Chapter - 9

Electromagnetic Induction

In previous chapter we have seen that a magnetic field is associated with moving charges and current. The above invention by orested raised questions to scientiests, is the converse effect possible i.e. can a magnetic field produce electric field which establishe electric current in a closed circuit? The answer is yes.

Experinments conducted independently by Michael Fraday and Joseph Henry conclusively showed that electric currents were induced in closed coils when subjected to varying magnetic fields. This phenomenon is called electro magnetic induction.

Experinments by Henry not only confirms the above inter-relationship but gave many practical utilities of phenomenon of electro magnetic induction. For example the electric generations which supply electric power to our homes and work places works on electro magnetic induction. The electric furnaces in which metals are melts in large amounts safely, works on electro magnetic induction. Now a days use of induction stoves is popular in kitchens, rapidly replacing the traditional stoves. In this chapter we will study the principles related to electro magnetic induction.

9.1 Magnetic Flux

Before studying the experinments performed by Faraday and Henry, we must get familiar with a physical quantity magnetic flux, which helps qualitative and quantitative explanation of these experinments. We will define magnetic flux as we have defined electric flux in chapter 2. Magnetic flux is measure of number of magnetic field lines passing through a surface. The magnetic flux $d\phi_B$ through an small area element dA placed in uniform magnetic field B can be given as

$$d\phi_B = \vec{B}\cdot d\vec{A} \qquad \ldots 9.1(a)$$

It is a scaler quantity. The area elements $d\vec{A}$ is a vector normal to its geomatrical area.

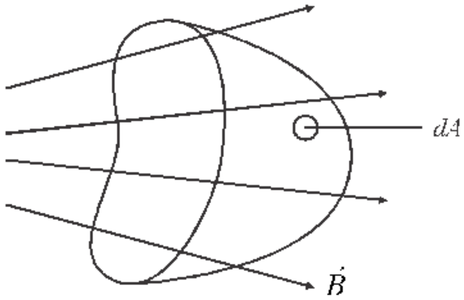


Fig. 9.1 (a) : An area placed in magnetic field are magnetic flux

The total magnetic flux passing through a surface placed in magnetic field $\vec{B}$ is summation of magnetic flux passing through all such small area elements.

Hence flux passing through a surface is given by

$$\phi_B = \int d\phi_B = \int \vec{B}\cdot d\vec{A} \qquad \ldots 9.1(b)$$

Here integral is considered over the surface area of surface under consideration.

As a special case magnetic flux passing through a plane surface placed in uniform magnetic $\vec{B}$ is :-

$$\phi_B = BA\cos\theta \qquad \ldots (6.2)$$

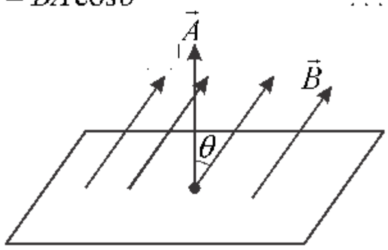


Fig. 9.1 (b) Plane surface of area $\vec{A}$ placed in uniform magnetic field, area vector $\vec{A}$ is normal to surface and outward

Here θ is angle between directions of normal to the surface and magnetic field $\vec{B}$. For a given area ϕ_B is maximum when $\cos\theta$ is maximum i.e. $\cos\theta = 1$ or $\theta = 0^\circ$. In this situation $\vec{B}$ and $\vec{A}$ are parallel and

$$\phi_{B\max} = BA \quad 1$$

In the case of $\theta = 90^\circ$ (here $\vec{B}$ and $\vec{A}$ are normal to each other) the magnetic flux is minimum and zero. Hence

$$\phi_{B\min} = 0$$

Outgoing flux to the surface is taken as positive and flux entering the surface is taken as negative.

Because magnetic field lines are closed curve or loop, so the total magnetic flux associated to a closed surface is always zero.

$$\oint \vec{B} \cdot d\vec{A} = 0$$

This is called Gauss law for magnetism. Magnetic flux is a scalar quantity, its dimensions are $\left[ML^2T^{-2}A^{-1}\right]$. Its S.I. unit is weber (Wb) or Tesla - meter^2. (Tm^2) as Joule/Ampere = $\frac{\text{Joule x sec}}{\text{Coulumb}}$ = volt × sec, these unit can also be used

Unit of magnetic flux in CGS is maxwel (Mx). maxwel and weber are related as follows :-
1 Wb = 1 V × s = 10^8 emu of potential = 10^8 Mx

Example 9.1 : A circular coil of area $(3\hat{i} + \hat{j} + 2\hat{k}) \times 10^{-2}\text{m}^2$ is placed in a magnetic field $(2\hat{i} - 2\hat{k}) \times 10^{-4}$ T. Find out magnetic flux passing through the coil.

Solution : Magnetic flux is given as :-

$$\phi_B = \vec{B} \cdot \vec{A}$$

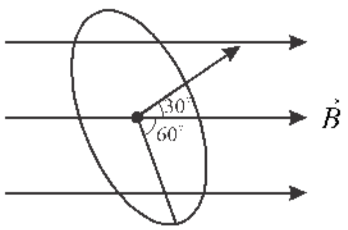
$$\therefore \quad \phi = (2\hat{i} - 2\hat{k}) \times 10^{-4} \cdot (3\hat{i} + \hat{j} + 2\hat{k}) \times 10^{-2}$$

$$= 8\,(6 - 4) \times 10^{-6}\,\text{Wb}$$

$$\phi_B = 2 \times 10^{-6}\ \text{Wb}$$

Example 9.2 : A circular coil is placed in a magentic field 5×10^{-3} T at 60^0 angle to the field. If the area of coil is 4m^2 than find the amount of magnetic flux through the coil.

Solution :



$$\phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$$

Here θ is angle between normal to the plane of coil and direction of magnetic field $\vec{B}$.

$$\theta = 30°$$

Putting values of B, A and θ we have

$$\phi_B = 5 \times 10^{-3}$$

$$= 5 \times 10^{-3} \times 4 \times \frac{\sqrt{3}}{2} = 10\sqrt{3} \times 10^{-3}\ \text{Wb}$$

9.2 Electroc Magnetic Induction

For the description of electro magnetic induction we will study first the three experiments carried out independently by Faraday and Henry. Understanding of different phenomenon related to electro magnetic induction are based on these experiments.

Experiment 1 : Let us consider a closed circuit of Galvano meter G and a coil C as shown in fig 9.2. Here coil is not connected with any source of e.m.f. (i.e. cell or battery). North pole of a bar magnet is kept towards the coil. As we move north pole of the magnet towards the coil, the pointer of the galvanometer shows deflection. But as the north pole of bar magnet moves away from the coil, the pointer of galvanometer deflects in opposite direction.

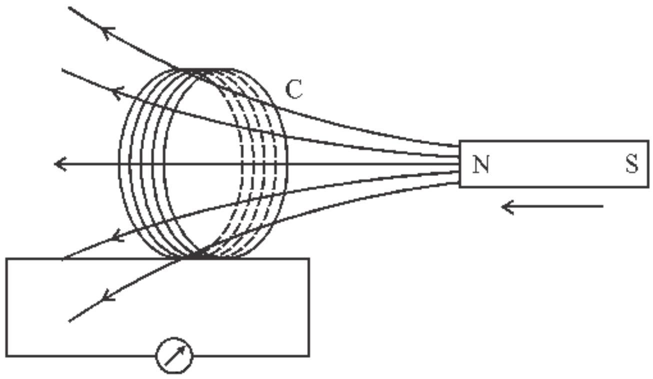


Fig. 9.2 : Electromagnetic induction north pole of bar magnet moving towards coil

If south pole of bar magnet moves toward or away from the coil, the direction of deflection in galvanometer is opposite to the direction of deflection for motion of north pole. If the magnet moves faster towards or away from the magnet, relatively large deflection is observed. If the bar magnet is held fixed and coil C is moved towards or away from the coil same effects are observed. However, when coil and bar magnet both remain stationary galvanometer shows no deflection.

Experiment - 2 : In this experiment bar magnet is replaced by another coil C_2 connected with a cell. When coil C_2 having current moves towrds or away from the coil C_1, the same effects are observed as in experiment one.

When coil C_2 is brought towrds C_1 galvanometer shows some deflection and when coil C_2 is brought away from C_1 galvanometer shows deflection in opposite direction. If coil C_2 is held fixed and coil C_1 moves deflection is observed in galvanometer. When both coil C_1 and C_2 are held fixed no deflection is observed in galvanometer.

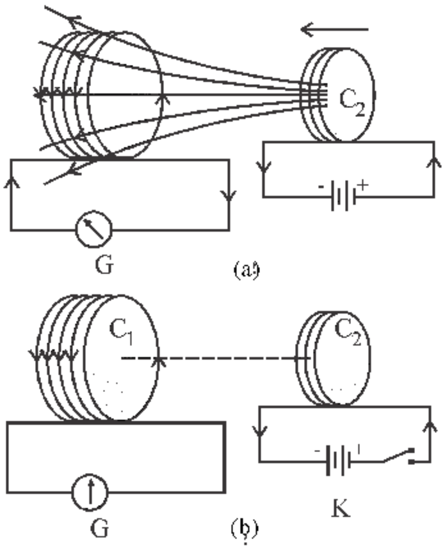


Fig. 9.3 (a) Induced current due to motion of C_2
(b) Induced current in C_1 due to change in current in C_2

Experiment - 3 : The above two experiment involve relative motion between magnet and coil and between two coils. But Faraday showed that for electro magnetic induction relative motion is not an absolute requirement. As shown in fig. 9.3 (b) if both coils C_1 and C_2 remains stationary and the key connected to coil C_2 is pressed, there is a momentary deflection in galvanometer. When the key is released there is again momentary deflection in galvanometer but now in opposite direction. When key is pressed continuously there is no deflection in galvanometer.

