Chapter 1

Transformers

LEARNING OBJECTIVES

After reading this chapter, you will be able to understand:

- Single-phase transformer
- Windings
- Bushings
- Breather
- Voltage transformation ratio
- Ideal two-winding transformer
- Phasor diagram of transformer under load
- Equivalent circuit
- Transformer tests
- Sumpner's test
- Losses and efficiency
- Voltage regulation of a transformer
- Auto-transformer
- Parallel operation of single-phase transformers

Introduction

Single-Phase Transformer

The transformer is a device that

1. Transfers electrical energy from one electrical circuit to another
2. Does so without a change in the frequency
3. Works on the principle of mutual induction
4. Has electric circuits that are linked by a common magnetic circuit

The energy transfer usually takes place with a change of voltage. If the energy transfer occurs at the same voltage, the purpose of transformer is merely to isolate the two electric circuits

- Transformer is a static device
- No rotating or moving parts
- Single-phase transformer has two circuits which are electrically isolated but magnetically coupled
- Transformer is an electromagnetic energy conversion device, since the energy received by the primary is first converted to magnetic energy and it is then reconverted to useful electrical energy in the secondary winding circuit
- Voltage and current levels change depending on transformation ratio
- Due to the absence of moving parts, transformers require very little maintenance and it has the highest possible efficiency out of all electrical machines
- Transformer is the main reason for the widespread popularity of AC systems over DC systems

Constructional Details

- Core: Sheet steel or silicon steel with 4% silicon is used
- The core is laminated and the laminations are insulated from each other by a coat of varnish or by an oxide layer on the surface to minimize eddy current losses
- The thickness of laminations varies from 0.35 mm for a frequency of 50 Hz to 0.5 mm for a frequency of 25 Hz
- The core provides a continuous magnetic path of low reluctance. The relative permeability of the core material is of the order of 1000
- Core stepping gives high space factor and also results in reduced length of mean turn and the consequent I^2R loss

Types of Transformers

1. **Core type:** In this type of transformers, the windings surround a considerable part of the core
2. **Shell type:** In this type of transformers, the core surrounds a considerable portion of the windings
3. **Spiral-core or wound-core (trade name—Spirakore):** In this type of transformers, the core is assembled of a continuous strip or ribbon of transformer steel wound in the form of a circular or elliptical cylinder

Advantages of Spiral—Core Construction

- High flux densities are used which reduces the weight/kVA
- Lower iron losses, lesser weight and size per kVA, lower cost of manufacture and a relatively more rigid core

Windings

Conventional transformer has two windings, which are made of high-grade copper or aluminium.

The winding in which electrical energy is fed from the AC supply mains is called primary winding and the other which delivers electrical energy is called secondary winding.

- Depending on voltage level, the windings can also be classified into LV and HV
- Low voltage winding is of higher cross section as it carries higher current compared to high voltage winding
- The winding is insulated from core and other parts of transformers

During the operation of transformer, two types of losses occur, namely core (iron) and Cu losses. These losses result in increase in temperature of both core and winding. To keep the temperature of core and winding within limits, it is necessary to radiate the heat produced due to losses in the transformer. Different cooling methods are used, depending upon the rating of the transformer.

Conservator

In an oil-filled transformer, due to variations in load and change in climatic conditions the oil expands or contracts. To allow room for expansion of the oil, conservator is used. In the absence of a conservator tank, high pressures may develop in transformer tank which may lead to bursting of tank.

Bushings

The purpose of bushings is to provide proper insulation to the output leads of the transformer tank. Porcelain bushings are used up to 33 kV. For higher voltages beyond 33 kV, condenser type or oil-filled type bushings are used.

Breather

Absorption of moisture and dust by the transformer oil will deteriorate the dielectric strength of transformer oil. To prevent moisture and dust from entering the transformer tank, breather is provided

E.m.f. Equation

- Transformer works on the principle of mutual induction. Voltage applied to the primary and the magnetic flux set up in the core assumed to be sinusoidal

Core flux $\phi = \phi_m \sin \omega t$

The primary induced e.m.f.

$$e_1 = -N_1 \frac{d\phi}{dt} = -N_1 \phi_m \omega \cos \omega t$$

$$= -N_1 \omega \phi_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

e_1 is maximum when $\sin\left(\omega t - \frac{\pi}{2}\right) = 1$

$$E_{1_{\max}} = N_1 \omega \phi_m$$

The RMS value of induced e.m.f.

$$E_1 = \frac{1}{\sqrt{2}} E_{\max} = \frac{1}{\sqrt{2}} N_1 \cdot 2\pi f \phi_m$$

$$= \sqrt{2}\,\pi N_1 f \phi_m = 4.44\, N_1 f \phi_m$$

Similarly $E_1 = \sqrt{2}\,\pi N_2 f \phi_m = 4.44\, N_2 f \phi_m$

$$\therefore \frac{E_1}{E_2} = \frac{N_1}{N_2}$$

- The voltage per turn of the primary and secondary windings is the same, since the same mutual flux is linking with both the windings
- E_1 and E_2 are in phase and lag behind the core flux by an angle of 90°
- In a loaded transformer, the primary draws a current to compensate for the secondary ampere turns so that core flux is maintained constant
- Core flux is constant and is independent of the load

If no load, primary ampere turns are neglected, then

$$N_1 I_1 = N_2 I_2$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{V_2}{V_1}$$

Voltage Transformation Ratio (*K*)

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

(i) If $N_2 > N_1$, i.e., $K > 1$, then the transformer is called step-up transformer

(ii) If $N_2 < N_1$, i.e., $K < 1$, then the transformer is known as step down transformer

For an ideal transformer, input VA = output VA

$$V_1 I_1 = V_2 I_2 \quad \text{or} \quad \frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{1}{K}$$

Hence, currents are in the inverse ratio of the transformation ratio

Solved Examples

Example 1: If the applied voltage of a certain transformer is increased by 50% and the frequency is reduced to 50% (assuming that the magnetic circuit remains unsaturated), the maximum core flux density will. ______

Solution: Induced e.m.f. $E \propto B_m f$

neglecting the impedance drops

$$\frac{V_2}{V_1} = \frac{E_2}{E_1}; \quad \frac{V_2}{V_1} = \frac{B_{m_2}}{B_{m_1}} \frac{f_2}{f_1}$$

$$B_{m_2} = \frac{V_2}{V_1} \times \frac{f_1}{f_2} \times B_{m_1}$$

$$= 1.5 \times \frac{1}{0.5} \times B_{m_1} = 3B_{m_1}$$

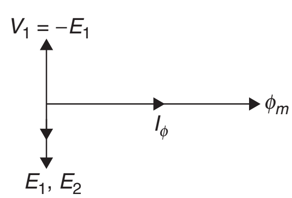
Ans: change to three times the original value.

Ideal Two-winding Transformer

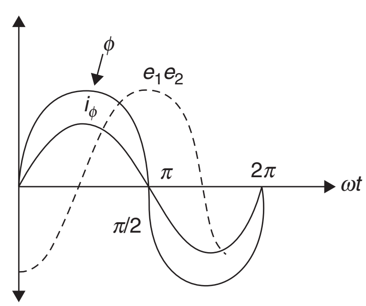
When the transformer is assumed to be ideal,

1. Winding resistances are negligible
2. No flux leakage. All the flux is confined to the magnetic core
3. The core losses are negligible
4. The core has constant permeability, i.e., the magnetization curve for the core is linear

(a) Phasor diagram



(b) Waveforms



When secondary of transformer is loaded, the load will draw secondary current I_2, which sets up N_2I_2, the secondary MMF and it opposes the main flux ϕ_m. The secondary MMF F_2 weakens the main flux ϕ_m. But any reduction in ϕ_m reduces E_1 and in case of ideal transformer $V_1 = -E_1$. Hence, an extra load component of current I_1' is drawn by primary to compensate the secondary ampere-turns to maintain constant core flux.

$$I_1'N_1 = N_2I_2 \Rightarrow \frac{I_1'}{I_2} = \frac{N_2}{N_1}$$

As I_ϕ is negligible

$$\overline{I_1} = \overline{I_1'} + \overline{I_\phi} \Rightarrow \frac{I_1}{I_2} = \frac{N_2}{N_1}$$

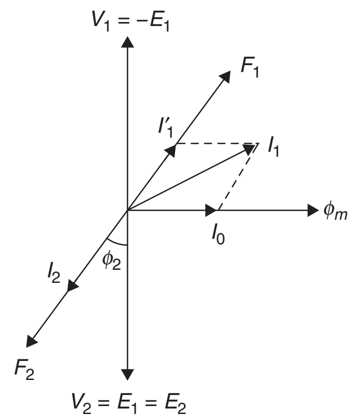
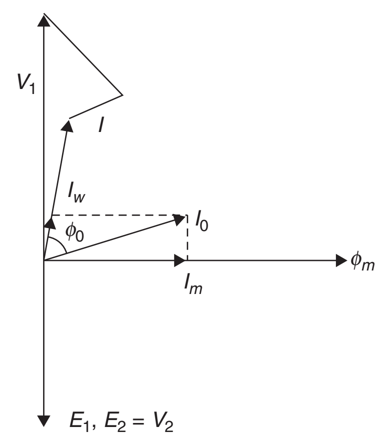


Figure 1 Phasor diagram of ideal transformer for inductive load

Phasor Diagram at no Load



The no-load primary current I_0 is called the exciting current of the transformer and can be resolved into two components.

1. One, in phase with V_1, known as active or working or iron loss component I_w because it mainly supplies the iron loss plus small quantity of primary Cu loss

$$I_w = I_0 \cos \phi_0$$

2. The other component is in quadrature with V_1 and is known as magnetizing component I_μ because its function is to sustain the alternating flux in the core. It is wattless.

$$I_\mu = I_0 \sin \phi_0$$

Even when the transformer is on no-load, the primary input current is not wholly reactive. The no-load primary current lags V_1 by an angle ϕ_0

$$W_0 = V_1 I_0 \cos \phi_0$$

This no-load power supplies the losses in transformer, i.e., a small amount of Cu loss and iron losses in the magnetic core. As it is principally the core-loss which is responsible for the shift in current vector from 90° position, angle ϕ_0 is known as hysteresis angle of advance

$$I_0 = \sqrt{I_w^2 + I_\mu^2}$$

Phasor Diagram of Transformer Under Load

The transformer is said to be loaded, when its secondary circuit is completed through an impedance or load. The magnitude and phase of secondary current I_2 with respect to secondary terminals depends on the characteristics of the load, i.e., I_2 will be in phase, lags behind and lead the terminal voltage V_2, respectively, when load is resistive, inductive and capacitive. The net flux in the core remains constant irrespective of the load.

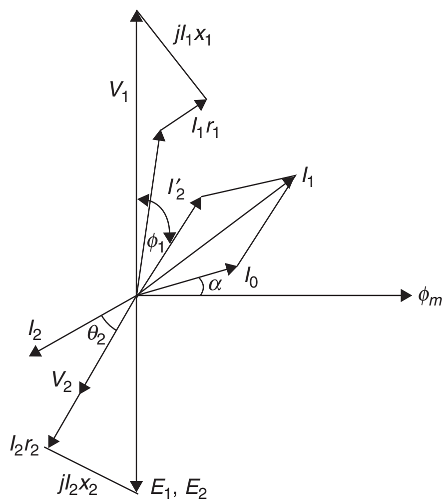


Figure 2 Phasor diagram for inductive load

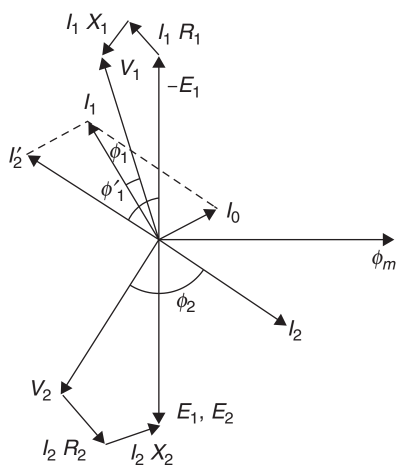


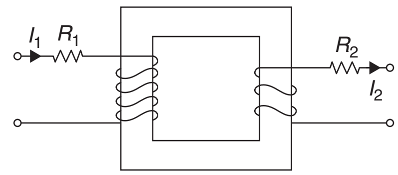
Figure 3 Phasor diagram for capacitive load

$$\overline{E_2} = \overline{V_2} + \overline{I_2}(r_2 + jx_2) = \overline{V_2} + \overline{I_2}Z_2$$

$$V_1 = \overline{V_1'} + \overline{I_1}\,(r_1 + jx_1) = \overline{V_1'} + \overline{I_1}Z_1$$

Impedance Transformation

1. Equivalent resistance



In the above figure, a transformer is shown whose primary and secondary windings have resistances of R_1 and R_2, respectively.

The copper loss in secondary is $I_2'R_2$. This loss is supplied by primary which takes a current I_1. Hence if R_2' is the equivalent resistance in primary which would have caused the same loss as R_2 in secondary, then

$$I_1^2R_2' = I_2^2R_2 \text{ or } R_2' = \left(\frac{I_2}{I_1}\right)^2 R_2$$

If we neglect no-load current I_0, then $\dfrac{I_2}{I_1} = \dfrac{1}{K}$

Hence $R_2' = \dfrac{R_2}{K^2}$

Similarly equivalent primary resistance as referred to secondary is

$$R_1' = K^2 R_1$$

Total or effective resistance of the transformer as referred to primary is

$$R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{K^2}$$

Similarly total transformer resistance as referred to secondary is

$$R_{02} = R_2 + R_1' = R_2 + K^2 R_1$$

The leakage **reactance** can also be transferred from one winding to the other in the same way as resistance

$$X_2' = \frac{X_2}{K^2} \text{ and } X_1' = K^2 X_1$$

and $X_{01} = X_1 + X_2' = X_1 + \dfrac{X_2}{K^2}$

and $X_{02} = X_2 + X_1' = X_2 + K^2 X_1$

The total impedance of the transformer referred to primary and secondary is given by

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} \text{ and } Z_{02} = \sqrt{R_{02}^2 + X_{02}^2}$$

Example 2: A 2400 V/400 V single-phase transformer takes a no-load current of 0.5 A and core loss = 400 W. Determine the values of the magnetizing and core loss components of no-load current. Draw the no load phasor diagram for the transformer.

Solution: $V_1 = 2400$; $V_2 = 400$ V and $I_o = 0.5$ A

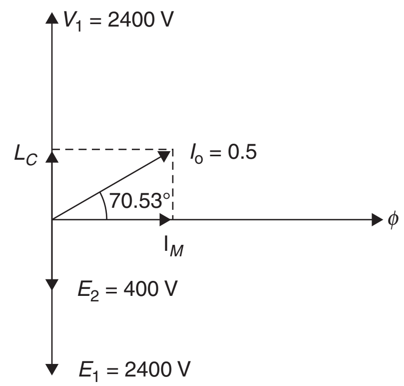
Core loss = Iron loss = 400

$= V_1 I_o \cos\phi_o$

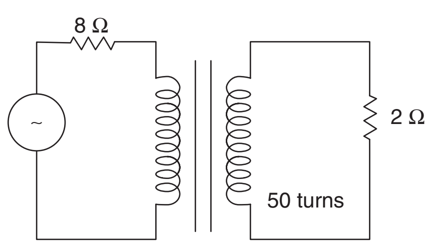
$\Rightarrow 400 = 2400 \times 0.5 \cos\phi_o$

$\phi_o = 70.53°$

∴ Phasor



Example 3: If the secondary winding of the transformer of the circuit shown in the given figure has 50 turns, then for maximum power transfer to the 2-ohm resistor, the number of turns required in the primary winding will be



Solution: For maximum power transfer 2

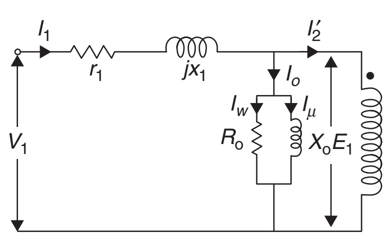
$$= 8 \cdot \left(\frac{N_2}{N_1}\right)^2 \quad \text{(or)} \quad \frac{N_2}{N_1} = \frac{1}{2}$$

Or $N_1 = 2N_2 = 2 \times 50 = 100$ turns

EQUIVALENT CIRCUIT

By making use of equivalent circuit, different performance indices such as efficiency, voltage regulation can be determined.

