3. Geometry

Exercise 3.1

1 A. Question

Choose the correct answer:

Which of the following will be the angles of a triangle?

A. 35°, 45°, 90°

B. 26°, 58°, 96°

C. 38°, 56°, 96°

D. 30°, 55°, 90°

Answer

Formula used: Angle Sum Property = ∠A + ∠B + ∠C = 180°

Option A: 35° + 45° + 90° = 170°. Hence, this cannot be correct because sum of all the angles of triangle.

Option B: 26° + 58° + 96° = 180°. Hence, this is correct because sum of all the angles of triangle.

Option C: 38° + 56° + 96° = 190°. Hence, this cannot be correct because sum of all the angles of triangle.

Option D: 35° + 45° + 90° = 175°. Hence, this cannot be correct because sum of all the angles of triangle.

1 B. Question

Choose the correct answer:

Which of the following statement is correct?

A. Equilateral triangle is equiangular.

B. Isosceles triangle is equiangular.

C. Equiangular triangle is not equilateral.

D. Scalene triangle is equiangular

Answer

Reason: All angles of an equilateral triangle are equal.

1 C. Question

Choose the correct answer:

The three exterior angles of a triangle are 130°, 140°, x° then x° is

A. 90°

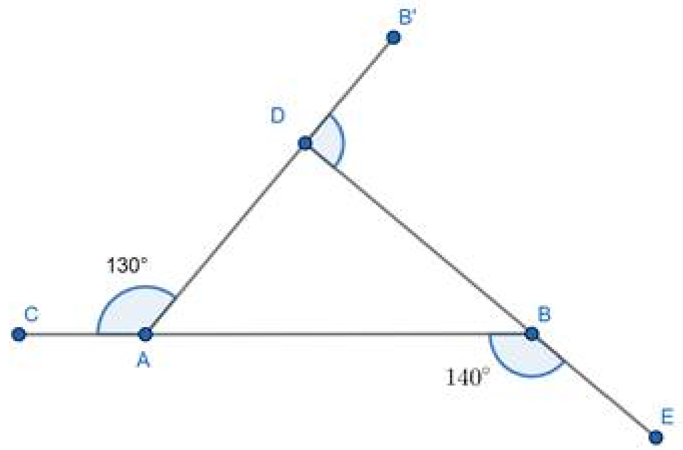
B. 100°

C. 110°

D. 120°

Answer

Angle Sum Property = ∠A + ∠B + ∠C = 180°[T1]



∠ DAC = 130° and ∠ ABE = 140°

∠ DAC + ∠ DAB = 180° (sum of the angle on a straight line at point is 180°)

∴ ∠DAB = 180° - ∠ DAC

⇒ ∠ DAB = 180° - 130° = 50°

Similarly,

∠ ABE + ∠ ABD = 180° (sum of the angle on a straight line at point is 180°)

∴ ∠ABD = 180° - ∠ ABE

⇒ ∠ DAB = 180° - 140° = 60°

∠ BDB' = ∠ DAB + ∠ DBA

(if a side of a triangle is produced, the exterior angle so formed, is equal to the sum of the two interior opposite angles)

∠ x° = 50° + 60° = 110°

Hence, option C is correct.

1 D. Question

Choose the correct answer:

Which of the following set of measurements will form a triangle?

A. 11 cm, 4 cm, 6 cm

B. 13 cm, 14 cm, 25 cm

C. 8 cm, 4 cm, 3 cm

D. 5 cm, 16 cm, 5 cm

Answer

Any Two sides of triangle together are greater than the third side.

Option A: AB + BC > AC = 11 + 4 > 6 = 15 > 6true

BC + AC > AB = 4 + 6 > 11 = 10 > 11not true

AC + AB > BC = 11 + 6 > 4 = 17 > 4 true.

∴, this cannot be true because only condition is satisfied.

Option B: AB + BC > AC = 13 + 14 > 25 = 27 > 25 true

BC + AC > AB = 14 + 25 > 13 = 39 > 13 true

AC + AB > BC = 25 + 13 > 14 = 38 > 13 true.

∴, this is true because all condition is satisfied.

