CHAPTER

5.4

THE Z-TRANSFORM

Statement forQ.1-12:

Determine the z-transform and choose correct option.

1. $x[n] = \delta[n-k],\ k > 0$

(A) z^{k}, $z > 0$

(B) z^{-k}, $z > 0$

(C) z^{k}, $z \neq 0$

(D) z^{-k}, $z \neq 0$

2. $x[n] = \delta[n+k],\ k > 0$

(A) z^{-k}, $z \neq 0$

(B) z^{k}, $z \neq 0$

(C) z^{-k}, all z

(D) z^{k}, all z

3. $x[n] = u[n]$

(A) $\frac{1}{1-z^{-1}}$, $|z| > 1$

(B) $\frac{1}{1-z^{-1}}$, $|z| < 1$

(C) $\frac{z}{1-z^{-1}}$, $|z| < 1$

(D) $\frac{z}{1-z^{-1}}$, $|z| > 1$

4. $x[n] = \left(\frac{1}{4}\right)^{n}(u[n] - u[n-5])$

(A) $\frac{z^5 - 0.25^5}{z^4(z-0.25)}$, $z > 0.25$

(B) $\frac{z^5 - 0.25^5}{z^4(z-0.25)}$, $z > 0$

(C) $\frac{z^5 - 0.25^5}{z^3(z-0.25)}$, $z < 0.25$

(D) $\frac{z^5 - 0.25^5}{z^4(z-0.25)}$, all z

5. $x[n] = \left(\frac{1}{4}\right)^{4} u[-n]$

(A) $\frac{4z}{4z-1}$, $|z| > \frac{1}{4}$

(B) $\frac{4z}{4z-1}$, $|z| < \frac{1}{4}$

(C) $\frac{1}{1-4z}$, $|z| > \frac{1}{4}$

(D) $\frac{1}{1-4z}$, $|z| < \frac{1}{4}$

6. $x[n] = 3^{n} u[-n-1]$

(A) $\frac{z}{3-z}$, $|z| > 3$

(B) $\frac{z}{3-z}$, $|z| < 3$

(C) $\frac{3}{3-z}$, $|z| > 3$

(D) $\frac{3}{3-z}$, $|z| < 3$

7. $x[n] = \left(\frac{2}{3}\right)^{|n|}$

(A) $\frac{-5z}{(2z-3)(3z-2)}$, $-\frac{3}{2} < z < -\frac{2}{3}$

(B) $\frac{-5z}{(2z-3)(3z-2)}$, $\frac{2}{3} < |z| < \frac{3}{2}$

(C) $\frac{5z}{(2z-3)(3z-2)}$, $\frac{2}{3} < |z| < \frac{2}{3}$

(D) $\frac{5z}{(2z-3)(3z-2)}$, $-\frac{3}{2} < z < -\frac{2}{3}$

8. $x[n] = \left(\frac{1}{2}\right)^{n} u[n] + \left(\frac{1}{4}\right)^{n} u[-n-1]$

(A) $\frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-\frac{1}{4}z^{-1}}$, $\frac{1}{4} < |z| < \frac{1}{2}$

(B) $\frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{1-\frac{1}{4}z^{-1}}$, $\frac{1}{4} < |z| < \frac{1}{2}$

(C) $\frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-\frac{1}{4}z^{-1}}$, $|z| > \frac{1}{2}$

