Chapter 1

Introduction to Control Systems

LEARNING OBJECTIVES

After reading this chapter, you will be able to understand:

- Basic definitions
- Classification of control systems
- Transfer function
- Poles and zeros of transfer function
- Effect of feedback on disturbance
- Block diagram
- Block diagram reduction techniques
- Signal-flow graph
- Signal-flow graph algebra
- Mason's gain formula

Basic Definitions

System

A set of components/elements connected in a proper sequence to perform a specific task.

Controller

Controller is an element/subsystem inside or outside the system which regulates the operating condition/response of the system.

Disturbance

Disturbance is a signal that tends to adversely affect the value of the output of the system. If a disturbance is generated within the system, it is called internal disturbance, while an external disturbance is generated outside the system.

Control System

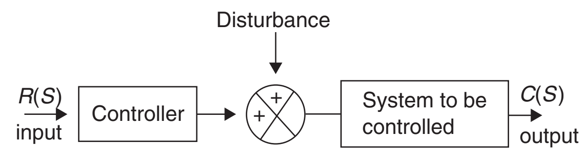
A set of components connected in a proper sequence to form a system which provides the desired response.
Control system is mainly classified into two types:

1. Open loop control system.
2. Closed loop control system.

Open Loop Control System (OLCS)

Any physical system in which the output is controlled directly by a controller/actuator without help of feedback is known as open loop control system. The input of the OLCS is independent of the output.

The output of an open loop control system is affected not only by input but also by disturbance in it. Open loop system does not automatically correct the disturbances.



Advantages

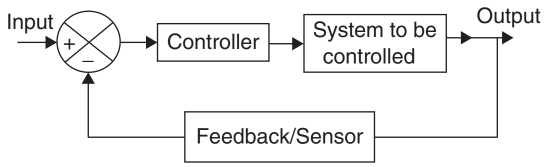
- Simple and easy to design.
- Cheap due to less number of components.
- Generally open loop systems are stable.

Disadvantages

- More sensitive to disturbances and unable to correct the disturbances.
- Inaccurate and Unreliable.

Closed Loop Control System (CLCS)

A Control System in which input is a function of output in order to maintain the desired value of output is called closed loop control system.



Since the feedback facilitates the system to automatically correct the system input to meet the desired response, it is also called as 'Automatic Control System.'

Advantages

- Accurate and reliable.
- Less sensitive to disturbances.
- Accurate even with presence of non-linearities.

Disadvantages

- Design is complex and costly gain of the system.
- Feedback presence reduces the overall.
- Feedback may lead to oscillatory response.
- Improper design of the Controller may cause the system to become unstable.

Comparison Between Open Loop and Closed Loop System

	Open loop control system		Closed loop control system
1.	Input is independent of output	1.	Input is dependent on output.
2.	Design is easy and cheap.	2.	Design is complex and costly.
3.	Inaccurate and Unreliable	3.	Accurate and reliable
4.	More sensitive to disturbances	4.	Less sensitive to disturbances
5.	Feedback does not exist.	5.	Feedback exists and reduces the gain of the system. It may also cause oscillations.
6.	Generally stable in operation	6.	Improper design of the controller may cause unstable operation of the system.

Other Classification of Control Systems

Depending on the nature of the system

- Linear and non-linear control systems.
- Time-variant and time-invariant systems.

Depending on the type of signals present at various parts of a feedback control system,

- Continuous and Discrete line control systems.

Linear and Non-Linear Control Systems

A system which obeys superposition and homogeneity principle is said to be a linear system.

Let $x_1(t)$ and $x_2(t)$ be two inputs to a system and $y_1(t)$ and $y_2(t)$ be the corresponding outputs. For arbitrary real constants k_1 and k_2, for input $k_1x_1(t) + k_2x_2(t)$, if the output of the system is given by $k_1y_1(t) + k_2y_2(t)$, then the system is said to be linear.

Any system which does not obey superposition and homogeneity principle is said to be 'non-linear'.

Physical systems are in general non-linear and analysis of such systems is very complicated. Hence these systems are usually linearized and analysed using linear techniques.

Time-variant and Time-invariant Control Systems

A system is said to be 'time variant' if its characteristics explicitly depend upon time.

A 'time-invariant' system is one whose output does not depend explicitly on time.

If an input signal $x(t)$ produces an output $y(t)$, then any time shifted input $x(t + \delta)$, results in a time shifted output $y(t + \delta)$, then system is time-invariant.

Continuous Time and Discrete Time Systems

If the signals in all parts of a control system are functions of time, the system is said to be continuous time control system.

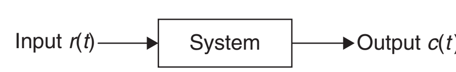
A system in which one or more parts of control systems signals are of the pulse form, it is said to be discrete time control system.

Transfer Function

Transfer function of a linear time-invariant system is defined to be the ratio of the Laplace transform of the output variables to the Laplace transform of the input variables under the assumption that all initial conditions are zero.

(or)

Transfer function is defined as the Laplace transform function of an impulse response of the system when all initial conditions are assumed to be zero.



Transfer function = $\dfrac{L[c(t)]}{L[r(t)]} = \dfrac{C(s)}{R(s)}$ with zero initial conditions

Transfer Function = $L[C(t)]$ zero initial conditions and impulse input

Note 1: Transfer function gives mathematical model of all components and individual analysis of various components.
Note 2: Transfer function is independent of input and output of the system.
Note 3: Transfer function is useful in determining response of the system for any type of input applied.
Note 4: Transfer function is helpful to obtain differential equations related to the system.
Note 5: Transfer function is helpful in stability analysis.

Limitations of Transfer Function

1. TF is applicable only for linear time-invariant systems.
2. Does not provide any information about the physical structure of the system.
3. Initial condition's effects are completely neglected. So, initial conditions lose their importance.

Solved Examples

Example 1: The transfer function of the system described by $5\frac{d^2y}{dt^2} + 4\frac{dy}{dt} = 2\frac{du}{dt} + 4u$, with u as input and y as output is

(A) $\frac{2s+4}{(5s^2+4s)}$ (B) $\frac{2s+4}{5s+4}$

(C) $\frac{2s+2}{5s^2+4s}$ (D) $\frac{2s}{5s^2+4}$

Solution: (A)
Apply Laplace transform on both sides for the given differential equation with zero initial condition,

$$\Rightarrow \quad 5s^2\,Y(s) + 4s.Y(s) = 2s\,U(s) + 4U(s)$$

$$(5s^2 + 4s)\,Y(s) = (2s + 4)\,U(s)$$

$$\frac{Y(s)}{U(s)} = \frac{2s+4}{(5s^2+4s)}$$

Example 2: The impulse response of the system is given as $c(t) = -4e^{-2t} + 6e^{-4t}$. The step response of the same system for $t \geq 0$ is equal to

(A) $\frac{1}{2}\left[-1+e^{-2t}\right]$ (B) $\frac{1}{2}\left[1-e^{-2t}\right]$

(C) $\frac{1}{4}\left[1-e^{-2t}+2e^{-4t}\right]$ (D) $\frac{1}{2}\left[-1+4.e^{-2t}-3.e^{-4t}\right]$

Solution: (D)

Transfer function of the system = L {Impulse response} = $\frac{-4}{s+2} + \frac{6}{s+4}$

Step response of the system $C(s) = \text{T.F} \times R(S) = \frac{2s-4}{(s+2)(s+4)} \times \frac{1}{s}$

$$c(t) = L^{-1}\left\{\frac{2s-4}{s(s+2)(s+4)}\right\}$$

$$L^{-1}\left\{\frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4}\right\}$$

$$8A = -4$$

$$\Rightarrow \quad A = \frac{-1}{2};$$

$$B = C(s) \times (s + 2)$$

at $S = -2$

$$B = 2 \text{ and } C = C(S) \times (S + 4)$$

at $S = -4$

$$C = \frac{-3}{2}$$

$$C(s) = L^{-1}\left\{\frac{-1}{2s} + \frac{2}{s+2} - \frac{3}{2(s+4)}\right\}$$

$$C(t) = \frac{-1}{2} + 2.e^{-2t} - \frac{3}{2}.\,e^{-4t}$$

Example 3: A control system is defined by the following differential mathematical relationship

$$9\frac{d^2x}{dt^2} + 16\frac{dx}{dt} + 5x = 12\,(1 - e^{-2t})$$

The response of the system as $t \to \infty$ is

(A) $x = 16$ (B) $x = 5$
(C) $x = 2.4$ (D) $x = -9$

Solution: (C)
Taking Laplace transform on both sides with zero initial conditions

$$X(s)\,[9s^2 + 16s + 5] = 12\left[\frac{1}{s} - \frac{1}{s+2}\right] = 12\left[\frac{2}{s(s+2)}\right]$$

$$\Rightarrow \quad X(s) = \frac{24}{s(s+2)(9s^2+16s+5)}$$

Response of the system $x(t)$ at $t \to \infty = \underset{t\to\infty}{\text{Lt}}\, x(t)$

$$= \underset{s\to 0}{\text{Lt}}\; sX(s) \quad [\because \text{ Final value theorem}]$$

$$= \underset{s\to 0}{\text{Lt}}\; s\left[\frac{24}{s(s+2)(9s^2+16s+5)}\right] = \frac{12}{5} = 2.4$$