Option C: AB + BC > AC = 8 + 4 > 3 = 12 > 3 true

BC + AC > AB = 4 + 3 > 8 = 7 > 8 not true

AC + AB > BC = 8 + 3 > 4 = 11 > 4 true.

∴, this cannot be true because only condition is satisfied.

Option D: AB + BC > AC = 5 + 16 > 5 = 21 > 5true

BC + AC > AB = 16 + 5 > 11 = 21 > 5 true

AC + AB > BC = 5 + 5 > 16 = 10 > 16 not true.

∴, this cannot be true because only condition is satisfied.

1 E. Question

Choose the correct answer:

Which of the following will form a right-angled triangle, given that the two angles are

A. 24°, 66°

B. 36°, 64°

C. 62°, 48°

D. 68°, 32°

Answer

For the triangle to be right-angled triangle one angle is 90° and sum of the two angles should be 90°

Option A: 24° + 66° = 90°. This option is correct.

Option B: 36° + 64° = 100°. This option is not correct.

Option C: 62° + 48° = 110°. This option is not correct.

Option D: 68° + 32° = 100°. This option is not correct.

2. Question

The angles of a triangle are (x – 35) °, (x – 20) ° and (x + 40) °.

Find the three angles.

Answer

Theorem 1: The sum of three angles is 180°.

i.e. ∠A + ∠B + ∠C = 180°

⇒ (x – 35)° + (x – 20)° + (x + 40)° = 180°

⇒ x – 35 + x – 20 + x + 40 = 180

⇒ 3x – 55 + 40 = 180

⇒ 3x – 15 = 180

⇒ 3x = 180 + 15

⇒ 3x = 195

⇒ $x = \frac{195}{3}$

⇒ x = 15

3. Question

In Δ ABC, the measure of ∠A is greater than the measure of ∠ B by 24° . If exterior angle ∠C is 108°. Find the angles of the Δ ABC.

Answer

Theorem 2:

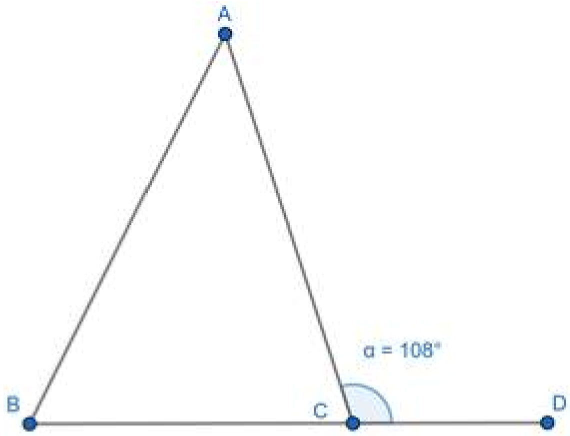
If a side of a triangle is produced, the exterior angle so formed, is equal to the sum of the two interior angle.

Let the ∠B be x.

∴ ∠ B = x

∴ ∠ A = ∠ B + 24°

= x + 24°



Ext. [T2] ∠ C = ∠ B + ∠ A

⇒ 108° = x + x + 24°

⇒ 108° = 2x + 24°

⇒ 2x = 108° - 24°

⇒ 2x = 84°

⇒ $x = \frac{84°}{2}$

⇒ x = 42°

4. Question

The bisectors of ∠ B and ∠ C of a Δ ABC meet at O.

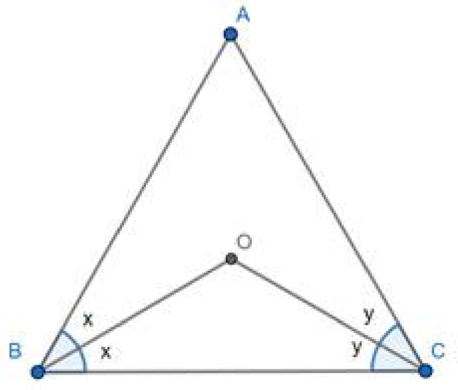
Show that BOC = $90^o + \frac{\angle A}{2}$.

Answer

Theorem 1:

The sum of all three angles is 180°.

Let the ∠ B and ∠ C be 2x and 2y respectively.



