Exercise 7.7

Answer 1e.

We know that a set of more than two points (x, y) fits an exponential pattern only when the set of transformed points $(x, \ln y)$ fits a linear pattern.

Given a set of more than two data pairs (x, y), you can decide whether a(n) **exponential** function fits the data well by making a scatter plot of the points $(x, \ln y)$.

Answer 1gp.

STEP 1 Replace x with 1, and y with 6 in $y = ab^x$ to obtain the first equation.

$6 = ab^1$

Substitute 3 for x, and 24 for y in the above function to obtain the second equation.

$24 = ab^3$

STEP 2 Solve the first equation for a.

Divide both the sides by b^1.

$$\frac{6}{b^1} = \frac{ab^1}{b^1}$$

$$\frac{6}{b} = a$$

Replace a with $\frac{6}{b}$ in the second equation and simplify.

$$24 = \left(\frac{6}{b}\right)b^3$$

$$= 6b^2$$

Divide both the sides by 6.

$$\frac{24}{6} = \frac{6b^2}{6}$$

$$4 = b^2$$

Take positive square root of both the sides since $b > 0$.

$$\sqrt{4} = \sqrt{b^2}$$

$$2 = b$$

STEP 3 Substitute 2 for b in $\frac{6}{b} = a$ and simplify.

$$\frac{6}{2} = a$$

$$3 = a$$

Replace a with 3, and b with 2 in $y = ab^x$.

$y = 3(2)^x$

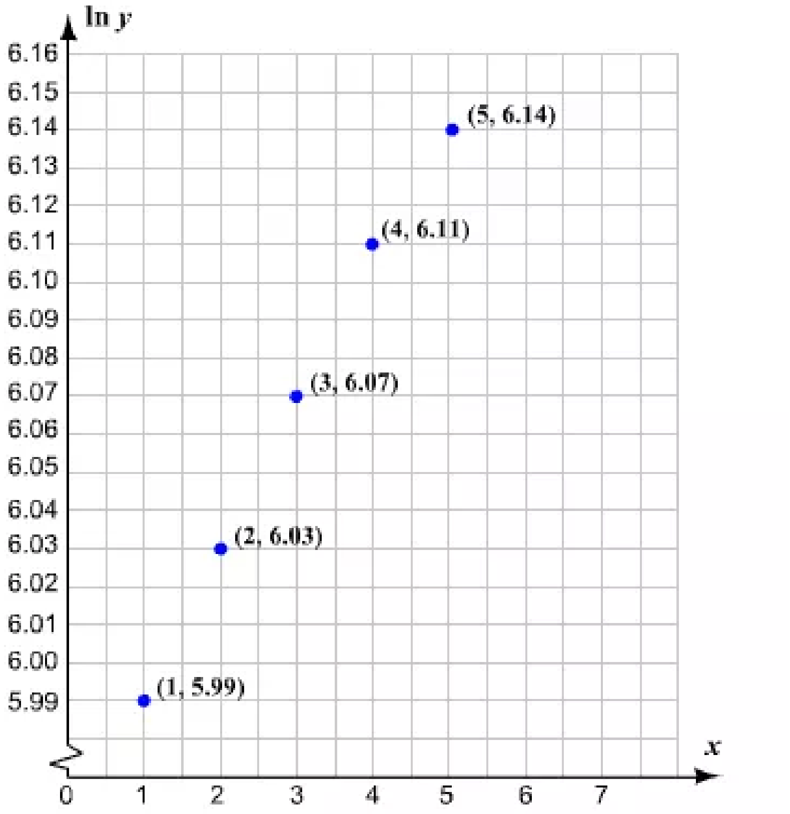
Therefore, the exponential function is $y = 3(2)^x$.

Answer 1mr.

a. Use a calculator to create a table of data pairs $(x, \ln y)$.

x	1	2	3	4	5
$\ln y$	5.99	6.03	6.07	6.11	6.14

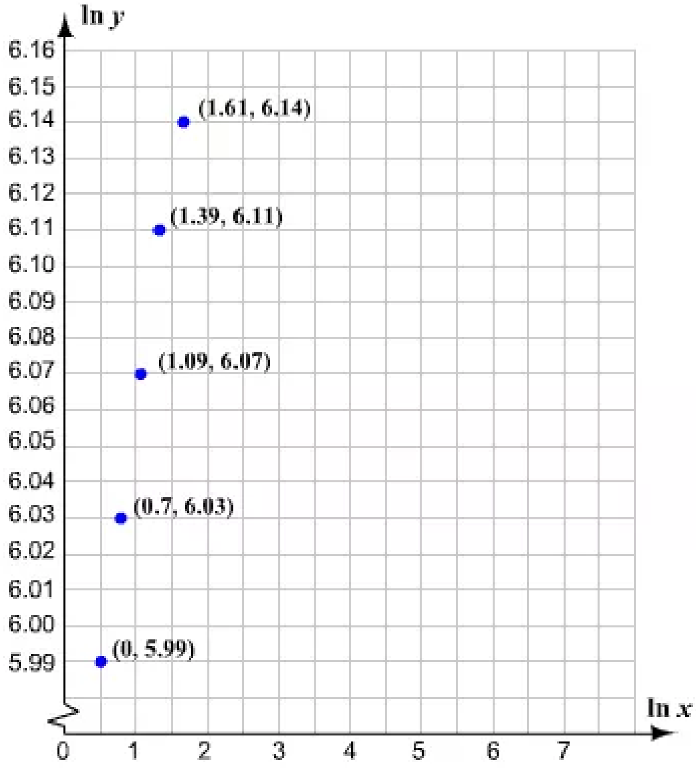
Plot the points on a graph to get the scatter plot.



b. Use a calculator to create a table of data pairs $(\ln x, \ln y)$.

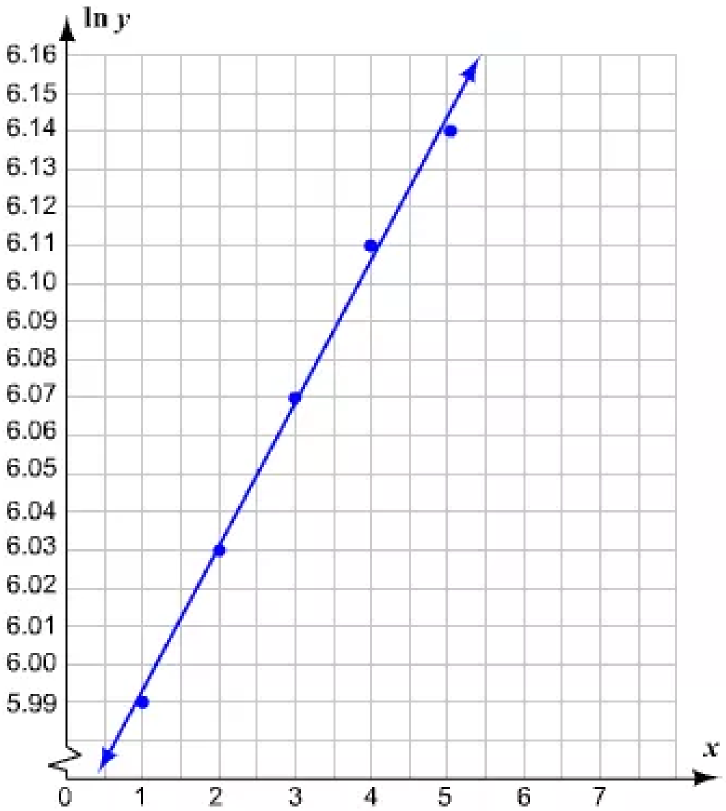
$\ln x$	0	0.7	1.09	1.39	1.61
$\ln y$	5.99	6.03	6.07	6.11	6.14

Plot the points on a graph to get the scatter plot.



c. We have the scatter plots of $(x, \ln y)$ and $(\ln x, \ln y)$ from part **a**. Compare the graphs. The points for $(x, \ln y)$ appear more linear than the points $(\ln x, \ln y)$. Thus, an exponential model appears to be the best fit for the data.

Consider the scatter plot of $(x, \ln y)$. Sketch the line that best fits the data.



Choose two data points that appear to lie on the line. Let the points be (2, 6.03) and (3, 6.07).

Find the slope, m, using these points.

$$\begin{aligned} m &= \frac{6.07 - 6.03}{3 - 2} \\ &= \frac{0.04}{1} \\ &= 0.04 \end{aligned}$$

The point slope form of an equation is $y - y_1 = m(x - x_1)$. Choose (2, 6.03) as the point (x_1, y_1).

Substitute 0.04 for m, 2 for x_1, and 6.03 for y_1 in the above equation.
$y - 6.03 = 0.04(x - 2)$

Use the distributive property to open the parentheses.
$y - 6.03 = 0.04x - 0.08$

Add 6.03 to both sides of the equation.

$$\begin{aligned} y - 6.03 + 6.03 &= 0.04x + 6.03 \\ y &= 0.04x + 6.03 \end{aligned}$$

An approximation of the best fitting line for the original data is
$\ln y = 0.04x + 6.03$.

d. An approximation of the best fitting line for the original data is
$\ln y = 0.04x + 6.03$.

In order to find the total expenditure in 2005, substitute 9 for x in the equation.
$\ln y = 0.04(9) + 6.03$

Simplify.

$$\begin{aligned} \ln y &= 0.36 + 6.03 \\ &= 6.39 \end{aligned}$$

Exponentiate each side and evaluate.

$$\begin{aligned} e^{\ln y} &= e^{6.39} \\ y &\approx 596 \end{aligned}$$

Therefore, the total expenditure in the year 2005 is about $596 billion.

Answer 1q.

First, we need to rewrite 16 as a power with base 2.

$$(2)^{x+1} = \left(2^4\right)^{x+2}$$

Apply the power of a power property.

$$2^{x+1} = 2^{4x+8}$$

Now, use the property of equality for exponential equations. If b is any positive number other than 1, then $b^x = b^y$ if and only if $x = y$.

Thus,

$x + 1 = 4x + 8.$

Subtract $4x$ from each side to isolate the variable term on one side.

$$x + 1 - 4x = 4x + 8 - 4x$$
$$-3x + 1 = 8$$

Subtract 1 from each side.

$$-3x + 1 - 1 = 8 - 1$$
$$-3x = 7$$

Divide each side by -3.

$$\frac{-3x}{-3} = \frac{7}{-3}$$
$$x = -\frac{7}{3}$$

CHECK

Substitute $-\frac{7}{3}$ for x in the original equation and check the solution.

$$2^{x+1} = 16^{x+2}$$
$$2^{-\frac{7}{3}+1} \stackrel{?}{=} 16^{-\frac{7}{3}+2}$$
$$2^{-\frac{4}{3}} \stackrel{?}{=} 16^{-\frac{1}{3}}$$
$$2^{-\frac{4}{3}} \stackrel{?}{=} \left(2^4\right)^{-\frac{1}{3}}$$
$$2^{-\frac{4}{3}} = 2^{-\frac{4}{3}} \quad \checkmark$$

The solution is $-\frac{7}{3}$.

Answer 2e.

To determine, whether a power function is a good model, for a set of order pairs (x, y).

First find the order pairs $(\ln x, \ln y)$ for each order pair (x, y).

Next graph the points $(\ln x, \ln y)$ from the data.

If the points lie very close to one line then a power function is a good model.

Answer 2gp.

Consider the exponential function $y = ab^x$, whose graph is passes through the points,

$(2,8)$ and $(3,32)$.

Step 1: Substitute the coordinate of the two given points $(2,8)$ and $(3,32)$ into $y = ab^x$.

$8 = ab^2$	Substitute 8 for y and 2 for x
$32 = ab^3$	Substitute 32 for y and 3 for x

Step 2: Solve for a in the first equation to obtain,

$$a = \frac{8}{b^2}.$$

Next substitute this expression for a in the second equation.

$32 = \left(\frac{8}{b^2}\right)b^3$	Substitute $a = \frac{8}{b^2}$
$32 = 8b$	Simplify
$4 = b$	Divide both sides by 8.

Therefore,

$$b = 4.$$

To find a, substitute 4 for b in $a = \frac{8}{b^2}$, then,

$$\begin{aligned} a &= \frac{8}{b^2} \\ &= \frac{8}{4^2} \\ &= \frac{8}{16} \\ &= 0.5 \end{aligned}$$

Hence, the exponential equation is,

$$\boxed{y = (0.5)\cdot(4)^x}.$$

Answer 2mr.

It is given that the number c of cents by which two notes differ in pitch is:

$$c = 1200 \log_2 \frac{a}{b}$$

where a and b are the frequencies of the notes a and b.

(a)

Consider the notes on the standard scale are:

$C4, E4$ and $G4$.

We need to compare the difference in the number of cents from $C4$ to $E4$ and with the difference from $E4$ to $G4$ by evaluating the expression

$$1200 \log_2 \frac{E4}{C4} - 1200 \log_2 \frac{G4}{E4}$$

From the above expression, we have

$$1200 \log_2 \frac{E4}{C4} - 1200 \log_2 \frac{G4}{E4} = \log_2 \frac{\left(\frac{E4}{C4}\right)}{\left(\frac{G4}{E4}\right)} \qquad \left[\text{Since } \log a - \log b = \log \frac{a}{b}\right]$$

$$= \log_2 \left(\frac{E4}{C4}\right)\left(\frac{E4}{G4}\right)$$

$$\boxed{1200 \log_2 \frac{E4}{C4} - 1200 \log_2 \frac{G4}{E4} = \log_2 \frac{(E4)^2}{C4 \cdot G4}} \qquad \ldots\ldots (1)$$

(b)

The given values are:

$C4 = 264$ hertz, $E4 = 330$ hertz and $G4 = 396$ hertz

Using above values we need to evaluate the expression in part (a).

From the expression (1), we have

$$\log_2 \frac{(E4)^2}{C4 \cdot G4} = \log_2 \frac{330^2}{264 \times 396}$$

$$= \log_2 1.04$$

$$\boxed{\log_2 \frac{(E4)^2}{C4 \cdot G4} \approx 0.0566}$$

Answer 2q.

Consider the equation,

$$e^{-x} = 4.$$

Solve the equation:

$e^{-x} = 4$	
$-x = \ln 4$	Use the natural logarithm
$-x = 1.3863$	Use calculator
$x = -1.3863$	Multiply both sides by -1.

Check the solution by substituting it into the original equation.

$e^{-x} = 4$	
$e^{-(-1.386)} \overset{?}{=} 4$	Substitute -1.3863 for x
$e^{1.3863} \overset{?}{=} 4$	Simplify
$4 = 4$	TRUE

Therefore, the solution is,

$$\boxed{-1.3863}.$$

Answer 3e.

STEP 1 Replace x with 1, and y with 3 in $y = ab^x$ to obtain the first equation.

$$3 = ab^1$$

Substitute 2 for x, and 12 for y in the above function to obtain the second equation.

$$12 = ab^2$$

STEP 2 Solve the first equation for a.

Divide both the sides by b^1.

$$\frac{3}{b^1} = \frac{ab^1}{b^1}$$

$$\frac{3}{b} = a$$

Replace a with $\frac{3}{b}$ in the second equation and simplify.

$$12 = \left(\frac{3}{b}\right)b^2$$

$$= 3b$$

Divide both the sides by 3.

$$\frac{12}{3} = \frac{3b}{3}$$

$$4 = b$$

STEP 3 Substitute 4 for b in $\frac{3}{b} = a$.

$\frac{3}{4} = a$

Replace a with $\frac{3}{4}$, and b with 4 in $y = ab^x$.

$y = \frac{3}{4} \cdot 4^x$

Therefore, the exponential function is $y = \frac{3}{4} \cdot 4^x$.

Answer 3gp.

STEP 1 Replace x with 3, and y with 8 in $y = ab^x$ to obtain the first equation.

$8 = ab^3$

Substitute 6 for x, and 64 for y in the above function to obtain the second equation.

$64 = ab^6$

STEP 2 Solve the first equation for a.

Divide both the sides by b^3.

$\frac{8}{b^3} = \frac{ab^3}{b^3}$

$\frac{8}{b^3} = a$

Replace a with $\frac{8}{b^3}$ in the second equation and simplify.

$64 = \left(\frac{8}{b^3}\right)b^6$

$= 8b^3$

Divide both the sides by 8.

$\frac{64}{8} = \frac{8b^3}{8}$

$8 = b^3$

Take positive square root of both the sides since $b > 0$.

$\sqrt{8} = \sqrt{b^2}$

$2.8 = b$

STEP 3 Substitute 2.8 for b in $\frac{8}{b^3} = a$ and simplify.

$$\frac{8}{(2.8)^3} = a$$

$$0.36 = a$$

Replace a with 0.36, and b with 2.8 in $y = ab^x$.

$y = 0.36(2.8^x)$

Therefore, the exponential function is $y = 0.36(2.8^x)$.

Answer 3mr.

Replace x with 2, and y with 7 in $y = ab^x$.

$7 = ab^2$

Assume that the value of b is 2. Substitute 2 for b and simplify.

$$7 = a(2)^2$$

$$7 = 4a$$

Solve the equation for a. For this, divide both the sides by 4.

$$\frac{7}{4} = \frac{4a}{4}$$

$$\frac{7}{4} = a$$

Now, replace a with $\frac{7}{4}$, and b with 2 in $y = ab^x$.

$$y = \frac{7}{4} \cdot 2^x$$

Therefore, an exponential function whose graph passes through the given point can be

$y = \frac{7}{4} \cdot 2^x$.

Answer 3q.

First, subtract 5 from each side of the equation.

$$3^{2x} + 5 - 5 = 13 - 5$$

$$3^{2x} = 8$$

Take $\log_3$ of each side.

$$\log_3 3^{2x} = \log_3 8$$

We know that $\log_b b^x = x$. Thus,

$2x = \log_3 8$.

Divide each side by 2.

$$\frac{2x}{2} = \frac{\log_3 8}{2}$$

$$x = \frac{\log_3 8}{2}$$

Apply the change-of-base formula.

$$x = \frac{\frac{\log 8}{\log 3}}{2}$$

Use a calculator to evaluate.

$x \approx 0.9464$

CHECK

Substitute 0.946 for x in the original equation and check the solution.

$$3^{2x} + 5 = 13$$

$$3^{2(0.9464)} + 5 \stackrel{?}{=} 13$$

$$3^{1.9292} + 5 \stackrel{?}{=} 13$$

$$13 \approx 13 \quad ✓$$

Thus, the solution is about 0.9464.

Answer 4e.

Consider the exponential function $y = ab^x$, whose graph is passes through the points,

$(2,24)$ and $(3,144)$.

Step 1: Substitute the coordinate of the two given points $(2,24)$ and $(3,144)$ into $y = ab^x$.

$24 = ab^2$ Substitute 24 for y and 2 for x

$144 = ab^3$ Substitute 32 for y and 3 for x

Step 2: Solve for a in the first equation as follows,

$24 = ab^2$

$a = \frac{24}{b^2}$. Divide both sides by b^2

Next substitute this expression for a in the second equation.

$144 = ab^3$

$144 = \left(\frac{24}{b^2}\right)b^3$ Substitute $a = \frac{24}{b^2}$

$144 = 24b$ Simplify

$6 = b$ Divide both sides by24.

Therefore,

$b = 6$.

Step 3: To find a, substitute 6 for b in $a = \frac{24}{b^2}$, then,

$$\begin{aligned} a &= \frac{24}{b^2} \\ &= \frac{24}{(6)^2} \\ &= \frac{24}{36} \\ &= \frac{2}{3} \end{aligned}$$

Hence, the exponential equation is,

$$\boxed{y = \left(\frac{2}{3}\right)\cdot(6)^x}.$$

Answer 4gp.

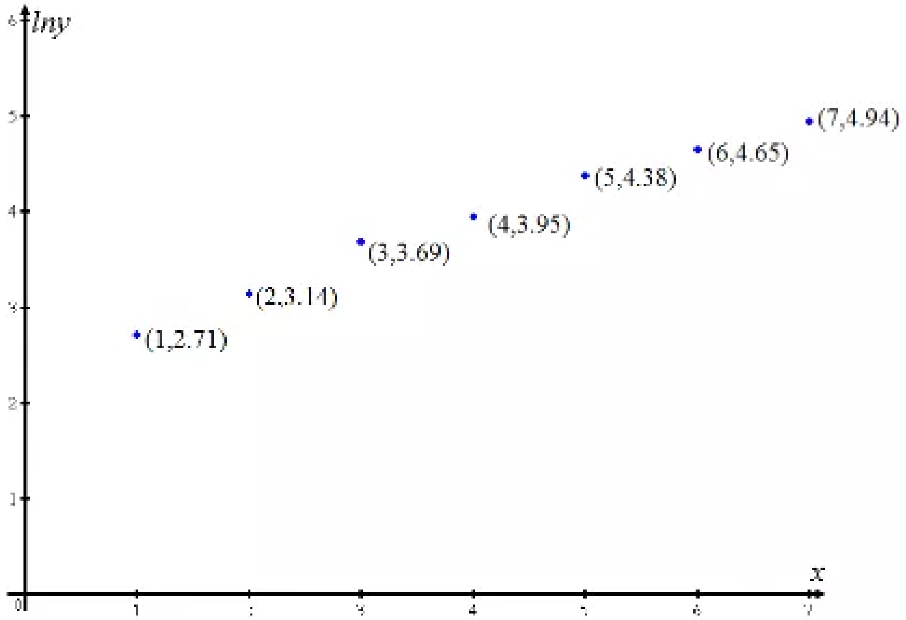
Consider the scooter sales were as shown in the table below:

Year, x	1	2	3	4	5	6	7
Number of scooters sold, y	15	23	40	52	80	105	140

Step 1: Use a calculator to create a table of data pairs $(x, \ln y)$.

x	1	2	3	4	5	6	7
$\ln y$	2.71	3.14	3.69	3.95	4.38	4.65	4.94

Step 2: Plot the order pairs $(x, \ln y)$ on the graph. Then they will be displayed as follows:



Step 3: The exponential models change, if the scooter sales will change.
Because axis are x and ln$y$, the point-slope form is rewritten as,

$$\ln y - y_1 = m(x - x_1) \text{ .}$$

Next find the slope of the line through $(1, 2.71)$ and $(7, 4.94)$ is,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4.94 - 2.71}{7 - 1} \qquad \text{Substitute } 4.94 \text{ for } y_2, 2.71 \text{ for } y_1, 7 \text{ for } x_2, 1 \text{ for } x_1$$

$$\approx 0.37$$

Find the exponential model $y = ab^x$ by choosing a point on the line, such as $(1, 2.71)$.
Use this point to write an equation of the line. Then solve for y.

