CBSE Board
Class IX Mathematics

Time: 3 hrs **Total Marks: 80**

General Instructions:

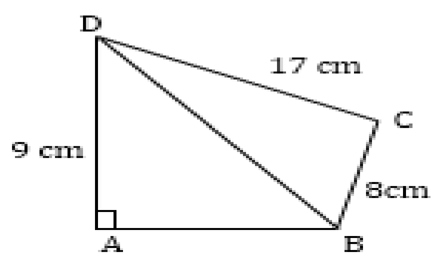
1. **All** questions are **compulsory**.
2. The question paper consists of **30** questions divided into **four sections** A, B, C, and D. **Section A** comprises of **6** questions of 1 mark each, **Section B** comprises of **6** questions of 2 marks each, **Section C** comprises of **10** questions of 3 marks each and **Section D** comprises of **8** questions of 4 marks each.
3. Question numbers **1 to 6** in **Section A** are multiple choice questions where you are to select **one** correct option out of the given four.
4. Use of calculator is **not** permitted.

Section A
(Questions 1 to 6 carry 1 mark each)

1. If $\left(\sqrt{5}+\sqrt{6}\right)^2$ = a + b $\sqrt{30}$, Find the respective values of a and b.

2. If for one of the solutions of the equation ax + by + c = 0, x is negative and y is positive, then a portion of the above line definitely lies in which Quadrant?

3. In the given figure, find the length of AB, if Area of □ ABCD is 122 cm^2 and the area of ΔBCD is 68 cm^2.



4. $7x^3 - 2x^2 + 3\sqrt{x} - 4$, Is given expression is a Polynomial?

5. The ages of ten students of a group are given below. The ages have been recorded in years and months:

   8 – 6, 9 – 0, 8 – 4, 9 – 3, 7 – 8, 8 – 11, 8 – 7, 9 – 2, 7 – 10, 8 – 8

   Determine the range?

6. Can we say every Rectangle is a Square?

Section B

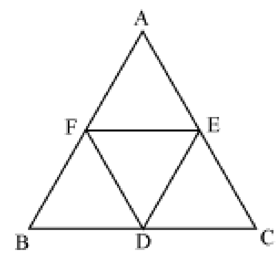
(Questions 7 to 12 carry 2 marks each)

7. If $a = 2 + \sqrt{3}$, find the value of $a + \frac{1}{a}$.

8. Without actually calculating the cubes, find the value of $75^3 - 25^3 - 50^3$.

9. Find the value of k, if x = 1, y = 1 is a solution of the equation $9kx + 12ky = 63$.

10. D, E and F are respectively the mid-points of the sides BC, CA and AB of ΔABC. Show that BDEF is a parallelogram.



11. It is required to make a closed cylindrical tank of height 1 m and base diameter 140 cm from a metal sheet. How many square metres of the sheet are required for the same?

12. Write two solutions for $\pi x + y = 9$.

Section C

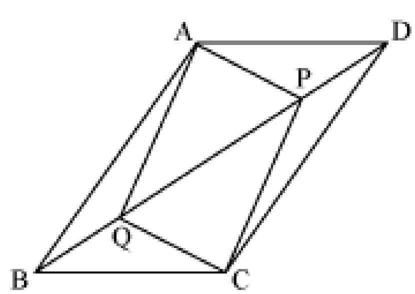
(Questions 13 to 22 carry 3 marks each)

13. Represent $\sqrt{3.2}$ on the number line.

14. Factorise: $b^2 + c^2 + 2(ab + bc + ca)$.

15. If $x = (3 + \sqrt{8})$, find the value of $\left(x^2 + \frac{1}{x^2}\right)$

16. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see the given figure). Show that (i) $\Delta APD \cong \Delta CQB$ (ii) AP = CQ



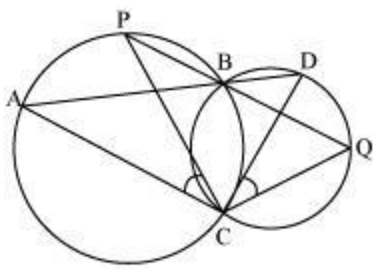
17. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

18. A survey was conducted by a group of students as a part of their Environment Awareness Programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

No of plants	0-2	2-4	4-6	6-8	8-10	10-12	12-14
No of houses	1	2	1	5	6	2	3

19. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively.