In Δ ABC

∠A + ∠B + ∠C = 180° (Angle Sum Property)

⇒ ∠ A + 2x + 2y = 180°

⇒ 2x + 2y = 180° - ∠A

⇒ 2(x + y) = 180° - ∠ A

⇒ $x + y = \frac{180° - \angle A}{2}$

⇒ $x + y = 90° - \frac{\angle A}{2}$ ...(1)

In Δ OBC

∠OBC + ∠ OCB + ∠ BOC = 180° (Angle Sum Property)

⇒ x + y + ∠ BOC = 180°

⇒ $90 - \frac{\angle A}{2} + \angle BOC = 180°$

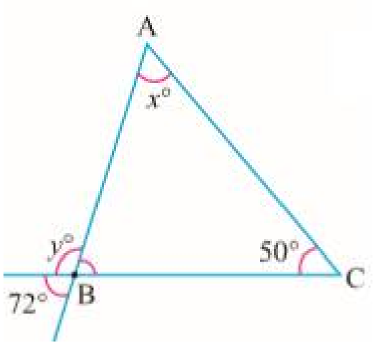
⇒ $\angle BOC = 180° - 90° + \frac{\angle A}{2}$

⇒ $\angle BOC = 90° + \frac{\angle A}{2}$

Hence Proved.

5 A. Question

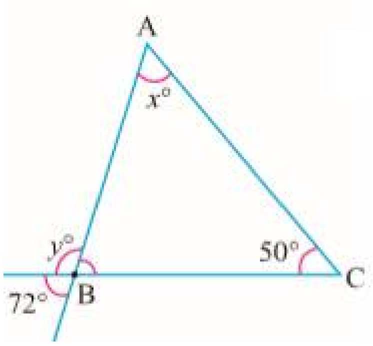
Find the value of x° and y° from the following figures:



Answer

Theorem 2:

If a side of a triangle is produced, the exterior angle so formed, is equal to the sum of the two-interior angle.



Ext. ∠ B = ∠ A + ∠ C

⇒ y° = x° + 50° ...(1)

Ext. ∠ B + 72° = 180°

⇒ Ext. ∠ B = 180° - 72°

⇒ Ext. ∠ B = 108°

⇒ y° = 108°

Put y = 108° in (1)

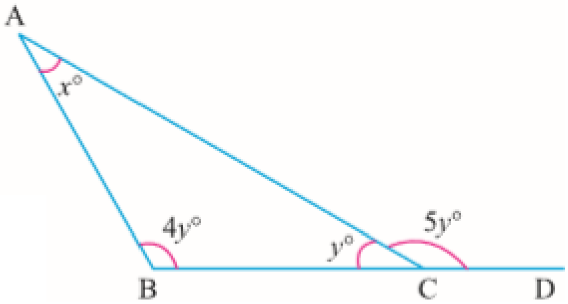
⇒ 108° = x° + 50°

⇒ x° = 108° - 50°

⇒ x° = 58°

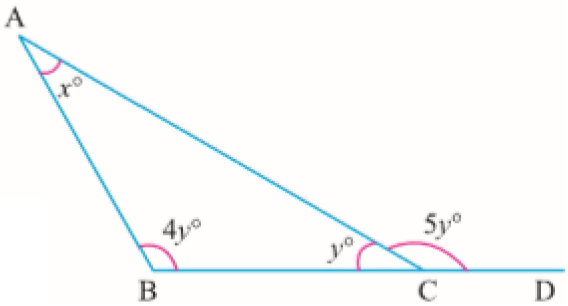
5 B. Question

Find the value of x° and y° from the following figures:



