Chapter 4

Quadratic Equations and Inequations

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Euclid

The Babylonians *knew of quadratic equations some 4000 years ago. The Greek mathematician* Euclid (300 B.C.) *gives several quadratic equation while solving geometrical problems,.*

Aryabhatta (476 A.D.) *gives a rule to sum the geometric series which involves the solution of the quadratic equations* Brahmagupta (598 A.D.) *provides a rule for the solution of the quadratic equations which is very much the quadratic formula.* Mahavira *around 850 A.D. proposed a problem involving the use of quadratic equation and its solution.*

It was Sridhara, *an Indian mathematician, around* 900 A.D. *who was the first to give an algebraic solution of the general equation* $ax^2 + bx + c = 0$ $a \neq 0$, *showing the roots to be* $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The first important treatment of a quadratic equation, by factoring, is found in Harriot's works in approximately 1631 A.D.

Quadratic Equations and Inequations

4.1 Polynomial

Algebraic expression containing many terms of the form cx^n, n being a non-negative integer is called a polynomial. *i.e.*, $f(x)=a_0+a_1x+a_2x^2+a_3x^3+......+a_{n-1}x^{n-1}+a_nx^n$, where x is a variable, $a_0, a_1, a_2 ....a_n$ are constants and $a_n \neq 0$

Example : $4x^4+3x^3-7x^2+5x+3$, $3x^3+x^2-3x+5$.

(1) **Real polynomial :** Let $a_0, a_1, a_2 ....a_n$ be real numbers and x is a real variable.

Then $f(x)=a_0+a_1x+a_2x^2+a_3x^3+......+a_nx^n$ is called real polynomial of real variable x with real coefficients.

Example : $3x^3-4x^2+5x-4, x^2-2x+1$ etc. are real polynomials.

(2) **Complex polynomial :** If $a_0, a_1, a_2 ....a_n$ be complex numbers and x is a varying complex number.

Then $f(x)=a_0+a_1x+a_2x^2+a_3x^3+......+a_nx^n$ is called complex polynomial of complex variable x with complex coefficients.

Example : $3x^2-(2+4i)x+(5i-4), x^3-5ix^2+(1+2i)x+4$ etc. are complex polynomials.

(3) **Degree of polynomial :** Highest power of variable x in a polynomial is called degree of polynomial.

Example : $f(x)=a_0+a_1x+a_2x^2+......+a_{n-1}x^{n-1}+a_nx^n$ is a n degree polynomial.

$f(x)=4x^3+3x^2-7x+5$ is a 3 degree polynomial.

$f(x)=3x-4$ is single degree polynomial or linear polynomial.

$f(x)=bx$ is an odd linear polynomial.

A polynomial of second degree is generally called a quadratic polynomial. Polynomials of degree 3 and 4 are known as cubic and biquadratic polynomials respectively.

(4) **Polynomial equation :** If $f(x)$ is a polynomial, real or complex, then $f(x) = 0$ is called a polynomial equation.

4.2 Types of Quadratic Equation

A quadratic polynomial $f(x)$ when equated to zero is called quadratic equation.

Example : $3x^2+7x+5=0, -9x^2+7x+5=0, x^2+2x=0, 2x^2=0$

or

An equation in which the highest power of the unknown quantity is two is called quadratic equation.

Quadratic equations are of two types :

(1) **Purely quadratic equation :** A quadratic equation in which the term containing the first degree of the unknown quantity is absent is called a purely quadratic equation.

i.e. $ax^2 + c = 0$ where $a, c \in C$ and $a \neq 0$

(2) **Adfected quadratic equation :** A quadratic equation which contains terms of first as well as second degrees of the unknown quantity is called an adfected quadratic equation.

i.e. $ax^2 + bx + c = 0$ where $a, b, c \in C$ and $a \neq 0$, $b \neq 0$.

(3) **Roots of a quadratic equation :** The values of variable x which satisfy the quadratic equation is called roots of quadratic equation.

Important Tips

- ☞ *An equation of degree n has n roots, real or imaginary.*
- ☞ *Surd and imaginary roots always occur in pairs in a polynomial equation with real coefficients i.e. if 2 – 3i is a root of an equation, then 2 + 3i is also its root. Similarly if $2+\sqrt{3}$ is a root of given equation, then $2-\sqrt{3}$ is also its root.*
- ☞ *An odd degree equation has at least one real root whose sign is opposite to that of its last term (constant term), provided that the coefficient of highest degree term is positive.*
- ☞ *Every equation of an even degree whose constant term is negative and the coefficient of highest degree term is positive has at least two real roots, one positive and one negative.*

4.3 Solution of Quadratic Equation

(1) **Factorization method :** Let $ax^2 + bx + c = a(x-\alpha)(x-\beta) = 0$. Then $x = \alpha$ and $x = \beta$ will satisfy the given equation.

Hence, factorize the equation and equating each factor to zero gives roots of the equation.

Example : $3x^2 - 2x + 1 = 0 \Rightarrow (x-1)(3x+1) = 0$

$x = 1, -1/3$

(2) **Hindu method (Sri Dharacharya method) :** By completing the perfect square as

$ax^2 + bx + c = 0 \Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

Adding and subtracting $\left(\frac{b}{2a}\right)^2$, $\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2}\right] = 0$ which gives, $\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

Hence the quadratic equation $ax^2 + bx + c = 0$ ($a \neq 0$) has two roots, given by

$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$, $\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

Note : ❑ Every quadratic equation has two and only two roots.

4.4 Nature of Roots

In quadratic equation $ax^2 + bx + c = 0$, the term $b^2 - 4ac$ is called discriminant of the equation, which plays an important role in finding the nature of the roots. It is denoted by Δ or D.

(1) **If *a*, *b*, *c* ∈ *R* and *a* ≠ 0, then :** (i) If $D < 0$, then equation $ax^2 + bx + c = 0$ has non-real complex roots.

(ii) If $D > 0$, then equation $ax^2 + bx + c = 0$ has real and distinct roots, namely $\alpha = \frac{-b + \sqrt{D}}{2a}$, $\beta = \frac{-b - \sqrt{D}}{2a}$

and then $ax^2 + bx + c = a(x - \alpha)(x - \beta)$ .....(i)

(iii) If $D = 0$, then equation $ax^2 + bx + c = 0$ has real and equal roots $\alpha = \beta = \frac{-b}{2a}$

and then $ax^2 + bx + c = a(x - \alpha)^2$ .....(ii)

To represent the quadratic expression $ax^2 + bx + c$ in form (i) and (ii), transform it into linear factors.

(iv) If $D \geq 0$, then equation $ax^2 + bx + c = 0$ has real roots.

(2) **If *a*, *b*, *c* ∈ *Q*, *a* ≠ 0, then :** (i) If $D > 0$ and D is a perfect square ⇒ roots are unequal and rational.

(ii) If $D > 0$ and D is not a perfect square ⇒ roots are irrational and unequal.

(3) **Conjugate roots :** The irrational and complex roots of a quadratic equation always occur in pairs. Therefore

(i) If one root be $\alpha + i\beta$ then other root will be $\alpha - i\beta$.

(ii) If one root be $\alpha + \sqrt{\beta}$ then other root will be $\alpha - \sqrt{\beta}$.

(4) **If D_1 and D_2 be the discriminants of two quadratic equations,then**

(i) If $D_1 + D_2 \geq 0$, then

(a) At least one of D_1 and $D_2 \geq 0$. (b) If $D_1 < 0$ then $D_2 > 0$

(ii) If $D_1 + D_2 < 0$, then

(a) At least one of D_1 and $D_2 < 0$. (b) If $D_1 > 0$ then $D_2 < 0$.

4.5 Roots Under Particular Conditions

For the quadratic equation $ax^2 + bx + c = 0$.

(1) If $b = 0$ ⇒ roots are of equal magnitude but of opposite sign.

(2) If $c = 0$ ⇒ one root is zero, other is $-b/a$.

(3) If $b = c = 0$ ⇒ both roots are zero.

(4) If $a = c$ ⇒ roots are reciprocal to each other.

(5) If $\left.\begin{matrix} a>0 & c<0 \\ a<0 & c>0 \end{matrix}\right\}$ $\Rightarrow$ roots are of opposite signs.

(6) If $\left.\begin{matrix} a>0 & b>0 & c>0 \\ a<0 & b<0 & c<0 \end{matrix}\right\}$ $\Rightarrow$ both roots are negative, provided $D \geq 0$.

(7) If $\left.\begin{matrix} a>0 & b<0 & c>0 \\ a<0 & b>0 & c<0 \end{matrix}\right\}$ $\Rightarrow$ both roots are positive, provided $D \geq 0$.

(8) If sign of a = sign of $b \neq$ sign of $c \Rightarrow$ greater root in magnitude, is negative.

(9) If sign of b = sign of $c \neq$ sign of $a \Rightarrow$ greater root in magnitude, is positive.

(10) If $a+b+c=0$ $\Rightarrow$ one root is 1 and second root is c/a.

(11) If $a=b=c=0$, then equation will become an identity and will be satisfied by every value of x.

(12) If $a=1$ and $b, c \in I$ and the root of equation $ax^2+bx+c=0$ are rational numbers, then these roots must be integers.

Important Tips

- ☞ *If an equation has only one change of sign, it has one +ve root and no more.*
- ☞ *If all the terms of an equation are +ve and the equation involves no odd power of x, then all its roots are complex.*

Example: 1 Both the roots of given equation $(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$ are always

[MNR 1986; IIT 1980; Kurukshetra CEE 1998]

(a) Positive (b) Negative (c) Real (d) Imaginary

Solution: (c) Given equation $(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$ can be re-written as $3x^2-2(a+b+c)x+(ab+bc+ca)=0$

$D=4[(a+b+c)^2-3(ab+bc+ca)]=4[a^2+b^2+c^2-ab-bc-ac]=2[(a-b)^2+(b-c)^2+(c-a)^2]\geq 0$

Hence both roots are always real.

Example: 2 If the roots of $(b-c)x^2+(c-a)x+(a-b)=0$ are equal then $a+c=$ **[Kurukshetra CEE 1992]**

(a) $2b$ (b) b^2 (c) $3b$ (d) b

Solution: (a) $b-c+c-a+a-b=0$

Hence one root is 1. Also as roots are equal, other root will also be equal to 1.

Also $\alpha.\beta=\frac{a-b}{b-c} \Rightarrow 1.1=\frac{a-b}{b-c} \Rightarrow a-b=b-c \Rightarrow 2b=a+c$

Example: 3 If the roots of equation $\frac{1}{x+p}+\frac{1}{x+q}=\frac{1}{r}$ are equal in magnitude but opposite in sign, then $(p+q)=$

[Rajasthan PET 1999]

(a) $2r$ (b) r (c) $-2r$ (d) None of these

Solution: (a) Given equation can be written as $x^2+(p+q-2r)x+[pq-(p+q)r]=0$

Since the roots are equal and of opposite sign, $\therefore$ Sum of roots = 0

$\Rightarrow -(p+q-2r)=0 \Rightarrow p+q=2r$

Example: 4 If 3 is a root of $x^2+kx-24=0$, it is also a root of **[EAMCET 2002]**

(a) $x^2+5x+k=0$ (b) $x^2-5x+k=0$ (c) $x^2-kx+6=0$ (d) $x^2+kx+24=0$

Solution: (c) Equation $x^2 + kx - 24 = 0$ has one root as 3,

$\Rightarrow 3^2 + 3k - 24 = 0 \Rightarrow k = 5$

Put $x = 3$ and $k = 5$ in option

Only (c) gives the correct answer *i.e.* $\Rightarrow 3^2 - 15 + 9 = 0 \Rightarrow 0 = 0$

Example: 5 For what values of k will the equation $x^2 - 2(1+3k)x + 7(3+2k) = 0$ have equal roots **[MP PET 1997]**

(a) 1, –10/9 (b) 2, –10/9 (c) 3, –10/9 (d) 4, –10/9

Solution: (b) Since roots are equal then $[-2(1+3k)]^2 = 4.1.7(3+2k) \Rightarrow 1 + 9k^2 + 6k = 21 + 14k \Rightarrow 9k^2 - 8k - 20 = 0$

Solving, we get $k = 2, -10/9$

4.6 Relations between Roots and Coefficients

(1) **Relation between roots and coefficients of quadratic equation :** If α and β are the roots of quadratic equation $ax^2 + bx + c = 0$, ($a \neq 0$) then

Sum of roots $= S = \alpha + \beta = \dfrac{-b}{a} = -\dfrac{\text{coefficient of } x}{\text{coefficient of } x^2}$

Product of roots $= P = \alpha.\beta = \dfrac{c}{a} = \dfrac{\text{constant term}}{\text{coefficient of } x^2}$

If roots of quadratic equation $ax^2 + bx + c = 0$ ($a \neq 0$) are α and β then

(i) $(\alpha - \beta) = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \pm\dfrac{\sqrt{b^2 - 4ac}}{a} = \dfrac{\pm\sqrt{D}}{a}$

(ii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \dfrac{b^2 - 2ac}{a^2}$

(iii) $\alpha^2 - \beta^2 = (\alpha+\beta)\sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = -\dfrac{b\sqrt{b^2-4ac}}{a^2} = \dfrac{\pm b\sqrt{D}}{a^2}$

(iv) $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = -\dfrac{b(b^2 - 3ac)}{a^3}$

(v) $\alpha^3 - \beta^3 = (\alpha-\beta)^3 + 3\alpha\beta(\alpha-\beta) = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}\{(\alpha+\beta)^2 - \alpha\beta\} = \dfrac{\pm(b^2 - ac)\sqrt{b^2-4ac}}{a^3}$

(vi) $\alpha^4 + \beta^4 = \{(\alpha+\beta)^2 - 2\alpha\beta\}^2 - 2\alpha^2\beta^2 = \left(\dfrac{b^2-2ac}{a^2}\right)^2 - 2\dfrac{c^2}{a^2}$

(vii) $\alpha^4 - \beta^4 = (\alpha^2 - \beta^2)(\alpha^2 + \beta^2) = \dfrac{\pm b(b^2 - 2ac)\sqrt{b^2-4ac}}{a^4}$

(viii) $\alpha^2 + \alpha\beta + \beta^2 = (\alpha+\beta)^2 - \alpha\beta = \dfrac{b^2 - ac}{a^2}$

(ix) $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{\alpha^2+\beta^2}{\alpha\beta} = \dfrac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \dfrac{b^2 - 2ac}{ac}$

(x) $\alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha+\beta) = -\dfrac{bc}{a^2}$

(xi) $\left(\dfrac{\alpha}{\beta}\right)^2 + \left(\dfrac{\beta}{\alpha}\right)^2 = \dfrac{a^4 + \beta^4}{\alpha^2\beta^2} = \dfrac{(\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2}{\alpha^2\beta^2} = \dfrac{b^2D + 2a^2c^2}{a^2c^2}$

(2) **Formation of an equation with given roots :** A quadratic equation whose roots are α and β is given by $(x-\alpha)(x-\beta)=0$

$\therefore\ x^2-(\alpha+\beta)x+\alpha\beta=0$ *i.e.* $x^2-(\text{sum of roots})x+(\text{product of roots})=0$

$\therefore\ x^2-Sx+P=0$

(3) **Equation in terms of the roots of another equation :** If $\alpha,\ \beta$ are roots of the equation $ax^2+bx+c=0$, then the equation whose roots are

(i) $-\alpha,\ -\beta\ \Rightarrow\ ax^2-bx+c=0$ (Replace x by $-x$)

(ii) $1/\alpha, 1/\beta\ \Rightarrow\ cx^2+bx+a=0$ (Replace x by $1/x$)

(iii) α^n, β^n; $n\in N \Rightarrow a(x^{1/n})^2+b(x^{1/n})+c=0$ (Replace x by $x^{1/n}$)

(iv) $k\alpha,\ k\beta \Rightarrow ax^2+kbx+k^2c=0$ (Replace x by x/k)

(v) $k+\alpha$, $k+\beta\ \Rightarrow\ a(x-k)^2+b(x-k)+c=0$ (Replace x by $(x-k)$)

(vi) $\dfrac{\alpha}{k}, \dfrac{\beta}{k}\ \Rightarrow\ k^2ax^2+kbx+c=0$ (Replace x by kx)

(vii) $\alpha^{1/n}, \beta^{1/n}$; $n\in N \Rightarrow a(x^n)^2+b(x^n)+c=0$ (Replace x by x^n)

(4) **Symmetric expressions :** The symmetric expressions of the roots $\alpha,\ \beta$ of an equation are those expressions in α and β, which do not change by interchanging α and β. To find the value of such an expression, we generally express that in terms of $\alpha+\beta$ and $\alpha\beta$.