Prove that $\angle ACP = \angle QCD$.



20. Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

21. 1500 families with 2 children were selected randomly and the following data was recorded:

Number of girls in a family	2	1	0
Number of families	475	814	211

Compute the probability of a family, chosen at random, having

i. 2 girls ii. 1 girl iii. No girl

22. A storehouse measures 40 m × 25 m × 10 m. Find the maximum number of wooden crates each measuring 1.5 m × 1.25 m × 0.5 m that can be stored in the storehouse.

Section D

(Questions 23 to 30 carry 4 marks each)

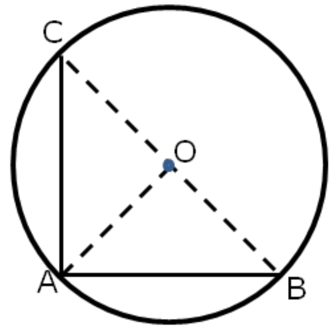
23. Simplify:

$$\left(\frac{16}{9}\right)^{-\frac{1}{2}} \div \left[\left(\frac{256}{81}\right)^{-\frac{1}{4}} + \frac{\sqrt{3}}{\sqrt{27}}\right]$$

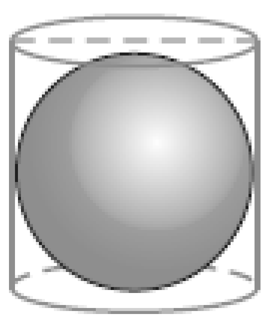
24. Find the value of $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$

25. Without actual division, prove that $2x^4 + x^3 - 14x^2 - 19x - 6$ is exactly divisible by $x^2 + 3x + 2$.

26. In the given figure, AB and AC are two equal chords of a circle with centre O. Show that O lies on the bisectors of $\angle BAC$.



27. A right circular cylinder just encloses a sphere of radius r.



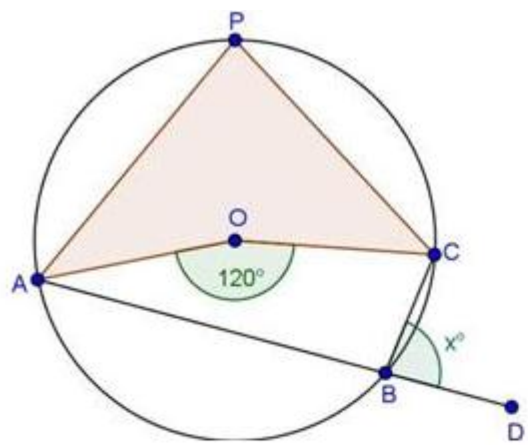
Find the

i. Surface area of the sphere,

ii. Curved surface area of the cylinder,

iii. Ratio of the areas obtained in i. and ii.

28. If O is the centre of the circle, find the value of x in the following figure.



29. Construct a right triangle in which one side is of length 4 cm and the difference between the hypotenuse and the other side is 2 cm.

30. Draw the graph of the linear equation x + 2y = 8. From the graph, check whether (−1, −2) is a solution of this equation.

CBSE Board
Class IX Mathematics
Solution

Time: 3 hrs **Total Marks: 80**

Section A

1. $\left(\sqrt{5}+\sqrt{6}\right)^2 = 5+6+2\sqrt{30} = 11+2\sqrt{30}$

   On comparing $a+b\sqrt{30}$ and $11+2\sqrt{30}$, we get $a=11$ and $b=2$.

2. Since, In the II Quadrant x-axis contains positive numbers and y-axis contains negative numbers.
   If for one of the solutions of the equation ax + by + c = 0, x is negative and y is positive, then a portion of the above line definitely lies in the **II Quadrant**.

3. Here, Area of □ ABCD = Area of ΔABD + Area of ΔBCD
   ∴ Area of ΔABD = 122 – 68 = 54
   Area of ΔABD = $\frac{1}{2} \times AB \times AD$
   ∴ $54 = \frac{1}{2} \times AB \times 9 \Rightarrow AB = \frac{54 \times 2}{9} = 12$ cm

4. $7x^3 - 2x^2 + 3\sqrt{x} - 4$ is not a Polynomial as the exponent of x in $3\sqrt{x}$ is not a positive integer.