Answer

theorem 1: Sum of all the angles of triangle is 180°



∠ ACD + ∠ ACB = 180°

⇒ 5y° + y° = 180°

⇒ 6y° = 180°

⇒ $y° = \frac{180°}{6}$

⇒ y° = 30°

In Δ ABC

∠ A + ∠ B + ∠ C = 180°

⇒ x° + 4y° + y° = 180°

⇒ x° + 4 × 30° + 30° = 180°

⇒ x° + 120° + 30° = 180°

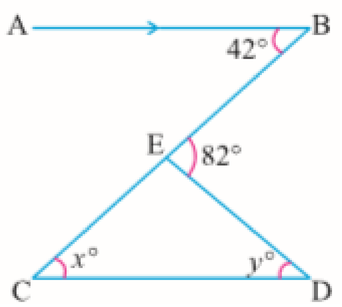
⇒ x° + 150° = 180°

⇒ x° = 180° - 150°

⇒ x° = 30°

5 C. Question

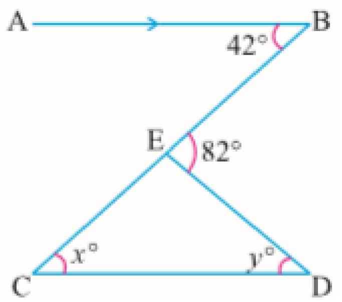
Find the value of x° and y° from the following figures:



Answer

Theorem 2:

If a side of a triangle is produced, the exterior angle so formed, is equal to the sum of the two interior angle.



∠ C = ∠ B (Alternate interior angles)

⇒ x° = 42°

In Δ CDE

Ext. ∠ E = ∠ C + ∠ D

⇒ 82° = 42° + y°

⇒ y° = 82° - 42°

⇒ y° = 40°

6. Question

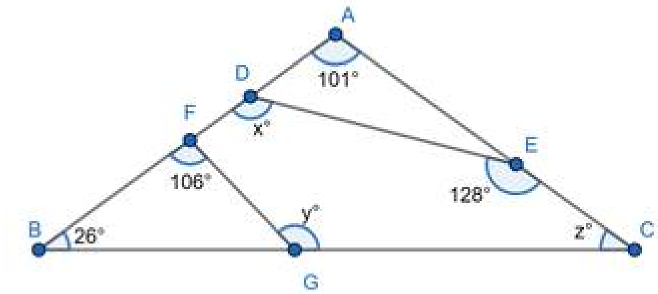
Find the angles x°, y° and z° from the given figure.

Answer

Theorem 1: Sum of all the angles of triangles is 180°

Theorem 2:

If a side of a triangle is produced, the exterior angle so formed, is equal to the sum of the two interior angle.



In Δ ABC

∠ A + ∠ B + ∠ C = 180° (Angle Sum Property)

⇒ 101° + 26° + z° = 180°

⇒ 127° + z° = 180°

⇒ z° = 180° - 127°

⇒ z° = 53°

In Δ FBG

Ext. ∠ G = ∠ B + ∠ F

⇒ y° = 26° + 106°

⇒ y° = 132°

∠ AED + ∠ DEC = 180°

⇒ ∠ AED + 128° = 180°

⇒ ∠ AED = 180° - 128°

⇒ ∠ AED = 52°

In AED

Ext. ∠ D = ∠ A + ∠ C

⇒ x° = 101° + 52°

⇒ x° = 153°

Exercise 3.2

1 A. Question

Choose the correct answer:

In the isosceles ΔXYZ, given XY = YZ then which of the following angles are equal?

A. ∠X and ∠Y

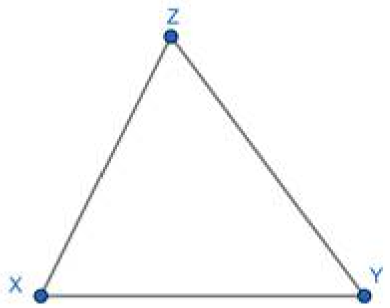
B. ∠Y and ∠Z

C. ∠Z and ∠X

D. ∠X, ∠Y and ∠Z

Answer

Reason: Angles opposite XY and YZ are angle X and Z respectively.[T3]



1 B. Question

Choose the correct answer:

In Δ ABC and ΔDEF, ∠B = ∠E, AB = DE, BC = EF. The two triangles are congruent under _____ axiom