Some examples of symmetric expressions are :

(i) $\alpha^2+\beta^2$ (ii) $\alpha^2+\alpha\beta+\beta^2$ (iii) $\dfrac{1}{\alpha}+\dfrac{1}{\beta}$ (iv) $\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}$

(v) $\alpha^2\beta+\beta^2\alpha$ (vi) $\left(\dfrac{\alpha}{\beta}\right)^2+\left(\dfrac{\beta}{\alpha}\right)^2$ (vii) $\alpha^3+\beta^3$ (viii) $\alpha^4+\beta^4$

4.7 Biquadratic Equation

If $\alpha,\ \beta,\ \gamma,\ \delta$ are roots of the biquadratic equation $ax^4+bx^3+cx^2+dx+e=0$, then

$S_1=\alpha+\beta+\gamma+\delta=-b/a$, $S_2=\alpha.\beta+\alpha.\gamma+\alpha\delta+\beta\gamma+\beta.\delta+\gamma\delta=(-1)^2\dfrac{c}{a}=\dfrac{c}{a}$

or $S_2=(\alpha+\beta)(\gamma+\delta)+\alpha\beta+\gamma\delta=c/a$, $S_3=\alpha\beta\gamma+\beta\gamma\delta+\gamma\delta\alpha+\alpha\beta\delta=(-1)^3\dfrac{d}{a}=-d/a$

or $S_3=\alpha\beta(\gamma+\delta)+\gamma\delta(\alpha+\beta)=-d/a$ and $S_4=\alpha.\beta.\gamma.\delta=(-1)^4\dfrac{e}{a}=\dfrac{e}{a}$

Example: 6 If the difference between the corresponding roots of $x^2+ax+b=0$ and $x^2+bx+a=0$ is same and $a\neq b$, then

[AIEEE 2002]

(a) $a+b+4=0$ (b) $a+b-4=0$ (c) $a-b-4=0$ (d) $a-b+4=0$

Solution: (a) $\alpha+\beta=-a$, $\alpha\beta=b\ \Rightarrow\ \alpha-\beta=\sqrt{a^2-4b}$ and $\gamma+\delta=-b$, $\gamma\delta=a\ \Rightarrow\ \gamma-\delta=\sqrt{b^2-4a}$

According to question, $\alpha-\beta=\gamma-\delta\ \Rightarrow\ \sqrt{a^2-4b}=\sqrt{b^2-4a}\ \Rightarrow\ a+b+4=0$

Example: 7 If the sum of the roots of the quadratic equation $ax^2+bx+c=0$ is equal to the sum of the squares of their reciprocals, then $a/c, b/a, c/b$ are in **[AIEEE 2003; DCE 2000]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

Solution: (c) As given, if α, β be the roots of the quadratic equation, then

$$\Rightarrow \alpha+\beta=\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha^2\beta^2} \Rightarrow -\frac{b}{a}=\frac{b^2/a^2-2c/a}{c^2/a^2}=\frac{b^2-2ac}{c^2}$$

$$\Rightarrow \frac{2a}{c}=\frac{b^2}{c^2}+\frac{b}{a}=\frac{ab^2+bc^2}{ac^2} \Rightarrow 2a^2c=ab^2+bc^2 \Rightarrow \frac{2a}{b}=\frac{b}{c}+\frac{c}{a}$$

$\frac{c}{a},\frac{a}{b},\frac{b}{c}$ are in A.P. $\Rightarrow \frac{a}{c},\frac{b}{a},\frac{c}{b}$ are in H.P.

Example: 8 Let α, β be the roots of $x^2-x+p=0$ and γ, δ be root of $x^2-4x+q=0$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then the integral value of p and q respectively are **[IIT Screening 2001]**

(a) – 2, – 32 (b) – 2, 3 (c) – 6, 3 (d) – 6, – 32

Solution: (a) $\alpha+\beta=1$, $\alpha\beta=p$, $\gamma+\delta=4$, $\gamma\delta=q$

Since $\alpha, \beta, \gamma, \delta$ are in G.P.

$r=\beta/\alpha=\gamma/\beta=\delta/\gamma$

$\alpha+\alpha r=1 \Rightarrow \alpha(1+r)=1$, $\alpha(r^2+r^3)=4 \Rightarrow \alpha.r^2(1+r)=4$

So $r^2=4 \Rightarrow r=\pm 2$

If $r=2$, $\alpha+2\alpha=1 \Rightarrow \alpha=1/3$ and $r=-2$, $\alpha-2\alpha=1 \Rightarrow \alpha=-1$

But $p=\alpha\beta \in I \;\therefore\; r=-2, \alpha=-1$

$\therefore\; p=-2$, $q=\alpha^2 r^5=1(-2)^5=-32$

Example: 9 If 1 – i is a root of the equation $x^2+ax+b=0$, then the values of a and b are **[Tamil Nadu Engg. 2002]**

(a) 2, 1 (b) – 2, 2 (c) 2, 2 (d) 2, – 2

Solution: (b) Since $1-i$ is a root of $x^2+ax+b=0$. $\therefore$ $1+i$ is also a root.

Sum of roots $\Rightarrow 1-i+1+i=-a \Rightarrow a=-2$

Product of roots $\Rightarrow (1-i)(1+i)=b \Rightarrow b=2$

Hence $a=-2$, $b=2$

Example: 10 If the roots of the equation $x^2-5x+16=0$ are α, β and the roots of equation $x^2+px+q=0$ are $\alpha^2+\beta^2$, $\alpha\beta/2$, then **[MP PET 2001]**

(a) $p=1, q=-56$ (b) $p=-1, q=-56$ (c) $p=1, q=56$ (d) $p=-1, q=56$

Solution: (b) Since roots of the equation $x^2-5x+16=0$ are α, β.

$$\Rightarrow \alpha+\beta=5, \alpha\beta=16 \text{ and } \alpha^2+\beta^2+\frac{\alpha\beta}{2}=-p \Rightarrow (\alpha+\beta)^2-2\alpha\beta+\frac{\alpha\beta}{2}=-p \Rightarrow 25-2(16)+\frac{16}{2}=-p \Rightarrow p=-1$$

and $(\alpha^2+\beta^2)\left(\frac{\alpha\beta}{2}\right)=q \Rightarrow [(\alpha+\beta)^2-2\alpha\beta]\frac{\alpha\beta}{2}=q \Rightarrow (25-32)8=q \Rightarrow q=-56$

Example: 11 If $\alpha \neq \beta$, but $\alpha^2=5\alpha-3, \beta^2=5\beta-3$, then the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is **[EAMCET 1989; AIEEE 2002]**

(a) $x^2-5x-3=0$ (b) $3x^2-19x+3=0$ (c) $3x^2+12x+3=0$ (d) None of these

Solution: (b) $S=\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{5\alpha-3+5\beta-3}{\alpha\beta}$ $\left[\because \alpha^2=5\alpha-3,\ \beta^2=5\beta-3\right]$

$S=\frac{5(\alpha+\beta)-6}{\alpha\beta}$, $p=\frac{\alpha}{\beta}\cdot\frac{\beta}{\alpha}=1 \Rightarrow p=1$. α, β are roots of $x^2-5x+3=0$. Therefore $\alpha+\beta=5$, $\alpha\beta=3$

$S=\frac{5(5)-6}{3}=\frac{19}{3}$

$\because\ x^2 - \frac{19}{3}x + 1 = 0 \Rightarrow 3x^2 - 19x + 3 = 0$

Example: 12 Let α, β be the roots of the equation $(x-a)(x-b) = c$, $c \neq 0$, then the roots of the equation $(x-\alpha)(x-\beta) + c = 0$ are

[IIT 1992; DCE 1998, 2000; Roorkee 2000]

(a) a, c (b) b, c (c) a, b (d) a, d

Solution: (c) Since α, β are the roots of $(x-a)(x-b) = c$ *i.e.* of $x^2 - (a+b)x + ab - c = 0$

$\therefore\ \alpha + \beta = a + b \Rightarrow a + b = \alpha + \beta$ and $\alpha\beta = ab - c \Rightarrow ab = \alpha\beta + c$

$\therefore$ a, b are the roots of $x^2 - (\alpha+\beta)x + \alpha\beta + c = 0 \Rightarrow (x-\alpha)(x-\beta) + c = 0$

Hence (c) is the correct answer

Example: 13 If α and β are roots of the equation $x^2 - ax + b = 0$ and $V_n = \alpha^n + \beta^n$, then **[Rajasthan PET 1995; Karnataka CET 20**

(a) $V_{n+1} = aV_n - bV_{n-1}$ (b) $V_{n+1} = bV_n - aV_{n-1}$ (c) $V_{n+1} = aV_n + bV_{n-1}$ (d) $V_{n+1} = bV_n + aV_{n-1}$

Solution: (a) Since α and β are roots of equation, $x^2 - ax + b = 0$, therefore $\alpha + \beta = a$, $\alpha\beta = b$

Now, $V_{n+1} = \alpha^{n+1} + \beta^{n+1} = (\alpha+\beta)(\alpha^n + \beta^n) - \alpha\beta(\alpha^{n-1} + \beta^{n-1}) \Rightarrow V_{n+1} = a.V_n - b.V_{n-1}$

Example: 14 If one root of the equation $x^2 + px + q = 0$ is the square of the other, then **[IIT Screening 2004]**

(a) $p^3 + q^2 - q(3p+1) = 0$ (b) $p^3 + q^2 + q(1+3p) = 0$

(c) $p^3 + q^2 + q(3p-1) = 0$ (d) $p^3 + q^2 + q(1-3p) = 0$

Solution: (d) Let α and α^2 be the roots then $\alpha + \alpha^2 = -p$, $\alpha.\alpha^2 = q$

Now $(\alpha+\alpha^2)^3 = \alpha^3 + \alpha^6 + 3\alpha^3(\alpha+\alpha^2) \Rightarrow -p^3 = q + q^2 - 3pq \Rightarrow p^3 + q^2 + q(1-3p) = 0$

Example: 15 Let α and β be the roots of the equation $x^2 + x + 1 = 0$, the equation whose roots are α^{19}, β^7 is**[IIT 1994; Pb. CET**

(a) $x^2 - x - 1 = 0$ (b) $x^2 - x + 1 = 0$ (c) $x^2 + x - 1 = 0$ (d) $x^2 + x + 1 = 0$

Solution: (d) Roots of $x^2 + x + 1 = 0$ are $x = \frac{-1 \pm \sqrt{1-4}}{2}, = \frac{-1 \pm \sqrt{3}i}{2} = \omega, \omega^2$

Take $\alpha = \omega, \beta = \omega^2$

$\therefore\ \alpha^{19} = w^{19} = w, \beta^7 = (w^2)^7 = w^{14} = w^2$

$\therefore$ Required equation is $x^2 + x + 1 = 0$

Example: 16 If one root of a quadratic equation is $\frac{1}{2+\sqrt{5}}$, then the equation is **[Rajasthan PET 1987]**

(a) $x^2 + 4x + 1 = 0$ (b) $x^2 + 4x - 1 = 0$ (c) $x^2 - 4x + 1 = 0$ (d) None of these

Solution: (b) Given root $= \frac{1}{2+\sqrt{5}} = \frac{2-\sqrt{5}}{-1} = -2 + \sqrt{5}$, $\therefore$ other root $= -2 - \sqrt{5}$

Again, sum of roots = – 4 and product of roots = – 1. The required equation is $x^2 + 4x - 1 = 0$

4.8 Condition for Common Roots

(1) **Only one root is common** : Let α be the common root of quadratic equations $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$.