(D) None of the above

33. $X(z) = \frac{1}{1-4z^{-2}}, \quad |z| < \frac{1}{4}$

(A) $-\sum_{k=0}^{\infty} 2^{2(k+1)} \delta[-n-2(k+1)]$

(B) $-\sum_{k=0}^{\infty} 2^{2(k+1)} \delta[-n+2(k+1)]$

(C) $-\sum_{k=0}^{\infty} 2^{2(k+1)} \delta[n+2(k+1)]$

(D) $-\sum_{k=0}^{\infty} 2^{2(k+1)} \delta[n-2(k+1)]$

34. $X(z) = \ln(1+z^{-1}), \quad |z| > 0$

(A) $\frac{(-1)^{k-1}}{k} \delta[n-1]$ (B) $\frac{(-1)^{k-1}}{k} \delta[n+1]$

(C) $\frac{(-1)^{k}}{k} \delta[n-1]$ (D) $\frac{(-1)^{k}}{k} \delta[n+1]$

35. If z-transform is given by

$$X(z) = \cos(z^{-3}), \ |z| > 0,$$

The value of $x[12]$ is

(A) $-\frac{1}{24}$ (B) $\frac{1}{24}$

(C) $-\frac{1}{6}$ (D) $\frac{1}{6}$

36. $X(z)$ of a system is specified by a pole zero pattern in fig. P.5.4.36.

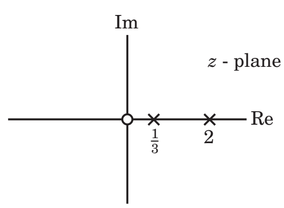

Fig. P.5.4.36

Consider three different solution of $x[n]$

$$x_1[n] = \left[2^n - \left(\frac{1}{3}\right)^n\right] u[n]$$

$$x_2[n] = -2^n u[n-1] - \frac{1}{3^n} u[n]$$

$$x_3[n] = -2^n u[n-1] + \frac{1}{3^n} u[-n-1]$$

Correct solution is

(A) $x_1[n]$ (B) $x_2[n]$

(C) $x_3[n]$ (D) All three

37. Consider three different signal

$$x_1[n] = \left[2^n - \left(\frac{1}{2}\right)^n\right] u[n]$$

$$x_2[n] = -2^n u[-n-1] + \frac{1}{2^n} u[-n-1]$$

$$x_3[n] = -2^n u[-n-1] - \frac{1}{2^n} u[n]$$

Fig. P.5.4.37 shows the three different region. Choose the correct option for the ROC of signal

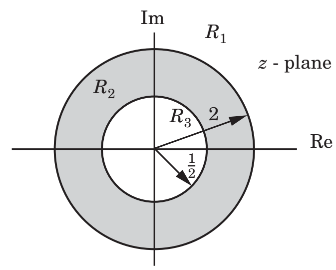

Fig. P5.4.37

	R_1	R_2	R_3
(A)	$x_1[n]$	$x_2[n]$	$x_3[n]$
(B)	$x_2[n]$	$x_3[n]$	$x_1[n]$
(C)	$x_1[n]$	$x_3[n]$	$x_2[n]$
(D)	$x_3[n]$	$x_2[n]$	$x_1[n]$

38. Given

$$X(z) = \frac{1 + \frac{7}{6} z^{-1}}{\left(1 - \frac{1}{2} z^{-1}\right)\left(1 + \frac{1}{3} z^{-1}\right)}$$

For three different ROC consider there different solution of signal $x[n]$:

(a) $|z| > \frac{1}{2}, \ x[n] = \left[\frac{1}{2^{n-1}} - \left(\frac{-1}{3}\right)^n\right] u[n]$

(b) $|z| < \frac{1}{3}, \ x[n] = \left[\frac{-1}{2^{n-1}} + \left(\frac{-1}{3}\right)^n\right] u[-n+1]$

(c) $\frac{1}{3} < |z| < \frac{1}{2}, \ x[n] = -\frac{1}{2^{n-1}} u[-n-1] - \left(\frac{-1}{3}\right)^n u[n]$

Correct solutions are

(A) (a) and (b) (B) (a) and (c)

(C) (b) and (c) (D) (a), (b), (c)

39. $X(z)$ has poles at $z=1/2$ and $z=-1$. If $x[1]=1$ $x[-1]=1$, and the ROC includes the point $z=3/4$. The time signal $x[n]$ is

(A) $\frac{1}{2^{n-1}}u[n]-(-1)^n u[-n-1]$

(B) $\frac{1}{2^n}u[n]-(-1)^n u[-n-1]$

(C) $\frac{1}{2^{n-1}}u[n]+u[-n+1]$

(D) $\frac{1}{2^n}u[n]+u[-n+1]$

40. $x[n]$ is right-sided, $X(z)$ has a signal pole, and $x[0]=2$, $x[2]=1/2$. $x[n]$ is

