

Solution: Let X denote the number of sixes.

P(getting a six when a die is thrown) = p = $\frac{1}{6}$

$\therefore$ q = 1 – p = $1-\frac{1}{6}=\frac{5}{6}$

Given, n = 6

$\therefore$ X ~ B$\left(6, \frac{1}{6}\right)$

The p.m.f. of X is given by

P(X = x) = $^6C_x\left(\frac{1}{6}\right)^x\left(\frac{5}{6}\right)^{6-x}, x$ = 0, 1,...,6

P(getting at most 2 sixes)

= P(X ≤ 2)

= P(X = 0 or X = 1 or X = 2)

= P(X = 0) + P(X = 1) + P(X = 2)

= $^6C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^6+{}^6C_1\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)5+{}^6C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{4}\right)^4$

= $\left(\frac{5}{6}\right)^4+6\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^5+\frac{6!}{2!\times 4!}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^4$

= $\left(\frac{5}{6}\right)^4\left[\left(\frac{5}{6}\right)^2+\left(\frac{5}{6}\right)+\frac{6\times 5\times 4!}{2\times 1\times 4}\times\left(\frac{1}{6}\right)^2\right]$

$$= \left(\frac{5}{6}\right)^4 \left[\frac{25}{6^2} + \frac{30}{6^+} \frac{15}{6^2}\right]$$

$$= \left(\frac{5}{6}\right)^4 \left(\frac{70}{36}\right)$$

$$= \left(\frac{5}{6}\right)^4 \left(\frac{35}{18}\right)$$

$$= \left(\frac{5}{4}\right)^4 \left(\frac{5 \times 7}{6 \times 3}\right)$$

$$= \frac{7}{3}\left(\frac{5}{6}\right)^5.$$

Exercise 8.3 | Q 1.09 | Page 151

Given that X ~ B(n,p), if n = 10 and p = 0.4, find E(X) and Var(X).

Solution: X ~ B(n,p)

Here, n = 10, p = 0.4

∴ q = 1 – p = 1 – 0.4 = 0.6

E(X) = np = 10 x 0.4 = 4

Var(X) = npq = 10 x 0.4 x 0.6 = 2.4

Exercise 8.3 | Q 1.09 | Page 151

Given that X ~ B(n,p), if p = 0.6 and E(X) = 6, find n and Var(X).

Solution: X ~ B(n,p)

Here, p = 0.6

∴ q = 1 – p = 1 – 0.6 = 0.4

and E(X) = 6

∴ n = 6

∴ n = 6/p - 60./6 = 60/6 = 10

∴ Var(X) = pq = 10 x 0.6 x 0.4 = 2.4

Exercise 8.3 | Q 1.09 | Page 151

Given that X ~ B(n,p), if n = 25, E(X) = 10, find p and Var (X).

Solution: X ~ B(n,p)

Here, n = 25, E(X) = 10

∴ np = 10

$$\therefore p = \frac{10}{n} = \frac{10}{25} = \frac{2}{5}$$

$$\therefore q = 1 - p = 1 - \frac{2}{5} = \frac{3}{5}$$

∴ Var(X) = npq

$$= 25 \times \frac{2}{5} \times \frac{3}{5}$$

= 2 x 3

= 6.

Exercise 8.3 | Q 1.09 | Page 151

Given that X ~ B(n,p), if n = 10, E(X) = 8, find Var(X).

Solution: X ~ B(n,p)

Here, n = 10, E(X) = 8

∴ np = 8

$$\therefore p = \frac{8}{n} = \frac{8}{10} = \frac{4}{5}$$

$$\therefore q = 1 - p = 1 - \frac{4}{5} = \frac{1}{5}$$

∴ Var(X) = npq

$$= 10 \times \frac{4}{5} \times \frac{1}{5}$$

$= 2 \times \frac{4}{5}$

$= \frac{8}{5}$

= 1.6

EXERCISE 8.4, EXERISE 8.4 [PAGE 152]

Exercise 8.4 | Q 1.01 | Page 152

If X has Poisson distribution with m = 1, then find P(X ≤ 1) given $e^{-1} = 0.3678$.

Solution: Given, m = 1 and e–1 = 0.3678

∴ X ~ P(m) = X ~ P(1)

The p.m.f. of X is given by

$P(X = x) = \frac{e^{-m} m^x}{x!}$

$\therefore P(X = x) = \frac{e^{-1} 1^x}{x!}, x = 0,1,2,.$

∴ P(X ≤ 1)

= P(X = 0 or X = 1)

= P(X = 0) + P(X = 1)

$= \frac{e^{-1} 1^0}{0!} + \frac{e^{-1} 1^1}{1!}$

$= \frac{0.3678 \times 1}{1} + \frac{0.3678 \times 1}{1}$

= 0.7356

Exercise 8.4 | Q 1.02 | Page 152

If X~P(0.5), then find P(X = 3) given $e^{-0.5} = 0.6065$.

Solution: Given, X ~ P(0.5) and $e^{-0.5} = 0.6065$

∴ m = 0.5

The p.m.f. of X is given by

$$P(X = x) = \frac{e^{-m}m^x}{x!}$$

$$\therefore P(X = x) = \frac{e^{-0.5}(0.5)^x}{x!}, x = 0, 1, 2,...$$

$$\therefore P(X = 3) = \frac{e^{-0.5}(0.5)^3}{3!}$$

$$= \frac{0.6065 \times 0.125}{3 \times 2 \times 1}$$

$= 0.0126$

Exercise 8.4 | Q 1.03 | Page 152

If X has Poisson distribution with parameter m and P(X = 2) = P(X = 3), then find P(X ≥ 2). Use $e^{-3} = 0.0497$.

Solution: The p.m.f. of X is given by

$$P(X = x) = \frac{e^{-m}m^x}{x!}$$

Given, P(X = 2) = P(X = 3)

$$\therefore \frac{e^{-m}m^2}{2!} = \frac{e^{-m}m^3}{3!}$$

$$\therefore \frac{m^2}{2} = \frac{m^3}{6}$$

$$\therefore \frac{m^2}{m^3} = \frac{6}{2}$$

∴ m = 3

∴ P(X ≥ 2) = 1 – P(X < 2)

= 1 – P(X = 0 or X = 1)

= 1 – [P(X = 0) + P(X = 1)]

$$= 1 - \left[\frac{e^{-3}(3)^0}{0!} + \frac{e^{-3}(3)^1}{1!}\right]$$

= 1 – [0.0497 + 3 x 0.0497]

= 0.8012

Exercise 8.4 | Q 1.04 | Page 152

The number of complaints which a bank manager receives per day follows a Poisson distribution with parameter m = 4. Find the probability that the manager receives only two complaints on a given day

Solution: Let X denote the number of complaints which a bank manager receives per day.

Given, m = 4 and e^{-4} = 0.0183

∴ X ~ P(m) = X ~ p(4)

The p.m.f. of X is given by

$$P(X = x) = \frac{e^{-m}m^x}{x!}$$

$$\therefore P(X = x) \ \frac{e^{-4}(4)^x}{x!}, x = 0, 1, 2, \ldots$$

P(only two complaints on a given day)

= P(X = 2)

$$= \frac{e^{-4}(4)^2}{2!}$$

$$= \frac{0.0183 \times 16}{2}$$

= 0.1464

Exercise 8.4 | Q 1.04 | Page 152

The number of complaints which a bank manager receives per day follows a Poisson distribution with parameter m = 4. Find the probability that the manager receives at most two complaints on a given day. Use $e^{-4} = 0.0183$.

Solution: Let X denote the number of complaints which a bank manager receives per day.

Given, m = 4 and $e^{-4} = 0.0183$

∴ X ~ P(m) = X ~ p(4)

The p.m.f. of X is given by

$$P(X = x) = \frac{e^{-m}m^x}{x!}$$

$$\therefore P(X = x) \frac{e^{-4}(4)^x}{x!}, x = 0, 1, 2, \ldots$$

P(at most two complaints)

= P(X ≤ 2)

= P(X = 0 or X = 1 or X = 2)

= P(X = 0) + P(X = 1) + P(X = 2)

$$= \frac{e^{-4}(4)^2}{2!} + \frac{e^{-4}(4)^1}{1!} + 0.1464 \quad \text{...[From (i)]}$$

= 0.0183 + 4 x 0.0183 + 0.1464

= 0.2379

Exercise 8.4 | Q 1.05 | Page 152

A car firm has 2 cars, which are hired out day by day. The number of cars hired on a day follows Poisson distribution with mean 1.5. Find the probability that (i) no car is used on a given day, (ii) some demand is refused on a given day, given $e^{-1.5} = 0.2231$.

Solution: Let X denote the number of cars hired on a day.

Given, m = 1.5 and $e^{-1.5} = 0.2231$

∴ X ~ P(m) = X ~ P(1.5)

The p.m.f. of X is given by

$$P(X = x) = \frac{e^{-m} \hat{} x}{x!}$$

$$\therefore P(X = x) = \frac{e^{-1.5}(1.5)^x}{x!}$$

i. P(no car is used on a given day)

= P(X = 0)

$$= \frac{e^{-1.5}(1.5)^0}{0!}$$

= 0.2231

ii. P(some demand is refused on a given day)

= P(X > 2)

= 1 – P(X ≤ 2)

= 1 – P(X = 0 or X = 1 or X = 2)

= 1 – [P(X = 0) + P(X = 1) + P(X = 2)]

$$= 1 - \left[\frac{e^{-1.5}(1.5)^0}{0!} + \frac{e^{-1.5}(1.5)^1}{1!} + \frac{e^{-1.5}(1.5)^2}{2!}\right]$$

$$= 1 - \left[0.2231 + 0.2231 \times 1.5 + \frac{0.2231 \times 2.25}{2 \times 1}\right]$$

= 1 – 0.8087

= 0.1913

Exercise 8.4 | Q 1.06 | Page 152

Defects on plywood sheet occur at random with the average of one defect per 50 sq. ft. Find the probability that such a sheet has no defect

Solution: Let X denote the number of defects on a plywood sheet.

Given, m = 1, e^{-1} = 0.3678

$\therefore$ X ~ P(m) ≅ X ~ P(1)

The p.m.f. of X is given by

$$P(X = x) = \frac{e^{-m} m^x}{x!}$$

$$\therefore P(X = x) = \frac{e^{-1}(1)^x}{x!}$$

P(no defects on a plywood)

$= P(X = 0)$

$$= \frac{e^{-1}(1)^0}{0!}$$

$$= \frac{0.3678 \times 1}{1}$$

$= 0.3678$

Exercise 8.4 | Q 1.06 | Page 152

Defects on plywood sheet occur at random with the average of one defect per 50 sq. ft. Find the probability that such a sheet has at least one defect. Use $e^{-1} = 0.3678$.

Solution: Let X denote the number of defects on a plywood sheet.

Given, m = 1, $e^{-1} = 0.3678$

$\therefore$ X ~ P(m) ≅ X ~ P(1)

The p.m.f. of X is given by

$$P(X = x) = \frac{e^{-m} m^x}{x!}$$

$$\therefore P(X = x) = \frac{e^{-1}(1)^x}{x!}$$

P (at least one defect)

= P (X ≥ 1)

= 1 – P (X = 0)

= 1 – 0.3678

= 0.6322

Exercise 8.4 | Q 1.07 | Page 152

It is known that, in a certain area of a large city, the average number of rats per bungalow is five. Assuming that the number of rats follows Poisson distribution, find the probability that a randomly selected bungalow has exactly 5 rats.

Solution: Let X denote the number of rats per bunglow.