$\therefore\ a_1\alpha^2 + b_1\alpha + c_1 = 0$, $a_2\alpha^2 + b_2\alpha + c_2 = 0$

By Crammer's rule : $\frac{\alpha^2}{\begin{vmatrix} -c_1 & b_1 \\ -c_2 & b_2 \end{vmatrix}} = \frac{\alpha}{\begin{vmatrix} a_1 & -c_1 \\ a_2 & -c_2 \end{vmatrix}} = \frac{1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$ or $\frac{\alpha^2}{b_1c_2 - b_2c_1} = \frac{\alpha}{a_2c_1 - a_1c_2} = \frac{1}{a_1b_2 - a_2b_1}$

$\therefore\ \alpha = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1} = \frac{b_1c_2 - b_2c_1}{a_2c_1 - a_1c_2}$, $\alpha \neq 0$

$\therefore$ The condition for only one root common is $(c_1a_2 - c_2a_1)^2 = (b_1c_2 - b_2c_1)(a_1b_2 - a_2b_1)$

(2) **Both roots are common:** Then required condition is $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

Important Tips

- ☞ *To find the common root of two equations, make the coefficient of second degree term in the two equations equal and subtract. The value of x obtained is the required common root.*
- ☞ *Two different quadratic equations with rational coefficient can not have single common root which is complex or irrational as imaginary and surd roots always occur in pair.*

Example: 17 If one of the roots of the equation $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ is coincident. Then the numerical value of $(a+b)$ is

[IIT 1986; Rajasthan PET 1992; EAMCET 2002]

(a) 0 (b) – 1 (c) (d) 5

Solution: (b) If α is the coincident root, then $\alpha^2 + a\alpha + b = 0$ and $\alpha^2 + b\alpha + a = 0$

$\Rightarrow \frac{\alpha^2}{a^2 - b^2} = \frac{\alpha}{b-a} = \frac{1}{b-a}$

$\alpha^2 = -(a+b)$, $\alpha = 1 \Rightarrow -(a+b) = 1 \Rightarrow (a+b) = -1$

Example: 18 If *a*, *b*, *c* are in G.P. then the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in

[IIT 1985; Pb. CET 2000; DCE 2000]

(a) A.P. (b) G.P. (c) H.P. (d) None of these

Solution: (a) As given, $b^2 = ac \Rightarrow ax^2 + 2bx + c = 0$ can be written as $ax^2 + 2\sqrt{ac}x + c = 0 \Rightarrow (\sqrt{a}x + \sqrt{c})^2 = 0 \Rightarrow x = -\sqrt{\frac{c}{a}}$

This must be common root by hypothesis

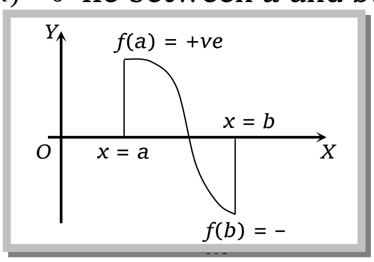
So it must satisfy the equation, $dx^2 + 2ex + f = 0 \Rightarrow d\left(\frac{c}{a}\right) - 2e\sqrt{\frac{c}{a}} + f = 0$

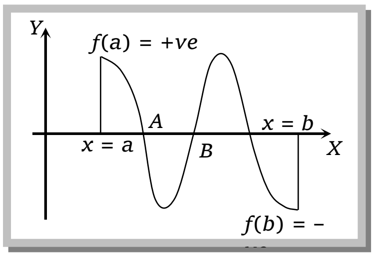
$\frac{d}{a} + \frac{f}{c} = \frac{2e}{c}\sqrt{\frac{c}{a}} = \frac{2e}{\sqrt{c}.\sqrt{a}} \Rightarrow \frac{d}{a} + \frac{f}{c} = \frac{2e}{b}$

Hence $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

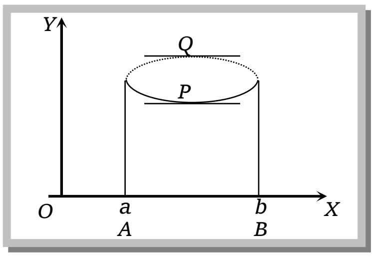
4.9 Properties of Quadratic Equation

(1) If *f*(*a*) and *f*(*b*) are of opposite signs then at least one or in general odd number of roots of the equation $f(x) = 0$ lie between *a* and *b*.





(2) If $f(a) = f(b)$ then there exists a point *c* between *a* and *b* such that $f'(c) = 0$, $a < c < b$.



As is clear from the figure, in either case there is a point P or Q at $x = c$ where tangent is parallel to *x-axis*

i.e. $f'(x) = 0$ at $x = c$.

(3) If α is a root of the equation $f(x) = 0$ then the polynomial $f(x)$ is exactly divisible by $(x - \alpha)$ or $(x - \alpha)$ is factor of $f(x)$.

(4) If the roots of the quadratic equations $ax^2 + bx + c = 0$, $a_2x^2 + b_2x + c_2 = 0$ are in the same ratio $\left(i.e. \frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} \right)$ then $b_1^2 / b_2^2 = a_1c_1 / a_2c_2$.

(5) If one root is k times the other root of the quadratic equation $ax^2 + bx + c = 0$ then $\frac{(k+1)^2}{k} = \frac{b^2}{ac}$.

Example: 19 The value of 'a' for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other is

[AIEEE 2003]

(a) 2/3 (b) – 2/3 (c) 1/3 (d) – 1/3

Solution: (a) Let the roots are α and 2α

Now, $\alpha + 2\alpha = \frac{1 - 3a}{a^2 - 5a + 3}$, $\alpha . 2\alpha = \frac{2}{a^2 - 5a + 3}$ $\Rightarrow 3\alpha = \frac{1 - 3a}{a^2 - 5a + 3}$, $2\alpha^2 = \frac{2}{a^2 - 5a + 3}$

$\Rightarrow 2\left[\frac{1}{9} \frac{(1 - 3a)^2}{(a^2 - 5a + 3)^2}\right] = \frac{2}{a^2 - 5a + 3} \Rightarrow \frac{(1 - 3a)^2}{a^2 - 5a + 3} = 9 \Rightarrow 9a^2 - 45a + 27 = 1 + 9a^2 - 6a \Rightarrow 39a = 26 \Rightarrow a = 2/3$

4.10 Quadratic Expression

An expression of the form $ax^2 + bx + c$, where a, b, $c \in R$ and $a \neq 0$ is called a quadratic expression in x. So in general, quadratic expression is represented by $f(x) = ax^2 + bx + c$ or $y = ax^2 + bx + c$.

(1) **Graph of a quadratic expression :** We have $y = ax^2 + bx + c = f(x)$

$$y = a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{D}{4a^2}\right] \Rightarrow y + \frac{D}{4a} = a\left(x + \frac{b}{2a}\right)^2$$

Now, let $y + \frac{D}{4a} = Y$ and $X = x + \frac{b}{2a}$

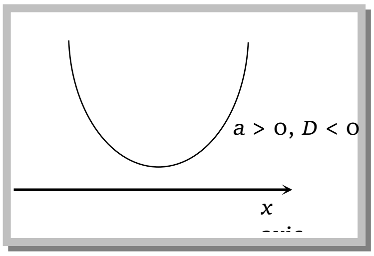
$$Y = a.X^2 \Rightarrow X^2 = \frac{1}{a}Y$$

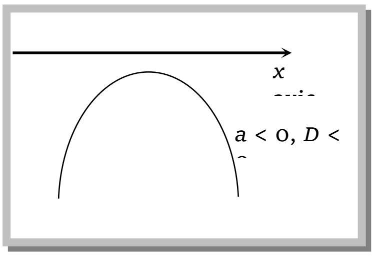
(i) The graph of the curve $y = f(x)$ is parabolic.

(ii) The axis of parabola is $X = 0$ or $x + \frac{b}{2a} = 0$ *i.e.* (parallel to y-axis).

(iii) (a) If $a > 0$, then the parabola opens upward.

(b) If $a < 0$, then the parabola opens downward.



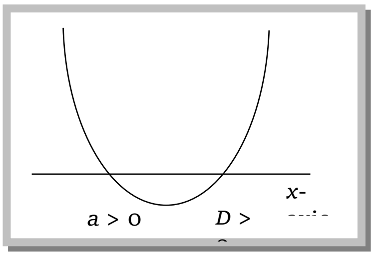


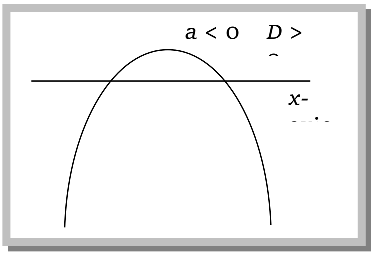
(iv) **Intersection with axis**

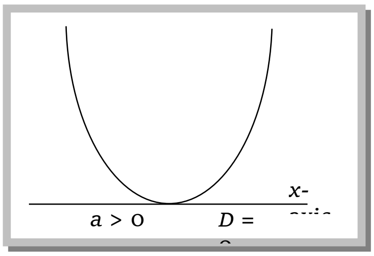
(a) ***x-axis*:** For x axis, $y = 0 \Rightarrow ax^2 + bx + c = 0 \Rightarrow x = \dfrac{-b \pm \sqrt{D}}{2a}$

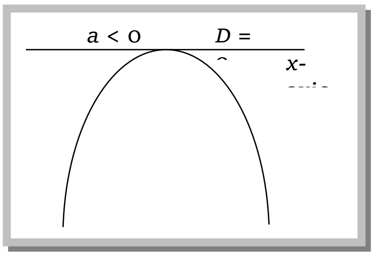
For $D > 0$, parabola cuts x-axis in two real and distinct points *i.e.* $x = \dfrac{-b \pm \sqrt{D}}{2a}$.

For $D = 0$, parabola touches x-axis in one point, $x = -b/2a$.

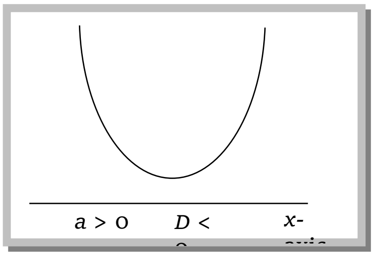


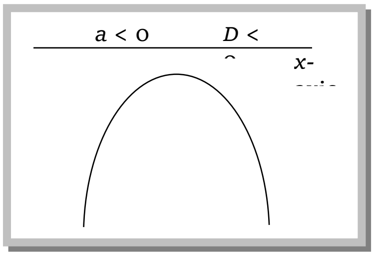






For $D < 0$, parabola does not cut *x-axis*(*i.e.* imaginary value of x).





(b) ***y*-axis :** For y axis $x = 0$, $y = c$

(2) **Maximum and minimum values of quadratic expression :** Maximum and minimum value of quadratic expression can be found out by two methods :

(i) **Discriminant method :** In a quadratic expression $ax^2 + bx + c$.

(a) If $a > 0$, quadratic expression has least value at $x = -b/2a$. This least value is given by $\frac{4ac-b^2}{4a} = -\frac{D}{4a}$.

(b) If $a < 0$, quadratic expression has greatest value at $x = -b/2a$. This greatest value is given by $\frac{4ac-b^2}{4a} = -\frac{D}{4a}$.

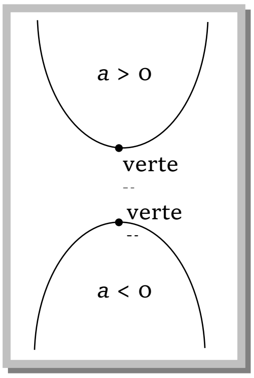
(ii) **Graphical method :** Vertex of the parabola $Y = aX^2$ is $X = 0$, $Y = 0$

i.e., $x + \frac{b}{2a} = 0$, $y + \frac{D}{4a} = 0 \Rightarrow x = -b/2a$, $y = -D/4a$

Hence, vertex of $y = ax^2 + bx + c$ is $(-b/2a, -D/4a)$

(a) For $a > 0$, $f(x)$ has least value at $x = -\frac{b}{2a}$. This least value is given by $f\left(-\frac{b}{2a}\right) = -\frac{D}{4a}$.

(b) For $a < 0$, $f(x)$ has greatest value at $x = -b/2a$. This greatest value is given by $f\left(-\frac{b}{2a}\right) = -\frac{D}{4a}$.



(3) **Sign of quadratic expression :** Let $f(x) = ax^2 + bx + c$ or $y = ax^2 + bx + c$

Where $a, b, c \in R$ and $a \neq 0$, for some values of x, $f(x)$ may be positive, negative or zero. This gives the following cases :

(i) $a > 0$ and $D < 0$, so $f(x) > 0$ for all $x \in R$ *i.e.*, $f(x)$ is positive for all real values of x.

(ii) $a < 0$ and $D < 0$, so $f(x) < 0$ for all $x \in R$ *i.e.*, $f(x)$ is negative for all real values of x.

(iii) $a > 0$ and $D = 0$ so, $f(x) \geq 0$ for all $x \in R$ *i.e.*, $f(x)$ is positive for all real values of x except at vertex, where $f(x) = 0$.

(iv) $a < 0$ and $D = 0$ so, $f(x) \leq 0$ for all $x \in R$ *i.e.* $f(x)$ is negative for all real values of x except at vertex, where $f(x) = 0$.

(v) $a > 0$ and $D > 0$

Let $f(x) = 0$ have two real roots α and β $(\alpha < \beta)$, then $f(x) > 0$ for all $x \in (-\infty, \alpha) \cup (\beta, \infty)$ and $f(x) < 0$ for all $x \in (\alpha, \beta)$.

(vi) $a < 0$ and $D > 0$

Let $f(x) = 0$ have two real roots α and β $(\alpha < \beta)$,

Then $f(x) < 0$ for all $x \in (-\infty, \alpha) \cup (\beta, \infty)$ and $f(x) > 0$ for all $x \in (\alpha, \beta)$

Example: 20 If x be real, then the minimum value of $x^2 - 8x + 17$ is **[MNR 1980]**

(a) – 1 (b) 0 (c) 1 (d) 2

Solution: (c) Since $a = 1 > 0$ therefore its minimum value is $= \frac{4ac-b^2}{4a} = \frac{4(1)(17)-64}{4} = \frac{4}{4} = 1$

Example: 21 If x is real, then greatest and least values of $\frac{x^2-x+1}{x^2+x+1}$ are **[IIT 1968; Rajasthan PET 1988]**

(a) 3, –1/2 (b) 3, 1/3 (c) – 3, –1/3 (d) None of these

Solution: (b) Let $y = \frac{x^2 - x + 1}{x^2 + x + 1}$

$x^2(y-1) + (y+1)x + (y-1) = 0$

$\because$ x is real, therefore $b^2 - 4ac \geq 0$

$\Rightarrow (y+1)^2 - 4(y-1)(y-1) \geq 0 \Rightarrow 3y^2 - 10y + 3 \leq 0 \Rightarrow (3y-1)(y-3) \leq 0 \Rightarrow \left(y - \frac{1}{3}\right)(y-3) \leq 0 \Rightarrow \frac{1}{3} \leq y \leq 3$

Thus greatest and least values of expression are 3, 1/3 respectively.

Example: 22 If $f(x)$ is quadratic expression which is positive for all real value of x and $g(x) = f(x) + f'(x) + f''(x)$. Then for any real value of x **[IIT 1990]**

(a) $g(x) < 0$ (b) $g(x) > 0$ (c) $g(x) = 0$ (d) $g(x) \geq 0$

Solution: (b) Let $f(x) = ax^2 + bx + c$, then $g(x) = ax^2 + bx + c + 2ax + b + 2a = ax^2 + (b+2a)x + (b+c+2a)$

$\because$ $f(x) > 0$. Therefore $b^2 - 4ac < 0$ and $a > 0$

Now for $g(x)$,

Discriminant $= (b+2a)^2 - 4a(b+c+2a) = b^2 + 4a^2 + 4ab - 4ab - 4ac - 8a^2 = (b^2 - 4ac) - 4a^2 < 0$ as $b^2 - 4ac < 0$

Therefore sign of $g(x)$ and a are same *i.e.* $g(x) > 0$.