(A) $\frac{u[-n]}{2^{n-1}}$ (B) $\frac{u[n]}{2^{n-1}}$

(C) $\frac{u[-n]}{2^{n+1}}$ (D) $\frac{u[-n]}{2^{n+1}}$

41. The z-transform function of a stable system is given as

$$H(z)=\frac{2-\frac{3}{2}z^{-1}}{(1-2z^{-1})(1+\frac{1}{2}z^{-1})}$$

The impuse response $h[n]$ is

(A) $2^n u[-n+1]-\left(\frac{1}{2}\right)^n u[n]$

(B) $2^n u[-n-1]+\left(\frac{-1}{2}\right)^n u[n]$

(C) $-2^n u[-n-1]+\left(\frac{-1}{2}\right)^n u[n]$

(D) $2^n u[n]-\left(\frac{1}{2}\right)^n u[n]$

42. Let $x[n]=\delta[n-2]+\delta[n+2]$. The unilateral z-transform is

(A) z^{-2} (B) z^2

(C) $-2z^{-2}$ (D) $2z^2$

43. The unilateral z-transform of signal $x[n]=u[n+4]$ is

(A) $1+z+z^2+3z+z^4$ (B) $\frac{1}{1-z}$

(C) $1+z^{-1}+z^{-2}+z^{-3}+z^{-4}$ (D) $\frac{1}{1-z^{-1}}$

44. The transfer function of a causal system is given as

$$H(z)=\frac{5z^2}{z^2-z-6}$$

The impulse response is

(A) $(3^n+(-1)^n 2^{n+1})u[n]$

(B) $(3^{n+1}+2(-2)^n)u[n]$

(C) $(3^{n-1}+(-1)^n 2^{n+1})u[n]$

(D) $(3^{n-1}-(-2)^{n+1})u[n]$

45. A causal system has input

$x[n]=\delta[n]+\frac{1}{4}\delta[n-1]-\frac{1}{8}\delta[n-2]$ and output

$y[n]=\delta[n]-\frac{3}{4}\delta[n-1]$.

The impulse response of this system is

(A) $\frac{1}{3}\left[5\left(\frac{-1}{2}\right)^n-2\left(\frac{1}{4}\right)^n\right]u[n]$

(B) $\frac{1}{3}\left[5\left(\frac{1}{2}\right)^n+2\left(\frac{-1}{4}\right)^n\right]u[n]$

(C) $\frac{1}{3}\left[5\left(\frac{1}{2}\right)^n-2\left(\frac{-1}{4}\right)^n\right]u[n]$

(D) $\frac{1}{3}\left[5\left(\frac{1}{2}\right)^n+2\left(\frac{1}{4}\right)^n\right]u[n]$

46. A causal system has input $x[n]=(-3)^n u[n]$ and output

$$y[n]=\left[4(2)^n-\left(\frac{1}{2}\right)^n\right]u[n].$$

The impulse response of this system is

(A) $\left[7\left(\frac{1}{2}\right)^n-10\left(\frac{1}{2}\right)^n\right]u[n]$ (B) $\left[7(2^n)-10\left(\frac{1}{2}\right)^n\right]u[n]$

(C) $\left[10\left(\frac{1}{2}\right)^2-7(2)^n\right]u[n]$ (D) $\left[10(2^n)-7\left(\frac{1}{2}\right)^n\right]u[n]$

47. A system has impulse response

$$h[n]=\frac{1}{2^n}u[n]$$

The output $y[n]$ to the input $x[n]$ is given by $y[n]=2\delta[n-4]$. The input $x[n]$ is