$\ln y - y_1 = m(x - x_1)$	
$\ln y - 2.71 = 0.37(x-1)$	Equation of line
$\ln y - 2.71 = 0.37x - 0.37$	Apply distributive property
$\ln y = 0.37x + 2.34$	Add 2.71 on both sides
$y = e^{0.37x+2.34}$	Exponentiate each side using base e.
$y = e^{2.34}\left(e^{0.37}\right)^x$	Use properties of exponents
$y = 10.38(1.45)^x$	Exponential model

Thus the exponential model is $\boxed{y = 10.38(1.45)^x}$.

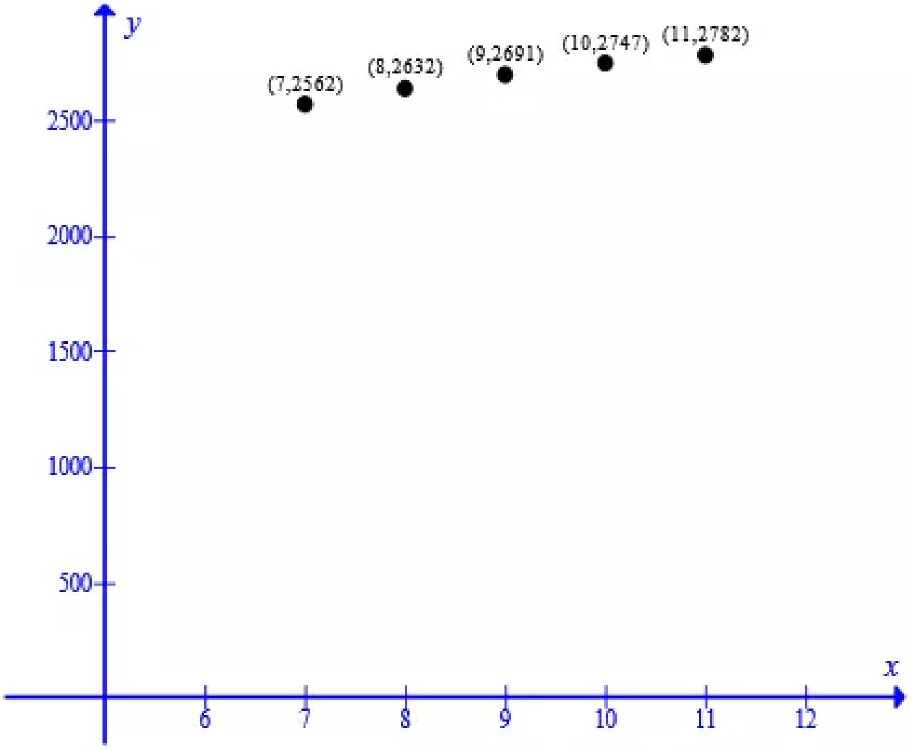
Answer 4mr.

The total number of miles traveled by motor vehicles in the United States is given by:

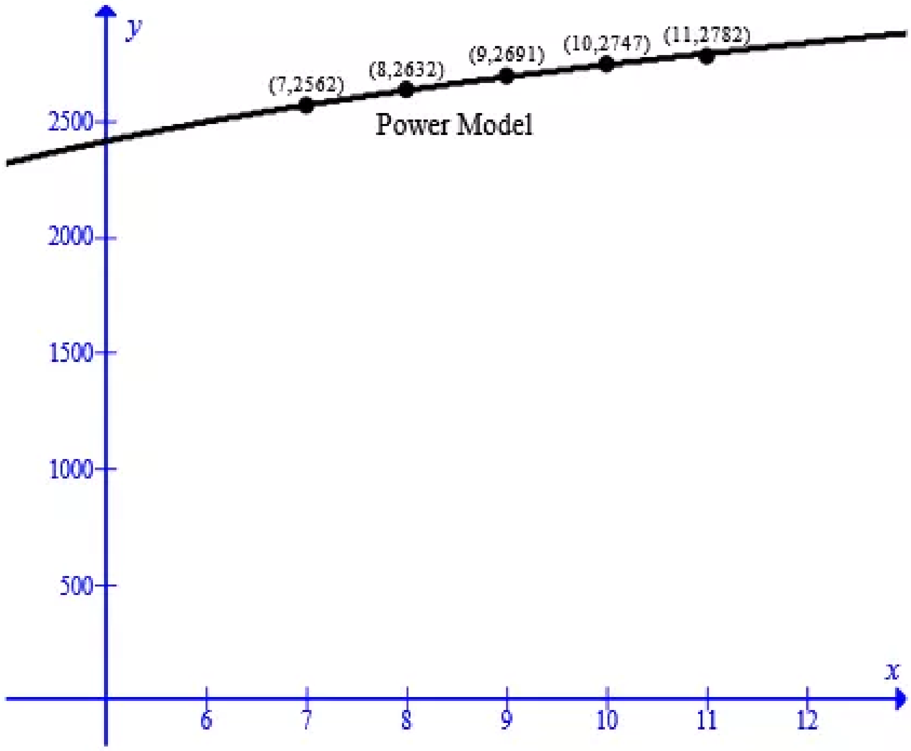
Years since 1990, x	Miles (billions), y
7	2562
8	2632
9	2691
10	2747
11	2782

We need to determine whether an exponential model or a power model is best fit for data.

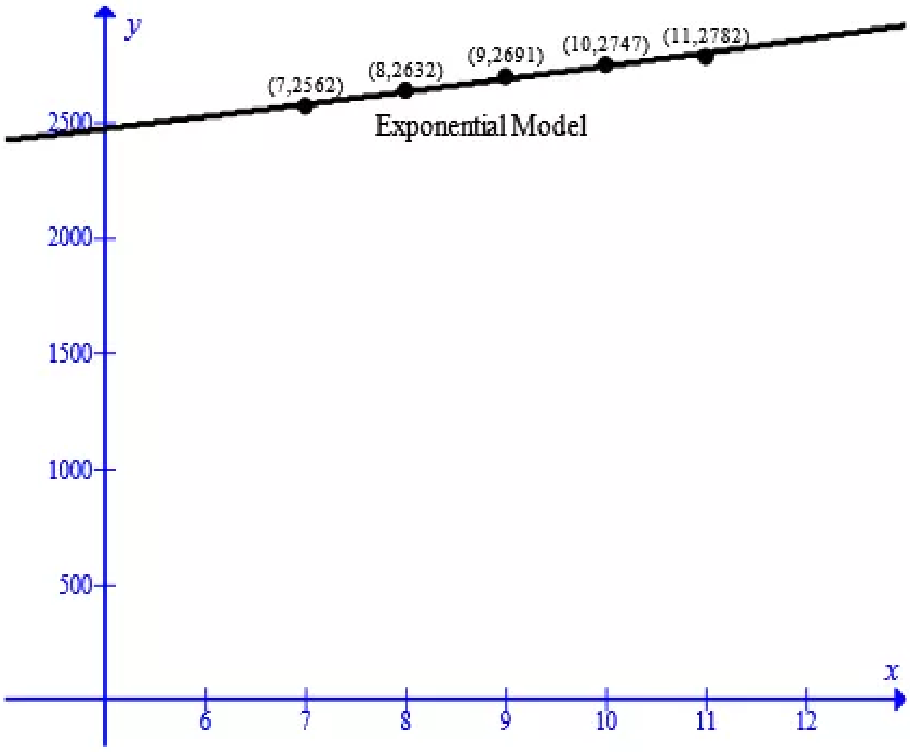
Scatter plots of the points $(7,2562)$, $(8,2632)$, $(9,2691)$, $(10,2747)$ and $(11,2782)$ in the xy - plane are shown in the graph below:



The power model for the given data is shown below:



The exponential model for the given data is shown below:



From the above graphs we can verify that in the power model of the data, the curves passes through all the points. But in the exponential model, though the curves passes through some of the points still it touches some points also. Therefore the power model will be the best fit for the data.

Answer 4q.

Consider the equation,

$$3^{x+1} - 5 = 10.$$

Solve the equation:

$3^{x+1} - 5 = 10$	
$3^{x+1} = 15$	Add 5 on both sides
$\log 3^{x+1} = \log 15$	Apply log on both sides
$(x+1)\log 3 = \log(3\cdot 5)$	Use power property; $\log_b m^n = n\log_b m$
$(x+1)\log 3 = \log 3 + \log 5$	Use product property; $\log_b mn = \log_b m + \log_b n$
$x\log 3 = \log 5$	Subtract log3 from both sides
$x = \dfrac{\log 5}{\log 3}$	Divide both sides by log3
$x = 1.46$	Use calculator

Check the solution by substituting it into the original equation.

$$3^{x+1}-5=10$$

$$3^{1.46+1}-5\stackrel{?}{=}10 \qquad \text{Substitute 1.46 for } x$$

$$3^{2.46}-5\stackrel{?}{=}10 \qquad \text{Simplify}$$

$$15-5\stackrel{?}{=}10$$

$$10=10 \qquad \text{TRUE}$$

Therefore, the solution is,

$\boxed{1.46}$.

Answer 5e.

STEP 1 Replace x with 3, and y with 1 in $y=ab^x$ to obtain the first equation.

$$1=ab^3$$

Substitute 5 for x, and 4 for y in the above function to obtain the second equation.

$$4=ab^5$$

STEP 2 Solve the first equation for a.

Divide both the sides by b^3.

$$\frac{1}{b^3} = \frac{ab^3}{b^3}$$

$$= a$$

Replace a with $\frac{1}{b^3}$ in the second equation and simplify.

$$4 = \left(\frac{1}{b^3}\right)b^5$$

$$= b^2$$

Take positive square root of both the sides since $b > 0$.

$$\sqrt{4} = \sqrt{b^2}$$

$$2 = b$$

STEP 3 Substitute 2 for b in $\frac{1}{b^3} = a$ and simplify.

$$\frac{1}{2^3} = a$$

$$\frac{1}{8} = a$$

Replace a with $\frac{1}{8}$, and b with 2 in $y = ab^x$.

$y = \frac{1}{8} \cdot 2^x$

Therefore, the exponential function is $y = \frac{1}{8} \cdot 2^x$.

Answer 5gp.

STEP 1 Replace x with 2, and y with 1 in $y = ax^b$ to obtain the first equation.

$1 = a \cdot 2^b$

Substitute 7 for x, and 6 for y in the above function to obtain the second equation.

$6 = a \cdot 7^b$

STEP 2 Solve the first equation for a.

Divide both the sides by 2^b and simplify.

$$\frac{1}{2^b} = \frac{a \cdot 2^b}{2^b}$$

$$= a$$

Replace a with $\frac{1}{2^b}$ in the second equation and simplify.

$$6 = \frac{1}{2^b} \cdot 7^b$$

$$= 3.5^b$$

Take $\log_{3.5}$ of each side.

$\log_{3.5} 6 = b$

Apply the change-of-base formula and simplify.

$$\frac{\log 6}{\log 3.5} = b$$

$$1.43 \approx b$$

STEP 3 Substitute 1.43 for b in $\frac{1}{2^b} = a$ and simplify.

$$\frac{1}{2^{1.43}} \approx a$$

$$0.372 \approx a$$

Replace a with 0.372, and b with 1.43 in $y = ax^b$.

$$y \approx 0.372x^{1.43}$$

Therefore, the exponential function is $y \approx 0.372x^{1.43}$.

Answer 5mr.

a. Substitute 0.05 for N in the given formula.

$$0.05 = \ln(E + 1)$$

Exponentiate each side with base e.

$$e^{0.05} = e^{\ln}(E + 1)$$

We know that $e^{\ln x} = x$. Thus,

$$E + 1 = e^{0.05}.$$

Subtract 1 from both the sides

$$E + 1 - 1 = e^{0.05} - 1$$

$$E = e^{0.05} - 1$$

Thus, the effective interest rate for an account that has a nominal interest of 5% is $e^{0.05} - 1$.

b. Substitute 0.1 for N in the given formula.

$$0.1 = \ln(E + 1)$$

Exponentiate each side with base e.

$$e^{0.1} = e^{\ln}(E + 1)$$

We know that $e^{\ln x} = x$. Thus,

$$E + 1 = e^{0.1}.$$

Subtract 1 from both the sides

$$E + 1 - 1 = e^{0.1} - 1$$

$$E = e^{0.1} - 1$$

Thus, the effective interest rate for an account that has a nominal interest of 10% is $e^{0.1} - 1$.

c. The effective interest rate for an account that has a nominal interest of 5% is $e^{0.05} - 1$ and that for 10% is $e^{0.1} - 1$.

Divide the effective interest rate for an account that has a nominal interest of 10% by that for an account that has a nominal interest of 5%.

$$\frac{e^{0.1} - 1}{e^{0.05} - 1}$$

The effective interest rate for an account that has a nominal interest of 10% is $\frac{e^{0.1} - 1}{e^{0.05} - 1}$ times greater than that for an account that has a nominal interest of 5%.

d. We know that the ratio in terms of e is $\frac{e^{0.1} - 1}{e^{0.05} - 1}$.

Simplify.

$$\begin{aligned}\frac{e^{0.1} - 1}{e^{0.05} - 1} &= \frac{\left(e^{0.05}\right)^2 - 1^2}{e^{0.05} - 1} \\ &= \frac{\left(e^{0.05} + 1\right)\left(e^{0.05} - 1\right)}{e^{0.05} - 1} \\ &= e^{0.05} + 1\end{aligned}$$

Thus, the ratio is equal to $e^{0.05} + 1$.

Answer 5q.

Apply the property of equality for logarithmic equations. We know that if b, x and y are positive numbers with $b \neq 1$, then $\log_b x = \log_b y$ if and only if $x = y$.

Thus,

$4x + 7 = 11x$.

Subtract $11x$ from each side.

$$\begin{aligned}4x + 7 - 11x &= 11x - 11x \\ -7x + 7 &= 0\end{aligned}$$

Subtract 7 from each side.

$$\begin{aligned}-7x + 7 - 7 &= 0 - 7 \\ -7x &= -7\end{aligned}$$

Divide each side by 7.

$$\begin{aligned}\frac{-7x}{-7} &= \frac{-7}{-7} \\ x &= 1\end{aligned}$$

CHECK

Substitute 1 for x in the original equation and check the solution.

$$\log_4(4x+7) = \log_4 11x$$
$$\log_4[4(1)+7] \stackrel{?}{=} \log_4 11(1)$$
$$\log_4(4+7) \stackrel{?}{=} \log_4 11$$
$$\log_4 11 = \log_4 11 \quad \checkmark$$

The solution is 1.

Answer 6e.

Consider the exponential function $y = ab^x$, whose graph is passes through the points,

$(3,27)$ and $(5,243)$.

Step 1: Substitute the coordinate of the two given points $(3,27)$ and $(5,243)$ into $y = ab^x$.

$27 = ab^3$ Substitute 27 for y and 3 for x

$243 = ab^5$ Substitute 243 for y and 5 for x

Step 2: Solve for a in the first equation as follows,

$27 = ab^3$

$a = \frac{27}{b^3}$. Divide both sides by b^3

Next substitute this expression for a in the second equation.

$243 = ab^5$

$243 = \left(\frac{27}{b^3}\right)b^5$ Substitute $a = \frac{27}{b^3}$

$243 = 27b^2$ Simplify

$9 = b^2$ Divide both sides by27.

$3 = b$

Therefore,

$b = 3$.

Step 3: To find a, substitute 3 for b in $a=\frac{27}{b^3}$, then,

$$\begin{aligned} a &= \frac{27}{b^3} \\ &= \frac{27}{(3)^3} \\ &= \frac{27}{27} \\ &= 1 \end{aligned}$$

Hence, the exponential equation is,

$$\boxed{y=1\cdot(3)^x}.$$

Answer 6gp.

Consider the power function $y=ax^b$, whose graph is passes through the points, $(3,4)$ and $(6,15)$.

Step 1: Substitute the coordinate of the two given points $(3,4)$ and $(6,15)$ into,

$y=ax^b$.

$4=a\cdot 3^b$ Substitute 4 for y and 3 for x

$15=a\cdot 6^b$ Substitute 15 for y and 6 for x

Step 2: Solve for a in the first equation as follows,

$4=a\cdot 3^b$

$a=\frac{4}{3^b}$. Divide both sides by 3^b

Next substitute this expression for a in the second equation.

$15=a\cdot 6^b$

$15=\left(\frac{4}{3^b}\right)6^b$ Substitute $a=\frac{4}{3^b}$

$15=\left(\frac{4}{3^b}\right)\cdot(3\cdot 2)^b$ Write $6^b=(3\cdot 2)^b$

$15=\left(\frac{4}{3^b}\right)\cdot 3^b\cdot 2^b$ Since, $(a\cdot b)^m=a^m b^m$

$15=4\cdot 2^b$ Simplify

$3.75=2^b$ Divide both sides by 4.

$\log_2 3.75=b$ Take $\log_2$ on each side

$\frac{\log 3.75}{\log 2}=b$ Use Change base formula

$1.9=b$ Use calculator

Therefore,

$b=1.9$.

Step 3: To find a, substitute 1.9 for b in $a=\frac{4}{3^b}$, then,

$$a=\frac{4}{3^b}$$
$$=\frac{4}{3^{1.9}}$$
$$=0.49$$

Hence, the power function equation is,

$$\boxed{y=(0.5)\cdot x^{1.9}}.$$

Answer 6mr.

It is given that the $4000 is invested in an account that pays 2% annual interest compounded continuously.

We need to determine the nearest year which pays $1000 interest.

Consider the number of year that gives $1000 interest is n.

If P amount is invested at compound rate of interest r for n years then the total amount A after n years is given by:

$$A=P\left(1+\frac{r}{100}\right)^n \qquad \text{...... (1)}$$

Here $P=\$4000$, $r=2\%$ and $A=\$4000+\$1000=\$5000$

Substituting the values in (1), we have

$$5000=4000\left(1+\frac{2}{100}\right)^n$$

$$4000\left(1+\frac{2}{100}\right)^n=5000$$

$$\left(1+\frac{2}{100}\right)^n=\frac{5000}{4000} \qquad \text{[Dividing both sides by 4000]}$$

$$(1+0.02)^n=\frac{5}{4}$$

$$(1.02)^n=1.25$$

$$\ln 1.02^n=\ln 1.25 \qquad \text{[Taking log]}$$

$$n\times 0.0198=0.223$$

$$n=\frac{0.223}{0.0198} \qquad \text{[Dividing both sides by 0.0198]}$$

$$n\approx 11.27$$

Therefore it will take approximately $\boxed{\text{11 years}}$ to earn $1000 interest.

Answer 6q.

Consider the equation,

$$\ln(3x-2)=\ln 6x.$$

Solving the equation:

$$\ln(3x-2)=\ln 6x$$

$3x-2=6x$	Since, if $\log_b M=\log_b N$, then, $M=N$
$3x=-2$	Subtract $3x$ from both sides
$x=-2/3$	Divide both sides by 3.

Check the solution by substituting it into the original equation.

$$\ln(3x-2)=\ln 6x$$

$\ln\left[3\left(-\frac{2}{3}\right)-2\right]-5\stackrel{?}{=}\ln 6\left(-\frac{2}{3}\right)$	Substitute $-\frac{2}{3}$ for x
$\ln(-2-2)-5\stackrel{?}{=}\ln 2(-2)$	Simplify
$\ln(-4)=\ln(-4)$	TRUE

Therefore, the solution is,

$$\boxed{-2/3}.$$

Answer 7e.

STEP 1 Replace x with 1, and y with 2 in $y=ab^x$ to obtain the first equation.

$2=ab^1$

Now, substitute 3 for x, and 50 for y in the above function to obtain the second equation.

$50=ab^3$

STEP 2 Solve the first equation for a.

