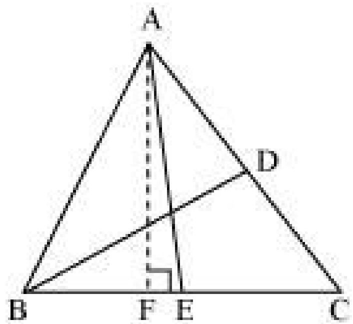
Mensuration

- Area of a triangle:
  - Area of a triangle = $\frac{1}{2} \times \text{Base} \times \text{Altitude}$
  - All the congruent triangles are equal in area, but the triangles having equal areas may or may not be congruent.

Example: ΔABC is isosceles with AC = BC = 6 cm. AE and BD are the medians and AF = 4 cm. What is the area of ΔABD?



Solution: In ΔABE and ΔBAD, we have

BE = AD $\left[AC = BC \Rightarrow \frac{1}{2}AC = \frac{1}{2}BC\right]$]

$\angle ABE = \angle BAD$ [Angles opposite to equal sides]

AB = AB [Common]

$\Rightarrow \Delta ABE \cong \Delta BAD$ [By SAS congruency criterion]

Area (ΔABE) = Area (ΔBAD)

Now, Area $\Delta ABE = \frac{1}{2} \times \text{Base} \times \text{Altitude}$

$= \frac{1}{2} \times BE \times AF$

$= \frac{1}{2} \times \left(\frac{6\text{ cm}}{2}\right) \times 4\text{ cm}$

$= 6\text{ cm}^2$

$\Rightarrow$ Area $\Delta ABD = 6\text{ cm}^2$

- **Area of triangle using Heron's formula:**

When all the three sides of a triangle are given, its area can be calculated using Heron's formula, which is given by:

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Here, s is the semi-perimeter of the triangle and is given by, $s=\frac{a+b+c}{2}$

Example:
Find the area of a triangle whose sides are 9 cm, 28 cm and 35 cm.
Solution:
Let a = 9 cm, b = 28 cm and c = 35 cm
Semi-perimeter, $s=\frac{a+b+c}{2}=\frac{9+28+35}{2}\text{ cm}=36\text{ cm}$
Area of triangle = $\sqrt{36(36-9)(36-28)(36-35)}\text{ cm}^2$

$=\sqrt{36\times 27\times 8\times 1}\text{ cm}^2$
$=36\sqrt{6}\text{ cm}^2$

- Perimeter of a rectangle = 2 (length + breadth)

  Example:

  What is the perimeter of a rectangular field whose length and breadth are 15 m and 8 m respectively?

  Solution:

  Perimeter of rectangular field = 2 (15 m + 8 m) = (2 × 23) m = 46 m

- Area of a rectangle is given by the formula:

  Area of a rectangle = length × breadth
  Example: How much carpet is required to cover a rectangular floor of length 25 m and breadth 18 m?

  Solution: Area of the carpet required = Area of rectangular floor

  = 25 m × 18 m = 450 m^2

- Area of a square is given by the formula:

Area of a square = side × side

Example: What is the area of a square park of side 10 m 20 cm?

Solution: Length of park = 10 m 20 cm = 10.2 m

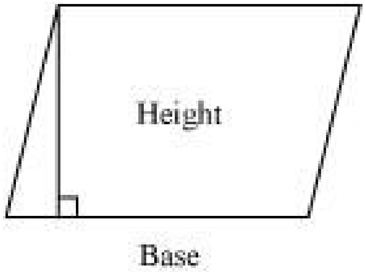
Area of park = 10.2 m × 10.2 m = 104.04 m^2

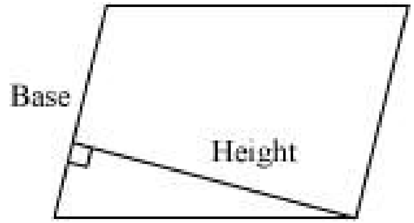
- Area and perimeter of various shapes:

Shape	Area	Perimeter
1. Rectangle with adjacent sides *a* and *b*	$a \times b$	$2(a + b)$
2. Square with side *a*	a^2	$4a$
3. Circle with radius *r*	πr^2	$2\pi r$
4. Triangle with base *b* and its corresponding height *h*	$\frac{1}{2} \times b \times h$	Sum of the three sides
5. Parallelogram with base *b* and its corresponding height *h*	$b \times h$	Sum of the four sides

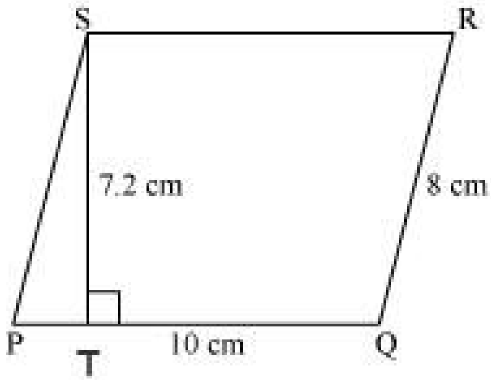
Area of trapezium = $\frac{1}{2}$ (Sum of the lengths of the parallel sides) × (Perpendicular distance between them)

- Area of a parallelogram:
  - The perpendicular dropped on a side from its opposite vertex is known as the height and the side is known as the base.
  - Area of a parallelogram = Base × Height





Example:
Find the height of the parallelogram PQRS corresponding to the base RQ.