Deflection in galvanometer increases when number of turns of the coil increases and a soft iron rod is placed inside the coil.

9.2.1 Conclusions From Experiments

Faraday explained the results of experiments on the basis of magnetic flux. According to Faraday when magnetic flux associated with coil varies an emf is induced in coil due to which a current is also induced.

When coil and magnet remains fixed relative to each other or steady current flows through the secondary coil C_2 then magnetic flux through the first coil remains unchanged. When coil and magnet have relative motion or when varible current flows through the secondary coil,the magnetic flux varies through the first coil.

By moving the magnet near to the coil C_1 or by increasing current in secondary coil C_2, magnetic flux through coil C_1 increases. According to Faraday, variation of magnetic flux associated with coil C_1 develops induced emf in the coil.

Induced e.m.f is as larger as the rate of change of magnetic flux is faster.

The phenomenon of developing induced emf in coil due to variation in magnetic flux through the coil is called electromagnetic induction.

9.2.2 Faraday Laws of Electromagnetic Induction

From the experimental observations Faraday gave two laws for electro magnetic induction which are called Faraday's law of electro magnetic induction laws.

First Law :- When magnetic flux assoicated with a closed circuit varies, an emf is induced in the circuit. If the circuit is closed an induced current develops due to this induced e.m.f in the circuit, the induced current persists as long as the magnetic flux is varied.

Second Law :- According to this law "the magnitude of the induced emf is equal to rate of change of magnetic flux" associated with coil. If induced emf is denoted by ε , than mathematically it is given as

$$\varepsilon = \frac{d\phi_B}{dt} \qquad ...9.3\ (a)$$

For a closely wound coil of N turns, variation of flux assoicated with each turn is same. The total induced emf is given by :-

$$\varepsilon = -N\frac{d\phi_B}{dt} \qquad ...9.3\ (b)$$

(the negative sign in this equation is due to the Lenz's law discussed later)

From formula $\phi_B = BA\cos\theta$ flux can be changed by changing following processes-

(i) By changing magnetic field B.

(ii) By changing total area of the coil or part of area associated with in the magnetic field. For example by stretching or by shrinking the coil (by changing the shape of coil) or by pushing the coil inside the field or pushing it out of field.

(iii) By varing the angle between the direction of magnetic field B and normal to the plane of the coil (or plane of the coil itself).

For example rotating the coil in a magnetic field B in such a way that initially B remains normal to the plane of coil and afterwards it is in the plane of coil.

Example 9.3 : A coil is placed in magnetic field $\vec{B}$ so that its plane is normal to the field. If magnetic flux associated with coil is $\phi_B = (2t^2 - 6t + 9)$ mWb then find the induced emf at $t = 5$ sec.

Solution : Induced e.m.f $\varepsilon = -\frac{d\phi_B}{dt}$

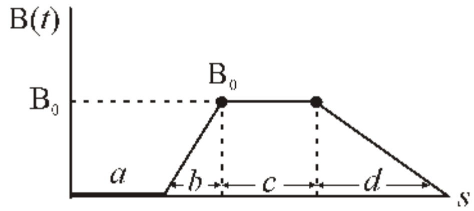
given $\phi_B = (2t^2 - 6t + 9)$ mWb

$$\varepsilon = -\frac{d}{dt}(2t^2 - 6t + 9) = -(4t - 6)\,\text{mV}$$

$t = 5$ s

$$\varepsilon = -(4 \times 5 - 6) = -14\,\text{mV}$$

Example 9.4 : Graph given below shows a time dependent magnetic field B (t) that exists uniformly over a conducting loop. Direction of magnetic field is normal to the plane of loop. Arrange four parts a,b,c and d of figure in order of induced emf first greatest.



Solution : Magnitude of induced e.m.f is given by

$$|\varepsilon| = \left|\frac{d\phi_B}{dt}\right|$$

Here $\phi_B = B(t)A$

As B(t) is normal to the plane of loop and A is constant

$$\therefore \quad |\varepsilon| = A\left|\frac{dB}{dt}\right| \propto \left|\frac{dB}{dt}\right|$$

(i) In part (a) B(t) = o

$\varepsilon = 0$

(ii) In part (b) $\left|\frac{dB}{dt}\right| = \left|\frac{\Delta B}{dt}\right| = \frac{B_0 - 0}{2T - T} = \frac{B_0}{T}$

(iii) In part (c) $\frac{dB}{dt} = 0$ (here B = Bo = constant)

(iv) In part (d) $\left|\frac{dB}{dt}\right| = \left|\frac{b_0 - 0}{5T - 3T}\right| = \frac{B_0}{2T}$

Hence induced emf is in decreasing order as follows -

$$\varepsilon_b > \varepsilon_d > \varepsilon_a = \varepsilon_0 (= 0)$$

9.3 Lenz's Laws

By Faraday's law we can find the magnitude of induced emf but the direction of induced emf and induced current is given by Lenz's law.

"According to lenz's law the polarity of induced e.m.f and direction of induced current in the circuit is such that it opposes the change in magnetic flux that produced it."

From Faraday and Lenz's law :-

$$\varepsilon = -\frac{d\phi_B}{dt} \quad \ldots (9.3)$$

For coil having N turns : -

$$\varepsilon = -N\frac{d\phi_B}{dt} \quad \ldots (9.4)$$

When magnetic flux through the coil increases the direction of magnetic field lines due to induced current is opposite to the original magnetic field lines. When magnetic flux through the coil decreases the direction of magnetic field lines due to induced current is in the direction of original magnetic filed lines.

In fig. 9.4 north pole of a bar magnet moves towards on face of a coil, direction of induced current in the coil is such that this face of coil behaves like a north pole. There is a repulsion between bar magnet and coil so induced current in coil opposes the motion of bar magnet.

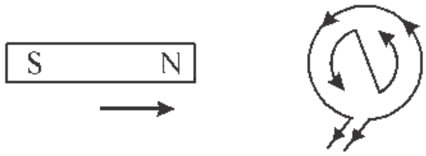


Fig. 9.4 : Moving magnet towards stationery coil

In fig. 9.5 when north pole of a bar magnet moves away form the coil, the direction of induced current in the coil is such that the face towards the magnet behaves as south pole. There is attraction between coil and magnet, so the induced induced current opposes the motion of magnet.

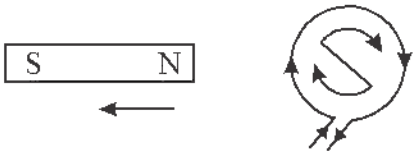


Fig. 9.5 Magnet moving away from stationery coil

In Lenz's law

$$\varepsilon = -d\phi_B / dt$$

The negative sign gives the direction of induced emf.

What will happen if an open circuit or open coil is used in place of a closed circuit. In this case emf is induced due to variation in magnetic flux but there is no induced current.

9.3.1 Lenz's Law and Conservation of Energy

Lenz's law is based on the law of conservation of energy. Let us imagine that the north pole of a magnet moves towards the coil and the direction of induced current is such that plane of coil towards magnet behaves as south pole and not as a north pole as discussed . The coil attracts the magnet and it is accelerated. Due to acceleration the induced current in the coil also increases which produces greater force on magnet and acceleration also increases. Hence kintetic energy of magnet and rate of heat i^2R, in the coil increases. Hence if initially we push magnet slightly towards the coil its velocity and kinetic energy increases continuously without spending any energy further which violets the law of consevation of energy. Hence our imgination is wrong.

In the experiments for electromagnetic induction we have seen that for keeping the magnet in motion external work is to be done against induced emf. Where does the energy spent in work goes? The mechnical energy spent in work converts in the form of electrical energy. Mechanical work done by external source is equal to energy dissipated by Joule heating due to induced current. Hence Lenz's law follow the law of conservation of energy.

9.3.2 Induced Current and Induced Charge

According to Faraday's law induced emf in coil having N turns is

$$\varepsilon = -N\frac{d\phi_B}{dt}$$

If area vector $\vec{A}$ of coil is along magnetic field $\vec{B}$ than magnetic flux $\phi = BA$

$$\varepsilon = -N\frac{d}{dt}(BA)$$

If $\vec{A}$ is fixed and $\vec{B}$ is variable than

$$\varepsilon = -NA\frac{dB}{dt} \qquad \ldots (9.4)$$

If $\vec{B}$ is fixed and $\vec{A}$ is variable than

$$\varepsilon = -NB\frac{dA}{dt} \qquad \ldots (9.5)$$

If the total resistance of circuit is R than induced current is

$$I = \frac{\varepsilon}{R} = -\frac{N}{R}\frac{d\phi_B}{dt} \qquad \ldots (9.6)$$

Induced charge in time interval dt is

$$dq = I\,dt$$

$$dq = -\frac{N}{R}d\phi_B$$

If flux changes from ϕ_{B_1} to ϕ_{B_2} than induced charge is

$$\int dq = -\frac{N}{R}\int_{\phi_{B1}}^{\phi_{B_2}} d\phi$$

$$q = -\frac{N}{R}(\phi_{B_2} - \phi_{B_1})$$

$$q = \frac{N}{R}(\phi_{B_1} - \phi_{B_2}) \qquad \ldots (9.7)$$

From above equation it is clear that magnitude of induced charge depend on change of flux but does not depends on rate of change of flux.

Example 9.5 : A coil of area 1.6 cm^2 having 50 turns placed in magntic field of 1.8 T in 0.3 sec. The plane of coil is normal to the direction of magnetic field lines. Find the amount of charge flown in coil if its resistance is 10 Ω .