Response of the system at $t \to \infty = 2.4$

Example 4: The impulse response of an initially relaxed linear system is $e^{-3t}u(t)$. To produce a response of $te^{-3t}u(t)$, the input must be equal to

(A) $3e^{-t}u(t)$ (B) $\frac{1}{3}e^{-3t}u(t)$

(C) $e^{-3t}u(t)$ (D) $te^{-3t}u(t)$

Solution: (C)
Impulse response = $e^{-3t}u(t)$
Transfer function = L [Impulse Response]

$$= L\left[e^{-3t}u(t)\right] = \frac{1}{s+3}$$

Response of the system

$$[C(s)] = L[t\,e^{-3t}u(t)]$$

$$= \frac{1}{(s+3)^2}$$

$$\text{Input } R(s) = \frac{C(s)}{\text{Transfer function}}$$

$$= \frac{\frac{1}{(s+3)^2}}{\frac{1}{s+3}} = \frac{1}{s+3}$$

Input $r(t) = L^{-1}[R(s)] = e^{-3t}u(t)$

Example 5: A linear time-invariant system has an impulse response e^{2t}, $t > 0$. If the initial conditions are zero and input is e^{8t}, then output for $t > 0$ is

(A) $\frac{1}{6}\left[e^{8t} - e^{2t}\right]$ (B) e^{5t}
(C) $e^{8t} + e^{2t}$ (D) None

Solution: (A)
Transfer Function = L [Impulse response]

$$= L\,[e^{2t}] = \frac{1}{s-2}$$

Input applied $R(s) = L[e^{8t}] = \frac{1}{s-8}$

Response of the system = T.F × R(s)

$$= \frac{1}{s-2} \times \frac{1}{s-8}$$

Output of the System

$$C(t) = L^{-1}\left[\frac{1}{(s-2)(s-8)}\right] = L^{-1}\left[\frac{\frac{1}{6}}{s-8} - \frac{\frac{1}{6}}{s-2}\right]$$

$$C(t) = \left(\frac{e^{8t} - e^{2t}}{6}\right)$$

Example 6: Let $x(t)$ be the input to a linear, time-invariant system. The required output is $8x(t-4)$. The transfer function of the system should be

(A) $8e^{j8\pi f}$ (B) $4e^{-j8\pi f}$
(C) $8e^{-j8\pi f}$ (D) $4e^{j8\pi f}$

Solution: (C)
Required output of the System

$$y(t) = 8x(t-4)$$

$$\Rightarrow \quad Y(s) = 8e^{-4s}X(s)$$

$$\text{Transfer Function } \frac{Y(s)}{X(s)} = 8e^{-4s}$$

$$= 8e^{-4\omega j} \quad [\because s = j\omega]$$

Transfer Function = $8e^{-j8\pi f}$ $[\because \omega = 2\pi f]$

Poles and Zeros of Transfer Function

Transfer Function

It is defined as the ratio of Laplace transform of the response to the Laplace transform of the excitation or input, with all initial conditions are zero.

$$G(s) = \frac{C(s)}{R(s)}$$

The impulse response of the system

$$H(s) = C(s) = G(s)$$

The transfer function of a linear control system can be expressed as

$$G(s) = \frac{K(s+b_0)(s+b_1)(s+b_2)\ldots}{(s+a_0)(s+a_1)(s+a_2)\ldots}$$

Characteristic equation $1 + G(s)\,H(s) = 0$

$$(s+a_0)(s+a_1)(s+a_2)\ldots + k(s+b_0)(s+b_1)(s+b_2)\ldots = 0$$

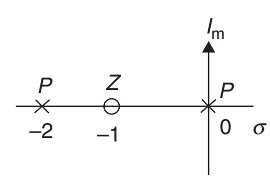
If $K = 0$
Poles of the system are $S = -a_0, -a_1, -a_2 \ldots$ etc.
If $K = \infty$
Zeros of the system is

$$s = -b_0, -b_1, -b_2 \ldots \text{ etc.}$$

where k is gain factor of the transfer function.

Example 7: Consider the unity feedback open loop system transfer function $G(s) = \frac{s+1}{s(s+2)}$. Draw the p–z location and explain about the stability.

Solution: Poles are at $S = 0, -2$
Zero at $S = -1$



$$\text{Impulse response of the system } H(s) = \frac{s+1}{s(s+2)}$$

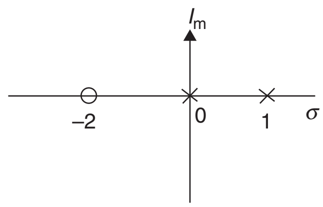
$$h(t) = L^{-1}\left\{\frac{A}{S} + \frac{B}{S+2}\right\}$$

$$\therefore \quad h(t) = \{A + B.e^{-2t}\}u(t)$$

It is a stable system [exponentially decaying function]

Example 8: Consider the impulse response of the system is $H(s) = \dfrac{(s+2)}{s(s-1)}$ Explain about the stability of system.

Solution:



$$h(t) = \{A + Be^{t}\}u(t)$$

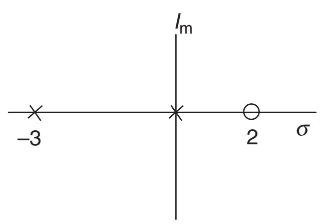
If $t \to \infty$

$$h(t) = \infty \text{ \{unbounded for bounded input\}}$$

So, the system is unstable. [exponentially increasing function]

Example 9: If $H(s) = \dfrac{(s-2)}{s(s+3)}$ then the system stability is ______

Solution: P–Z location:



$$h(t) = (A + B.e^{-3t})\, u(t)$$

It is a exponentially decaying function.

If $t \to \infty;\ h(t) = A$

For bounded input bounded output,
So, the system is stable.

Note:
System stability depends on the poles location but not zeros.

If system poles located only in the LHS of S-plane (non-repeated at origin and Imaginary axis), then the system is called stable system otherwise it is unstable.

Sensitivity Analysis

Sensitivity: Sensitivity is the ratio of the percentage change in the function to the percentage change in the Parameter.

$$\text{Sensitivity} = \frac{\text{Percentage change in } F(s)}{\text{Percentage change in } P(s)}$$

where $F(s)$ is the function and $P(s)$ is the parameter.

Transfer Function Sensitivity with Respect to Parameter Variation

Transfer function = $T(s)$.

Forward path gain = $G(s)$

Transfer function sensitivity with respect to parameter variation = S_G^T

$$S_G^T = \frac{\frac{\partial T}{T}\times 100}{\frac{\partial G}{G}\times 100} = \frac{\partial T}{\partial G}\times\frac{G}{T}$$

For open loop system: $S_G^T = 1$

For closed loop system: $S_G^T = \dfrac{1}{1+GH}$ (Negative feedback)

Note 1: The sensitivity of closed loop system with respect to variation in G is reduced by a factor $(1+GH)$ as compared to that of an open loop system.

Sensitivity of *T* with Feedback Parameters

Feedback gain = H

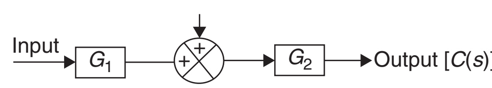
Sensitivity of T with feedback parameters = S_H^T

$$S_H^T = \frac{\partial T}{\partial H}\times\frac{H}{T} = \frac{-GH}{1+GH}.$$

Note 2: Closed loop system is more sensitive to the feedback parameters variation than that of the forward path parameters variation.

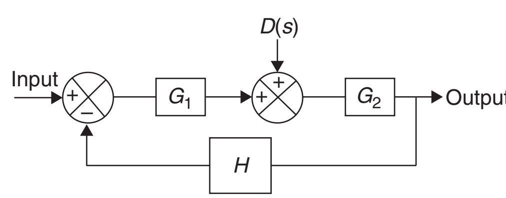
Effect of Feedback on Disturbance

Open loop system:



Output due to disturbance = $G_2D(s)$.

Closed loop system:



Output due to disturbance = $\dfrac{G_2}{1+G_1G_2H}\,.\,D(s)$.

Note: Negative feedback reduces the effect of noise on output by a factor of $1+G_1G_2H$ as compared to that of open loop systems.