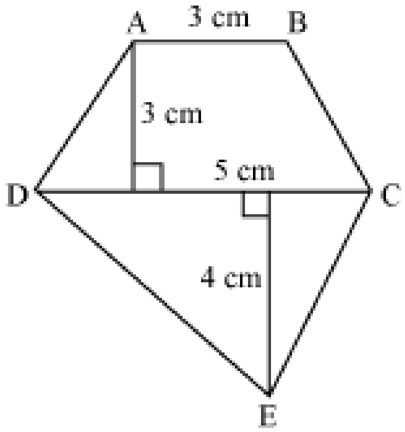


Solution:
Let the height corresponding to the base RQ be x cm.
Area of the parallelogram PQRS = PQ × ST
= 10 cm × 7.2 cm
= 72 cm^2
Area of the parallelogram = RQ × x
= 8 cm × x cm
= $8x$ cm^2
$\therefore 8x = 72$
$\Rightarrow x = 9$
Thus, the height of the parallelogram corresponding to the base RQ is 9 cm.

- Area of rhombus $= \frac{1}{2}$ (Product of its diagonals)

- Area of a polygon can be calculated by breaking the polygon into triangles or any types of a quadrilateral.

  Example: Find the area of the given polygon, where ABCD is a trapezium.



Solution:

Area of ABCED = Area of trapezium ABCD + Area of ΔCDE

$=12\times3+5\times3 + 12\times5\times4$

$= 12 + 10$

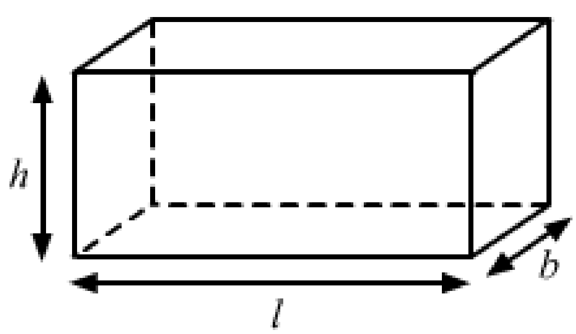
$= 22\ \text{cm}^2$

- Area of a circle = $\pi \times (\text{Radius})^2$

  Example: What is the area of a circle whose circumference is 44 cm? $\left(\pi = \frac{22}{7}\right)$

  Solution:
  Circumference = $2\pi r$ = 44 cm
  $\Rightarrow 2 \times \frac{22}{7} \times r = 44\,\text{cm}$
  $\Rightarrow r = 44 \times \frac{7}{22\times2} = 7\ \text{cm}$
  $\therefore$ Area of the circle $= \pi r^2 = \frac{22}{7} \times 7 \times 7 = 154\ \text{cm}^2$

- **Surface areas of cuboid:**



Lateral surface area of the cuboid = $2h\,(l + b)$

Total surface area of the cuboid = $2\,(lb + bh + hl)$

Note: Length of the diagonal of a cuboid = $\sqrt{l^2 + b^2 + h^2}$

Example:

Find the edge of a cube whose surface area is 294 m^2.

Solution:

Let the edge of the given cube be a.

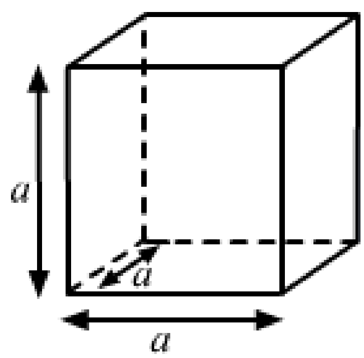
$\therefore$ Surface area of the cube = $6a^2$

Given, $6a^2 = 294$

$\Rightarrow a^2 = 49\text{ m}^2$

$\therefore a = \sqrt{49}\text{ m} = 7\text{ m}$

- **Surface areas of cube:**



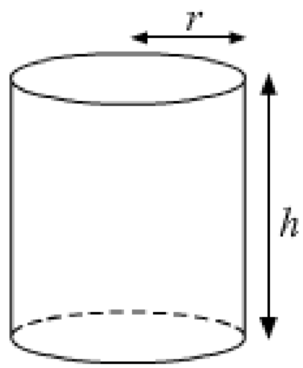
Lateral surface area of the cube = $4a^2$

Total surface area of the cube = $6a^2$

Note: Length of the diagonal of a cube = $\sqrt{a^2 + a^2 + a^2} = \sqrt{3a^2} = \sqrt{3}a$

- **Surface areas of solid cylinder**
  - Curved surface area = $2\pi rh$, where r and h are the radius and height

- Total surface area = $2\pi r\,(r + h)$, where r and h are the radius and height



Example :

What is the curved surface area of a cylinder of radius 2 cm and height 14 cm?