Given, m = 5 and e–5 = 0.0067

∴ X ~ P(m) ≡ X ~ P(5)

The p.m.f. of X is given by

$$P(X = x) = \frac{e^{-m} m^x}{x!}$$

$$\therefore P(X = x) = \frac{e^{-5} \cdot (5)^x}{x!}, x = 0, 1, ..., 5$$

P(exactly five rats)

= P(X = 5)

$$= \frac{e^{-5} \cdot (5)^5}{5!}$$

$$= \frac{0.0067 \times 5^5}{5 \times 4 \times 3 \times 2 \times 1}$$

$$= \frac{0.0067 \times 625}{24}$$

$$= \frac{4.1875}{24}$$

= 0.1745

Exerise 8.4 | Q 1.07 | Page 152

It is known that, in a certain area of a large city, the average number of rats per bungalow is five. Assuming that the number of rats follows Poisson distribution, find the probability that a randomly selected bungalow has more than 5 rats.

Solution: Let X denote the number of rats per bunglow.

Given, m = 5 and e–5 = 0.0067

∴ X ~ P(m) ≡ X ~ P(5)

The p.m.f. of X is given by

$$P(X = x) = \frac{e^{-m} m^x}{x!}$$

$$\therefore P(X = x) = \frac{e^{-5} \cdot (5)^x}{x!}, x = 0, 1, ..., 5$$

P(more than five rats)

= P(X > 5)

= 1– P(X ≤ 5)

= 1– P[X = 0 or X = 1 or X = 2 or X = 3 or X = 4 or X = 5]

= 1– [P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P (X = 4) + P(X = 5)]

$$= 1 - \left[\frac{e^{-5}(5)^0}{0!} + \frac{e^{-5}(5)^1}{1!} + \frac{e^{-5}(5)^2}{2!} + \frac{e^{-5}(5)^3}{3!} + \frac{e^{-5}(5)^4}{4!} + \frac{e^{-5}(5)^5}{5!}\right]$$

$$= 1 - \left[e^{-5}\left(\frac{5^0}{0!} + \frac{5^1}{1!} + \frac{5^2}{2!} + \frac{5^3}{3!} + \frac{5^4}{4!} + \frac{5^5}{5!}\right)\right]$$

$$= 1 - \left[e^{-5}\left(\frac{1}{1} + \frac{5}{1} + \frac{25}{2} + \frac{125}{6} + \frac{625}{24} + \frac{3125}{120}\right)\right]$$

= 1 – [e^{-5} (1 + 5 + 12.5 + 20.83 + 26.04 + 26.04)]

= 1 – [0.0067 (91.417)]

= 1 – 0.6125

= 0.3875

Exercise 8.4 | Q 1.07 | Page 152

It is known that, in a certain area of a large city, the average number of rats per bungalow is five. Assuming that the number of rats follows Poisson distribution, find the probability that a randomly selected bungalow has between 5 and 7 rats, inclusive. Given e^{-5} = 0067.

Solution: Let X denote the number of rats per bunglow.

Given, m = 5 and e–5 = 0.0067

∴ X ~ P(m) ≡ X ~ P(5)

The p.m.f. of X is given by

$$P(X = x) = \frac{e^{-m}m^x}{x!}$$

$$\therefore P(X = x) = \frac{e^{-5} \cdot (5)^x}{x!}, x = 0, 1, ..., 5$$

P(between 5 and 7 rats, inclusive)

= P(5 ≤ X ≤ 7)

= P(X = 5 or X = 6 or X = 7)

= P(X = 5) + P(X = 6) + P(X = 7)

$$= \frac{e^{-5}(5)^5}{5!} + \frac{e^{-5}(5)^6}{6!} + \frac{e^{-5}(5)^7}{7!}$$

$$= \frac{e^{-5}(5)^5}{5!} + \frac{e^{-5}(5)^6}{6 \times 5!} + \frac{e^{-5}(5)^7}{7 \times 6 \times 5!}$$

$$= \frac{e^{-5} \times 5^5}{5!}\left[1 + \frac{5}{6} + \frac{5^2}{7 \times 6}\right]$$

$$= \frac{e^{-5} \times 5^5}{5 \times 4 \times 3 \times 2 \times 1}[1 + 0.833 + 0.595]$$

$$= \frac{0.0067 \times 5^4}{24}(2.428)$$

$$= \frac{0.0067 \times 625 \times 2.428}{24}$$

$$= \frac{10.1673}{24}$$

= 0.4236

MISCELLANEOUS EXERCISE 8 [PAGES 153 - 154]

Miscellaneous Exercise 8 | Q 1.01 | Page 153

Choose the correct alternative :

F(x) is c.d.f. of discrete r.v. X whose p.m.f. is given by P(x) = $k\begin{bmatrix}4\\x\end{bmatrix}$,

for x = 0, 1, 2, 3, 4 and P(x) = 0 otherwise then F(5) = ______

1. 1/16
2. 1/8
3. 1/4
4. **1**

Solution: Given,

$P(x) = k\begin{bmatrix}4\\x\end{bmatrix}$, for x = 0, 1, 2, 3, 4.

= 0, otherwise.

$\therefore$ f(5) = P(X ≤ 5)

= P(X ≤ 4) + P(X = 5)

= 1 + 0

= 1.

Miscellaneous Exercise 8 | Q 1.02 | Page 153

Choose the correct alternative :

F(x) is c.d.f. of discrete r.v. X whose distribution is

X_i	– 2	– 1	0	1	2
P_i	0.2	0.3	0.15	0.25	0.1

Then F(– 3) = ________ .

1. **0**
2. 1
3. 0.2
4. 0.15

Solution: F(x) is c.d.f. of discrete r.v. X whose distribution is

X_i	– 2	– 1	0	1	2
P_i	0.2	0.3	0.15	0.25	0.1

Then F(– 3) = **0**.

Miscellaneous Exercise 8 | Q 1.03 | Page 153

Choose the correct alternative :

X: is number obtained on upper most face when a fair die….thrown then E(X) = ________.

1. 3.0
2. **3.5**

3. 4.0
4. 4.5

Solution: X : number obtained on upper most face.

∴ Possible values of X are {1, 2, 3, 4, 5, 6}

∴ $P(X = x) = \frac{1}{6}$, for x = 1, 2, ..., 6

∴ $E(X) = \sum_{x=1}^{6} xP(x)$

$= \frac{1 \times 1}{6} + \frac{2 \times 1}{6} + \frac{3 \times 1}{6} + \frac{4 \times 1}{6} + \frac{5 \times 1}{6} + \frac{6 \times 1}{6}$

= **3.5**

Miscellaneous Exercise 8 | Q 1.04 | Page 153

Choose the correct alternative :

If p.m.f. of r.v.X is given below.

x	0	1	2
P(x)	q^2	2pq	p^2

Then Var(X) = ________

1. p^2
2. q^2
3. pq
4. **2pq**

Solution: Since given data is p.m.f. of r.v. X, we get

$q^2 + 2pq + p^2 = 1$

∴ $(q + p)^2 = 1$

∴ (q + p) = 1 ...(i)

$E(X) = \sum_{x=0}^{2} xP(x)$

$= 0 \times q^2 + 1 \times 2pq + 2 \times p^2$

$= 2pq + 2p^2$

$= 2p (q + p)$

$= 2p$ ...[From (i)]

$$E(X^2) = \sum_{x=0}^{2} x^2 P(x)$$

$= (0)^2 \times q^2 + (1)^2 \times 2pq + (2)^2 \times p^2$

$= 2pq + 4p^2$

$\therefore Var(X) = E(X^2) - [E(X)]^2$

$= 2pq + 4p^2 - (2p)^2$

$= 2pq + 4p^2 - 4p^2$

$=$ **$2pq$**.

Miscellaneous Exercise 8 | Q 1.05 | Page 153

Choose the correct alternative :

The expected value of the sum of two numbers obtained when two fair dice are rolled is ________.

1. 5
2. 6
3. **7**
4. 8

Solution: The sample space of the experiment consists of 36 elementary events in the form of ordered pairs (x_i, y_i), where $x_i = 1, 2, 3, 4, 5, 6$ and $y_i = 1, 2, 3, 4, 5, 6$.
The random variable X, i.e., the sum of the numbers on the two dice takes the values 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 or 12.

$X = x_i$	**$P(x_i)$**	**$x_iP(x_i)$**
2	1/36	2/36

3	2/36	6/36
4	3/36	12/36
5	4/36	20/36
6	5/36	30/36
7	6/36	42/36
8	5/36	40/36
9	4/36	36/36
10	3/36	30/36
11	2/36	22/36
12	1/36	12/36
		E(X) = $\sum_{i=1}^{n} x_i \mathrm{P}(x_i)$ = $\frac{252}{36}$ = **7**

Miscellaneous Exercise 8 | Q 1.06 | Page 153

Choose the correct alternative :

Given p.d.f. of a continuous r.v.X as f(x) = $x^2/3$ for –1 < x < 2 = 0 otherwise then F(1) = _______.

1. 1/9
2. **2/9**
3. 3/9
4. 4/9

Solution: F(1) = P(X ≤ 1)

$= \int_{-1}^{1} \frac{x^2}{3} \cdot dx$

$= \frac{1}{3} \int_{-1}^{1} x^2 \cdot dx$

$= \frac{1}{9} \left[x^3\right]_{-1}^{1}$

$= \frac{2}{9}$.

Miscellaneous Exercise 8 | Q 1.07 | Page 153

Choose the correct alternative :

X is r.v. with p.d.f. f(x) = $\frac{\text{k}}{\sqrt{x}}$, 0 < x < 4 = 0 otherwise then x E(X) = _______

1. 1/3
2. **4/3**
3. 2/3
4. 1

Solution:

Since E(X) = $\int_{-\infty}^{\infty} x f(x) \cdot dx$

Since f(x) is a p.d.f. of r.v.X

$\therefore \int_{0}^{4} \frac{\text{k}}{\sqrt{x}} \cdot dx = 1$

$\therefore \text{k}\left[2\sqrt{x}\right]_{0}^{4} = 1$

$\therefore 2k\left[\sqrt{x}\right]_0^4 = 1$

$\therefore 2k\left[\sqrt{4} - \sqrt{0}\right] = 1$

∴ 2k [2 – 0] = 1

∴ 4k = 1

$\therefore k = \frac{1}{4}$

$\therefore E(X) = \int_0^4 x\left(\frac{\frac{1}{4}}{\sqrt{x}}\right) \cdot dx$

$= \frac{1}{4}\int_0^4 \sqrt{x} \cdot dx$

$= \frac{1}{4}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$

$= \frac{1}{4} \times \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^4$

$= \frac{1}{6}\left[(4)^{\frac{3}{2}} - (0)^{\frac{3}{2}}\right]$

$= \frac{1}{6}[8 - 0]$

$= \frac{8}{6}$

$\therefore E(X) = \frac{4}{3}.$

Miscellaneous Exercise 8 | Q 1.08 | Page 153

Choose the correct alternative :

If $X \sim B\left(20, \frac{1}{10}\right)$ then E(X) = ______

1. **2**
2. 5
3. 4
4. 3

Solution: Given,

$$X \sim B\left(20, \frac{1}{10}\right)$$

$$\therefore \text{ Here, } n = 20 \text{ and } p = \frac{1}{10}$$

$$\therefore E(x) = np = 20 \times \frac{1}{10} = \mathbf{2}.$$

Miscellaneous Exercise 8 | Q 1.09 | Page 153

Choose the correct alternative :

If E(X) = m and Var (X) = m then X follows ______ .

1. Binomial distribution
2. **Poisson distribution**
3. Normal distribution
4. Normal distribution

Solution: If E(X) = m and Var (X) = m then X follows **Poisson distribution**.

Miscellaneous Exercise 8 | Q 1.1 | Page 154

Choose the correct alternative :

If E(X) > Var (X) then X follows _______ .

1. **Binomial distribution**
2. Poisson distribution
3. Normal distribution

4. None of the above

Solution: If E(X) > Var (X) then X follows **Binomial distribution** .