Divide both the sides by b^1.

$$\frac{2}{b^1}=\frac{ab^1}{b^1}$$

$$\frac{2}{b}=a$$

Replace a with $\frac{2}{b}$ in the second equation and simplify.

$$50 = \left(\frac{2}{b}\right)b^3$$
$$= 2b^2$$

Divide both the sides by 2.

$$\frac{50}{2} = \frac{2b^2}{2}$$
$$25 = b^2$$

Take the positive square root of both the sides as $b > 0$.

$$\sqrt{25} = \sqrt{b^2}$$
$$5 = b$$

STEP 3 Substitute 5 for b in $\frac{2}{b} = a$ and simplify.

$$\frac{2}{5} = a$$

Replace a with $\frac{2}{5}$, and b with 2 in $y = ab^x$.

$$y = \frac{2}{5} \cdot 5^x$$

Therefore, the exponential function is $y = \frac{2}{5} \cdot 5^x$.

Answer 7gp.

STEP 1 Replace x with 5, and y with 8 in $y = ax^b$ to obtain the first equation.

$$8 = a \cdot 5^b$$

Substitute 10 for x, and 34 for y in the above function to obtain the second equation.

$$34 = a \cdot 10^b$$

STEP 2 Solve the first equation for a.

Divide both the sides by 5^b and simplify.

$$\frac{8}{5^b} = \frac{a \cdot 5^b}{5^b}$$
$$= a$$

Replace a with $\frac{8}{5^b}$ in the second equation and simplify.

$$34 = \left(\frac{8}{5^b}\right)10^b$$
$$= 8 \cdot 2^b$$

Divide both the sides by 8.

$$\frac{34}{8} = \frac{8 \cdot 2^b}{8}$$
$$4.25 = 2^b$$

Take $\log_2$ of each side.

$\log_2 4.25 = b$

Apply the change-of-base formula and simplify.

$$\frac{\log 4.25}{\log 2} = b$$
$$2.09 \approx b$$

STEP 3 Substitute 2.09 for b in $\frac{8}{5^b} = a$ and simplify.

$$\frac{8}{5^{2.09}} = a$$
$$0.277 \approx a$$

Replace a with 0.277, and b with 2.09 in $y = ax^b$.

$y = 0.277x^{2.09}$

Therefore, the exponential function is $y = 0.277x^{2.09}$.

Answer 7q.

Exponentiate each side with base 3.

$$3^{\log_3 x} = 3^{-1}$$

We know that by the inverse property of logarithms, $b^{\log_b x} = x$. Thus,

$x = 3^{-1}$

Use a calculator to evaluate.

$$x = \frac{1}{3}$$

CHECK

Substitute $\frac{1}{3}$ for x in the original equation and check the solution.

$$\log_3 x = -1$$

$$\log_3\left(\frac{1}{3}\right) \stackrel{?}{=} -1$$

$$\frac{\log \frac{1}{3}}{\log 3} \stackrel{?}{=} -1$$

$$-1 \approx -1 \quad \checkmark$$

The solution is $\frac{1}{3}$.

Answer 8e.

Consider the exponential function $y = ab^x$, whose graph is passes through the points, $(1, 40)$ and $(3, 640)$.

Step 1: Substitute the coordinate of the two given points $(1, 40)$ and $(3, 640)$ into $y = ab^x$.

$40 = ab^1$ Substitute 40 for y and 1 for x

$640 = ab^3$ Substitute 640 for y and 3 for x

Step 2: Solve for a in the first equation as follows,

$40 = ab^1$

$a = \frac{40}{b}$. Divide both sides by b

Next substitute this expression for a in the second equation.

$640 = ab^3$

$640 = \left(\frac{40}{b}\right)b^3$ Substitute $a = \frac{40}{b}$

$640 = 40b^2$ Simplify

$16 = b^2$ Divide both sides by 40.

$4 = b$

Therefore,

$b = 4$.

Step 3: To find a, substitute 4 for b in $a = \frac{40}{b}$, then,

$$\begin{aligned} a &= \frac{40}{b} \\ &= \frac{40}{4} \\ &= 10 \end{aligned}$$

Hence, the exponential equation is,

$$\boxed{y = 10 \cdot (4)^x}.$$

Answer 8gp.

Consider the power function $y = ax^b$, whose graph is passes through the points,

$(3,5)$ and $(3,7)$.

Step 1: Substitute the coordinate of the two given points $(3,5)$ and $(3,7)$ into,

$y = ax^b$.

$5 = a \cdot 3^b$ Substitute 5 for y and 3 for x

$7 = a \cdot 3^b$ Substitute 7 for y and 3 for x

Step 2: Solve for a in the first equation as follows,

$5 = a \cdot 3^b$

$a = \frac{5}{3^b}$. Divide both sides by 3^b

Next substitute this expression for a in the second equation.

$7 = a \cdot 3^b$

$7 = \left(\frac{5}{3^b}\right)3^b$ Substitute $a = \frac{5}{3^b}$

$7 = 5$ Simplify

Since, $7 = 5$ so, we cannot find constants a and b.

Thus the conclusion is we cannot find a power function whose graph passes through $(3,5)$ and $(3,7)$

Answer 8q.

Consider the equation,

$6 \ln x = 30$.

Solving the equation:

$6 \ln x = 30$

$\ln x^6 = 30$ Use Power property; $\log_b m^n = n \log_b m$

$x^6 = e^{30}$ Exponentiate both sides using base e

$x = e^5$ Apply sixth root on both sides

$x = 148.4$ Use calculator

Check the solution by substituting it into the original equation.

$$6\ln x = 30$$

$6\ln 148.4 \overset{?}{=} 30$	Substitute 148.4 for x
$\ln(148.4)^6 \overset{?}{=} 30$	Use Power property; $\log_b m^n = n\log_b m$
$30 = 30$	TRUE

Therefore, the solution is $\boxed{148.4}$.

Answer 9e.

STEP 1 Replace x with -1, and y with 10 in $y = ab^x$ to obtain the first equation.

$$10 = ab^{-1}$$

Substitute 4 for x, and 0.31 for y in the above function to obtain the second equation.

$$0.31 = ab^4$$

STEP 2 Solve the first equation for a.

Divide both the sides by b^{-1} and simplify.

$$\frac{10}{b^{-1}} = \frac{ab^{-1}}{b^{-1}}$$

$$\frac{10}{b^{-1}} = a$$

$$10b = a$$

Replace a with $10b$ in the second equation and simplify.

$$0.31 = (10b)b^4$$

$$= 10b^5$$

Divide both the sides by 10.

$$\frac{0.31}{10} = \frac{10b^5}{10}$$

$$0.031 = b^5$$

Take fifth root of both the sides.

$$\sqrt[5]{0.031} = \sqrt[5]{b^5}$$

$$0.499 = b$$

STEP 3 Substitute 0.499 for b in $10b = a$ and simplify.

$$10 \cdot 0.499 = a$$

$$4.99 = a$$

Replace a with 4.99, and b with 0.499 in $y = ab^x$.

$y = 4.99 \cdot 0.499^x$

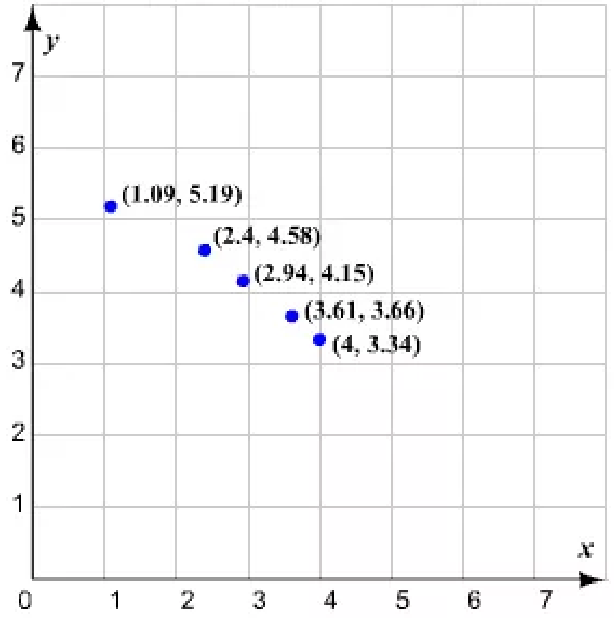
Therefore, the exponential function is $y = 4.99 \cdot 0.499^x$.

Answer 9gp.

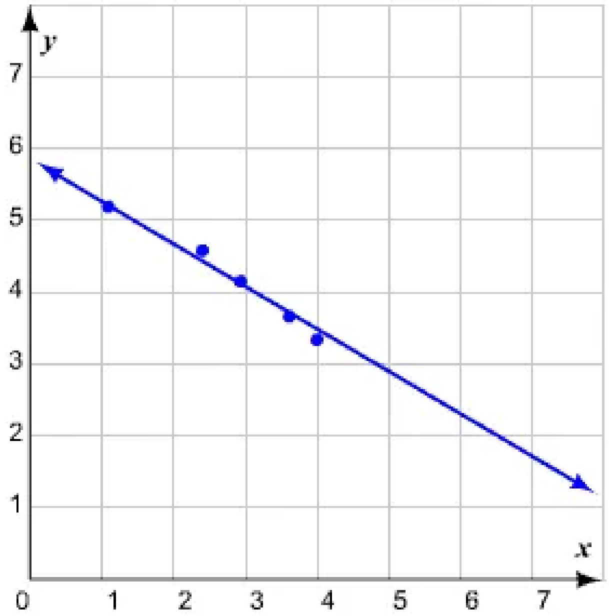
STEP 1 Find the natural logarithm of both x and y values in the given data pairs and organize them in a table.

ln *x*	1.09	2.4	2.94	3.61	4
ln *y*	5.19	4.58	4.15	3.66	3.34

Plot these points to get the scatter plot.



STEP 2 Since the points lie close to a line, a power model will be good fit for the original data.



STEP 3 In order to find a power model, first write an equation for the line using these points and the point-slope form.

We know the point-slope form is $y - y_1 = m(x - x_1)$, where m is the slope. Rewrite the equation by replacing x with $\ln x$ and y with $\ln y$.
$\ln y - y_1 = m(\ln x - x_1)$

Find the slope by choosing two points that lie on the line. Let the points be (1.09, 5.19) and (2.94, 4.15).

$$m = \frac{4.15 - 5.19}{2.94 - 1.09}$$
$$= \frac{-1.04}{1.85}$$
$$= -0.56$$

Substitute the value for m, 1.09 for x_1, and 5.19 for y_1 in $\ln y - y_1 = m(\ln x - x_1)$ and simplify.

$$\ln\ y - 5.19 = -0.56(\ln\ x - 1.09)$$
$$\ln\ y - 5.19 = -0.56 \ln\ x + 0.6104$$

Add 5.19 to both the sides.

$$\ln y - 5.19 + 5.19 = -0.56\ln x + 0.6104 + 5.19$$
$$\ln y = -0.56\ln x + 5.8004$$

Use the power property of logarithms to rewrite the above expression.

$$\ln y = \ln x^{-0.56} + 5.8004$$

Exponentiate each side using base e and simplify.

$$\ln y = e^{\ln x^{-0.56} + 5.8004}$$
$$= e^{5.8004} \cdot e^{\ln x^{-0.56}}$$
$$\approx 330.43\, x^{-0.56}$$

Thus, the power model is $y = 330.43x^{-0.56}$.

Answer 9q.

Exponentiate each side with base 2.

$$2^{\log_2(x+4)} \approx 2^5$$

We know that by the inverse property of logarithms, $b^{\log_b x} = x$. Thus,

$$x + 4 = 2^5$$

Subtract 4 from each side.

$$x + 4 - 4 = 2^5 - 4$$
$$x = 32 - 4$$
$$= 28$$

CHECK

Substitute 28 for x in the original equation and check the solution.

$$\log_2(x+4) = 5$$
$$\log_2(28+4) \stackrel{?}{=} 5$$
$$\log_2 32 \stackrel{?}{=} 5$$
$$\frac{\log 32}{\log 2} \stackrel{?}{=} 5$$
$$5 = 5 \quad \checkmark$$

Thus, the solution is 28.

Answer 10e.

Consider the exponential function $y = ab^x$, whose graph is passes through the points,

$(2, 6.4)$ and $(5, 409.6)$.

Step 1: Substitute the coordinate of the two given points $(2, 6.4)$ and $(5, 409.6)$ into,

$y = ab^x$.

$6.4 = ab^2$ Substitute 6.4 for y and 2 for x

$409.6 = ab^5$ Substitute 409.6 for y and 5 for x

Step 2: Solve for a in the first equation as follows,

$6.4 = ab^2$

$a = \frac{6.4}{b^2}$. Divide both sides by b^2

Next substitute this expression for a in the second equation.

$409.6 = ab^5$

$409.6 = \left(\frac{6.4}{b^2}\right)b^5$ Substitute $a = \frac{6.4}{b^2}$

$409.6 = 6.4b^3$ Simplify

$64 = b^3$ Divide both sides by 6.4.

$4 = b$

Therefore,

$b = 4$.

Step 3: To find a, substitute 4 for b in $a = \frac{6.4}{b^2}$, then,

$$\begin{aligned} a &= \frac{6.4}{b^2} \\ &= \frac{6.4}{4^2} \\ &= \frac{6.4}{16} \\ &= 0.4 \end{aligned}$$

Hence, the exponential equation is,

$\boxed{y = (0.4)\cdot(4)^x}$.

Answer 10q.

Consider the exponential function $y = ab^x$, whose graph is passes through the points, $(1,5)$ and $(2,30)$.

Step 1: Substitute the coordinate of the two given points $(1,5)$ and $(2,30)$ into $y = ab^x$.

$5 = ab^1$ Substitute 5 for y and 1 for x

$30 = ab^2$ Substitute 30 for y and 2 for x

Step 2: Solve for a in the first equation to obtain,

$5 = ab^1$

$a = \frac{5}{b}$. Divide both sides by b

Next substitute this expression for a in the second equation.

$30 = \left(\frac{5}{b}\right)b^2$ Substitute $a = \frac{5}{b}$

$30 = 5b$ Simplify

$6 = b$ Divide both sides by 5.

Therefore,

$b = 6$.

To find a, substitute 6 for b in $a = \frac{5}{b}$, then,

$$a = \frac{5}{b}$$
$$= \frac{5}{6}$$

Hence, the exponential equation is,

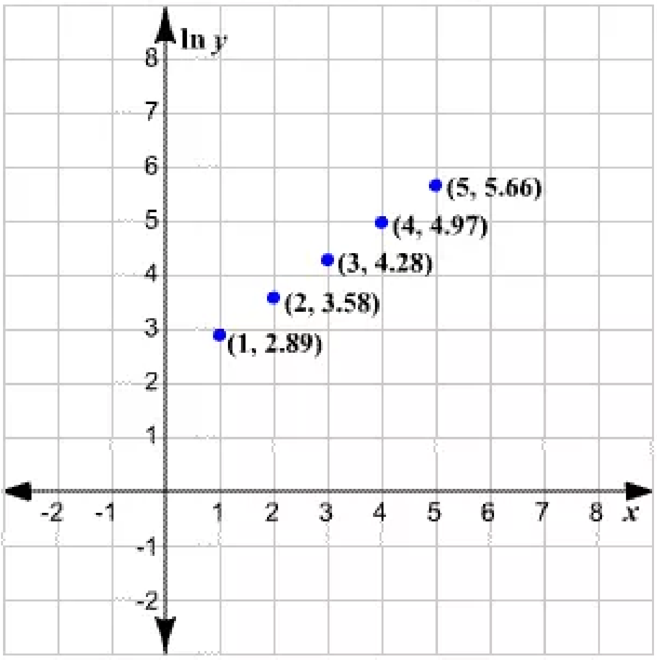
$$\boxed{y = \frac{5}{6} \cdot 6^x}.$$

Answer 11e.

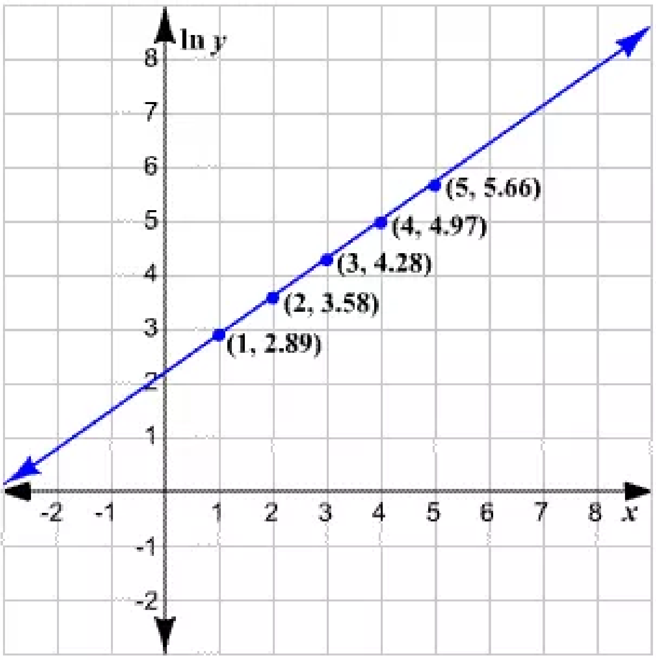
STEP 1 Find the natural logarithm of the y values in the given data pairs and organize them in a table.

x	1	2	3	4	5
$\ln\ y$	2.89	3.58	4.28	4.97	5.66

Plot these points to get the scatter plot.



STEP 2 Since the points lie close to a line, an exponential model will be good fit for the original data.



STEP 3 In order to find an exponential model, first write an equation for the line using these points and the point-slope form.

We know the point-slope form is $y - y_1 = m(x - x_1)$, where m is the slope. Rewrite the equation by replacing y by $\ln y$.

$\ln y - y_1 = m(x - x_1)$

Find the slope by choosing two points that lie on the line. Let the points be (1, 2.89) and (2, 3.58).

$$\frac{3.58 - 2.89}{2 - 1} = 0.69$$

Substitute 0.69 for m, 1 for x_1, and 2.89 for y_1 in $\ln y - y_1 = m(x - x_1)$ and simplify.

$$\begin{aligned} \ln y - 2.89 &= 0.69(x - 1) \\ &= 0.69x - 0.69 + 2.89 \\ &= 0.69x + 2.2 \end{aligned}$$

Exponentiate each side using base e and simplify.

$$\begin{aligned} y &= e^{0.69x + 2.2} \\ &= e^{2.2}\left(e^{0.69}\right)^x \\ &\approx 9(2)^x \end{aligned}$$

Thus, the exponential model is $y = 9(2)^x$.

Answer 11q.

STEP 1 Replace x with 1, and y with 4 in $y = ab^x$ to obtain the first equation.

$4 = ab^1$

Substitute 2 for x, and 32 for y in the above function to obtain the second equation.

$32 = ab^2$

STEP 2 Solve the first equation for a.

Divide both the sides by b^1.

$$\frac{4}{b^1} = \frac{ab^1}{b^1}$$

$$\frac{4}{b} = a$$

Replace a with $\frac{4}{b}$ in the second equation and simplify.

$$32 = \left(\frac{4}{b}\right)b^2$$
$$= 4b$$

Divide both the sides by 4.

$$\frac{32}{4} = \frac{4b}{4}$$
$$8 = b$$

STEP 3 Substitute 8 for b in $\frac{4}{b} = a$ and simplify.

$$\frac{4}{8} = a$$
$$\frac{1}{2} = a$$

Replace a with $\frac{1}{2}$, and b with 8 in $y = ab^x$.

$$y = \frac{1}{2} \cdot 8^x$$

Therefore, the exponential function is $y = \frac{1}{2} \cdot 8^x$.

Answer 12e.

Consider the following the points (x, y):

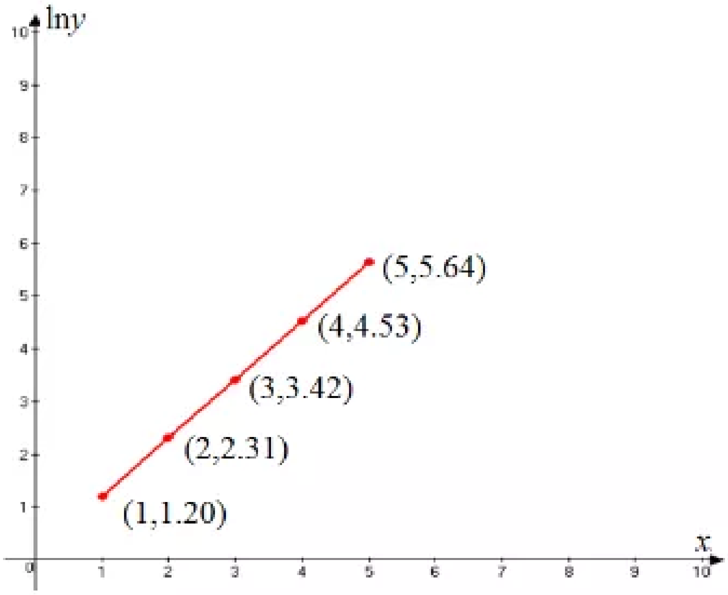
$$(1,3.3),(2,10.1),(3.30.6),(4.92.7),(5,280.9)$$

Now draw a scatter plot of the points $(x, \ln y)$:

Step 1: Use a calculator to create a table of data pairs $(x, \ln y)$.

x	1	2	3	4	5
$\ln y$	1.20	2.31	3.42	4.53	5.64

Step 2: Plot the new points as shown below.
The points lie close to a line, so an exponential model should be a good fit for the original data.