The equivalent circuit of a transformer having voltage transformation ratio $K = \frac{E_2}{E_1}$ is given in the following figure



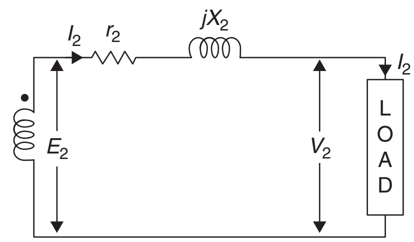


Figure 4(a)

To make calculations simpler, it is preferable to transfer voltage, current and impedance either to the primary or to the secondary. In that case we would have to work in one winding only which is more convenient.

The secondary circuit is shown in figure 4(a) and its equivalent primary values are shown in figure 4(b).

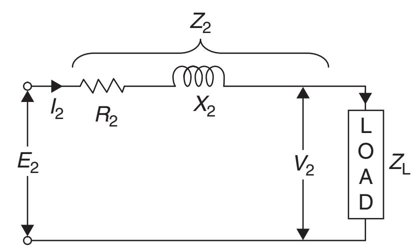
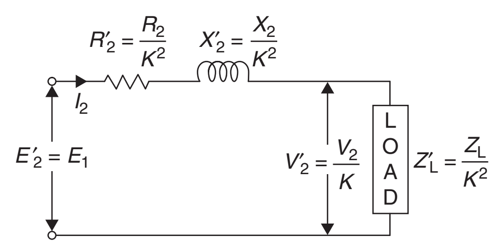
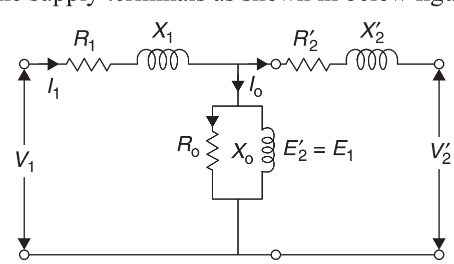


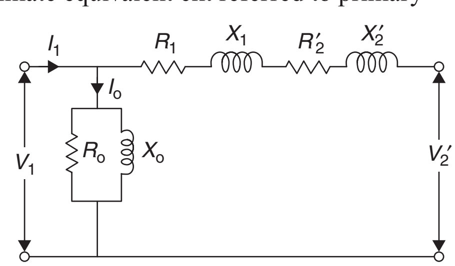
Figure 4(b)



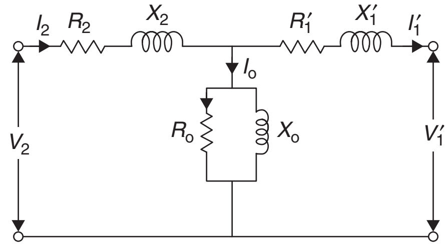
The total equivalent circuit of the transformer is obtained by adding in the primary impedance as shown in the following figure. This is known as exact equivalent circuit. A simplification can be made by transferring the existing circuit across the supply terminals as shown in below figure.



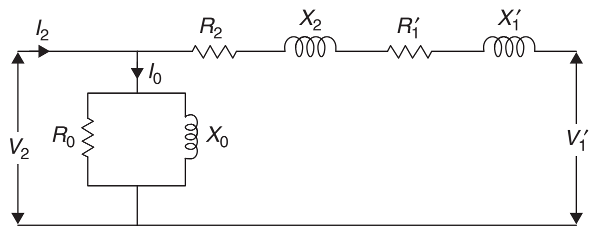
Approximate equivalent ckt referred to primary



In a similar manner, the equivalent circuit referred to the secondary can be obtained. Which is shown in the following figure?



Approximate equivalent circuit referred to the secondary is shown in figure.



Example 4: A 15 kVA, 2300/230 V, 50 Hz single-phase transformer gave the following test data.

Open-circuit test (H. V. side)

$$V_0 = 2300 \text{ V}, I_0 = 0.21 \text{ A}$$
$$W_0 = 50 \text{ W}$$

Short-circuit test (H. V. side)

$$V_{SC} = 47\text{V}, I_{SC} = 6.0 \text{ A}$$
$$W_{SC} = 160 \text{ W}$$

Obtain the equivalent circuit referred to high voltage side.

Solution: As both O.C. test and S.C. test carried out on high voltage side the results will directly lead to equivalent circuit referred to high voltage side.

Shunt branch elements

$$V_0 = 2300 \text{ V}, I_0 = 0.21 \text{ A}, W_0 = 50 \text{ W}$$

$$R_0 = \frac{V_0^{\,2}}{W_0} = \frac{2300^2}{50}$$
$$= 105.8 \times 10^3 \ \Omega$$

$$I_W = I_0 \cos\phi_0 = \frac{W_0}{V_0} = \frac{50}{2300}$$
$$= 0.02174 \text{ A}$$

$$I_\mu = \sqrt{I^2_{\ 0} - I^2_{\ W}}$$
$$= \sqrt{0.21^2 - (0.02174)^2} = 0.20887 \text{ A}$$

$$\therefore X_0 = \frac{V_0}{I_\mu} = \frac{2300}{0.20887} = 11.011 \times 10^3 \ \Omega$$

Series branch elements

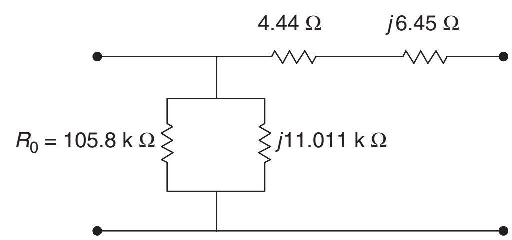
$$V_{SC} = 47 \text{ V}, I_{SC} = 6.0 \text{ A},$$
$$W_{SC} = 160 \text{ W}$$

$$R = \frac{160}{6^2} = 4.44 \ \Omega$$

$$Z = \frac{47}{6.0} = 7.833 \ \Omega$$

$$X = \sqrt{Z^2 - R^2} = \sqrt{41.608}$$
$$= 6.45 \ \Omega$$

Equivalent circuit referred to h.v. side.



Example 5: A 3300/230 V, 50-kVA transformer is found to have impedance of 4% and a Cu. loss of 1.8% at full load. Find its percentage reactance and also the ohmic values of resistance, reactance and impedance as referred to primary. What would be the value of primary short-circuit current if primary voltage is assumed constant?

Solution: $\% X = \sqrt{\left(\% Z^2 - \% R^2\right)}$

$$= \sqrt{4^2 - 1.8^2} = 3.57\%$$

($\therefore$ % Cu loss = % R)

$$I_1, \text{ full load} = \frac{50 \times 10^3}{3300} = 15.2 \text{ A}$$

Considering primary winding, we have % R = 1.8

$$\% R = \frac{R_{01} \times I_1}{V_L} \times 100$$

$$R_{01} = \frac{\% R \times V_L}{I_I \times 100} = 3.91$$

$$\text{Similarly } \% X = \frac{X_{01} \ \ I_1 \ \times 100}{V_1} = 3.57$$

$$\therefore X_{01} = \frac{3.57 \times 3300}{100 \times 15.2} = 7.76 \ \Omega$$

$$\text{Similarly } Z_{01} = \frac{4 \times 3300}{100 \times 15.2} = 8.7 \ \Omega$$

Short-circuit current

$$I_{SC} = \frac{V_1}{Z_{01}}$$

Now $Z_{01} = \dfrac{V_1 \times \%Z}{100 \times I_1}$

$$I_{SC} = \frac{V_I \times 100 \times I_I}{V_I \times \%Z} = \frac{100 \times I_1}{\%Z}$$

∴ Short-circuit current

$$= \frac{100 \times 15.2}{4} = 380 \text{ A}$$

Example 6: Calculate the regulation of a transformer in which the ohmic loss is 1% of the output and the reactance drop is 5% of the output voltage, when the power factor is 0.8 lead.

Solution: Per unit regulation at 0.8 leading
p.f. = 0.01 × 0.8 − 0.05 × 0.6
= –0.022 or –2.2%

Transformer Tests

The usual tests conducted on the transformer are

1. Open-circuit or no-load test
2. Short-circuit or impedance test
3. Sumpner's or back-to-back test

Open-circuit Test

The purpose of this test is to determine no load loss or core loss and no-load current I_0, which is helpful in finding shunt branch parameters of the equivalent circuit, i.e., R_0 and X_0.

Also this test is helpful in determining the magnetizing current I_μ and core loss component (working component) I_w.

- It is preferable to apply the rated voltage to the LV winding keeping the HV winding open. This is from the point of view of the safety of the testing personnel and also for reading the meters with great accuracy.

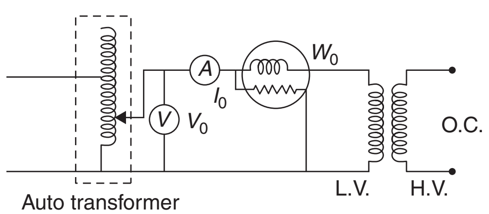


Figure 5 O.C. test circuit diagram

If W_0 is the no-load power input, (V_0 rated voltage), and I_0 the no-load current we have

No-load p.f., $\cos\phi_0 = \dfrac{W_0}{V_0 I_0}$.

Active component of no-load current, $I_w = I_0 \cos\phi_0 = \dfrac{W_0}{V_0}$

The magnetizing component, $I_\mu = \sqrt{I_0^2 - I_w^2}$

Shunt branch parameters of the equivalent circuit are

$$R_0 = \frac{V_0}{I_w} \text{ and } X_0 = \frac{V_0}{I_\mu}$$

Note: Since I_0 is only of the order of 5% of the rated current, the no-load Cu loss of the primary winding will be $\left(\dfrac{1}{20}\right)^2$ times the full-load copper loss. Hence copper loss can be neglected. So, the no load power input W_0 is equal to the iron or core loss P_i.

S.C. Test

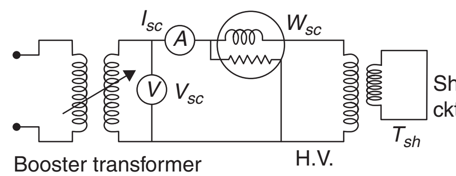


Figure 6 S.C. test circuit diagram

- Since the H.V. rated current is less, short-circuit test is conducted preferably on H.V. side keeping L.V. short circuited.
- For conducting short-circuit test only a small percentage (of the order 5% to 10%) of the rated voltage is sufficient to cause rated current to flow through the windings. As the voltage is less, the core flux density is very much less compared to normal value, so the iron loss which depends on flux density (voltage) can be neglected. The input is therefore equal to total copper loss.
- Through this test, the full-load Cu loss and the equivalent resistance and leakage reactance are determined.
- Let I_{sc} be the current, V_{sc} be the voltage and W_{sc} be the power input, respectively, on short circuit.

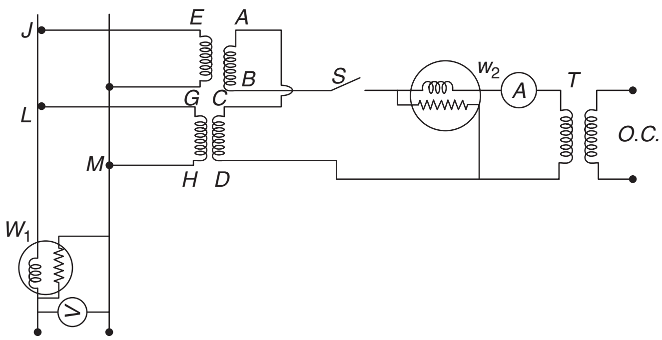
  Then, $R_{01} = \dfrac{W_{sc}}{I_{sc}^2}$ and $Z_{01} = \dfrac{V_{sc}}{I_{sc}}$ and $X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$

  If the H.V. winding is energized and L.V. is short circuited as is the usual case, the values of R and X obtained above are the values referred to the H.V. side.
- For drawing the equivalent circuit referred to H.V. or L.V. side, one set of parameters obtained from either O.C. side or S.C. side are to be referred to the other side.
- Limitations of O.C. and S.C. tests

  In the O.C. test, though the rated voltage is applied to one of the windings, the other winding carries no current, so that the effect of temperature rise is not taken care of. In the short-circuit test, though the windings are made to carry the rated currents, yet the voltage applied is only a small fraction of a rated voltage, with the result, the flux density is not at its usual value.
- When the transformer is operating under load conditions, both core loss and ohmic loss exists. But in O.C. and S.C. tests, each of them exist separately. Hence from O.C.

and S.C. tests the effect of temperature rises because the losses in the temperature are not taken care of, which is possible with Sumpner's or back-to-back test.

Sumpner's Test



This test is mainly performed to determine temperature rise and the drawbacks of the O.C. and S.C. tests are overcome. For conducting this test two identical transformers are required. The primaries are connected in parallel across the supply voltage and rated voltage of the winding is impressed. The secondaries are connected in series opposition, so that the resultant voltage acting around the closed circuit formed by the secondaries is zero. To circulate the current through the secondaries (and hence in primaries), a voltage is injected into the circuit.

By suitably adjusting the value of the injected voltage, both the secondaries can be made to carry the rated (or any desired value) of the current.

Before closing the switch 'S' to complete the local circuit formed by the secondaries, the correctness of the polarities is to be ensured. If zero reading of the voltmeter connected across the switch ensures that the polarities are correct.

- The wattmeter connected on the primary side reads twice the iron loss of each transformer. The watt meter connected in the secondary circuit reads twice the copper loss corresponding to the short-circuit current I_{sc} and V_{sc} equals twice the impedance drop of each transformer.

All-day Efficiency

The all-day efficiency is defined as the ratio of output power in kWh to the input in kWh during the day.

Transformers used for distribution have their primaries energized all 24 hours, although their secondaries supply little or no-load much of the time during the day except during the house lighting period. Hence, it is considered a good practice to design such transformers so that core losses are very low.

The Cu losses are relatively less important, because they depend on the load. The performance of distribution transformer is compared on the basis of energy consumed during a certain time period, usually a day of 24 hours.

$$\eta_{\text{all-day}} = \frac{\text{Output in kWh}}{\text{Input in kWh}} \text{ (for 24 hours)}$$

This efficiency is always less than the commercial efficiency of a transformer.

The Per Unit (p.u.) System

Per unit quantity

$$= \frac{\text{Actual value of the quantity in any unit}}{\text{Base value of the quantity in the same unit}}$$

Percentage Resistance, Reactance and Impedance

These quantities are usually measured by the voltage drop at full-load current expressed as a percentage of the normal voltage of the winding on which calculations are made.