Solution : Flux passing through each turn of coil having area A placed normal to magnetic field is

$$\Phi_B = BA$$

$$\phi_B = 1.8 \times 1.6 \times 10^{-4} = 2.88 \times 10^{-4} \text{ Wb}$$

From Faraday's law induced e.m.f

$$\varepsilon = -N\frac{d\phi_B}{dt} = -50 \times \frac{2.88 \times 10^{-4}}{0.3} = -4.8 \times 10^{-2} \text{ V}$$

Magnitude of induced e.m.f

$$|\varepsilon| = 4.8 \text{ x } 10^{-2} \text{ V}$$

is Induced current in the coil

$$I = \frac{|\varepsilon|}{R} = \frac{4.8 \times 10^{-3}}{10} = 4.8 \times 10^{-3} \text{ A}$$

Induced charge in coil

$$q = I\Delta t = 4.8 \times 10^{-3} \times 0.3 = 1.44 \times 10^{-3} \text{ C}$$

9.4 Flemings Right Hand Rule

Fleming's right hand rule gives the the direction of induced current. According to this law if the fore finger, central finger and thumb of right hand are held perpendicular to each other as in fig (9.6). If fore finger shows direction of magnetic field and thumb shows direction of motion than central finger shows direction of induced current.

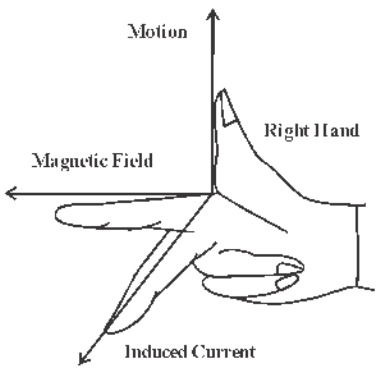
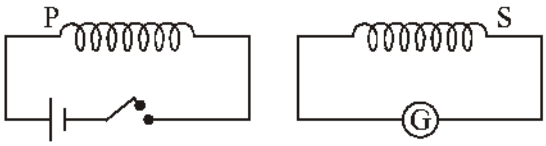
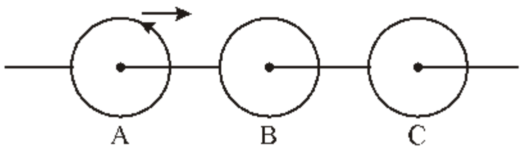


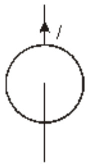
Fig. 9.6

Example 9.6 : Explain direction of induced current under following situations :-

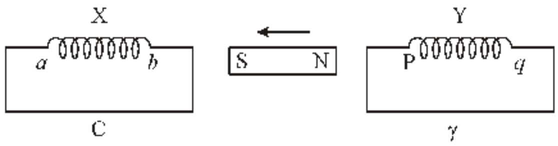
(i) 

(ii) 

(iii) 

(iv) 

direction of induced current in the loop when current decreases uniformly.

(v) 

Solution :

(i) According to Lenz's law direction of induced current in loop is from B to A. Hence plate A will be positive and palte B negative.

(ii) Looking from the side of coil P direction of induced current in coil S is clock wise.

(iii) There is no induced current in coil B due to current in coil C because C coil is fixed. Looking from the side of coil A the direction of indcued current in B is anti clockwise.

(iv) The rate of change of magnetic flux associated with circular loop is constant hence current will not be induced in the loop.

(v) As south pole moves towards coil X the induced current is in the sense a c b and as north pole moves away from coil y the induced current in it is the sense q r p.

9.5 Induced emf in a Conductor Rod Moving in a Uniform Magnetic Field

In fig. 9.7 uniform magnetic field B is shown by dots. Its direction is normal to the plane of paper and outward. A conducting rod ab of length l is placed perpendicular to the field, and the rod is moved with a constant velocity $\vec{v}$ perpendicular to the direction of both ℓ and magnetic field $\vec{B}$.

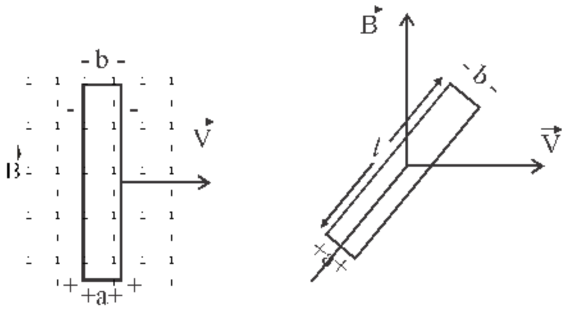


Fig. 9.7 : Motion of conducting rod in perpendicular magnetic field

The free electrons present in conducting rod also move with velocity $\vec{v}$ in magnetic field. Magnetic force on moving free electrons is

$$\vec{F}_m = q(\vec{v} \times \vec{B}) \qquad \ldots (9.8)$$

here is q charge of an electron. According to figure for positive charge the direction of $\vec{v} \times \vec{B}$ is from b to a. Hence magnetic force on electrons because of negative charge is from a to b along the length of conductor. Due to drift of electrons there is excess of electrons's at and b and deficiency of electrons at and a. Hence there is excess of negative charges at end 'b' and excess of positive charge at end 'a'.

Due to accumulation of opposite charge at both ends of the rod a static electric field get developed between the edges. The drift process due to motion of conductor continuous till the force on electron due to electric field is balanced by the force of magnetic field. Force on electron of charge q due to electric field E is

$$\vec{F}_e = q\vec{E} \qquad \ldots (9.9)$$

In the state of equilibirium

$$q\vec{E} + q(\vec{v} \times \vec{B}) = 0$$

$$\vec{E} = -(\vec{v} \times \vec{B})$$

i.e. the direction of $\vec{E}$ is opposite to direction of $\vec{v} \times \vec{B}$ or inside conductor it is from end a to end b ends magnitude of electric field $E = vB$.

Due to this electric field E, an emf ε is induced between two ends of conductor.

Hence

ε = work done in displacing a unit positive charge against electric field from one end to another end.

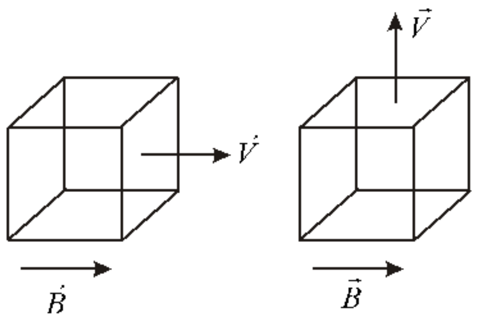
= Force on unit positive charge × displacement.

$$\varepsilon = E\ell$$

or $$\varepsilon = vB\ell \qquad \ldots (9.10)$$

If the direction of ℓ is taken from negatively charge end to positively charged end and the conducting rod is moving at an angle θ with the direction of magnetic field lines than component of v perpendicular to the direction of B is $v\sin\theta$. In such a case induced emf between the ends of conductor is given by $Bv\ell\sin\theta$. The induced emf is zero when conductor moves along the direction of magnetic field.

Example 9.7 : A cube is made by joining twelve straight conducting wires. Length of each arm is 5 cm. Cube is moving in a magnetic field of 0.5 T with velocity 5m/sec.



(i) If cube moves along the direction of magnetic field than what is induced emf in each arm.

(ii) If cube moves perpendicular to the direction of magnetic field than what is induced emf on each arm.

Soltuion : (i) In first case the velocity of conductor is parallel to magentic field hence induced emf along each arm is zero.

(ii) Induced emf will not develop along arms AB,CD,EF and GH because they are parallel to magnetic field. Arms AE, DH, BF and CG ara parallel to direction of velocity, hence induced emf along these arms is zero. Arms AD, BC, EH amd FG are perpendicular to both velocity v and magnetic field B, induced emf along each arm is given by -

$$\varepsilon = Bv\ell$$

$$= 0.05 \times 5 \times 5 \times 10^{-2}$$

$$= 1.25 \times 10^{-2} \text{ V}$$

Example 9.8 : A conducting rod of length 40 cm is placed perpendicular to a magnetic field of 0.5 T. The rod is moving with velocity 15m/s at an angle of 30^0 with magnetic field. Find induced emf across the rod.

Solution : $\varepsilon = BVl \sin\theta$

$$= 0.5 \times 15 \times 0.4 \sin 30^{\circ}$$

$$= 0.5 \times 15 \times 0.4 \times \frac{1}{2} = 1.5 \text{ V}$$

Example 9.9 : Two lines of a railway track are separated from each other and also from earth. They are connected by a milivoltmeter. When a train moves on this track with speed 180 km/hr then what is the reading in milivoltmeter. Distance between the railway lines is 1 m and vertical component of earths magnetic field is 0.2×10^{-4}T.

Solution : Induced potential difference between railway lines - $\varepsilon = Bvl$

$$\text{given } v = 180 \text{ km/h} = \frac{180 \times 5}{8} = 50 \text{ m/s}$$

$$B = 0.2 \times 10^{-4} \text{ T and } l = 1 \text{ m}$$

$$\varepsilon = 0.2 \times 10^{-4} \times 50 \times 1$$

$$= 1 \times 10^{-3} \text{ V}$$

$$= 1 \text{ mV}$$

9.6 Induced emf and Current in a Rectangular Loop Moving in a Non Uniform Magnetic Field

In fig 9.8 a conducing rectangular loop or coil is placed perpendicular to a non uniform magnetic field. Suppose magnetic is B_1 of arm ab and B_2 on arm cd.

Coil is moved with *e* velocity *v* perpendicular to magnetic field in such a way that direction of velocity is perpendicular to arm ab and cd.