5. Range = Highest age – Lowest age
   = (9 years, 3 months) – (7 years, 8 months)
   = (12 × 9 + 3) months – (7 × 12 + 8) months.........(1 year = 12 months)
   = (111 – 92) months
   = 19 months
   = (12 × 1 + 7) months
   = 1 year, 7 months

6. No, Every Rectangle is not a Square.

Because in Square all sides are equal but in Rectangle opposite sides equal.

Section B

7. a = 2 + $\sqrt{3}$

$$\Rightarrow \frac{1}{a} = \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{2^2-\left(\sqrt{3}\right)^2} = 2 - \sqrt{3}$$

So, $a + \frac{1}{a} = (2 + \sqrt{3}) + (2 - \sqrt{3}) = 4$

8. Let x = 75, y = –25, z = –50

x + y + z = 75 – 25 – 50 = 0

We know, if x + y + z = 0 then $x^3 + y^3 + z^3 = 3xyz$

$\Rightarrow 75^3 - 25^3 - 50^3 = 3(75)(-25)(-50) = 281250$

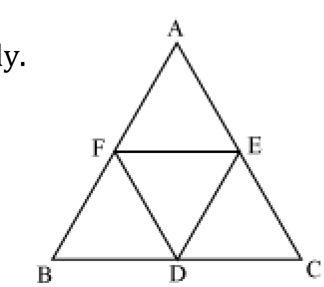
9. Since x = 1, y = 1 is the solution of 9kx + 12ky = 1, it will satisfy the equation.

$\therefore$ 9k(1) + 12k(1) = 63

$\therefore$ 9k + 12k = 63

$\therefore$ 21k = 63 $\Rightarrow$ k = 3

10. In ΔABC, E and F are midpoints of side AC and AB respectively.



By using the mid-point theorem we get,

EF || CB and $EF = \frac{1}{2}(CB)$

As D is the midpoint of CB $\Rightarrow BD = \frac{1}{2}(CB)$

So, BD = EF

The line segments BF and DE join two parallel lines EF and BD of the same length.

Hence the line segments BF and DE will also be parallel to each other and also equal in length. Therefore BDEF is a parallelogram.

11. Height (h) of cylindrical tank is 1 m

Base radius (r) of cylindrical tank = $\left(\dfrac{140}{2}\right)\text{cm} = 70\text{ cm}$

Base radius (r) of cylindrical tank = 0.7 m

Surface area of cylinder $= 2\pi r[h+r] = 2\times\dfrac{22}{7}\times 0.7[1+0.7] = 7.48\text{ m}^2$

Therefore, it will require 7.48 m^2 of sheet.

12. For x = 0,

$\pi(0) + y = 9 \Rightarrow y = 9$

So (0, 9) is a solution of this equation.

For x = 1,

$\pi(1) + y = 9 \Rightarrow y = 9 - \pi$

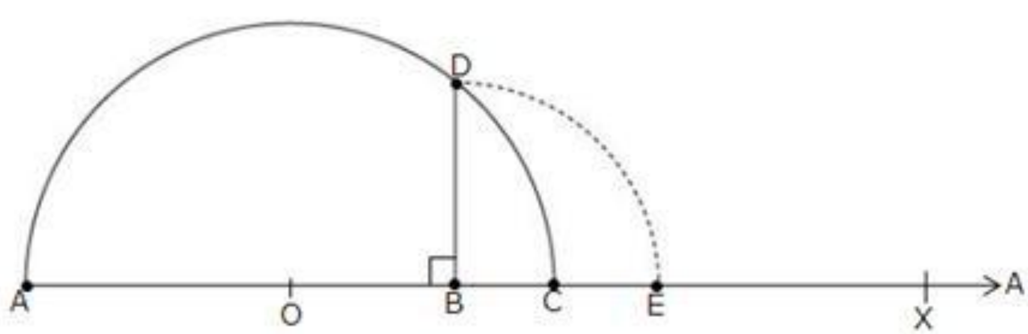
So, $(1, 9 - \pi)$ is a solution of this equation.

Section C

13. Steps of construction:

1. Draw a line segment AB = 3.2 units and extend it to C such that BC = 1 units.
2. Find the midpoint O of AC. With O as centre and OA as radius, draw a semicircle.
3. Now, draw BD $\perp$ AC, intersecting the semicircle at D. Then, BD = $\sqrt{3.2}$ units.
4. With B as centre and BD as radius, draw an arc meeting AC produced at E.