Step 3: Because axes are x and ln$y$, the point-slope form is rewritten as,

$$\ln y - y_1 = m(x - x_1).$$

Next find the slope of the line through $(1,1.20)$ and $(5,5.64)$ as follows,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5.64 - 1.20}{5 - 1} \quad \text{Substitute } 5.64 \text{ for } y_2, 1.20 \text{ for } y_1, 5 \text{ for } x_2, 1 \text{ for } x_1$$

$$\approx 1.11$$

Find the exponential model $y = ab^x$ by choosing a point on the line, such as $(1,1.20)$.
Use this point to write an equation of the line. Then solve for y.

$$\ln y - y_1 = m(x - x_1)$$

$$\ln y - 1.20 = 1.11(x - 1) \quad \text{Substitute 1.20 for } y_1\text{, 1 for } x\text{, and, 1.11 for } m$$

$$\ln y - 1.20 = 1.11x - 1.11 \quad \text{Apply distributive property}$$

$$\ln y = 1.11x + 0.09 \quad \text{Add 1.20 on both sides}$$

$$y = e^{1.11x + 0.09} \quad \text{Exponentiate each side using base } e.$$

$$y = e^{0.09}\left(e^{1.11}\right)^x \quad \text{Use properties of exponents}$$

$$y = 1.09(3.03)^x \quad \text{Exponential model}$$

Thus the exponential model is $\boxed{y = 1.09(3.03)^x}$.

Answer 12q.

Consider the exponential function $y = ab^x$, whose graph is passes through the points,

$(2,15)$ and $(3,45)$.

Step 1: Substitute the coordinate of the two given points $(2,15)$ and $(3,45)$ into $y = ab^x$.

$15 = ab^2$ Substitute 15 for y and 2 for x

$45 = ab^3$ Substitute 45 for y and 3 for x

Step 2: Solve for a in the first equation to obtain,

$15 = ab^2$

$a = \frac{15}{b^2}$. Divide both sides by b^2

Next substitute this expression for a in the second equation.

$45 = \left(\frac{15}{b^2}\right)b^3$ Substitute $a = \frac{15}{b^2}$

$45 = 15b$ Simplify

$3 = b$ Divide both sides by 15.

Therefore,

$b = 3$.

To find a, substitute 3 for b in $a = \frac{15}{b^2}$, then,

$$\begin{aligned} a &= \frac{15}{b^2} \\ &= \frac{15}{3^2} \\ &= \frac{15}{9} \\ &= \frac{5}{3} \end{aligned}$$

Hence, the exponential equation is,

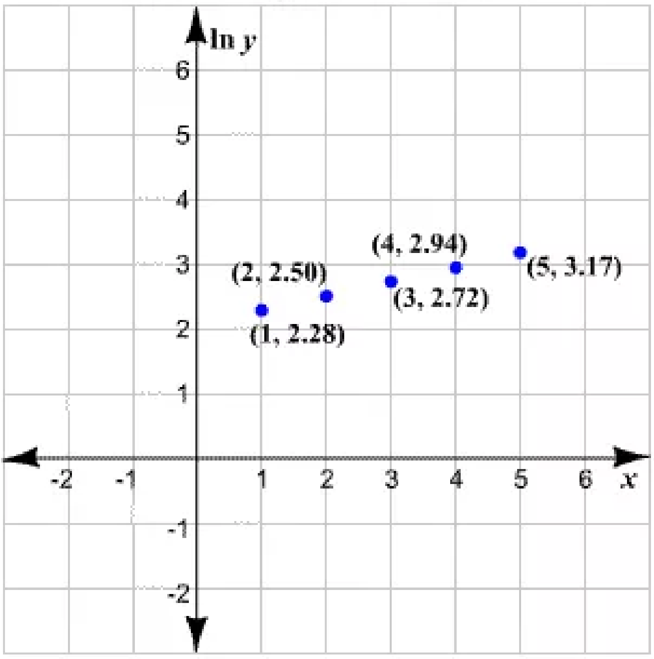
$$\boxed{y = \frac{5}{3} \cdot 3^x}.$$

Answer 13e.

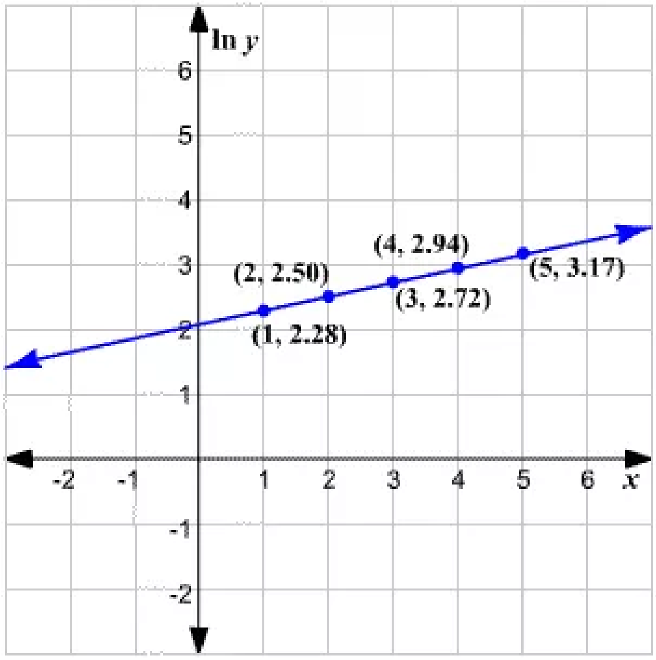
STEP 1 Find the natural logarithm of the y values in the given data pairs and organize them in a table.

x	1	2	3	4	5
$\ln y$	2.28	2.50	2.72	2.94	3.17

Plot these points to get the scatter plot.



STEP 3 Since the points lie close to a line, an exponential model will be good fit for the original data.



STEP 3 In order to find an exponential model, first write an equation for the line using these points and the point-slope form.

We know the point-slope form is $y - y_1 = m(x - x_1)$, where m is the slope.
Rewrite the equation by replacing y by $\ln y$.
$\ln y - y_1 = m(x - x_1)$

Find the slope by choosing two points that lie on the line. Let the points be (1, 2.28) and (2, 2.50).

$$\frac{2.50 - 2.28}{2 - 1} = 0.22$$

Substitute 0.22 for m, 1 for x_1, and 2.28 for y_1 in $\ln y - y_1 = m(x - x_1)$ and simplify.

$$\begin{aligned} \ln y - 2.28 &= 0.22(x - 1) \\ &= 0.22x - 0.22 + 2.28 \\ &= 0.22x + 2.06 \end{aligned}$$

Exponentiate each side using base e and simplify.

$$\begin{aligned} y &= e^{0.22x + 2.06} \\ &= e^{2.06}\left(e^{0.22}\right)^x \\ &\approx 7.83(1.25)^x \end{aligned}$$

Thus, the exponential model is $y = 7.83(1.25)^x$.

Answer 13q.

STEP 1 Replace x with 4, and y with 8 in $y = ax^b$ to obtain the first equation.

$$8 = a \cdot 4^b$$

Substitute 9 for x, and 23 for y in the above function to obtain the second equation.

$$23 = a \cdot 9^b$$

STEP 2 Solve the first equation for a.

Divide both the sides by 4^b and simplify.

$$\begin{aligned} \frac{8}{4^b} &= \frac{a \cdot 4^b}{4^b} \\ &= a \end{aligned}$$

Replace a with $\frac{8}{4^b}$ in the second equation and simplify.

$$\begin{aligned} 23 &= \left(\frac{8}{4^b}\right)9^b \\ &= 8 \cdot 2.25^b \end{aligned}$$

Divide both the sides by 8 and simplify.

$$\frac{23}{8} = \frac{8 \cdot 2.25^b}{8}$$

$$2.875 = 2.25^b$$

Take $\log_{2.25}$ of each side.

$\log_{2.25} 2.875 = b$

Apply the change-of-base formula and simplify.

$$\frac{\log 2.875}{\log 2.25} = b$$

$$1.30 \approx b$$

STEP 3 Substitute 1.30 for b in $\frac{8}{4^b} = a$ and simplify.

$$\frac{8}{4^{1.30}} = a$$

$$1.32 \approx a$$

Replace a with 1.32, and b with 1.30 in $y = ax^b$.

$y = 1.32x^{1.30}$

Therefore, the power function is $y = 1.32x^{1.30}$.

Answer 14e.

Consider the following the points (x, y):

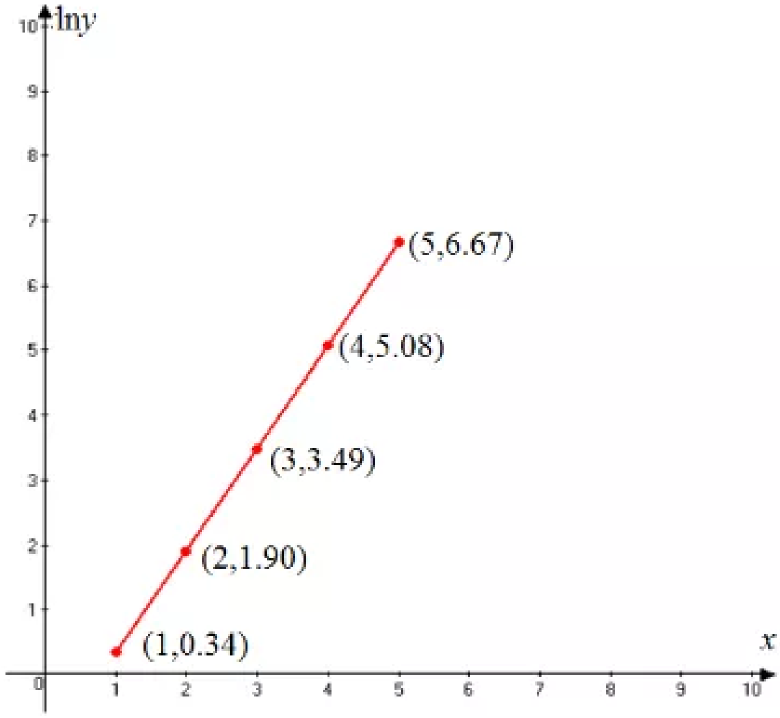
$$(1,1.4),(2,6.7),(3,32.9),(4,161.4),(5,790.9)$$

Now draw a scatter plot of the points $(x, \ln y)$:

Step 1: Use a calculator to create a table of data pairs $(x, \ln y)$.

x	1	2	3	4	5
$\ln y$	0.34	1.90	3.49	5.08	6.67

Step 2: Plot the new points as shown below.
The points lie close to a line, so an exponential model should be a good fit for the original data.



Step 3: Because axes are x and ln$y$, the point-slope form is rewritten as,

$$\ln y - y_1 = m(x - x_1).$$

Next find the slope of the line through $(1, 0.34)$, and, $(5, 6.67)$ is,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{6.67 - 0.34}{5 - 1} \quad \text{Substitute, 6.67 for } y_2, 0.34 \text{ for } y_1, 5 \text{ for } x_2, 1 \text{ for } x_1$$

$$\approx 1.58$$

Find the exponential model $y = ab^x$ by choosing a point on the line, such as $(1, 0.34)$. Use this point to write an equation of the line. Then solve for y.

$\ln y - y_1 = m(x - x_1)$

$\ln y - 0.34 = 1.58(x-1)$ Substitute 0.34 for y_1, 1 for x, and, 1.58 for m

$\ln y - 0.34 = 1.58x - 1.58$ Apply distributive property

$\ln y = 1.58x - 1.24$ Add 0.34 on both sides

$y = e^{1.58x - 1.24}$ Exponentiate each side using base e.

$y = e^{-1.24}\left(e^{1.58}\right)^x$ Use properties of exponents

$y = 0.29(4.85)^x$ Exponential model

Thus, the exponential model is,

$\boxed{y = 0.29(4.85)^x}$.

Answer 14q.

Consider the power function $y = ax^b$, whose graph is passes through the points,

$(3,12)$ and $(10,36)$.

Step 1: Substitute the coordinate of the two given points $(3,12)$ and $(10,36)$ into,

$y = ax^b$.

$12 = a \cdot 3^b$ Substitute 12 for y and 3 for x

$36 = a \cdot 10^b$ Substitute 36 for y and 10 for x

Step 2: Solve for a in the first equation as follows,

$12 = a \cdot 3^b$

$a = \frac{12}{3^b}$. Divide both sides by 3^b

Next substitute this expression for a in the second equation.

$36 = a \cdot 10^b$

$36 = \left(\frac{12}{3^b}\right)10^b$ Substitute $a = \frac{12}{3^b}$

$3 = \frac{10^b}{3^b}$ Divide both sides by 12.

$\log 3 = \log\left(\frac{10^b}{3^b}\right)$ Apply logarithm on both sides

$\log 3 = \log 10^b - \log 3^b$ Use Quotient property, $\log_b \frac{m}{n} = \log_b m - \log_b n$

$\log 3 = b\log 10 - b\log 3$ Use power property, $\log_b m^n = n\log_b m$

$\log 3 = b(\log 10 - \log 3)$ Simplify

$0.4771 = b(0.52289)$ Use calculator

$0.91 = b$ Divide both sides by 0.52289

Therefore,

$b = 0.91$.

Step 3: To find a, substitute 0.91 for b in $a = \frac{12}{3^b}$, then,

$$a = \frac{12}{3^b}$$
$$= \frac{12}{3^{0.91}}$$
$$= 4.4$$

Hence, the power function equation is $\boxed{y = 4.4 \cdot x^{0.91}}$.

Answer 15e.

STEP 1 Replace x with 4, and y with 3 in $y = ax^b$ to obtain the first equation.

$$3 = a \cdot 4^b$$

Substitute 8 for x, and 15 for y in the above function to obtain the second equation.

$$15 = a \cdot 8^b$$

STEP 2 Solve the first equation for a.

Divide both the sides by 4^b and simplify.

$$\frac{3}{4^b} = \frac{a \cdot 4^b}{4^b}$$
$$= a$$

Replace a with $\frac{3}{4^b}$ in the second equation and simplify.

$$15 = \left(\frac{3}{4^b}\right)8^b$$
$$= 3 \cdot 2^b$$

Divide both the sides by 3.

$$\frac{15}{3} = \frac{3 \cdot 2^b}{3}$$
$$5 = 2^b$$

Take $\log_2$ of each side.

$$\log_2 5 = b$$

Apply the change-of-base formula and simplify.

$$\frac{\log 5}{\log 2} = b$$
$$2.32 \approx b$$

STEP 3 Substitute 2.32 for b in $\frac{3}{4^b} = a$ and simplify.

$$\frac{3}{4^{232}} = a$$

$$0.12 \approx a$$

Replace a with 0.12, and b with 2.32 in $y = ax^b$.

$$y = 0.12x^{232}$$

Therefore, the power function is $y = 0.12x^{232}$.

Answer 15q.

STEP 1 Replace x with 5, and y with 4 in $y = ax^b$ to obtain the first equation.

$$4 = a \cdot 5^b$$

Substitute 11 for x, and 51 for y in the above function to obtain the second equation.

$$51 = a \cdot 11^b$$

STEP 2 Solve the first equation for a.

Divide both the sides by 5^b and simplify.

$$\frac{4}{5^b} = \frac{a \cdot 5^b}{5^b}$$

$$= a$$

Replace a with $\frac{4}{5^b}$ in the second equation and simplify.

$$51 = \left(\frac{4}{5^b}\right)11^b$$

$$= 4 \cdot 2.2^b$$

Divide both the sides by 4 and simplify.

$$\frac{51}{4} = \frac{4 \cdot 2.2^b}{4}$$

$$12.75 = 2.2^b$$

Take $\log_{2.2}$ of each side.

$$\log_{2.2} 12.75 = b$$

Apply the change-of-base formula and simplify.

$$\frac{\log 12.75}{\log 2.2} = b$$

$$3.23 \approx b$$

STEP 3 Substitute 3.23 for b in $\frac{4}{5^b} = a$ and simplify.

$$\frac{4}{5^{3.23}} = a$$

$$0.022 \approx a$$

Replace a with 0.022, and b with 3.23 in $y = ax^b$.

$$y = 0.022x^{3.23}$$

Therefore, the power function is $y = 0.022x^{3.23}$

Answer 16e.

Consider the power function $y = ax^b$, whose graph is passes through the points, $(5,9)$ and $(8,34)$.

Step 1: Substitute the coordinate of the two given points $(5,9)$ and $(8,34)$ into, $y = ax^b$.

$9 = a \cdot 5^b$ Substitute 9 for y and 5 for x

$34 = a \cdot 8^b$ Substitute 34 for y and 8 for x

Step 2: Solve for a in the first equation as follows,

$9 = a \cdot 5^b$

$a = \frac{9}{5^b}$. Divide both sides by 5^b

Next substitute this expression for a in the second equation.

$34 = a \cdot 8^b$

$34 = \left(\frac{9}{5^b}\right)8^b$ Substitute $a = \frac{9}{5^b}$

$34 = \frac{9}{5^b} \cdot 5^b (1.6)^b$ Write $8^b = 5^b (1.6)^b$

$34 = 9(1.6)^b$ Simplify

$3.8 = (1.6)^b$ Divide both sides by 9.

$\log_{1.6} 3.8 = b$ Take $\log_{1.6}$ on each side

$\frac{\log 3.8}{\log 1.6} = b$ Use Change base formula

$2.8 = b$ Use calculator

Therefore,

$b = 2.8$.

Step 3: To find a, substitute 2.8 for b in $a=\frac{9}{5^b}$, then,

$$\begin{aligned}a&=\frac{9}{5^b}\\&=\frac{9}{5^{2.8}}\\&=0.9\end{aligned}$$

Hence, the power function equation is,

$$\boxed{y=(0.9)\cdot(x)^{2.8}}.$$

Answer 16q.

Consider the following equation, which gives the average weight y (in kilograms) of an Atlantic cod from the Gulf Maine.

$$y=0.51(1.46)^x \qquad \ldots\ldots (1)$$

Where, x is the age of the cod (in year).

Determine that age of the cod that weights 15 kilograms.

Substitute 15 for y in the equation (1), then,

$$y=0.51(1.46)^x$$

$$15=0.51(1.46)^x \qquad \text{Substitute 15 for } y$$

$$29.41=(1.46)^x \qquad \text{Divide both sides by 0.51}$$

$$\log_{1.46} 29.41=x \qquad \text{Take } \log_{1.46} \text{ on each side}$$

$$\frac{\log 29.41}{\log 1.46}=x \qquad \text{Use the change base formula}$$

$$8.93=x \qquad \text{Use calculator}$$

Therefore, the age of cod is $\boxed{8.93}$ years.

Answer 17e.

STEP 1 Replace x with 2, and y with 3 in $y = ax^b$ to obtain the first equation.

$3 = a \cdot 2^b$

Substitute 6 for x, and 12 for y in the above function to obtain the second equation.

$12 = a \cdot 6^b$

STEP 2 Solve the first equation for a.

Divide both the sides by 2^b and simplify.

$$\frac{3}{2^b} = \frac{a \cdot 2^b}{2^b}$$
$$= a$$

Replace a with $\frac{3}{2^b}$ in the second equation and simplify.

$$12 = \left(\frac{3}{2^b}\right)6^b$$
$$= 3 \cdot 3^b$$

Divide both the sides by 3.

$$\frac{12}{3} = \frac{3 \cdot 3^b}{3}$$
$$4 = 3^b$$

Take $\log_3$ of each side.

$\log_3 4 = b$

Apply the change-of-base formula and simplify.

$$\frac{\log 4}{\log 3} = b$$
$$1.26 \approx b$$

STEP 3 Substitute 1.26 for b in $\frac{3}{2^b} = a$ and simplify.

$$\frac{3}{2^{1.26}} = a$$
$$1.25 \approx a$$

Replace a with 1.25, and b with 2.26 in $y = ax^b$.

$y = 12.5x^{1.26}$

Therefore, the power function is $y = 12.5x^{1.26}$.

Answer 18e.

Consider the power function $y = ax^b$, whose graph is passes through the points,

$(3,14)$ and $(9,44)$.

Step 1: Substitute the coordinate of the two given points $(3,14)$ and $(9,44)$ into,

$y = ax^b$.

$14 = a \cdot 3^b$ Substitute 14 for y and 3 for x

$44 = a \cdot 9^b$ Substitute 44 for y and 9 for x

Step 2: Solve for a in the first equation as follows,

$14 = a \cdot 3^b$

$a = \frac{14}{3^b}$. Divide both sides by 3^b

Next substitute this expression for a in the second equation.

$44 = a \cdot 9^b$

$44 = \left(\frac{14}{3^b}\right)9^b$ Substitute $a = \frac{14}{3^b}$

$44 = \left(\frac{14}{3^b}\right) \cdot \left(3^2\right)^b$ Write $9^b = \left(3^2\right)^b$

$44 = \left(\frac{14}{3^b}\right) \cdot (3)^{2b}$ Since, $\left(a^m\right)^n = a^{mn}$

$44 = 14 \cdot 3^b$ Since, $\frac{a^m}{a^n} = a^{m-n}$

$3.14 = 3^b$ Divide both sides by 14.