Miscellaneous Exercise 8 | Q 2.01 | Page 154

Fill in the blank :

The values of discrete r.v. are generally obtained by _______

Solution: The values of discrete r.v. are generally obtained by **Counting**.

Miscellaneous Exercise 8 | Q 2.02 | Page 154

Fill in the blank :

The value of continuous r.v. are generally obtained by _______

Solution: The value of continuous r.v. are generally obtained by **Measurement**.

Miscellaneous Exercise 8 | Q 2.03 | Page 154

Fill in the blank :

If X is discrete random variable takes the value $x_1, x_2, x_3, \ldots,$ xn then

$$\sum_{i=1}^{n} \mathrm{P}(x_i) = \_\_\_\_\_\_$$

Solution:

If X is discrete random variable takes the value $x_1, x_2, x_3, \ldots,$ xn then

$$\sum_{i=1}^{n} \mathrm{P}(x_i) = \underline{\mathbf{1}}.$$

Miscellaneous Exercise 8 | Q 2.04 | Page 154

Fill in the blank :

If F(x) is distribution function of discrete r.v.x with p.m.f. P(x) = $\frac{x-1}{3}$ for x = 0, 1 2, 3, and P(x) = 0 otherwise then F(4) = ______

Solution:

If F(x) is distribution function of discrete r.v.x with p.m.f. P(x) = $\frac{x-1}{3}$ for x = 0, 1 2, 3, and P(x) = 0 otherwise then F(4) = **<u>1</u>**.

Miscellaneous Exercise 8 | Q 2.05 | Page 154

Fill in the blank :

If F(x) is distribution function of discrete r.v.X with p.m.f. P(x) = $\mathrm{k}\begin{bmatrix}4\\x\end{bmatrix}$ for x = 0, 1, 2, 3, 4 and P(x) = 0 otherwise then F(–1) = _______

Solution:

If F(x) is distribution function of discrete r.v.X with p.m.f. P(x) = $\mathrm{k}\begin{bmatrix}4\\x\end{bmatrix}$ for x = 0, 1, 2, 3, 4 and P(x) = 0 otherwise then F(–1) = **<u>0</u>**.

Miscellaneous Exercise 8 | Q 2.06 | Page 154

Fill in the blank :

E(x) is considered to be ________ of the probability distribution of x.

Solution: E(x) is considered to be **<u>Centre of gravity</u>** of the probability distribution of x.

Miscellaneous Exercise 8 | Q 2.07 | Page 154

Fill in the blank :

If x is continuous r.v. and F(x_i) = P(X ≤ x_i) = $\int_{-\infty}^{\infty} f(x)\cdot dx$ then F(x) is called _______

Solution:

If x is continuous r.v. and F(x_i) = P(X ≤ x_i) = $\int_{-\infty}^{\infty} f(x)\cdot dx$ then F(x) is called **<u>Distribution function</u>**.

Miscellaneous Exercise 8 | Q 2.08 | Page 154

Fill in the blank :

In Binomial distribution probability of success **Remains constant / independent** from trial to trial.

Solution: In Binomial distribution probability of success _______ from trial to trial.

Miscellaneous Exercise 8 | Q 2.09 | Page 154

Fill in the blank :

In Binomial distribution if n is very large and probability success of p is very small such that np = m (constant) then _______ distribution is applied.

Solution: In Binomial distribution if n is very large and probability success of p is very small such that np = m (constant) then **Poisson** distribution is applied.

Miscellaneous Exercise 8 | Q 3.01 | Page 154

State whether the following is True or False :

If P(X = x) = $k\begin{bmatrix}4\\x\end{bmatrix}$ for x = 0, 1, 2, 3, 4 , then F(5) = $\frac{1}{4}$ when F(x) is c.d.f.

1. True
2. **False**

Solution: False

F (5) = 1**.**

Miscellaneous Exercise 8 | Q 3.02 | Page 154

State whether the following is True or False :

x	– 2	– 1	0	1	2
P(X = x)	0.2	0.3	0.15	0.25	0.1

If F(x) is c.d.f. of discrete r.v. X then F(–3) = 0

1. **True**
2. False

Solution: If F(x) is c.d.f. of discrete r.v. X then F(–3) = 0 **True**.

Miscellaneous Exercise 8 | Q 3.03 | Page 154

State whether the following is True or False :

X is the number obtained on upper most face when a die is thrown then E(X) = 3.5.

1. **True**
2. False

Solution: X : number obtained on upper most face.

∴ Possible values of X are {1, 2, 3, 4, 5, 6}

$\therefore P(X = x) = \frac{1}{6}$, for x = 1, 2, ..., 6

$$\therefore E(X) = \sum_{x=1}^{6} xP(x)$$

$$= \frac{1 \times 1}{6} + \frac{2 \times 1}{6} + \frac{3 \times 1}{6} + \frac{4 \times 1}{6} + \frac{5 \times 1}{6} + \frac{6 \times 1}{6}$$

= **3.5 is True**.

Miscellaneous Exercise 8 | Q 3.04 | Page 154

State whether the following is True or False :

If p.m.f. of discrete r.v. X is

x	0	1	2
P(X = x)	q^2	2pq	p^2

then E(x) = 2p.

1. **True**
2. False

Solution: Since given data is p.m.f. of r.v. X, we get

$q^2 + 2pq + p^2 = 1$

$\therefore (q + p)^2 = 1$

$\therefore (q + p) = 1$ ...(i)

$$E(X) = \sum_{x=0}^{2} x P(x)$$

$= 0 \times q^2 + 1 \times 2pq + 2 \times p^2$

$= 2pq + 2p^2$

$= 2p\ (q + p)$

$\therefore (q + p) = 1$ ...(i)

$$E(X) = \sum_{x=0}^{2} x P(x)$$

$= 0 \times q^2 + 1 \times 2pq + 2 \times p^2$

$= 2pq + 2p^2$

$= 2p\ (q + p)$

$= 2p$ ...[From (i)]

$$E(X^2) = \sum_{x=0}^{2} x^2 P(x)$$

$= (0)^2 \times q^2 + (1)^2 \times 2pq + (2)^2 \times p^2$

$= 2pq + 4p^2$

$\therefore Var(X) = E(X^2) - [E(X)]^2$

$= 2pq + 4p^2 - (2p)^2$

$= 2pq + 4p^2 - 4p^2$

= **2pq is <u>True</u>**.

Miscellaneous Exercise 8 | Q 3.05 | Page 154

State whether the following is True or False :

The p.m.f. of a r.v. X is P(x) = $\dfrac{2x}{n(n+1)}$, x = 1, 2, ... n

= 0 ,otherwise

Then E(x) = $\dfrac{2n+1}{3}$

1. **True**
2. False

Solution:

X	1	2	3	....	n
P(X)	$\dfrac{2}{n(n+1)}$	$\dfrac{4}{n(n+1)}$	$\dfrac{6}{n(n+1)}$	....	$\dfrac{2n}{n(n+1)}$

E(X) = $\sum x_1 \cdot p(x_i)$

=

$$1. \frac{2}{n(n+1)} + 2. \frac{4}{n(n+1)} + 3. \frac{6}{n(n+1)} + ... + n. \frac{2n}{n(n+1)}$$

$$= \frac{2n}{n(n+1)} (1+4+9+...+n^2)$$

$$= \frac{2n}{n(n+1)} (1^2+2^2+3^2+...+ n^2)$$

$$= \frac{2n}{n(n+1)} \frac{n(n+1)(2n+1)}{6}$$

= $\dfrac{2n+1}{3}$ is **True**.

Miscellaneous Exercise 8 | Q 3.06 | Page 154

State whether the following is True or False :

If f(x) = k x (1 – x) for 0 < x < 1 = 0 otherwise k = 12

1. True
2. **False**

Solution: False

Since the function represents a p.d.f. 1

$$\therefore \int_0^1 f(x) \cdot dx = 1$$

$$\therefore \int_0^1 f(x) \cdot dx = 1$$

$$\therefore \int_0^1 \mathrm{k}x(1-x) \cdot dx = 1$$

$$\therefore \mathrm{k} \int_0^1 \left(x - x^2\right) \cdot dx = 1$$

$$\therefore \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{\mathrm{k}}$$

$$\therefore \frac{1}{2} - \frac{1}{3} = \frac{1}{\mathrm{k}}$$

$$\therefore \frac{1}{6} = \frac{1}{\mathrm{k}}$$

$\therefore$ k = 6.

Miscellaneous Exercise 8 | Q 3.07 | Page 154

State whether the following is True or False :

If X ~ B(n,p) and n = 6 and P(X = 4) = P(X = 2) then p = 1/2

1. **True**
2. False

Solution: True

Given, n = 6, P(X = 4) = P(X = 2)

X ~ B(n, p) ≡ X ~ B (6, p)

The p.m.f. of X is given by

$p(X = x) = {}^{n}C_{x}p^{x}q^{n-x}$

$\therefore p(X = x) = {}^{6}C_{x}p^{x}q^{6-x}, x = 0, 1, 2, ..., 6$

Now, P(X = 4) = P(X = 2)

$\therefore {}^{6}C_{4}p^{4}q^{2} = {}^{6}C_{2}p^{2}q^{4}$

$\therefore \frac{6!}{4!2!}p^{2} = \frac{6!}{2!4!}q^{2}$

$\therefore p^{2} = q^{2}$

$\therefore p = \pm q$

$\therefore p = q$

$\therefore p = 1 - p$ ...[$\because q = 1 - p$]

$\therefore p + p = 1$

$\therefore 2p = 1$

$\therefore p = \frac{1}{2}$.

Miscellaneous Exercise 8 | Q 3.08 | Page 154

State whether the following is True or False :

If r.v. X assumes values 1, 2, 3, ……. n with equal probabilities then E(X) = n+1 / 2

1. **True**
2. False

Solution: True

X	1	2	3	....	n
P(X)	$\frac{1}{n}$	$\frac{1}{n}$	$\frac{1}{n}$		$\frac{1}{n}$

E(X) = $\sum x_i \cdot P(x_i)$

= $1\left(\frac{1}{n}\right) + 2\left(\frac{1}{n}\right) + 3\left(\frac{1}{n}\right) + ... + n\left(\frac{1}{n}\right)$

= $\frac{1}{n}(1 + 2 + 3 + ... + n)$

= $\frac{1}{n} \cdot \frac{n(n+1)}{2}$

= $\frac{n+1}{2}$.

Miscellaneous Exercise 8 | Q 3.09 | Page 154

State whether the following is True or False :

If r.v. X assumes the values 1, 2, 3, ……. 9 with equal probabilities, E(x) = 5.

1. **True**
2. False

Solution: If r.v. X assumes the values 1, 2, 3, ……. 9 with equal probabilities, E(x) = 5 is **True**.

PART I [PAGES 155 - 156]

Part I | Q 1.01 | Page 155

Solve the following problem :

Identify the random variable as discrete or continuous in each of the following. Identify its range if it is discrete.

An economist is interested in knowing the number of unemployed graduates in the town with a population of 1 lakh.

Solution: Let X = number of unemployed graduates in a town.

Here, X takes only finite values.

∴ X is a discrete r.v.

Range of X = {0, 1, 2, ..., 100000}.

Part I | Q 1.01 | Page 155

Solve the following problem :

Identify the random variable as discrete or continuous in each of the following. Identify its range if it is discrete.

Amount of syrup prescribed by a physician.

Solution: Let X = amount of syrup prescribed by a physician.

Here, X can take any positive or fractional value, i.e, X takes uncountably infinite values.

∴ X is a continuous r.v.

Part I | Q 1.01 | Page 155

Solve the following problem :

Identify the random variable as discrete or continuous in each of the following. Identify its range if it is discrete.

A person on high protein diet is interested in the weight gained in a week.