1. Percentage resistance at full load

$$\% R = \frac{I_1 R_{01}}{V_1} \times 100$$

$$= \frac{I_1^2 R_{01}}{V_1 I_1} \times 100$$

$$= \frac{I_2^2 R_{02}}{V_2 I_2} \times 100 = \% \text{ Cu loss at full load}$$

$$\% R = \% \text{ Cu loss} = V_r$$

2. Percentage reactance at full load

$$\% X = \frac{I_1 X_{01}}{V_1} \times 100$$

$$= \frac{I_2 X_{02}}{V_2} \times 100 = V_x$$

3. Percentage impedance at full-load

$$\% Z = \frac{I_1 Z_{01}}{V_1} \times 100$$

$$= \frac{I_2 Z_{02}}{V_2} \times 100$$

4. $\% Z = \sqrt{\% R^2 + \% X^2}$

The ohmic values of resistance and reactance can be obtained from % values as given below.

$$R_{01} = \frac{\% R \times V_1}{100 \times I_1} = \frac{\% \text{ Cu loss} \times V_1}{100 \times I_1}$$

Similarly

$$R_{02} = \frac{\% R \times V_2}{100 \times I_2} = \frac{\% \text{Cu loss} \times V_2}{100 \times I_2}$$

$$X_{01} = \frac{\% X \times V_1}{100 \times I_1} = \frac{V_x \times V_1}{100 \times I_1}$$

Similarly

$$X_{02} = \frac{\%X \times V_2}{100 \times I_2} = \frac{V_x \times V_2}{100 \times I_2}$$

Note: The percentage resistance and impedance are the same value whether referred to primary or secondary.

Example 7: A 3300/230 V, 50-kVA transformer is found to have impedance of 4% and a Cu loss of 1.8% at full load. Find its percentage reactance and also the ohmic values of resistance, reactance and impedance as referred to primary. What would be the value of primary short-circuit current if primary voltage is assumed constant?

Solution: $\% X = \sqrt{\left(\%Z^2 - \%R^2\right)}$

$$= \sqrt{4^2 - 1.8^2} = 3.57\%$$

($\therefore$ % Cu loss = %R)

$$I_1\text{, full load} = \frac{50 \times 10^3}{3300} = 15.2 \text{ A}$$

Considering primary winding, we have

$$\% R = 1.8$$

$$\% R = \frac{R_{01} \times I_1}{V_L} \times 100$$

$$R_{01} = \frac{\%R \times V_L}{I_1 \times 100} = 3.91$$

Similarly

$$\% X = \frac{X_{01} \quad I_1 \times 100}{V_1} = 3.57$$

$$\therefore X_{01} = \frac{3.57 \times 3300}{100 \times 15.2} = 7.76\ \Omega$$

$$\text{Similarly } Z_{01} = \frac{4 \times 3300}{100 \times 15.2} = 8.7\ \Omega$$

Short-circuit current

$$I_{SC} = \frac{V_1}{Z_{01}}$$

Now $$Z_{01} = \frac{V_1 \times \%Z}{100 \times I_1}$$

$$I_{SC} = \frac{V_1 \times 100 \times I_1}{V_1 \times \%Z} = \frac{100 \times I_1}{\%Z}$$

$\therefore$ Short-circuit current

$$= \frac{100 \times 15.2}{4} = 380 \text{ A}$$

Losses and Efficiency

Transformer is a static device; hence there are no mechanical losses. Losses in the transformer are

1. Iron loss or core loss, i.e., hysteresis and eddy current losses, which occur in magnetic core.
2. Copper losses or I^2R loss which occurs in the windings.

The core loss of a transformer depends upon the frequency and the maximum flux density when the volume and the thickness of the core laminations are given. The core loss is made up of two parts.

1. Hysteresis loss, $W_h = PB_{max}^{1.6} f$ as given by Steinmetz empirical relation and
2. Eddy current loss, $W_e = QB_{max}^{2} f^2$,
   where P and Q are constants.
   The total core loss is given by
   $$W_i = W_h + W_e = PB_{max}^{1.6} f + QB_{max}^{2} f^2$$

Separation of Core Losses

- If we carry out two experiments using two different frequencies but the same maximum flux density, we should be able to find the constants P and Q and hence calculate hysteresis and eddy current losses separately.
- The flow of current through the windings gives rise to the copper losses, i.e., $I_1^2 r_1$ and $I_2^2 r_2$.
- The magnetic losses are present as long as primary is energized.
- The no-load current is only of the order of 5% of the rated or full-load current. Hence no load Cu loss can be neglected. So, the no-load input to a transformer is considered as Iron or core or magnetic loss. The iron loss is assumed to be the same under all operating conditions, i.e., from no load to full load. It is denoted by P_i.
- The Cu loss varies with the secondary (and hence primary) current. The copper loss corresponding to the rated value of current is called full-load Cu loss. It is denoted by P_{cu}.
- The efficiency (or commercial efficiency) of a transformer is given by

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{02}}$$

where P_i = iron loss

$I_2^2 R_{02}$ = ohmic loss or Cu loss.

$V_2 I_2$ = Output kVA and $\cos\phi_2$ = load pf.

Condition for Maximum Efficiency

Maximum efficiency occurs when variable losses (i.e., Cu losses) are equal to the fixed loss (i.e., core loss).

$$I_2^2 R_{02} = P_i$$

$$\text{or } I_2 = \sqrt{\frac{P_i}{R_{02}}}$$

$$= I_{FL}\sqrt{\frac{P_i}{\text{Full-load Cu loss}}}$$

kVA corresponding to maximum efficiency.

$$(\text{kVA})_{\max\,\eta} = \text{Full-load kVA}\sqrt{\frac{P_i}{I_{FL}^2 R_{02}}}$$

Example 8: A 50 kVA transformer has a core loss of 400 W and a full-load copper loss of 800 W. what is the load corresponding to maximum efficiency.

Solution: Iron loss $P_i = 400$ W
Full-load copper loss P_{Cu}, = 800 W
kVA corresponding to max efficiency

$$= \text{full-load kVA} \times \sqrt{\frac{P_i}{P_{CU}}}$$

$$= 50 \times \sqrt{\frac{400}{800}} = 35.35 \text{ kVA}$$

Example 9: In a transformer, the core loss is found to be 52 W at 40 Hz and 90 W at 60 Hz measured at same flux density. Calculate the hysteresis and eddy current losses at 50 Hz.

Solution: Since the flux density is the same in both cases, we can use the relation
Total core loss, $W_i = Af + Bf^2$ or $W_i/f = A + Bf$,

$$\therefore \quad \frac{52}{40} = A + 40\,B$$

$$\text{And} \quad \frac{90}{60} = A + 60\,B$$

$$\therefore \quad A = 0.9 \text{ and } B = 0.01$$

At 50 Hz,
The hysteresis loss $W_h = Af$

$$= 0.9 \times 50 = 45 \text{ W}$$

The eddy current loss

$$W_e = Bf^2 = 0.01 \times 50^2 = 25 \text{ W}$$

Common Data for Examples 10 to 12
A transformer with normal voltage impressed has a flux density of 1.4 wb/m^2 and a core loss comprising of 1000 W eddy current loss and 3000 W hysteresis loss. What do these losses become under the following conditions?

Example 10: Increasing the applied voltage by 10% at rated frequency

Solution: Hysteresis loss $W_h \propto B^{1.6}_{\max}$

$$= PB^{1.6}_{\max} f$$

Eddy current loss $W_e \propto B^2_{\max} f^2$

$$= QB^2_{\max} f^2$$

From the relation

$$E = 4.44 f\,NB_{\max}\,A.$$

We get $B_{\max} \propto E/f$
Putting this value of $B_{\max}$ in the above equations, we have

$$W_h = P\left(\frac{E}{f}\right)^2 f = PE^{1.6} f^{-0.6}$$

$$\text{And} \quad W_e = Q\left[\frac{E}{f}\right]^2 f^2 = QE^2$$

From the given data, we have

$$3000 = PE^{1.6} f^{-0.6} \quad (1)$$

$$\text{And} \quad 1000 = QE^2 \quad (2)$$

where E and f are the normal values of primary voltage and frequency, respectively.

If the voltage is increased by 10% at rated frequency, the new hysteresis loss is

$$W_h = P(1.1E)^{1.6} f^{-0.6} \quad (3)$$

Dividing Eq. (3) by (1),

$$\text{We get} \quad \frac{W_h}{3000} = 1.1^{1.6}$$

$$W_h = 3000 \times 1.165 = 3495 \text{ W}$$

The new eddy current loss is

$$W_e = Q(1.1E)^2 \Rightarrow \therefore \frac{W_e}{1000} = 1.1^2$$

$$W_e = 1000 \times 1.21 = 1210 \text{ W}$$

Example 11: Reducing the frequency by 10% with normal voltage impressed.

Solution: Since $W_e = QE^2$, eddy current loss is unaffected with change in the supply frequency. The new hysteresis loss is,

$$W_h = PE^{1.6}(0.9)f^{-0.6} \quad (4)$$

From Eq. (1) and (4), we get

$$\frac{w_h}{3000} = 0.9^{-0.6}$$

$$W_h = 3000 \times 1.065 = 3196 \text{ W}$$

Example 12: Increasing both impressed voltage and frequency by 10 per cent

Solution: In this case, both E and f are increased by 10%. The new losses are as under;

$$W_h = P(1.1E)^{1.6}\,(1.1f)^{-0.6}$$

$$\frac{W_h}{3000} = 1.1^{1.6} \times 1.1^{-0.6}$$
$$= 1.165 \times 0.944$$
$$W_h = 3000 \times 1.165 \times 0.944$$
$$= 3299 \text{ W}$$

The eddy current loss, W_e depends only on applied voltage and is unaffected by changes in f,

$$W_e = Q(1.1)E^2$$

$$\therefore \quad \frac{W_e}{1000} = 1.1^2$$
$$= 1000 \times 1.21 = 1210 \text{ W.}$$

Voltage Regulation of a Transformer

When a transformer is supplied with a constant primary voltage, the secondary terminal voltage varies as the load current and p.f. of the load vary. The voltage regulation is defined as the change in secondary voltage, expressed as a percentage (or p.u.) of the secondary rated voltage, when load at a given power factor is reduced to zero, with primary applied voltage held constant.

The per unit voltage regulation =

$$\frac{(\text{No load secondary voltage} - \text{Secondary voltage at rated load})}{\text{Secondary voltage at rated load}}$$

$$\%\text{Regulation} = (\text{p.u. regulation} \times 100)$$

The magnitude of change in secondary voltage depends on the load power factor, load current total resistance and total reactance of the transformer.

$$\text{Voltage regulation} = \frac{E_2 - V_2}{E_2} \text{ in p.u.}$$

$$= \frac{E_2 - V_2}{E_2} \times 100 \text{ in percentage}$$

From the approximate equivalent circuit, the expression for voltage regulation can be given as follows.

$$E_2 - V_2 = I_2 R_{02} \cos\phi \pm I_2 X_{02} \sin\phi$$

[+ve and –ve signs are taken for lagging and leading p.f.'s, respectively]
where $\cos\phi$ is load power factor.
Therefore per unit voltage regulation

$$\varepsilon = \varepsilon_r \cos\phi \pm \varepsilon_x \sin\phi$$

where ε – p.u. voltage regulation
ε_r – p.u. resistance
ε_x – p.u. reactance
+ for lagging p.f. and
– for leading p.f.

- condition for maximum voltage regulation:

Since, $|I|R_{02}\cos\theta - |I|X_{02}\sin\theta = 0$

$$\phi = \tan^{-1}\left[\frac{X_{01}}{R_{01}}\right] = \tan^{-1}\left[\frac{X_{02}}{R_{02}}\right]$$

Maximum regulation occurs at a lagging p.f.

Condition for Zero Voltage Regulation

$$\phi = \tan^{-1}\left[\frac{R_{01}}{X_{01}}\right] = \tan^{-1}\left[\frac{R_{02}}{X_{02}}\right]$$

Zero voltage regulation occurs at a leading p.f.

Example 13: The equivalent resistance and reactance of a transformer are, respectively, 0.6 Ω and 0.8 Ω. At what power factor the voltage regulation will be zero?

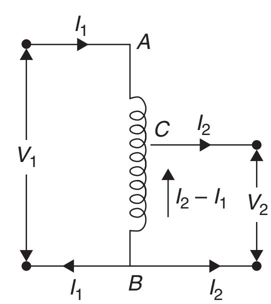
Solution: % Regulation is zero at a p.f. of

$$\phi = \tan^{-1}\left[\frac{R}{X}\right] \text{leading} \Rightarrow \phi = \tan^{-1}\left[\frac{0.6}{0.8}\right]$$

$$\cos\phi = \cos\left(\tan^{-1}\left(\frac{6}{8}\right)\right)_{\text{lead}}$$

Auto-transformer

It is a transformer with one winding only, part of this being common to both primary and secondary. The primary and secondary are connected electrically as well as coupled magnetically.



- There is a superimposition of the input and output currents in the part of the winding common to primary and secondary.
- As an ordinary transformer, the transformer ratio,

$$\frac{V_2}{V_1} = \frac{E_{BC}}{E_{AB}} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K$$

Power is transferred from the primary to the secondary both inductively and conductively.

Power delivered to load $P = V_2 I_2$

Power transformed or power transferred inductively $= V_2 I_2 (1 - K)$

Power transferred conductively $= K\,V_2 I_2$

- As compared to an ordinary two-winding transformer of same output, an auto-transformer uses less copper because of one winding, has higher efficiency, smaller in size and cheaper.

- Since the HV and LV windings are electrically connected, a fault on the HV side may cause a high voltage to appear on LV side.
- Weight of Cu in auto-transformer
  $= (1-K) \times$ weight of Cu in two-winding transformer

Saving $= K \times$ weight of Cu in two-winding transformer.

- $$\frac{\text{kVA rating as an auto-transformer}}{\text{kVA rating as two-winding transformer}} = \frac{1}{1-K}$$
- $$\frac{\text{Full load losses as an auto-transformer}}{\text{Full load losses as a two-winding transformer}} = (1-K)$$

$$\frac{\text{p.u. impedance drop in auto-transformer}}{\text{p.u. impedance drop in two-winding transformer}} = 1-K$$

$$\frac{\text{Voltage regulation as an auto-transformer}}{\text{Voltage regulation in two-winding transformer}} = 1-K$$

Applications

1. It is used as an auto starter for induction motor.
2. It is used to give small boost to a distribution line.
3. It is used in testing laboratories.

Conversion of Two-winding Transformer into Auto-transformer

Any two-winding transformer can be converted into an auto-transformer either step-down or step-up.

There are two possibilities of such conversion.

1. Connecting two windings with additive polarity to get a step-up auto-transformer.
2. Connecting two windings with subtractive polarity to get a step-down auto-transformer.

For example consider a 20-kVA, 2400/240 V two-winding transformer with its polarity markings shown in figure 7(a).

(a) Additive polarity

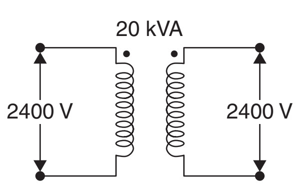


Figure 7(a)

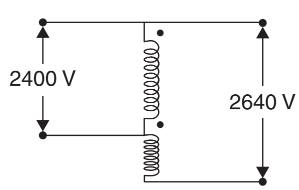


Figure 7(b)

Because of additive polarity

$V_2 = 2400 + 240 = 2640$ and

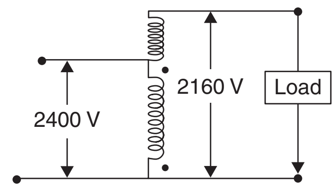
$V_1 = 2400$ V

There is a marked increase in the kVA rating of the auto-transformer.

kVA rating of auto-transformer

$$= \frac{V_2 I_2}{1000} = \frac{2640 \times 83.3}{1000} = 220 \text{ kVA}$$

(b) Subtractive polarity



Here $V_2 = 2400 - 240 = 2160$ V

In this case also there is a large increase in the kVA rating of auto-transformer.

kVA rating of auto-transformer

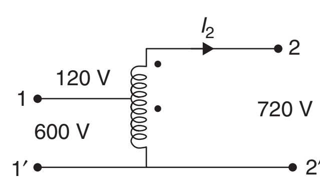
$$= \frac{V_2 I_2}{1000} = \frac{2160 \times 83.3}{1000} = 180 \text{ kVA.}$$

When two-winding transformer is converted to an auto-transformer some portion (or common portion) of winding is overloaded. The increase in kVA capacity is due to power transfer both by conduction and induction.