$\log_3 3.14 = b$ Take $\log_3$ on each side

$\frac{\log 3.14}{\log 3} = b$ Use Change base formula

$1.04 = b$ Use calculator

Therefore,

$b = 1.04$.

Step 3: To find a, substitute 1.04 for b in $a = \frac{14}{3^b}$, then,

$$
\begin{aligned}
a &= \frac{14}{3^b} \\
&= \frac{14}{3^{1.04}} \\
&= 4.46
\end{aligned}
$$

Hence, the power function equation is,

$\boxed{y = (4.4) \cdot x^{1.04}}$.

Answer 19e.

STEP 1 Replace x with 4 and y with 8 in $y = ax^b$ to obtain the first equation.

$$8 = a \cdot 4^b$$

Substitute 8 for x and 30 for y in the above function to obtain the second equation.

$$30 = a \cdot 8^b$$

STEP 2 Solve the first equation for a.

Divide both the sides by 4^b and simplify.

$$\frac{8}{4^b} = \frac{a \cdot 4^b}{4^b}$$
$$= a$$

Replace a with $\frac{8}{4^b}$ in the second equation and simplify.

$$30 = \left(\frac{8}{4^b}\right)8^b$$
$$= 8 \cdot 2^b$$

Divide both the sides by 8.

$$\frac{30}{8} = \frac{8 \cdot 2^b}{8}$$
$$3.75 = 2^b$$

Take $\log_2$ of each side.

$$\log_2 3.75 = b$$

Apply the change-of-base formula and simplify.

$$\frac{\log 3.75}{\log 2} = b$$
$$1.91 \approx b$$

STEP 3 Substitute 1.91 for b in $\frac{8}{4^b} = a$ and simplify.

$$\frac{8}{4^{1.91}} = a$$
$$0.569 \approx a$$

Replace a with 0.569, and b with 1.91 in $y = ax^b$.

$$y = 0.569x^{1.91}$$

Therefore, the power function is $y = 0.569x^{1.91}$.

Answer 20e.

Consider the power function $y = ax^b$, whose graph is passes through the points,

$(5,10)$ and $(12,81)$.

Step 1: Substitute the coordinate of the two given points $(5,10)$ and $(12,81)$ into,

$y = ax^b$.

$10 = a \cdot 5^b$ Substitute 10 for y and 5 for x

$81 = a \cdot 12^b$ Substitute 81 for y and 12 for x

Step 2: Solve for a in the first equation as follows,

$10 = a \cdot 5^b$

$a = \frac{10}{5^b}$. Divide both sides by 5^b

Next substitute this expression for a in the second equation.

$81 = a \cdot 12^b$

$81 = \left(\frac{10}{5^b}\right) 12^b$ Substitute $a = \frac{10}{5^b}$

$8.1 = \frac{12^b}{5^b}$ Divide both sides by 10

$\log 8.1 = \log\left(\frac{12^b}{5^b}\right)$ Apply logarithm on both sides

$\log 8.1 = \log 12^b - \log 5^b$ Use Quotient property, $\log_b \frac{m}{n} = \log_b m - \log_b n$

$\log 8.1 = b \log 12 - b \log 5$ Use power property, $\log_b m^n = n \log_b m$

$\log 8.1 = b(\log 12 - \log 5)$ Factor out b

$0.9085 = b(0.38025)$ Use calculator

$2.4 = b$ Divide both sides by 0.38025

Therefore,

$b = 2.4$.

Step 3: To find a, substitute 2.4 for b in $a = \frac{10}{5^b}$, then,

$$a = \frac{10}{5^b} = \frac{10}{5^{2.4}} = 0.21$$

Hence, the power function equation is,

$\boxed{y = 0.21 \cdot x^{2.4}}$.

Answer 21e.

STEP 1 Replace x with 4, and y with 6.2 in $y = ax^b$ to obtain the first equation.

$$6.2 = a \cdot 4^b$$

Substitute 7 for x, and 23 for y in the above function to obtain the second equation.

$$23 = a \cdot 7^b$$

STEP 2 Solve the first equation for a.

Divide both the sides by 4^b and simplify.

$$\frac{6.2}{4^b} = \frac{a \cdot 4^b}{4^b}$$
$$= a$$

Replace a with $\frac{6.2}{4^b}$ in the second equation and simplify.

$$23 = \left(\frac{6.2}{4^b}\right)7^b$$
$$= (6.2)\left(1.75^b\right)$$

Divide both the sides by 6.2.

$$\frac{23}{6.2} = \frac{(6.2)\left(1.75^b\right)}{6.2}$$
$$3.71 \approx 1.75^b$$

Take $\log_{1.75}$ of each side.

$$\log_{1.75} 3.71 = b$$

Apply the change-of-base formula and simplify.

$$\frac{\log 3.71}{\log 1.75} = b$$
$$2.34 \approx b$$

STEP 3 Substitute 2.34 for b in $\frac{6.2}{4^b} = a$ and simplify.

$$\frac{6.2}{4^{2.34}} = a$$
$$0.241 \approx a$$

Replace a with 0.241. and b with 2.34 in $y = ax^b$.

$$y = 0.241x^{2.34}$$

Therefore, the power function is $y = 0.241x^{2.34}$.

Answer 22e.

Consider the power function $y = ax^b$, whose graph is passes through the points,

$(3.1, 5)$ and $(6.8, 9.7)$.

Step 1: Substitute the coordinate of the two given points $(3.1, 5)$ and $(6.8, 9.7)$ into,

$y = ax^b$.

$5 = a\cdot(3.1)^b$ Substitute 5 for y and 3.1 for x

$9.7 = a\cdot(6.8)^b$ Substitute 9.7 for y and 6.8 for x

Step 2: Solve for a in the first equation as follows,

$5 = a\cdot(3.1)^b$

$a = \frac{5}{(3.1)^b}$. Divide both sides by $(3.1)^b$

Next substitute this expression for a in the second equation.

$9.7 = a\cdot(6.8)^b$

$9.7 = \left(\frac{5}{(3.1)^b}\right)(6.8)^b$ Substitute $a = \frac{5}{(3.1)^b}$

$1.94 = \frac{(6.8)^b}{(3.1)^b}$ Divide both sides by 5

$\log 1.94 = \log\frac{(6.8)^b}{(3.1)^b}$ Apply logarithm on both sides

$\log 1.94 = \log 6.8^b - \log 3.1^b$ Use Quotient property, $\log_b \frac{m}{n} = \log_b m - \log_b n$

$\log 1.94 = b\log 6.8 - b\log 3.1$ Use power property, $\log_b m^n = n\log_b m$

$\log 1.94 = b(\log 6.8 - \log 3.1)$ Factor out b

$0.28780 = b(0.34144)$ Use calculator

$b = 0.84$

Therefore,

$b = 0.84$.

Step 3: To find a, substitute 0.84 for b in $a = \frac{5}{(3.1)^b}$, then,

$$a = \frac{5}{(3.1)^b} = \frac{5}{(3.1)^{0.84}} = 1.93$$

Hence, the power function equation is,

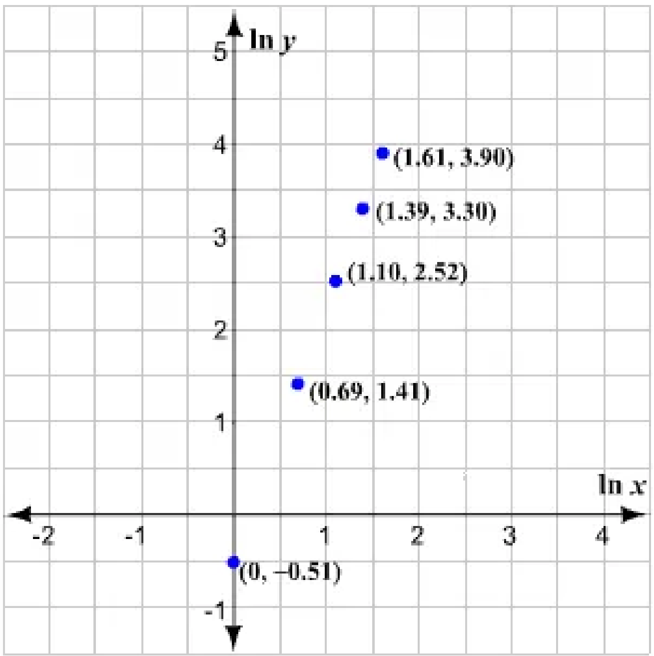
$\boxed{y = (1.93) \cdot x^{0.84}}$.

Answer 23e.

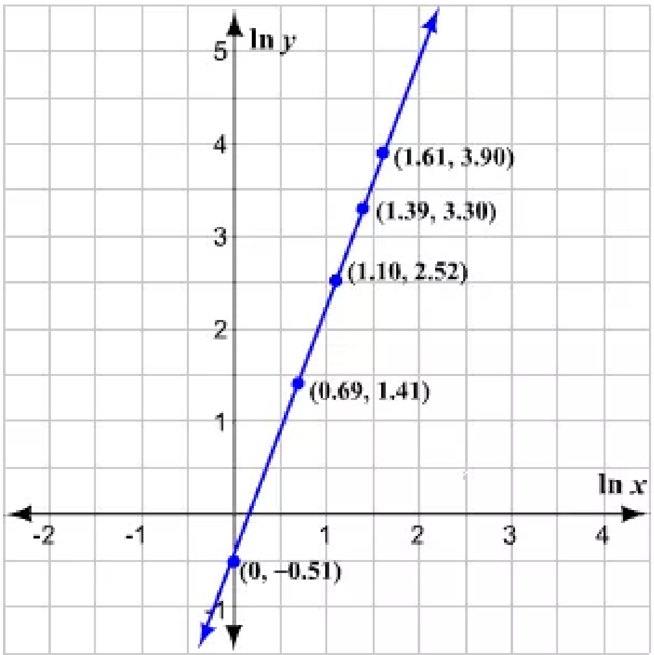
STEP 1 Find the natural logarithm of both x and y values in the given data pairs and organize them in a table.

$\ln x$	0	0.69	1.10	1.39	1.61
$\ln y$	−0.51	1.41	2.52	3.30	3.90

Plot these points to get the scatter plot.



STEP 2 Since the points lie close to a line, a power model will be good fit for the original data.



STEP 3 In order to find a power model, first write an equation for the line using these points and the point-slope form.

We know the point-slope form is $y - y_1 = m(x - x_1)$, where m is the slope. Rewrite the equation by replacing x with $\ln x$ and y with $\ln y$.
$\ln y - y_1 = m(\ln x - x_1)$

Find the slope by choosing two points that lie on the line. Let the points be $(0, -0.51)$ and $(0.69, 1.41)$.

$$\begin{aligned} m &= \frac{1.41 - (-0.51)}{0.69 - 0} \\ &= \frac{1.41 + 0.51}{0.69} \\ &= \frac{1.92}{0.69} \\ &\approx 2.78 \end{aligned}$$

Substitute 2.78 for m, 0 for x_1, and -0.51 for y_1 in $\ln y - y_1 = m(\ln x - x_1)$ and simplify.

$$\begin{aligned} \ln y - (-0.51) &= 2.78(\ln x - 0) \\ \ln y + 0.51 &= 2.78 \ln x \end{aligned}$$

Subtract 0.51 from both the sides.

$$\ln\ y + 0.51 - 0.51 = 2.78 \ln\ x - 0.51$$

$$\ln\ y = 2.78 \ln\ x - 0.51$$

Use the power property of logarithms to rewrite the above expression.

$\ln y = \ln x^{2.78} - 0.51$

Exponentiate each side using base e and simplify.

$$\begin{aligned} y &= e^{\ln x^{2.78} - 0.51} \\ &= e^{-0.51} \cdot e^{\ln x^{2.78}} \\ &\approx 0.606\ x^{2.78} \end{aligned}$$

Thus, the power model is $y = 0.605x^{2.78}$.

Answer 24e.

Consider the following the points (x, y):

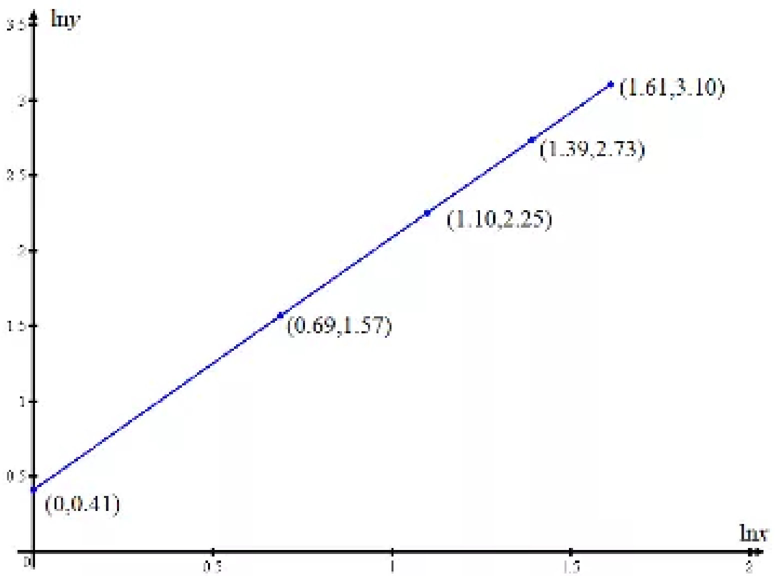
$$(1,1.5),(2,4.8),(3,9.5),(4,15.4),(5,22.3)$$

Now draw a scatter plot of the points $(\ln x, \ln y)$:

Step 1: Use a calculator to create a table of data pairs $(\ln x, \ln y)$.

$\ln x$	0	0.69	1.10	1.39	1.61
$\ln y$	0.41	1.57	2.25	2.73	3.10

Step 2: Plot the new points as shown below:
The points lie close to a line, so a power model should be a good fit for the original data.



Step 3: Because axis are ln$x$, and, ln$y$, the point-slope form is rewritten as,

$$\ln y - y_1 = m(\ln x - x_1).$$

Next find the slope of the line through $(0, 0.41)$, and, $(1.61, 3.10)$ is,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3.10 - 0.41}{1.61 - 0} \quad \text{Substitute, } 3.10 \text{ for } y_2, 0.41 \text{ for } y_1, 1.61 \text{ for } x_2, 0 \text{ for } x_1$$

$$\approx 1.67$$

Find the power model $y = ax^b$ by choosing a point on the line, such as $(0, 0.41)$.

Use these points to write an equation of the line. Then solve for y.

$$\ln y - y_1 = m(\ln x - x_1)$$

$$\ln y - 0.41 = 1.67(\ln x - 0) \quad \text{Substitute } 0.14 \text{ for } y_1, 0 \text{ for } x, \text{ and, } 1.67 \text{ for } m$$

$$\ln y - 0.41 = 1.67 \ln x - 0 \quad \text{Apply distributive property}$$

$$\ln y = \ln x^{1.67} + 0.41 \quad \text{Add } 0.41 \text{ on both sides}$$

$$y = e^{\ln x^{1.67} + 0.41} \quad \text{Exponentiate each side using base } e.$$

$$y = e^{0.41} \cdot e^{\ln x^{1.67}} \quad \text{Use properties of exponents}$$

$$y = 1.51x^{1.67} \quad \text{Simplify}$$

Thus, the power model is,

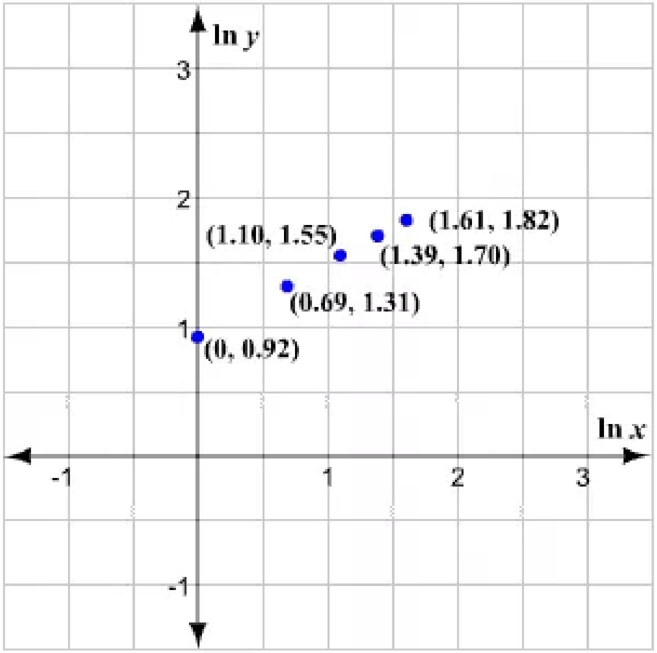
$$\boxed{y = 1.51x^{1.67}}.$$

Answer 25e.

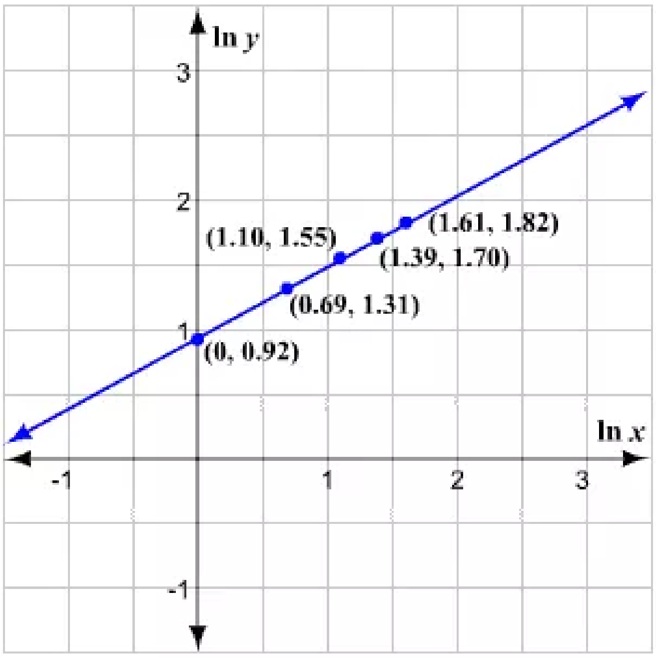
STEP 1 Find the natural logarithm of both x and y values in the given data pairs and organize them in a table.

ln x	0	0.69	1.10	1.39	1.61
ln y	0.92	1.31	1.55	1.70	1.82

Plot these points to get the scatter plot.



STEP 2 Since the points lie close to the line, a power model will be good fit for the original data.



STEP 3 In order to find a power model, first write an equation for the line using these points and the point-slope form.

We know the point-slope form is $y - y_1 = m(x - x_1)$, where m is the slope.
Rewrite the equation by replacing x with $\ln x$, and y with $\ln y$.
$\ln y - y_1 = m(\ln x - x_1)$

Find the slope by choosing two points that lie on the line. Let the points be (0, 0.92) and (1.39, 1.70).

$$m = \frac{1.70 - 0.92}{1.39 - 0}$$
$$= \frac{0.78}{1.39}$$
$$\approx 0.56$$

Substitute 0.39 for m, 0 for x_1, and 0.92 for y_1 in $\ln y - y_1 = m(\ln x - x_1)$ and simplify.

$$\ln y - 0.92 = 0.56(\ln x - 0)$$
$$= 0.56 \ln x$$

Add 0.92 to both the sides.

$$\ln y - 0.92 + 0.92 = 0.56 \ln x + 0.92$$
$$\ln y = 0.56 \ln x + 0.92$$

Use the power property of logarithms to rewrite the above expression.

$\ln y = \ln x^{0.56} + 0.92$

Exponentiate each side using base e and simplify.

$$y = e^{\ln x^{0.56} + 0.92}$$
$$= e^{0.92} \cdot e^{\ln x^{0.56}}$$
$$\approx 2.50\, x^{0.56}$$

Thus, the power model is $y = 2.50x^{0.56}$.

Answer 26e.