Solution: Let X = gain in weight in a week.

Here, X takes uncountably infinite values.

∴ X is a continuous r.v.

Part I | Q 1.01 | Page 155

Solve the following problem :

Identify the random variable as discrete or continuous in each of the following. Identify its range if it is discrete.

Twelve of 20 white rats available for an experiment are male. A scientist randomly selects 5 rats and counts the number of female rats among them.

Solution: There are 12 male rats and 8 female rats, i.e., finite number of female rats.

∴ X is a discrete r.v.

Range of X = {0, 1, 2, 3, 4, 5}.

Part I | Q 1.01 | Page 155

Solve the following problem :

Identify the random variable as discrete or continuous in each of the following. Identify its range if it is discrete.

A highway safety group is interested in the speed (km/hrs) of a car at a check point.

Solution: Let X = speed of the car in km/hr.

X takes uncountably infinite values.

∴ X is a continuous r.v.

Part I | Q 1.02 | Page 155

Solve the following problem :

The probability distribution of a discrete r.v. X is as follows.

X	1	2	3	4	5	6
(X = x)	k	2k	3k	4k	5k	6k

Determine the value of k.

Solution: Since P(X) is the probability distribution of X,

$$\sum_{x=1}^{6} \mathrm{P}(X = x) = 1$$

∴ k + 2k + 3k + 4k + 5k + 6k = 1

∴ 21k = 1

∴ k = $\frac{1}{21}$.

Part I | Q 1.02 | Page 155

Solve the following problem :

The probability distribution of a discrete r.v. X is as follows.

X	1	2	3	4	5	6
(X = x)	k	2k	3k	4k	5k	6k

Find P(X ≤ 4), P(2 < X < 4), P(X ≤ 3).

Solution: a. P(X ≤ 4) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)
= k + 2k + 3k + 4k

= 10k
= 10/21

b. P(2 < X < 4) = P(X = 3) = 3k = 3/21=1/7

c. P(X ≥ 3) = 1 – P(X < 3)
= 1 – [P(X = 1) + P(X = 2)]
= 1 – (k + 2k) = 1 – 3k

$$= 1 - \frac{3}{21}$$

$$= 1 - \frac{1}{7}$$

$$= \frac{6}{7}.$$

Part I | Q 1.03 | Page 155

Solve the following problem :

Following is the probability distribution of a r.v.X.

x	– 3	– 2	–1	0	1	2	3
P(X = x)	0.05	0.1	0.15	0.20	0.25	0.15	0.1

Find the probability that X is positive.

Solution: P(X is positive)

= P(X = 1 or X = 2 or X = 3)

= P(X = 1) + P(X = 2) + P(X = 3)

= 0.25 + 0.15 + 0.10 = 0.50

Part I | Q 1.03 | Page 155

Solve the following problem :

Following is the probability distribution of a r.v.X.

x	– 3	– 2	–1	0	1	2	3
P(X = x)	0.05	0.1	0.15	0.20	0.25	0.15	0.1

Find the probability that X is non-negative

Solution: P(X is non-negative)

= P(X = 0 or X = 1 or X = 2 or X = 3)

= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)

= 0.20 + 0. 25 + 0.15 + 0.10 = 0.70

Part I | Q 1.03 | Page 155

Solve the following problem :

Following is the probability distribution of a r.v.X.

x	– 3	– 2	–1	0	1	2	3
P(X = x)	0.05	0.1	0.15	0.20	0.25	0.15	0.1

Find the probability that X is odd.

Solution: P(X is odd)

= P(X = –3 or X = –1 or X = 1 or X = 3)

= P(X = –3) + P(X = – 1) + P(X = 1) + P(X = 3)

= 0.05 + 0.15 + 0.25 + 0.10 = 0.55

Part I | Q 1.03 | Page 155

Solve the following problem :

Following is the probability distribution of a r.v.X.

x	– 3	– 2	–1	0	1	2	3
P(X = x)	0.05	0.1	0.15	0.20	0.25	0.15	0.1

Find the probability that X is even.

Solution: P(X is even)

= P(X = –2 or X = 0 or X = 2)

= P(X = –2) + P(X = 0) + P(X = 2)

= 0.10 + 0.20 + 0.15 = 0.45

Part I | Q 1.04 | Page 155

Solve the following problem :

The p.m.f. of a r.v.X is given by

$$P(X = x) = \begin{cases} \binom{5}{x} \frac{1}{2^5}, & x = 0,\ \ 1,\ \ 2,\ \ 3,\ \ 4,\ \ 5. \\ 0 & \text{otherwise} \end{cases}$$

Show that P(X ≤ 2) = P(X ≤ 3).

Solution: P(X ≤ 2) = P(X = 0 or X = 1 or X = 2)

= P(X = 0) + P(X = 1) + P(X = 2)

$$= \frac{^5C_0}{2^5} + \frac{^5C_1}{2^5} + \frac{^5C_2}{2^5}$$

$$= \frac{^5C_0 + {}^5C_1 + {}^5C_2}{2^5}$$

$$= \frac{1 + 5 + 10}{2^5}$$

$$= \frac{16}{32}$$

$$= \frac{1}{2} \qquad \text{...(i)}$$

P(X ≥ 3) = P (X = 3 or X = 4 or X = 5)

= P(X = 3) + P(X = 4) + P(X = 5)

$= \frac{^5C_3}{2^5} + \frac{^5C_4}{2^5} + \frac{^5C_5}{2^5}$

$= \frac{10 + 5 + 1}{2}$

$= \frac{16}{32}$

$= \frac{1}{2}$ ...(ii)

From (i) and (ii), we get

$P(X \leq 2) = P(X \geq 3)$.

Part I | Q 1.05 | Page 155

Solve the following problem :

In the following probability distribution of a r.v.X.

x	1	2	3	4	5
P (x)	1/20	3/20	a	2a	1/20

Find a and obtain the c.d.f. of X.

Solution: Since the given table represents a p.m.f. of r.v. X,

$$\sum_{x=1}^{5} P(x) = 1$$

$\therefore P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) = 1$

$\therefore \frac{1}{20} + \frac{3}{20} + a + 2a + \frac{1}{20} = 1$

$\therefore 3a = 1 - \frac{5}{20}$

$\therefore$ 3a = $\frac{3}{4}$

$\therefore$ a = $\frac{1}{4}$

By definition of c.d.f.,

F(x) = P(X ≤ x)

F(1) = P(X ≤ 1) = P(1) = $\frac{1}{20}$

F(2) = P(X ≤ 2) = F(1) + P(2)

= $\frac{1}{20}+\frac{3}{20}=\frac{4}{20}$

F(3) = P(X ≤ 3)

= F(2) + P(3)

= $\frac{4}{20}+\mathrm{a}=\frac{4}{20}+\frac{1}{4}=\frac{4}{20}+\frac{5}{20}=\frac{9}{20}$

F(4) = P(X ≤ 4)

= F(3) + P(4)

= $\frac{4}{20}+\mathrm{a}=\frac{4}{20}+\frac{1}{4}=\frac{4}{20}+\frac{5}{20}=\frac{9}{20}$

F(4) = P(X ≤ 4)

= F(3) + P(4)

= $\frac{9}{20}+2\mathrm{a}=\frac{9}{20}+\frac{1}{2}=\frac{9}{20}+\frac{10}{20}=\frac{19}{20}$

F(5) = P(X ≤ 5)

= F(4) + P(5)

$= \frac{19}{20} + \frac{1}{20} = 1$

∴ c.d.f. of X is as follows:

x_i	1	2	3	4	5
$F(x_i)$	$\frac{1}{20}$	$\frac{4}{20}$	$\frac{9}{20}$	$\frac{19}{20}$	1

Part I | Q 1.06 | Page 155

Solve the following problem :

A fair coin is tossed 4 times. Let X denote the number of heads obtained. Identify the probability distribution of X and state the formula for p. m. f. of X.

Solution 1: When a fair coin is tossed 4 times then the sample space is

S = {HHHH,HHHT,HHTH, HTHH, THHH, HHTT, HTHT, HTTH, THHT, THTH, TTHH, HTTT, THTT, TTHT, TTTH, TTTT}

∴ n (S) = 16

X denotes the number of heads.

∴ X can take the value 0, 1, 2, 3, 4 When X = 0,

then X= {TTTT}

∴ n (X) = 1

∴ P (X=0) = $\frac{n(x)}{n(s)} = \frac{1}{16} = \frac{{}^4C_0}{16}$

When X = 1, then

X = {HTTT, THTT, TTHT, TTTH}

∴ n (X) = 4

∴ P (X=1) = $\frac{n(x)}{n(s)} = \frac{4}{16} = \frac{^4C_1}{16}$

When X = 2, then

X ={ HHTT, HTHT, HTTH, THHT, THTH, TTHH}

∴ n (X) = 6

∴ P (X=2) = $\frac{n(x)}{n(s)} = \frac{6}{16} = \frac{^4C_2}{16}$

When X = 3, then

X ={ HHHT, HHTH, HTHH, THHH}

∴ n (X) = 4

∴ P (X=3) = $\frac{n(x)}{n(s)} = \frac{4}{16} = \frac{^4C_3}{16}$

When X = 4, then

X = {HHHH}

∴ n (X) = 1

∴ P (X=4) = $\frac{n(x)}{n(s)} = \frac{1}{16} = \frac{^4C_4}{16}$

∴ the probability distribution of X is as follows :

x	0	1			
p(x)	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

Also, the formula for p.m.f. of X is

$$P(x) = \frac{^4C_x}{16}$$

x = 0,1,2,3,4

= 0 otherwise.

Solution 2: When a fair coin is tossed 4 times then the sample space is

S = {HHHH,HHHT,HHTH, HTHH, THHH, HHTT, HTHT, HTTH, THHT, THTH, TTHH, HTTT, THTT, TTHT, TTTH, TTTT}

∴ n (S) = 16

X denotes the number of heads.

∴ X can take the value 0, 1, 2, 3, 4 When X = 0,

then X= {TTTT}

∴ n (X) = 1

$$\therefore P(X=0) = \frac{n(x)}{n(s)} = \frac{1}{16} = \frac{^4C_0}{16}$$

When X = 1, then

X = {HTTT, THTT, TTHT, TTTH}

∴ n (X) = 4

$$\therefore P(X=1) = \frac{n(x)}{n(s)} = \frac{4}{16} = \frac{^4C_1}{16}$$

When X = 2, then

X ={HHTT, HTHT, HTTH, THHT, THTH, TTHH}

∴ n (X) = 6

$\therefore$ P (X=2) = $\frac{n(x)}{n(s)} = \frac{6}{16} = \frac{^4C_2}{16}$

When X = 3, then

X ={ HHHT, HHTH, HTHH, THHH}

$\therefore$ n (X) = 4

$\therefore$ P (X=3) = $\frac{n(x)}{n(s)} = \frac{4}{16} = \frac{^4C_3}{16}$

When X = 4, then

X = {HHHH}

$\therefore$ n (X) = 1

$\therefore$ P (X=4) = $\frac{n(x)}{n(s)} = \frac{1}{16} = \frac{^4C_4}{16}$

$\therefore$ the probability distribution of X is as follows :

x	0	1	2	3	4
p(x)	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

Also, the formula for p.m.f. of X is

P (x) = $\frac{^4C_x}{16}$

x = 0,1,2,3,4

= 0 otherwise.

Solution 3: A coin is tossed 4 times.
$\therefore$ n(S) = 2^4 = 16

Let X be the number of heads.
Thus, X can take values 0, 1, 2, 3, 4

When X = 0, i.e., all tails {TTTT},

$n(X) = {}^4C_0 = 1$

$\therefore P(X = 0) = \frac{1}{16}$

When X = 1, i.e., only one head.