Example 14: A single-phase transformer has rating of 15 kVA, 600/120 V. It is recommended as an auto-transformer to supply at 720 V from a 600 V primary source. The maximum load it can supply is ________

Solution: Current rating of L.V. winding

$$= \frac{15 \times 10^3}{120} = 125 \text{ A}$$



Maximum load can be supplied, if the two windings are connected in additive polarity as shown in above figure.

$$\text{Output kVA} = \frac{720 \times 125}{1000} = 90 \text{ kVA}$$

Parallel Operation of Single-Phase Transformers

For supplying a load in excess of the rating of an existing transformer, a second transformer may be connected in parallel with it. If the demand is more than present supply, two or more transformers are placed in parallel instead of a single large unit, due to which the power system becomes more reliable. Transformers are switched off or added as per load requirement.

There are certain definite conditions which must be satisfied in order to avoid any local circulating currents and to ensure that transformers share the common load in proportion to their kVA ratings. The conditions are

1. Primary windings of the transformers should be suitable for the supply system voltage and frequency
2. The transformers should be properly connected with regard to polarity.
3. The voltage ratings of both primaries and secondaries should be identical. In other words, the transformers should have the same turn ratio, i.e., transformation ratio.
4. The percentage impedance should be equal in magnitude and have the same X/R ratio in order to avoid circulating currents and operation at different power factors.
5. With transformers having different kVA ratings, the equivalent impedances should be inversely proportional to the individual kVA rating if circulating currents are to be avoided.

Of these conditions, condition (2) is absolutely essential (otherwise paralleling with incorrect polarities will result in dead short-circuit).

If condition (3) is not exactly satisfied, i.e., the two transformers have slightly different transformation or voltage ratios, even then parallel operation is possible. But due to inequality of induced e.m.f.'s in secondaries, there will be some circulating current between the secondaries even on no-load. When secondaries are loaded, this localized circulating current will tend to produce unequal loading condition. Hence, it may be impossible to take full kVA output from the parallel connected group without one of the transformers becoming overheated.

If condition (4) is not exactly satisfied, i.e., impedance triangles are not identical in shape and size, parallel operation will still be possible, but the power factors at which the transformers operate will be different from the power factor of common load. Therefore in this case, the two transformers will not share the load in proportion to their kVA ratings.

The currents carried by the two transformers are proportional to their ratings, provided the numerical impedances are inversely proportional to these ratings and their percentage impedances are identical.

If E_A and E_B are the no-load induced e.m.f.'s of the two transformers connected in parallel, 'I_A' and 'I_B' are the currents delivered and 'Z_A' and 'Z_B' are the ohmic impedances referred to the L.V side of the two transformers and Z_L is the Load impedance then

$$I_A = \frac{\left[E_A Z_B + (E_A - E_B)Z_L\right]}{(Z_A+Z_B)Z_L + Z_A Z_B} \quad I_B = \frac{\left[E_B Z_A + (E_A - E_B)Z_L\right]}{(Z_A+Z_B)Z_L + Z_A Z_B}$$

Let 'S', 'S_A' and 'S_B' be the total kVA, kVA delivered by the transformer 'A' and that delivered by the transformer 'B', respectively, then we have

$$S_A = S\frac{Z_B}{Z_A+Z_B} \quad S_B = S\frac{Z_A}{Z_A+Z_B}$$

Example 15: Two 1-ϕ transformers are connected in parallel at no-load. One has a turn ratio of 5000/440 and a rating of 200 kVA, the other has a ratio of 5000/480 and a rating of 350 kVA. The leakage reactance of each is 3.5%. What is the no-load circulation current expressed as a percentage of the nominal current of the 200 kVA transformer.

Solution: Assume that per turn voltage is 1V. the normal currents are

$$\frac{200\times10^3}{440} = 455 \text{ A}$$

And
$$\frac{350\times10^3}{480} = 730 \text{ A}$$

Reactances seen from the secondary side are

$$\frac{3.5}{100}\times\frac{440}{455} = 0.034\ \Omega$$

$$\frac{3.5}{100}\times\frac{480}{730} = 0.023\ \Omega$$

The difference of induced voltages is 40 V. The circulating current is

$$I_C = \frac{40}{(0.034+0.023)} = 704 \text{ A}$$

= 1.55 times or 155% the normal current of 200 kVA unit.

Example 16: A 500-kVA single-phase transformer with 0.012 p.u resistance and 0.06 p.u. reactance, is connected in parallel with a 250 kVA single-phase transformer with 0.014 p.u. resistance and 0.045 p.u. reactance to share a load of 600 kVA at 0.8 p.f. lagging. Find the load shared by each transformer.

Solution: The p.u. resistance and reactance given above refer to different ratings. These should be adjusted to some basic kVA.

Let Base kVA = 500

$$Z_A \text{ p.u.} = 0.012 + j0.06 = 0.061\angle 78.69°$$

$$Z_B \text{ p.u.} = (0.014 + j0.045)\times\frac{500}{250} = 0.094\angle 72.7°$$

Load shared by transformer A

$$S_A = \frac{Z_B}{Z_A+Z_B}\cdot S$$

$$= \frac{0.094\angle 72.7 \times 600\angle -36.87}{0.1552\angle 75.07}$$

$$= 364.37\angle -39.22°$$

364 kVA at a p.f. of 0.774 lag

Load shared by transformer B

$$S_B = \frac{Z_A \cdot S}{Z_A + Z_B}$$

$$= \frac{0.0612\angle 78.69 \times 600\angle -36.87°}{0.1552\angle 75.07}$$

$$= 236.60\angle -33.25$$

236.6 kVA at a p.f. of 0.836 lag.

Example 17: Two transformers A and B are connected in parallel to a load of 2 + *j*1.5 ohms. Their impedances in secondary terms are $Z_A = (0.15 + j0.5)$ ohm and $Z_B = (0.1 + j0.6$ ohm). Their no-load terminal voltages are $E_A = 207\angle 0°$ V and $E_b = 205\angle 0°$ V. Find the power output and power factor of each transformer.

Solution: $$I_A = \frac{E_A Z_B + (E_A - E_B) Z_L}{Z_A Z_B + Z_L (Z_A + Z_B)}$$

$$Z_A = (0.15 + j0.5)\ \Omega$$

$$Z_B = (0.1 + j0.6)\ \Omega;$$

$$Z_L = (2 + j1.5) = 2.5\angle 36.9°$$

$$I_A = \frac{207(0.1 + j0.6) + (207 - 205)(2 + j1.5)}{(0.15 + j0.5) \times (0.1 + j0.6) + (2 + j1.5)(0.25 + j1.1)}$$

$$= \frac{24.7 + j127.2}{-1.435 + j2.715} = \frac{129.7\angle 79°}{3.07\angle 117.9°}$$

$$= 42.26\angle -38.9°$$

$$= (32.89 - j26.55)\text{ A}$$

$$I_B = \frac{E_B Z_A - (E_A - E_B) Z_L}{Z_A Z_B + Z_L (Z_A + Z_B)}$$

$$= \frac{205(0.15 + j0.5) - 2(2 + j1.5)}{-1.435 + j2.715}$$

$$= \frac{103\angle 75°}{3.07\angle 117.9°} = 33.56\angle -42.9°$$

$$= (24.58 - j22.84)\text{ A}$$

$$V_2 = IZ_L = (I_A + I_B) Z_L$$

$$= (57.47 - j49.39)(2 + j1.5)$$

$$= 189 - j12.58 = 189.4\angle -3.9°$$

Power factor angle of transformer A

$$= -3.9° - (-38.9°) = 35°$$

$\therefore$ p.f. of A = cos35° = 0.818 lag

p.f. of B = cos(–3.9°) – (–42.9°)

= 0.776 lag

Power output of transformer

A is $P_A = 189.4 \times 42.26 \times 0.818 = 6548$ W

Similarly $P_B = 189.4 \times 33.56 \times 0.776 = 4900$ W

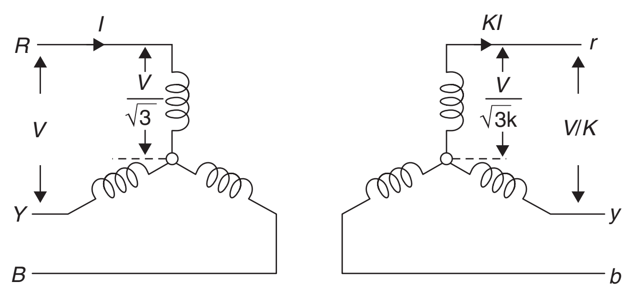
Three-phase Transformers

Three-phase transformation can be obtained by having a single, three-phase transformer or by suitable inter connection of three single-phase transformers.

As compared to a bank of single-phase transformers, the main advantages of a three-phase transformer are that it occupies less floor space for equal rating, weight less, costs about 15% less and further, that only one unit is to be handled and connected. However, if a fault occurs, all the loads connected to the three-phase transformer will be interrupted till the repair works are carried out. But, in the case of 3-phase bank of single-phase transformers if one transformer goes out of order, the system can still be run open-Δ at reduced capacity or the faulty transformer can be readily replaced by a single spare.

Three-phase Transformer Connections

Star/Star: 1:1



This connection is most economical for small, high-voltage transformers because the number of turns/phase and the amount of insulation required is minimum (as phase voltage is only $1/\sqrt{3}$ of line voltage). The ratio of line voltages on the primary and secondary is the same as the transformation ratio of each transformer.

There is a phase shift of 30° between the phase voltages and line voltages both on primary and secondary sides.

This connection works only if the load is balanced. With the unbalanced load, the neutral point shifts thereby making the three line-to-neutral voltages unequal. This difficulty of shifting (or floating) neutral can be obviated by connecting the primary neutral to neutral of generator. Star-Star transformers are operated with grounded neutrals.

With an isolated neutral, any unbalanced load causes the shifting of the neutral.

For delivering a sine wave of voltage, it is necessary to have a sine wave of flux in the core but on account of the characteristics of iron, a sine wave of flux requires a third harmonic component in the exciting current. At any instant the third harmonic component tends to flow towards or away from neutral. If primary neutral is isolated, the third harmonic components of the magnetizing current do not have a path. Hence, the flux in the core cannot be a sine wave and so the phase voltages become non-sinusoidal, though the line voltages are sinusoidal.

Another way of avoiding this trouble of oscillating neutral is to provide each of the transformers with a third or tertiary winding of relatively low kVA rating. This tertiary winding is connected in Δ and provides a circuit in which the third harmonic component can flow.

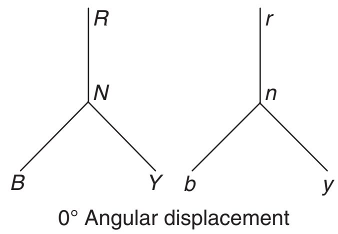
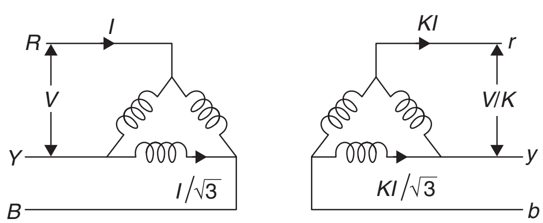


Figure 8 Phasor diagram

Delta/Delta



This connection is economical for large, low voltage transformers in which insulation problem is not so urgent, because it increases number of turns/phase.

The ratio of transformation between primary and secondary line voltage is exactly same as that of each transformer and there is no angular displacement between them. This connection has the following advantages:

1. In order that the output voltage be sinusoidal, it is necessary that the magnetizing current of the transformer must contain a third order harmonic component. In this case, the third harmonic component of the magnetizing current can flow in the Δ-connected transformer primaries without flowing in the line wires. The three phases are 120° apart, which is 3 × 120° = 360° with respect to the third harmonic, hence it merely circulates in the delta. Therefore the flux is sinusoidal, which results in sinusoidal voltages.
2. No difficulty is experienced from unbalanced loading as was the case in star-star connection. The three-phase voltages remain practically constant regardless of the load impedance.
3. An added advantage of this connection is that if one of the transformer phases becomes disabled, the system can continue to operate in open-delta or in *V–V* although with reduced available capacity. The reduced capacity is 58% and not 66.7% of the normal capacity.

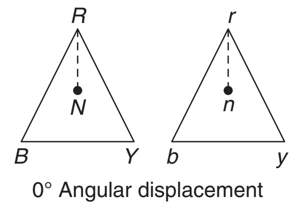
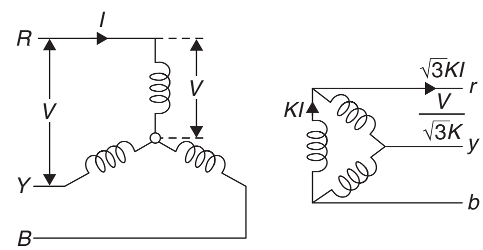


Figure 9 Phasor diagram

Star/Delta



The main use of this connection is at the substation end of the transmission line where the voltage is to be stepped down. The primary winding is star connected with grounded neutral.

The ratio between the secondary and primary line voltage is $1/\sqrt{3}$ times the transformation ratio of each transformer. There is a 30° shift between the primary and secondary voltages which means that a star-delta transformer bank cannot be paralleled with either a star-star or a delta-delta bank.

The third harmonic currents flow in the delta to provide a sinusoidal flux.

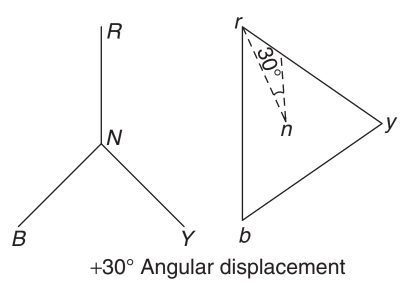
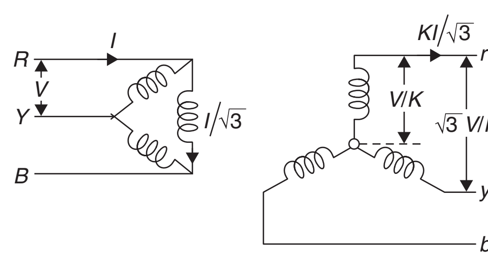


Figure 10 Phasor diagram

Delta/Star



This connection is generally employed where it is necessary to step up the voltage as for example, at the beginning of high tension transmission system. The neutral of the secondary is grounded for providing 3-phase, 4-wire service.

- This is the most popular connection as it can be used to serve both the 3-phase power equipment and single-phase lighting circuits.
- This connection is not open to the objection of a floating neutral and voltage distortion because the existence of a Δ-connection allows a path for the third-harmonic currents.
- The primary and secondary line voltages and line currents are out of phase with each other by 30°. Because of this 30° shift, it is impossible to parallel such a bank of transformers even though the voltage ratios are correctly adjusted. The ratio of secondary to primary voltage is $\sqrt{3}$ times the transformation ratio of each transformer.

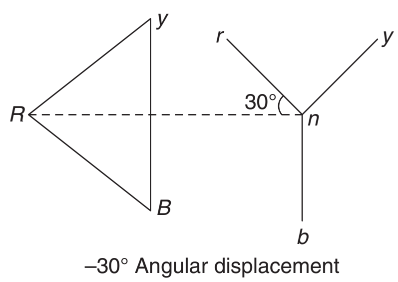


Figure 11 Phasor diagram

Example 18: The percentage impedance of a 100 kVA, 11 kV /400 V, delta/wye, 50 Hz transformer is 4.55 for the circulation of half the full-load current during short-circuit test, with low voltage terminals shorted, what should be the applied voltage on high voltage side?