Note: For positive feedback

Output due to disturbance = $\dfrac{G_2}{1-G_1G_2H}\,.\,D(s)$.

Effect of disturbance on output of the system increases as compared to open loop and negative feedback closed loop control systems.

Example 10: A negative feedback system has an amplifier of gain 10 with ±1% tolerance in the forward path, and an alternator of a value $\frac{9}{10}$ in the feedback path. The overall system gain is approximately.

(A) 10 ±1% (B) 9±1%

(C) 1±0.1% (D) 9±0.1%

Solution: (C)

Overall gain of the system without tolerance $= \frac{10}{1+\frac{90}{10}} = \frac{1}{10}$

Sensitivity of gain with tolerance $\left(S_G^T\right) = \frac{1}{1+GH}$ (For negative feedback) $= \frac{1}{1+\frac{90}{10}} = \frac{1}{10}$

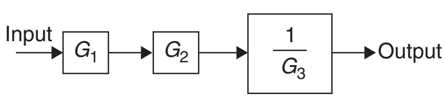
$$\left(S_G^T\right) = \frac{\text{\% Change in gain}}{\text{\% Change in } G}$$

$$\text{\% Change in gain} = \frac{1}{10} \times \text{Change in G}$$

$$= \frac{1}{10} \times 1\% = 0.1\%$$

Overall gain with tolerance = 1±0.1%

Example 11: Errors associated with each respective sub-systems G_1, G_2 and G_3 are 4, 5 and 8. The error associated with the output is



(A) 160 (B) 17 (C) 1 (D) $\frac{5}{2}$

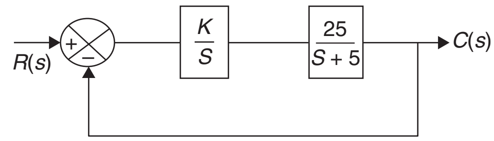
Solution: (C)

Total error $= 4 \times S_{G_1}^T + 5 \times S_{G_2}^T + 8 \times S_{G_3}^T$

$$= 4 + 5 - 8 \qquad \left[\because\ S_{G_1}^T = 1,\ S_{G_2}^T\ 1,\ S_{G_3}^T = -1\right]$$

$$= 1$$

Example 12: The sensitivity of transfer function $\frac{C(s)}{R(s)}$ to variation in parameter K if system operating frequency $\omega = 5$ rad/sec and $K = 1$ is



(A) 1.41 (B) 2.82 (C) 0 (D) $\frac{1}{5}$

Solution: (A)

Transfer function T.F $= \frac{C(s)}{R(s)} = \frac{25K}{5^2 + 5s + 25K}$

Sensitivity of T.F with $K\left(S_K^T\right) = \frac{\partial T}{\partial K} \times \frac{K}{T}$

$$= \frac{s(s+5)}{s^2 + 5s + 25K}$$

If $K = 1$ and $W = 5$ rad/sec

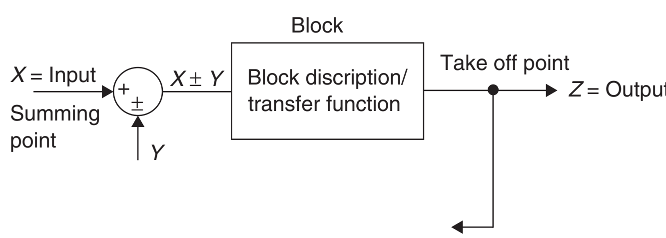
$$\left|S_K^T\right| = 1.41.$$

Block Diagram

Block diagram is a pictorial representation of system between input and output. Block diagram together with transfer function is used to describe the cause- and -effect relationship throughout the system.

Different elements in a block diagram are:

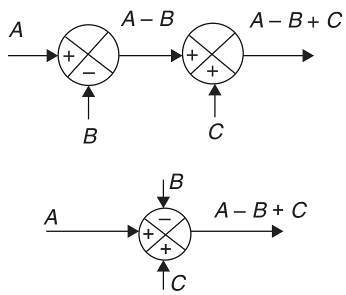
1. Block: Physical description or transfer function of a subsystem.
2. Summing point: Addition or subtraction of all incoming signals.
3. Take-off point: Measurement or sensing a signal.
4. Line with arrow: Represents unidirectional signal-flow and connectivity between subsystem.

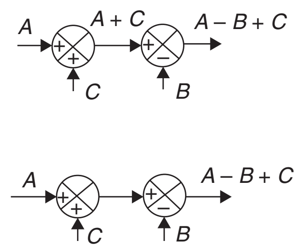


Block Diagram Reduction Techniques

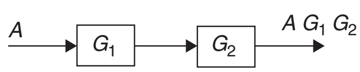
Original diagram | **Equivalent diagram**

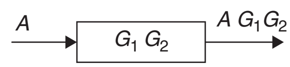
1. Associative law



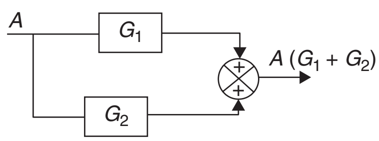


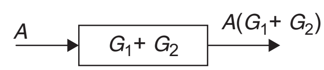
2. Blocks in series (or) combining blocks in cascade.



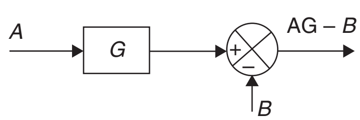


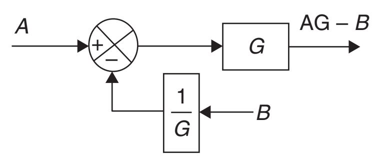
3. Blocks in parallel



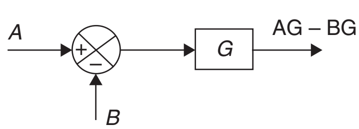


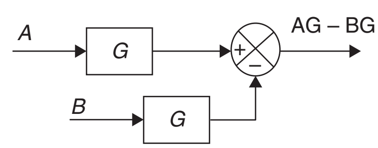
4. Shifting summing point behind the block



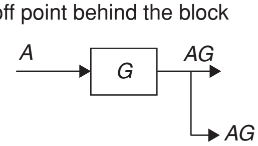


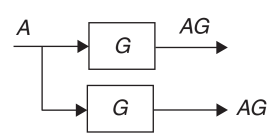
5. Shifting summing point beyond the block



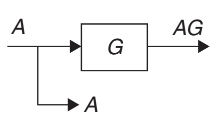


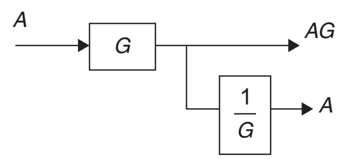
6. Shifting a take-off point behind the block



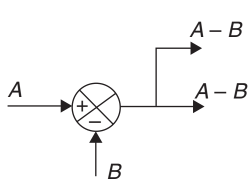


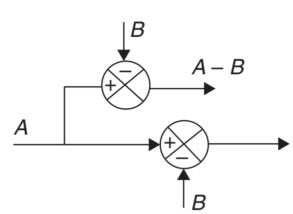
7. Shifting a take-off point beyond the block



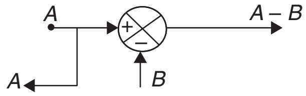
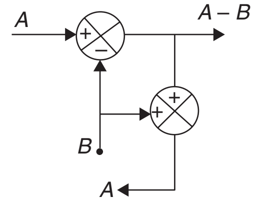
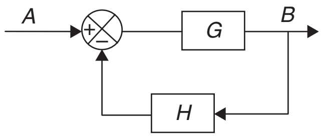
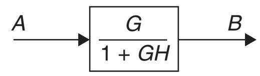
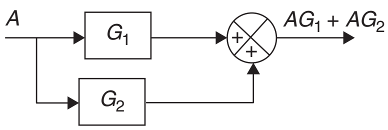
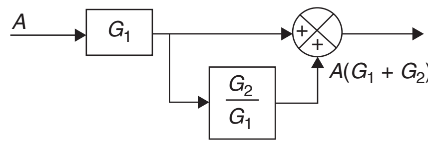
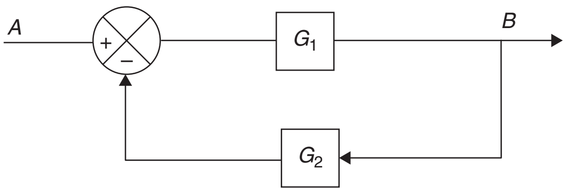
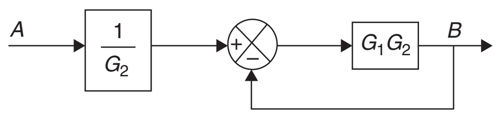