$n(X) = {}^4C_1 = 4$

$\therefore P(X = 1) = \frac{4}{16}$

When X = 2, i.e., two heads.

$n(X) = {}^4C_2 = \frac{4!}{2!2!} = 6$

$\therefore P(X = 2) = \frac{6}{16}$

When X = 3, i.e., three heads.

n(X) = $^4C_3$ = 4

∴ P(X = 3) = $\frac{4}{16} = \frac{1}{14}$

When X = 4, i.e., all heads ≅ {HHHH},

n(X) = $^4C_4$ = 1

∴ P(X = 4) = $\frac{1}{16}$

Then,

X	0	1	2	3	4
P(X)	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

∴ Formula for p.m.f. of X is

P(X) = $\frac{\binom{4}{x}}{16}, x$ = 0, 1, 2, 3, 4

= 0, otherwise.

Part I | Q 1.07 | Page 155

Solve the following problem :

Find the probability of the number of successes in two tosses of a die, where success is defined as number greater than 4.

Solution: Success is defined as a number greater than 4 appears on at least one die.

Let X denote the number of successes.

∴ Possible values of X and 0, 1, 2.

Let, P(getting a number greater than 4) = p = $\frac{2}{6} = \frac{1}{3}$

∴ q = 1 – p = $1 - \frac{1}{3} = \frac{2}{3}$

∴ P(X = 0) = P(no success) = qq = q^2 = $\frac{4}{9}$

9 P(X = 1) = P(one success) = qp + qp = 2pq

= $\frac{2 \times 1}{3} \times \frac{2}{3}$

= $\frac{4}{9}$

P(X = 2) = P(two successes) = pp = p^2 = $\frac{1}{9}$

∴ Probability distribution of X is as follows:

X	0	1	2
P(X = x)	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

Part I | Q 1.07 | Page 155

Solve the following problem :

Find the probability of the number of successes in two tosses of a die, where success is defined as six appears in at least one toss.

Solution: Success is defined as a number six appears on at least one die.

Let X denote the number of successes.

∴ possible values of X are 0, 1, 2.

Let P(getting six) = p = $\frac{1}{6}$

$\therefore$ q = 1 – p = $1 - \frac{1}{6} = \frac{5}{6}$

$\therefore$ P(X = 0) = P(no success) = qq = q^2 = $\frac{25}{36}$

P(X =1) = P(one success) pq + qp = 2pq

= $2 \times \frac{1}{6} \times \frac{5}{6}$

= $\frac{10}{36}$

P(X = 2) = P(two successes) = pp = p^2 = $\frac{1}{36}$

$\therefore$ Probability distribution of X is as follows:

X	0	1	2
P(X = x)	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

Part I | Q 1.08 | Page 155

Solve the following problem :

A random variable X has the following probability distribution.

x	1	2	3	4	5	6	7
F (x)	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Determine k

Solution:

The table gives a probability distribution and therefore $\sum_{i=1}^{3} P_i = 1$

$\therefore$ k + 2k + 2k + 3k + k^2 + $2k^2$ + $7k^2$ + k = 1

$\therefore$ $10k^2 + 9k - 1 = 0$

$\therefore 10k^2 + 10k - k - 1 = 0$

$\therefore 10k(k + 1) - 1(k + 1) = 0$

$\therefore (10k - 1)(k + 1) = 0$

$\therefore k = \frac{1}{10}$ or $k = -1$

But k cannot be negative

$\therefore k = \frac{1}{10}$.

Part I | Q 1.08 | Page 155

Solve the following problem :

A random variable X has the following probability distribution.

x	1	2	3	4	5	6	7
F (x)	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Determine P(X < 3)

Solution: P(X < 3)

= P(X = 1 or X = 2)

= P(X = 1) + P(X = 2)

= k + 2k

= 3k = 3/10.

Part I | Q 1.08 | Page 155

Solve the following problem :

A random variable X has the following probability distribution.

x	1	2	3	4	5	6	7
F (x)	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Determine P(X > 6)

Solution: P(X > 6)

$= P(X = 7)$

$= 7k^2 + k$

$$= \frac{7}{(10)^2} + \frac{1}{10}$$

$$= \frac{17}{100}.$$

Part I | Q 1.08 | Page 155

Solve the following problem :

A random variable X has the following probability distribution.

x	1	2	3	4	5	6	7
F (x)	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Determine P(0 < X < 3)

Solution:

$P(0 < X < 3)$

$= P(X = 1 \text{ or } X = 2)$

$= P(X = 1) + P(X = 2)$

$= k + 2k$

$= 3k$

$$= \frac{3}{10}.$$

Part I | Q 1.09 | Page 155

Solve the following problem :

The following is the c.d.f of a r.v.X.

x	– 3	– 2	– 1	0	1	2	3	4
F (x)	0.1	0.3	0.5	0.65	0.75	0.85	0.9	1

Find the probability distribution of X and P(–1 ≤ X ≤ 2).

Solution: P(X = –3) = F(–3) = 0.1
P(X = –2) = F(–2) – F(–3) = 0.3 – 0.1 = 0.2
P(X = –1) = F(–1) – F(–2) = 0.5 – 0.3 = 0.2
P(X = 0) = F(0) – F(–1) = 0.65 – 0.5 = 0.15
P(X = 1) = F(1) – F(0) = 0.75 – 0.65 = 0.1
P(X = 2) = F(2) – F(1) = 0.85 – 0.75 = 0.1
P(X = 3) = F(3) – F(2) = 0.9 – 0.85 = 0.05
P(X = 4) = F(4) – F(3) = 1 – 0.9 = 0.1
∴ The probability distribution of X is as follows:

X = x	– 3	– 2	– 1	0	1	2	3	4
P(X = x)	0.1	0.2	0.2	0.15	0.1	0.1	0.05	0.1

P(–1 ≤ X ≤ 2)
= P(X = –1 or X = 0 or X = 1 or X = 2)
= P(X = –1) + P(X = 0) + P(X = 1) + P(X = 2)
= 0.2 + 0.15 + 0.1 + 0.1
= 0.55

Part I | Q 1.1 | Page 155

Solve the following problem :

Find the expected value and variance of the r. v. X if its probability distribution is as follows.

x	1	2	3
P(X = x)	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

Solution:

$$E(X) = \sum_{i=1}^{3} x_i \cdot P(x_i)$$

$$= 1\left(\frac{1}{5}\right) + 2\left(\frac{2}{5}\right) + 3\left(\frac{2}{5}\right)$$

$$= \frac{1+4+6}{5}$$

$$= \frac{11}{5}$$

$$E(X^2) = \sum_{i=1}^{3} x_i^2 \cdot P(x_i)$$

$$= 1^2\left(\frac{1}{5}\right) + 2^2\left(\frac{2}{5}\right) + 3^2\left(\frac{2}{5}\right)$$

$$= \frac{1+8+18}{5}$$

$$= \frac{27}{5}$$

$\therefore$ Var(X) = $E(X^2) - [E(X)]^2$

$$= \frac{27}{5} - \left(\frac{11}{5}\right)^2$$

$$= \frac{14}{25}.$$

Part I | Q 1.1 | Page 156

Solve the following problem :

Find the expected value and variance of the r. v. X if its probability distribution is as follows.

x	– 1	0	1
P(X = x)	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

Solution:

E(X) = $\sum_{i=1}^{3} x_i \cdot \mathrm{P}(x_i)$

= $-1\left(\frac{1}{5}\right)+0\left(\frac{2}{5}\right)+1\left(\frac{2}{5}\right)$

= $\frac{1}{5}$

E(X^2) = $\sum_{i=1}^{3} x_i^2 \mathrm{P}(x_i)$

= $(-1)^2\left(\frac{1}{5}\right)+0^2\left(\frac{2}{5}\right)+1^2\left(\frac{2}{5}\right)$

= $\frac{3}{5}$

∴ Var(X) = E(X^2) – $[E(X)]^2$

= $\frac{3}{5}-\left(\frac{1}{5}\right)^2$

= $\frac{14}{25}$ ..

Part I | Q 1.1 | Page 156

Solve the following problem :

Find the expected value and variance of the r. v. X if its probability distribution is as follows.

x	1	2	3	...	n
P(X = x)	$\frac{1}{n}$	$\frac{1}{n}$	$\frac{1}{n}$	...	$\frac{1}{n}$

Solution:

$$E(X) = \sum_{i=1}^{n} x_i \cdot P(x_i)$$

$$= 1\left(\frac{1}{n}\right) + 2\left(\frac{1}{n}\right) + 3\left(\frac{1}{n}\right) + \ldots + n\left(\frac{1}{n}\right)$$

$$= \frac{1 + 2 + 3 + \ldots. + n}{n}$$

$$= \frac{1}{n} \times \frac{n(n+1)}{2}$$

$$= \frac{n+1}{2}$$

$$E(X^2) = \sum_{i=1}^{n} x_i^2 P(x_i)$$

$$= 1^2\left(\frac{1}{n}\right) + 2^2\left(\frac{1}{n}\right) + 3^2\left(\frac{1}{n}\right) + \ldots + n^2\left(\frac{1}{n}\right)$$

$$= \frac{1^2 + 2^2 + 3^2 + \ldots. + n^2}{n}$$

$$= \frac{1}{n} \times \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{(n+1)(2n+1)}{6}$$

$\therefore$ Var(X) = E(X^2) – [E(X)]^2

$$= \frac{(n+1)(2n+1)}{2 \times 3} - \frac{(n+1)^2}{4}$$

$$= \frac{n+1}{2}\left(\frac{2n+1}{3} - \frac{n+1}{2}\right)$$

$$= \frac{n+1}{2}\left(\frac{4n+2-3n-3}{6}\right)$$

$$= \frac{(n+1)(n-1)}{12}$$

$$= \frac{n^2-1}{12}.$$

Part I | Q 1.1 | Page 156

Solve the following problem :

Find the expected value and variance of the r. v. X if its probability distribution is as follows.

X	0	1	2	3	4	5
P(X = x)	$\frac{1}{32}$	$\frac{5}{32}$	$\frac{10}{32}$	$\frac{10}{32}$	$\frac{5}{32}$	$\frac{1}{32}$

Solution:

E(X) = $\sum_{i=1}^{6} x_i \cdot \mathrm{P}(x_i)$

= $0\left(\frac{1}{32}\right)+1\left(\frac{5}{32}\right)+2\left(\frac{10}{32}\right)+3\left(\frac{10}{32}\right)+4\left(\frac{5}{32}\right)+5\left(\frac{1}{32}\right)$

= $\frac{0+5+20+30+20+5}{32}$

= $\frac{80}{32}$

= $\frac{5}{2}$

= 2.5

E(X^2) = $\sum_{i=1}^{6} x_i^2 \mathrm{P}(x_i)$

= $0^2\left(\frac{1}{32}\right)+1^2\left(\frac{5}{32}\right)+2^2\left(\frac{10}{32}\right)+3^2\left(\frac{10}{32}\right)+4^2\left(\frac{5}{32}\right)+5^2\left(\frac{1}{32}\right)$

= $\frac{0+5+40+90+80+25}{32}$

= $\frac{240}{32}$

= $\frac{15}{2}$

∴ Var(X) = E(X^2) – $[E(X)]^2$

$$= \frac{15}{2} - \left(\frac{5}{2}\right)^2$$

$$= \frac{5}{4}$$

= 1.25

Part I | Q 1.11 | Page 156

Solve the following problem :

A player tosses two coins. He wins ₹ 10 if 2 heads appear, ₹ 5 if 1 head appears, and ₹ 2 if no head appears. Find the expected value and variance of winning amount.

Solution 1: When a coin is tossed twice, the sample space is

S = {HH, HT, TH, HH}

Let X denote the amount he wins.

Then X takes values 10, 5, 2.