Consider the following the points (x, y):

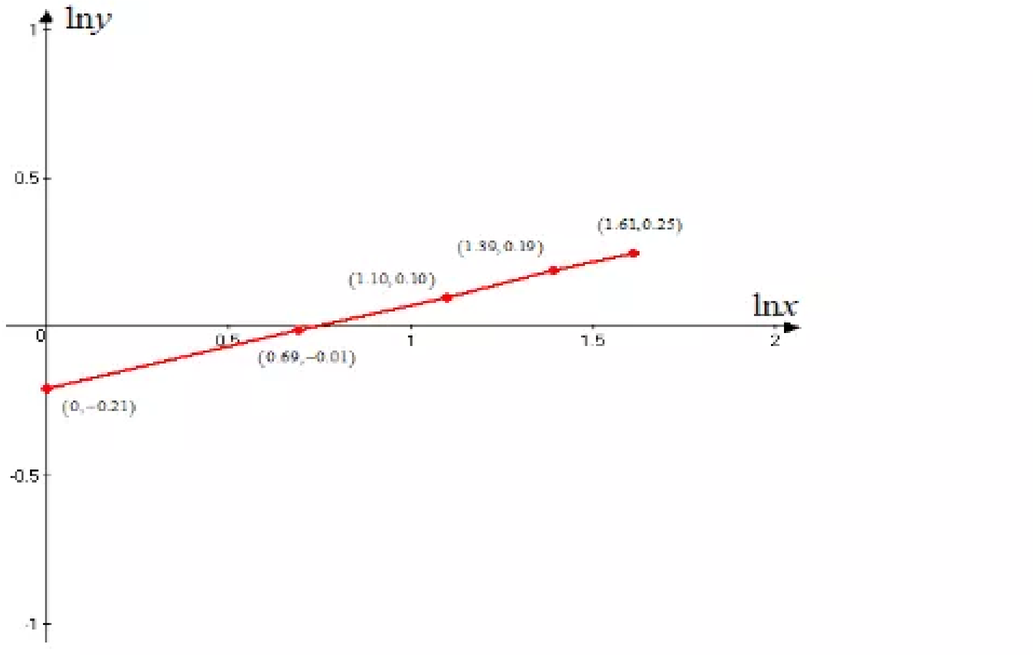
$$(1, 0.81), (2, 0.99), (3, 1.11), (4, 1.21), (5, 1.29)$$

Now draw a scatter plot of the points $(\ln x, \ln y)$:

Step 1: Use a calculator to create a table of data pairs $(\ln x, \ln y)$.

$\ln x$	0	0.69	1.10	1.39	1.61
$\ln y$	-0.21	-0.01	0.10	0.19	0.25

Step 2: Plot the new points as shown below:
The points lie close to a line, so a power model should be a good fit for the original data.



Step 3: Because axis are ln$x$, and, ln$y$, the point-slope form is rewritten as,

$$\ln y - y_1 = m(\ln x - x_1).$$

Next find the slope of the line through $(0, -0.21)$ and $(1.61, 0.25)$ is,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0.25 - (-0.21)}{1.61 - 0} \quad \text{Substitute, 0.25 for } y_2, -0.21 \text{ for } y_1, 1.61 \text{ for } x_2, 0 \text{ for } x_1$$

$$\approx 0.29$$

Find the power model $y = ax^b$ by choosing a point on the line, such as $(0, -0.21)$.

Use these points to write an equation of the line. Then solve for y.

$$\ln y - y_1 = m(\ln x - x_1)$$

$$\ln y - (-0.21) = 0.29(\ln x - 0) \quad \text{Substitute -0.21 for } y_1, \text{ 0 for } x, \text{ and, 0.29 for } m$$

$$\ln y + 0.21 = 0.29 \ln x - 0 \quad \text{Apply distributive property}$$

$$\ln y = \ln x^{0.29} - 0.21 \quad \text{Subtract 0.21 from both sides}$$

$$y = e^{\ln x^{0.29} - 0.21} \quad \text{Exponentiate each side using base } e.$$

$$y = e^{-0.21} \cdot e^{\ln x^{0.29}} \quad \text{Use properties of exponents}$$

$$y = 0.81x^{0.29} \quad \text{Simplify}$$

Thus, the power model is,

$$\boxed{y = 0.81x^{0.29}}.$$

Answer 27e.

Exponentiate each side of the given expression using base 10.

$y = 10^{2x+1}$

Rewrite the above expression using the product of powers property.

$y = (10^{2x})(10)^1$

Use the power of a power property to rewrite the expression.

$$\begin{aligned} y &= \left(10^2\right)^x 10 \\ &= 10(100)^x \end{aligned}$$

Thus, the solution is the equation in choice **A**.

Answer 28e.

Describing and correct the error in writing y as a function of x.

The given solution is,

$$\begin{aligned} \ln y &= 2x+1 \\ y &= e^{2x+1} \\ y &= e^{2x} + e^1 \\ y &= \left(e^2\right)^x + e \\ y &= 7.39^x + 2.72 \end{aligned}$$

In the third step the error is $e^{2x+1} = e^{2x} + e^1$, because from the properties of exponents,

$a^{m+n} = a^m \cdot a^n$.

So, the correct solution is,

$$\begin{aligned} \ln y &= 2x+1 \\ y &= e^{2x+1} \\ y &= e^{2x} \cdot e^1 \\ y &= \left(e^2\right)^x \cdot e \\ y &= (7.39)^x (2.72) \\ y &= (2.72)(7.39)^x \end{aligned}$$

From the law of exponents, $a^{m+n} = a^m \cdot a^n$

Therefore, the correct answer is,

$\boxed{y = (2.72)(7.39)^x}$.

Answer 29e.

According to the power property of logarithms, x should be raised to 3. In the given simplification, x is multiplied by 3 and thus the error is identified.

Rewrite the expression using the power property of logarithms.

$\ln y = \ln x^3 - 2$

Exponentiate each side using base e.

$$y = e^{\ln x^3 - 2}$$

Rewrite the above expression using the product of powers property.

$$y = e^{\ln x^3} \cdot e^{-2}$$

Use the inverse property of logarithms to rewrite the above expression and simplify.

$$y = x^3 \cdot e^{-2}$$
$$= 0.135x^3$$

Therefore the correct solution is $y = 0.135x^3$.

Answer 30e.

Take the natural logarithm of both sides of the equation $y = ab^x$

$$y = ab^x$$

$\ln y = \ln\left(ab^x\right)$	Take the natural logarithm
$\ln y = \ln a + \ln b^x$	Apply, $\ln(ab) = \ln a + \ln b$
$\ln y = \ln a + x \ln b$	Apply, $\ln(M)^r = r \ln M$
$\ln y = (\ln b)x + \ln a$	Rearrange

So, for the exponential form, $\ln b$ is the slope and $\ln a$ is the $\ln y$ intercept.

Take the natural logarithm of both sides of the equations $y = ax^b$

$$y = ax^b$$

$\ln y = \ln\left(ax^b\right)$	Take the natural logarithm
$\ln y = \ln a + \ln x^b$	Apply, $\ln(ab) = \ln a + \ln b$
$\ln y = \ln a + b \ln x$	Apply, $\ln(M)^r = r \ln M$
$\ln y = b(\ln x) + \ln a$	Rearrange

So, if we plot $\ln y$ versus $\ln x$ the slope is b and the $\ln y$ intercept is $\ln a$.

Answer 31e.

Consider the following table, which shows the femur circumference C (in millimeters) and the weight W (in kilograms) for several animals.
Scientists use the circumference of an animal's femur to estimate the animal's weight.

Animal	Giraffe	Polar bear	Lion	Squirrel	Otter
C(mm)	173	135	93.5	13	28
W(kg)	710	448	143	0.399	9.68

Table1

(a)

Use a calculator to create a table of data pairs $(\ln C, \ln W)$.

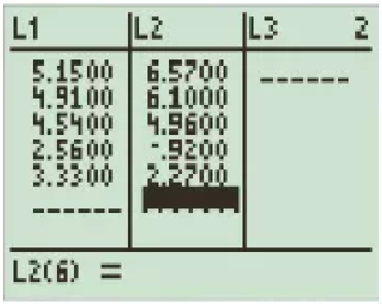
Animal	Giraffe	Polar bear	Lion	Squirrel	Otter
$\ln C$	5.15	4.91	4.54	2.56	3.33
$\ln W$	6.57	6.10	4.96	-0.92	2.27

Table 2

Enter the data points,

$(5.15, 6.57), (4.91, 6.10), (4.54, 4.96), (2.56, -0.92)$, and, $(3.33, 2.27)$, into the graphing calculator as follows:

Step 1: Press STAT and select edit option by pressing 1. Now enter each ln*C*, in the list L1, followed by ENTER , and, enter the corresponding each weight ln*W*, in the List L2, followed by ENTER . Then it will be displayed as follows:



Step 2: Next press WINDOW , and set the window as Xmin=0, Xmax=8, Xscl=2, Ymin=-2, Ymax=8, and Yscl=2.

Then it will be displayed as follows.

Step 3: First set the STAT PLOT on as press [2nd] [Y=] and then select PLOTS 1 by pressing the key 1.
And set the PLOTS ON option and the Type option as shown in below figure. Next press [GRAPH] , to graph the scatter plot.
Then it will be displayed as follows,

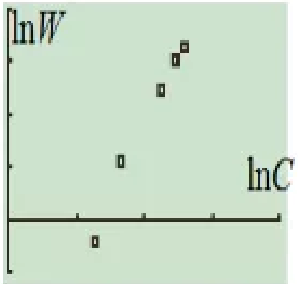
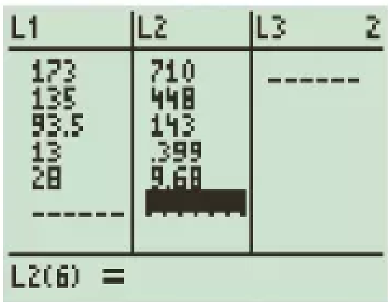


Figure 1

A scatterplot of the Table2 data is shown in the Figure1.

(b)
Now find the Power model for the original data.
Enter the data points, $(173,710)$, $(135,448)$, $(93.5,143)$, $(13,0.399)$, and, $(28,9.68)$, into the graphing calculator as follows:

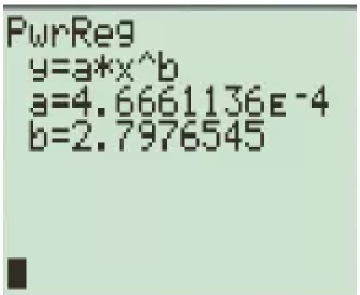
Step 1: Press [STAT] and select edit option by pressing 1. Now enter each circumference C, in the list L1, followed by [ENTER] , and, enter the corresponding each weight W, in the List L2, followed by [ENTER] .
Then it will be displayed as follows:



Step 2: To find the Power model, $W = aC^b$ use the PwrReg command.

Press STAT and then select the CALC Option by pressing the right arrow key. Then, from the displayed menu select the PwrReg by pressing arrow keys and press ENTER .
Then it will be displayed as follows:

Finally press ENTER 2times. Then it will be displayed as follows,



The above Figure show that, the values of $a \approx 0.000466$, and, $b \approx 2.80$, are obtained from Power regression, where $W = aC^b$.
Therefore, the Power model for the original data is,

$$\boxed{W = 0.000466C^{2.80}}.$$

c)
Now find the weight of a cheetah if the circumference of its femur is 68.7 millimeters.
Substitute 68.7 for C into the model $W = 0.000466C^{2.80}$, then,

$$\begin{aligned} W &= 0.000466C^{2.80} \\ &= 0.000466(68.7)^{2.80} \\ &\approx 64.84 \end{aligned}$$

Thus, the weight of a cheetah is $\boxed{64.84 \text{ kg}}$.

Answer 32e.

Consider the following table, which shows the mean distance x from the sun (in astronomical units) and the period y (in years) of six planets.

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn
x	0.387	0.723	1.000	1.524	5.203	9.539
y	0.241	0.615	1.000	1.881	11.862	29.458

Table1

Now draw a scatter plot of the data pairs $(\ln x, \ln y)$:

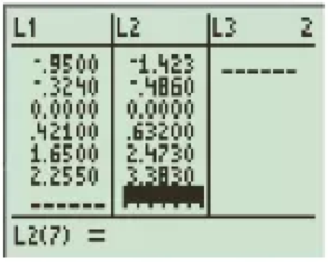
Use a calculator to create a table of data pairs $(\ln x, \ln y)$.

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn
$\ln x$	-0.950	-0.324	0	0.421	1.650	2.255
$\ln y$	-1.423	-0.486	0	0.632	2.473	3.383

Table 2

Enter the data points, $(-0.950,-1.423)$, $(-0.324,-0.486)$, $(0,0)$, $(0.421,0.632)$, $(1.650,2.473)$ and, $(2.255,3.383)$, into the graphing calculator as follows:

Step 1: Press STAT and select edit option by pressing 1. Now enter each lnx, in the list L1, followed by ENTER , and, enter the corresponding each weight lny, in the List L2, followed by ENTER . Then it will be displayed as follows:



Step 2: Next press WINDOW , and set the window as Xmin=-1, Xmax=3, Xscl=1, Ymin=-2, Ymax=4, and Yscl=1. Then it will be displayed as follows.

Step 3: First set the STAT PLOT on as press [2nd] [Y=] and then select PLOTS 1 by pressing the key 1. And set the PLOTS ON option and the Type option as shown in below figure. Next press [GRAPH], to graph the scatter plot. Then it will be displayed as follows,

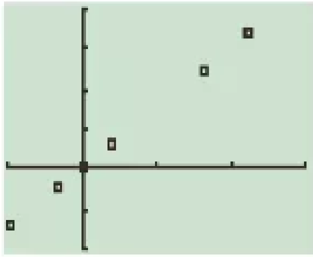
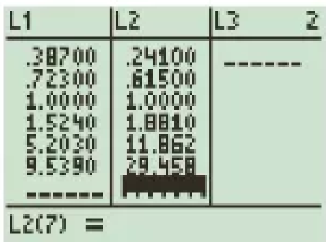


Figure 1

A scatterplot of the Table2 data is shown in the Figure 1.

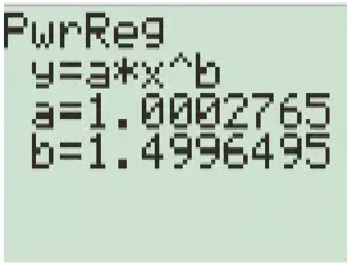
(b) Now find the Power model for the original data.
Enter the data points, $(0.387, 0.241), (0.723, 0.615), (1.000, 1.000)$, $(1.524, 1.881), (5.203, 11.862)$ and, $(9.539, 29.458)$, into the graphing calculator as follows:

Step 1: Press [STAT] and select edit option by pressing 1. Now enter each mean distance *x*, in the list L1, followed by [ENTER], and, enter the corresponding each period *y*, in the List L2, followed by [ENTER]. Then it will be displayed as follows:



Step 2: To find the Power model, $y = ax^b$ use the PwrReg command.
Press [STAT] and then select the CALC Option by pressing the right arrow key. Then, from the displayed menu select the PwrReg by pressing arrow keys and press [ENTER].
Then it will be displayed as follows:

Finally press ENTER 2times. Then it will be displayed as follows,



The above Figure show that, the values of $a \approx 1$, and, $b \approx 1.5$, are obtained from Power regression, where $y = ax^b$.

Therefore, the Power model for the original data is,

$\boxed{y = x^{1.5}}$.

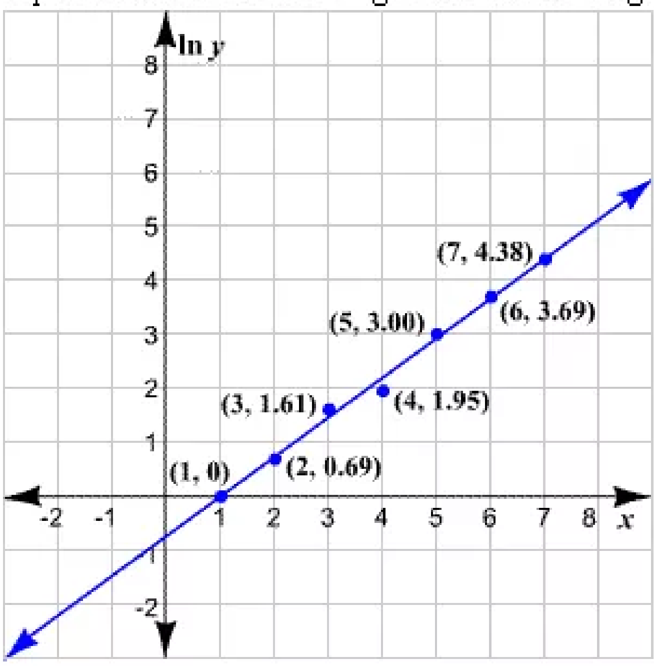
Answer 33e.

(a) Let x represent the number of years since 1997 and y represent the number of business users.

Find the natural logarithm of the y values in the given data and organize them in a table.

x	1	2	3	4	5	6	7
$\ln y$	0	0.69	1.69	1.95	3	3.69	4.38

Plot the above points on a graph. Since the points lie close to the line, an exponential model will be good fit for the original data.



In order to find an exponential model, first write an equation for the line using these points and the point-slope form.

We know the point-slope form is $y - y_1 = m(x - x_1)$, where m is the slope. Rewrite the equation by replacing y by $\ln y$.
$\ln y - y_1 = m(x - x_1)$

Find the slope by choosing two points that lie on the line. Let the points be (1, 0) and (7, 4.38).

$$\frac{4.38 - 0}{7 - 1} = 0.73$$

Substitute 0.73 for m, 1 for x_1, and 0 for y_1 in $\ln y - y_1 = m(x - x_1)$ and simplify.

$$\ln y - 0 = 0.73(x - 1)$$
$$\ln y = 0.73x - 0.73$$

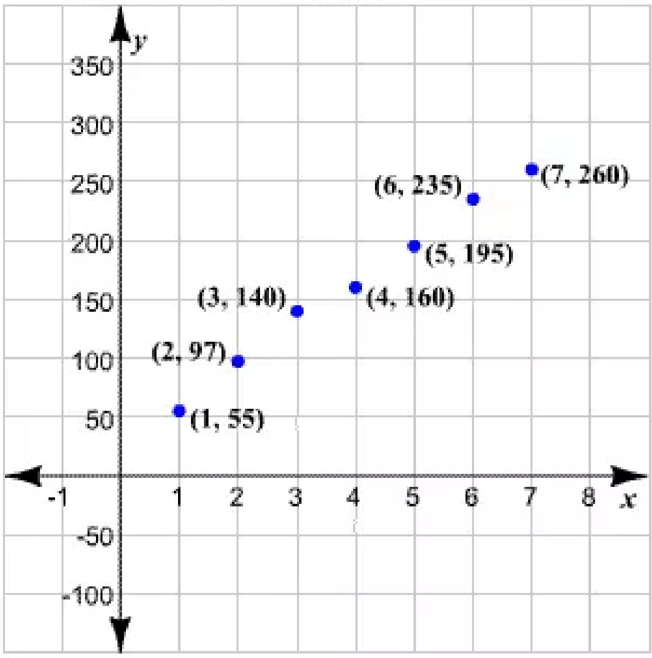
Exponentiate each side using base e and simplify.

$$y = e^{0.73x - 0.73}$$
$$= e^{-0.73}\left(e^{0.73}\right)^x$$
$$\approx 0.48(2.08)^x$$

Thus, the exponential model is $y = 0.48(2.08)^x$.

(b) Let x represent the number of years since 1997 and y represent the number of non-business users.

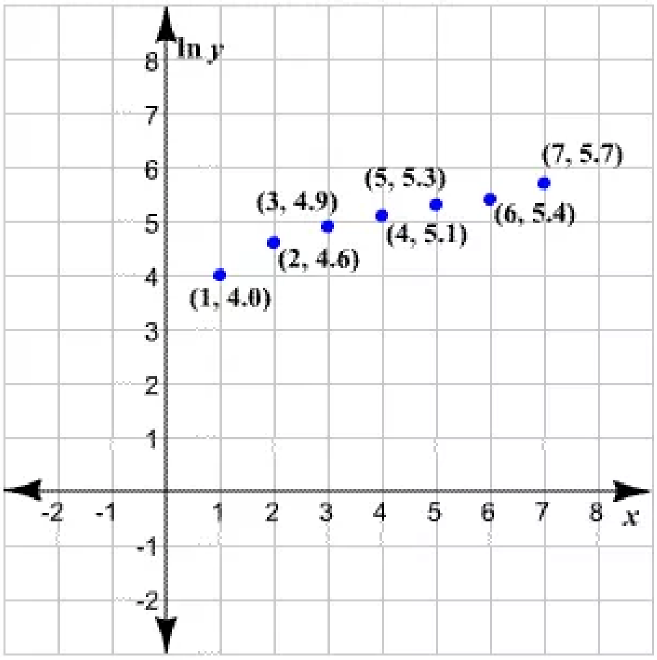
First, graph the points (x, y).



Next, find the natural logarithm of the y values in the given data, and organize them in a table.

x	1	2	3	4	5	6	7
$\ln\ y$	4	4.6	4.9	5.1	5.3	5.5	5.6

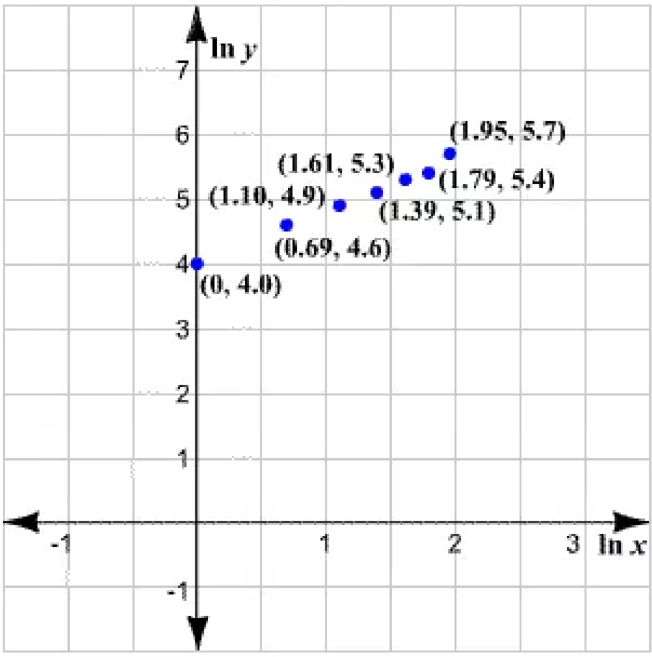
Plot the above points on a graph.