Solution: Full-load primary current

$$I_1 = \frac{100\times10^3}{11\times10^3} = 9.09 \text{ A}$$

$$Z_{01} = \% \frac{Z\times V_1}{100\ I_1}$$

$$= \frac{4.5\times11\times10^3}{100\times9.09} = 55\ \Omega$$

For half full-load current during short-circuit test

$$Z_{01} = \% \frac{Z\times V_1}{100\times I_1/2}$$

$$Z_{01} = \frac{4.5\times11\times10^3}{100\times\frac{9.09}{2}} = 110\ \Omega$$

$$\text{Half full-load current} = \frac{9.09}{2} = 4.5$$

$$\text{Applied voltage} = 4.5\times55$$
$$= 247.5 \text{ V}$$

Open-Delta or V–V Connection

If one of the transformers of Δ–Δ is removed and 3-phase supply is connected to the primaries as shown in fig, then three equal 3-phase voltages will be available at the secondary terminals on no-load. This method of transforming 3-phase power by means of only two transformers is called the open-Δ or V–V connection.

The total load that can be carried by a V–V bank is not two-third of the capacity of a Δ–Δ bank but it is only 57.7% of it.

$$\frac{V-V \text{ capacity}}{\Delta-\Delta \text{ capacity}} = \frac{\sqrt{3}V_L I_S}{3V_L I_S}$$

$$= \frac{1}{\sqrt{3}} = 0.577 \text{ or } 58\%$$

i.e., only 86.6% of the rated capacity of the two remaining transformers is available. In other words, ratio of operating capacity to available capacity of an open Δ is 0.866 or utility factor is 0.866.

Each transformer will supply 57.7% of load and not 50% when operating in V–V.

If three transformers in a Δ–Δ bank are delivering their rated load and one transformer is removed, the overload on each of the two remaining transformers is 73.2% because

$$\frac{\text{Total load in } V-V}{\text{VA of transformer}} = \frac{\sqrt{3}V_L I_S}{V_L I_S} = \sqrt{3} = 1.732$$

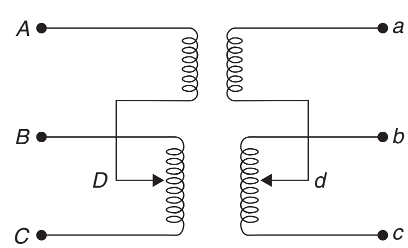
The disadvantages of this connection are

1. The average power factor is 86.6% of the balanced load power factor. One of two transformers operates at a p.f. of cos(30 + θ) and the other at cos(30 – θ).
2. The secondary terminal voltages tend to become unbalanced to a great extent when the load is increased, this happens even when the load is perfectly balanced.

SCOTT Connection or T-T Connection

This is a connection by which 3-phase to 3-phase transformation is accomplished using two transformers as shown in below figure.

This connection can also be used for 3-phase to 2-phase transformation.



If the two transformers are of identical voltage ratings, one of the transformers has centre taps both on the primary and secondary windings and is known as the main transformer. The other has 0.866 tap and is known as teaser transformer.

- Alternatively teaser may be designed for 86.6% voltage of the main transformer.
- If the load power factor is $\cos\theta$ lag then
  - (a) Teaser transformer operators at a lagging p.f. of $\cos\theta$
  - (b) one half of the main transformer operates a p.f. of $\cos(30 + \theta)$ lag
- If V is the line voltage and I is the rated line current, the combined VA rating of the two transformers = VI (1 + 0.866) if both the teaser primary and secondary windings are designed for 0.866 times the voltage on the main transformer.

$$\text{Output} = \sqrt{3}\, VI$$

$$\text{Utilization factor} = \frac{\sqrt{3}VI}{(1+0.866)VI} = 0.928$$

If the two transformers are of equal voltage rating (and with taps at 86.6% points), 13.4% of the winding is unutilized.

$$\text{Then utility factor of the winding} = \frac{\sqrt{3}VI}{2\ VI} = 0.866$$

Example 19: A 5000 kVA, three-phase transformer 6.6/33-kV, Δ/Y, has a no-load loss of 15 kW and a full-load loss of 50 kW. The impedance drop at full load is 7%. Calculate the primary voltage when a load of 3,200 kW at 0.8 p.f. is delivered at 33 kV.

Solution: Full-load $I_2 = 5 \times 10^6/\sqrt{3} \times 33{,}000 = 87.5$ A

Impedance drop /phase

$$= 7\% \text{ of } \left(33/\sqrt{3}\right)\text{kV}$$

$$= 1330 \text{ V}$$

$$\therefore Z_{02} = \frac{1330}{87.5} = 15.3\ \Omega/\text{phase}$$

F. L. Cu loss = 50 – 15 = 35 kW

$$\therefore 3I^2_2 R_{02} = 35{,}000$$

$$R_{02} = 35{,}000/3 \times (87.52)^2 = 1.53\ \Omega/\text{phase}$$

$$X_{02} = \sqrt{15.3^2 - 1.53^2} = 15.23\ \Omega$$

When the load is 3200 kW at 0.8 p.f.

$$I_2 = 3{,}200 / \sqrt{3} \times 33 \times 0.8 = 70 \text{ A}$$

$$\text{Voltage drop} = 70(1.53 \times 0.8 + 15.23 \times 0.6) = 725 \text{ V/phase}$$

$$\therefore \%\ \text{regulation} = \frac{725\times100}{19{,}000} = 3.8\%$$

Primary voltage will have to be increased by 3.8%

$\therefore$ Primary voltage = 6.6 + 3.8% of 6.6 = 6.58 kV = 6580 V

Common data for Examples 20 and 21

A Δ–Δ bank consisting of three 30-kVA 2300/230-V transformer supplies a load of 60 kVA. If one transformer is removed, find for the resulting V–V connecting.

Example 20: (a) KVA load carried by each transformer (b) Per cent of rated load carried by each transformer

Solution: (a) $$\frac{\text{Total kVA load in } V-V \text{ bank}}{\text{VA/transformer}} = \sqrt{3}$$

$\therefore$ kVA load supplied by each of the two transformers = $\dfrac{60}{\sqrt{3}}$

$$= 34.64 \text{ kVA}$$

Obviously, each transformer in V–V bank does not carry 50% of the original load but 57.7%

(b) Percentage of rated load

$$= \frac{\text{KVA load/ transformer}}{\text{kVA rating/transformer}}$$

$$= \frac{34.64}{30} = 115.5\%$$

Carried by each transformer obviously, in this case, each transformer is overloaded to the extent of 15.5 per cent.

Example 21: (a) Total kVA rating of the V–V bank (b) Ratio of the V–V bank to Δ–Δ bank transformer ratings (c) Percentage increase in load on each transformer

Solution: (a) kVA rating of the V–V bank = (2 × 30) × 0.8666 = 51.96 kVA

(b) $$\frac{V-V \text{ rating}}{\Delta-\Delta \text{ rating}} = \frac{51.96}{90} = 0.577 = 57.7\%$$

As seen, the rating is reduced to 57.7% of the original rating.

(c) Load supplied by each transformer in Δ–Δ bank = 60/3 = 20 kVA

$\therefore$ Percentage increase in load supplied by each transformer

$$= \frac{\text{kVA load/transformer in } V-V \text{ bank}}{\text{kVA load/transforemer in } \Delta-\Delta \text{ bank}}$$

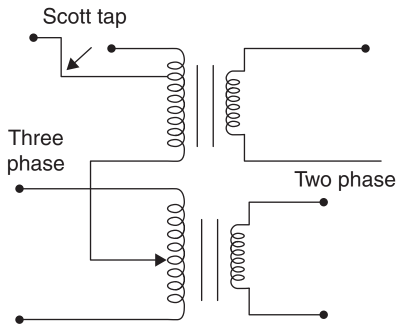
$$= \frac{34.64}{20} = 1.732 = 173.2\%$$

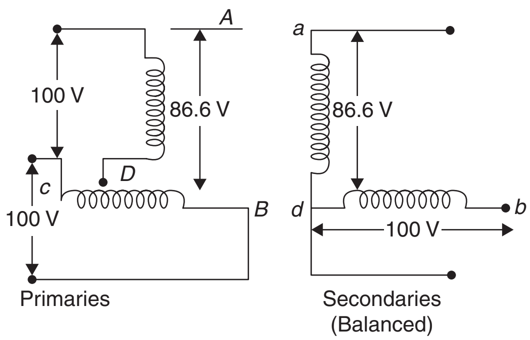
It is obvious that each transformer in the Δ–Δ bank supplying 60 kVA was running underloaded (20 vs. 30 kVA) but runs over loaded (34.64 vs. 30) in V–V connection.

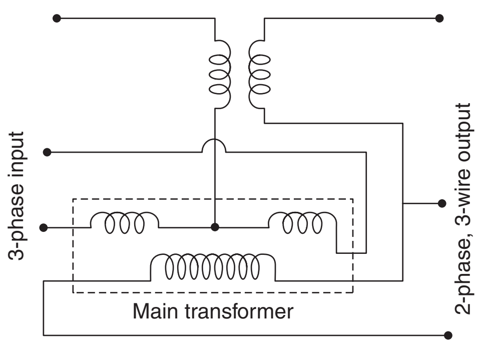
Three-phase to Two-phase Conversion

This conversion is required (i) to supply 2-phase furnaces. (ii) to interconnect 2-phase systems with 3-phase systems. (iii) to supply a 3-phase apparatus from 2-phase supply source.

For this purpose, Scott connection as shown in following fig is employed.







- Teaser transformer primary has $\sqrt{3}/2$ times the turns of main primary. But volt /turn is the same. Their secondaries have the same turns which results in equal secondary terminal voltages.
- If main primary has N_1 turns and main secondary has N_2 turns then main transformation ratio is N_2/N_1. However the transformation ratio of teaser is

$$\frac{N_2}{\left(\sqrt{3}N_1/2\right)} = 1.15N_2/N_1 = 1.15\text{ K}$$

- If the load is balanced on one side, it is balanced on the other side as well.
- Under balanced condition, main transformer rating is 15% greater than that of the teaser.
- The currents in either of the two halves of main primary are the vector sum of KI_{2M} and $0.58\,KI_{2T}$ (or) $\left(\frac{1}{2}I_{1T}\right)$.

Parallel Operations of Three-phase Transformers

All the conditions which apply to the parallel operation of single-phase transformer also apply to the parallel operation of three-phase transformer but with following conditions.

1. The voltage ratio must refer to the terminal voltage of primary and secondary. It is obvious that this ratio may not be equal to the ratio of the number of turns per phase. For example if V_1, V_2 are the primary and secondary terminal voltages then for star/delta connection, this turns ratio is $V_2/(V_1/\sqrt{3})$
$$= \sqrt{3}\,V_2/V_1.$$
2. The phase displacement between primary and secondary voltages must be the same for all transformers which are to be connected for parallel operation.
3. The phase sequence must be the same.
4. All the three transformers in the three-phase transformer bank will be of the same construction either core or shell.

Exercises

Practice Problems I

Directions for questions 1 to 45: Select the correct alternative from the given choices.

1. In a transformer ratio of the copper losses at half load to that at full load will be
(A) 2 times (B) one fourth
(C) half (D) 4 times

2. The eddy current loss of a transformer at 120 V, 60 Hz is 150 W. The eddy current loss at 120 V, 30 Hz will be
(A) 150 W (B) 75 W
(C) 37.5 W (D) 300 W

3. The ratio of primary to secondary turns of a transformer is 4. The primary resistance is 1 Ω and the secondary resistance is 0.5 Ω. The resistance of the transformer referred to primary side is
(A) 9 Ω (B) $\frac{1}{9}$ Ω
(C) 3 Ω (D) $\frac{1}{3}$ Ω

4. At 50 Hz operation, a single-phase transformer has hysteresis and eddy current losses of 300 W and 150 W, respectively. Its core loss at 60 Hz will be
(A) 450 W (B) 576 W
(C) 648 W (D) 540 W

5. When the iron and full-load copper losses in a transformer are 900 W and 1600 W, respectively, the maximum efficiency occurs at ______ of full load.
(A) 25% (B) 50%
(C) 75% (D) 100%

6. A transformer has 600 primary turns and 120 secondary turns. If the load current is 10 A, its primary load component of current is ______
(A) 20 A (B) 10 A
(C) 2 A (D) 5 A

7. A transformer possesses a percentage resistance and percentage reactance of Z2% and 4%, respectively. Its voltage regulation at power factor of 0.6 leading and 0.6 lagging would be, respectively,
(A) –2% and –2% (B) 4.4% and –2%
(C) –2% and 4.4% (D) 4.4% and –4.4%

8. If the applied voltage of a certain transformer is increased by 100% and the frequency is reduced to 25% (assuming that the magnetic circuit remains unsaturated) the maximum flux density will change to _______ times the original value
(A) 2 (B) 4
(C) 6 (D) 8

9. A 50 kVA transformer has a core loss of 500 W and a full-load copper loss of 1000 W. Maximum efficiency occurs at ________ per cent of full load
(A) 50 (B) 37.5
(C) 70.7 (D) 100

10. A single-phase transformer has rating of 10 kVA, 400/100 V. It is recommended as an auto-transformer to supply at 500 V from a 400 V primary source. The maximum load which it can supply is
(A) 50 kVA (B) 25 kVA
(C) 100 kVA (D) 12.5 kVA

11. An auto-transformer having a transformation ratio of 0.5, supplies a load of 20 kW. The power transferred inductively from the primary to secondary is
(A) 20 kW (B) 10 kW
(C) 5 kW (D) zero

Common Data for Questions 12 and 13:
A single-phase transformer on full load has an impedance drop of 25 V and resistance drop of 15 V.

12. The reactance drop is ___ V
(A) 15 (B) 20
(C) 25 (D) 30

13. The p.f. at which regulation will be zero is _______
(A) zero (B) 0.6
(C) 0.8 (D) 0.707

Common Data for Questions 14 and 15:
The exciting current was found to be 4A when measured on the LV side of a 10 – kVA, 1000/100 V transformer. Its equivalent impedance referred to the HV side is $8.5 + j12.5$ (Assume that the transformer rating as the base.)

14. The exciting current in p.u. on the LV as well as HV side, respectively _______
(A) 0.04, 0.02 (B) 0.02, 0.04
(C) 0.04, 0.04 (D) 0.08, 0.08

15. The equivalent impedance in p.u. on the HV side is
(A) $0.105 + j0.085$ (B) $0.21 + j0.17$
(C) $0.085 + j0.105$ (D) None of these

16. Two transformers of the same type, using the same grade of iron and conductor materials, are designed to work at the same flux and current densities, but the linear dimensions, of one are two times those of the other in all respects. The ratio of kVA of the two transformers closely equals
(A) 2 (B) 4
(C) 8 (D) 16

17. Two transformers of different kVA ratings working in parallel share the load In reply to: proportion to their ratings when their
(A) ohmic values of the leakage impedances are inversely proportional to their ratings.
(B) ohmic values of the leakage magnetizing reactances are the same.
(C) per unit leakage impedances on the same kVA base are the same.
(D) per unit leakage impedances on their respective ratings are equal.

18. The percentage impedance of a 100 kVA, 11 kV/400 V, delta/wye, 50 Hz transformer is 5%. For the circulation of half the full-load current during short-circuit test, with low voltage terminals, shorted, the applied voltage on the high voltage side will be
(A) 245 V (B) 275 V
(C) 315 V (D) 550 V

19. The laws of electromagnetic induction are summarized in which of the following equation?
(A) $e = iR$ (B) $e = \frac{d\psi}{dt}$
(C) $e = -\frac{Ldi}{dt}$ (D) $e = \frac{Nd\phi}{dt}$

20. Keeping in view the requirement of parallel operation, which of the 3-phase connections given below are possible?
(A) star–star to delta–star
(B) delta–delta to star–delta
(C) delta–delta to delta– star
(D) delta–star to star–delta

21. If an AC voltage wave is corrupted with an arbitrary number of harmonics, then the overall voltage wave form differs from its fundamental frequency component in terms of
(A) only the peak values.
(B) only the average values.