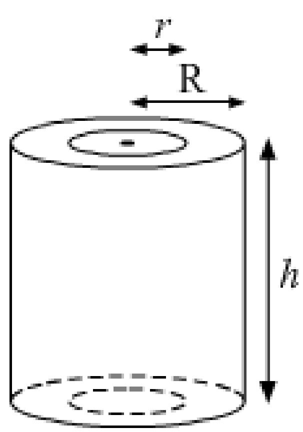
Solution:

Curved surface area of cylinder = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 2 \times 14\ \text{cm}^2$$
$$= 176\ \text{cm}^2$$

- **Surface areas of hollow cylinder**
  - Curved surface area = $2\pi h\,(r + R)$, where r, R and h are the inner radius, outer radius and height
  - Total surface area = CSA of outer cylinder + CSA of inner cylinder + 2 × Area of base

    = $2\pi\,(r + R)\,(h + R - r)$, where r, R and h are the inner radius, outer radius and height



• Space occupied by a solid shape is its volume while the maximum quantity of liquid that it can hold shows its capacity.

• When each of the length, breadth and height of a cube measures 1 cm, its volume is said to be 1 cubic centimeter. It is written as c.c. or cm^3, which is the fundamental unit of volume. This cube is called the unit cube of side 1 cm.

Example:

By using the small cube in figure (a), find the volume of the solid in figure (b).

Figure (a)

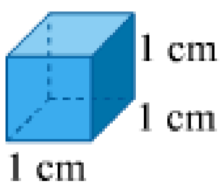
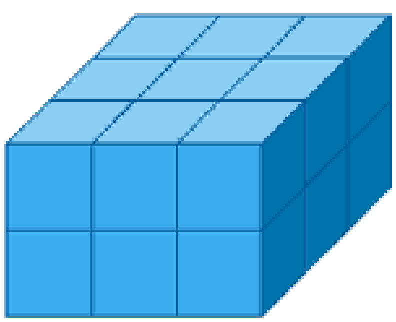


Figure (b)



Solution:

Volume of smaller cube in figure (a) = 1 cm^3

It can be observed that the solid in figure (b) consists of 18 cubes like figure (a).

∴ Volume of solid = (18 × 1) cm^3 = 18 cm^3

- **Volume of cube and cuboid**

- Volume of cube = a^3, where a is the side of the cube
- Volume of cuboid = $l \times b \times h$, where l, b and h are respectively the length, breadth and height of the cuboid.

Example:

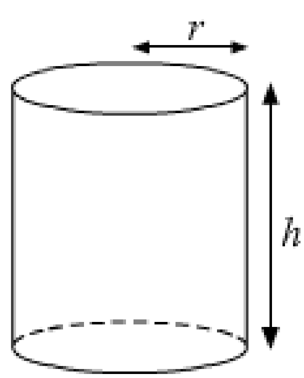
What is the side of a cube of volume 512 cm^3?

Solution:

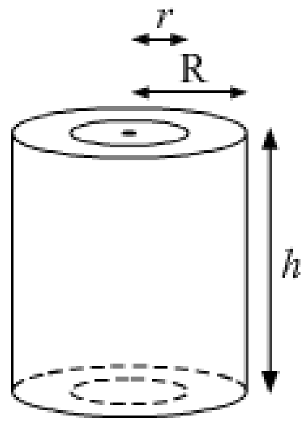
Volume of cube = 512 cm^3

$$\Rightarrow a^3 = 512 \text{cm}^3$$
$$\Rightarrow a = \sqrt[3]{512} \text{cm}^3$$
$$\Rightarrow a = 8 \text{cm}$$

- **Volume of the solid cylinder and hollow cylinder**
  - Volume of solid cylinder = $\pi r^2 h$, where r and h are the radius and height of the solid cylinder



  - Volume of the hollow cylinder = $\pi (R^2 - r^2) h$, where r, R and h are the inner radius, outer radius and height of hollow cylinder



Example:

Find the volume of the pillar of radius 70 cm and height 10 m.

Solution:

Radius of the pillar (r) = 70 cm = $\frac{70 \text{ m}}{100}$ = 0.7 m

Height of the pillar (h) = 10 m

$$\text{Volume of the pillar} = \pi r^2 h$$

$$= \frac{22}{7} \times (0.7)^2 \times 10 \text{ m}^3$$

$$= 15.4 \text{ m}^3$$