P(X = 10) = P(2 heads appear)= $\frac{1}{4}$

P(X = 5) = P(1 head appears) = $\frac{2}{4} = \frac{1}{2}$

P(X = 2) = P(no head appears) = $\frac{1}{4}$

We construct the following table to calculate the mean and the variance of X :

x_i	P (x_i)	xi P (x_i)	x_i^2 P (x_i)
10	$\frac{1}{4}$	$\frac{5}{2}$	25
5	$\frac{1}{2}$	$\frac{5}{2}$	25/2
2	$\frac{1}{4}$	$\frac{1}{2}$*	1
Total	**1**	**5.5**	**38.5**

From the table $\sum x_i P(x_i) = 5.5$, $\sum x_i 2 \cdot P(x_i) = 38.5$

$E(X) = \sum x_i P(x_i) = 5.5$

$Var (X) = x_i{}^2 P(x_i) - [E(X)]^2$

$= 38.5 - (5.5)^2$

$= 38.5 - 30.25 = 8.25$

∴ Hence, expected winning amount ₹ 5.5 and variance of winning amount ₹8.25

Solution 2: Let X denote the winning amount.

∴ Possible values of X are 2, 5, 10

Let P(getting head) = $p = \frac{1}{2}$

∴ $q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

∴ P(X = 2) = P(no head) = qq= q^2 = $\frac{1}{4}$

P(X = 5) = P(one head) = pq + qp = 2pq

= $2 \times \frac{1}{2} \times \frac{1}{2}$

= $\frac{2}{4}$

P(X = 10) = P(two heads) = pp = p^2 = $\frac{1}{4}$

∴ The probability distribution of X is as follows:

X = x	2	5	10
P(X = x)	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

Expected winning amount

= E(X) = $\sum_{i=1}^{3} x_i \mathrm{P}(x_i)$

= $2 \times \frac{1}{4} + 5 \times \frac{2}{4} + 10 \times \frac{1}{4}$

= $\frac{2+10+10}{4}$

= $\frac{22}{4}$

= ₹ 5.5

E(X^2) = $\sum_{i=1}^{3} x_i^2 \mathrm{P}(x_i)$

$= (2)^2 \times \frac{1}{4} + (5)^2 \times \frac{2}{4} + (10)^2 \times \frac{1}{4}$

$= \frac{4 + 50 + 100}{4}$

$= \frac{154}{4}$

= 38.5

Variance of winning amount

= Var(X) = E(X^2) – [E(X)]^2

= 38.5 – $(5.5)^2$

= 38.5 – 30.25

= ₹ 8.25

Part I | Q 1.12 | Page 156

Solve the following problem :

Let the p. m. f. of the r. v. X be

$$P(x) = \begin{cases} \frac{3-x}{10}, & \text{for } x = -1,\ 0,\ 1,\ 2. \\ 0 & \text{otherwise.} \end{cases}$$

Calculate E(X) and Var(X).

Solution:

E(X) = $\sum_{i=1}^{4} x_i P(x_i)$

$= -1 \times \left(\frac{3-(-1)}{10}\right) + 0 \times \left(\frac{3-0}{10}\right) + 1 \times \left(\frac{3-1}{10}\right) 2 \times \left(\frac{3-2}{10}\right)$

$= \frac{-4+0+2+2}{10} = 0$

E(X^2) = $\sum_{i=1}^{4} x_i^2 \mathrm{P}(x_i)$

= $(-1)^2 \times \frac{4}{10} + (0)^2 \times \frac{3}{10} + (1)^2 \times \frac{2}{10} + (2)^2 \times \frac{1}{10}$

= $\frac{4+0+2+4}{10}$ = 1

Var(X) = E(X^2) – $[E(X)]^2$

= 1 – $(0)^2$

=1.

Part I | Q 1.13 | Page 156

Solve the following problem :

Suppose error involved in making a certain measurement is a continuous r.v.X with p.d.f.

f(x) = $\begin{cases} \mathrm{k}\left(4-x^2\right) & \text{for } -2 \leq x \leq 2 \\ 0 & \text{otherwise.} \end{cases}$

Compute P(X > 0)

Solution: Given that f(x) represents a p.d.f. of r.v. X.

$\therefore \int_{-2}^{2} f(x) \cdot dx$ = 1

$\therefore \int_{-2}^{2} \mathrm{k}\left(4-x^2\right) \cdot dx$ = 1

$\therefore \mathrm{k}\left[4x - \frac{x^3}{3}\right]_{-2}^{2}$ = 1

$\therefore \mathrm{k}\left[\left(8-\frac{8}{3}\right)-\left(-8+\frac{8}{3}\right)\right]$ = 1

$\therefore \text{k}\left(\frac{16}{3} + \frac{16}{3}\right) = 1$

$\therefore \text{k}\left(\frac{32}{3}\right) = 1$

$\therefore \text{k} = \frac{3}{32}$

$\text{F(x)} = \int_{-2}^{2} f(x) \cdot dx$

$= \int_{-2}^{2} \text{k}\left(4 - x^2\right) \cdot dx$

$= \frac{3}{32}\left[4x - \frac{x^3}{3}\right]_{-2}^{2}$

$= \frac{3}{32}\left[4x - \frac{x^3}{3} + 8 - \frac{8}{3}\right]$

$\therefore \text{F(x)} = \frac{3}{32}\left[4x - \frac{x^3}{3} + \frac{16}{3}\right]$

P(X > 0) = 1 – P(X ≤ 0)

= 1 – F(0)

$= 1 - \frac{3}{32}\left(0 - 0 + \frac{16}{3}\right)$

$= 1 - \frac{1}{2}$

$= \frac{1}{2}.$

Part I | Q 1.13 | Page 156

Solve the following problem :

Suppose error involved in making a certain measurement is a continuous r.v.X with p.d.f.

$$f(x) = \begin{cases} k\left(4 - x^2\right) & \text{for } -2 \le x \le 2 \\ 0 & \text{otherwise.} \end{cases}$$

Compute P(–1 < X < 1)

Solution: Given that f(x) represents a p.d.f. of r.v. X.

$$\therefore \int_{-2}^{2} f(x) \cdot dx = 1$$

$$\therefore \int_{-2}^{2} k\left(4 - x^2\right) \cdot dx = 1$$

$$\therefore k\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = 1$$

$$\therefore k\left[\left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)\right] = 1$$

$$\therefore k\left(\frac{16}{3} + \frac{16}{3}\right) = 1$$

$$\therefore k\left(\frac{32}{3}\right) = 1$$

$$\therefore k = \frac{3}{32}$$

$$F(x) = \int_{-2}^{2} f(x) \cdot dx$$

$$= \int_{-2}^{2} k\left(4 - x^2\right) \cdot dx$$

$= \frac{3}{32}\left[4x - \frac{x^3}{3}\right]_{-2}^{2}$

$= \frac{3}{32}\left[4x - \frac{x^3}{3} + 8 - \frac{8}{3}\right]$

$\therefore$ F(x) = $\frac{3}{32}\left[4x - \frac{x^3}{3} + \frac{16}{3}\right]$

P(–1 < X < 1) = F(1) – F(–1)

$= \frac{3}{32}\left(4 - \frac{1}{3} + \frac{16}{3}\right) - \frac{3}{32}\left(-4 + \frac{1}{3} + \frac{16}{3}\right)$

$= \frac{3}{32}\left(9 - \frac{5}{3}\right)$

$= \frac{3}{32}\left(\frac{22}{3}\right)$

$= \frac{11}{16}$.

Part I | Q 1.13 | Page 156

Solve the following problem :

Suppose error involved in making a certain measurement is a continuous r.v.X with p.d.f.

f(x) = $\begin{cases} \mathrm{k}\left(4 - x^2\right) & \text{for } -2 \le x \le 2 \\ 0 & \text{otherwise.} \end{cases}$

Compute P(X < – 0.5 or X > 0.5)

Solution: Given that f(x) represents a p.d.f. of r.v. X.

$\therefore \int_{-2}^{2} f(x) \cdot dx = 1$

$\therefore \int_{-2}^{2} \mathrm{k}\left(4 - x^2\right) \cdot dx = 1$

$$\therefore \mathrm{k}\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = 1$$

$$\therefore \mathrm{k}\left[\left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)\right] = 1$$

$$\therefore \mathrm{k}\left(\frac{16}{3} + \frac{16}{3}\right) = 1$$

$$\therefore \mathrm{k}\left(\frac{32}{3}\right) = 1$$

$$\therefore \mathrm{k} = \frac{3}{32}$$

$$\mathrm{F(x)} = \int_{-2}^{2} f(x) \cdot dx$$

$$= \int_{-2}^{2} \mathrm{k}\left(4 - x^2\right) \cdot dx$$

$$= \frac{3}{32}\left[4x - \frac{x^3}{3}\right]_{-2}^{2}$$

$$= \frac{3}{32}\left[4x - \frac{x^3}{3} + 8 - \frac{8}{3}\right]$$

$$\therefore \mathrm{F(x)} = \frac{3}{32}\left[4x - \frac{x^3}{3} + \frac{16}{3}\right]$$

P(X < – 0.5 or X > 0.5)

= 1 – P(– 0.5 ≤ X ≤ 0.5)

= 1 – [F(0.5) – F(– 0.5)]

$$= 1 - \left\{\frac{3}{32}\left[4(0.5) - \frac{(0.5)^3}{3} + \frac{16}{3}\right] - \frac{3}{32}\left[4(-0.5) - \frac{(0.5)^3}{3} + \frac{16}{3}\right]\right.$$

$$= 1 - \frac{3}{32}\left(2 - \frac{1}{24} + \frac{16}{3} + 2 - \frac{1}{24} - \frac{16}{3}\right)$$

$$= 1 - \frac{3}{32}\left(4 - \frac{1}{12}\right)$$

$$= 1 - \frac{3}{32} \times \frac{47}{12}$$

$$= 1 - \frac{47}{128}$$

$$= \frac{81}{128}.$$

Part I | Q 1.14 | Page 156

Solve the following problem :

The p.d.f. of the r.v. X is given by

$$f(x) = \begin{cases} \frac{1}{2a}, & \text{for } 0 < \ x = 2a. \\ 0 & \text{otherwise.} \end{cases}$$

Show that $P\left(X < \frac{a}{2}\right) = P\left(X > \frac{3a}{2}\right)$

Solution:

$$P\left(X < \frac{a}{2}\right) = \int_0^{\frac{a}{2}} f(x) \cdot dx$$

$$= \int_0^{\frac{a}{2}} \frac{1}{2a} \cdot dx$$

$$= \frac{1}{2a}[x]_0^{\frac{a}{2}}$$

$= \frac{1}{2a}\left(\frac{a}{2} - 0\right)$

$= \frac{1}{4}$ ...(i)

$$P\left(X > \frac{3a}{2}\right) = \int_{\frac{3a}{2}}^{2a} f(x) \cdot dx$$

$= \int_{\frac{3a}{2}}^{2a} \frac{1}{2a} \cdot dx$

$= \frac{1}{2a}[x]_{\frac{3a}{2}}^{2a}$

$= \frac{1}{2a}\left[2a - \frac{3a}{2}\right]$

$= \frac{1}{2a} \times \frac{a}{2}$

$= \frac{1}{4}$ ...(ii)

From (i) and (ii), we get

$$P\left(X < \frac{a}{2}\right) = P\left(X > \frac{3a}{2}\right)$$

Part I | Q 1.15 | Page 156

Solve the following problem :

Determine k if the p.d.f. of the r.v. is

f(x) = $\begin{cases} ke^{-\theta x} & \text{for } 0 \le x < \infty \\ 0 & \text{otherwise.} \end{cases}$

Find $P\left(X > \frac{1}{\theta}\right)$ and determine M is P(0 < X < M) = $\frac{1}{2}$