Now, find the natural logarithm of both x and y values in the given data and organize them in a table.

$\ln\ x$	0	0.69	1.10	1.39	1.61	1.79	1.95
$\ln\ y$	4	4.6	4.9	5.1	5.3	5.5	5.6

Plot the above points on a graph.



On comparing the above three graphs we note that most of the points in the first graph appear to lie on the line.

Thus, we can say that linear function is the best fitting model for the given data.

In order to find the linear model, first write an equation for the line using the above points and the point-slope form.

We know the point-slope form is $y - y_1 = m(x - x_1)$, where m is the slope.

Find the slope by choosing two points that appear to lie on the line. Let the points be (1, 55) and (7, 260).

$$\frac{260 - 55}{7 - 1} = 34.2$$

Substitute 34.2 for m, 1 for x_1, and 55 for y_1 in $y - y_1 = m(x - x_1)$ and simplify.

$$\begin{aligned} y - 55 &= 34.2(x - 1) \\ &= 34.2x - 34.2 + 55 \\ &= 34.2x + 20.8 \end{aligned}$$

Therefore, the linear model is $y = 34.2x + 20.8$.

Answer 34e.

Consider the following table, which shows the boiling point T of water (in degree Celsius) for several different values of atmospheric pressure P (in millimeters of mercury).
Here, the boiling point of water increases with atmospheric pressure, also at sea level, where the atmospheric pressure is about 760 millimeters of mercury, water boils at $100^{\circ}C$.

Table 1

P	T
149	60
234	70
355	80
526	90
760	100
1075	110

Use a calculator to create a table of data pairs $(\ln P, \ln T)$

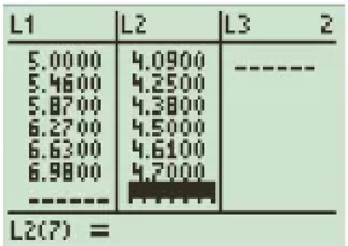
Table 2

$\ln P$	$\ln T$
5.00	4.09
5.46	4.25
5.87	4.38
6.27	4.50
6.63	4.61
6.98	4.70

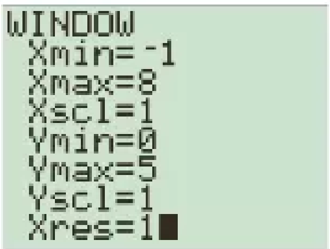
a) Now draw a scatter plot for the data pairs $(\ln P, \ln T)$:

Enter the data points, $(5.00, 4.09), (5.46, 4.25), (5.87, 4.38)$, $(6.27, 4.50), (6.63, 4.61)$ and, $(6.98, 4.70)$, into the graphing calculator as follows:

Step 1: Press STAT and select edit option by pressing 1. Now enter each ln*P*, in the list L1, followed by ENTER , and, enter the corresponding each weight ln*T*, in the List L2, followed by ENTER . Then it will be displayed as follows:



Step 2: Next press WINDOW, and set the window as Xmin=0, Xmax=8, Xscl=1, Ymin=0, Ymax=5, and Yscl=1. Then it will be displayed as follows.



Step 3: First set the STAT PLOT on as press 2nd Y= and then select PLOTS 1 by pressing the key 1. And set the PLOTS ON option and the Type option as shown in below figure. Next press GRAPH, to graph the scatter plot. Then it will be displayed as follows,

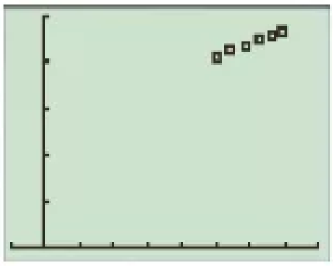
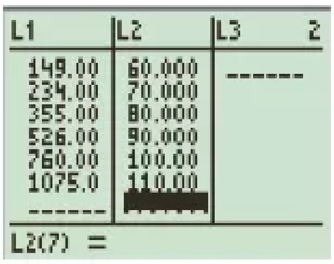


Figure 1

A scatterplot of the Table2 data is shown in the Figure 1.

(b) Now find the Power model for the original data.
Enter the data points, $(149,60),(234,70),(355,80),$
$(526,90),(760,100)$, and, $(1075,110)$, into the graphing calculator as follows:

Step 1: Press STAT and select edit option by pressing 1. Now enter each atmospheric pressure P, in the list L1, followed by ENTER, and, enter the corresponding each boiling point T of water, in the List L2, followed by ENTER. Then it will be displayed as follows:

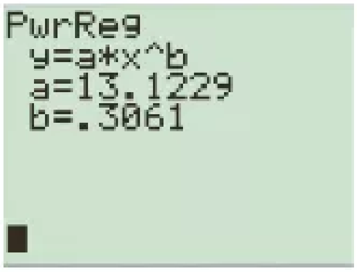


Step 2: To find the Power model, $T = aP^b$ use the PwrReg command.

Press STAT and then select the CALC Option by pressing the right arrow key. Then, from the displayed menu select the PwrReg by pressing arrow keys and press ENTER .

Then it will be displayed as follows:

Finally press ENTER 2times. Then it will be displayed as follows,



The above Figure show that, the values of $a \approx 13.12$, and, $b \approx 0.31$ are obtained from Power regression, where $T = aP^b$.

Therefore, the Power model for the original data is,

$\boxed{T = 13.12P^{0.31}}$.

c)

Now predict the boiling point T of water, when atmospheric pressure P is 620millimeters of mercury.

Substitute 620 for P into the model $T = 13.12P^{0.31}$, then,

$$\begin{aligned} T &= 13.12P^{0.31} \\ &= 13.12(620)^{0.31} \\ &\approx 96.3 \end{aligned}$$

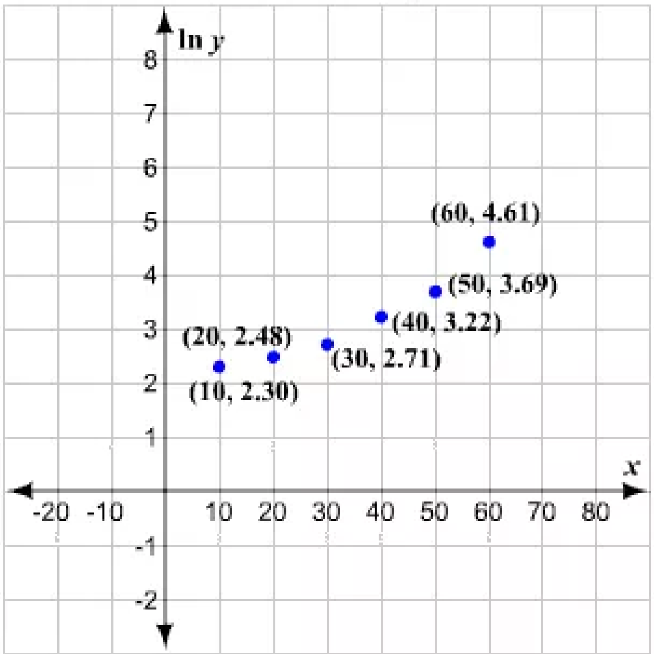
Thus, the temperature is $\boxed{96.3\text{`}C}$.

Answer 35e.

(a) Find the natural logarithm of the y values in the given data and organize them in a table.

x	10	20	30	40	50	60
$\ln\ y$	2.30	2.48	2.71	3.22	3.69	4.61

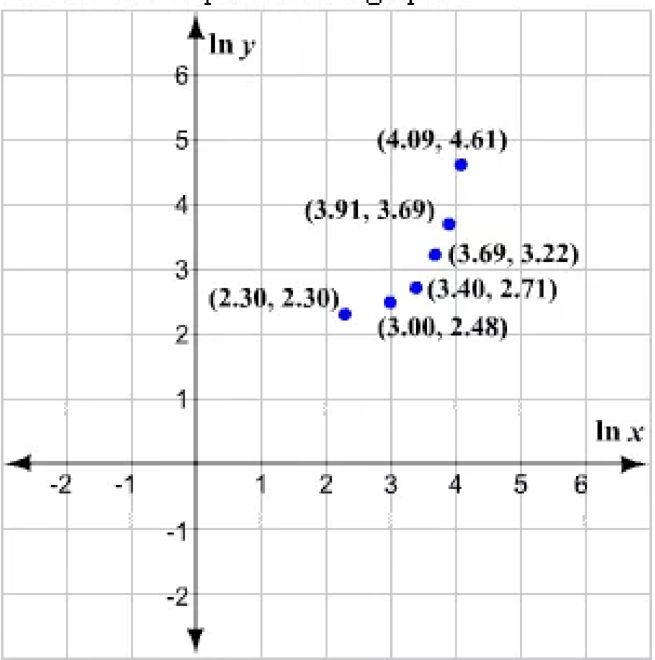
Plot the above points on a graph.



(b) Find the natural logarithm of both x and y values in the given data and organize them in a table.

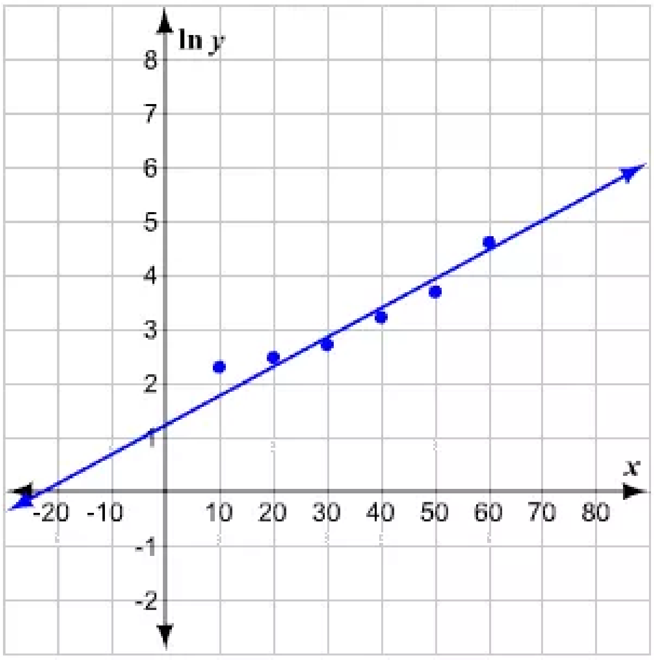
$\ln x$	2.30	3	3.40	3.69	3.91	4.09
$\ln y$	2.30	2.48	2.71	3.21	3.69	4.61

Plot the above points on a graph.



(c) On comparing the above two scatter plots, we note that the points for $(x, \ln y)$ appear more linear than the points for $(\ln x, \ln y)$. Thus, the exponential model appears to be best fit for the given data.

(d) In order to find an exponential model, first, draw a line that best fits the data pairs.



Write an equation for the line using these points and the point-slope form. We know the point-slope form is $y - y_1 = m(x - x_1)$, where m is the slope. Rewrite the equation by replacing y by $\ln y$.

$\ln y - y_1 = m(x - x_1)$

Find the slope by choosing two points that lie on the line. Let the points be (20, 2.48) and (60, 4.61).

$$\frac{4.61 - 2.48}{60 - 20} = 0.054$$

Substitute 0.054 for m, 20 for x_1, and 2.48 for y_1 in $\ln y - y_1 = m(x - x_1)$ and simplify.

$$\begin{aligned} \ln y - 2.48 &= 0.054(x - 20) \\ &= 0.054x - 1.08 + 2.48 \\ &= 0.054x + 1.4 \end{aligned}$$

Exponentiate each side using base e and simplify.

$$\begin{aligned} y &= e^{0.054x+1.4} \\ &= e^{1.4}\left(e^{0.054}\right)^x \\ &\approx 4.05(1.05)^x \end{aligned}$$

Thus, the exponential model is $y = 4.05(1.05)^x$.

In order to predict the visual near point for an 80 year old person, replace x with 80 in the above equation.

$$\begin{aligned} y &= 4.05(1.05)^{80} \\ &= 4.05 \cdot 49.6 \\ &= 200 \end{aligned}$$

Thus, the visual near point for an 80 year old person is 200 cm.

Answer 36e.

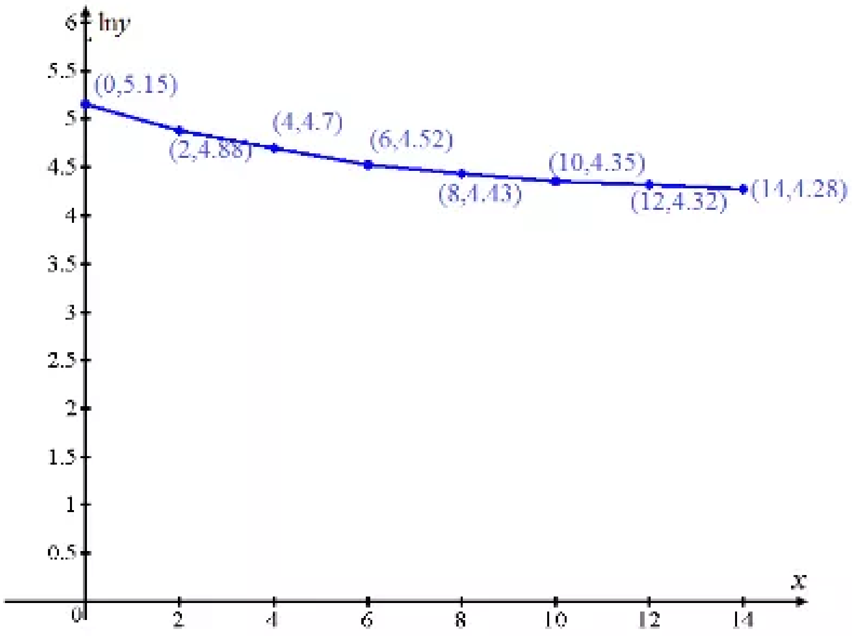
Consider the following table, which shows the results of an astronaut's pulse rate y (in beats per minute) at various times x (in minutes), after the astronaut has finished exercising.

x	0	2	4	6	8	10	12	14
y	172	132	110	92	84	78	75	72

Step 1: Use a calculator to create a table of data pairs $(x, \ln y)$.

x	0	2	4	6	8	10	12	14
$\ln y$	5.15	4.88	4.70	4.52	4.43	4.35	4.32	4.28

Step 2: Plot the new points as shown below. The points lie close to a line, so an exponential model should be a good fit for the original data.



Step 3: Because axes are x and $\ln y$, the point-slope form is rewritten as,

$$\ln y - y_1 = m(x - x_1).$$

Next find the slope of the line through $(0, 5.15)$ and $(14, 4.28)$ as follows:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4.28 - 5.15}{14 - 0} && \text{Substitute } 4.28 \text{ for } y_2, 5.15 \text{ for } y_1, 14 \text{ for } x_2, 0 \text{ for } x_1 \\ &\approx -0.06 \end{aligned}$$

Find the exponential model $y = ab^x$ by choosing a point on the line, such as $(0, 5.15)$.
Use this point to write an equation of the line. Then solve for y.

$$\begin{aligned} \ln y - y_1 &= m(x - x_1) \\ \ln y - 5.15 &= -0.06(x - 0) && \text{Substitute } 5.15 \text{ for } y, \text{and}, 0 \text{ for } x \\ \ln y - 5.15 &= -0.06x - 0 && \text{Apply distributive property} \\ \ln y &= -0.06x + 5.15 && \text{Add 5.15 on both sides} \\ y &= e^{-0.06x + 5.15} && \text{Exponentiate each side using base } e. \\ y &= e^{5.15}\left(e^{-0.06}\right)^x && \text{Use properties of exponents} \\ y &= 172.43(0.94)^x && \text{Exponential model} \end{aligned}$$

Thus the exponential model is $\boxed{y = 172.43(0.94)^x}$.

Answer 37e.

The direct variation equation for the variables x and y is $y = ax$.

In order to find the value of a, substitute for x and y.

$48 = a(6)$

Divide each term by 6 to solve for a.

$$\frac{48}{6} = \frac{a(6)}{6}$$

$$8 = a$$

Replace a with 8 in $y = ax$.

$y = 8x$

Thus, the direct variation equation that relates the given values is $y = 8x$.

Answer 38e.

Consider that the variables x and y are vary directly, and,

$x = -7$ and $y = 28$

Now write an equation that relates x and y.

The equation $y = ax$ represents Direct variation between x and y.

So, substitute $x = -7$ and $y = 28$ in $y = ax$, then,

$$y = ax$$

$$28 = a(-7) \quad \text{Substitute 28 for } y\text{, and, } -7 \text{ for } x$$

$$\frac{28}{-7} = \frac{-7a}{-7} \quad \text{Divide both sides by } -7$$

$$-4 = a$$

Now substitute -4 for a in the equation $y = ax$, then,

$$y = ax$$

$$y = -4x$$

Therefore, the equation that relates x and y is $\boxed{y = -4x}$.

Answer 39e.

The direct variation equation for the variables x and y is $y = ax$.

In order to find the value of a, substitute the given values for x and y.

$6 = a(10)$

Divide each term by 10 to solve for a.

$$\frac{6}{10} = \frac{a(10)}{10}$$

$$0.6 = a$$

Replace a with 0.6 in $y = ax$.

$y = 0.6x$

Thus, the direct variation equation that relates the given values is $y = 0.6x$.

Answer 40e.

Consider that the variables x and y are vary directly, and,

$x = 35$ and $y = 15$

Now write an equation that relates x and y.

The equation $y = ax$ represents Direct variation between x and y.

So, substitute $x = 35$ and $y = 15$ in $y = ax$, then,

$$y = ax$$

$$15 = a(35)$$ Substitute 15 for y, and, 35 for x

$$\frac{15}{35} = \frac{35a}{35}$$ Divide both sides by 35

$$\frac{3}{7} = a$$

Now substitute $\frac{3}{7}$ for a in the equation $y = ax$, then,

$$y = ax$$

$$y = \frac{3}{7}x$$

Therefore, the equation that relates x and y is $\boxed{y = \frac{3}{7}x}$.

Answer 41e.

The direct variation equation for the variables x and y is $y = ax$. In order to find the value of a, substitute the given values for x and y.

$1.2 = a(0.3)$

Divide each term by 0.3 to solve for a.

$$\frac{1.2}{0.3} = \frac{a(0.3)}{0.3}$$

$$4 = a$$

Replace a with 4 in $y = ax$.

$y = 4x$

Thus, the direct variation equation that relates the given values is $y = 4x$.

Answer 42e.

Consider that the variables x and y are vary directly, and,

$x = 12$ and $y = 15$

Now write an equation that relates x and y.

The equation $y = ax$ represents Direct variation between x and y.

So, substitute $x = 12$ and $y = 15$ in $y = ax$, then,

$$y = ax$$

$$15 = a(12) \quad \text{Substitute 15 for } y\text{, and, 12 for } x$$

$$\frac{15}{12} = \frac{12a}{12} \quad \text{Divide both sides by 12}$$

$$\frac{5}{4} = a$$

Now substitute $\frac{5}{4}$ for a in the equation $y = ax$, then,

$$y = ax$$

$$y = \frac{5}{4}x$$

Therefore, the equation that relates x and y is $\boxed{y = \frac{5}{4}x}$.

Answer 43e.

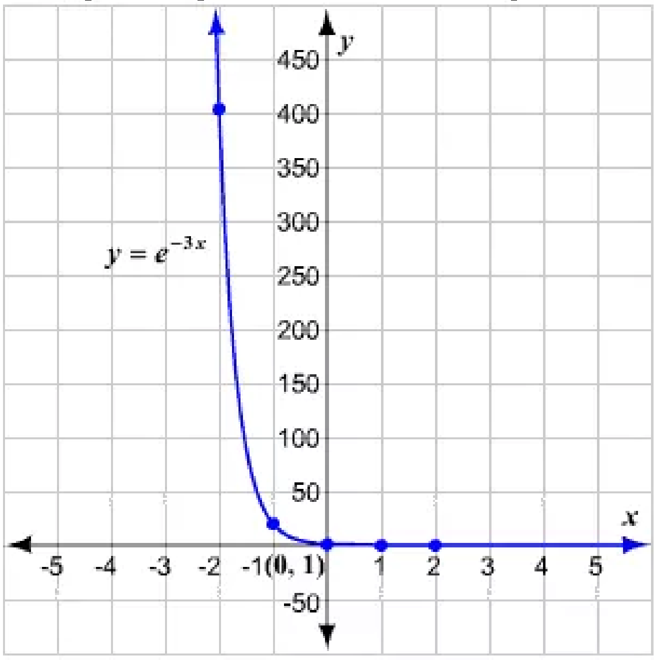
First, we have to find some points on the graph. Choose any value for x, say, -2 and find the corresponding value of y.

$$\begin{aligned} y &= e^{-3(-2)} \\ &= e^{6} \\ &\approx 403.43 \end{aligned}$$

Organize the results in a table.

x	-2	-1	0	1	2
y	403.43	20.09	1	0.05	0.002

Now, plot the points on a coordinate plane and connect them with a smooth curve.