8. Shifting take-off point after summing point

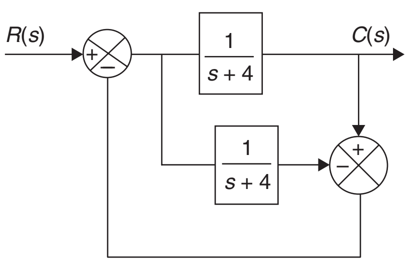




(*Continued*)

Original diagram	Equivalent diagram
9. Shifting take-off point before summing point 	
10. Removing minor feedback loop 	
11. Removing block in forward path 	
12. Removing block in feedback path 	

Example 13: The transfer function $\frac{C(s)}{R(s)}$ of the system shown is



(A) $\frac{1}{s+4}$ (B) $\frac{1}{s+8}$ (C) $\frac{1}{s+3}$ (D) $\frac{2}{s+8}$

Solution: (A)

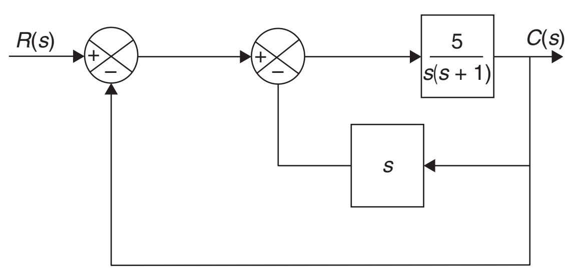
$$\text{Forward path} = \frac{1}{s+4}$$

$$\text{Loops} = \frac{1}{s+4}, -\frac{1}{s+4}$$

$$\text{Transfer function} = \frac{P_1}{1-L_1-L_2}$$

$$= \frac{\frac{1}{s+4}}{1-\frac{1}{s+4}+\frac{1}{s+4}} = \frac{1}{s+4}$$

Example 14: For the system shown in figure the transfer function $\frac{C(s)}{R(s)}$ is equal to



(A) $\frac{5}{s^2+4s+5}$ (B) $\frac{5}{s^2-4s+5}$

(C) $\frac{s^2}{s^2+6s+5}$ (D) $\frac{5}{s^2+6s+5}$

Solution: (D)

$$\text{Forward path} = \frac{5}{s(s+1)}$$

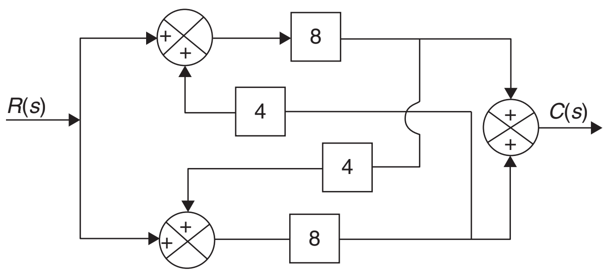
$$\text{Loops} = \frac{-5s}{s(s+1)}, \frac{-5}{s(s+1)}$$

$$\text{Transfer function} = \frac{\frac{5}{s(s+1)}}{1+\frac{5s}{s(s+1)}+\frac{5}{s(s+1)}}$$

$$= \frac{5}{s^2+s+5s+5}$$

$$\text{Transfer function} = \frac{5}{s^2+6s+5}$$

Example 15: The overall transfer function of the system in figure is



(A) $\frac{31}{16}$ (B) $\frac{16}{31}$

(C) $\frac{-16}{31}$ (D) $-\frac{32}{15}$

Solution: (C)

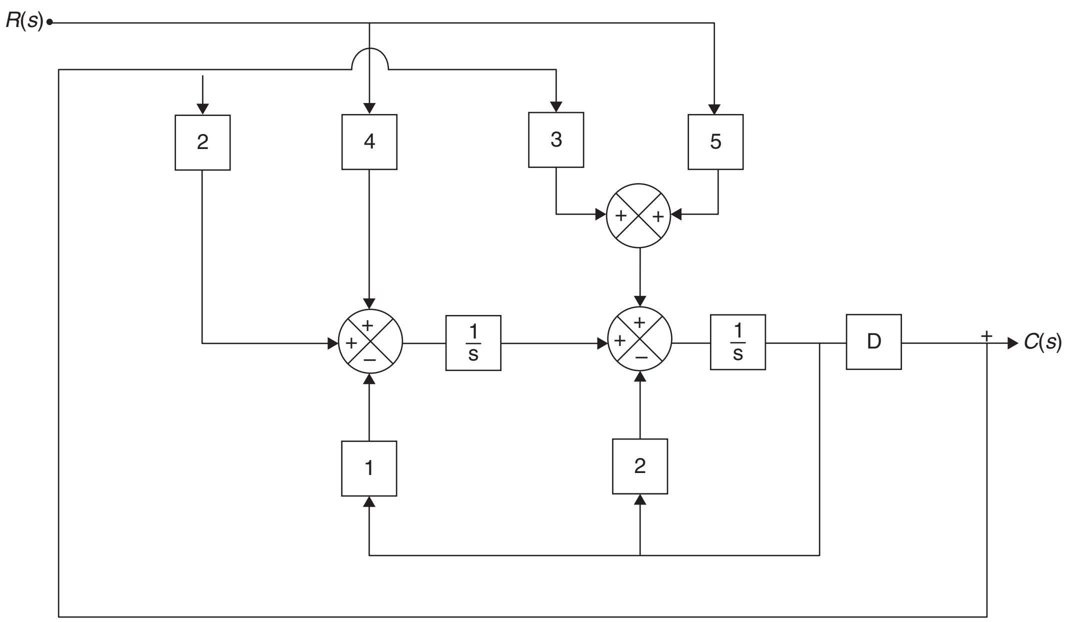
Forward paths → 8, 8 – 4 – 8, 8 – 4 – 8, 8

$$\text{Transfer function} = \frac{8+(8\times4\times8)+8+(8\times4\times8)}{1-(8\times4\times8\times4)}$$

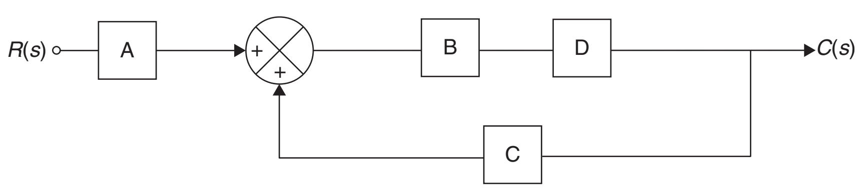
$$= \frac{16(1+32)}{1-32\times32}$$

$$\text{Transfer function} = \frac{16}{1-32} = \frac{-16}{31}$$

Example 16: The system shown in the figure below.



Can be reduced to the form



(A) $A = 2 + 3s$, $B = \frac{1}{s^2+2s+1}$, $C = 4s + 5$

(B) $A = 4s + 5$, $B = \frac{1}{s^2+2s+1}$, $C = 2 + 3s$

(C) $A = 2 + 3s$, $B = \frac{4s+5}{s^2+2s+5}$, $C = 4s + 5$

(D) $A = 4s + 5$, $B = 2 + 3s$, $C = 1/s^2 + 2s + 5$

Solution: (B)

$$\text{Forward paths} = 4 - \frac{1}{s} - \frac{1}{s} - D \left[\frac{4D}{s^2}\right]$$

$$= 5 - \frac{1}{s} - D \left[\frac{5D}{s}\right]$$

$$\text{Loops} = \frac{1}{s} - 2 \;\; [-2/s]$$

$$\frac{1}{s} - 1 - \frac{1}{s} \;\; [-1/s^2]$$

$$\frac{1}{s} - D - 2 - \frac{1}{s} \;\; [2D/s^2]$$

$$\frac{1}{s} - D - 3 \;\; [3D/s]$$

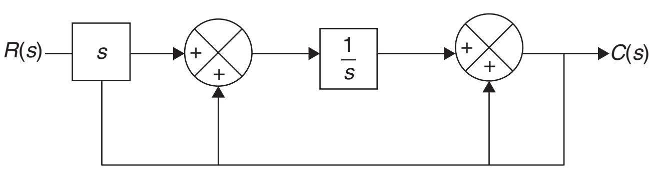
$$\text{Transfer function} = \frac{\frac{4D}{s^2} + \frac{5D}{s}}{1 + \frac{2}{s} + \frac{1}{s^2} - \frac{2D}{s^2} - \frac{3D}{s}}$$

$$= \frac{(4s+5)D}{(s^2+2s-1)-(2D+3Ds)}$$

$$B\,\frac{AD}{1-BCD} = \frac{1}{s^2+2s+1} \cdot \frac{(4s+5)D}{1 - \frac{(2+3s)}{s^2+2s+1}.D}$$

$$\therefore \;\; A = (4s+5),\; B = \frac{1}{s^2+2s+1},\; C = (2+3s)$$

Example 17: For the block diagram shown in figure, the transfer function is equal to



(A) $\frac{s}{2s+1}$ (B) $s + \frac{1}{s}$

(C) $\frac{s}{s^2+1}$ (D) $\frac{2s+1}{s}$

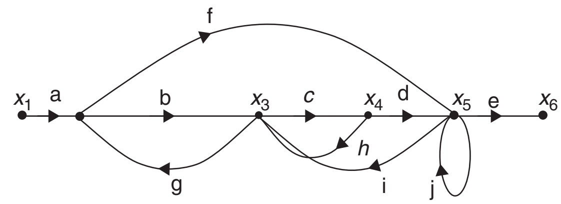
$$\text{Forward paths} = 1, \frac{1}{s}, 1$$

$$\text{Transfer function} = 1 + \frac{1}{s} + 1 = \frac{2s+1}{s}$$

Solution: (D)

Signal-flow Graph

Signal-flow graph is a graphical representation of simultaneous algebraic equations.