(A) $2\delta[-n-4]-\delta[-n-5]$ (B) $2\delta[n+4]-\delta[n+5]$

(C) $2\delta[-n+4]-\delta[-n+5]$ (D) $2\delta[n-4]-\delta[n-5]$

48. A system is described by the difference equation

$$y[n]-\frac{1}{2}y[n-1]=2x[n-1]$$

The impulse response of the system is

(A) $\frac{-1}{2^{n-2}}u[n-1]$ (B) $\frac{1}{2^{n-2}}u[n+1]$

(C) $\frac{1}{2^{n-2}}u[n-2]$ (D) $\frac{-1}{2^{n-2}}u[n-2]$

49. A system is described by the difference equation

$$y[n]=x[n]-x[n-2]+x[n-4]-x[n-6]$$

The impulse response of system is

(A) $\delta[n]-2\delta[n+2]+4\delta[n+4]-6\delta[n+6]$

(B) $\delta[n]+2\delta[n-2]-4\delta[n-4]+6\delta[n-6]$

(C) $\delta[n]-\delta[n-2]+\delta[n-4]-\delta[n-6]$

(D) $\delta[n]-\delta[n+2]+\delta[n+4]-\delta[n+6]$

50. The impulse response of a system is given by

$$h[n]=\frac{3}{4^n}u[n-1].$$

The difference equation representation for this system is

(A) $4y[n]-y[n-1]=3x[n-1]$

(B) $4y[n]-y[n+1]=3x[n+1]$

(C) $4y[n]+y[n-1]=-3x[n-1]$

(D) $4y[n]+y[n+1]=3x[n+1]$

51. The impulse response of a system is given by

$$h[n]=\delta[n]-\delta[n-5]$$

The difference equation representation for this system is

(A) $y[n]=x[n]-x[n-5]$ (B) $y[n]=x[n]-x[n+5]$

(C) $y[n]=x[n]+5x[n-5]$ (D) $y[n]=x[n]-5x[n+5]$

52. The transfer function of a system is given by

$$H(z)=\frac{z(3z-2)}{z^2-z-\frac{1}{4}}$$

The system is

(A) Causal and Stable

(B) Causal, Stable and minimum phase

(C) Minimum phase

(D) None of the above

53. The z-transform of a signal $x[n]$ is given by

$$X(z)=\frac{3}{1-\frac{10}{3}z^{-1}+z^{-2}}$$

If $X(z)$ converges on the unit circle, $x[n]$ is

(A) $-\frac{1}{3^{n-1}8}u[n]-\frac{3^{n+3}}{8}u[-n-1]$

(B) $\frac{1}{3^{n-1}8}u[n]-\frac{3^{n+3}}{8}u[-n-1]$

(C) $\frac{1}{3^{n-1}8}u[n]-\frac{3^{n+3}}{8}u[-n]$

(D) $-\frac{1}{3^{n-1}8}u[n]-\frac{3^{n+3}}{8}u[-n]$

54. The transfer function of a system is given as

$$H(z)=\frac{4z^{-1}}{\left(1-\frac{1}{4}z^{-1}\right)^2},\quad |z|>\frac{1}{4}$$

The $h[n]$ is

(A) Stable (B) Causal

(C) Stable and Causal (D) None of the above

55. The transfer function of a system is given as

$$H(z)=\frac{2\left(z+\frac{1}{2}\right)}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)}.$$

Consider the two statements

Statement(i) : System is causal and stable.

Statement(ii) : Inverse system is causal and stable.

The correct option is

(A) (i) is true

(B) (ii) is true

(C) Both (i) and (ii) are true

(D) Both are false

56. The impulse response of a system is given by

$$h[n]=10\left(\frac{-1}{2}\right)^n u[n]-9\left(\frac{-1}{4}\right)^n u[n]$$

For this system two statement are

Statement (i): System is causal and stable

Statement (ii): Inverse system is causal and stable.