A. SSS

B. AAA

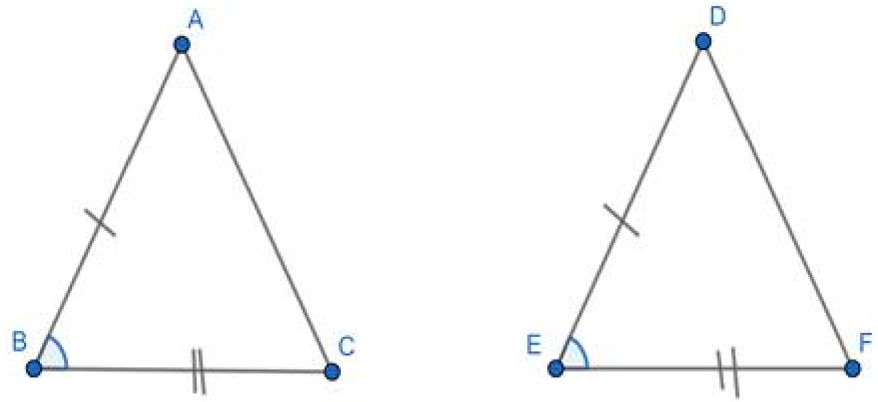
C. SAS

D. ASA

Answer

option C.

If any two sides and the included angle of a triangle are respectively equal to any two sides and the included angles of another triangle then the two triangles are congruent.



1 C. Question

Choose the correct answer:

Two plane figures are said to be congruent if they have

A. the same size

B. the same shape

C. the same size and the same shape

D. the same size but not same shape

Answer

Reason: If two geometrical figures are identical in shape and size then they are said to be congruent.

1 D. Question

Choose the correct answer:

In a triangle ABC, $\angle A = 40^{0}$ and AB = AC, then ABC is _____ triangle.

A. a right angled

B. an equilateral

C. an isosceles

D. a scalene

Answer

In a triangle, When two sides are equal then triangle are said to be an isosceles triangle.

1 E. Question

Choose the correct answer:

In the triangle ABC, when ∠A = 90^{o} the hypotenuse is _____

A. AB

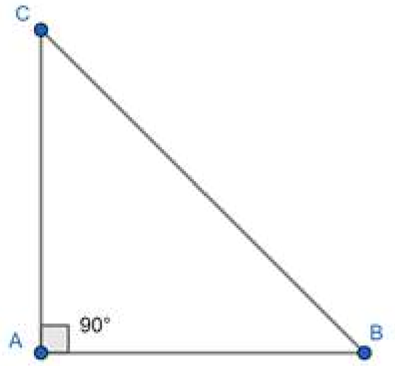
B. BC

C. CA

D. None of these

Answer

option B

[T4] 

1 F. Question

Choose the correct answer:

In the Δ PQR the angle included by the sides PQ and PR is

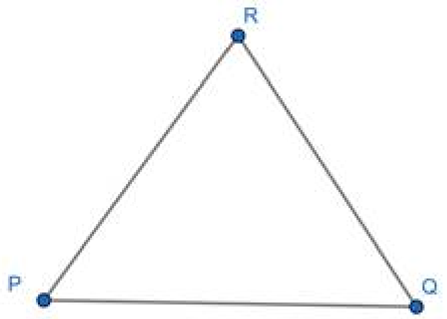
A. ∠P

B. ∠Q

C. ∠R

D. None of these

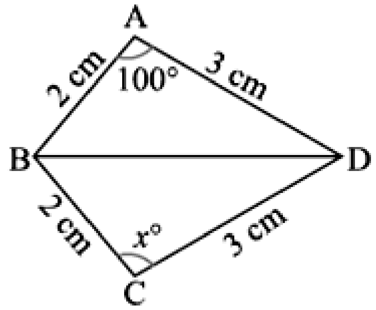
Answer

[T5] 

1 G. Question

Choose the correct answer:

In the figure, the value of x° is______



A. 80^o

B. 100^o

C. 120^o

D. 200^o

Answer

In Δ ABD and Δ CBD

AB = CB = 2cm

AD = CD = 3cm

BD = BD = common

∴ Δ ABD ≅ Δ CBD

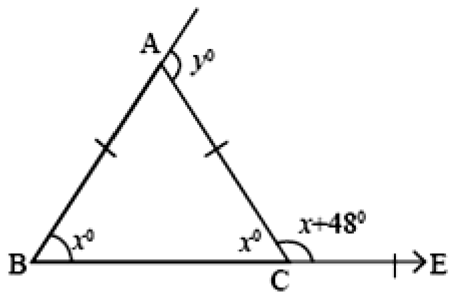
∴ ∠ A = ∠ C

⇒ 100° = x°

Hence, option B is correct.