Node: A node is a point representing a variable or signal.

Transmittance: This is a real gain or complex gain between two nodes.

Branch: A branch is a directed line segment joining two nodes.

Input node or source: A source is a node that has only outgoing variables. (node x_1)

Output node or sink: A sink is a node that has only incoming branches. (node x_6)

Mixed node: A mixed node has both incoming and outgoing branches. (nodes x_2, x_3, x_4, x_5)

Forward path: A forward path is a path from an input node (source) to an output node (sink) that does not cross any nodes or branch more than once. ($x_1 - x_2 - x_3 - x_4 - x_5 - x_6$ and $x_1 - x_2 - x_5 - x_6$)

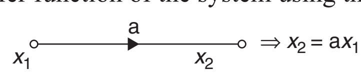
Loop: A loop is a closed path with no node or branch repeated more than once. ($x_2 - x_3 - x_2$, $- x_3 - x_4 - x_3$, $x_3 - x_4 - x_5 - x_3$, $x_5 - x_5$) and ($x_2 - x_5 - x_2$)

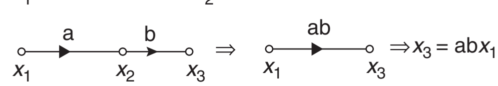
Loop gain: The loop gain is the product of the branch transmittances of a loop. (bg, ch, cdi,f, fig)

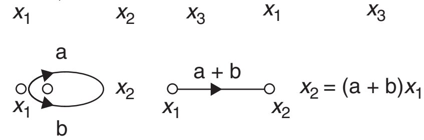
Non touching loops: Loops are non-touching if they do not possess any common nodes.

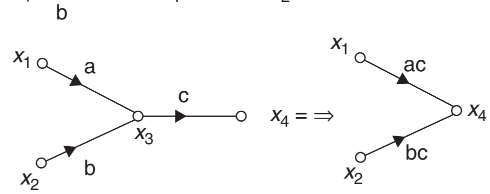
Signal-flow Graph Algebra

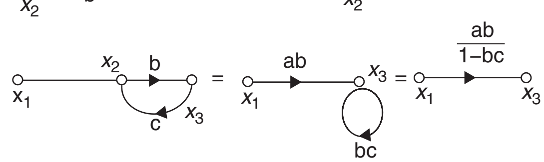
Signal-flow graph for a system can be reduced to obtain the transfer function of the system using the following rules.

1. 

2. 

3. 

4. 

5. 

Mason's Gain Formula

The transfer function of a signal-flow graph can be found from mason's gain formula as follows.

$$M = \frac{1}{\Delta} \Sigma_k P_k \Delta_k$$

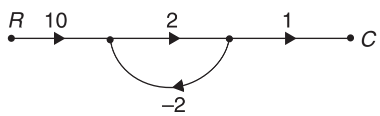
M = Overall gain of the system

P_k = Gain of the k^{th} forward path

Δ = 1 – (Sum of all individual loop gains) + (Sum of gain products of all possible combinations of two non-touching loops) – (Sum of gain products of all possible combinations of three non-touching loops) + …

Δ_k = Same as Δ but formed by loops not touching the k^{th} forward path

Example 18: In the signal-flow graph of figure $\frac{C}{R}$ is equals



(A) $\frac{-20}{3}$ (B) 4

(C) 2 (D) 18

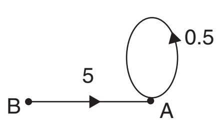
Solution: (B)

Forward paths = 10 – 2 – 1 (20)

Loops = 2 – –2 (–4)

$$\text{Transfer function} = \frac{20}{1+4} = 4$$

Example 19: In the signal-flow graph shown in the figure, if $A = TB$, where T is equal to



(A) 2.5 (B) 5.5

(C) 5 (D) 10

Solution: (D)

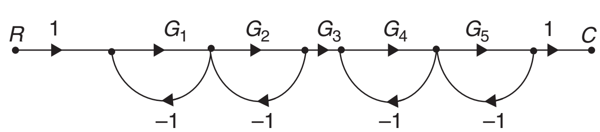
Forward path = 5,

Loop = 0.5

$$\text{Transfer function} = \frac{A}{B} = \frac{5}{1-0.5} = 10$$

$\Rightarrow$ $A = 10\,B$

Example 20: The $\frac{C}{R}$ for the signal-flow graph in figure is



(A) $\frac{G_1\ G_2\ G_3\ G_4\ G_5}{1+G_1+G_2+G_3+G_4}$

(B) $\frac{G_1\ G_2\ G_3\ G_4\ G_5}{(1+G_1+G_2+G_3+G_4+G_1\ G_2\ G_4\ G_5)}$

(C) $\frac{G_1\ G_2\ G_3\ G_4\ G_5}{(1+G_1+G_2)(1+G_4+G_5)}$

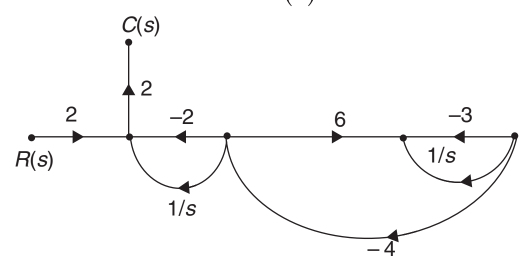
(D) $\frac{G_1\ G_2\ G_4\ G_5}{(1+G_1)(1+G_2)(1+G_4)(1+G_5)}$

Solution: (C)

Given signal-flow graph can be considered as two series signal-flow graphs

$$\text{Transfer function} = \frac{G_1\ G_2\ G_3\ G_4\ G_5}{(1+G_1+G_2)(1+G_4+G_5)}$$

Example 21: The signal flow graph of a system is shown in figure. The transfer function $\frac{C(s)}{R(s)}$ of the system is



(A) $\frac{s+27}{s^2+29s+6}$ (B) $\frac{6s}{s^2+29s+6}$

(C) $\frac{27s}{s^2+6s+6}$ (D) $\frac{4(s+27)}{s^2+29s+6}$

Solution: (D)

Forward path = 2 – 2 (4)

$$\text{Loops} = -2 - \frac{1}{s}\left(-\frac{2}{s}\right)$$

$$6 - \frac{1}{s} - -4\left(\frac{-24}{s}\right) = \frac{1}{s} - -3\left(\frac{-3}{s}\right)$$

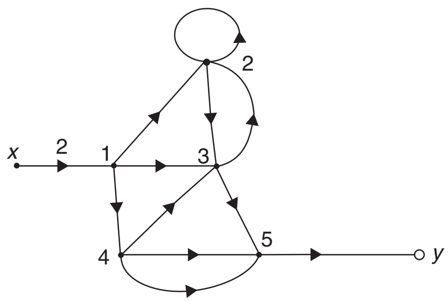
$$\text{Non-touching loops pair} = \left(\frac{-2}{s}, \frac{-3}{s}\right)$$

$$\text{Non-touching loops to forward path} = \frac{-24}{s}, \frac{-3}{s}$$

$$\text{Transfer function} = \frac{4\left(1+\frac{24}{s}+\frac{3}{s}\right)}{1+\frac{2}{s}+\frac{24}{s}+\frac{3}{s}+\frac{6}{s^2}}$$

$$= \frac{4(s+27)}{(s^2+29s+6)}$$

Example 22: The signal-flow graph is shown in the figure, has—forward paths and—self-loops



(A) 4, 4 (B) 4, 1 (C) 3, 3 (D) 3, 1

Solution: (B)