The correct option is

(A) (i) is true
(B) (ii) is true
(C) Both are true
(D) Both are false

57. The system

$$y[n] = cy[n-1] - 0.12y[n-2] + x[n-1] + x[n-2]$$

is stable if

(A) $c < 1.12$
(B) $c > 1.12$
(C) $|c| < 1.12$
(D) $|c| > 1.12$

58. Consider the following three systems

$$y_1[n] = 0.2y[n-1] + x[n] - 0.3x[n-1] + 0.02x[n-2]$$

$$y_2[n] = x[n] - 0.1x[n-1]$$

$$y_3[n] = 0.5y[n-1] + 0.4x[n] - 0.3x[n-1]$$

The equivalent system are

(A) $y_1[n]$ and $y_2[n]$
(B) $y_2[n]$ and $y_3[n]$
(C) $y_3[n]$ and $y_1[n]$
(D) all

59. The z-transform of a causal system is given as

$$X(z) = \frac{2 - 1.5z^{-1}}{1 - 1.5z^{-1} + 0.5z^{-2}}$$

The $x[0]$ is

(A) -1.5
(B) 2
(C) 1.5
(D) 0

60. The z-transform of a anti causal system is

$$X(z) = \frac{12 - 21z}{3 - 7z + 12z^2}$$

The value of $x[0]$ is

(A) $-\frac{7}{4}$
(B) 0
(C) 4
(D) Does not exist

61. Given the z-transforms

$$X(z) = \frac{z(8z-7)}{4z^2 - 7z + 3}$$

The limit of $x[\infty]$ is

(A) 1
(B) 2
(C) ∞
(D) 0

62. The impulse response of the system shown in fig. P5.4.62 is

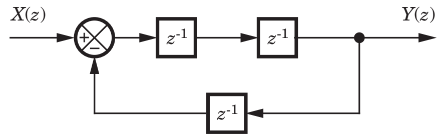

Fig. P5.4.62

(A) $2^{\left(\frac{n}{2}-2\right)}(1+(-1)^n)u[n] + \frac{1}{2}\delta[n]$

(B) $\frac{2^n}{2}(1+(-1)^n)u[n] + \frac{1}{2}\delta[n]$

(C) $2^{\left(\frac{n}{2}-2\right)}(1+(-1)^n)u[n] - \frac{1}{2}\delta[n]$

(D) $\frac{2^n}{2}\,[1+(-1)^n]\,u[n] - \frac{1}{2}\delta[n]$

63. The system diagram for the transfer function

$$H(z) = \frac{z}{z^2 + z + 1}$$

is shown in fig. P5.4.63. This system diagram is a

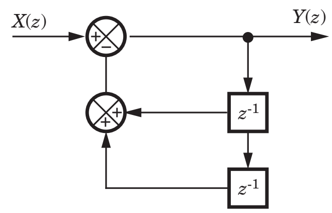

Fig. P5.4.63

(A) Correct solution

(B) Not correct solution

(C) Correct and unique solution

(D) Correct but not unique solution

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24. (A) $X(z)=\frac{z^2-3z}{z^2+\frac{3}{2}z^{-1}}=\frac{1-3z^{-1}}{1+\frac{3}{2}z^{-1}-z^{-2}}$

$=\frac{2}{1+2z^{-1}}-\frac{1}{1-\frac{1}{2}z^{-1}}$, ROC $:\frac{1}{2}<|z|<2$

$\Rightarrow \quad x[n]=-2(2)^n u[-n-1]-\frac{1}{2^n}u[n]$

25. (A) $x[n]$ is right sided

$X(z)=\frac{z-\frac{1}{4}z^{-1}}{1-16z^{-1}}=\frac{\frac{49}{32}}{1+4z^{-1}}+\frac{\frac{47}{32}}{1-4z^{-1}}$

$\Rightarrow \quad x[n]=\left[\frac{49}{32}(-4)^n+\frac{47}{32}4^n\right]u[n]$

26. (C) $x[n]$ is right sided

$X(z)=\left(2+\frac{1}{1+z^{-1}}+\frac{-1}{1-z^{-1}}\right)z^2$

$\Rightarrow \quad x[n]=2\delta[n+2]+((-1)^n-1)u[n+2]$

27. (A) $\delta[n]+2\delta[n-6]+4\delta[n-8]$

28. (B) $x[n]$ is right sided, $x[n]=\sum_{k=5}^{10}\frac{1}{k}\delta[n-k]$

29. (D) $x[n]$ is right sided signal

$X(z)=1+3z^{-1}+3z^{-2}+z^{-3}$

$\Rightarrow \quad x[n]=\delta[n]+3\delta[n-1]+3\delta[n-2]+\delta[n-3]$

30. (A)