Solution: Since f(x) is the p.d.f. of X.

$$\therefore \int_0^\infty f(x) \cdot dx = 1$$

$$\therefore \int_0^\infty \left(\mathrm{k}\mathrm{e}^{-\theta x}\right) \cdot dx = 1$$

$$\therefore \mathrm{k} \cdot \left[\frac{\mathrm{e}^{-\theta x}}{-\theta}\right]_0^\infty = 1$$

$$\therefore -\frac{\mathrm{k}}{\theta}\left[\mathrm{e}^{-\theta x})\right]_0^\infty = 1$$

$$\therefore -\frac{\mathrm{K}}{\theta}\left[\frac{1}{\mathrm{e}^{\theta x}}\right]_0^\infty = 1$$

$$\therefore -\frac{\mathrm{k}}{\theta}\left(\frac{1}{\infty} - \frac{1}{1}\right) = 1$$

$$\therefore -\frac{\mathrm{k}}{\theta}(0 - 1) = 1$$

$$\therefore \mathrm{k} = \theta$$

$$\mathrm{F(x)} = \mathrm{k}\int_0^x \mathrm{e}^{-\theta x} \cdot dx$$

$$= \theta \int_0^x e^{-\theta x} \cdot dx \qquad \ldots[\because \mathrm{k} = \theta]$$

$$= \theta\left[\frac{\mathrm{e}^{-\theta x}}{-\theta}\right]_0^x$$

$$= -\left[\mathrm{e}^{-\theta x}\right]_0^x$$

$$= -\left(\mathrm{e}^{-\theta x} - 1\right)$$

$= 1 - \frac{1}{e^{\theta x}}$ ...(i)

$\therefore P\left(X > \frac{1}{\theta}\right) = 1 - P\left(X \leq \frac{1}{\theta}\right)$

$= 1 - F\left(\frac{1}{\theta}\right)$

$= 1 - \left[1 - \frac{1}{e^{\theta\left(\frac{1}{\theta}\right)}}\right]$ ...[From (i)]

$= \frac{1}{e}$

Given that, P(0 < X < M) = $\frac{1}{2}$

$\therefore$ F(M) – F(0) = $\frac{1}{2}$

$\therefore 1 - \frac{1}{e^{\theta M}} - 0 = \frac{1}{2}$ ...[From (i)]

$\therefore \frac{1}{2} = \frac{1}{e^{\theta M}}$

$\therefore e^{\theta M} = 2$

$\therefore \theta M = \log 2$

$\therefore M = \frac{1}{\theta}\log 2.$

Part I | Q 1.16 | Page 156

Solve the following problem :

The p.d.f. of the r.v. X is given by

$$f(x) = \begin{cases} \frac{k}{\sqrt{x}} & \text{for } 0 < x < 4. \\ 0 & \text{otherwise.} \end{cases}$$

Determine k, the c.d.f. of X, and hence find P(X ≤ 2) and P(X ≥ 1).

Solution: Given that f(x) represents p.d.f. of r.v. X.

$$\therefore \int_0^4 \frac{k}{\sqrt{x}} \cdot dx = 1$$

$$\therefore k \cdot \left[2\sqrt{x}\right]_0^4 = 1$$

$$\therefore 2k\left[\sqrt{x}\right]_0^4 = 1$$

$$\therefore 2k\,(2 - 0) = 1$$

$$\therefore k = \frac{1}{4}$$

By definition of c.d.f.,

$$F(x) = P(X \leq x)$$

$$= \int_0^4 \frac{k}{\sqrt{x}} \cdot dx$$

$$= k\left[2\sqrt{x}\right]_0^x$$

$$= \frac{1}{4}\left[2\sqrt{x}\right]_0^x$$

$$= \frac{\sqrt{x}}{2}$$

$$P(X \leq 2) = F\,(2) = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$P(X \geq 1) = 1 - P(X < 1)$$

$$= 1 - F(1)$$

$$= 1 - \frac{\sqrt{1}}{2}$$

$$= \frac{1}{2}.$$

Part I | Q 1.17 | Page 156

Solve the following problem :

Let X denote the reaction temperature in Celsius of a certain chemical process. Let X have the p. d. f.

$$f(x) = \begin{cases} \frac{1}{10} & \text{for } -5 \leq x < 5 \\ 0 & \text{otherwise.} \end{cases}$$

Compute P(X < 0).

Solution:

$$P(X < 0) = \int_{-5}^{0} f(x) \cdot dx$$

$$= \frac{1}{10} \int_{-5}^{0} 1 \cdot dx$$

$$= \frac{1}{10} [x]_{-5}^{0}$$

$$= \frac{1}{10} (0 + 5)$$

$$= \frac{1}{2}.$$

PART II [PAGES 156 - 157]

Part II | Q 1.01 | Page 156

Solve the following problem :

Let X ~ B(10,0.2). Find P(X = 1)

Solution: X ~ B(10, 0.2) …[Given]

∴ n = 10, p = 0.2

∴ q = 1 – p = 1 – 0.2 = 0.8

The p.m.f. of X is given by

P(X = x) = $^{10}C_x(0.2)^x(0.8)^{10-x}, x$ = 0, 1,...,10

P(X = 1) = $^{10}C_1(0.2)^1(0.8)^9$

= 10 (0.2) $(0.8)^9$

= 2 $(0.8)^9$.

Part II | Q 1.01 | Page 156

Solve the following problem :

Let X ~ B(10,0.2). Find P(X ≥1)

Solution: X ~ B(10, 0.2) …[Given]

∴ n = 10, p = 0.2

∴ q = 1 – p = 1 – 0.2 = 0.8

The p.m.f. of X is given by

P(X = x) = $^{10}C_x(0.2)^x(0.8)^{10-x}, x$ = 0, 1,...,10

P(X ≥1) = 1 – P(X < 1) = 1 – P(X = 0)

= $1 - {}^{10}C_0(0.2)^0(0.8)^{10}$

= 1 – $(0.8)^{10}$.

Part II | Q 1.01 | Page 156

Solve the following problem :

Let X ~ B(10,0.2). Find P(X ≤ 8).

Solution: X ~ B(10, 0.2) …[Given]

∴ n = 10, p = 0.2

∴ q = 1 – p = 1 – 0.2 = 0.8

The p.m.f. of X is given by

P(X = x) = $^{10}C_x(0.2)^x(0.8)^{10-x}$, x = 0, 1,...,10

P(X ≤ 8) = 1 – P (X > 8) = 1 – P(X = 9 or X = 10)

= 1 – [P(X = 9) + P(X = 10)]

$= 1 - \left[{}^{10}C_9(0.2)^9(0.8) + {}^{10}C_{10}(0.2)^{10}(.8)^0\right]$

= 1 – $(0.2)^9$ [10 x 0.8 + 0.2]

= 1 – (8.2) $(0.2)^9$.

Part II | Q 1.02 | Page 156

Solve the following problem :

Let X~B(n,p) If n = 10 and E(X)= 5, find p and Var(X).

Solution: Let X ~ B(n, p)

n = 10, E(X) = 5 ...[Given]

But E(X) = np

∴ 5 = 10(p)

∴ p = $\frac{1}{2}$

∴ q = 1 – p = $1 - \frac{1}{2} = \frac{1}{2}$

Var (X) = npq

= $10 \times \frac{1}{2} \times \frac{1}{2}$

= $1 \times \frac{1}{4}$

= 2.5

Part II | Q 1.02 | Page 156

Solve the following problem :

Let X~B(n,p) If E(X) = 5 and Var(X) = 2.5, find n and p.

Solution: Let X ~ B(n, p)

E(X) = 5 and Var (X) = 2.5 ...[Given]

But E(X) = np = 5 and

Var (X) = npq = 2.5

$\therefore$ 5(q) = 2.5

$\therefore q = \frac{2.5}{5} = \frac{1}{2}$

$\therefore p = 1 - q = 1 - \frac{1}{2} = \frac{1}{2}$

Now, np = 5

$\therefore n\left(\frac{1}{2}\right) = 5$

$\therefore$ n = 10.

Part II | Q 1.03 | Page 156

Solve the following problem :

If a fair coin is tossed 4 times, find the probability that it shows 3 heads

Solution: Let X denote the number of heads.

$P(\text{getting head}) = p = \frac{1}{2}$

$\therefore q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

Given, n = 4

$\therefore X \sim B\left(4, \frac{1}{2}\right)$

The p.m.f of X is given by

$P(X = x) = {}^4C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{4-x}, x = 0, 1, \ldots, 4$

$P(\text{getting 3 heads}) = P(X = 3) = {}^4C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)$

$= \frac{4}{2^4}$

$= \frac{1}{4}.$

Part II | Q 1.03 | Page 156

Solve the following problem :

If a fair coin is tossed 4 times, find the probability that it shows head in the first 2 tosses and tail in last 2 tosses.

Solution: Let X denote the number of heads.

$P(\text{getting head}) = p = \frac{1}{2}$

$\therefore q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

Given, n = 4

$\therefore X \sim B\left(4, \frac{1}{2}\right)$

The p.m.f of X is given by

P(X = x) = $^4C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{4-x}$, x = 0, 1,....,4

P(getting head in the first 2 tosses and tail in last 2 tosses.)

= $\left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2$

= $\frac{1}{2^4}$

= $\frac{1}{16}$.

Part II | Q 1.04 | Page 156

Solve the following problem :

The probability that a bomb will hit the target is 0.8. Find the probability that, out of 5 bombs, exactly 2 will miss the target.

Solution: Let X denote the number of bombs hitting the target.

P(bomb hits the target) = p = 0.8

∴ q = 1 – p = 1 – 0.8 = 0.2

Given, n = 5

∴ X ~ B(5, 0.8)

The p.m.f. of X is given by

P(X = x) $^5C_x(0.8)^x(0.2)^{5-x}$, x = 0, 1,...,5

∴ P(exactly two will miss the target)

= P(exactly three will hit the target)

= P(X = 3)

= $^5C_3(0.8)^3(0.2)^2$

$$= \frac{5!}{3! \times 2!} \left(\frac{4}{5}\right)^3 \left(\frac{1}{5}\right)^2$$

$$= \frac{5 \times 4 \times 3!}{2 \times 1 \times 3!} \times \left(\frac{4^3}{5^5}\right)$$

$$= 10\left(\frac{4^3}{5^5}\right).$$

Part II | Q 1.05 | Page 156

Solve the following problem :

The probability that a lamp in the classroom will burn is 0.3. 3 lamps are fitted in the classroom. The classroom is unusable if the number of lamps burning in it is less than 2. Find the probability that the classroom cannot be used on a random occasion.

Solution: Let X denote the number of burning lamps.

P(lamp will burn) = p = 0.3

$\therefore$ q = 1 – p = 1 – 0.3 = 0.7

Given, n = 3

$\therefore$ X ~ B(3, 0.3)

$\therefore$ The p.m.f. of X is given by

$P(X = x) = {}^3C_x (0.3)^x (0.7)^{3-x}, x = 0, 1, 2, 3$

P(Classroom cannot be used)

= P(X < 2)

= P(X = 0 or X = 1)

= P(X = 0) + P(X = 1)

$= {}^3C_0 (0.3)^0 (0.7)^3 + {}^3C_1 (0.3)(0.7)^2$

$= (0.7)^3 + 3 \times (0.3) \times (0.7)^2$

= 0.784

Part II | Q 1.06 | Page 156

Solve the following problem :

A large chain retailer purchases an electric device from the manufacturer. The manufacturer indicates that the defective rate of the device is 10%. The inspector of the retailer randomly selects 4 items from a shipment. Find the probability that the inspector finds at most one defective item in the 4 selected items.

Solution: Let X be the number of defective items.