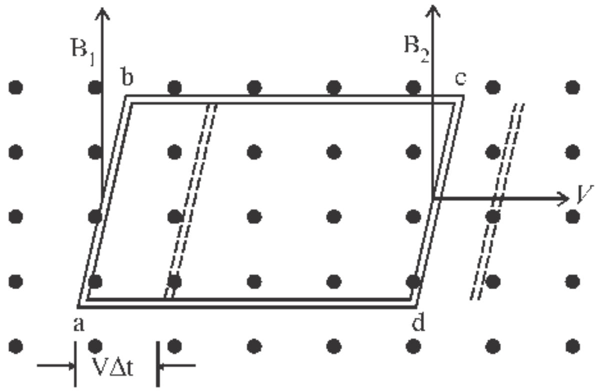


Fig. 9.8 Motion of rectangular loop in non uniform magnetic field

Suppose length of arm ab and cd is *l*. Distance traversed by coil in small time interval Δt is $v\Delta t$. Area crossed by arm ab and cd is $\Delta A = \ell v \Delta t$. Magnetic fields in these small areas can be taken as B_1 and B_2. From fig 9.8 it is clear that the area which comes out of magnetic field B_1 on left side and same amount of area enters in magnetic field B_2 on right side. Due to motion of coil decrease in magnetic flux crossing through coil on left side is

$$\phi_{B_1} = B_1 \times \Delta A = B_1 \ell v \Delta t \qquad \ldots(9.12)$$

Increase in magnetic flux crossing through coil on right side is

$$\phi_{B_1} = B_1 \times \Delta A = B_1 \ell v \Delta t \qquad \ldots(9.13)$$

Change in magnetic flux passing through coil

$$\Delta\phi_B = \phi_{B_2} - \phi_{B_1} = (B_2 - B_1)\ell v \Delta t \qquad \ldots(9.14)$$

$$\text{Hence } \frac{\Delta\phi_B}{\Delta t} = (B_2 - B_1)\ell v$$

According to Faraday's law, the induced emf

$$\varepsilon = -\frac{\Delta\phi_B}{\Delta t} = \frac{-d\phi_B}{dt}$$

or $\varepsilon = -(B_2 - B_1)\ell v$

$$\varepsilon = (B_1 - B_2)\ell v \qquad \ldots(9.15a)$$

If resistance of coil is R than induced current in the coil

$$I = \frac{E}{R} = \frac{(B_1 - B_2)\ell V}{R} \quad \ldots (9.15b)$$

9.7 Energy Conversation

In fig 9.9 (a) conducting loop abcd is placed perpemdicular to non uniform magnetic field. Suppose loop moves with velocity v. Due to motion of wire 'ab' in magnetic field positive charges accumulate at end a and negative at end b. In the same way on wire cd positive charges accumulate at end b and negative at c.

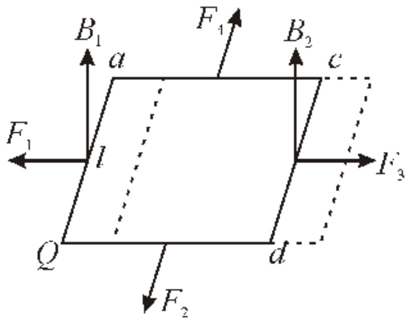


Fig. 9.9 Force on current carrying conducting in magnetic field

If $B_1 > B_2$ than amount of positive charge at end a is greater than at end d.

Induced current in loop flows in direction adcba. Let this induced current be I.

By force on current carrying conductor in magnetic field, the force on length l of arm ab is

$F_1 = IlB_1$ (towards left side)

In the same way froce on arm cd

$F_3 = IlB_2$ (towards right side)

Resultant of these two forces

$F = F_1 - F_3$

$F = IlB_1 - IlB_2$ (toward left side)

Forces F_2 and F_4 on arms bc and cd are equal in magnitude and opposite in direction. Hence they cancel each other.

When loop travels a distance $v\Delta\tau$ in time Δt towards right side work done against force F is

$$W = F\Delta x = I\ell(B_1 - B_2)v\Delta t$$

$$I = \frac{E}{R} = \frac{(B_1 - B_2)\ell V}{R}$$

$$W = (B_1 - B_2)^2 \frac{\ell^2 v^2}{R}\Delta t \quad \ldots (9.16)$$

Energy spent in this work done is converted into electrical energy in the circuit and ultimately dissipated in the form of heat.

$$H = I^2 R\Delta t$$

On putting value of I

$$H = \frac{(B_1 - B_2)^2 \ell^2 v^2}{R^2} \times R\Delta t$$

$$H = \frac{(B_1 - B_2)^2 l^2 v^2}{R}\Delta t \quad \ldots (9.17)$$

From equations 9.16 and 9.17

W = H

We see that power delivered by the external force is equal to the thermal power developed in the loop. This is consistent with the low of energy conservation.

Example 9.10 : Length of arm of a square loop is 1.5 m. Half of the loop is in magnetic field 2.5 T and remaining half part is in 1 T. It is moved with a velocity 7.2 km/hr perpendiculatr to magnetic field. Find induced emf.

Solution : Induced emf in a loop moving in a non uniform magnetic field

$$\varepsilon = (B_1 - B_2)v\ell$$

Here $B_1 = 2.5\,\text{T},\ B_2 = 1\,\text{T},\ \ell = 1.5\,\text{m}$

$$V = \frac{7.2 \times 5}{18} = 2\,\text{m/s}$$

$$\varepsilon = (2.5 - 1) \times 2 \times 1.5 = 4.5V$$

Example 9.11 : 2m long conducting rod is placed perpendicular to a magnetic field of 1 T. Rod is moved with a velocity 0.6 m/s perpendicular to its length and magnetic field. If conducting rod is connected across a resistance wire of 12 Ω. What is the required force and power for motion of rod. What is rate of heat production in the circuit.

Solution : Force on current carrying conducitng palced in magnetic filed

$$F = I(\ell \times B)$$

$$F = I\ell B \sin 90^\circ = I\ell B$$

or $$F = 0.1 \times 2 \times 1 = 0.2 \text{ N}$$

For uniform motion of rod an opposite force of same magnitude as of above force is required.

Required power for motion of rod

$$P = Fv = 0.2 \times 0.6 = 0.12 \text{ W}$$

Rate of heat produced in circuit

$$H = I^2 R = (0.1)^2 \times 12 = 0.12 \text{ W}$$

9.8 Induced emf in a metal rod rotating in a Uniform Magnetic Field

In fig. 9.10 uniform magnetic field B is shown by cross (x) its direction is inward perpendicular to the plane of paper. A conducting rod OA of length L is rotating with uniform angular velocity ω anti clock wise in this magnetic field. Let us consider a small element $d\ell$ of rod moving perpendicular to magnetic field with velocity v. Magnitude of induced emf in this small element.

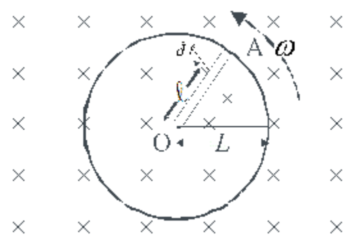


Fig. 9.10 Conducting rod rotating perpendicular to a uniform magnetic field

$$d\varepsilon = Bvd\ell$$

If small element is at distance l from centre than

$$v = \omega\ell$$

Hence $$d\varepsilon = B\omega\ell . d\ell$$

Induced emf across the rod is equal to the integration of above equation from 0 to L.

$$\int d\varepsilon = \int_0^L B\omega\ell \, d\ell$$

$$\varepsilon = \frac{1}{2} B\omega L^2 \qquad \ldots (9.18)$$

By Fleming's right hand rule and in view of direction of magetic field and direction of rotation the induced current is directed from A to O and the end O of rod if positively charged and A is negatively charged for this case.

If frequency of rotation of rod is f than

$$\omega = 2\pi f$$

$$\varepsilon = \frac{1}{2} B \times 2\pi f \times L^2$$

$$= B \times \pi L^2 \times f$$

Suppose area of circle traversed by rod in magnetic field is A

$$\pi L^2 = A$$

and $$\varepsilon = BAf \qquad \ldots (9.19)$$

9.9 Induced emf in a metal Disc Rotating in a Uniform Magnetic Field

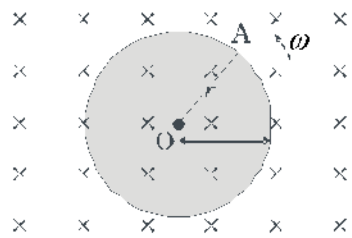


Fig. 9.11 Conducting disc rotating perpendicular to magnetic field

A uniform magnetic field B perpendicular to plane of paper and inward is shown in fig 9.11 by cross (x). A metallic disc of radius r is rotating in this magnetic field with angular velocity ω in the plane of paper anticlock wise. Disc can be considered as made up of many rods having one common end at the centre O of the disc and other end at the circumference. Length of each such rod L is equal to radius r. Due to rotation in magnetic field emf will induced across each rod. The end at the centre is positive and at circumference is negative.

emf across each rod is given by -

$$\varepsilon = \frac{1}{2} B\omega r^2 = BAf \qquad \ldots (9.20)$$

Because emf across each rod is same and these are connected in parallel. Hence resultant emf across centre and circumference of disc is $\varepsilon = BAf$

$$|\varepsilon| = B \times \pi r^2 \times \frac{\omega}{2\pi}$$

or $\varepsilon = \frac{1}{2}B\omega r^2$ . . . (9.21)

Example 9.12 : A 0.5 m long conductor rod is placed in uniform magnetic field of 0.04 T. The rod id rotating about its one end perpendicular to the plane of magnetic field with angular velocity 40 revolutions per second. Find emf induced across the rod.