Forward path = $X - 1 - 2 - 3 - 5 - y$

$X - 1 - 3 - 5 - y$

$X - 1 - 4 - 3 - 5 - y$

$X - 1 - 4 - 5 - y$

Total = 4

Self-loop = 2 – 2

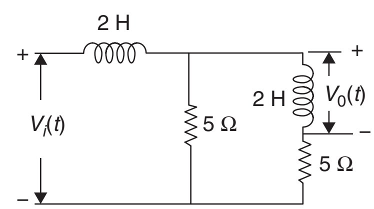
Total = 1

Exercises

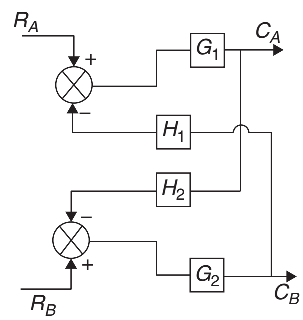
Practice Problems I

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. Which of the following statements are true?
   (i) In a closed loop system the effect of non-linearities is reduced.
   (ii) Feedback in closed loop may lead to oscillatory response.
   (iii) Feedback cannot control dynamics of the system.
   (iv) Open loop systems are stable.
   (A) i, ii and iv (B) i, ii and iii
   (C) ii, iii and iv (D) i and iii
2. The transfer function of a linear system is
   (A) Ratio of two quantities which have the same units.
   (B) Ratio of the output to the input.
   (C) Ratio of the Laplace transform of the output to that of the input.
   (D) Ratio of the Laplace transform of the output to that of the input with all initial conditions zeros.
3. Which of the following is not a characteristic of negative feedback system?
   (A) Rejection of disturbance signal
   (B) High sensitivity to parameter variations
   (C) Reduction in gain
   (D) Accuracy in tracking steady-state value
4. Which one of the following is not a closed loop system?
   (A) Respiratory system of an animal
   (B) Execution of a program by a computer
   (C) Air-conditioning system
   (D) Driving a car
5. If $\frac{dy}{dx} = x$ represents as the equation of an integrator then which of the following is true
   (A) The system is stable
   (B) The system is unstable
   (C) The system is marginally stable
   (D) Cannot be determined
6. The speed of response of the given three systems will be in the order
   Given $G_1(s) = \frac{5}{0.5s+1}$
   $G_2(s) = \frac{5}{2s+1}$
   $G_3(s) = \frac{5}{5s+1}$
   (A) $G_1(s) > G_2(s) > G_3(s)$
   (B) $G_1(s) = G_2(s) = G_3(s)$
   (C) $G_1(s) < G_2(s) < G_3(s)$
   (D) Cannot be determined
7. Derive the transfer function of the network shown



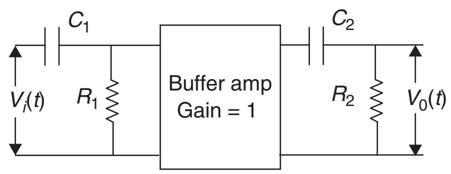
   (A) $\frac{s}{4s^2+30s+25}$ (B) $\frac{10s}{4s^2+30s+25}$
   (C) $\frac{1}{s(s^2+6s+5)}$ (D) $\frac{10}{s^2+3s+5}$
8. Find $\frac{C_A}{R_B}$ of the given system if $G_1 = \frac{20}{S}$, $G_2 = \frac{S}{S+1}$, $H_1 = 50s + 1$ and $H_2 = 0.5s + 1$.



(A) $\dfrac{20s+1}{25s^2+100s+2}$ (B) $\dfrac{5s+1}{25s^2+40s+2}$

(C) $\dfrac{20}{25s^2+100s+10}$ (D) $\dfrac{(5s+1)}{s^2+4s+25}$

9. Find the transfer function:



Given $R_1 = R_2 = 10\,\Omega$

$C_1 = C_2 = 0.1\ \mu F$

(A) $\dfrac{s}{s^2+2s+1}$ (B) $=1$

(C) $\dfrac{s^2}{(s+1)^2}$ (D) $\dfrac{(1s)^2}{(s+10)^2}$

10. The Laplace transform of a function is given by $\dfrac{s}{(s+1)^2}$. Find its impulse response

(A) t^{-t} e cos t (B) $\dfrac{te^{-t}}{1-t}$

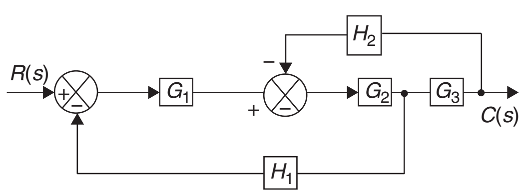
(C) $t\,e^{-t}$ (D) $e^{-t}(1-t)$

11. A system is described by $3\dfrac{dc(t)}{dt}+c(t)=r(t-3)$, where $r(t)$ and $c(t)$ are the input (ramp) and output, respectively. The transfer function of the system is

(A) $\dfrac{1+3s}{s^2(1+6s)}$ (B) $\dfrac{1-3s}{(1+3s)}$

(C) $\dfrac{1-3s}{s^2(1-6s)}$ (D) $\dfrac{1-3s}{s(1+3s)}$

12. The transfer function of the system whose block diagram is shown in fig is given by



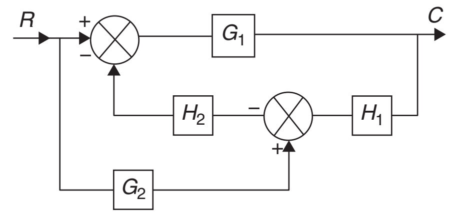
(A) $\dfrac{1+G_1G_2G_3}{1+G_1G_2H_1+G_2G_3H_2}$

(B) $\dfrac{G_1G_2G_3}{1+G_1G_2H_1+G_2G_3H_2}$

(C) $\dfrac{G_1G_2G_3}{1+G_1G_2G_3H_1H_2}$

(D) $\dfrac{G_1G_2G_3}{1+G_1G_3H_1+G_1G_2H_2}$

13. Using the block diagram given below, find the transfer function:



(A) $\dfrac{G_2}{H_2}\left[\dfrac{G_1}{(1+G_1H_1)}\right]$

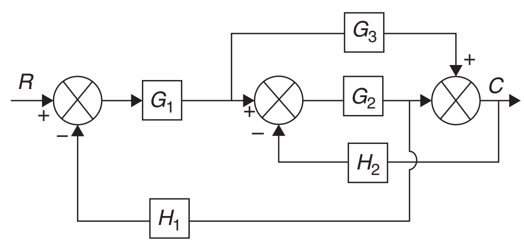
(B) $\dfrac{G_1[1+G_2]}{1+G_1H_1\ H_2}$

(C) $\dfrac{[1+G_2H_2]G_1}{1+G_1H_1H_2}$

(D) $\dfrac{G_1G_2}{H_2+G_1H_1}$

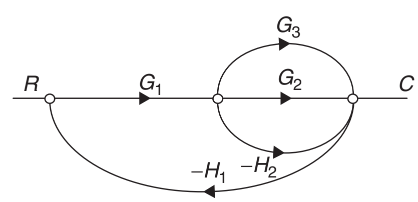
Common Data for Questions 14 and 15:

Given the block diagram representation of a closed loop control system

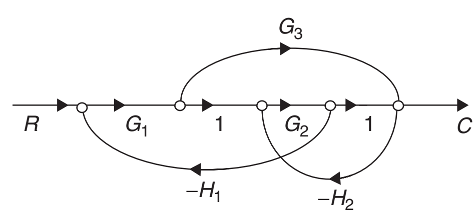


14. Draw the signal-flow graph of the given system

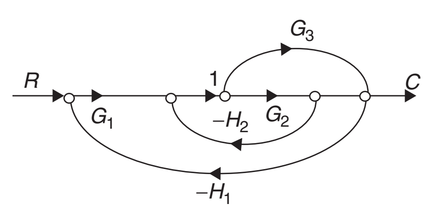
I.



II.



III.