Example: 23 If α, β $(\alpha < \beta)$ are roots of the equation $x^2 + bx + c = 0$ where $(c < 0 < b)$ then **[IIT Screening 2000]**

(a) $0 < \alpha < \beta$ (b) $\alpha < 0 < \beta < |\alpha|$ (c) $\alpha < \beta < 0$ (d) $\alpha < 0 < |\alpha| < \beta$

Solution: (b) Since $f(0) = 0 + 0 + c = c < 0$

$\therefore$ Roots will be of opposite sign, $\alpha + \beta = -b = -ve$ $(b > 0)$

It is given that $\alpha < \beta$

So, $\alpha + \beta = -ve$ is possible only when $|\alpha| > \beta$

$\Rightarrow \alpha < 0, \beta > 0, |\alpha| > \beta \Rightarrow \alpha < 0 < \beta < |\alpha|$

4.11 Wavy Curve Method

Let $f(x) = (x - a_1)^{k_1}(x - a_2)^{k_2}(x - a_3)^{k_3} ....(x - a_{n-1})^{k_{n-1}}(x - a_n)^{k_n}$ .....(i)

Where $k_1, k_2, k_3 ..., k_n \in N$ and $a_1, a_2, a_3, ..., a_n$ are fixed natural numbers satisfying the condition

$$a_1 < a_2 < a_3 ..... < a_{n-1} < a_n$$

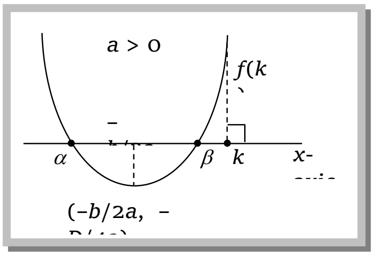
First we mark the numbers $a_1, a_2, a_3, ..., a_n$ on the real axis and the plus sign in the interval of the right of the largest of these numbers, *i.e.* on the right of a_n. If k_n is even then we put plus sign on the left of a_n and if k_n is odd then we put minus sign on the left of a_n. In the next interval we put a sign according to the following rule :

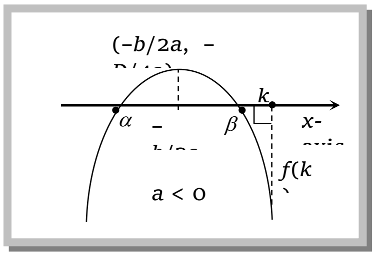
When passing through the point a_{n-1} the polynomial $f(x)$ changes sign if k_{n-1} is an odd number and the polynomial $f(x)$ has same sign if k_{n-1} is an even number. Then, we consider the next interval and put a sign in it using the same rule. Thus, we consider all the intervals. The solution of $f(x) > 0$ is the union of all intervals in which we have put the plus sign and the solution of $f(x) < 0$ is the union of all intervals in which we have put the minus sign.

4.12 Position of Roots of a Quadratic Equation

Let $f(x) = ax^2 + bx + c$, where a, b, $c \in R$ be a quadratic expression and k, k_1, k_2 be real numbers such that $k_1 < k_2$. Let α, β be the roots of the equation $f(x) = 0$ *i.e.* $ax^2 + bx + c = 0$. Then $\alpha = \frac{-b + \sqrt{D}}{2a}$, $\beta = \frac{-b - \sqrt{D}}{2a}$ where D is the discriminant of the equation.

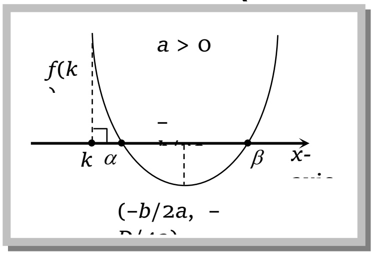
(1) **Condition for a number *k* (If both the roots of *f*(*x*) = 0 are less than *k*)**

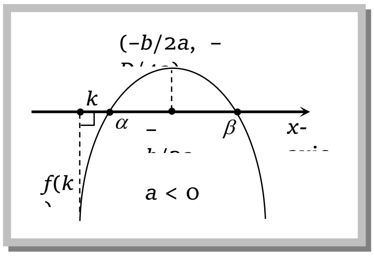




(i) $D \geq 0$ (roots may be equal) (ii) $a\,f(k) > 0$ (iii) $k > -b/2a$, where $\alpha \leq \beta$

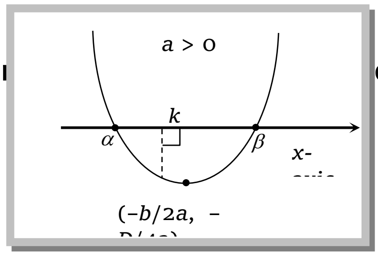
(2) **Condition for a number *k* (If both the roots of *f*(*x*) = 0 are greater than *k*)**

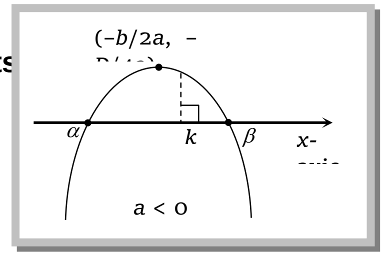




(i) $D \geq 0$ (roots may be equal) (ii) $a\,f(k) > 0$ (iii) $k < -b/2a$, where $\alpha \leq \beta$

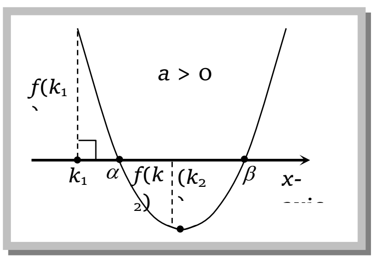
(3) **Conditio** **etween the roots**

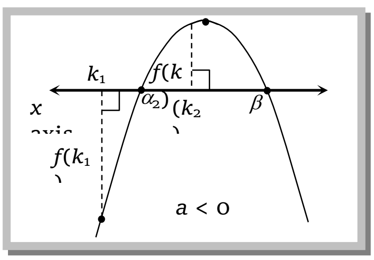




(i) $D > 0$ (ii) $a\,f(k) < 0$, where $\alpha < \beta$

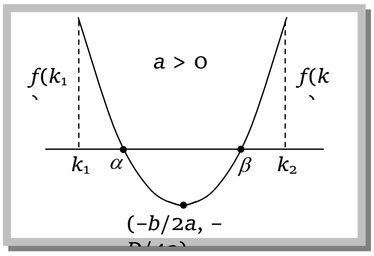
(4) **Condition for numbers k_1 and k_2 (If exactly one root of *f*(*x*) = 0 lies in the interval (k_1, k_2))**

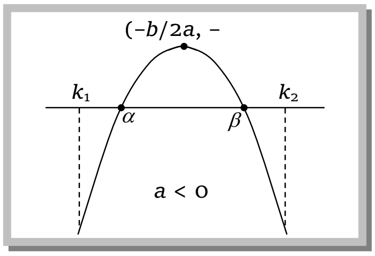




(i) $D > 0$ (ii) $f(k_1)f(k_2) < 0$, where $\alpha < \beta$.

(5) **Condition for numbers k_1 and k_2 (If both roots of $f(x) = 0$ are confined between k_1 and k_2)**

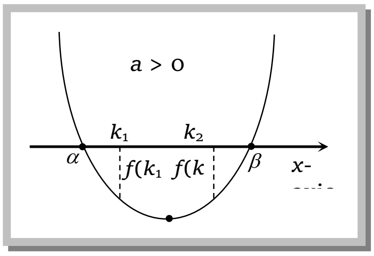


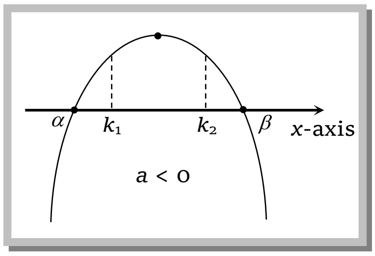


(i) $D \geq 0$ (roots may be equal) (ii) $a\,f(k_1) > 0$ (iii) $a\,f(k_2) > 0$

(iv) $k_1 < -b/2a < k_2$, where $\alpha \leq \beta$ and $k_1 < k_2$

(6) **Condition for numbers k_1 and k_2 (If k_1 and k_2 lie between the roots of $f(x) = 0$)**





(i) $D > 0$ (ii) $a\,f(k_1) < 0$ (iii) $a\,f(k_2) < 0$, where $\alpha < \beta$

Example: 24 If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3, then **[IIT 1999; MP PET 2000]**

(a) $a < 2$ (b) $2 \leq a \leq 3$ (c) $3 < a \leq 4$ (d) $a > 4$

Solution: (a) Given equation is $x^2 - 2ax + a^2 + a - 3 = 0$

If roots are real, then $D \geq 0$

$\Rightarrow 4a^2 - 4(a^2 + a - 3) \geq 0 \Rightarrow -a + 3 \geq 0 \Rightarrow a - 3 \leq 0 \Rightarrow a \leq 3$

As roots are less than 3, hence $f(3) > 0$

$9 - 6a + a^2 + a - 3 > 0 \Rightarrow a^2 - 5a + 6 > 0 \Rightarrow (a-2)(a-3) > 0 \Rightarrow a < 2, a > 3$. Hence $a < 2$ satisfy all the conditions.

Example: 25 The value of a for which $2x^2 - 2(2a+1)x + a(a+1) = 0$ may have one root less than a and another root greater than a are given by **[UPSEAT 2001]**

(a) $1 > a > 0$ (b) $-1 < a < 0$ (c) $a \geq 0$ (d) $a > 0$ or $a < -1$

Solution: (d) The given condition suggest that a lies between the roots. Let $f(x) = 2x^2 - 2(2a+1)x + a(a+1)$

For 'a' to lie between the roots we must have Discriminant ≥ 0 and $f(a) < 0$

Now, Discriminant ≥ 0

$4(2a+1)^2 - 8a(a+1) \geq 0 \Rightarrow 8(a^2 + a + 1/2) \geq 0$ which is always true.

Also $f(a) < 0 \Rightarrow 2a^2 - 2a(2a+1) + a(a+1) < 0 \Rightarrow -a^2 - a < 0 \Rightarrow a^2 + a > 0 \Rightarrow a(1+a) > 0 \Rightarrow a > 0$ or $a < -1$

4.13 Descarte's Rule of Signs

The maximum number of positive real roots of a polynomial equation $f(x)=0$ is the number of changes of sign from positive to negative and negative to positive in $f(x)$.

The maximum number of negative real roots of a polynomial equation $f(x)=0$ is the number of changes of sign from positive to negative and negative to positive in $f(-x)$.

Example: 26 The maximum possible number of real roots of equation $x^5-6x^2-4x+5=0$ is **[EAMCET 2002]**

(a) 0 (b) 3 (c) 4 (d) 5

Solution: (b) $f(x)=x^5-6x^2-4x+5=0$

+ − − +

2 changes of sign ⇒ maximum two positive roots.

$f(-x)=-x^5-6x^2+4x+5$

− − + +

1 changes of sign ⇒ maximum one negative roots.

⇒ total maximum possible number of real roots = 2 + 1 = 3.

4.14 Rational Algebraic Inequations

(1) **Values of rational expression *P*(*x*)/*Q*(*x*) for real values of *x*, where *P*(*x*) and *Q*(*x*) are quadratic expressions :** To find the values attained by rational expression of the form $\frac{a_1x^2+b_1x+c_1}{a_2x^2+b_2x+c_2}$ for real values of x, the following algorithm will explain the procedure :

Algorithm

Step I: Equate the given rational expression to y.

Step II: Obtain a quadratic equation in x by simplifying the expression in step I.

Step III: Obtain the discriminant of the quadratic equation in Step II.

Step IV: Put Discriminant ≥ 0 and solve the inequation for y. The values of y so obtained determines the set of values attained by the given rational expression.

(2) **Solution of rational algebraic inequation:** If $P(x)$ and $Q(x)$ are polynomial in x, then the inequation $\frac{P(x)}{Q(x)}>0, \frac{P(x)}{Q(x)}<0, \frac{P(x)}{Q(x)}\geq 0$ and $\frac{P(x)}{Q(x)}\leq 0$ are known as rational algebraic inequations.

To solve these inequations we use the sign method as explained in the following algorithm.

Algorithm

Step I: Obtain $P(x)$ and $Q(x)$.

Step II: Factorize $P(x)$ and $Q(x)$ into linear factors.

Step III: Make the coefficient of x positive in all factors.

Step IV: Obtain critical points by equating all factors to zero.

Step V: Plot the critical points on the number line. If there are n critical points, they divide the number line into $(n+1)$ regions.

Step VI: In the right most region the expression $\frac{P(x)}{Q(x)}$ bears positive sign and in other regions the expression bears positive and negative signs depending on the exponents of the factors.

4.15 Algebraic Interpretation of Rolle's Theorem

Let $f(x)$ be a polynomial having α and β as its roots, such that $\alpha < \beta$. Then, $f(\alpha) = f(\beta) = 0$. Also a polynomial function is everywhere continuous and differentiable. Thus $f(x)$ satisfies all the three conditions of Rolle's theorem. Consequently there exists $\gamma \in (\alpha, \beta)$ such that $f'(\gamma) = 0$ *i.e.* $f'(x) = 0$ at $x = \gamma$. In other words $x = \gamma$ is a root of $f'(x) = 0$. Thus algebraically Rolle's theorem can be interpreted as follows.

Between any two roots of polynomial $f(x)$, there is always a root of its derivative $f'(x)$.

Lagrange's theorem : Let $f(x)$ be a function defined on $[a\ b]$ such that

(i) $f(x)$ is continuous on $[a\ b]$ and

(ii) $f(x)$ is differentiable on (a, b), then $c \in (a, b)$, such that $f'(c) = \frac{f(b) - f(a)}{b - a}$

Lagrange's identity : If $a_1, a_2, a_3, b_1, b_2, b_3 \in R$ then :

$$(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2) - (a_1b_1 + a_2b_2 + a_3b_3)^2 = (a_1b_2 - a_2b_1)^2 + (a_2b_3 - a_3b_2)^2 + (a_3b_1 - a_1b_3)^2$$

Example: 27 If $\frac{2x}{2x^2 + 5x + 2} > \frac{1}{x+1}$, then **[IIT 1987]**

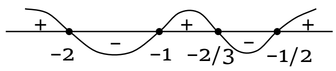
(a) $-2 > x > -1$ (b) $-2 \geq x \geq -1$ (c) $-2 < x < -1$ (d) $-2 < x \leq -1$

Solution: (c) Given $\frac{2x}{2x^2 + 5x + 2} - \frac{1}{x+1} > 0 \Rightarrow \frac{2x^2 + 2x - 2x^2 - 5x - 2}{(2x+1)(x+2)(x+1)} > 0 \Rightarrow \frac{-3x - 2}{(2x+1)(x+2)(x+1)} > 0$

$\Rightarrow \frac{-3(x + 2/3)}{(x+1)(x+2)(2x+1)} > 0 \Rightarrow \frac{(x + 2/3)}{(x+1)(x+2)(2x+1)} < 0$

Equating each factor equal to 0,

We get $x = -2, -1, -2/3, -1/2$



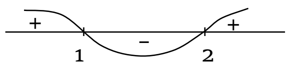
$\therefore\ x \in]-2, -1[\cup]-2/3, -1/2[\Rightarrow -2/3 < x < -1/2$ or $-2 < x < -1$

Example: 28 If for real values of x, $x^2 - 3x + 2 > 0$ and $x^2 - 3x - 4 \leq 0$, then **[IIT 1983]**

(a) $-1 \leq x < 1$ (b) $-1 \leq x < 4$ (c) $-1 \leq x < 1$ or $2 < x \leq 4$ (d) $2 < x \leq 4$

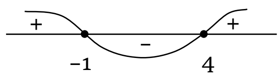
Solution: (c) $x^2 - 3x + 2 > 0$ or $(x-1)(x-2) > 0$

$\therefore\ x \in (-\infty, 1) \cup (2, \infty)$ .....(i)



Again $x^2 - 3x - 4 \leq 0$ or $(x-4)(x+1) \leq 0$

$\therefore\ x \in [-1, 4]$ .....(ii)



From eq. (i) and (ii), $x \in [-1, 1) \cup (2\ 4] \Rightarrow -1 \leq x < 1$ or $2 < x \leq 4$

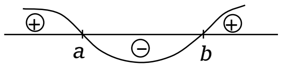
Example: 29 If $b > a$, then the equation $(x - a)(x - b) - 1 = 0$, has **[IIT Screening 2000]**

(a) Both roots in $[a\ b]$ (b) Both roots in $(-\infty, a)$

(c) Both roots in (b, ∞) (d) One root in $(-\infty, a)$ and other in $(b, +\infty)$

Solution: (d) We have, $(x-a)(x-b)-1=0$

$(x-a)(x-b)=1>0 \Rightarrow (x-a)(x-b)>0$ [$\because b > a$]



$x \in]-\infty, a[\cup]b, +\infty[$, *i.e.* $(-\infty, a)$ and (b, ∞).