P(an item is defective) = p = $\frac{10}{100}$ = 0.1

∴ q = 1 – p = 1 – 0.1 = 0.9

Given, n = 4

∴ X ~ B(4, 0.1)

The p.m.f. of X is given by

$P(X = x) = {}^4C_x(0.1)^x(0.9)^{4-x}, x = 0, 1, ..., 4$

P(at most one defective item)

= P(X ≤ 1)

= P(X = 0 or = 1)

= P(X = 0) + P(X = 1)

$= {}^4C_0(0.1)^0(0.9)^4 + {}^4C_1(0.1)(0.9)^3$

$= (0.9)^4 + 4(0.1)(0.9)^3$

= 0.9477

Part II | Q 1.07 | Page 157

Solve the following problem :

The probability that a component will survive a check test is 0.6. Find the probability that exactly 2 of the next 4 components tested survive.

Solution: Let X denote the number of tested components survive.

P(component survive the check test) = p = 0.6 ...[Given]

∴ q = 1 – p = 1 – 0.6 = 0.4

Given, n = 4

∴ X ~ B (4, 0.6)

The p.m.f. of X is given by

P(X = x) = $^4C_x(0.6)^x(0.4)^{4-x}, x$ = 0, 1, ...4

∴ P(exactly 2 components tested survive)

= P(X = 2)

= $^4C_2(0.6)^2(0.4)^2$

= 6(0.36) (0.16)

= 0.3456

Part II | Q 1.08 | Page 157

Solve the following problem :

An examination consists of 5 multiple choice questions, in each of which the candidate has to decide which one of 4 suggested answers is correct. A completely unprepared student guesses each answer completely randomly. Find the probability that this student gets 4 or more correct answers.

Solution: Let X denote the number of correct questions. Since only one of 4 suggested answers is correct

P(answer is correct) = p = $\frac{1}{4}$

∴ q = 1 – p = $1 - \frac{1}{4} = \frac{3}{4}$

Given, n = 5

$\therefore$ X ~ B$\left(5, \frac{1}{4}\right)$

The p.m.f. of X is given by

P(X = x) = ${}^5C_x \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{5-x}$, x = 0, 1,...,5

P(student gets 4 or more correct answers)

= P(X ≥ 4)

= P(X = 4 or X = 5)

= P(X = 4) + P(X = 5)

$= {}^5C_4 \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right) + {}^5C_5 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^0$

$= 5 \times \frac{1}{4^4} \times \frac{3}{4} + \frac{1}{4^5}$

$= \frac{15 + 1}{4^5}$

= 1/64

Part II | Q 1.09 | Page 157

Solve the following problem :

The probability that a machine will produce all bolts in a production run within the specification is 0.9. A sample of 3 machines is taken at random. Calculate the probability that all machines will produce all bolts in a production run within the specification.

Solution: Let X denote the number of machines that run within specification.

P(a machine will produce all bolts in production run within the specification) = p = 0.9

$\therefore$ q = 1 – p = 1 – 0.9 = 0.1

Given, n = 3

$\therefore$ X ~ B(3, 0.9)

The p.m.f. of X is given by

P(X = x) = $^3C_x(0.9)^x(0.1)^{3-x}$, x = 0, 1, 2, 3.

P(all machines will produce all bolts in a production run within the specification)

= P(X = 3)

= $^3C_3(0.9)^3(0.1)^0$

= 0.729

Part II | Q 1.1 | Page 157

Solve the following problem :

A computer installation has 3 terminals. The probability that any one terminal requires attention during a week is 0.1, independent of other terminals. Find the probabilities that 0

Solution: Let X denote the number of terminals that will require attention.

P(a terminal that will require attention during a week) = p = 0.1

$\therefore$ q = 1 – p = 1 – 0.1 = 0.9

Given, n = 3

$\therefore$ X ~ B(3, 0.1)

The p.m.f. of X is given by

P(X = x) = $^3C_x(0.1)^x(0.9)^{3-x}$, x = 0, 1, 2, 3.

P(0 Terminal requires attention during a week)

= P(X = 0)

= $^3C_0(0.1)^0(0.9)^3$

= $(0.9)^3$.

Part II | Q 1.1 | Page 157

Solve the following problem :

A computer installation has 3 terminals. The probability that any one terminal requires attention during a week is 0.1, independent of other terminals. Find the probabilities that 1 terminal requires attention during a week.

Solution: Let X denote the number of terminals that will require attention.

P(a terminal that will require attention during a week) = p = 0.1

∴ q = 1 – p = 1 – 0.1 = 0.9

Given, n = 3

∴ X ~ B(3, 0.1)

The p.m.f. of X is given by

P(X = x) = $^3C_x(0.1)^x(0.9)^{3-x}, x$ = 0, 1, 2, 3.

P(1 terminal requires attention during a week)

= P(X = 1)

= $^3C_1(0.1)^1(0.9)^2$

= 3 x 0.1 x $(0.9)^2$

= 0.3 x $0.9)^2$.

Part II | Q 1.11 | Page 157

Solve the following problem :

In a large school, 80% of the students like mathematics. A visitor asks each of 4 students, selected at random, whether they like mathematics.

Calculate the probabilities of obtaining an answer yes from all of the selected students.

Solution: Let X denote the number of pupils who like mathematics.

P(pupils like mathematics) = p = $\frac{8}{100} = \frac{4}{5}$ ...[Given]

q = 1 – p = $1 - \frac{4}{5} = \frac{1}{5}$

Given, n = 4

$\therefore X \sim B\left(4, \frac{4}{5}\right)$

The p.m.f. of X is given by

P(X = x) = ${}^4Cx \frac{4}{5^x} \left(\frac{1}{5}\right)^{4-x}$, x = 0, 1, ...,4

P(obtaining an answer yes form all of the selected students)

= P(X = 4)

= ${}^4C_4 \left(\frac{4}{5}\right)^4 \left(\frac{1}{5}\right)^0$

= $\frac{4^4}{5^4}$

= $\frac{256}{5^4}$.

Part II | Q 1.11 | Page 157

Solve the following problem :

In a large school, 80% of the students like mathematics. A visitor asks each of 4 students, selected at random, whether they like mathematics.

Find the probability that the visitor obtains the answer yes from at least 3 students.

Solution: Let X denote the number of pupils who like mathematics.

P(pupils like mathematics) = p = $\frac{8}{100} = \frac{4}{5}$ ...[Given]

q = 1 – p = $1 - \frac{4}{5} = \frac{1}{5}$

Given, n = 4

∴ X ~ B $\left(4, \frac{4}{5}\right)$

The p.m.f. of X is given by

P(X = x) = ${}^4\mathrm{C}x \frac{4}{5^x} \left(\frac{1}{5}\right)^{4-x}, x$ = 0, 1, ...,4

P(the visitor obtains the answer yes from at least 3 students)

= P(X ≥ 3)

= P(X = 3 or X = 4)

= P(X = 3) + P(X = 4)

= ${}^4\mathrm{C}_3 \left(\frac{4}{5}\right)^3 \left(\frac{1}{5}\right)^1 + \frac{256}{5^4}$ ...[From (i)]

= $\frac{4^4}{5^4} + \frac{256}{5^4}$

= $\frac{256}{5^4} + \frac{256}{5^4}$

= $\frac{512}{5^4}$.

Part II | Q 1.12 | Page 157

Solve the following problem :

It is observed that it rains on 10 days out of 30 days. Find the probability that it rains on exactly 3 days of a week.

Solution: Let X denote the number of days it rains in a week.

P(it rains) = p = $\frac{10}{30} = \frac{1}{3}$

∴ q = 1 – p = $1 - \frac{1}{3} = \frac{2}{3}$

Given, n = 7

∴ X ~ B$\left(7, \frac{1}{3}\right)$

The p.m.f. of X is given by

P(X = x) = $^7C_x \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{7-x}$, x = 0, 1, ...,7

P(it rains on exactly 3 days of a week)

= P(X = 3)

= $^7C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^4$

= $\frac{7!}{3! \times 4!} \times \frac{1}{3^3} \times \frac{2^4}{3^4}$

= $\frac{7 \times 6 \times 5 \times 4!}{3 \times 2 \times 1 \times 4!} \times \frac{16}{3!}$

= $\frac{35 \times 16}{3^7}$

= $\frac{560}{3^7}$.

Part II | Q 1.12 | Page 157

Solve the following problem :

It is observed that it rains on 10 days out of 30 days. Find the probability that it rains on at most 2 days of a week.

Solution: Let X denote the number of days it rains in a week.

P(it rains) = p = $\frac{10}{30} = \frac{1}{3}$

$\therefore$ q = 1 – p = $1 - \frac{1}{3} = \frac{2}{3}$

Given, n = 7

$\therefore$ X ~ B$\left(7, \frac{1}{3}\right)$

The p.m.f. of X is given by

P(X = x) = ${}^{7}C_{x}\left(\frac{1}{3}\right)^{x}\left(\frac{2}{3}\right)^{7-x}$, x = 0, 1, ...,7

P(it rains at most 2 days of a week)

= P(X ≤ 2)

= P(X = 0 or X = 1 or X = 2)

= P(X = 0) + P(X = 1) + P(X = 2)

= ${}^{7}C_{0}\left(\frac{1}{3}\right)^{0}\left(\frac{2}{3}\right)^{7} + {}^{7}C\_1\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^{6} + {}^{7}C_{2}\left(\frac{1}{3}\right)^{2}\left(\frac{2}{3}\right)^{5}$

= $\frac{2^5}{3^7}\left(4 + 14 + \frac{7 \times 6 \times 5!}{2 \times 1 \times 5!}\right)$

= $\frac{2^5}{3^7}(18 + 21)$

= $39\left(\frac{2^5}{3^7}\right)$.

Part II | Q 1.13 | Page 157

Solve the following problem :

If X follows Poisson distribution such that P(X = 1) = 0.4 and P(X = 2) = 0.2, find variance of X.

Solution: Given, P[X = 1] = 0.4, P[X = 2] = 0.2,

$e^{-1} = 0.3678$

For Poisson distribution,

$X \sim P(m)$

The p.m.f. of X is given by

$$P[X = x] = \frac{e^{-m}m^{x}}{x!}$$

Now,

$$P[X = 1] = \frac{e^{-m}m^{1}}{1!} = me^{-m}$$

$\therefore 0.4 = me^{-m}$ ...(i)

$$P[X = 2] = \frac{e^{-m}m^{2}}{2!} = \frac{m^{2}e^{-m}}{2}$$

$$\therefore 0.2 = \frac{m^{2}e^{-m}}{2}$$

$\therefore 0.4 = m^{2} e^{-m}$ ...(ii)

$$\therefore \frac{0.4}{0.4} = \frac{m^{2}e^{-m}}{me^{-m}}$$ ...[From (i) and (ii)]

$\therefore$ m=1

$\therefore X \sim P(1)$

$\therefore$ Var (X) = m = 1.

Part II | Q 1.14 | Page 157

Solve the following problem :

If X follows Poisson distribution with parameter m such that

$$\frac{P(X = x + 1)}{P(X = x)} = \frac{2}{x + 1}$$

Find mean and variance of X.

Solution:

Given, X ~ P(m) and $\frac{P(X = x + 1)}{P(X = x)} = \frac{2}{x + 1}$

The p.m.f. of X is given by

P(X = x) = $\frac{e^{-m} m^x}{x!}$

∴ According to the given condition, we get

$$\frac{\frac{e^{-m} m^{x+1}}{(x+1)!}}{\frac{e^{-m} m^x}{x!}} = \frac{2}{x + 1}$$

$$\therefore \frac{e^{-m} \times m^x \times m}{(x + 1) \times x!} \times \frac{x!}{e^{-m} \times m^x} = \frac{2}{x + 1}$$

$$\therefore \frac{m}{x + 1} = \frac{2}{x + 1}$$

∴ m = 2

∴ Mean = Variance = m = 2.