2. Question

In the figure, ABC is a triangle in which AB = AC. Find x° and y°.



Answer

Theorem 2:

If a side of a triangle is produced, the exterior angle so formed, is equal to the sum of the two interior angle.

∠ ACE + ∠ ACB = 180°

⇒ x + 48° + x° = 180°

⇒ 2x + 48° = 180°

⇒ 2x = 180° - 48°

⇒ 2x = 132°

⇒ $x = \left(\frac{132}{2}\right)^{\circ}$

⇒ x = 66°

Ext. ∠ A = ∠ B + ∠ C

⇒ y° = x° + x°

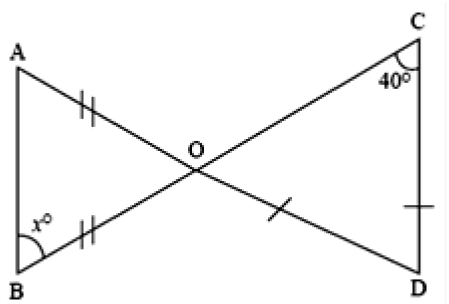
⇒ y° = 2x°

⇒ y° = 2 × 66°

⇒ y° = 132°

3. Question

In the figure, Find x°.



Answer

Theorem 1: Sum of all the angles of the triangles is 180°

In Δ COB

OD = DC

∴ ∠ COD = ∠DCO = 40°

In Δ AOB

OA = OB (Given)

∴ ∠ OAB = ∠ OBA = x°

∠ OAB + ∠ OBA + ∠ AOB = 180° (Angle Sum Property)

⇒ x° + x° + ∠ AOB = 180°

⇒ 2x° + ∠ AOB = 180°

⇒ ∠ AOB = 180° - 2x°

∠ AOB = ∠ COD (opposite angles are equal)

180 – 2x° = 40°

⇒ 180° = 40 + 2x°

⇒ 180° - 40° = 2x°

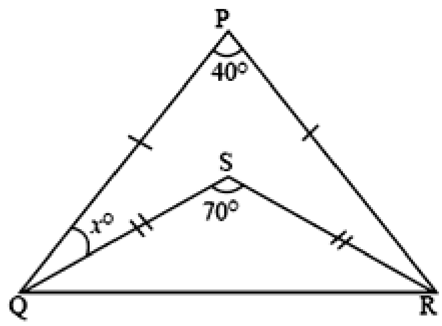
⇒ 140° = 2x°

⇒ $x = \frac{140^\circ}{2}$

⇒ x = 70°

4. Question

In the figure ΔPQR and ΔSQR are isosceles triangles. Find x°.



Answer

Theorem 1: Sum of all the angles of triangles is 180°

Let ∠ PQR be y

In Δ SQR,

SQ = SR

∴ ∠ SQR = ∠ SRQ

∠ SQR + ∠ SRQ + ∠ QSR = 180° (Angle Sum Property)

⇒ 2 ∠ SQR + 70° = 180°

⇒ 2 ∠ SQR = 180° - 70°

⇒ 2 ∠ SQR = 110°

⇒ $\angle SQR = \frac{110^\circ}{2}$

⇒ ∠ SQR = 55° = ∠ SRQ

In Δ PQR,

∠ PQR + ∠ PRQ + ∠ RPQ = 180°

⇒ y + y + 40° = 180°

⇒ 2y + 40° = 180°

⇒ 2y = 180° - 40°

⇒ 2y = 140°

⇒ $y = \frac{140°}{2}$

⇒ y = 70°

∠ PQR = ∠ PQS + ∠ SQR

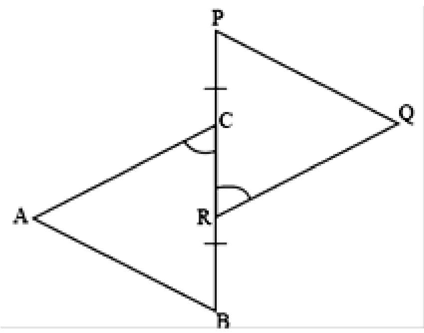
⇒ 70° = x° + 55°

⇒ x° = 70° - 55°

⇒ x° = 15°

5. Question

In the figure, it is given that BR = PC and ∠ACB= ∠QRP and AB ∥ PQ. Prove thatAC = QR.