Example: 30 The number of integral solution of $\frac{x+1}{x^2+2} > \frac{1}{4}$ is **[Orissa JEE 2002]**

(a) 1 (b) 2 (c) 5 (d) None of these

Solution: (c) $\frac{x+1}{x^2+2} - \frac{1}{4} > 0 \Rightarrow \frac{x^2-4x-2}{x^2+2} < 0 \Rightarrow (x-(2+\sqrt{6}))(x-(2-\sqrt{6})) < 0$

$\Rightarrow 2-\sqrt{6} < x < 2+\sqrt{6}$

Approximately, $-0.4 < x < 4.4$

Hence, integral values of x are 0, 1, 2, 3, 4

Hence, number of integral solution = 5

Example: 31 If $2a+3b+6c=0$ then at least one root of the equation $ax^2+bx+c=0$ lies in the interval

[Kurukshetra CEE 2002; AIEEE 2002]

(a) (0, 1) (b) (1, 2) (c) (2, 3) (d) (3, 4)

Solution: (a) Let $f'(x) = ax^2+bx+c$

$\therefore \; f(x) = \int f'(x)\,dx = \frac{ax^3}{3} + \frac{bx^2}{2} + cx$

Clearly $f(0)=0$, $f(1) = \frac{a}{3} + \frac{b}{2} + c = \frac{2a+3b+6c}{6} = \frac{0}{6} = 0$

Since, $f(0)=f(1)=0$. Hence, there exists at least one point c in between 0 and 1, such that $f'(x)=0$, by Rolle's theorem.

Trick: Put the value of $a=-3, b=2, c=0$ in given equation

$-3x^2+2x=0 \Rightarrow 3x^2-2x=0 \Rightarrow x(3x-2)=0$

$x=0, x=2/3$, which lie in the interval (0, 1)

4.16 Equation and Inequation containing Absolute Value

(1) Equations containing absolute values

By definition, $|x| = \begin{cases} x, \text{ if } x \geq 0 \\ -x, \text{ if } x < 0 \end{cases}$

Important forms containing absolute value :

Form I: The equation of the form $|f(x)+g(x)| = |f(x)| + |g(x)|$ is equivalent of the system $f(x).g(x) \geq 0$.

Form II: The equation of the form $|f_1(x)| + |f_2(x)| + |f_3(x)| + \ldots\ldots |f_n(x)| = g(x)$ .....(i)

Where $f_1(x), f_2(x), f_3(x) \ldots\ldots f_n(x), g(x)$ are functions of x and $g(x)$ may be a constant.

Equations of this form can be solved by the method of interval. We first find all critical points of $f_1(x), f_2(x).....f_n(x)$. If coefficient of x is $+ve$, then graph starts with $+ve$ sign and if it is negative, then graph starts with negative sign. Then using the definition of the absolute value, we pass form equation (i) to a collection of system which do not contain the absolute value symbols.

(2) **Inequations containing absolute value**

By definition, $|x| < a \Rightarrow -a < x < a$ $(a > 0)$, $|x| \le a \Rightarrow -a \le x \le a$,

$$|x| > a \Rightarrow x < -a \text{ or } x > a \text{ and } |x| \ge a \Rightarrow x \le -a \text{ or } x \ge a$$

Example: 32 The roots of $|x-2|^2 + |x-2| - 6 = 0$ are **[UPSEAT 2003]**

(a) 0, 4 (b) –1, 3 (c) 4, 2 (d) 5, 1

Solution: (a) We have $|x-2|^2 + |x-2| - 6 = 0$

Let $|x-2| = X$

$X^2 + X - 6 = 0$

$\Rightarrow X = \dfrac{-1 \pm \sqrt{1+24}}{2} = 2, -3 \Rightarrow X = 2$ and $X = -3$

$\therefore |x-2| = 2$ and $|x-2| = -3$, which is not possible.

$\Rightarrow x - 2 = 2$ or $x - 2 = -2$

$\therefore x = 4$ or $x = 0$

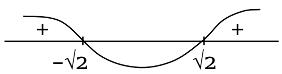
Example: 33 The set of all real numbers x for which $x^2 - |x+2| + x > 0$, is **[IIT Screening 2002]**

(a) $(-\infty, -2) \cup (2, \infty)$ (b) $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$ (c) $(-\infty, -1) \cup (1, \infty)$ (d) $(\sqrt{2}, \infty)$

Solution: (b) **Case I:** If $x + 2 \ge 0$ *i.e.* $x \ge -2$, we get

$x^2 - x - 2 + x > 0 \Rightarrow x^2 - 2 > 0 \Rightarrow (x - \sqrt{2})(x + \sqrt{2}) > 0$

$\Rightarrow x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$



But $x \ge -2$

$\therefore x \in [-2, -\sqrt{2}) \cup (\sqrt{2}\ \infty)$ .....(i)

Case II: $x + 2 < 0$ *i.e.* $x < -2$, then

$x^2 + x + 2 + x > 0 \Rightarrow x^2 + 2x + 2 > 0 \Rightarrow (x+1)^2 + 1 > 0$. Which is true for all x

$\therefore x \in (-\infty, -2)$ .....(ii)

From (i) and (ii), we get, $x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

Example: 34 Product of real roots of the equation $t^2x^2 + |x| + 9 = 0$ $(t \ne 0)$ **[AIEEE 2002]**

(a) Is always $+ve$ (b) Is always $-ve$ (c) Does not exist (d) None of these

Solution: (c) Expression is always $+ve$, so $t^2x^2 + |x| + 9 \ne 0$. Hence roots of given equation does not exist.

Example: 35 The number of solution of $\log_4(x-1) = \log_2(x-3)$ **[IIT Screening 2001]**

(a) 3 (b) 1 (c) 2 (d) 0

Solution: (b) We have $\log_4(x-1)=\log_2(x-3)$

$(x-1)=(x-3)^2 \Rightarrow x-1=x^2+9-6x \Rightarrow x^2-7x+10=0 \Rightarrow (x-5)(x-2)=0$

$x=5$ or $x=2$

But $x-3<0$, when $x=2$. $\therefore$ Only solution is $x=5$.

Hence number of solution is one.

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Assignment

Solution of Quadratic equations

Basic Level

1. A real root of the equation $\log_4\{\log_2(\sqrt{x+8}-\sqrt{x})\}=0$ is **[AMU 1999]**

(a) 1 (b) 2 (c) 3 (d) 4

2. The roots of the equation $7^{\log_7(x^2-4x+5)}=x-1$ are

(a) 4, 5 (b) 2, – 3 (c) 2, 3 (d) 3, 5

3. The solution set of the equation $\log_x 2.\log_{2x} 2=\log_{4x} 2$ is

(a) $\{2^{-\sqrt{2}},2^{\sqrt{2}}\}$ (b) $\left\{\frac{1}{2},\ 2\right\}$ (c) $\left\{\frac{1}{4},\ 2^2\right\}$ (d) None of these

4. The solution of the equation $3^{\log_a x}+3x^{\log_a 3}=2$ is given by

(a) $3^{\log_2 a}$ (b) $3^{-\log_2 a}$ (c) $2^{\log_3 a}$ (d) $2^{-\log_3 a}$

5. If $3^{x+1}=6^{\log_2 3}$, then x is

(a) 3 (b) 2 (c) $\log_3 2$ (d) $\log_2 3$

6. The solution of $|x/(x-1)|+|x|=x^2/|x-1|$ is

(a) $x\geq 0$ (b) $x>0$ (c) $x\in(1,\alpha)$ (d) None of these

7. If $2\log(x+1)-\log(x^2-1)=\log 2$, then x equals

(a) 1 (b) 0 (c) 2 (d) 3

8. The real roots of the equation $x^2+5|x|+4=0$ are **[MNR 1993]**

(a) $\{-1,-4\}$ (b) $\{1,4\}$ (c) $\{-4,4\}$ (d) None of these

9. If $|x^2-x-6|=x+2$, then the values of x are **[Roorkee 1982; Rajasthan PET 1992]**

(a) $-2,2,-4$ (b) $-2,2,4$ (c) $3,2,-2$ (d) $4,4,3$

10. $\{x\in R:|x-2|=x^2\}=$ **[EAMCET 2000]**

(a) $\{-1,2\}$ (b) $\{1,2\}$ (c) $\{-1,-2\}$ (d) $\{1,-2\}$

11. If $ax^2+bx+c=0$, then $x=$ **[MP PET 1995]**

(a) $\frac{b\pm\sqrt{b^2-4ac}}{2a}$ (b) $\frac{-b\pm\sqrt{b^2-ac}}{2a}$ (c) $\frac{2c}{-b\pm\sqrt{b^2-4ac}}$ (d) None of these

12. If $x^{2/3}-7x^{1/3}+10=0$, then $x=$ **[BIT Ranchi 1992]**

(a) $\{125\}$ (b) $\{8\}$ (c) ϕ (d) $\{125,8\}$

13. The roots of the given equation $(p-q)x^2+(q-r)x+(r-p)=0$ are **[Rajasthan PET 1986; MP PET 1999]**

(a) $\frac{p-q}{r-p},1$ (b) $\frac{q-r}{p-q},1$ (c) $\frac{r-p}{p-q},1$ (d) $1,\frac{q-r}{p-q}$

14. The solution of the equation $x+\frac{1}{x}=2$ will be **[MNR 1983]**

(a) 2, −1 (b) $0,-1,-\frac{1}{5}$ (c) $-1,-\frac{1}{5}$ (d) None of these

15. One root of the following given equation $2x^5-14x^4+31x^3-64x^2+19x+130=0$ is **[MP PET 1985]**

(a) 1 (b) 3 (c) 5 (d) 7

16. The roots of the equation $x^4-4x^3+6x^2-4x+1=0$ are **[MP PET 1986]**

(a) 1, 1, 1, 1 (b) 2, 2, 2, 2 (c) 3, 1, 3, 1 (d) 1, 2, 1, 2

17. One root of the equation $(x+1)(x+3)(x+2)(x+4)=120$ is **[T.S. Rajendra 1991]**

(a) −1 (b) 2 (c) 1 (d) 0

18. If $9^x-4\times 3^{x+2}+3^5=0,$ then the solution pair is

(a) (1, 2) (b) (2, 3) (c) (2, 4) (d) (1, 3)

19. In the equation $4^{x+2}=2^{2x+3}+48,$ the value of x will be

(a) $-\frac{3}{2}$ (b) − 2 (c) − 3 (d) 1

20. The roots of the equation $4^x-3.2^{x+3}+128=0$ are **[AMU 1985]**

(a) 1 and 2 (b) 2 and 3 (c) 3 and 4 (d) 4 and 5

21. The root of the equation $\sqrt{2x-2}+\sqrt{x-3}=2$ is **[Roorkee 1979]**

(a) 3 (b) 19 (c) 3, 19 (d) 3, −19

22. The solution of the equation $\sqrt{x+1}+\sqrt{x-1}=0$ is **[IIT 1978]**

(a) 1 (b) − 1 (c) 5/4 (d) None of these

23. If $x=\sqrt{6+\sqrt{6+\sqrt{6+.....\text{to}\infty}}}$, then **[Pb.CET 1999]**

(a) x is an irrational number (b) $2<x<3$ (c) $x=3$ (d)

24. The real values of x which satisfy the equation $(5+2\sqrt{6})^{x^2-3}+(5-2\sqrt{6})^{x^2-3}=10$ are

[Kurukshetra CEE 1995; Karnataka CET 1993]

(a) ± 2 (b) $\pm\sqrt{2}$ (c) $\pm 2,\pm\sqrt{2}$ (d) $2,\sqrt{2}$

25. If one root of the equation $a(b-c)x^2+b(c-a)x+c(a-b)=0$ is 1 then, its other roots is **[Rajasthan PET 1986]**

(a) $\frac{a(b-c)}{b(c-a)}$ (b) $\frac{c(a-b)}{a(b-c)}$ (c) $\frac{b(c-a)}{a(b-c)}$ (d) None of these

26. The imaginary roots of the equation $(x^2+2)^2+8x^2=6x(x^2+2)$ are **[Roorkee 1986]**

(a) $1\pm i$ (b) $2\pm i$ (c) $-1\pm i$ (d) None of these

27. GM of the roots of the equation $x^2-18x+9=0$ is **[Rajasthan PET 1997]**

(a) 6 (b) 3 (c) − 3 (d) ± 3

28. The solution set of the equation $(x+1)^2+[x-1]^2=(x-1)^2+[x+1]^2$ is

(a) $x\in R$ (b) $x\in N$ (c) $x\in I$ (d) $x\in Q$

29. $\left[\frac{1}{4}\right]+\left[\frac{1}{4}+\frac{1}{200}\right]+\left[\frac{1}{4}+\frac{1}{100}\right]+.....\left[\frac{1}{4}+\frac{199}{200}\right]$ is

(a) 49 (b) 50 (c) 51 (d) None of these

30. The value of $x=\sqrt{2+\sqrt{2+\sqrt{2+.....}}}$ is **[Karnataka CET 2001]**

(a) −1 (b) 1 (c) 2 (d) 3

31. If $x^2-x+1=0,$ then value of x^{3n} is **[DCE 1995]**

(a) −1,1 (b) 1 (c) −1 (d) 0

32. For what value of a the curve $y=x^2+ax+25$ touches the x- axis

(a) 0 (b) ± 5 (c) ± 10 (d) None of these

33. Let α,β be the roots of the quadratic equation $x^2+px+p^3=0$ $(p\neq 0)$. If (α,β) is a point on the parabola $y^2=x,$ then the roots of the quadratic equation are **[MP PET 2000]**

(a) 4, – 2 (b) – 4, – 2 (c) 4, 2 (d) – 4, 2

34. If expression $e^{\{(\sin^2 x+\sin^4 x+\sin^6 x+.....\infty)\ln 2\}}$ satisfies the equation $x^2-9x+8=0$, find the value of $\frac{\cos x}{\cos x+\sin x}$, $0<x<\frac{\pi}{2}$ **[IIT 199**