   Then, BE = BD = $\sqrt{3.2}$ units.



14. $b^2 + c^2 + 2(ab + bc + ca)$

$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - a^2$ [Adding and subtracting a^2]

$= [a^2 + b^2 + c^2 + 2ab + 2bc + 2ca] - a^2$

$= (a + b + c)^2 - (a)^2$ [Using $x^2 + y^2 + 2xy + 2yz + 2zx = (x + y + z)^2$]

$= (a + b + c + a)(a + b + c - a)$ [Because $a^2 - b^2 = (a+ b)\ (a - b)$]

$= (2a + b +c)(b + c)$

15. $x=\left(3+\sqrt{8}\right)\Rightarrow\frac{1}{x}=\frac{1}{\left(3+\sqrt{8}\right)}$

$$\frac{1}{x}=\frac{1}{\left(3+\sqrt{8}\right)}\times\frac{\left(3-\sqrt{8}\right)}{\left(3-\sqrt{8}\right)}=\frac{\left(3-\sqrt{8}\right)}{\left(3^2-\left(\sqrt{8}\right)^2\right)}=\frac{\left(3-\sqrt{8}\right)}{\left(9-8\right)}=\left(3-\sqrt{8}\right)$$

$$x+\frac{1}{x}=\left(3+\sqrt{8}\right)+\left(3-\sqrt{8}\right)=6$$

$$\Rightarrow\left(x+\frac{1}{x}\right)^2=6^2=36$$

$$\therefore\ x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2x.\frac{1}{x}=\left(x+\frac{1}{x}\right)^2-2=36-2=34$$

16. i. In ΔAPD and ΔCQB

$\angle ADP = \angle CBQ$ (alternate interior angles for BC || AD)

AD = CB (opposite sides of parallelogram ABCD)

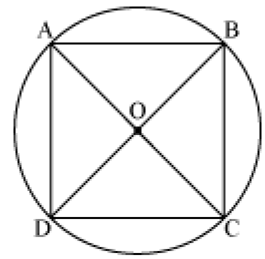
DP = BQ (given)

$\therefore \Delta APD \cong \Delta CQB$ (using SAS congruence rule)

ii. As observed $\Delta APD \cong \Delta CQ$

$\therefore$ AP = CQ (CPCT)

17. Let ABCD be a cyclic quadrilateral having diagonals BD and AC, intersecting each other at point O.



$$m\angle BAD = \frac{1}{2} m\angle BOD = \frac{180^\circ}{2} = 90^\circ \quad \text{(Consider BD as a chord)}$$

$m\angle BCD + m\angle BAD = 180^\circ$ (Cyclic quadrilateral)

$m\angle BCD = 180^\circ - 90^\circ = 90^\circ$

$$m\angle ADC = \frac{1}{2} m\angle AOC = \frac{1}{2}(180^\circ) = 90^\circ \text{ (Considering AC as a chord)}$$

$m\angle ADC + m\angle ABC = 180^\circ$ (Cyclic quadrilateral)

$90^\circ + m\angle ABC = 180^\circ$

$m\angle ABC = 90^\circ$

Here, each interior angle of cyclic quadrilateral is of 90°. Hence it is a rectangle.

18. Let us find the class marks x_i of each class by taking the average of the upper class limit and lower class limit and put them in a table.

We can use the Direct Method because numerical values of x_i and f_i are small.

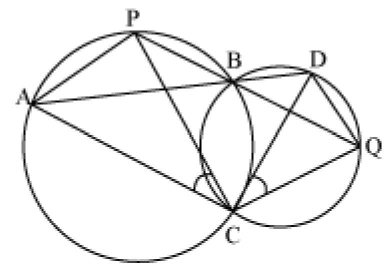
Class interval	No. of houses (f_i)	Class marks (x_i)	$f_i x_i$
0-2	1	1	1
2-4	2	3	6
4-6	1	5	5
6-8	5	7	35
8-10	6	9	54
10-12	2	11	22
12-14	3	13	39
Total	$\sum f_i = 20$		$\sum f_i x_i = 162$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{162}{20} = 8.1$$

Thus, the mean number of plants per house is 8.1 plants

19. Given, two circles intersect at two points B and C.

Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively.