(A) I and II are true
(B) All true
(C) II only
(D) I only

15. Obtain the transfer function representation of the above block diagram.

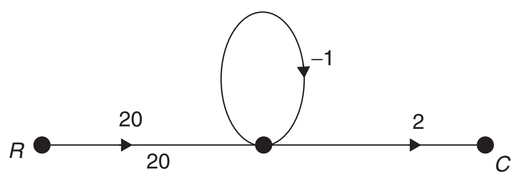
(A) $\dfrac{G_1G_2 + G_1G_3}{1 + G_1G_2H_2H_3 + G_2H_2}$

(B) $\dfrac{G_1G_2 + G_1G_3}{1 + G_2H_2 + G_1G_2H_1 + G_3H_1}$

(C) $\dfrac{G_1G_2 + G_1G_3}{1 + G_2H_2 + G_1G_2H_1}$

(D) $\dfrac{G_1G_2 + G_1G_3}{G_2H_2 + G_1G_2H_1 + G_2G_3H_2}$

16. Transfer function $\dfrac{C}{R}$ of the given signal-flow graph is

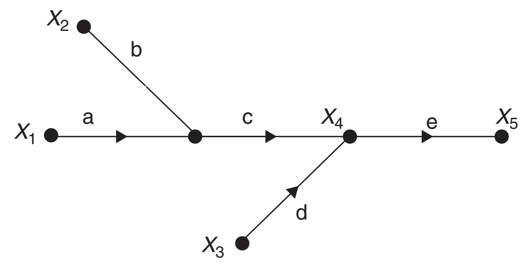


(A) 40
(B) 20
(C) 10
(D) 2

17. The dynamics equation related to a system is given by $\dfrac{d^2c(t)}{dt^2} + 5\dfrac{dc(t)}{dt} + 8c(t) = 7r(t)$ where $r(t)$ is input and $c(t)$ is output. Transfer function of the system is given by

(A) $\dfrac{8}{7s^2 + 5s + 1}$
(B) $\dfrac{7}{s^2 + 5s + 8}$
(C) $\dfrac{5}{s^2 + 7s + 8}$
(D) $\dfrac{7}{7s^2 + 5s + 8}$

18. The expression for the following signal-flow graph is



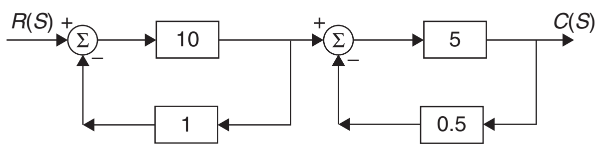
(A) $[(acX_1 + bcX_2) + dX_3]e$

(B) $[(aX_1 + bX_2)cdX_3]e$

(C) $[(aX_1 + bX_2) + dX_3]e$

(D) $[acX_1 + bcX_2 + dX_3]e$

19. The transfer function $\dfrac{C(s)}{R(s)}$ of the block diagram given bellow is



(A) 5
(B) 10
(C) 15
(D) 50

Practice Problems 2

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. In a time-variant system,
(A) The system parameters are independent of time.
(B) The system parameters are functions of time.
(C) The input and output are functions of time.
(D) The system parameters depend on time varying input and output.

2. The transfer function of a tachometer has
(A) A pole at origin, and a zero anywhere in the real axis
(B) A zero at origin, pole anywhere in the real axis
(C) Only a zero at origin
(D) Only a pole at origin

3. Match the following

	Type of roots		Nature of response term
1.	Single root at $s = \sigma$	I	$A\sin(\omega t + \beta)$
2.	Roots of multiplicity K at the origin	II	$(A_1 + a_2t + \ldots)e^{\sigma t}$
3.	Roots of multiplicity K, at $s = \sigma$	III	$Ae^{\sigma t}$
4.	Single complex conjugate root pair on the $j\omega$ axis	IV	$A + A_2t + \ldots + A_Kt^{K-1}$

(A) I – III, 2 – II, 3 – IV, 4 – I
(B) 1 – III, 2 – IV, 3 – II, 4 – I
(C) I – III, 2 – I, 3 – IV, 4 – II
(D) 1 – I, 2 – III, 3 – II, 4 – IV

4. A system $Y = f(x)$ is said to be linear if it satisfies the following properties.
(A) $f(x + y) = f(x) + f(y)$
(B) $f(\alpha x) = \alpha f(x)$
(C) $f(ax_1 + bx_2) = af(x_1) + bf(x_2)$
(D) None

5. The error transfer function of system is given by

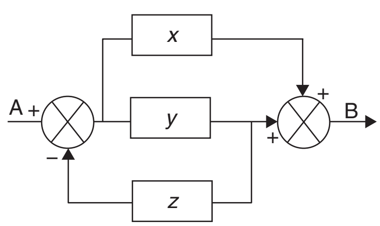
(A) $E(s) = \dfrac{G(s)}{1+G(s)\,H(s)} - R(s)$

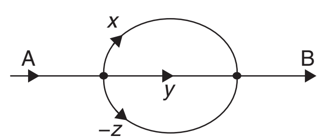
(B) $E(s) = \dfrac{R(s)}{1+G(s)\,H(s)}$

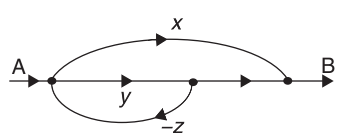
(C) $E(s) = \dfrac{1}{1+G(s)\,H(s)}$

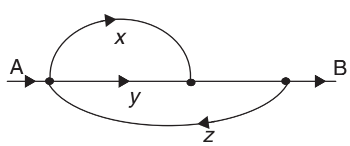
(D) $E(s) = R(s) - G(s)\,H(s)$

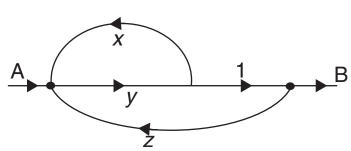
6. The signal-flow graph for the given system is



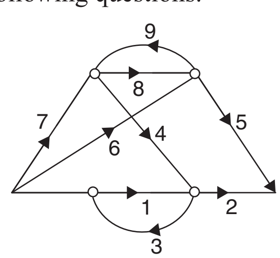








Common Data for Questions 7 and 8:
SFD of a system is shown below. Study the system to give answers to the following questions.



7. Find the number of forward paths in the given SFD.
(A) 5 (B) 4 (C) 6 (D) 7

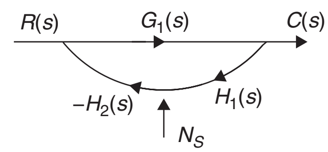
8. Find the number of independent loops in the system.
(A) 4 (B) 3 (C) 2 (D) 5

9. The impulse response of a system is given by $g(t) = e^{-2t}(1 - \cos 2t)$. Find the transfer function of the system.

(A) $\dfrac{1}{s+2}$ (B) $\dfrac{4}{(s+2)\,(s^2+4s+8)}$

(C) $\dfrac{8}{(s+2)\,(s^2+4s+8)}$ (D) $\dfrac{1}{(s^2+4s+8)}$

10. For the system shown if gain is very high, the transfer function $\dfrac{C(s)}{R(s)}$ will be



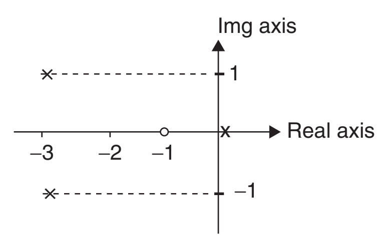
(A) $\dfrac{G_1(s)}{1+G_1(s)\,H_1(s)\,H_2(s)}$

(B) $\dfrac{G_1(s)}{1-H_1(s)\,H_2(s)\,N(s)}$

(C) $\dfrac{1}{H_1(s)\,H_2(s)}$

(D) $\dfrac{1}{1-H_1(s)\,H_2(s)}$

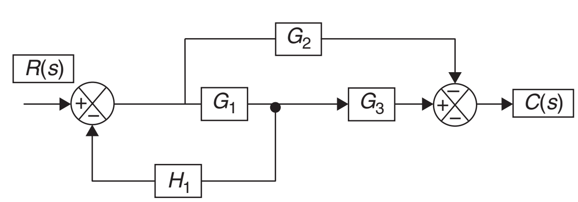
11. A transfer function $G(s)$ has the pole zero plot as shown in the figure. Given the steady-state gain as 10, find the transfer function.



(A) $\dfrac{10(s+1)}{s^2+6s+10}$ (B) $\dfrac{10(s-1)}{(s^2+6s+10)s}$

(C) $\dfrac{10(s+1)}{s^2(s^2+6s+10)}$ (D) $\dfrac{10(s+1)s}{s^2+6s+10}$

12.