(a) $\frac{1}{1+\sqrt{3}}$ (b) $\frac{1}{1-\sqrt{3}}$ (c) $\frac{2}{1-\sqrt{2}}$ (d) None of these

35. The roots of equation $\frac{2x+31}{9}+\frac{x^2+7}{x^2-7}=\frac{2x+47}{9}$ are **[Rajasthan PET 1994]**

(a) 3, – 3 (b) 5, – 5 (c) $\sqrt{3},-\sqrt{3}$ (d) $\sqrt{5},-\sqrt{5}$

36. If $x^2+y^2=25, xy=12$, then x = **[BIT Ranchi 1992]**

(a) {3, 4} (b) {3, – 3} (c) {3, 4, – 3, – 4} (d) {– 3, – 3}

37. The some of all real roots of the equation $|x-2|^2+|x-2|-2=0$ is **[IIT 1997; Himachal CET 2002]**

(a) 2 (b) 4 (c) 1 (d) None of these

38. A two digit number is four times the sum and three times the product of its digits. The number is **[MP PET 1994]**

(a) 42 (b) 24 (c) 12 (d) 21

39. The number of real solutions of the equation $|x^2+4x+3|+2x+5=0$ are **[IIT 1988]**

(a) 1 (b) 2 (c) 3 (d) 4

40. The number of the real values of x for which the equality $|3x^2+12x+6|=5x+16$ holds good is **[AMU 1999]**

(a) 4 (b) 3 (c) 2 (d) 1

41. The number of real solutions of the equation $\sin e^x=5^x+5^{-x}$ is **[IIT 1990, 2002]**

(a) 0 (b) 1 (c) 2 (d) Infinitely many

42. The number of the real solutions of the equation $-x^2+x-1=\sin^4 x$ is

(a) 1 (b) 2 (c) 0 (d) 4

43. The number of solutions of $\cos x=\frac{|x|}{80}$ is

(a) 50 (b) 52 (c) 53 (d) None of these

44. The equation $\sqrt{(x+1)}-\sqrt{(x-1)}=\sqrt{(4x-1)}$ has **[IIT 1997]**

(a) No solution (b) One solution (c) Two solutions (d) More than two solution

45. The number of real roots of $\sqrt{5x^2-6x+8}-\sqrt{5x^2-6x-7}=1$ is **[Roorkee 1984]**

(a) 1 (b) 2 (c) 3 (d) 4

46. The number of roots of the quadratic equation $8\sec^2\theta-6\sec\theta+1=0$ is **[Pb. CET 1989,94]**

(a) Infinite (b) 1 (c) 2 (d) 0

47. The number of values of x in the interval $[0,5\pi]$ satisfying the equation $3\sin^2 x-7\sin x+2=0$ is **[IIT 1998, MP PET 2000]**

(a) 0 (b) 5 (c) 6 (d) 10

48. The maximum number of real roots of the equation $x^{2n}-1=0$, is **[MP PET 2001]**

(a) 2 (b) 3 (c) n (d) $2n$

49. The equation $x+\frac{2}{1-x}=1+\frac{2}{1-x}$, has **[IIT 1983; MNR 1998; Kurukshetra CEE 1993]**

(a) No real root (b) One real root (c) Two equal roots (d) Infinitely many roots

50. The number of real roots of equation $(x-1)^2+(x-2)^2+(x-3)^2=0$ is **[IIT 1990; Karnataka CET 1998]**

(a) 2 (b) 1 (c) 0 (d) 3

51. The number of roots of the equation $\log(-2x)=2\log(x+1)$ are **[AMU 2001]**

(a) 3 (b) 2 (c) 1 (d) None of these

52. Number of real roots of the equation $\sum_{r=1}^{10}(x-r)^3=0$ is

(a) 0 (b) 1 (c) 2 (d) 3

53. The minimum value of $|x-3|+|x-2|+|x-5|$ is

(a) 3 (b) 7 (c) 5 (d) 9

54. Rationalised denominator of $\frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$ is

(a) $\frac{2\sqrt{3}+3\sqrt{2}-\sqrt{30}}{12}$ (b) $\frac{3\sqrt{2}-2\sqrt{3}-\sqrt{30}}{15}$ (c) $\frac{2\sqrt{3}-3\sqrt{2}+\sqrt{40}}{10}$ (d) $\frac{2\sqrt{3}+3\sqrt{2}-\sqrt{20}}{15}$

55. If $x=\sqrt{7+4\sqrt{3}}$, then $x+\frac{1}{x}=$ **[EAMCET 1994]**

(a) 4 (b) 6 (c) 3 (d) 2

56. If $\log_2 x+\log_x 2=\frac{10}{3}=\log_2 y+\log_y 2$ and $x\neq y$, then $x+y=$ **[EAMCET 1994]**

(a) 2 (b) 65/8 (c) 37/6 (d) None of these

57. The equation $\log_e x+\log_e(1+x)=0$ can be written as **[Kurukshetra CEE 1993; MP PET 1989]**

(a) $x^2+x-e=0$ (b) $x^2+x-1=0$ (c) $x^2+x+1=0$ (d) $x^2+xe-e=0$

58. If $f(x)=2x^3+mx^2-13x+n$ and 2, 3 are roots of the equation $f(x)=0$, then the value of m and n are **[Roorkee 1990]**

(a) – 5, – 30 (b) – 5, 30 (c) 5, 30 (d) None of these

59. The number of real solutions of the equation $e^x=x$ is

(a) 1 (b) 2 (c) 0 (d) None of these

60. The sum of the real roots of the equation $x^2+|x|-6=0$ is

(a) 4 (b) 0 (c) – 1 (d) None of these

61. The number of values of a for which $(a^2-3a+2)x^2+(a^2-5a+6)x+a^2-4=0$ is an identity in x is

(a) 0 (b) 2 (c) 1 (d) 3

62. The number of values of the pair (a, b) for which $a(x+1)^2+b(x^2-3x-2)+x+1=0$ is an identity in x is

(a) 0 (b) 1 (c) 2 (d) Infinite

63. If $(\sqrt{2})^x+(\sqrt{3})^x=(\sqrt{13})^{x/2}$ then the number of values of x is

(a) 2 (b) 4 (c) 1 (d) None of these

64. The number of real solutions of the equation $\frac{6-x}{x^2-4}=2+\frac{x}{x+2}$ is

(a) Two (b) One (c) Zero (d) None of these

65. The number of real solutions of $\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+6}$ is

(a) One (b) Two (c) Three (d) None of these

Advance Level

66. If $-1\leq x<0$, then solution of the equation $|x+1|-|x|+3|x-1|\,|x-2|=x+2$ is **[IIT 1976]**

(a) 1, 5/3 (b) 5/3 (c) 1/3 (d) None of these

67. The real roots of $|x|^3-3x^2+3|x|-2=0$ are **[DCE 1997]**

(a) 0, 2 (b) ± 1 (c) ± 2 (d) 1, 2

68. The number of real solutions of the equation $2^{x/2}+(\sqrt{2}+1)^x=(5+2\sqrt{2})^{x/2}$ is

(a) One (b) Two (c) Four (d) Infinite

69. The number of negative integral solutions of $x^2.2^{x+1}+2^{|x-3|+2}=x^2.2^{|x-3|+4}+2^{x-1}$ is **[DCE 1993]**

(a) 0 (b) 1 (c) 2 (d) 4

70. The equation $e^x-x-1=0$ has **[Kurukshetra CEE 1998]**

(a) Only one real root x = 0 (b) At least two real roots (c) Exactly two real roots(d)

71. The number of real roots of the equation $e^{\sin x}-e^{-\sin x}-4=0$ are **[IIT 1982]**

(a) 1 (b) 2 (c) Infinite (d) None of these

72. If a, b, c are positive real numbers, then the number of real roots of the equation $ax^2+b|x|+c=0$ is

[DCE 1998, UPSEAT 1999]

(a) 2 (b) 4 (c) 0 (d) None of these

73. The number of real solutions of equation $\log_{10}[98+\sqrt{[x^3-x^2-12x+36]}]=2$ are

(a) 4 (b) 1 (c) 2 (d) 3

74. The equation $x^{(3/4)(\log_2 x)^2+(\log_2 x)-5/4}=\sqrt{2}$ has **[IIT 1989]**

(a) At least one real solution (b) Exactly three real solutions

(c) Exactly one irrational solution (d) All the above

75. The number of solutions of $|[x]-2x|=4$, where $[x]$ is the greatest integer is $\le x$, is

(a) 2 (b) 4 (c) 1 (d) Infinite

76. Let $f(x)$ be a function defined by $f(x)=x-[x]$, $0\ne x\in R$, where $[x]$ is the greatest integer less than or equal to x. then the number of solutions of $f(x)+f\left(\frac{1}{x}\right)=1$

(a) 0 (b) Infinite (c) 1 (d) 2

77. If m be the number of integral solutions of equation $2x^2-3xy-9y^2-11=0$ and n be the number of real solutions of equation $x^3-[x]-3=0$, then m =

(a) n (b) $2n$ (c) $n/2$ (d) $3n$

78. The set of values of c for which x^3-6x^2+9x-c is of the form $(x-\alpha)^2(x-\beta)$ (α, β real) is given by

(a) {0} (b) {4} (c) {0, 4} (d) Null set

79. If $0<a_r<1$ for r = 1, 2, 3, ..., k and m be the number of real solutions of equation $\sum_{r=1}^{k}(a_r)^x=1$ and n be the number of real solution of equation $\sum_{r=1}^{k}(x-a_r)^{101}=0$, then

(a) $m=n$ (b) $m\le n$ (c) $m\ge n$ (d) $m>n$

80. Let $P_n(x)=1+2x+3x^2+.....+(n+1)x^n$ be a polynomial such that n is even. Then the number of real roots of $P_n(x)=0$ is **[DCE 1994]**

(a) 0 (b) n (c) 1 (d) None of these

81. The number of all possible triplets (a_1,a_2,a_3) such that $a_1+a_2\cos 2x+a_3\sin^2 x=0$ for all x is **[IIT 1987]**

(a) Zero (b) One (c) Three (d) Infinite

82. The solutions of the equation $2x-2[x]=1$, where $[x]$ = the greatest integer less than or equal to x, are

(a) $x=n+\frac{1}{2},\ n\in N$ (b) $x=n-\frac{1}{2},\ n\in N$ (c) $x=n+\frac{1}{2},\ n\in Z$ (d) $n<x<n+1,\ n\in Z$

83. The number of real solutions of $1+|e^x-1|=e^x(e^x-2)$ is

(a) 0 (b) 1 (c) 2 (d) 4

84. The equation $2\sin^2\frac{x}{2}\cdot\cos^2 x=x+\frac{1}{x}, 0<x\le\frac{\pi}{2}$ has

(a) One real solution (b) No real solution

(c) Infinitely many real solutions (d) None of these

85. If $y\ne 0$ then the number of values of the pair (x, y) such that $x+y+\frac{x}{y}=\frac{1}{2}$ and $(x+y)\frac{x}{y}=-\frac{1}{2}$, is

(a) 1 (b) 2 (c) 0 (d) None of these

86. The number of real solutions of the equation $\log_{0.5} x=|x|$ is

(a) 1 (b) 2 (c) 0 (d) None of these

87. The product of all the solutions of the equation $(x-2)^2-3|x-2|+2=0$ is

(a) 2 (b) – 4 (c) 0 (d) None of these

88. If $0 < x < 1000$ and $\left[\frac{x}{2}\right]+\left[\frac{x}{3}\right]+\left[\frac{x}{5}\right]=\frac{31}{30}x$, where [x] is the greatest integer less than or equal to x, the number of possible values of x is

(a) 34 (b) 32 (c) 33 (d) None of these

89. The solution set of $(x)^2+(x+1)^2=25$, where (x) is the least integer greater than or equal to x, is

(a) (2, 4) (b) (– 5, – 4] ∪ (2, 3] (c) [– 4, – 3) ∪ (3, 4] (d) None of these

90. If $[x]^2=[x+2]$, where [x] = the greatest integer less than or equal to x, then x must be such that

(a) $x=2,-1$ (b) $x\in[2,3)$ (c) $x\in[-1,0)$ (d) None of these

91. The solution set of $\left|\frac{x+1}{x}\right|+|x+1|=\frac{(x+1)^2}{|x|}$ is

(a) $\{x|\ x\ge 0\}$ (b) $\{x|\ x>0\}\cup\{-1\}$ (c) {–1, 1} (d) $\{x|\ x\ge 1$ or $x\le -1\}$

92. If $a.3^{\tan x}+a.3^{-\tan x}-2=0$ has real solutions, $x\ne\frac{\pi}{2}, 0\le x\le\pi$, then the set of possible values of the parameter a is

(a) [–1, 1] (b) [–1, 0) (c) (0, 1] (d) (0, +∞)

Nature of roots

Basic Level

93. The roots of the quadratic equation $2x^2+3x+1=0$, are **[IIT 1983]**

(a) Irrational (b) Rational (c) Imaginary (d) None of these

94. The roots of the equation $x^2+2\sqrt{3}x+3=0$ are **[Rajasthan PET 1986]**

(a) Real and equal (b) Rational and equal (c) Irrational and equal (d) Irrational and unequal

95. If l, m, n are real and l ≠ m, then the roots of the equation $(l-m)x^2-5(l+m)x-2(l-m)=0$ are**[IIT 1979; Rajasthan PET 1983**

(a) Complex (b) Real and distinct (c) Real and equal (d) None of these

96. If a and b are the odd integers, then the roots of the equation $2ax^2+(2a+b)x+b=0$, $a\ne 0$, will be **[Pb. CET 1988]**

(a) Rational (b) Irrational (c) Non-real (d) Equal

97. If $k\in(-\infty,-2)\cup(2,\infty)$, then the roots of the equation $x^2+2kx+4=0$ are **[DCE 2002]**

(a) Complex (b) Real and unequal (c) Real and equal (d) One real and one imaginary

98. Let a, b and c be real numbers such that $4a+2b+c=0$ and $ab>0$. Then the quadratic equation $ax^2+bx+c=0$ has **[IIT 1990]**

(a) Real roots (b) Complex roots (c) Purely imaginary roots (d) Only one root

99. If $a<b<c<d$, then the roots of the equation $(x-a)(x-c)+2(x-b)(x-d)=0$ are **[IIT 1984]**

(a) Real and distinct (b) Real and equal (c) Imaginary (d) None of these

100. If $b_1b_2=2(c_1+c_2)$, then at least one of the equations $x^2+b_1x+c_1=0$ and $x^2+b_2x+c_2=0$ has

(a) Real roots (b) Purely imaginary roots (c) Imaginary roots (d) None of these

101. In the equation $x^3+3Hx+G=0$, if G and H are real and $G^2+4H^3>0$, then the roots are **[Karnataka CET 2000]**

(a) All real and equal (b) All real and distinct (c) One real and two imaginary (d)