To Prove: ∠ACP = ∠QCD.

Construction: Join chords AP and DQ

Proof: Consider chord AP,

∠PBA = ∠ACP (angles in the same segment) ... (1)

Consider chord DQ,

∠DBQ =∠QCD (angles in the same segment) ... (2)

ABD and PBQ are line segments intersecting at B.

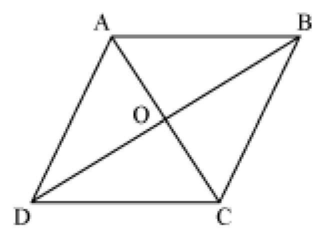
∴ ∠PBA = ∠DBQ (vertically opposite angles) ... (3)

From equations (1), (2), and (3), we obtain

∠ACP = ∠QCD

20. Let ABCD be a quadrilateral, whose diagonals AC and BD bisect each other at right angle i.e. OA = OC, OB = OD and m∠AOB = m∠BOC = m∠COD = m∠AOD = 90°

To prove that ABCD is a rhombus, we need to prove that ABCD is a parallelogram and all sides of ABCD are equal.



Now, in ΔAOD and ΔCOD,

OA = OC (Diagonal bisects each other)

∠AOD = ∠COD (given)

OD = OD (common)

∴ ΔAOD ≅ ΔCOD (by SAS congruence rule)

∴ AD = CD (1)

Similarly we can prove that

AD = AB and CD = BC (2)

From equations (1) and (2), we can say that

AB = BC = CD = AD

Since opposite sides of quadrilateral ABCD are equal, so, we can say that ABCD is a parallelogram. Since all sides of a parallelogram ABCD are equal, so we can say that ABCD is a rhombus.

21. Total number of families = 475 + 814 + 211 = 1500

i. Number of families with 2 girls = 475

$$\therefore \text{Required probability} = \frac{\text{Number of families with 2 girls}}{\text{Total number of families}} = \frac{475}{1500} = \frac{19}{60}$$

ii. Number of families with 1 girl = 814

$$\therefore \text{Required probability} = \frac{\text{Number of families with 1 girl}}{\text{Total number of families}} = \frac{814}{1500} = \frac{407}{750}$$

iii. Number of families with no girl = 211

$$\therefore \text{Required probability} = \frac{\text{Number of families with no girls}}{\text{Total number of families}} = \frac{211}{1500}$$

22. Length (l_1) of the storehouse = 40 m

Breadth (b_1) of the storehouse = 25 m

Height (h_1) of the storehouse = 10 m

Volume of storehouse = $l_1 \times b_1 \times h_1 = (40 \times 25 \times 10)\ m^3 = 10000\ m^3$

Length (l_2) of a wooden crate = 1.5 m

Breadth (b_2) of a wooden crate = 1.25 m

Height (h_2) of a wooden crate = 0.5 m

Volume of a wooden crate = $l_2 \times b_2 \times h_2 = (1.5 \times 1.25 \times 0.5)\ m^3 = 0.9375 m^3$

Let n wooden crates be stored in the storehouse.

Volume of n wooden crates = volume of storehouse

0.9375 × n = 10000

$$\therefore\ n = \frac{10000}{0.9375} = 10666.66$$

Since, the number of crates cannot be a decimal number, we take the whole number part.

Thus, 10666 numbers of wooden crates can be stored in the storehouse.

Section D

23.

$$\left(\frac{16}{9}\right)^{-\frac{1}{2}} \div \left[\left(\frac{256}{81}\right)^{-\frac{1}{4}} + \frac{\sqrt{3}}{\sqrt{27}}\right]$$

$$= \left(\frac{4^2}{3^2}\right)^{-\frac{1}{2}} \div \left[\left(\frac{4^4}{3^4}\right)^{-\frac{1}{4}} + \sqrt{\frac{3}{27}}\right]$$

$$= \left[\left(\frac{4}{3}\right)^2\right]^{-\frac{1}{2}} \div \left[\left\{\left(\frac{4}{3}\right)^4\right\}^{-\frac{1}{4}} + \sqrt{\frac{1}{9}}\right]$$

$$= \left(\frac{4}{3}\right)^{2\times\left(-\frac{1}{2}\right)} \div \left[\left(\frac{4}{3}\right)^{4\times\left(-\frac{1}{4}\right)} + \frac{1}{3}\right] \qquad \left[\left(a^x\right)^y = a^{xy}\right]$$

$$= \left(\frac{4}{3}\right)^{-1} \div \left[\left(\frac{4}{3}\right)^{-1} + \frac{1}{3}\right]$$

$$= \frac{3}{4} \div \left(\frac{3}{4} + \frac{1}{3}\right) \qquad \left[a^{-1} = \frac{1}{a}\right]$$

$$= \frac{3}{4} \div \frac{13}{12}$$

$$= \frac{3}{4} \times \frac{12}{13}$$

$$= \frac{9}{13}$$

24.