Answer

Given: BR = PC and ∠ ACB = ∠ QRP , AB || PQ

To Prove: AC = QR

Proof:

In Δ ABC, we have

BC = BR + RC

In Δ PQR

PR = PC + RC

But , BR = PC [Given]

So, BC = PC + RC and PR = BR + RC

⇒ BC = PR

So, in Δ ABC and Δ PQR, we have

∠ ACB = ∠ QRP [Given]

BC = PR [Proved Above]

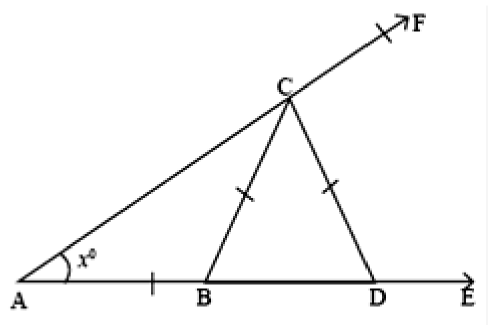
∠ ABC = ∠ QPR [AB || PQ, alternate interior angles]

Thus, Δ ABC ≅ Δ PQR [Angle – Side – Angle]

∴ AC = QR [C. P. C. T]

6. Question

In the figure, AB = BC = CD, ∠A = x°. Prove that ∠DCF = 3∠A.



Answer

Given: AB = BC = CD and ∠ A = x°

To Prove: ∠ DCF = 3 ∠ A

Proof:

In Δ ABC

AB = BC [Given]

∴ ∠ A = ∠ C = x°

Now,

∴ ext. ∠ B = ∠ A + ∠ C

⇒ Ext. ∠ B = x° + x°

⇒ Ext. ∠ B = 2x°

In Δ CBD

BC = CD [Given]

∴ ∠ B = ∠ D = 2x°

Now,

In Δ ADC,

Ext. ∠ DCF = ∠ CDA + ∠ CAD

⇒ ∠ DCF = 2x + x

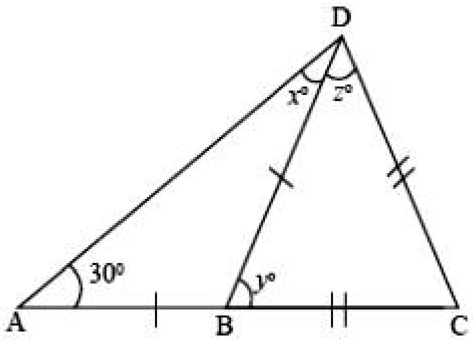
⇒ ∠ DCF = 3x

⇒ ∠ DCF = 3 ∠ A [∠ A = x°, Given]

Hence Proved.

7. Question

Find x°, y°, z° from the figure, where AB = BD, BC = DC and ∠DAC 30°.



Answer

Theorem 1: Sum of all the angles in the triangle is 180°.

In Δ ABD,

We know that, AB = BD

∴ ∠ A = ∠ D

⇒ 30° = x°

Hence, ∠ A + ∠ B + ∠ C = 180°

⇒ 30° + ∠ B + 30° = 180°

⇒ ∠ B + 60° = 180°

⇒ ∠ B = 180° - 60°

⇒ ∠ B = 120°

∠ DBA + ∠ DBC = 180° (Sum of adjacent angles is 180°)

⇒ 120° + y° = 180°

⇒ y° = 180° - 120°

⇒ y° = 60°

In Δ DBC

We know that, BC = DC

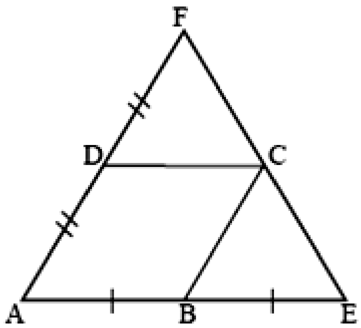
∴ ∠ B = ∠ D

⇒ y° = z°

⇒ 60° = z°

8. Question

In the figure, ABCD is a parallelogram. AB is produced to E such that AB = BE. AD produced to F such that AD = DF. Show that ΔFDC ≡ ΔCBE.