(C) only the rms values.
(D) all the three measures (Peak, rms and average values).

22. A single-phase transformer is to be switched to the supply to have minimum inrush current. The switch should be closed at
(A) zero supply voltage.
(B) $\frac{1}{2}$ maximum supply voltage.
(C) $\frac{1}{\sqrt{2}}$ maximum supply voltage.
(D) maximum supply voltage.

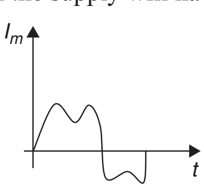
23. In the protection of transformers harmonic restraint is used to guard against
(A) switching.
(B) lightning.
(C) magnetizing in rush current.
(D) unbalanced operation.

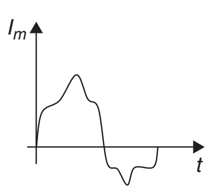
24. The efficiency of a 100 kVA transformer is 0.98 at full as well as at half full-load. For this transformer at full load, the copper loss
(A) is equal to core loss.
(B) is less than core loss.
(C) is more than core loss.
(D) None of the above

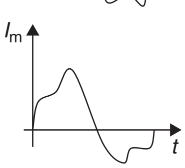
25. The core flux of a practical transformer with a resistive load
(A) increases linearly with load.
(B) increases as the square root of the load.
(C) is strictly constant with load changes.
(D) decreases with increased load.

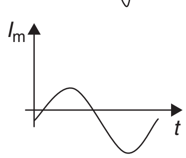
26. The single-phase, 50 Hz iron core transformer in the circuit has both the vertical arms of cross sectional area 20 cm^2 and both the horizontal arms of cross sectional area 10 cm^2. If the windings were wound instead on opposite horizontal arms, the mutual inductances will
(A) remain same.
(B) become one quarter.
(C) be halved.
(D) doubled.

27. A single-phase air core transformer, fed from a rated sinusoidal supply, is operating at no load. The steady state magnetizing current drawn by the transformers from the supply will have the wave form

(A) 

(B) 

(C) 

(D) 

28. In transformer, which of the following statements is valid?
(A) In an oc test, current is drawn at high power factor.
(B) In an oc test, copper losses are obtained, while in short-circuit test core losses are obtained.
(C) In a sc test, current is drawn at zero power factor.
(D) In an oc test, current is drawn at low power factor.

29. In a transformer, zero voltage regulation at full load is
(A) possible at unity power factor load.
(B) possible at leading power factor load.
(C) possible at lagging power factor load.
(D) Not possible.

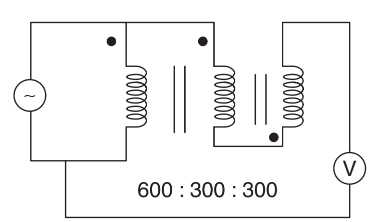
30. A 50 Hz transformer having equal hysteresis and eddy current losses at rated excitation is operated at 45 Hz at 90% of its rated voltage. Compared to rated operating point the core losses under this condition
(A) increase by 10%.
(B) reduce by 10%.
(C) reduce by 14.5%.
(D) remain unchanged.

31. The hysteresis loop of a magnetic material has an area of 10 cm^2 with the scales given as 1 cm = 2 AT and 1 cm = 50 mwb. At 50 Hz the total hysteresis loss is
(A) 10 W. (B) 25 W.
(C) 50 W. (D) 100 W.

32. The windings of a Q kVA, V_1/V_2 volt, three phase delta connected core type transformer are reconnected to work as a single-phase transformer. The maximum voltage and the power ratings are
(A) $\frac{V_1}{V_2}, Q/3$. (B) $\frac{\sqrt{3}V_1}{\sqrt{3}V_2}, 3Q$.
(C) $\frac{2V_1}{2V_2}, \frac{2Q}{3}$. (D) $\frac{\sqrt{3}V_1}{\sqrt{3}V_2}, Q$.

33. A 10 kVA 400 V/200V single-phase transformer with 5% impedance draws short-circuit line current of
(A) 50 A. (B) 100 A.
(C) 250 A. (D) 500 A.

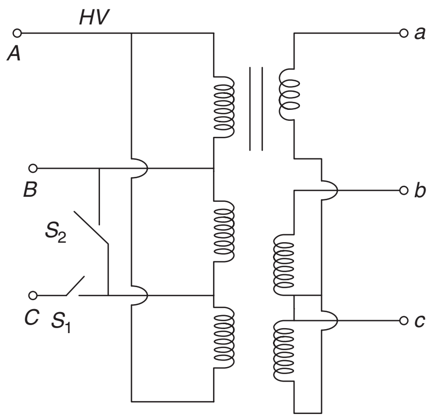
34. A 600 V / 300 V/ 300 V, 50 Hz three-winding transformer is connected as shown in figure. The reading of the voltmeter 'V' will be



(A) 0 V. (B) 300 V.
(C) 600 V. (D) 1200 V.

35. Figure shows Δ–Y connected 3-phase distribution transformer used to step down the voltage form 11 kV to 415 V line-to-line. It has two switches S_1 and S_2.

Under normal conditions S_1 is closed and S_2 is open. The magnitude of the voltage across LV terminals a and c with S_1 open and S_2 closed is

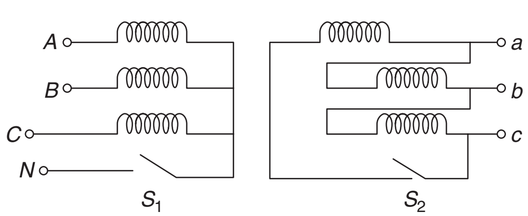


(A) 240 V. (B) 415 V.
(C) 830 V. (D) 0 V.

36. Two transformers are to be operated in parallel such that they share the load in proportion to their kVA ratings. The rating of the first transformer is 200 kVA and its p.u. leakage impedance is 0.05 p.u. If the second transformer is 100 kVA, its p.u. leakage impedance is
(A) 0.10. (B) 0.20.
(C) 0.05. (D) 0.025.

37. A single-phase 50 kVA, 250 V/500 V two-winding transformer has an efficiency of 95% at full load, unity power factor. It is reconfigured as a 500/750 V auto-transformer, its efficiency at its new rated load at unity power factor will be
(A) 93.275%. (B) 95.752%.
(C) 98.276%. (D) 99.241%.

Common Data for Questions 38 and 39:

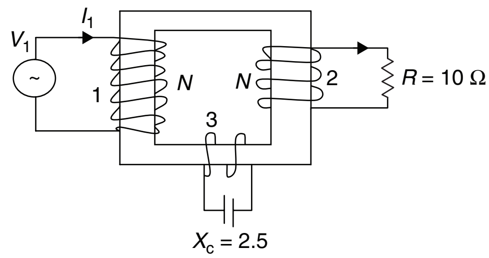


The star-delta transformer shown above is excited on the star side with balanced, 4-wire, 3-phase, sinusoidal voltage supply of rated magnitude. The transformer is under no load condition.

38. With both S_1 and S_2 open, the core flux waveform will be
(A) flat-topped with third harmonic.
(B) peaky with third harmonic.
(C) A sinusoid at fundamental frequency.
(D) flat-topped with fifth harmonic.

39. With S_2 closed and S_1 open, the current wave form in the delta winding will be
(A) flat-topped with fifth harmonic.
(B) a sinusoid with third harmonic.
(C) only third harmonic.
(D) flat-topped with third harmonic.

40. Figure shows an ideal three-winding transformer. The three windings 1, 2, and 3 of the transformer are wound as shown in fig. the turns ratio N_1:N_2:N_3 is 4:2:1, A resistor of 10 Ω is connected across winding-2. A capacitor of reactance 2.5 Ω is connected across winding – 3. Winding -1 is connected to a 200 V supply. If the supply voltage phasor $V_1 = 200 \angle 0°$, the supply current phasor I_1 is given by

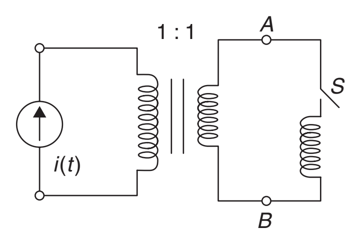


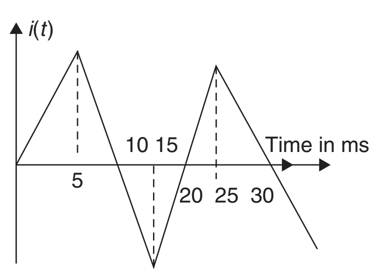
(A) $(-5 + j5)$ A. (B) $(5 + j5)$ A.
(C) $(5 - j5)$ A. (D) $(-5 - j5)$ A.

41. The resistance and reactance of a 100 kVA 11000/400 V, Δ–y distribution transformer are 0.02 and 0.07 p.u., respectively. The phase impedance of the transformer referred to the primary is
(A) $(0.55 + j1.925)$ Ω. (B) $(2.15 + j7.35)$ Ω.
(C) $(15.125 + j52.94)$ Ω. (D) $(72.6 + j254.1)$ Ω.

Common Data for Questions 42 and 43:

The circuit diagram shows a two-winding loss less transformer with no leakage flux, excited form a current source, $i(t)$, whose wave form is also shown. The transformer has a magnetizing inductance of $200/\pi$ mH.

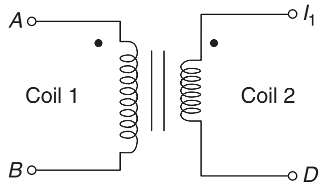




42. The peak voltage across A and B with S open is
(A) $200/\pi$ V. (B) 400 V.
(C) $\frac{2000}{\pi}$ V (D) $\frac{400}{\pi}$ V.

43. If the wave form of $i(t) = 10\sin(100\pi t)$A, the peak voltage across A and B with 'S' closed is
(A) 200 V. (B) $200 \times \sqrt{2}$ V.
(C) 400 V. (D) $400 \times \sqrt{2}$ V.

Common Data for Questions 44 and 45:



The figure above shows coils 1 and 2, with dot markings as shown, having 2000 and 3000 turns, respectively. Both the coils have a rated current of 25 A. Coil-1 is excited with single phase, 400 V, 50 Hz supply.

44. The coils are to be connected as a single phase, 400/1000 V auto-transformer to drive a load of 10 kVA. Which of the options given should be exercised to realize the required auto-transformer?
(A) connect A and C; common B.
(B) connect A and C; common D.
(C) connect A and D; common B.
(D) connect B and D; common C.

45. In the auto-transformer obtained in above question the current in each coil is
(A) coil-1 is 15 A and coil-2 is 10 A.
(B) coil-1 is 10 A and coil-2 is 25 A.
(C) coil-1 is 10 A and coil-2 is 15 A.
(D) coil-1 is 25 A and coil-2 is 10 A.

Practice Problems 2

Directions for questions 1 to 50: Select the correct alternative from the given choices.

1. The purpose of transformer oil is
(A) to provide lubrication
(B) to provide insulation and cooling
(C) to provide protection against short circuit
(D) to provide protection against lightning

2. A voltage $V = 400 \sin 314t$ is applied to a single-phase transformer on no load. If the no-load current of the transformer is $2\sin(314.16t - \angle 85°)$, then magnetization branch impedance will be approximately equal to
(A) $200 \angle -85°\ \Omega$ (B) $200 \angle 85°\ \Omega$
(C) $200 \angle 5°\ \Omega$ (D) $200 \angle -5°\ \Omega$

3. The efficiency of a 1000 kVA transformer is 95% at full load as well as at half the full- load both at u.p.f. The Cu loss at full load
(A) is less than core loss
(B) is equal to core loss
(C) is more than core loss
(D) none of these

4. The dominant harmonic in the magnetizing current of a transformer is
(A) 3^{rd} harmonic (B) 5^{th} harmonic
(C) 7^{th} harmonic (D) 13^{th} harmonic

5. A 60 Hz transformer having equal hysteresis and eddy current losses at rated excitation is operated at 48 Hz and 80% rated voltage. The core losses under this condition
(A) reduce by 20%
(B) reduce by 38%
(C) reduce by 28%
(D) remain unchanged

6. A 400 / 200 V 20 kVA two-winding transformer is reconnected as an auto-transformer. The maximum possible rating of auto-transformer could be
(A) 50 kVA (B) 15 kVA
(C) 40 kVA (D) 60 kVA

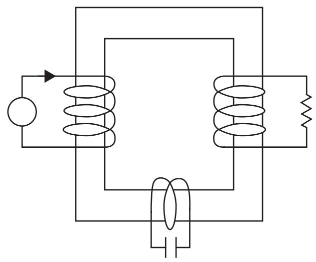
7. In a constant voltage transformer (CVT) output voltage remains constant because of
(A) saturation
(B) tapped windings
(C) capacitor across the terminals
(D) a DC voltage source across the terminals

8. Two transformers of different kVA rating working in parallel share the load in proportion to their rating when
(A) ohmic values of leakage impedances are inversely proportional to their ratings
(B) ohmic values of magnetizing reactances are same
(C) per unit leakage reactance on their respective kVA bases are equal
(D) per unit leakage impedance on same kVA base are equal

9. A transformer is subjected to a short circuit. The turns of the winding experience
(A) attractive force
(B) repulsive force
(C) no force
(D) can be attractive or repulsive

10. Maximum efficiency of a single-phase transformer is 90% at full load and u.p.f. Efficiency at half load and u.p.f. is
(A) 89.1% (B) 87.8%
(C) 90% (D) 86.8%

11.