$x[n]=\delta[n+6]+\delta[n+2]+3\delta[n]+2\delta[n-3]+\delta[n-4]$

31. (B) $X(z)=1+z+\frac{z^2}{2!}+\frac{z^3}{3!}+\ldots\ldots$

$+1+\frac{1}{z}+\frac{1}{2!z^2}+\frac{1}{3!z^3}\ldots\ldots$

$x[n]=\delta[n]+\delta[n+1]+\frac{\delta[n+2]}{2!}+\frac{\delta[n+3]}{3!}\ldots\ldots$

$+\delta[n]+\delta[n-1]+\frac{\delta[n-2]}{2!}+\frac{\delta[n-3]}{3!}\ldots\ldots$

$x[n]=\delta[n]+\frac{1}{n!}$

32. (A) $X(z)=1+\frac{z^{-2}}{4}+\left(\frac{z^{-2}}{4}\right)^2=\sum_{k=0}^{\infty}\left(\frac{1}{4}z^{-2}\right)^k$

$\Rightarrow \quad x[n]=\sum_{k=0}^{\infty}\left(\frac{1}{4}\right)^k\delta[n-2k]$

$$=\begin{cases}\left(\frac{1}{4}\right)^{\frac{n}{2}}, & n \text{ even and } n\ge 0\\ 0, & n \text{ odd}\end{cases}$$

$$=\begin{cases}2^{-n}, & n \text{ even and } n\ge 0\\ 0, & n \text{ odd}\end{cases}$$

33. (C) $X(z)=-4z^2\sum_{k=0}^{\infty}(2z)^{2k}=-\sum_{k=0}^{\infty}2^{2(k+1)}z^{2(k+1)}$

$\Rightarrow \quad x[n]=-\sum_{k=0}^{\infty}2^{2(k+1)}\delta[n+2(k+1)]$

34. (A) $\ln(1+\alpha)=\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}(\alpha)^k$

$X(z)=\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}(z^{-1})^k$

$\Rightarrow \quad x[n]=\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}\delta[n-1]$

35. (B) $\cos\alpha=\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}\alpha^{2k}$

$X(z)=\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}(z^{-3k})^{2k}$

$\Rightarrow \quad x[n]=\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}\delta[n-6k]$

$n=12 \quad\Rightarrow\quad 12-6k=0,\ k=2,$

$x[12]=\frac{(-1)^2}{4!}=\frac{1}{24}$

36. (D) All gives the same z transform with different ROC. So all are the solution.

37. (C) $x_1[n]$ is right-sided signal

$z_1>2,\ z_1>\frac{1}{2}$ gives $z_1>2$

$x_2[n]$ is left-sided signal

$z_2<2,\ z_2<\frac{1}{2}$ gives $z_2<\frac{1}{2}$

$x_3[n]$ is double sided signal

$z_3>\frac{1}{2}$ and $z_3<2$ gives $\frac{1}{2}<z_3<2$

38. (B) $X(z)=\frac{2}{1-\frac{1}{2}z^{-1}}+\frac{-1}{1+\frac{1}{z}z^{-1}}$,

$|z|>\frac{1}{2}$ (Right-sided) $\Rightarrow \quad x[n]=\frac{2}{2^n}u[n]-\left(\frac{-1}{3}\right)^n u[n]$

$|z|<\frac{1}{3}$ (Left-sided) $\Rightarrow \quad x[n]=\left[\frac{-2}{2^n}+\left(\frac{-1}{3}\right)^n\right]u[-n-1]$

$\frac{1}{3} < |z| < \frac{1}{2}$ (Two-sided) $x[n] = -\frac{2}{2^n} u[-n-1] - \left(\frac{-1}{3}\right)^n u[n]$

So (b) is wrong.