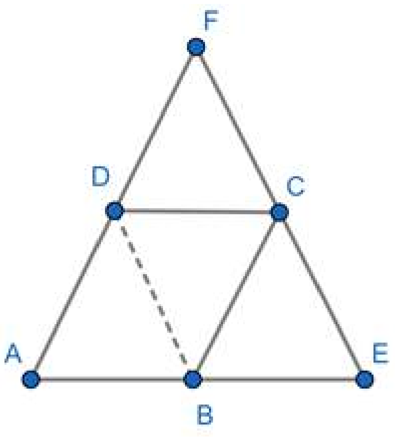


Answer

Given: Parallelogram ABCD and AB = BE and AD = FD

To prove: Δ FDC ≡ ΔCBE

Construction: Join DB



Proof:

We know that,

AB = DC [opposite sides of parallelogram]

BE = DC [AB = BE, because B is the midpoint of AE]

Similarly,

AD = BC [opposite sides of parallelogram]

DF = BC [AD = DF, because B is the midpoint of AE]

Now, AD||BC and AB

∠ A = ∠ B [corresponding angles] …(1)

Now, AB||CD and AD

∠ A = ∠ D [corresponding angles] …(2)

∴ ∠ B = ∠ D (From 1 and 2)

In Δ FDC and Δ CBE

FD = CB [Proved Above]

DC = BE [Proved Above]

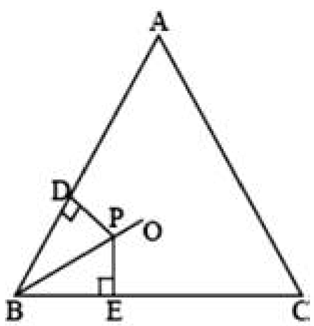
∠ D = ∠ B [Proved Above]

Thus, Δ FDC ≡ Δ CBE

Hence Proved.

9. Question

In figure, BO bisects ∠ABC of ΔABC. P is any point on BO. Prove that the perpendicular drawn from P to BA and BC are equal.



Answer

Given: A Δ ABC in which BO is bisector of ∠ABC

Also, we have PD ⊥ AB and PE ⊥ BC

To Prove: PD = PE

Proof:

In Δ PBD and Δ PBE

PB = PB [common]

∠ PBD = ∠ PBE [given]

∠ PDB = ∠ PEB = 90° [Given]

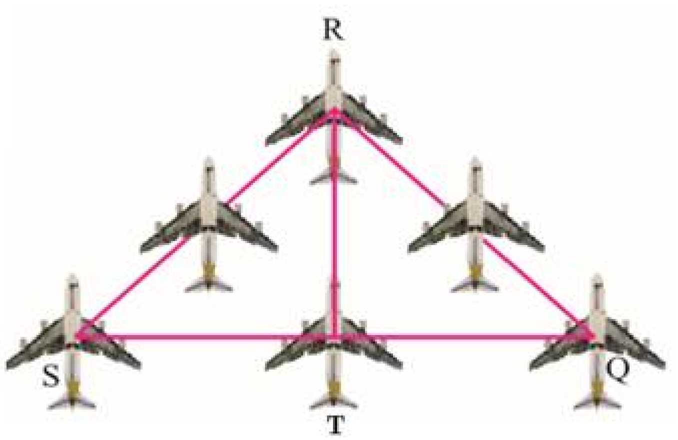
Thus, Δ PBD ≅ Δ PBE [Angle – Angle – Side]

∴ PD = PE

Hence Proved.

10. Question

The Indian Navy flights fly in a formation that can be viewed as two triangles with common side. Prove that ΔSRT ≡ ΔQRT, if T is the midpoint of SQ and SR = RQ.



Answer

Given: T is the mid-point of SQ and SR = RQ

To Prove: Δ SRT ≅ Δ QRT

Proof

In Δ SRT and QRT

RT = RT [common]

ST = QT [T is the mid-point of SQ]

SR = RQ [Given]

Thus, Δ SRT ≅ Δ QRT [Side – Side – Side]

Hence Proved.