$$\frac{1}{3-\sqrt{8}}=\frac{1}{3-\sqrt{8}}\times\frac{3+\sqrt{8}}{3+\sqrt{8}}=\frac{3+\sqrt{8}}{9-8}=3+\sqrt{8}$$
$$\frac{1}{\sqrt{8}-\sqrt{7}}=\frac{1}{\sqrt{8}-\sqrt{7}}\times\frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}}=\frac{\sqrt{8}+\sqrt{7}}{8-7}=\sqrt{8}+\sqrt{7}$$
$$\frac{1}{\sqrt{7}-\sqrt{6}}=\frac{1}{\sqrt{7}-\sqrt{6}}\times\frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}=\frac{\sqrt{7}+\sqrt{6}}{7-6}=\sqrt{7}+\sqrt{6}$$
$$\frac{1}{\sqrt{6}-\sqrt{5}}=\frac{1}{\sqrt{6}-\sqrt{5}}\times\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}=\frac{\sqrt{6}+\sqrt{5}}{6-5}=\sqrt{6}+\sqrt{5}$$
$$\frac{1}{\sqrt{5}-2}=\frac{1}{\sqrt{5}-2}\times\frac{\sqrt{5}\times 2}{\sqrt{5}+2}=\frac{\sqrt{5}+2}{5-4}=\sqrt{5}+2$$

$$\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$$
$$=3+\sqrt{8}-\left(\sqrt{8}+\sqrt{7}\right)+\left(\sqrt{7}+\sqrt{6}\right)-\left(\sqrt{6}+\sqrt{5}\right)+\left(\sqrt{5}+2\right)$$
$$=5$$

25. Let $p(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$ and $q(x) = x^2 + 3x + 2$

$q(x) = x^2 + 3x + 2 = (x + 1)(x + 2)$

Now, $p(-1) = 2(-1)^4 + (-1)^3 - 14(-1)^2 - 19(-1) - 6 = 2 - 1 - 14 + 19 - 6 = 21 - 21 = 0$

And, $p(-2) = 2(-2)^4 + (-2)^3 - 14(-2)^2 - 19(-2) - 6 = 32 - 8 - 56 + 38 - 6 = 70 - 70 = 0$

Therefore, $(x + 1)$ and $(x + 2)$ are the factors of $p(x)$, so $p(x)$ is divisible by $(x + 1)$ and $(x + 2)$.

Hence, $p(x)$ is divisible by $x^2 + 3x + 2$.

26. In $\triangle AOB$ and $\triangle AOC$,

OA = OA (common side)

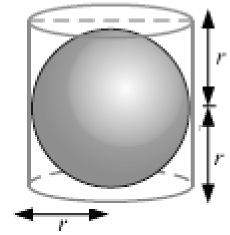
OB = OC (radius of the circle)

AB = AC (given)

$\therefore \triangle AOB \cong \triangle AOC$

Hence, $\angle OAC = \angle OAB$

27. A right circular cylinder just encloses a sphere of radius r then,



i. Surface area of sphere = $4\pi r^2$

ii. Height of cylinder = r + r = 2r

Radius of cylinder = r

C.S.A. of cylinder = $2\pi rh = 2\pi r\,(2r) = 4\pi r^2$

iii. Required ratio $= \dfrac{\text{Surface area of sphere}}{\text{CSA of cylinder}} = \dfrac{4\pi r^2}{4\pi r^2} = \dfrac{1}{1}$

28. We have $m\angle AOC = 120°$

By the degree measure theorem,

$m\angle AOC = 2m\angle APC$

$\therefore\ 120° = 2m\angle APC$

$\therefore\ m\angle APC = 60°$

Now, $m\angle APC + m\angle ABC = 180°$ ....(Opposite angles of a cyclic quadrilateral)