The domain of a function is the set of all input values and the range is the set of all output values.

From the figure, we can find that the input values include all real numbers whereas the output values include only positive real numbers. Therefore, the domain is the set of all real numbers and the range is $y > 0$.

Answer 44e.

Consider the following function,

$$y = 4e^x.$$

Now graph the natural base function as follows:

The function $y = 4e^x$ is in the form $y = ae^{rx}$.

Here, in this case, $a = 4$ is positive, and, $r = 1$ is positive.

So, the function is an exponential growth function.

Now find two points that satisfy the function $y = 4e^x$.

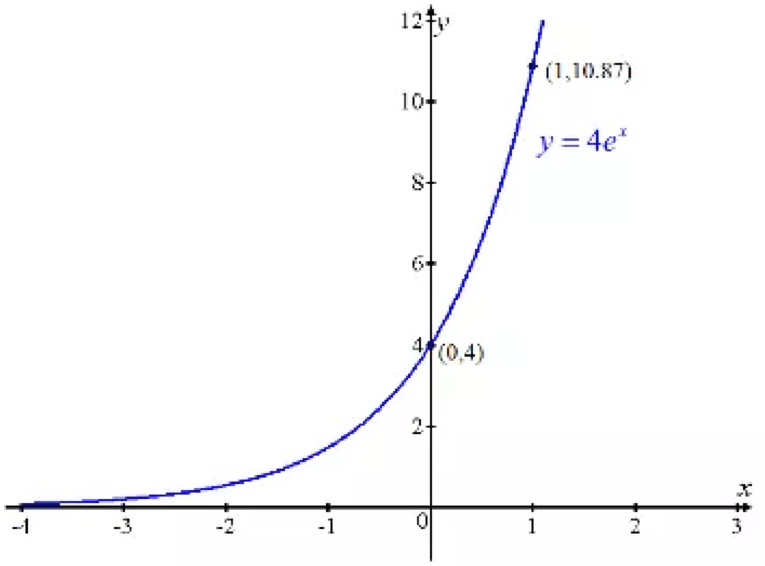
When $x = 0$,

$$y = 4e^x$$
$$y = 4e^0$$
$$= 4$$

When $x = 1$,

$$y = 4e^x$$
$$y = 4e^1$$
$$= 4e$$
$$= 10.87$$

Plot the points $(0,4)$, and, $(1,10.87)$ on the graph and then draw the curve:



It is known that the domain is the set of all input values, and, the range is the set of all output values.
Here, the input values are all x-values, and, the output values are all positive y values.
Form the graph, observe that the domain is all real numbers, and the range is $\boxed{y > 0}$.

Answer 45e.

The graph of the function $y = 2e^{2x} + 1$ is obtained by translating the graph of $y = 2e^{2x}$ one unit up.

For graphing the function $y = 2e^{2x}$, first we have to find some points on the graph. Choose any value for x, say, 1 and find the corresponding value of y.

$$y = 2e^{2(1)}$$
$$\approx 14.77$$

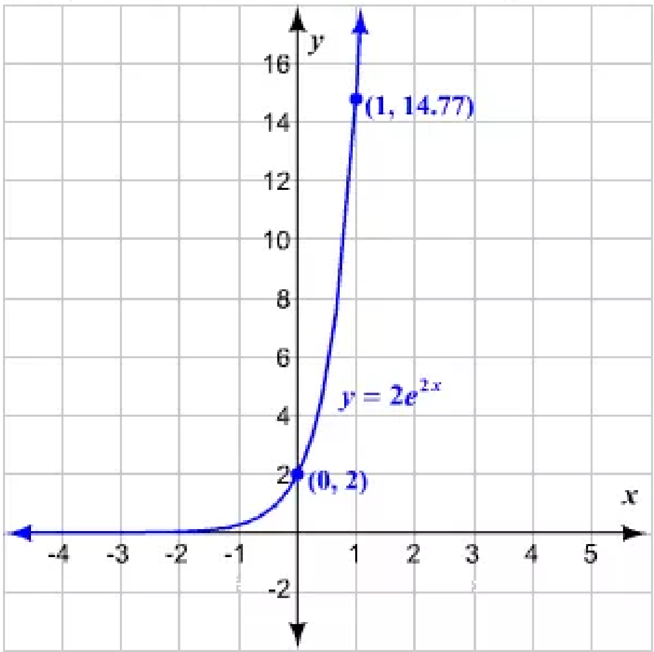
One point on the graph is (1, 14.77).

Put another value for x, say, 0 and find the y-value.

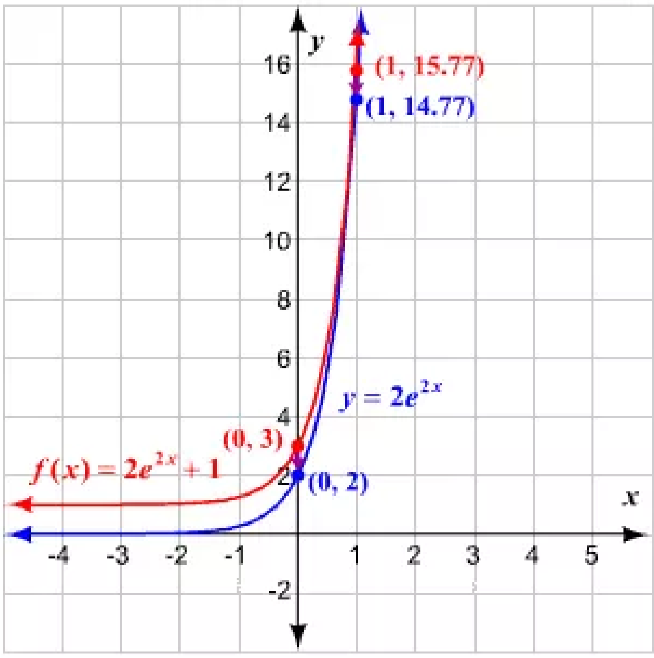
$$y = 2e^{0}$$
$$= 2$$

Another point on the graph is (0, 2).

Now, plot the points on a coordinate plane and connect them with a smooth curve.



Translate the graph of $y = 2e^{2x}$ one unit up to graph $y = 2e^{2x} + 1$.



The domain of a function is the set of all input values and the range is the set of all output values.

From the figure, we can find that the input values include all real numbers whereas the output values include only real numbers greater than 1. Therefore, the domain is the set of all real numbers and the range is $y > 1$.

Answer 46e.

Consider the following function,

$$y = e^{-x} - 4.$$

Now graph the natural base function as follows:

The function $y = e^{-x} - 4$ is in the form $y = ae^{rx}$.

Here, in this case, $a = 4$ is positive, and, and $r = -1$ is negative.

So, the function is an exponential decay function.

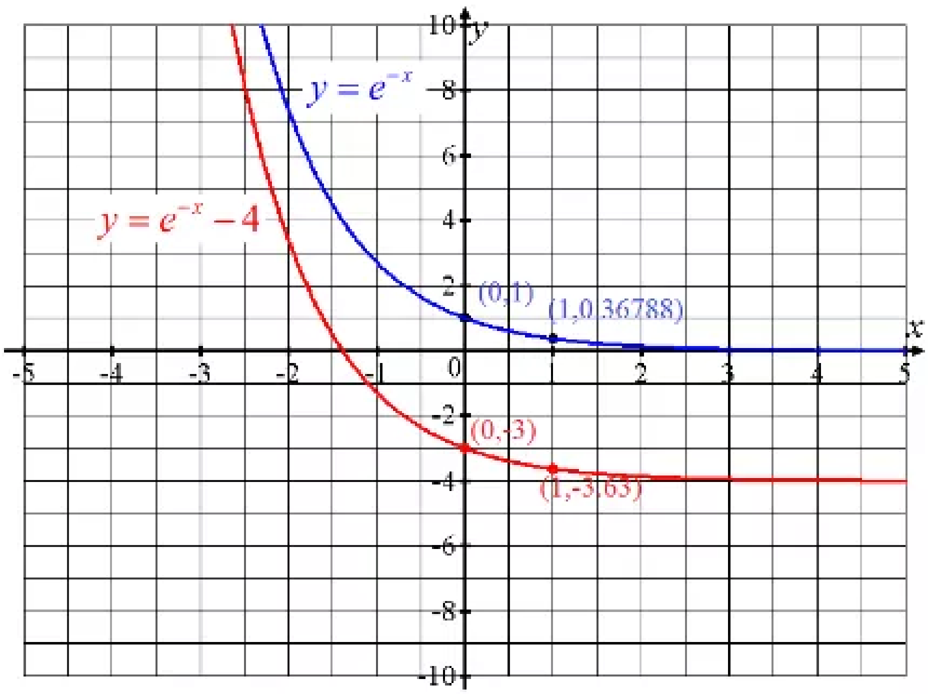
First draw the graph for $y = e^{-x}$, which passes through the points $(0,1)$, and, $(1, 0.36788)$

Plot the points $(0,-3)$, and, $(1,-3.63)$ on the graph and then draw the curve $y = e^{-x}$.

Then translate the graph down 4 units. Next observe that the translated graph passes through the points $(0,-3)$, and, $(1,-3.63)$.

The graph's asymptote is the line $y = -4$.

So, the graph of the function is shown below:



It is known that the domain is the set of all input values, and, the range is the set of all output values.

Here, the input values are all x-values, and, the output values are all positive y values.

Form the graph, observe that the domain of the function $y = e^{-x} - 4$ is all real numbers, and the range is $\boxed{y > -4}$.

Answer 47e.

The graph of the function $y = 2e^{x-2}$ is obtained by translating the graph of $y = 2e^x$ two units to the right.

For graphing the function $y = 2e^x$, first we have to find some points on the graph. Choose any value for x, say, 0 and find the corresponding value of y.

$$\begin{aligned} y &= 2e^0 \\ &= 2 \end{aligned}$$

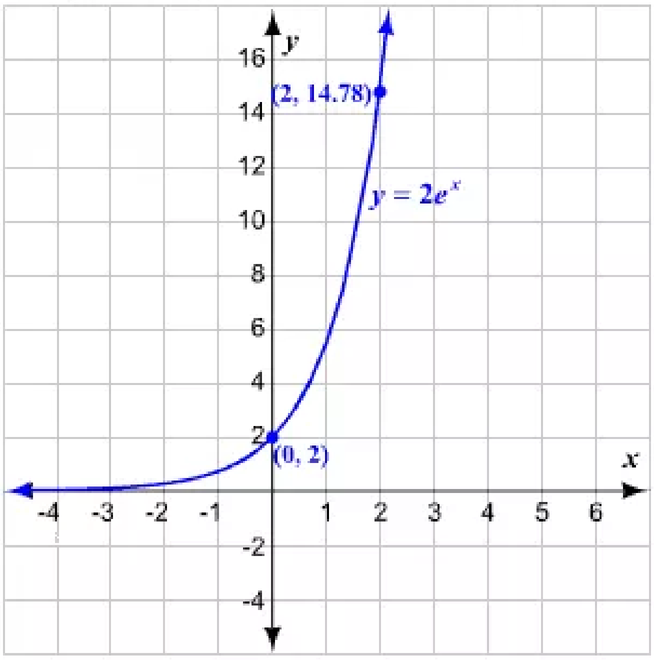
One point on the graph is (0, 2).

Put another value for x, say, 2 and find the y-value.

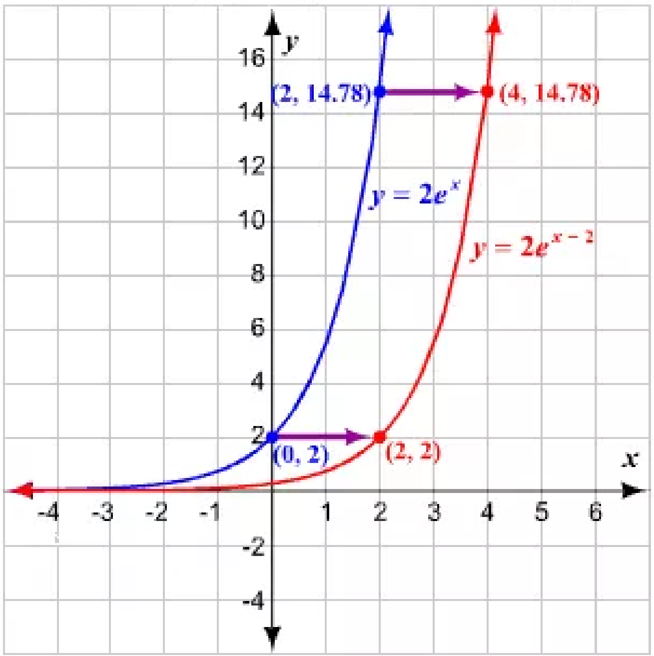
$$\begin{aligned} y &= 2e^2 \\ &\approx 14.78 \end{aligned}$$

Another point on the graph is (2, 14.78).

Now, plot the points on a coordinate plane and connect them with a smooth curve.



Translate the graph of $y = 2^x$ two units right to graph $y = 2e^{x-2}$.



The domain of a function is the set of all input values and the range is the set of all output values.

From the figure, we can find that the input values include all real numbers whereas the output values include only positive real numbers. Therefore, the domain is the set of all real numbers and the range is $y > 0$.

Answer 48e.

Consider the following function,

$$g(x) = 0.5e^{x+1} + 3.$$

The function $g(x) = 0.5e^{x+1} + 3$ is in the form $y = ae^{rx}$.

Here, in this case $a = 0.5$ is positive, and, $r = 1$ is positive.

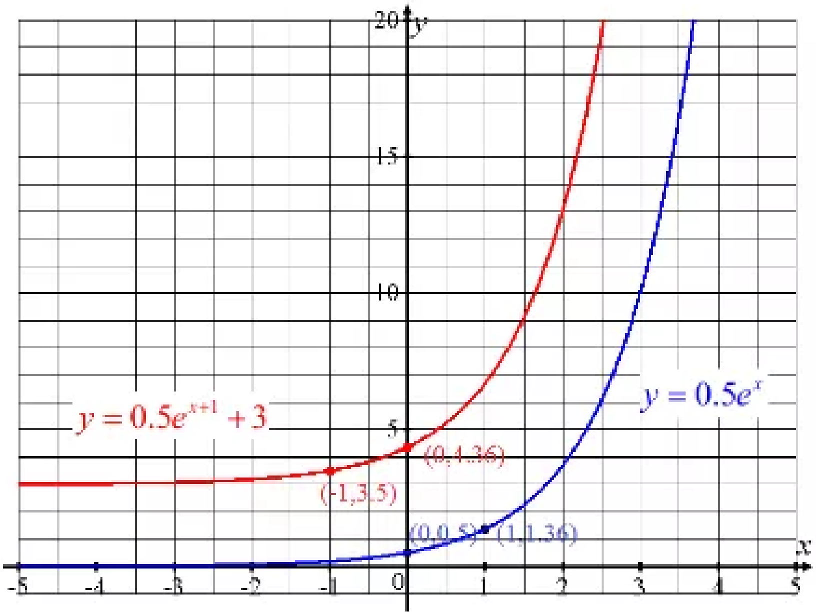
So, the function is an exponential growth function.

First draw the graph for $g(x) = 0.5e^x$, which passes through the points $(0, 0.5)$, and, $(1, 1.36)$

Then, translate left 1 unit and up 3 units to obtain the graph of the function $g(x) = 0.5e^{x+1} + 3$ which passes through the points $(-1, 3.5)$ and $(0, 4.36)$.

The graph's asymptote is the line $y = 3$.

So, the graph of the function is shown below:



It is known that the domain is the set of all input values, and, the range is the set of all output values.
Here, the input values are all x-values, and, the output values are all positive y values.
Form the graph, observe that the domain of the function $g(x) = 0.5e^{x+1} + 3$ is all real numbers, and the range is $\boxed{y > 3}$.

Answer 49e.

First, apply the power property of logarithm. This property is given as
$\log_b m^n = n \log_b m$, where m, n, and b are positive numbers such that $b \neq 1$.
$3\log_7 4 - \log_7 8 = \log_7 4^3 - \log_7 8$

Next, let us apply the quotient property of logarithm. This property is given as
$\log_b \frac{m}{n} = \log_b m - \log_b n$, where m, n, and b are positive numbers such that $b \neq 1$.

$$\log_7 4^3 - \log_7 8 = \log_7 \frac{4^3}{8}$$
$$= \log_7 \frac{64}{8}$$
$$= \log_7 8$$

Thus, we can condense the expression as $\log_7 8$.

Answer 50e.

Consider the following expression,

$2\log 5+\log 4$.

Let b, m and n be positive numbers such that $b\neq 1$.

$\log_b mn=\log_b m+\log_b n$ Product property

$\log_b m^n = n\log_b m$ Power property

Simplify the above expression as follows:

$$\begin{aligned} 2\log 5+\log 4 &= \log 5^2+\log 4 && \text{Use power property} \\ &= \log 25+\log 4 && \text{Simplify} \\ &= \log(25\cdot 4) && \text{Use product property} \\ &= \log 100 \\ &= \log 10^2 && \text{Simplify; Use product property} \\ &= 2 \end{aligned}$$

Therefore, the answer is $\boxed{2}$.

Answer 51e.

First, apply the power property of logarithm. This property is given as

$\log_b m^n = n\log_b m$, where m, n, and b are positive numbers such that $b \neq 1$.

$\ln x + 9\ln y = \ln x + \ln y^9$

Next, let us apply the product property of logarithm. This property is given as

$\log_b mn = \log_b m + \log_b m$, where m, n and b are positive numbers such that $b \neq 1$.

$\ln x + \ln y^9 = \ln xy^9$

Thus, we can condense the expression as $\ln xy^9$.

Answer 52e.

Consider the following expression,

$2\ln 6-\ln x$.

Let b, m and n be positive numbers, such that $b\neq 1$, then,

$\ln_b \frac{m}{n} = \ln_b m - \ln_b n$ Quotient property

$\ln_b m^n = n\ln_b m$ Power property

Simplify the above expression as follows:

$$\begin{aligned} 2\ln 6-\ln x &= \ln 6^2-\ln x && \text{Use power property} \\ &= \ln 36-\ln x && \text{Simplify} \\ &= \ln\frac{36}{x} && \text{Use quotient property} \end{aligned}$$

Therefore, the answer is,

$\boxed{\ln\frac{36}{x}}$.

Answer 53e.

First, apply the power property of logarithm. This property is given as

$\log_b m^n = n\log_b m$, where m, n, and b are positive numbers such that $b \neq 1$.

$\log_5 7 + 6\log_5 x - \log_5 3 = \log_5 7 + \log_5 x^6 - \log_5 3$

Next, let us apply the product property of logarithm. This property is given as

$\log_b mn = \log_b m + \log_b m$, where m, n, and b are positive numbers such that $b \neq 1$.

$$\log_5 7 + \log_5 x^6 - \log_5 3 = \log_5 \left(7 \cdot x^6\right) - \log_5 3$$
$$= \log_5 7x^6 - \log_5 3$$

Now, let us apply the quotient property of logarithm. This property is given as

$\log_b \frac{m}{n} = \log_b m - \log_b n$, where m, n, and b are positive numbers such that $b \neq 1$.

$$\log_5 7x^6 - \log_5 3 = \log \frac{7x^6}{3}$$

Thus, we can condense the expression as $\log \frac{7x^6}{3}$.

Answer 54e.

Consider the following expression,

$$\log 8 - 2\log 2 + 4\log x.$$

Let b, m and n be positive numbers such that $b \neq 1$.

$\log_b mn = \log_b m + \log_b n$	Product property
$\log_b m^n = n\log_b m$	Power property
$\log_b \frac{m}{n} = \log_b m - \log_b n$	Quotient property

Simplify the above expression as follows:

$\log 8 - 2\log 2 + 4\log x = \log 8 - \log 2^2 + \log x^4$	Use power property
$= \log 8 - \log 4 + \log x^4$	Simplify
$= \log \frac{8}{4} + \log x^4$	Use quotient property
$= \log 2 + \log x^4$	Simplify; Use product property
$= \log 2x^4$	

Therefore, the answer is,

$\boxed{\log 2x^4}$.

Answer 55e.

STEP 1 Let x be the number of toppings. Represent the problem as a verbal model. Then, translate it into an equation.

$$\begin{array}{ccccccc} \text{Cost for the pizza (dollars)} & + & \text{Cost per topping (dollars)} & \cdot & \text{Number of toppings} & \le & \text{Total fund (dollars)} \\ \Downarrow & & \Downarrow & & \Downarrow & & \Downarrow \\ 11 & + & 2 & \cdot & x & \le & 18 \end{array}$$

STEP 2 Solve the inequality.

$11 + 2x \le 18$

Subtract 11 from both the sides and simplify

$11 + 2x - 11 \le 18 - 11$

$2x \le 7$

Divide both the sides by 2.

$\frac{2x}{2} \le \frac{7}{2}$

$x \le 3.5$

Thus, you can order a maximum of 3 toppings.