Soltuion : Induced emf $E = BAf$

$$E = B\pi\ell^2 f$$

given $B = 0.04T$, $\ell = 0.5\,m$, f= 40 revolutions/sec

$$E = 0.040 \times 3.14 \times = 3.14 \times 0.4 = 1.256 \text{ V}$$

Example 9.13 : Diameter of a metallic gramo phone disc is 0.20 m, the disc is rotating at the rate 40 revolutions/minute in horizontal plane. Vertical component of earth magnetic field is 0.01 T. Find emf induced between centre and circumference of the disc.

Soltuion : Given in the question

$$B = 0.01T, \text{ radius } (r =) = \frac{0.20}{2} = 0.01\text{ m}$$

$$f = \frac{40}{60} \text{ rev./sec}$$

Induced emf $E = B\pi r^2 f$

$$= 0.01 \times 3.14 \times (0.1)^2 \times \frac{40}{60} = 2.09 \times 10^{-4} \text{ V}$$

9.10 Induced emf Due to Rotation of a rectangular Coil in Uniform Magnetic Field

In fig 9.12 (a) rectangular coil abcd is placed in uniform magnetic field such that its axis of rotation is perpendicualr to magnetic field. When coil rotates at angular velocity ω, the angle between plane of coil and direction of magnetic field changes continuously. Hence magnetic flux associated with coil also varies with time which produces induced emf in coil.

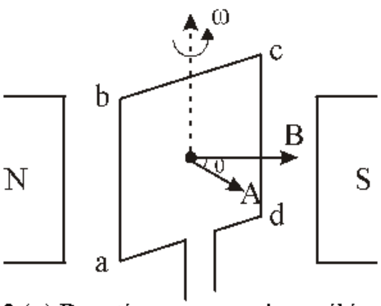


Fig. 9.12 (a) Rotating rectangular coil in uniform magnetic field

Suppose at any instant t the area vector $\vec{A}$ is at an angle θ with magnetic field $\vec{B}$. Number of turns in coil is N then flux passing through coil

$$\phi_B = N(\vec{B} \cdot \vec{A}) = NBA\cos\theta$$

$$\phi_B = NBA\cos\omega t \quad \text{. . . (9.22)}$$

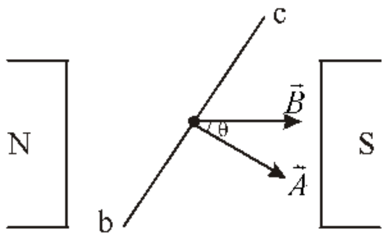


Fig. 9.12 (b) Position of coil at time t (top view)

Since magnituded of flux changes with time t the induced emf given by Faraday's law is-

$$\varepsilon = -\frac{d\phi_B}{dt}$$

$$\varepsilon = -N\frac{d}{dt}(BA\cos\omega t)$$

$$\varepsilon = NBA\omega\sin\omega t \quad \text{. . . (9.23)}$$

$$\varepsilon = \varepsilon_0 \sin\omega t \quad \text{. . . (9.24)}$$

Here ε_0 is maximum (Peak value of) induced emf.

$$\varepsilon_0 = NBA\omega \quad \text{. . . (9.25)}$$

A graph drawn between induced emf ε and time t is as shown in fig 9.13

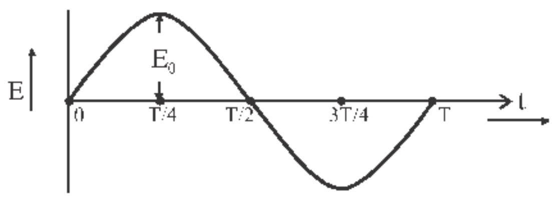


Fig. 9.13 Graph between induced emf and time

From fig 9.22 and 9.24 we see that induced emf is zero (minimum) when flux passing through coil is maximum and when flux passing through coil is minimum induced emf is maximum. If a resistor R is connected with coil then current in the circuit

$$I = \frac{\varepsilon}{R} = \frac{\varepsilon_0}{R}\sin\omega t$$

$$I = I_0 \sin\omega t \quad \text{. . . (9.26)}$$

Here I_0 is maximum (peak value of) induced emf.

The emf and current given by equations (9.23) and (9.26) are called alternating emf and alternating current respectively. This is also the principle of alternating current generator.

Example 9.14 : A coil of radius 0.15 m having 3000 turns is rotated at 250 rev/sec in the horizontal component of earth's magnetic field $B_H = 4\times10^{-5}$ T . Find out maximum induced emf.

Solution : Induced emf in a rotating coil

$$E = NBA\omega \sin \omega t$$

Maximum induced emf

$$E_0 = NBA\omega$$

We have given $N = 300, B = 4\times10^{-5}$ T, $r = 0.15$ m

$f = 250$ rev/sec

$$E_0 = 3000\times4\times10^{-5}\times3.14\times(0.15)^2\times2\times3.14\times250$$

$= 13.31$ V

Example 9.15 : If a conducting coil after rotating once on a frictionless axle continue to do so with angular frequency ω without any external torque. If coil is in magnetic field and not in a closed circuit than explain (i) Whether emf will be induced in the coil (ii) current will be induced in the coil (iii) is there a need of external torque of for continuous rotation. (iv) If the coil is in closed circuit than how its motion is effected.

Solution :

(i) Due to rotation of coil in magnetic field the angle between area of coil and manetic field changes, hence flux through coil also changes which produces induced emf.

(ii) In open circuit there is no induced current

(iii) When current is not flowing, energy is not spent and there is no need of torque for continuous rotation.

(iv) If circuit is closed induced current flows in the circuit hence according to Lenz's law angular velocity of coil decreases and external torque is required for continuous rotation.

9.11 Eddy Currents

When magnetic flux associated with a closed electric circuit varies with time current is induced in circuit. In the same way when bulk pieces of metallic conductor's are subjected to changing magnetic flux, induced currents are produced in them, their flow pattern resembles with swirling eddies in water. These plane of flow is normal to the direction magnetic field lines. These are called eddy currents. These currents opposes the motion of metallic pieces and also change in magnetic flux. These currents were discovered in 1895 by Focault so also called as Focault current.

9.11.1 Experimental Demonstration of Eddy Currents

Experiment - 1 In fig. 9.14 a metallic plane plate PQRS is placed perpendicular to uniform magnetic field B, extended in a limited region, when plate is pulled out of the field, the area of plate inside the field reduces hence flux associated with plate also reduces which produces eddy current, in the plate. Direction of eddy current is such that it oppose the motion of plate. This type of damping is called electromagnetic damping. In fig. 9.14 direction of eddy currents are according to fleming's right hand rule.

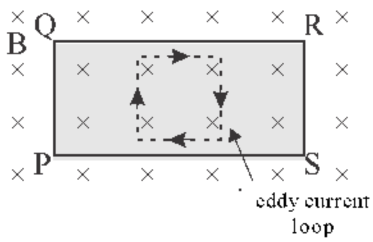


Fig. 9.14 Experimental demonstration of eddy currents

Experiment - 2 A copper or aluminium plate is allowed to swing like a pendulum between poles of a magnet. The area of plate associated with magnetic field changes with time. When plate enters and comes out of field, the flux through plate is minimum and when it is completely inside flux is maximum. Due to change in flux, eddy currents are induced in the plate, which damps its motion. When the plate swings into the region between the poles and when it swing out of the region some part of mechanical energy is converted into heat and plate stops after few oscillations. Here the oscillatory motion is damped. If rectangular slots are made in copper plate it swings comparatively more freely because this reduces the possible paths of the eddy currents considerably.

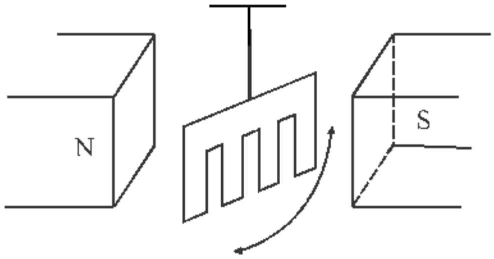


Fig. 9.15 Sloted copper plate to minimise eddy current

In electric devices, due to eddy current electrical energy is dissipated in the form of heat, hence eddy currents are reduced by laminations of core of electric devices. Core of transformer is made by thin plates placed one over other and separated by an insulating material like lacquer. Plane of thin plates of the core is kept parallel to magnetic field lines which increases resistance and reduces intensity of eddy currents.

9.11.2 Application of Eddy Currents

(i) Dead Beat Galvanometer : When current passes through a galvanometer its coil deflects and oscilates, when current is removed coil takes some time in coming to equilibirium position which is not required. So coil is made by winding its turns on a copper frame. Eddy currents are induced in the frame when coil rotates in magnetic field which damps the motion of coil and it comes to equilibirium position with in no time.

(ii) Brake in Electric Train :- The wheels of electric operated trains are joined with metallic drums. These drum rotates with wheels. Strong electro magnets are situated above the rails. When electro magnets are activiated, strong eddy current are induced in the drums which opposes the motion of drum which apply brake on trains. As there is no mechanical linkage the braking effect is smooth, free from wear-tear due to friction . Also the breaking action is efficient at high sbeeds as magnetic force increases with speed.

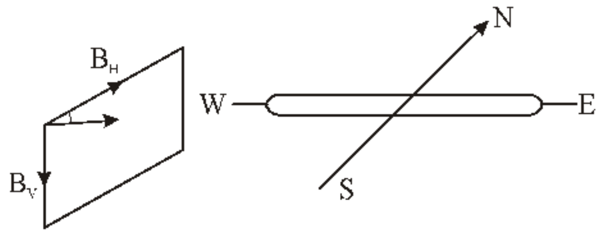
(iii) Diathermy :- Diathermy is production of heat in body tissuses for therapeutic purposes. A coil is wound on the part of the body when current flows through the coil which induced eddy current in tissues which produced heating of soft tissues.