102. The equation $(x-a)^3+(x-b)^3+(x-c)^3=0$, has

(a) All the roots real (b) One real and two imaginary roots

(c) Three real roots namely $x=a, x=b, x=c$ (d) None of these

103. For the equation $|x^2|+|x|-6=0$, the roots are **[EAMCET 1988, 93]**

(a) One and only one real number (b) Real with sum one

(c) Real with sum zero (d) Real with product zero

104. If $a>0, b>0, c>0$, then both the roots of the equation $ax^2+bx+c=0$ **[IIT 1980]**

(a) Are real and negative (b) Have negative real parts (c) Are rational numbers (d) None of these

105. Let one root of $ax^2+bx+c=0$, where a, b, c are integers be $3+\sqrt{5}$, then the other root is **[MNR 1982]**

(a) $3-\sqrt{5}$ (b) 3 (c) $\sqrt{5}$ (d) None of these

106. If $2+i$ is a root of the equation $x^3-5x^2+9x-5=0$, then the other roots are **[Kerala (Engg.) 2002]**

(a) 1 and $2-i$ (b) –1 and $3+i$ (c) 0 and 1 (d) –1 and $i-2$

107. If *a*, *b*, *c* are nonzero, unequal rational numbers then the roots of the equation $abc^2x^2+(3a^2+b^2)cx-6a^2-ab+2b^2=0$ are

(a) Rational (b) Imaginary (c) Irrational (d) None of these

108. The equation $x^2-6x+8+\lambda(x^2-4x+3)=0$, $\lambda \in R$, has

(a) Real and unequal roots for all λ (b) Real roots for $\lambda<0$ only

(c) Real roots for $\lambda>0$ only (d) Real and unequal roots for $\lambda=0$ only

109. If $a>1$, roots of the equation $(1-a)x^2+3ax-1=0$ are

(a) One positive and one negative (b) Both negative

(c) Both positive (d) Both nonreal complex

110. If the roots of the equation $ax^2+x+b=0$ be real, then the roots of the equation $x^2-4\sqrt{ab}\,x+1=0$ will be

(a) Rational (b) Irrational (c) Real (d) Imaginary

111. If the roots of the equation $x^2-8x+(a^2-6a)=0$ are real, then **[Rajasthan PET 1987, 97]**

(a) $-2<a<8$ (b) $2<a<8$ (c) $-2\le a\le 8$ (d) $2\le a\le 8$

112. If the roots of the given equation $(\cos p-1)x^2+(\cos p)x+\sin p=0$ are real, then **[IIT 1990; Rajasthan PET 1995]**

(a) $p\in(-\pi,0)$ (b) $p\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ (c) $p\in(0,\pi)$ (d) $p\in(0,2\pi)$

113. The greatest value of a non-negative real number λ for which both the equations $2x^2+(\lambda-1)x+8=0$ and $x^2-8x+\lambda+4=0$ have real roots is **[AMU 1990]**

(a) 9 (b) 12 (c) 15 (d) 16

114. If *p*, *q*, *r* are positive and are in A.P., then roots of the equation $px^2+qx+r=0$ are real if **[IIT 1995]**

(a) $\left|\frac{r}{p}-7\right|\ge 4\sqrt{3}$ (b) $\left|\frac{p}{r}-7\right|\ge 4\sqrt{3}$ (c) For all values of *p*, *r* (d) For no value of *p*, *r*

115. Let $p,\ q\in\{1,2,3,4\}$. The number of equations of the form $px^2+qx+1=0$ having real roots is **[IIT 1994]**

(a) 15 (b) 9 (c) 7 (d) 8

116. The least integer *k* which makes the roots of the equation $x^2+5x+k=0$ imaginary is **[Kerala (Engg.) 2002]**

(a) 4 (b) 5 (c) 6 (d) 7

117. If $0<a<b<c$, and the roots α, β of the equation $ax^2+bx+c=0$ are non-real complex numbers, then

(a) $|\alpha|=|\beta|$ (b) $|\alpha|>1$ (c) $|\beta|<1$ (d) None of these

118. If roots of the equation $a(b-c)x^2+b(c-a)x+c(a-b)=0$ are equal,, then *a*, *b*, *c* are in **[Roorkee 1993; Rajasthan PET 2001]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

119. If the equation $(m-n)x^2+(n-l)x+l-m=0$ has equal roots, then *l*, *m* and *n* satisfy **[DCE 2002; EAMCET 1990]**

(a) $2l=m+n$ (b) $2m=n+l$ (c) $m=n+l$ (d) $l=m+n$

120. The condition for the roots of the equation $(c^2-ab)x^2-2(a^2-bc)x+(b^2-ac)=0$ to be equal is **[TS Rajendra 1982]**

(a) $a=0$ (b) $b=0$ (c) $c=0$ (d) None of these

121. If the roots of the equation $(a^2+b^2)t^2-2(ac+bd)t+(c^2+d^2)=0$ are equal, then **[MP PET 1996]**

(a) $ab=dc$ (b) $ac=bd$ (c) $ad+bc=0$ (d) $\frac{a}{b}=\frac{c}{d}$

122. If one root of $x^2+px+12=0$ is 4 and roots of the equation $x^2+px+q=0$ are equal, then *q* is equal to **[Rajasthan PET 19**

(a) 49/4 (b) 4/49 (c) 4 (d) None of these

123. If the roots of the equation $x^2+2mx+m^2-2m+6=0$ are same, then the value of *m* will be **[MP PET 1986]**

(a) 3 (b) 0 (c) 2 (d) –1

124. If the roots of the equation $x^2 - 15 - m(2x - 8) = 0$ are equal then m is equal to **[Rajasthan PET 1985]**

(a) 3, – 5 (b) – 3, 5 (c) 3, 5 (d) – 3, – 5

125. For what value of k will the equation $x^2 - (3k - 1)x + 2k^2 - 11 = 0$ have equal roots **[Karnataka CET 1998]**

(a) 5 (b) 9 (c) Both the above (d) 0

126. The value of k for which the quadratic equation $kx^2 + 1 = kx + 3x - 11x^2 = 0$ has real and equal roots are **[BIT Ranchi 1993]**

(a) –11, – 3 (b) 5, 7 (c) 5, –7 (d) None of these

127. If the roots of $4x^2 + px + 9 = 0$ are equal, then absolute value of p is **[MP PET 1995]**

(a) 144 (b) 12 (c) – 12 (d) ± 12

128. The value of k for which $2x^2 - kx + x + 8 = 0$ has equal and real roots are **[BIT Ranchi 1990]**

(a) – 9 and – 7 (b) 9 and 7 (c) – 9 and 7 (d) 9 and – 7

129. The roots of $4x^2 + 6px + 1 = 0$ are equal, then the value of p is **[MP PET 2003]**

(a) $\frac{4}{5}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{4}{3}$

130. If the equation $x^2 - (2 + m)x + (m^2 - 4m + 4) = 0$ has coincident roots, then **[Roorkee 1991]**

(a) $m = 0, m = 1$ (b) $m = 0, m = 2$ (c) $m = \frac{2}{3}, m = 6$ (d) $m = \frac{2}{3}, m = 1$

131. If two roots of the equation $x^3 - 3x + 2 = 0$ are same, then the roots will be **[MP PET 1985]**

(a) 2, 2, 3 (b) 1, 1, – 2 (c) – 2, 3, 3 (d) – 2, – 2, 1

132. The equation $\|x - 1| + a| = 4$ can have real solutions for x if a belongs to the interval

(a) $(-\infty, 4]$ (b) $(-\infty, -4]$ (c) $(4, \infty)$ (d) $[-4, 4]$

133. The set of values of m for which both roots of the equation $x^2 - (m + 1)x + m + 4 = 0$ are real and negative consists of all m such that

[AMU 1992]

(a) $-3 < m \le -1$ (b) $-4 < m \le -3$ (c) $-3 \le m \le 5$ (d) $-3 \ge m$ or $m \ge 5$

134. Both the roots of the given equation $(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$ are always

[MNR 1986; IIT 1980; Kurukshetra CEE 1998]

(a) Positive (b) Negative (c) Real (d) Imaginary

135. If $P(x) = ax^2 + bx + c$ and $Q(x) = -ax^2 + dx + c$ where $ac \ne 0$, then $P(x).Q(x) = 0$, has at least **[IIT 1985]**

(a) Four real roots (b) Two real roots (c) Four imaginary roots (d) None of these

136. The conditions that the equation $ax^2 + bx + c = 0$ has both the roots positive is that **[SCRA 1990]**

(a) a, b and c are of the same sign (b) a and b are of the same sign

(c) b and c have the same sign opposite to that of a (d) a and c have the same sign opposite to that of b

137. If $[x]$ denotes the integral part of x and $k = \sin^{-1}\frac{1 + t^2}{2t} > 0$, then the integral value of α for which the equation $(x - [k])(x + \alpha) - 1 = 0$ has integral roots is

(a) 1 (b) 2 (c) 4 (d) None of these

138. If the roots of the equation $ax^2 + bx + c = 0$ are real and of the form $\frac{\alpha}{\alpha - 1}$ and $\frac{\alpha + 1}{\alpha}$, then the value of $(a + b + c)^2$ is **[AMU 2000]**

(a) $b^2 - 4ac$ (b) $b^2 - 2ac$ (c) $2b^2 - ac$ (d) None of these

Advance Level

139. Equation $\frac{a^2}{x-\alpha}+\frac{b^2}{x-\beta}+\frac{c^2}{x-\gamma}=m-n^2x$ ($a, b, c, m, n \in R$) has necessarily

(a) All the roots real (b) All the roots imaginary
(c) Two real and two imaginary roots (d) Two rational and two irrational roots

140. If cos θ, sin ϕ, sin θ are in G.P. then roots of $x^2+2\cot\phi\, x+1=0$ are always

(a) Equal (b) Real (c) Imaginary (d) Greater than 1

141. If $f(x)$ is a continuous function and attains only rational values and $f(0)=3$, then roots of equation $f(1)x^2+f(3)x+f(5)=0$ are

(a) Imaginary (b) Rational (c) Irrational (d) Real and equal

142. The roots of $ax^2+bx+c=0$, where $a\neq 0$ and coefficients are real, are non-real complex and $a+c<b$. Then

(a) $4a+c>2b$ (b) $4a+c<2b$ (c) $4a+c=2b$ (d) None of these

143. The equation $(a+2)x^2+(a-3)x=2a-1, a\neq -2$ has roots rational for

(a) All rational values of a except $a=-2$ (b) All real values of a except $a=-2$
(c) Rational values of $a>\frac{1}{2}$ (d) None of these

144. The quadratic equation $x^2-2x-\lambda=0, \lambda\neq 0$

(a) Cannot have a real root if $\lambda<1$
(b) Can have a rational root if λ is a perfect square
(c) Cannot have an integral root if $n^2-1<\lambda<n^2+2n$ where $n=0,1,2,3,....$
(d) None of these

145. If the roots of the equation $x^2+px+q=0$ are α and β and roots of the equation $x^2-xr+s=0$ are α^4,β^4, then the roots of the equation $x^2-4qx+2q^2-r=0$ will be **[IIT 1989]**

(a) Both negative (b) Both positive
(c) Both real (d) One negative and one positive

146. If equation $a(b-c)x^2+b(c-a)x+c(a-b)=0$ has equal roots, $a, b, c > 0$, $n\in N$, then

(a) $a^n+c^n\geq 2b^n$ (b) $a^n+c^n>2b^n$ (c) $a^n+c^n\leq 2b^n$ (d) $a^n+c^n<2b^n$

147. If $\frac{\sum_{r=0}^{k-1}x^{2r}}{\sum_{r=0}^{k-1}x^{r}}$ is a polynomial in x for two values of p and q of k, then roots of equation $x^2+px+q=0$ cannot be

(a) Real (b) Imaginary (c) Rational (d) Irrational

148. If for $x > 0$, $f(x)=(a-x^n)^{1/n}$, $g(x)=x^2+px+q, p,q\in R$ and equation $g(x)-x=0$ has imaginary roots, then number of real roots of equation $g(g(x))-f(f(x))=0$ is

(a) 0 (b) 2 (c) 4 (d) None of these

149. Let $p, q \in \{1, 2, 3, 4\}$. The number of equations of the form $px^2+qx+1=0$ having real and unequal roots is

(a) 15 (b) 9 (c) 7 (d) 8

150. If α_1,α_2 and β_1,β_2 are the roots of the equations $ax^2+bx+c=0$ and $px^2+qx+r=0$ respectively and system of equations $\alpha_1y+\alpha_2z=0$ and $\beta_1y+\beta_2z=0$ has a non-zero solution. Then **[IIT 1987]**

(a) $a^2qc=p^2br$ (b) $p^2br=q^2ac$ (c) $c^2ar=r^2pb$ (d) None of these

151. If a, b, c, d are four consecutive terms of an increasing AP then the roots of the equation $(x-a)(x-c)+2(x-b)(x-d)=0$ are

(a) Real and distinct (b) Nonreal complex (c) Real and equal (d) Integers

152. If a, b, c are three distinct positive real numbers then the number of real roots of $ax^2+2b|x|-c=0$ is

(a) 4 (b) 2 (c) 0 (d) None of these

153. If $a \in R$, $b \in R$ then the equation $x^2 - abx - a^2 = 0$ has
(a) One positive root and one negative root (b) Both roots positive
(c) Both roots negative (d) Non-real roots

154. The number of integral values of a for which $x^2 - (a-1)x + 3 = 0$ has both roots positive and $x^2 + 3x + 6 - a = 0$ has both roots negative is
(a) 0 (b) 1 (c) 2 (d) Infinite

155. The quadratic equations $x^2 + (a^2 - 2)x - 2a^2 = 0$ and $x^2 - 3x + 2 = 0$ have
(a) No common root for all $a \in R$ (b) Exactly one common root for all $a \in R$
(c) Two common roots for some $a \in R$ (d) None of these

156. If $f(x) = \frac{x^2 - 1}{x^2 + 1}$ for every real number x then the minimum value of f
(a) Does not exist because f is unbounded (b) Is not attained even though f is bounded
(c) Is equal to 1 (d) Is equal to –1

157. If x, y, z are real and distinct then $f(x, y) = x^2 + 4y^2 + 9z^2 - 6yz - 3zx - 2xy$ is always
(a) Non-negative (b) Nonpositive (c) Zero (d) None of these

158. If $a \in R$, $b \in R$ then the factors of the expression $a(x^2 - y^2) - bxy$ are
(a) Real and different (b) Real and identical (c) Complex (d) None of these

159. If a, b, c are in H.P. then the expression $a(b-c)x^2 + b(c-a)x + c(a-b)$
(a) Has real and distinct factors (b) Is a perfect square
(c) Has no real factor (d) None of these

160. If a, b, c are in G.P., where a, c are positive, then the equation $ax^2 + bx + c = 0$ has
(a) Real roots (b) Imaginary roots
(c) Ratio of roots = 1 : w where w is a nonreal cube root of unity (d) Ratio of roots = $b : ac$

161. The polynomial $(ax^2 + bx + c)(ax^2 - dx - c)$ $ac \neq 0$, has
(a) Four real zeros (b) At least two real zeros (c) At most two real zeros (d) No real zeros