39. (A) Since the ROC includes the $z = \frac{3}{4}$, ROC is

$\frac{1}{2} < |z| < 1,$

$$X(z) = \frac{A}{1 - \frac{1}{2}z^{-1}} + \frac{B}{1 + z^{-1}}$$

$$\Rightarrow \quad x[n] = \frac{A}{2^n} u[n] \cdot B\,(-1)^n u[-n-1]$$

$$1 = \frac{A}{2} \quad \Rightarrow \quad A = 2\ ,$$

$$x[-1] = 1 = (-1)B(-1) \quad \Rightarrow \quad B = 1$$

$$\Rightarrow \quad x[n] = \frac{1}{2^{n-1}} u[n] - (-1)^n u[-n-1]$$

40. (B) $x[n] = Cp^n u[n]\ ,\ x[0] = 2 = C$

$$x[2] = \frac{1}{2} = 2p^2 \quad \Rightarrow \quad p = \frac{1}{2}\ ,$$

$$x[n] = 2\left(\frac{1}{2}\right)^n u(n)$$

41. (B) $H(z) = \frac{1}{1 - 2z^{-1}} + \frac{1}{1 + \frac{1}{2}z^{-1}}$

$h[n]$ is stable, so ROC includes $|z| = 1$

ROC : $\frac{1}{2} < |z| < 2\ ,$

$$h[n] = (2)^n u[-n-1] + \left(\frac{-1}{2}\right)^n u\,[n]$$

42. (A) $X^+(z) = \sum_{n=0}^{\infty} x[n]z^{-n} = \delta[n-2]z^{-n} = z^{-2}$

43. (D) $X^+(z) = \sum_{n=0}^{\infty} x[n]z^{-n} = \sum_{n=0}^{\infty} z^{-n} = \frac{1}{1 - z^{-1}}$

44. (B) $H(z) = \frac{3}{1 - 3z^{-1}} + \frac{2}{1 + 2z^{-1}}$

$h[n]$ is causal so ROC is $|z| > 3$,

$$\Rightarrow \quad h[n] = [3^{n+1} + 2\,(-2)^n\,]u[n]$$

45. (A) $X(z) = 1 + \frac{z^{-1}}{4} - \frac{z^{-2}}{8},\quad Y(z) = 1 - \frac{3z^{-1}}{4}$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{-2}{3}}{1 - \frac{1}{4}z^{-1}} + \frac{\frac{5}{3}}{1 + \frac{1}{2}z^{-1}}\ ,$$

$$\Rightarrow \quad h[n] = \frac{1}{3}\left[5\left(\frac{-1}{2}\right)^n - 2\left(\frac{1}{4}\right)^n\right]u[n]$$

46. (D) $X(z) = \frac{1}{1 + 3z^{-1}}$

$$Y(z) = \frac{4}{1 - 2z^{-1}} - \frac{1}{1 - \frac{1}{2}z^{-1}} = \frac{3}{(1 - 2z^{-1})\left(1 - \frac{1}{2}z^{-1}\right)}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{10}{1 - 2z^{-1}} + \frac{-7}{1 - \frac{1}{2}z^{-1}}$$

$$\Rightarrow \quad h[n] = \left[10(2)^n - 7\left(\frac{1}{2}\right)^n\right]u(n)$$

47. (D) $H(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}\ ,\ Y(z) = 2z^{-4}$

$$X(z) = \frac{Y(z)}{H(z)} = 2z^{-4} - z^{-5}$$

$$\Rightarrow \quad x[n] = 2[\delta - 4] - \delta[n-5]$$

48. (A) $Y(z)\left[1 - \frac{z^{-1}}{2}\right] = 2z^{-1}X\,(z)$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2z^{-1}}{1 - \frac{z^{-1}}{2}}$$

$$\Rightarrow \quad h[n] = 2\left(\frac{1}{2}\right)^{n-1} u[n-1]$$

49. (C) $H(z) = \frac{Y(z)}{X(z)} = (1 - z^{-2} + z^{-4} - z^{-6})$

$$\Rightarrow \quad h[n] = \delta[n] - \delta[n-2] + \delta[n-4] - \delta[n-6]$$

50. (A) $h[n] = \frac{3}{4}\left(\frac{1}{4}\right)^{n-1} u[n-1]$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{3}{4}z^{-1}}{1 - \frac{1}{4}z^{-1}}$$