$N_1{:}N_2{:}N_3 = 4{:}2{:}1$, $V_1 = 400\ \angle 0$. The current phasor in N_1, I_1 is given by

(A) $-10 - 10j$ (B) $-10 + 10j$
(C) $10 - 10j$ (D) $10 + 10j$

12. Building steel core out of stampings reduces eddy current loss because it

(A) increases the effective length of eddy current paths thereby increasing effective resistance to flow of eddy currents
(B) reduces effective length of eddy current path, thereby reducing effective resistance to flow of eddy currents
(C) increases core resistivity
(D) increases core permeability

13. Assuming P_i is the iron loss and P_c is the copper loss, the maximum kVA delivered to the load corresponding to maximum efficiency is

(A) rated kVA $\times \sqrt{\dfrac{P_c}{P_i}}$ (B) rated kVA $\times \sqrt{\dfrac{P_i}{P_c}}$

(C) rated kVA $\times \left(\dfrac{P_c}{P_i}\right)^2$ (D) rated kVA $\times \left(\dfrac{P_i}{P_c}\right)^2$

14. A transformer of turns ratio 200/400, 50 Hz is to be excited from 40 Hz from 200 V side. For the exciting current to remain same, the applied voltage should be

(A) 300 V (B) 250 V
(C) 200 V (D) 60 V

15. The leakage reactance of a transformer can be found out using

(A) OC test only (B) SC test only
(C) both OC and SC (D) cannot be found

16. A transformer on no load, when switched on draws a current

(A) twice the steady-state magnetizing current provided core has no residual flux
(B) several times the steady-state magnetizing current independent of initial state of residual flux in the core
(C) Several times steady-state magnetizing current dependent on initial state of residual flux in core
(D) same as steady-state magnetizing current

17. If a two-winding 2/1 transformer is reconnected as auto-transformer, its kVA

(A) remains same (B) becomes 2 times
(C) becomes 3 times (D) becomes 1.5 times

18. A transformer has 95% efficiency at full load and 0.85 p.f. lag. Efficiency at full load 0.85 pf lead will be

(A) > 95% (B) < 95%
(C) = 95% (D) 100%

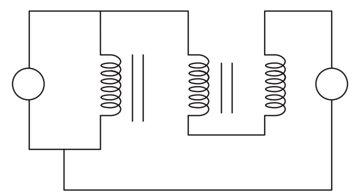
19. A single-phase transformer is to be switched to have minimum inrush current. Switch should be closed at

(A) $\dfrac{1}{\sqrt{2}}$ times maximum value of V

(B) $\dfrac{1}{2}$ times maximum value of V

(C) at maximum value of V
(D) at zero value of V

20. A 400 V/200 V/200 V is connected as shown. Reading in voltmeter V is



(A) 0 V (B) 200 V
(C) 400 V (D) 800 V

21. Auto-transformer is used in transmission and distribution when

(A) operator is not available
(B) iron losses are to be reduced
(C) efficiency consideration can be ignored
(D) when transformation ratio is small

22. Which of the following 3-phase connection can possibly be operated in parallel

(A) $\Delta - \Delta$ to $\Delta - Y$ (B) $\Delta - \Delta$ to $Y - \Delta$
(C) $Y - Y$ to $\Delta - \Delta$ (D) $\Delta - Y$ to $Y - \Delta$

23. A 2400/240 V, 200 kVA single-phase transformer has a core loss of 1.8 kW at rated voltage. $R = 1.1\%$. Transformer efficiency at 0.9 p.f. and full load is

(A) 97.57% (B) 98.05%
(C) 97.82% (D) 96.56%

24. Hysteresis loss and eddy current loss of 340 W and 120 W, respectively, for a 220 V, 60 Hz single-phase transformer. If the transformer is operated from 230 V, 50 Hz supply mains then core loss assuming (Steinmetz) constant equal to 1.6 is

(A) 539.3 W (B) 408 W
(C) 349.4 W (D) 308 W

25. Full-load voltage regulation of a power transformer is zero when p.f. of load is

(A) unity and leading
(B) zero and leading
(C) zero and lagging
(D) unity and lagging

26. Three single-phase transformers each with a 10 kVA rating are connected in a closed delta arrangement. If one transformer is taken out, o/p capacity of system will be

(A) 20 kVA (B) 8.66 kVA
(C) 17.39 kVA (D) 10 kVA

27. A three-phase transformer has rating at 20 mVA, 220 kV (Y), 33 kV (Δ) with leakage reactance at 12%. The transformer reactance (in Ω) referred to each phase of L.V delta is

(A) 23.5 (B) 19.6
(C) 18.5 (D) 8

28. While performing the O.C. and S.C. tests on a transformer

(A) in O.C. LV is open, in S.C LV is short
(B) in O.C. hv is open in SC LV is short
(C) in O.C. hv is open and in S.C hv is short
(D) in O.C. lv is open and in S.C hv is short

29. Iron loss and full-load copper loss in a transformer are 40.5 kW and 50 kW. Efficiency is maximum at load fraction equal to

(A) 0.57 (B) 0.8
(C) 0.9 (D) 0.75

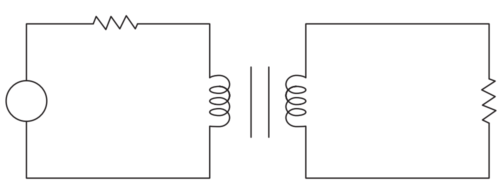
30. In a transformer, fed from a fundamental frequency voltage source, the source of harmonics is

(A) poor insulation
(B) iron loss
(C) overload
(D) saturation of core

31. In a 3 phase Δ–Y transformer phase displacement of secondary line voltages with primary line voltage will be

(A) 180° (B) zero
(C) 30° lead (D) 30° lag

32.



In the figure, transformer is ideal. For what number of turns in primary, maximum power is transferred.

(A) 40 (B) 80
(C) 160 (D) 320

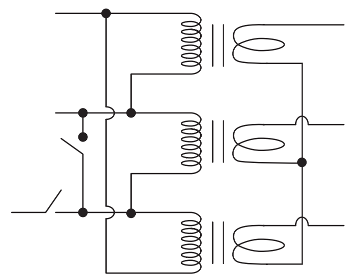
33. A 5 kVA transformer has iron loss of 200 W and full-load copper loss of 300 W. At maximum efficiency, total loss is

(A) 200 W (B) 400 W
(C) 600 W (D) 800 W

34. A transformer has turns ratio 4:1. Reactance referred to secondary side is 0.05 p.u. Reactance referred to primary is

(A) 0.2 ohm (B) 0.05 ohm
(C) 0.2 p.u. (D) 0.05 p.u.

35.



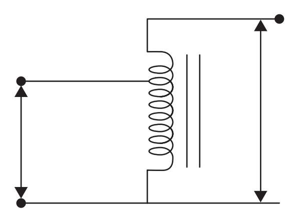
Transformer is used to step down voltage from 11000 V to 415 V line to line. Normally S_1 is closed and S_2 is open. If S_1 is open and S_2 is closed, what is the voltage across a and c?

(A) 415 V (B) 240 V
(C) 480 V (D) 0 V

36. A 500 kVA, three-phase transformer has iron losses of 300 W and full-load copper loss of 600 W. Percentage load at which transformer is expected to have maximum efficiency?

(A) 50.0% (B) 70.7%
(C) 141.4% (D) 200%

37.



50 kVA 3300/230 V transformer is reconnected as an auto-transformer. The nominal rating of auto-transformer would be

(A) 50 kVA (B) 53.5 kVA
(C) 717.4 kVA (D) 767.4 kVA

38. A 100 kVA, 11000/400 V, Δ–Y distribution transformer has resistance and reactance 0.02 and 0.07 p.u., respectively, at the secondary side. The phase impedance at primary side is

(A) $24.2 + j\,84.7\ \Omega$ (B) $0.02 + j\,0.07\ \Omega$
(C) $41.91 + j146.70\ \Omega$ (D) $72.6 + j254.1\ \Omega$

39. The equivalent circuit of a transformer has leakage reactance X_1 and magnetizing reactance X_m. their magnitudes satisfy

(A) $X_1 << X_2' << X_m$
(B) $X_1 >> X_2' >> X_m$
(C) $X_1 \cong X_2 << X_m$
(D) $X_1 \cong X_2 >> X_m$

40. To introduce a phase difference of 30° between input and output line voltages, which of the 3 phase connections can be used

(A) Y−Y (B) Y−Δ
(C) Δ−Δ (D) Δ−zigzag

41. Which of the following statements are valid with respect to transformers?

(A) In O.C. Cu losses are obtained, while in S.C. core losses are obtained
(B) In O.C. current is drawn at high power factor
(C) In O.C. current is drawn at low p.f.
(D) In O.C. current is drawn at zero p.f.

42. Two transformers are operated in parallel such that they share load in proportion to their kVA ratings. The rating of first transformer is 500 kVA and its p.u. leakage impedance is 0.05 p.u. If rating of second transformer is 250 kVA leakage impedance is

(A) 0.05 (B) 0.025
(C) 0.1 (D) 0.2

43. A bank of three identical transformers of 11 kV /230 V is used to provide 400 V from a 11 kV 3 phase substation. The kVA rating of the bank will be

(A) $250\sqrt{3}$ (B) 250
(C) 500 (D) 750

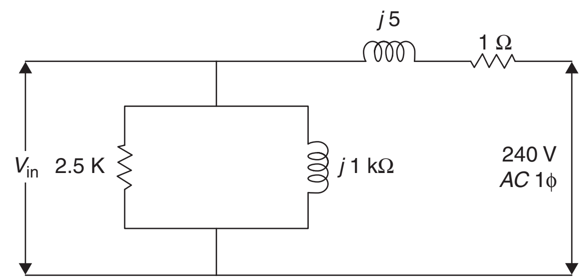
44. The frequency of power supply of a transformer designed for 60 Hz is changed to 50 Hz with voltage and current ratings remaining the same. The change in efficiency

(A) increases marginally
(B) increases by a factor 1.2
(C) remains unaltered
(D) decreases marginally

45. If the height-to-width ratio of the window core type transformer is increased then

(A) leakage reactance and % voltage regulation decreases
(B) leakage reactance and % voltage regulation increases
(C) leakage reactance will be unaffected
(D) efficiency will increase temperature rise decreases

Common Data for Questions 46 and 47:



46. Equivalent circuit of 240/110 V 1.2 kVA single-phase transformer referred to primary. Delivers full load at rated voltage and u.p.f. Efficiency will be

(A) 98% (B) 96%
(C) 94% (D) 92%

47. Regulation will be

(A) 1.546% (B) 2.546%
(C) 3.546% (D) 4.546%

48. A 250 kVA, 230/115 V 50 Hz transformer has $r_1 = 0.12\ \Omega$ $r_2 = 0.04\ \Omega$,
$X_1 = 0.2\ \Omega$ $X_2 = 0.05\ \Omega$
At what loading will the primary induced emf be equal to primary terminal voltage when transformer is carrying full-load current? Neglect magnetizing current

(A) 15.57 kW (B) 16.57 kW
(C) 17.57 kW (D) 19.57 kW

49. The leakage flux in a transformer depends on

(A) applied voltage (B) load current
(C) frequency (D) mutual flux

50. 15 kVA 600/120 V transformer is reconnected as an auto-transformer to supply 720 V from a 600 V source. Its new rating is

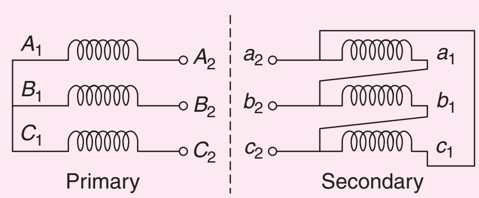
(A) 90 kVA (B) 18 kVA
(C) 15 kVA (D) 12 kVA

PREVIOUS YEARS' QUESTIONS

1. The equivalent circuit of a transformer has leakage reactance X_1, X'_2 and magnetizing reactance X_M. Their magnitudes satisfy **[2005]**

(A) $X_1 >> X'_2 >> X_M$ (B) $X_1 << X'_2 << X_M$
(C) $X_1 = X'_2 >> X_M$ (D) $X_1 = X'_2 << X_M$

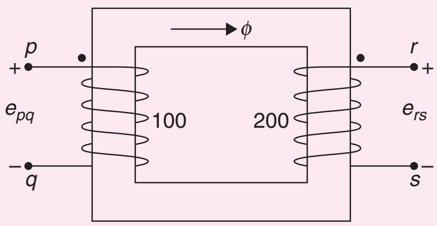
2. Three single-phase transformers are connected to form a three-phase transformer bank. The transformers are connected in the following manner

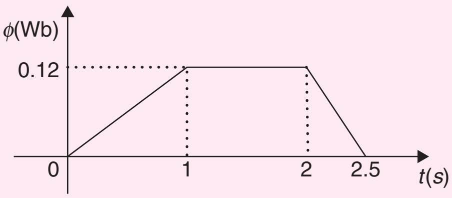


The transformer connection will be represented by **[2008]**

(A) Y d0 (B) Y d1
(C) Y d6 (D) Y d11

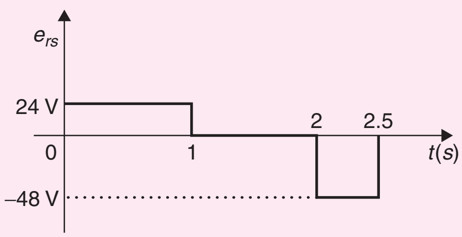
3. The core of a two-winding transformer is subjected to a magnetic flux variation as indicated in the figure. **[2008]**



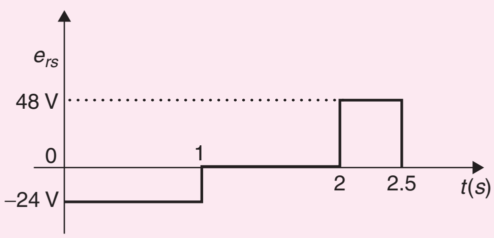


The induced emf (e_{rs}) in the secondary winding as a function of time will be of the form

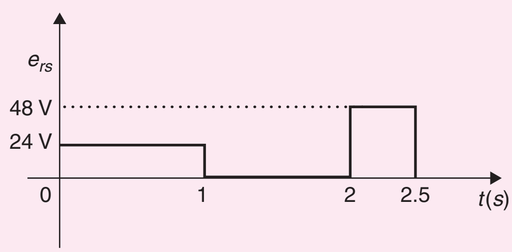
(A)



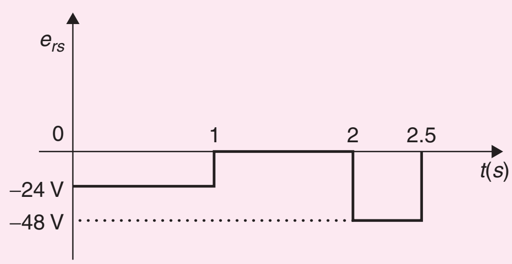
(B)



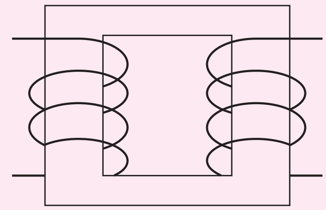
(C)



(D)

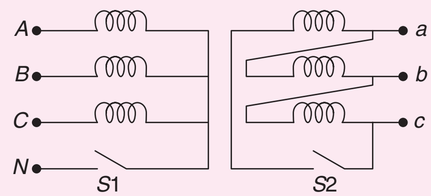


4. The single phase, 50 Hz, iron core transformer in the circuit has both the vertical arms of cross-sectional area 20 cm^2 and both the horizontal arms of cross sectional area 10 cm^2. If the two windings shown were wound instead on opposite horizontal arms, the mutual inductance will **[2009]**



(A) double (B) remain same
(C) be halved (D) become one quarter

Common Data for Questions 5 and 6



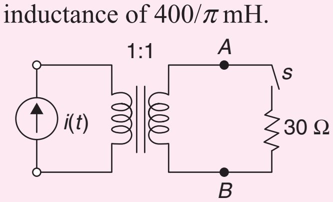
The star-delta transformer shown above is excited on the star side with a balanced, 4-wire, 3-phase, sinusoidal voltage supply of rated magnitude. The transformer is under no load condition.

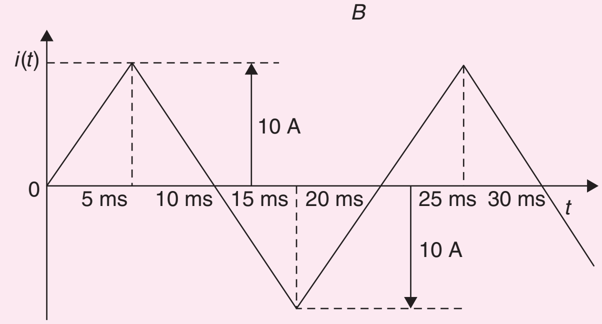
5. With both S1 and S2 open, the core flux waveform will be **[2009]**
(A) a sinusoid at fundamental frequency
(B) flat topped with third harmonic
(C) peaky with third harmonic
(D) none of these

6. With S2 closed and S1 open, the current waveform in the delta winding will be **[2009]**
(A) a sinusoid at fundamental frequency
(B) flat topped with third harmonic
(C) only third harmonic
(D) None of these

Common Data for Questions 7 and 8

The circuit diagram shows a two-winding, lossless transformer with no leakage flux, excited from a current source, $i(t)$, whose waveform is also shown. The transformer has a magnetizing inductance of $400/\pi$ mH.