(iv) Induction Furnace :- In induction furnace we get metals from ore. The metals to be melted are placed in high frequency variable magnetic field, which induces strong eddy currents in metals. These eddy currents produces large amount of heat which melts the metals.

9.11.3 Motion of Conducting Rod in Earth's Magnetic Field

When a conducting rod of length l moves with velocity v in earths magnetic field following cases are worth considering :-

Case - I Motion of conducting rod when placed in East-West direction and moves on perpendicular direction :-



(i) For the motion along East or West direction the conductor moves along its length hence area generated A = 0

$$\therefore \ \varepsilon = 0$$

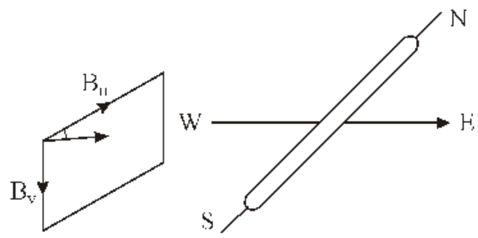
(ii) For the motion along north or south direction the conductor cut's earths vertical component B_v perpendicularly

$$\therefore \ \varepsilon = B_v Vl$$

(iii) The conductor crosses horizontal component of earth magnetic field B_{II} when it moves vertically upward or downward.

$$\varepsilon = B_H vl$$

Case II : When conductor is placed horizontal and moves in north-south direction and moves -



(i) While moving along east or west directions the conductor crosses vertical component of earth's magnetic field B_v

$$\varepsilon = B_v Vl$$

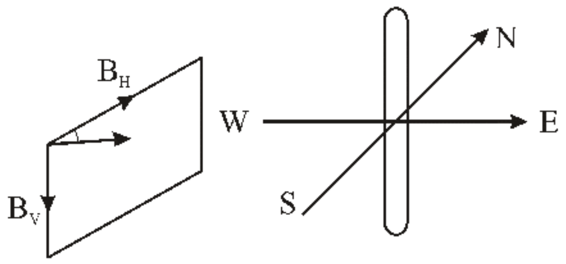
(ii) Along its length in north or south directions, then

$\therefore \ \varepsilon = 0$

(iii) Vertically upward or downward in magnetic meridian no component of earth's magnetic field is cut.

$\therefore \ \varepsilon = 0$

Case III : When conducting rod placed vertically and moves



(i) Along east or west direction the conductor cuts horizontal compnent of earths magnetic field B_H.

$\therefore \ \varepsilon = 0$

(ii) Along north and south direction, conductor moves in magnetic meridian

$$\varepsilon = B_H V \ell$$

(iii) The conductor moves along its length in vertically upward or downward directions, then

$\therefore \ \varepsilon = 0$

9.12 Self Induction

When current in a circuit or coil changes with time the magnetic field produced and flux asociated with the coil also changes. Due to this an emf is induced in the coil. This phenomenon is called self induction. According to Lenz's law the direction of induced emf is such that it opposes the change in linked magnetic flux.

Experimental Demonstration

In fig 9.16 a conducting coil is connected with a battery and key in series. As the key is made on a current flows throught the coil and the magnetic flux is associated with coil. Initially as key is made on current rises with time and induced magnetic flux also rises and emf is induced in the coil and induced current opposes the battery current in coil. when key is made off current in coil reduces to zero, the flux associated with coil also reduces and induced current flows in the direction of battery current. Direction of induced emf is shown in fig 9.16 when key is made on or off.

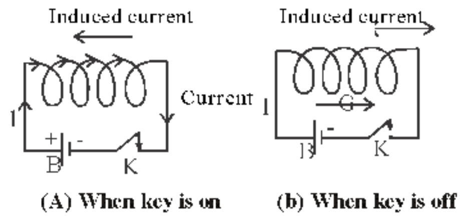


Fig 9.16 Phenomenon of self induction

9.12.1 Coefficient of self induction

Suppose current I flows through a coil. The magnetic field due to current is proportional to current and there by the magnetic flux associated with coil is directly proportional to current I, i.e.

$$\phi_B \propto I$$

$$\phi_B = LI$$

Here L is e constant of proportionality called coefficient of self induction or self inductance. Self inductance (L) depends upon shape and size of coil, marterial of core, medium and number of turns.

If I = 1 amp then $\phi_B = L$

Hence self inductance of a coil is equal to the magnetic flux linked with the coil when unit current flows through the coil. Due to variation of current, magnetic flux linked with coil changes hence by Faraday's law of electro magnetic induction -

Induced emf $\varepsilon = -\dfrac{d\phi_B}{dt}$

$$\varepsilon = -\frac{LdI}{dt} \quad \ldots (9.28)$$

negative sign shows that induced emf opposes the change in current

If $-\dfrac{dI}{dt} = 1$ than $|\varepsilon| = L$

Numberically, the self induction of a coil is equal to magnitude of induced emf due to unit rate of change of current.

When key is made on current flows in the coil, induced emf opposes the variation in current hence for the continuation of current work is required to be done against induced emf. This work is stored in coil in the form of magnetic potential energy.

Work done in time dt for maintaining current I :-

$$dW = |\varepsilon| I\,dt = \left(L\frac{dI}{dt}\right) I\,dt = LIdI$$

Hence work done to raise current from 0 to I :-

$$W = \int LIdI$$

$$W = \frac{1}{2}LI^2$$

If $I = 1$ than $L = 2W$

Hence self inductance of a circuit is equal to twice the work done against induced emf to maintaine unit current. Inductance is a scalar quantity its SI unit is Henery (H) or Vs/A or Wb/A and its dimensions are $[M^1L^2T^{-2}A^{-2}]$

9.12.2 Self Inductance of a Plane Circular Coil

Suppose a current I flows in a plane circular coil of radius r having N turns. Magnetic field at the centre of coil

$$B = \frac{\mu_0 NI}{2r}$$

magnetic flux linked with each turn of coil due to its own current

$$\phi'_B = BA$$

Total magnetic flux linked with coil is then

$$\phi_B = N\phi'_B = NBA = \frac{\mu_0 NI}{2r} \times N(\pi r^2)$$

$$\phi_B = \frac{\mu_0 N^2 I \pi r}{2}$$

From definition of self inductance

$$\phi_B = LI$$

$$L = \frac{\mu_0 \pi N^2 r}{2}$$

If some material of magnetic permeability μ is filled in the coil then

$$L = \frac{\mu \pi N^2 r}{2} \qquad ...(9.30)$$

9.12.3 Self Inductance of a Current Carrying Solenoid

Suppose I current flow through a solenoid of area of cross section A, length l and having number of turns N, than magnetic field inside the solenoid at its axis is

$$B = \mu_0 nI$$

Where $n = \frac{N}{\ell}$ is number of turns per unit length.

Total magnetic flux linked with solenoid

$$\phi_B = N(BA)$$

$$\phi_B = \frac{\mu_0 NI}{\ell} \times NA$$

$$\phi_B = \frac{\mu_0 N^2 A}{\ell} I$$

If self inductance of solenoid is L than

$$\phi_B = LI$$

$$L = \frac{\mu_0 N^2 A}{\ell} = \mu_0 n^2 A\ell \qquad ...(9.31)$$

If solenoid is filled with material of magnetic permeability μ than

$$L = \mu n^2 A\ell \qquad ...(9.32)$$

In resistance box the resistance coil are doutly twisted to remove the effect of self induction.

In wheat stone bridge experiment, first we press the battery key and then galvanometer key to remove the effect of induced current.

9.13 Mutual Inductance

If a variable current flows in a circuit or coil, the flux linked with an other coil in its vicinity also changes and emf is incuded in the second coil or circuit. This phenomenon is called mutual induction.

Coil in which current varies is called primary coil and the coil in which emf is induced due to mutual induction is called secondary coil.

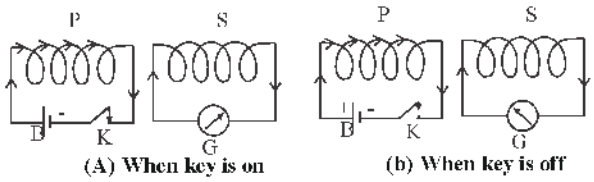


Fig. 9.17 Experiment for mutual induction

In fig. 9.17 coil P in primary circuit is connected to battery B through key K and in secondary circuit coil S is connected to galvanometer G. Both coils are placed near be each other. As key K in primary circuit is pressed there is momentary deflection in the galvanometer G in secondary circuit. When key K is released there is again momentary deflection in galvanometer but in opposite direction. when key K is kept pressed continuously constant current flows in primary coil P, there is no deflection in galvanometer.

Reason for the above results is that when intially key is on made current flows in primary coil magnetic flux around the coil increases and flux linked with secondary coil also increases, which induces emf and current in secondary coil. In the same way when key is made off the magnetic flux through secondary coil decays hence emf and current are induced in secondary coil. In both cases the direction of induced emf and current is such that they opposes the change in magnetic flux. The current is induced in secondary coil only when the magnetic field or magnetic flux of primary coil changes due to its current. This phenomenon is used in transformers and induction coils.

9.13.1 Coefficient of Mutual Inductance

If position, shape and orientation of primary coil C_1 and secondary coil C_2 remains unchanged and current in Coil C_1 is I_1 than magnetic flux ϕ_2 associated with coil C_2 is directly proportional to I_1.

$$\phi_2 \propto I_1$$

$$\phi_2 = MI_1 \qquad \ldots (9.33)$$

Here M is a constant of proportionality called coefficient of mutual induction or mutual inductance between the coils. It depends on number of turns in both the coils, area of secondary coil and nature of medium.