$$Y(z) - \frac{1}{4}z^{-1}Y(z) = \frac{3}{4}z^{-1}X(z)$$

$$\Rightarrow \quad y[n] - \frac{1}{4}y[n-1] = \frac{3}{4}x[n-1]$$

51. (A) $H(z)=\frac{Y(z)}{X(z)}=1-z^{-5}$

$\Rightarrow \quad y[n]=x[n]-x[n-5]$

52. (D) Zero at : $z=0,\ \frac{2}{3}$, poles at $z=\frac{1\pm\sqrt{2}}{2}$

(i) Not all poles are inside $|z|=1$, the system is not causal and stable.

(ii) Not are poles and zero are inside $|z|=1$, the system is not minimum phase.

53. (A) $X(z)=\frac{-\frac{3}{8}}{1-\frac{1}{3}z^{-1}}+\frac{\frac{27}{8}}{1-3z^{-1}}$

Since $X(z)$ converges on $|z|=1$. So ROC must include this circle.

ROC : $\frac{1}{3}<|z|<3$,

$x[n]=-\frac{1}{3^{n-1}8}u[n]-\frac{3^{n+3}}{8}u[-n-1]$

54. (C) $h[n]=16n\left(\frac{1}{4}\right)^n u[n]$. So system is both stable and causal. ROC includes $z=1$.

55. (C) Pole of system at : $z=-\frac{1}{2},\ \frac{1}{3}$

Pole of inverse system at : $z=-\frac{1}{2}$

For this system and inverse system all poles are inside $|z|=1$. So both system are both causal and stable.

56. (A) $H(z)=\frac{10}{1+\frac{1}{2}z^{-1}}-\frac{9}{1+\frac{1}{4}z^{-1}}$

$=\frac{1-2z^{-1}}{\left(1+\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{4}z^{-1}\right)}$

Pole of this system are inside $|z|=1$. So the system is stable and causal.

For the inverse system not all pole are inside $|z|=1$. So inverse system is not stable and causal.

57. (C) $|a_2|=0.12<1,\ a_1=|-c|<1+0.12,\ |c|<1.12$

58. (A) $Y_1(z)=1-0.1z^{-1}$, $Y_2(z)=1-0.1z^{-1}$

$Y_3(z)=\frac{0.4-0.3z^{-1}}{1-0.5z^{-1}}$

So y_1 and y_2 are equivalent.

59. (B) Causal signal $x[0]=\lim_{z\to\infty}X(z)=2$

60. (C) Anti causal signal, $x[0]=\lim_{z\to\infty}X(z)=4$

61. (A) The function has poles at $z=1,\frac{3}{4}$. Thus final value theorem applies.

$\lim_{n\to\infty}x(n)=\lim_{z\to 1}(z-1)X(z)=(z-1)\frac{z(2z-\frac{7}{4})}{(z-1)\left(z-\frac{3}{4}\right)}=1$

62. (C) $[2Y(z)+X(z)]z^{-2}=Y(z)$

$H(z)=\frac{z^{-2}}{1-2z^{-2}}$

$\Rightarrow \quad h[n]=-\frac{1}{2}+\frac{\frac{1}{4}}{1-\sqrt{2}z^{-1}}+\frac{\frac{1}{4}}{1+\sqrt{2}z^{-1}}$

$=-\frac{1}{2}\delta[n]+\frac{1}{4}\{(\sqrt{2})^n+(-\sqrt{2})^n\}u[n]$

63. (D) $Y(z)=X(z)z^{-1}-\{Y(z)z^{-1}+Y(z)z^{-2}\}$

$\frac{Y(z)}{X(z)}=\frac{z^{-1}}{1+z^{-1}+z^{-2}}=\frac{z}{z^2+z+1}$

So this is a solution but not unique. Many other correct diagrams can be drawn.

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