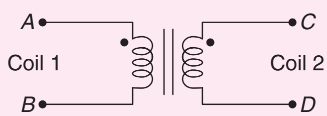
7. The peak voltage across A and B, with S open is **[2009]**

(A) $400/\pi$ V (B) 800 V
(C) $4000/\pi$ V (D) $800/\pi$ V

8. If the waveform of $i(t)$ is changed to $i(t) = 10\sin(100\pi t)$ A, the peak voltage across A and B with S closed is **[2009]**

(A) 400 V (B) 240 V
(C) 320 V (D) 160 V

Common Data for Questions 9 and 10:



The figure above shows coils 1 and 2, with dot markings as shown, having 4000 and 6000 turns, respectively. Both coils have a rated current of 25 A. Coil 1 is excited with single phase, 400 V, 50 Hz supply

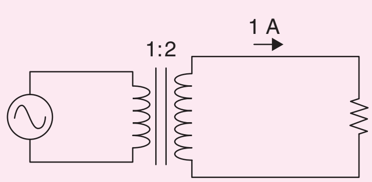
9. The coils are to be connected to obtain a single phase, 400/1000 V, auto-transformer to drive a load of 10 kVA. Which of the options given should be exercised to realize the required auto-transformer? **[2009]**

(A) Connect A and D; Common B
(B) Connect B and D; Common C
(C) Connect A and C; Common B
(D) Connect A and C; Common D

10. In the auto-transformer obtained in Question 57, the current in each coil is **[2009]**

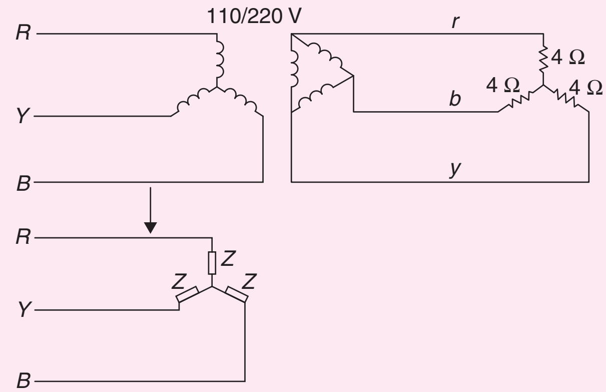
(A) Coil-1 is 25 A and Coil-2 is 10 A
(B) Coil-1 is 10 A and Coil-2 is 25 A
(C) Coil-1 is 10 A and Coil-2 is 15 A
(D) Coil-1 is 15 A and Coil-2 is 10 A

11. A single-phase transformer has a turns ratio of 1:2, and is connected to a purely resistive load as shown in the figure. The magnetizing current drawn is 1 A, and the secondary current is 1 A. If core losses and leakage reactances are neglected, the primary current is **[2010]**



(A) 1.41A (B) 2A
(C) 2.24 A (D) 3 A

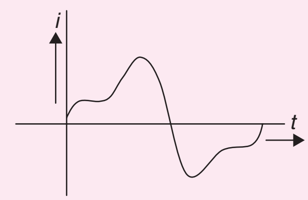
12. A balanced star-connected and purely resistive load is connected at the secondary of a star-delta transformer as shown in the figure. The line-to-line voltage rating of the transformer is 110 V/220 V. Neglecting the non-idealities of the transformer, the impedance 'Z' of the equivalent star-connected load, referred to the primary side of the transformer, is **[2010]**



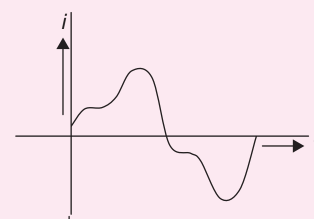
(A) $(3 + j0)$ Ω (B) $(0.866 - j0.5)$ Ω
(C) $(0.866 + j0.5)$ Ω (D) $(1 + j0)$ Ω

13. A single-phase air core transformer, fed from a rated sinusoidal supply, is operating at no load. The steady state magnetizing current drawn by the transformer from the supply will have the waveform. **[2011]**

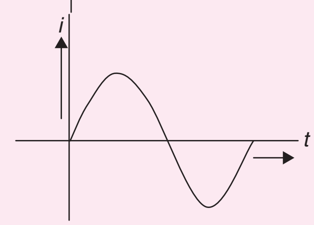
(A)



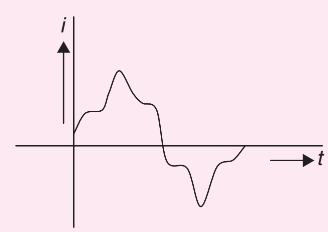
(B)



(C)



(D)



14. A single-phase 10 kVA, 50 Hz transformer with 1 kV primary winding draws 0.5 A and 55 W, at rated voltage and frequency, on no load. A second transformer has a core with all its linear dimensions $\sqrt{2}$ times the corresponding dimensions of the first transformer.

The core material and lamination thickness are the same in both transformers. The primary windings of both the transformers have the same number of turns. If a rated voltage of 2 kV at 50 Hz is applied to the primary of the second transformer, then the no-load current and power, respectively, are **[2012]**

(A) 0.7 A, 77.8 W (B) 0.7 A, 155.6 W
(C) 1 A, 110 W (D) 1 A, 220 W

15. A single-phase transformer has no-load loss of 64 W, as obtained from an open-circuit test. When a short-circuit test is performed on it with 90% of the rated currents flowing in its both LV and HV windings, the measured loss is 81 W. The transformer has maximum efficiency when operated at **[2013]**

(A) 50.0% of the rated current
(B) 64.0% of the rated current
(C) 80.0% of the rated current
(D) 88.8% of the rated current

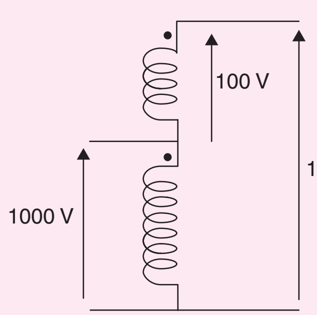
16. For a specified input voltage and frequency, if the equivalent radius of the core of a transformer is reduced by half, the factor by which the number of turns in the primary should change to maintain the same no-load current is **[2014]**

(A) 1/4 (B) 1/2
(C) 2 (D) 4

17. The core loss of a single phase, 230/115 V, 50 Hz power transformer is measured from 230 V side by feeding the primary (230 V side) from a variable voltage variable frequency source while keeping the secondary open circuited. The core loss is measured to be 1050 W for 230 V, 50 Hz input. The core loss is again measured to be 500 W for 138 V, 30 Hz input. The hysteresis and eddy current losses of the transformer for 230 V, 50 Hz input are, respectively, **[2014]**

(A) 508 W and 542 W (B) 468 W and 582 W
(C) 498 W and 552 W (D) 488 W and 562 W

18. A single-phase, 50 kVA, 1000 V/100 V two-winding transformer is connected as an auto-transformer as shown in the figure.

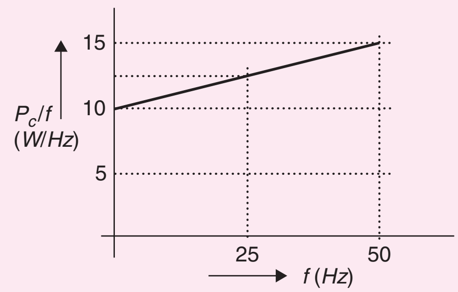


The kVA rating of the auto-transformer is ______. **[2014]**

19. For a single phase, two-winding transformer, the supply frequency and voltage are both increased by 10%. The percentage changes in the hysteresis loss and eddy current loss, respectively, are **[2014]**

(A) 10 and 21 (B) −10 and 21
(C) 21 and 10 (D) −21 and 10

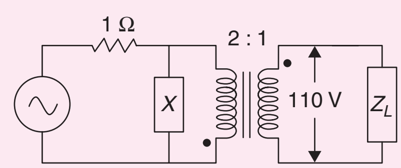
20. An open-circuit test is performed on 50 Hz transformer, using variable frequency source and keeping V/f ratio constant, to separate its eddy current and hysteresis losses. The variation of core loss/frequency as function of frequency is shown in the figure



The hysteresis and eddy current losses of the transformer at 25 Hz, respectively, are **[2014]**

(A) 250 W and 2.5 W
(B) 250 W and 62.5 W
(C) 312.5 W and 62.5 W
(D) 312.5 W and 250 W

21. The load shown in the figure absorbs 4 kW at a power factor of 0.89 lagging.



Assuming the transformer to be ideal, the value of the reactance X to improve the input power factor to unity is ________. **[2014]**

22. The parameters measured for a 220 V/110 V, 50 Hz, single-phase transformer are:

Self-inductance of primary winding = 45 mH

Self-inductance of secondary winding = 30 mH

Mutual inductance between primary and secondary windings = 20 mH

Using the above parameters, the leakage (L_{l1}, L_{l2}) and magnetizing (L_m) inductances as referred to primary side in the equivalent circuit, respectively, are **[2014]**

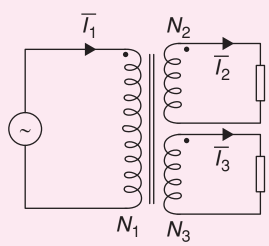
(A) 5 mH, 20 mH and 40 mH
(B) 5 mH, 80 mH and 40 mH
(C) 25 mH, 10 mH and 20 mH
(D) 45 mH, 30 mH and 20 mH

23. The primary mmf is least affected by the secondary terminal conditions in a **[2015]**
(A) power transformer
(B) potential transformer
(C) current transformer
(D) distribution transformer

24. A 200/400 V, 50 Hz, two-winding transformer is rated at 20 kVA. Its windings are connected as an auto transformer of rating 200/600 V. A resistive load of 12 Ω is connected to the high voltage (600 V) side of the auto-transformer. The value of equivalent load resistance (in Ohm) as seen from low voltage side is ______. **[2015]**

25. Two single-phase transformers T_1 and T_2 each rated at 500 kVA are operated in parallel. Percentage impedances of T_1 and T_2 are $(1 + j6)$ and $(0.8 + j4.8)$, respectively. To share a load of 1000 kVA at 0.8 lagging power factor, the contribution of T_2(in kVA) is _____. **[2015]**

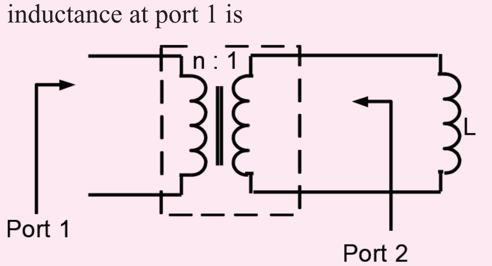
26. A three-winding transformer is connected to an AC voltage source as shown in the figure. The number of turns are as follows: $N_1 = 100$, $N_2 = 50$, $N_3 = 50$. If the magnetizing current is neglected, and the currents in two windings are $\overline{I_2} = 2\angle 30°$ A and $\overline{I_3} = 2\angle 150°$ A, then what is the value of the current $\overline{I_1}$ in Ampere? **[2015]**



(A) $1\angle 90°$ (B) $1\angle 270°$
(C) $4\angle 90°$ (D) $4\angle 270°$

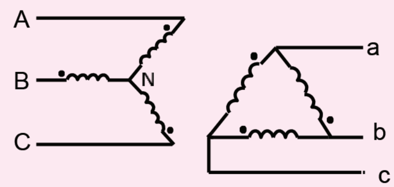
27. In a constant V/f induction motor drive, the slip at the maximum torque **[2016]**
(A) is directly proportional to the synchronous speed.
(B) remains constant with respect to the synchronous speed.
(C) has an inverse relation with the synchronous speed.
(D) has no relation with the synchronous speed.

28. If an ideal transformer has an inductive load element at port 2 as shown in the figure below, the equivalent inductance at port 1 is **[2016]**



(A) nL (B) n^2L
(C) $\frac{n}{L}$ (D) $\frac{n^2}{L}$

29. If the star side of the star-delta transformer shown in the figure is excited by a negative sequence voltage, then **[2016]**

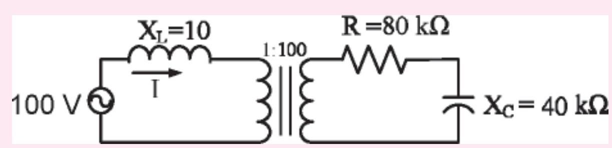


(A) V_{AB} leads V_{ab} by 60°
(B) V_{AB} lags V_{ab} by 60°
(C) V_{AB} leads V_{ab} by 30°
(D) V_{AB} lags V_{ab} by 30°

30. A single-phase 400 V, 50 Hz transformer has an iron loss of 5000 W at the rated condition. When operated at 200 V, 25 Hz, the iron loss is 2000 W. When operated at 416 V, 52 Hz, the value of the hysteretic loss divided by the eddy current loss is ______. **[2016]**

31. A single-phase, 22 kVA, 2200 V/ 220 V, 50 Hz, distribution transformer is to be connected as an auto-transformer to get an output voltage of 2420 V. Its maximum KVA rating as an auto- transformer is _______. **[2016]**
(A) 22 (B) 24.2
(C) 242 (D) 2420

32. The following figure shows the connection of an ideal transformer with primary to secondary turns ratio of 1: 100. The applied primary voltage is 100V (rms), 50Hz, AC. The rms value of the current I, in ampere, is ________. **[2016]**



33. A single-phase, 2kVA, 100/200V transformer is reconnected as an auto-transformer such that its kVA rating is maximum. The new rating, in kVA is _________. **[2016]**

34. Three single-phase transformers are connected to from a delta-star three-phase transformer of 110kV/11kV. The transformer supplies at 11kV a load of 8MW at 0.8p.f. lagging to a near by plant. Neglect the transformer losses. The ratio of phase currents in delta side to star side is ________. **[2016]**

(A) $1:10\sqrt{3}$ (B) $10\sqrt{3}:1$
(C) 1 : 10 (D) $\sqrt{3}:10$

Answer Keys

Exercises

Practice Problems 1

1. B	**2.** A	**3.** A	**4.** B	**5.** C	**6.** C	**7.** C	**8.** D	**9.** C	**10.** A
11. B	**12.** B	**13.** C	**14.** C	**15.** C	**16.** D	**17.** D	**18.** B	**19.** C	**20.** D
21. D	**22.** D	**23.** C	**24.** C	**25.** C	**26.** C	**27.** D	**28.** D	**29.** B	**30.** C
31. C	**32.** C	**33.** D	**34.** A	**35.** D	**36.** C	**37.** C	**38.** A	**39.** C	**40.** B
41. D	**42.** D	**43.** A	**44.** C	**45.** A					

Practice Problems 2

1. B	**2.** B	**3.** C	**4.** A	**5.** C	**6.** D	**7.** B	**8.** C	**9.** A	**10.** B
11. D	**12.** A	**13.** B	**14.** D	**15.** B	**16.** C	**17.** C	**18.** C	**19.** C	**20.** A
21. D	**22.** C	**23.** C	**24.** A	**25.** A	**26.** C	**27.** B	**28.** B	**29.** C	**30.** D
31. D	**32.** B	**33.** B	**34.** D	**35.** D	**36.** B	**37.** D	**38.** D	**39.** C	**40.** B
41. C	**42.** C	**43.** D	**44.** A	**45.** A	**46.** B	**47.** B	**48.** D	**49.** B	**50.** A

Previous Years' Questions

1. D	**2.** B	**3.** B	**4.** C	**5.** B	**6.** A	**7.** D	**8.** A	**9.** A	**10.** D
11. C	**12.** D	**13.** C	**14.** B	**15.** C	**16.** C	**17.** A	**18.** 550	**19.** A	**20.** B
21. 23.6	**22.** B	**23.** C	**24.** 1.3 Ω	**25.** 555.75 KVA		**26.** A	**27.** C	**28.** B	**29.** D
30. 1.44	**31.** C	**32.** 10	**33.** 6	**34.** A					