If current in coil C_1 changes with time than flux ϕ_2 linked with coil C_2 also changes, therefore induced emf in coil C_2 is

$$\varepsilon_2 = -\frac{d\phi_2}{dt}$$

or $$\varepsilon_2 = -M\frac{dI_1}{dt} \qquad \ldots (9.34)$$

Negative sign shows that the direction of induced emf in secondary coil is such that it opposes variation of current in primary coil -

From eq. 9.33 If $I_1 = 1$ than $M = \phi_2$, numerical value of mutual inductance between two coils is equal to the flux linked with secondary coil when unit current passes through primary coil.

From eq. 9.34

If $\frac{dI_1}{dt} = 1$ than $|\varepsilon| = M$

Hence numerical value of mutual inductance is equal to induced emf in secondary coil when there is unit rate of change of current in primary coil. SI unit of M is *Wb/A* or *Vs/A* or Henery (H) and its dimension are $[M^1L^2T^{-2}A^{-2}]$.

9.13.2 Mutual Inductance between Two Caoxial Solenoids1

Suppose there are two air cored coaxial solenoids S_1 and S_2. Number of turns in S_1 and S_2 are N_1 and N_2 and length and area of cross section of both the coils are and A respectively. Both the coils are wound in such a way that when current flows in coil S_1 it produces magnetic flux which is completely linked with coil S_2. Magnetic field at the axis of coil S_1 when I_1 current flows through it -

$$B_1 = \frac{\mu_0 N_1}{\ell} I_1 = \mu_0 n_1 I_1$$

Magnetic flux associated with S_2 due to field B_1 -

$$N_2\phi_2 = N_2 B_1 A = (\mu_0 n_1 I) N_2 A = \frac{\mu_0 N_1 N_2 A I_1}{\ell}$$

According to defination of mutual induction

$$N_2\phi_2 = MI_1$$

Hence mutual inductance

$$M = \frac{\mu_0 N_1 N_2}{\ell} A$$

Example 9.16 : Self inductance of a coil is 20 H. For obtaining 100 V induced emf to what value the current is to be reduced in it in 1 second from an initial value of 10 A.

Solution : Induced emf $\varepsilon = L - \frac{dI}{dt} = L\frac{\Delta I}{\Delta t}$

Here given $L = 20$ H

$I_1 = 10$ A, $I_2 = I$

E=100 V

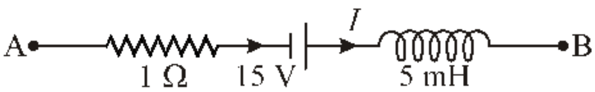
$dt = 1$ s

$$100 = 20\left[\frac{10 - I_2}{1}\right]$$

$10 - I_2 = 5$

$I_2 = 10 - 5 = 5$ A

Example 9.17 : In the fig shown current at some instant in circuit is $I - 5$ A and is decaying at a rate $10^3 A/S$. Then find $V_B - V_A$.



Solution : Rate of change of current in coil

$$\frac{dI}{dt} = -10^3 \text{ A/s}$$

Voltage accross resistance R

$V = IR = 5 \times 1 = 5$ V

Voltage accross termials of cell = 15 V

Voltage across indutance coil

$$= -L\frac{dI}{dt} = -\left(5 \times 10^{-3}\right) \times \left(-10^3\right) = 5 \text{ V}$$

terminal B is at higher potantial

$$V_B - V_A = 5\text{ V} + 15\text{ V} + (-5)V = 15V$$

Example 9.18 : An air cored solenoid of radius 1 cm has 100 number of turns. Its length is 60 cm. Find self inductance of solenoid.

Solution : Self inductance $L = \frac{\mu_0 N^2 A}{\ell}$

Given $N = 100,\ \ell = 0.60$ m,

$A = \pi r^2 = 3.14 \times (.01)^2 m^2$

$$L = \frac{4\pi \times 10^{7} \times (100)^2 \times \pi(0.01)^2}{0.60}$$

$= 65.73 \times 10^{7}$

$= 6.573 \times 10^{8}$ H

Important Points

1. When vector area $\vec{A}$ is placed in magnetic field $\vec{B}$, $\vec{A}$ is at an angle θ with $\vec{B}$, magnetic flux passing through $\vec{A}$ is given by

   $\phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$

2. Faraday Law :- According to Faraday's law of electro magnetic induction induced emf in a coil of N turns is equal to rate of change of magnetic flux passing through the coil

   Induced emf $\varepsilon = -N\frac{d\phi_B}{dt}$

3. When magnetic flux linked with a circuit changes, emf is induced in the circuit. If circuit is closed current is also induced. The phenomenon is called electro magnetic induction.

4. Lenz's law :- In electro magnetic induction, direction of induced emf and current is such that they oppose the cause due to which these are produced.

5. In electro magnetic induction

Induced current $I = \frac{E}{R} = -\frac{N}{R}\frac{d\phi_B}{dt}$

Induced charges $q = I\,dt = \frac{-N}{R} d\phi_B$

6. Right hand rule :- According to this rule the index finger, central finger and thumb of right hand are held out perpendicular to each other. If index finger showns direction of magnetic field and thumb shows direction of motion then central finger shows direction of induced current.

7. If conducting rod of length l moves with velocity v in uniform magnetic field B perpendicular to direction of field and its own length than induced emf accross the rod

ε = B v l

Induced emf in a rod moving at an angle θ with the direction of magnetic field is given by -

ε = B v l sin θ

8. Induced emf due to motion of rectangular loop with velocity v in non uniform magnetic field

$\varepsilon = (B_1 - B_2)\, vl$

here B_1 and B_2 are magnetic field on the two arms respectively.

9. Work done for moving a rectangular loop in magnetic field appears as electrical energy in the circuit and finally spent in the form of heat energy

$$W = H = \frac{(B_1 - B_2)^2 \ell^2 v^2 \Delta t}{R}$$

10. Due to rotation of a conducting rod of length L with with angular velocity ω in uniform magnetic field B, induced emf between the ends is given by -

$$E = \frac{1}{2} B\omega L^2 = BAf$$

11. Induced emf between centre and circumference of a metallic disc of radius r rotating with angular velocity ω in uniform magnetic field B

$\varepsilon = 1/2\, B\,\omega\, L^2 = BAf$

12. If a rectangular conducting coil of N turns and area of cross section A rotates with angular velocity ω in uniform magnetic field B then induced emf

$\varepsilon = NBA\omega \sin \omega t$

13. Circulating currents are induced in bulk metallic pieces placed in changing magnetic field. In these loops, electrical energy is spent in form of heat, these currents are called eddy currents. They are used in brakes of electric train, induction furnace, galvanometer etc.

14. Inductance is equal to ratio of linked magnetic flux and current. It is of two types (i) self inductance (ii) Mutual inductance.

15. Self inductance of a coil is equal to the magnetic flux associated with coil when unit current flows in it.

16. When current through a coil changes it produces an opposing emf which is given by

$$E = -L\frac{dI}{dt}$$

17. Work done against induced emf to maintain current I in the coil

$$W = \frac{1}{2}LI^2$$

18. Self inductance of a solenoid of length l and number of tunrs per unit length n is

$$L = \mu_0 n^2 Al$$

here A is area of cross section of solenoid.

19. When current in a coil or circuit changes then associated magnetic flux in another coil in its vicinity also changes due to this change, emf or current is induced in the second coil. This phenomenon is called mutual induction.

20. Induced emf due to phenomenon of mutual induction

$$E_2 = -M_{21}\frac{dI_1}{dt}$$

here M_{21} is mutual inductance of second coil relative to first coil.

21. Mutual inductance between two coaxil solenoid.

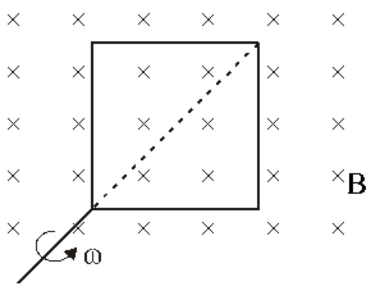
$$M_{21} = M_{12} = \frac{\mu_0 n_1 n_2 A}{l} \text{ H}$$

Questions for Practice

Multiple Choice Questions -

1. A conducting rod is moving with velocity V in a magnetic field B. An emf is induced across its ends when -
   (a) v and B are parallel
   (b) v and B perpendicular to each other
   (c) v and B are in opposite direction
   (d) All of the above

2. A square loop of length X. Loop is rotating with angular velocity ω about its diagonal in a perpendicular magnetic field as shown in fig. Find magnetic flux associated with loop at any moment number of turns in the loop is 20.