$\therefore\ 60° + m\angle ABC = 180°$

$\therefore\ m\angle ABC = 180° - 60° = 120°$

$m\angle ABC + m\angle DBC = 180°$ ....(Linear pair of angles)

$\therefore\ 120° + x° = 180°$

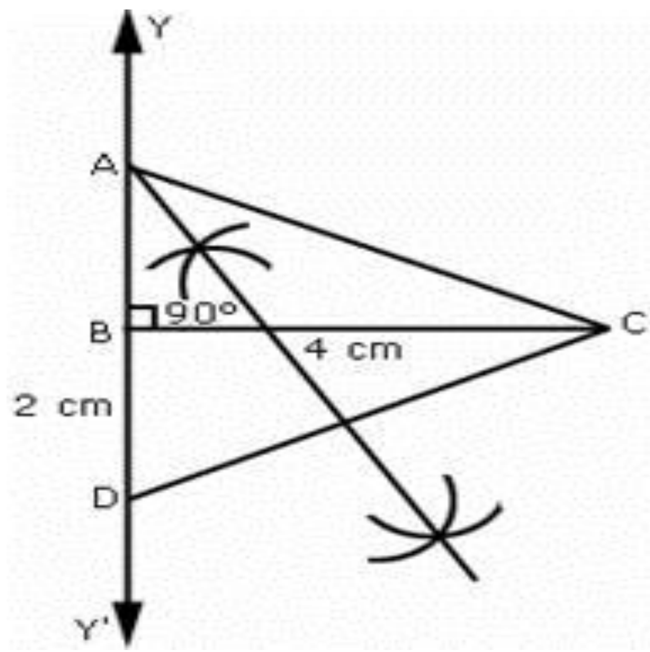
$\therefore\ x = 180° - 120° = 60°$

29. Given: In ΔABC $BC = 4$ cm, $m\angle ABC = 90^{\circ}$, $AC - AB = 2$ cm

Steps of Construction:

1. Draw $BC = 4$cm
2. Draw a ray BY such that $m\angle CBY = 90°$ and produce YB to form a line YBY'.
3. From ray BY' cut off $DB = 2$ cm
4. Join CD
5. Construct perpendicular bisector of CD intersecting BY at A
6. Join AC

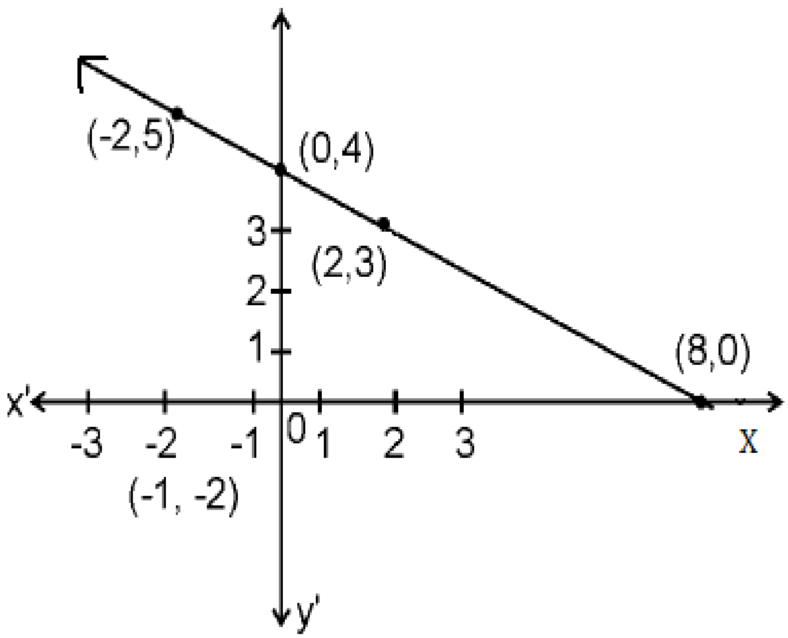
ΔABC is the required triangle.



30. $x + 2y = 8$

$\Rightarrow y = \frac{1}{2}(8 - x)$

x	−2	0	2
y	5	4	3



From the graph it is clear that (−1, −2) does not lie on the line.

Therefore, (−1, −2) is not a solution of line $x + 2y = 8$.