The transfer function of the block diagram shown in above figure is

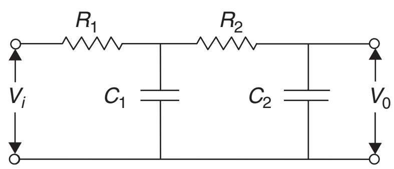
(A) $\dfrac{G_1G_2 - G_3}{1+G_1H}$

(B) $\dfrac{G_1G_3 + G_2}{1+G_1H}$

(C) $\dfrac{G_1G_2}{1+G_1H} - G_3$

(D) $\dfrac{G_1G_3 - G_2}{1+G_1H}$

13. The transfer function of the electrical network shown in the figure is



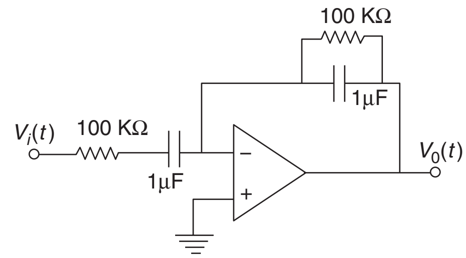
(A) $\dfrac{1}{1+s[R_1C_1+R_2C_2+R_1C_2]+s^2R_1R_2C_1C_2}$

(B) $\dfrac{1}{1+s^2[R_1C_1+R_2C_2+R_2C_1]+s^4R_1R_2C_1C_2}$

(C) $\dfrac{1}{1+s^2[R_1C_1+R_2C_2+R_1C_2]+s^4R_1R_2C_1C_2}$

(D) $\dfrac{1}{1+s[R_1C_1+R_2C_2+R_1C_2]+s^3R_1R_2C_1C_2}$

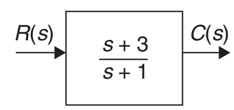
14. For the system given, find the transfer function:

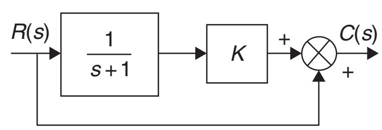


(A) $\dfrac{(0.1s+2)s}{(0.1s+1)^2}$ (B) $\dfrac{0.1s}{(0.1s+2)}$

(C) $\dfrac{-0.1s}{(0.1s+1)^2}$ (D) None of these

15. For what value of k, are the two block diagrams shown below equivalent?





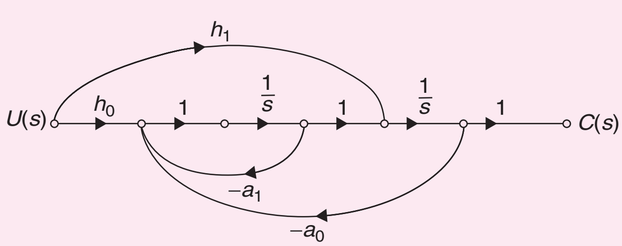
(A) 2 (B) 1
(C) $s+1$ (D) $s+2$

Previous Years' Questions

1. Two systems with impulse responses $h_1(t)$ and $h_2(t)$ are connected in cascade. Then the overall impulse response of the cascaded system is given by **[2013]**

(A) Product of $h_1(t)$ and $h_2(t)$
(B) Sum of $h_1(t)$ and $h_2(t)$
(C) Convolution of $h_1(t)$ and $h_2(t)$
(D) Subtraction of $h_2(t)$ from $h_1(t)$

2. The signal-flow graph of a system is shown below. $U(s)$ is the input and $C(s)$ is the output. **[2014]**



Assuming $h_1 = b_1$ and $h_0 = b_0 - b_1 a_1$, the input-output transfer function, $G(s) = \dfrac{C(s)}{U(s)}$ of the system is given by

(A) $G(s) = \dfrac{b_0 s + b_1}{s^2 + a_0 s + a_1}$

(B) $G(s) = \dfrac{a_1 s + a_0}{s^2 + b_1 s + b_0}$

(C) $G(s) = \dfrac{b_1 s + b_0}{s^2 + a_1 s + a_0}$

(D) $G(s) = \dfrac{a_0 s + a_1}{s^2 + b_0 s + b_1}$

3. A single-input single-output feedback system has forward transfer function $G(s)$ and feedback transfer function $H(s)$. It is given that $|G(s)\,H(s)| < 1$. Which of the following is true about the stability of the system? **[2014]**

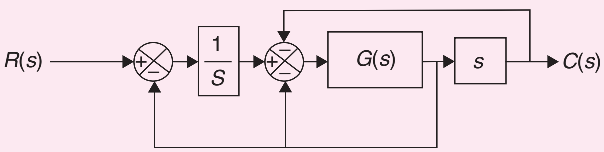
(A) The system is always stable

(B) The system is stable if all zeros of $G(s)H(s)$ are in left half of the S-plane.

(C) The system is stable if all poles of $G(s)H(s)$ are in left half of the S-plane.

(D) It is not possible to say whether or not the system is stable from the information given.

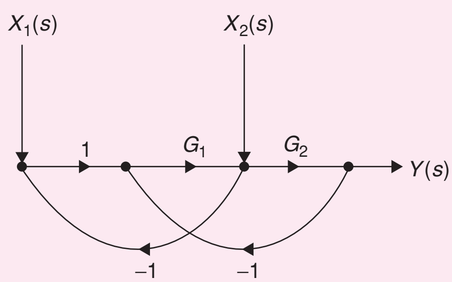
4. The block diagram of a system is shown in the figure **[2014]**



If the desired transfer function of the system is $\frac{C(s)}{R(s)} = \frac{s}{s^2+s+1}$ then $G(s)$ is

(A) 1
(B) s
(C) $1/s$
(D) $\frac{-s}{s^3+s^2-s-2}$

5. For the signal-flow graph shown in the figure, which one of the following expressions is equal to the transfer function $\left.\frac{Y(s)}{X_2(s)}\right|_{X_1(s)=0}$? **[2015]**



(A) $\frac{G_1}{1+G_2(1+G_1)}$
(B) $\frac{G_2}{1+G_1(1+G_2)}$

(C) $\frac{G_1}{1+G_1G_2}$
(D) $\frac{G_2}{1+G_1G_2}$

6. In the signal flow diagram given in the figure, u_1 and u_2 are possible inputs whereas y_1 and y_2 are possible outputs. When would the SISO system derived from this diagram be controllable and observable? **[2015]**

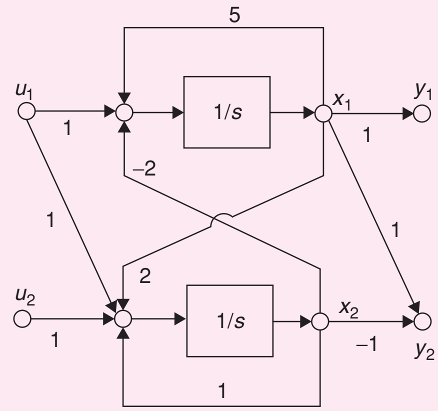
(A) When u_1 is the only input and y_1 is the only output.

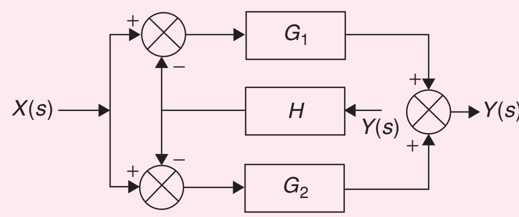
(B) When u_2 is the only input and y_1 is the only output.

(C) When u_1 is the only input and y_2 is the only output.

(D) When u_2 is the only input and y_2 is the only output.

7. Find the transfer function $\frac{Y(s)}{X(s)}$ of the system given below. **[2015]**





(A) $\frac{G_1}{1-HG_1}+\frac{G_2}{1-HG_2}$

(B) $\frac{G_1}{1+HG_1}+\frac{G_2}{1+HG_2}$

(C) $\frac{G_1+G_2}{1+H(G_1+G_2)}$

(D) $\frac{G_1+G_2}{1-H(G_1+G_2)}$

8. An open loop control system results in a response of $e^{-2t}(\sin 5t + \cos 5t)$ for a unit impulse input. The DC gain of the control system is ______. **[2015]**

9. For linear time invariant systems, that are Bounded Input Bounded Output stable, which one of the following statements is TRUE? **[2015]**

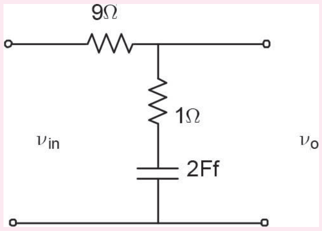
(A) The impulse response will be integrable, but may not be absolutely integrable.

(B) The unit impulse response will have finite support.

(C) The unit step response will be absolutely integrable.

(D) The unit step response will be bounded.

10. For the network shown in the figure below, the frequency (in rad/s) at which the maximum phase lag occurs, is ________. **[2016]**



11. A second-order real system has the following properties: **[2016]**

(a) the damping ratio $\delta = 0.5$ and undamped natural frequency $\omega_n = 10$rad/s

(b) the steady state value of the output, to a unit step input, is 1.02.

The transfer function of the system is

(A) $\frac{1.02}{S^2 + 5S + 100}$ (B) $\frac{102}{S^2 + 10S + 100}$

(C) $\frac{100}{S^2 + 10S + 100}$ (D) $\frac{102}{S2 + 5S + 100}$

Answer Keys

Exercises

Practice Problems 1

1. A **2.** D **3.** B **4.** B **5.** C **6.** A **7.** B **8.** C **9.** C **10.** D
11. B **12.** B **13.** C **14.** C **15.** C **16.** B **17.** B **18.** A **19.** D

Practice Problems 2

1. B **2.** C **3.** B **4.** C **5.** C **6.** B **7.** A **8.** A **9.** B **10.** C
11. C **12.** B **13.** A **14.** C **15.** A

Previous Years' Questions

1. C **2.** C **3.** A **4.** B **5.** B **6.** B **7.** C **8.** 0.23 to 0.25 **9.** D
10. 0.316 **11.** B