   (a) 20 B x (b) 10 Bx^2
   (c) 20 $Bx^2 \cos \omega t$ (d) 40 Bx^2

3. The unit of ratio of magnetic flux and resistance is same as which of the following quantity -
   (a) Charge (b) Potential difference
   (c) Current (d) Magnetic field

4. In electromagnetic induction the magnitude of induced emf depends only on -
   (a) Resistance of the conductor
   (b) Magnitude of magnetic field
   (c) Orientation of conductor realtive to direction of magnetic field
   (d) Rate of change of linked flux

5. When a bar magnet enters inside a coil, the induced emf in coil does not depends on
   (a) Velocity of magnet
   (b) Number of turns in coil

(c) Magnetic moment of magnet

(d) Specific resistance of wire of coil

6. A copper wire coil is moving in a uniform magnetic field parallel to the field then what is the value of induced current -

(a) Infinite (b) Zero

(c) Equal to magnetic field

(d) Equal to area of cross section of coil

7. Lenz's law gives -

(a) Magnitude of induced current

(b) Magnitude of induced emf

(c) Direction of induced current

(d) Magnitude and direction of induced current both

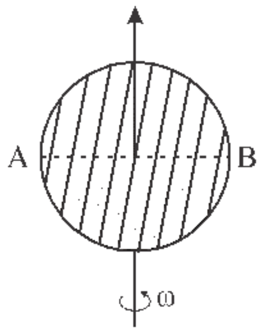
8. A copper wire coil C and a wire are place in the plane of paper as shown in fig. If current in wire increases from 1 A to 2 A along the direction shown in fig, than what is the direction of induced current in coil -

(a) Clockwise (b) Anticlock wise

(c) Current will not be induced

(d) None of above

9. If a disc is rotated about its axis and if magnetic field is uniform and along the axis of rotation than what the potential difference between the edges of diameter AB -



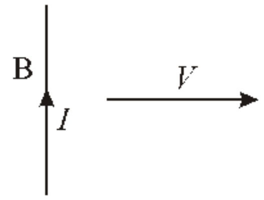
(a) Zero

(b) Half of potential difference between centre and circumference

(c) Double of potential difference between centre and circumference

(d) None of above

10. A conducting wire is moving towards right in magnetic field B. If direction of induced current is as shown in fig then the direction of magnetic field is-



(a) In the plane of paper towards left

(b) In the plane of paper towards right

(c) Perpendicular to the plane of paper, downward

(d) Perpendicular to the plane of paper, upward

11. In a electric transmission line current is flowing along north direction. On considering earth magnetic field negligible be find the direction of magnetic field above the electric line -

(a) Along east (b) Along west

(c) Along north (d) Along south

12. A coil is rotating in a uniform magnetic field. What is the phase differnce between induced emf and linked magnetic flux-

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) π

13. Current in a coil of self inductance 2×10^{-3} H rises uniformly in 0.1 sec 1 A than what is the magnitude of induced emf.

(a) 2 V (b) 0.2 V

(c) 0.02 V (d) Zero

14. If a coil having 100 turns produces a magnetic flux 5×10^{3} Maxwell, by 5 A current. What is its self inductance

(a) 0.5×10^{-3} H (b) 2×10^{-3} H

(c) Zero (d) 10^{-3} H

15. The magnetic flux passing perpendicularly through a coil changes with time as $\phi = 10t^2 + 5t + 1$ here t is in seconds and ϕ mWb then induced emf at t = 5s is -

(a) 1 V (b) 0.105 V

(c) 2 V (d) 0 V

Very Short Answer Type Questions -

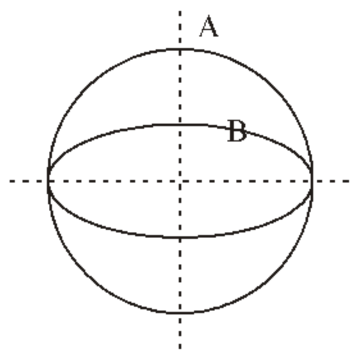
1. If current in inductance is doubled than how many times the stored energy increases?
2. When an electric circuit is suddenly breaked than why there is sparking?
3. How the mutual inductance between two coils can be increased?
4. On doubling the area of cross section while keeping the some number of turns in a coil, what will be the value of self inductance?
5. How we can reduce the effect of eddy currents in the core of transformer?
6. A metallic and a non metallic coins are droped from same height towards earth surface. Which coin reaches earlier on earth and why?
7. Why self induction is called electrical inertia?
8. On what factors and in what way the the self inductance of a solenoid depends?
9. In a wire current flows at high voltage. As current is swithed on in the wire why the bird sitting on wire fly?
10. Write down dimensions of L/R here L is self inductance and R is resistance.
11. When a rectangular loop moves in a uniform magnetic field with constant speed than what is the magnitude of induced emf?
12. In what way two coils are wound so that induced emf is maixmum?
13. If a coil or rectangular loop rotates in magnetic field what factors effects theinduced emf in it?
14. A straight and long wire is dropped freely in gravitional field keeping in north-south direction, why emf is induced in the wire?
15. How eddy currents are used to make the galvanometer dead beat?

Short Answer Type Questions -

1. What do you mean by electro magnetic induction? Write down Faraday's laws for electro magnetic induction and magnitude of induced emf?
2. If a coil is removed from magnetic field (i) with high rapidly (ii) slowly, then in which situation induced emf and work done is more.
3. Write down Lenz's law for electro magnetic induction and explain that Lenz's law follows the law of energy conservation.
4. When a metallic plate is pulled out of a uniform magnetic field or enters in a uniform magnetic field why we experience opposing force?
5. What is the reason

   (i) Resistance wire coils are doubly twisted in resistance boxes

   (ii) In wheatstone bridge why cell key is pressed first and then galvanometer key?
6. Write down Fleming's right hand rule for the direction of induced current?
7. Define mutual inductance and write its unit and dimensions.
8. A conducting rod of length L is rotating with uniform angular velocity ω in magnetic field B in such a way that the plane of rotation is perpendicular to magnetic field then find out induced emf between its ends?
9. Coils A and B are placed perpendicular to each other as shown in fig. If current in any one coil varies than will current be induced in other coil and why?



10. What factors effects the mutual inductance between two coil S_2 explain?
11. If the self inductance of a coil is 1 H, what do you understand by it.
12. Prove that induced charge $q = \frac{N}{R}(\phi_1 - \phi_2)$

    when flux associated with a coil changes from

ϕ_1 to ϕ_2. Here N is number of turns in coil, R is its resistance.

13. Prove that law of conservation of energy holds good when a rectangular coil moves perpendicular to a non uniform magnetic field with constant velocity?

Essay Type Questions -

1. Find out induced emf due to motion of conducting rod in uniform magnetic field with a constant velocity. How, we can find the direction of induced emf.
2. A rectangular loop is moving perpendicular to a non uniform magnetic field with constant velocity. Find out expression for induced emf and current and also prove that the law of conversation of energy holds good here.
3. If a rectangular coil of area A and number of turns N is rotating in a uniform magnetic field with a constant angular velocity ω. Prove that induced emf in the coil is $NBA\omega \sin \omega t$.
4. What is meant by self induction? Explain the phenomenon self induction through an experiment and find out self inductance of solenoid?
5. What are eddy currents? Write down thier two uses. How unwanted eddy currents are reduced in transformers?

Answer (Multi Choice Questions)

1. (b) 2. (c) 3. (a) 4. (d) 5. (d) 6. (b)

7. (c) 8. (a) 9. (c) 10. (c) 11. (a) 12. (b)

13. (c) 14. (d) 15. (b)

Numerical Questions

1. A window of metallic frame (120 x 50 cm) is on a wall which is parallel to magnetic meridian. Total resistance of frame is 0.01 Ω. When window is opened at 90^0 then find the amount of charge flown in the frame.

(If H = 0.36 G)

2. The magnetic flux passing through a coil of 50 turns is given by $\phi_B = 0.02 \cos 100\pi t$ Wb

Find out -

(a) Maximum induced emf

(b) Induced emf at time t = 0.01 S

(c) Induced current at t= 0.005 S (if external resistance is 100 Ω)

(3.14 V, zero 13.14 A)

3. A coil of 50 turns and area 0.2 m^2 in placed perpendicular to a 0.6 T magnetic field the resistance of circuit of coil is 10 Ω then find out induced charges - (a) When coil rotates by 180^0 (b) Coil is pulled out of magnetic field

(1.20 C, 0.60 C)

4. A conductor of length $-3\hat{k}$ m is moving with velocity $i + 2\hat{j} + 3\hat{k}$ m/s in $\hat{i} + 3\hat{j} + \hat{k}$ T magnetic field. Find out potential difference across the ends of conductor.
5. A rectangular coil of 1000 turns and 0.02 x 0.1 m^2 size is rotating with 4200 revolutions per mintues in 0.2 T magnetic field. Find the maximum induced emf in coil.

(1758.4 V)

6. One meter long conducting rod rotating with angular velocity 50 rotations/sec in a place perpendicular to a magnetic field of 0.001 T about its one of end. Find the induced emf across it ends.

(0.157 V)

7. Length and diameter of a solenoid are 1 m and 0.05 m respectively there are 500 turns/cm in the solenoid. Find the magnetic flux when 3 A current flows through it.
8. Length of a solenoid of radius 2cm and 100 number of turns is 50cm. Find the self indicates of solenoid in vacuume.
9. Two coils are wound on iron core. Their mutual inductance is 0.05 H. If current through one of the coil changes from 2 A to 3 A in 10^{-2} sec than find out induced emf in the other coil.

(-50 V)

10. Wires are wound on a soft iron rod of length 0.1 m and radius 0.01 m, to form a coil. If relative permeability of soft iron is 1200 then find out number of turns in coil.

(Self inductance of coil 0.25 H)

11. A metallic disc of diameter 15 cm is rotating in

horizontal plane with $\frac{100}{3}$ rotations per minutes. It vertical component of magnetic field is 0.01 Wb/m^2 than find out induced emf between centre and circumference of the coil.

$(9.75\times10^{-3}\ V)$

12. A 20 cm conducting long wire is placed perpendicular to $5\times10^{-4}\ Wb/m^2$ magnetic field. Wire is moving perpendicular to its length and magnetic field. If wire moves 1m is 4 s than find induced emf between its ends.

$(2.5\times10^{-5}\ V)$

13. A 12 m long metallic rod is moved from west to east with speed 15 km/h by keeping it (i) vertical (ii) horizontal. If horizontal component of earths magnetic field is $0.5\times10^{-5}\ Wb/m^2$ there find out induced emf across rod in each situation.

$(4.16\times10^{-5}\ V, 0)$

14. If current through primary coil is reduced from 5 A to zero is 2 ms than induced emf in the secondary coil is 25 kV. Find out mutual inductance of the coils.

(10 H)

15. Self inductance of a coils is 2 H, variation of current with time in the coil is shown in following graph. Draw graph for the variation of induced emf with time.