Relation between Roots and Coefficient

Basic Level

162. If α, β are roots of the equation $ax^2 + bx + c = 0$, then the value of $\alpha^3 + \beta^3$ is
[Kurukshetra CEE 1991; BIT Ranchi 1998; MP PET 1994; Rajasthan PET 1989, 96]
(a) $\frac{3abc + b^3}{a^3}$ (b) $\frac{a^3 + b^3}{3ab}$ (c) $\frac{3abc - b^3}{a^3}$ (d) $\frac{b^3 - 3abc}{a^3}$

163. If α, β are roots of the equation $x^2 - (1 + n^2)x + \frac{1}{2}(1 + n^2 + n^4) = 0$, then $\alpha^2 + \beta^2$ is equal to **[Rajasthan PET 1996]**
(a) $2n$ (b) n^2 (c) n^3 (d) $2n^2$

164. If α and β are the roots of the equation $ax^2 + bx + c = 0$ ($a \neq 0$; a, b, c being different), then $(1 + \alpha + \alpha^2)(1 + \beta + \beta^2) =$ **[DCE 20**
(a) Zero (b) Positive (c) Negative (d) None of these

165. If α, β are the roots of the equation $8x^2 - 3x + 27 = 0$, then the value of $\left(\frac{\alpha^2}{\beta}\right)^{\frac{1}{3}} + \left(\frac{\beta^2}{\alpha}\right)^{\frac{1}{3}}$ is **[AMU 1990]**
(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{7}{2}$ (d) 4

166. If α, β are the roots of the equation $x^2 + px + p^2 + q = 0$, then the value of $\alpha^2 + \alpha\beta + \beta^2 + q$ is equal to **[AMU 1993]**
(a) 0 (b) 1 (c) q (d) $2q$

167. If α, β are the roots of the equation $x^2 - p(x+1) - c = 0$, then $(\alpha+1)(\beta+1) =$ **[BITS Ranchi 2000; Him. CET 2001]**

(a) c (b) c – 1 (c) 1 – c (d) None of these

168. If α, β, γ are the roots of the equation $x^3 + 4x + 1 = 0$, then $(\alpha+\beta)^{-1} + (\beta+\gamma)^{-1} + (\gamma+\alpha)^{-1} =$ **[EAMCET 2003]**

(a) 2 (b) 3 (c) 4 (d) 5

169. If roots of $x^2 - 7x + 6 = 0$ are α, β then $\frac{1}{\alpha} + \frac{1}{\beta} =$ **[Rajasthan PET 1990, 95; MNR 1981]**

(a) 6/7 (b) 7/6 (c) 7/10 (d) 8/9

170. If α, β are the roots of $x^2 - 2x + 4 = 0$, then $\alpha^5 + \beta^5$ is equal to **[EAMCET 1990]**

(a) 16 (b) 32 (c) 64 (d) None of these

171. If the roots of the equation $ax^2 + bx + c = 0$ are α, β, then the value of $\alpha\beta^2 + \alpha^2\beta + \alpha$ will be **[EAMCET 1980; AMU 1984]**

(a) $\frac{c(a-b)}{a^2}$ (b) 0 (c) $-\frac{bc}{a^2}$ (d) None of these

172. If α, β be the roots of the equation $2x^2 - 35x + 2 = 0$, then the value of $(2\alpha-35)^3 . (2\beta-35)^3$ is equal to **[Bihar CEE 1994]**

(a) 1 (b) 64 (c) 8 (d) None of these

173. If α and β are roots of $ax^2 + 2bx + c = 0$, then $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}}$ is equal to **[BITS Ranchi 1990]**

(a) $\frac{2b}{ac}$ (b) $\frac{2b}{\sqrt{ac}}$ (c) $-\frac{2b}{\sqrt{ac}}$ (d) $-\frac{b}{\sqrt{2}}$

174. If α, β are the roots of the equation $x^2 + 2x + 4 = 0$, then $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$ is equal to **[Kerala (Engg.) 2002]**

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 32 (d) $\frac{1}{4}$

175. If α, β, γ are roots of equation $x^3 + ax^2 + bx + c = 0$, then $\alpha^{-1} + \beta^{-1} + \gamma^{-1} =$ **[EAMCET 2002]**

(a) a/c (b) $-b/c$ (c) b/a (d) c/a

176. If α, β are roots of $x^2 - 3x + 1 = 0$, then the value of $\alpha^3 + \beta^3$ is **[MP 1994; BIT Ranchi 1990]**

(a) 9 (b) 18 (c) – 9 (d) –18

177. If A.M. of the roots of a quadratic equation is 8/5 and A.M. of their reciprocals is 8/7, then the equation is **[AMU 2001]**

(a) $5x^2 - 16x + 7 = 0$ (b) $7x^2 - 16x + 5 = 0$ (c) $7x^2 - 16x + 8 = 0$ (d) $3x^2 - 12x + 7 = 0$

178. The quadratic in t, such that A.M. of its roots is A and G.M. is G, is **[IIT 1968, 74]**

(a) $t^2 - 2At + G^2 = 0$ (b) $t^2 - 2At - G^2 = 0$ (c) $t^2 + 2At + G^2 = 0$ (d) None of these

179. In a triangle ABC, the value of $\angle A$ is given by $5\cos A + 3 = 0$, then the equation whose roots are sin A and tan A will be **[Roorkee 1972]**

(a) $15x^3 - 8x + 16 = 0$ (b) $15x^2 + 8x - 16 = 0$ (c) $15x^2 - 8\sqrt{2}x + 16 = 0$ (d) $15x^2 - 8x - 16 = 0$

180. If $x^2 + px + q = 0$ is the quadratic whose roots are $a-2$ and $b-2$ where a and b are the roots of $x^2 - 3x + 1 = 0$, then **[Kerala (Engg.) 2002]**

(a) $p = 1, q = 5$ (b) $p = 1, q = -5$ (c) $p = -1, q = 1$ (d) None of these

181. The roots of the equation $x^2 + ax + b = 0$ are p and q, then the equation whose roots are p^2q and pq^2 will be **[MP PET 198**

(a) $x^2 + abx + b^3 = 0$ (b) $x^2 - abx + b^3 = 0$ (c) $bx^2 + x + a = 0$ (d) $x^2 + ax + ab = 0$

182. The equation whose roots are $\frac{1}{3+\sqrt{2}}$ and $\frac{1}{3-\sqrt{2}}$ is **[MP PET 1994]**

(a) $7x^2 - 6x + 1 = 0$ (b) $6x^2 - 7x + 1 = 0$ (c) $x^2 - 6x + 7 = 0$ (d) $x^2 - 7x + 6 = 0$

183. If α, β are the roots of the equation $lx^2 + mx + n = 0$ then the equation whose roots are $\alpha^3\beta$ and $\alpha\beta^3$ is **[MP PET 1997]**

(a) $l^4x^2 - nl(m^2 - 2nl)x + n^4 = 0$ (b) $l^4x^2 + nl(m^2 - 2nl)x + n^4 = 0$

(c) $l^4x^2 + nl(m^2 - 2nl)x - n^4 = 0$ (d) $l^4x^2 - nl(m^2 + 2nl)x + n^4 = 0$

184. If α, β are the roots of $9x^2+6x+1=0$, then the equation with the roots $\frac{1}{\alpha}, \frac{1}{\beta}$ is **[EAMCET 2000]**

(a) $2x^2+3x+18=0$ (b) $x^2+6x-9=0$ (c) $x^2+6x+9=0$ (d) $x^2-6x+9=0$

185. If α, β are the roots of the equation $ax^2+bx+c=0$, then the equation whose roots are $\alpha+\frac{1}{\beta}$ and $\beta+\frac{1}{\alpha}$, is**[Rajasthan PE**

(a) $acx^2+(a+c)bx+(a+c)^2=0$ (b) $abx^2+(a+c)bx+(a+c)^2=0$

(c) $acx^2+(a+b)cx+(a+c)^2=0$ (d) None of these

186. If α, β are the roots of $x^2-3x+1=0$, then the equation whose roots are $\frac{1}{\alpha-2}, \frac{1}{\beta-2}$ is **[Rajasthan PET 1999]**

(a) $x^2+x-1=0$ (b) $x^2+x+1=0$ (c) $x^2-x-1=0$ (d) None of these

187. If α, β are the roots of $ax^2+bx+c=0$, then the equation whose roots are $2+\alpha, 2+\beta$ is **[EAMCET 1994]**

(a) $ax^2+x(4a-b)+4a-2b+c=0$ (b) $ax^2+x(4a-b)+4a+2b+c=0$

(c) $ax^2+x(b-4a)+4a+2b+c=0$ (d) $ax^2+x(b-4a)+4a-2b+c=0$

188. If α, β are the roots of the equation $ax^2+bx+c=0$, then the equation with roots $1/\alpha$, $1/\beta$ will be **[MNR 1988; SCRA 1990; Rajasthan PET 1994]**

(a) $cx^2-bx+a=0$ (b) $cx^2+bx+a=0$ (c) $x^2+bx+a=0$ (d) $x^2+bx-a=0$

189. Let α, α^2 be the roots of $x^2+x+1=0$, then the equation whose roots are α^{31}, α^{62} is **[AMU 1999]**

(a) $x^2-x+1=0$ (b) $x^2+x-1=0$ (c) $x^2+x+1=0$ (d) $x^{60}+x^{30}+1=0$

190. If α, β are roots of the equation $x^2-2x\cos 2\theta+1=0$ then the equation with roots $\alpha^{n/2}, \beta^{n/2}$ will be**[Rajasthan PET 1998]**

(a) $x^2-2nx\cos\theta+1=0$ (b) $x^2+2nx\cos n\theta+1=0$ (c) $x^2+2x\cos n\theta+1=0$ (d) $x^2-2x\cos n\theta+1=0$

191. The equation whose roots are reciprocal of the roots of the equation $3x^2-20x+17=0$ is **[DCE 2002]**

(a) $3x^2+20x-17=0$ (b) $17x^2-20x+3=0$ (c) $17x^2+20x+3=0$ (d) None of these

192. The sum of the roots of a equation is 2 and sum of their cubes is 98, then the equation is **[MP PET 1986]**

(a) $x^2+2x+15=0$ (b) $x^2+15x+2=0$ (c) $2x^2-2x+15=0$ (d) $x^2-2x-15=0$

193. Sum of roots is –1 and sum of their reciprocals is $\frac{1}{6}$, then equation is **[Karnataka CET 1998]**

(a) $x^2+x-6=0$ (b) $x^2-x+6=0$ (c) $x^2+x+1=0$ (d) $x^2-6x+1=0$

194. If α, β are the roots of the quadratic equation $x^2+bx-c=0$, then the equation whose roots are b and c is**[Pb. CET 1989]**

(a) $x^2+\alpha x-\beta=0$ (b) $x^2-[(\alpha+\beta)+\alpha\beta]x-\alpha\beta(\alpha+\beta)=0$

(c) $x^2+[(\alpha+\beta)+\alpha\beta]x+\alpha\beta(\alpha+\beta)=0$ (d) $x^2+[\alpha\beta+(\alpha+\beta)]x-\alpha\beta(\alpha+\beta)=0$

195. If α, β are roots of $x^2-5x-3=0$, then the equation with roots $\frac{1}{2\alpha-3}$ and $\frac{1}{2\beta-3}$ is **[Rajasthan PET 1998]**

(a) $33x^2+4x-1=0$ (b) $33x^2-4x+1=0$ (c) $33x^2-4x-1=0$ (d) $33x^2+4x+1=0$

196. Given that $\tan\alpha$ and $\tan\beta$ are the roots of $x^2-px+q=0$, then the value of $\sin^2(\alpha+\beta)=$ **[Rajasthan PET 2000]**

(a) $\frac{p^2}{p^2+(1-q)^2}$ (b) $\frac{p^2}{p^2+q^2}$ (c) $\frac{q^2}{p^2+(1-q)^2}$ (d) $\frac{p^2}{(p+q)^2}$

197. If $2+i\sqrt{3}$ is a root of the equation $x^2+px+q=0$, then (p, q) is equal to **[IIT 1982; MP 1997]**

(a) (7, – 4) (b) (– 4, 7) (c) (4, 7) (d) (7, 4)

198. In the equation $x^2+px+q=0$, the coefficient of x was taken as 17 in place of 13 and its roots were found to be –2 and –15. The correct roots of the original equation are **[Rajasthan PET 1994; IIT 1979]**

(a) –10, – 3 (b) 10, 3 (c) –10, 3 (d) 10, – 3

199. Two students while solving a quadratic equation in x, one copied the constant term incorrectly and got the roots 3 and 2. The other copied the constant term and coefficient of x^2 correctly as – 6 and 1 respectively. The correct roots are **[EAAMCET 1991]**

(a) 3, – 2 (b) – 3, 2 (c) – 6, –1 (d) 6, –1

200. If 8, 2 are the roots of $x^2 + ax + \beta = 0$ and 3, 3 are the roots of $x^2 + \alpha x + b = 0$, then the roots of $x^2 + ax + b = 0$ are**[EAMCE**

(a) 8, –1 (b) –9, 2 (c) – 8, – 2 (d) 9, 1

201. The equation formed by decreasing each root of $ax^2 + bx + c = 0$ by 1 is $2x^2 + 8x + 2 = 0$, then **[EAMCET 2000]**

(a) $a = -b$ (b) $b = -c$ (c) $c = -a$ (d) $b = a + c$

202. If p and q are non-zero constants, the equation $x^2 + px + q = 0$ has roots u and v, then the equation $qx^2 + px + 1 = 0$ has roots **[MNR 1988]**

(a) u and $\frac{1}{v}$ (b) $\frac{1}{u}$ and v (c) $\frac{1}{u}$ and $\frac{1}{v}$ (d) None of these

203. If the sum of the roots of the equation $x^2 + px + q = 0$ is equal to the sum of their squares, then **[Pb. CET 1999]**

(a) $p^2 - q^2 = 0$ (b) $p^2 + q^2 = 2q$ (c) $p^2 + p = 2q$ (d) None of these

204. If the sum of the roots of the equation $x^2 + px + q = 0$ is three times their difference, then which one of the following is true **[Dhanbad Engg. 1968]**

(a) $9p^2 = 2q$ (b) $2q^2 = 9p$ (c) $2p^2 = 9q$ (d) $9q^2 = 2p$

205. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\frac{b^2}{ac} + \frac{bc}{a^2} =$ **[BITS Ranchi 1996]**

(a) 2 (b) – 2 (c) 1 (d) – 1

206. If the sum of the two roots of the equation $4x^3 + 16x^2 - 9x - 36 = 0$ is zero, then the roots are **[MP PET 1986]**

(a) 1, 2, – 2 (b) $-2, \frac{2}{3}, -\frac{2}{3}$ (c) $-3, \frac{3}{2}, -\frac{3}{2}$ (d) $-4, \frac{3}{2}, -\frac{3}{2}$

207. If the roots of the equation $ax^2 + bx + c = 0$ are l and $2l$, then **[MP PET 1986]**

(a) $b^2 = 9ac$ (b) $2b^2 = 9ac$ (c) $b^2 = -4ac$ (d) $a^2 = c^2$

208. If α, β are the roots of the equation $x^2 - px + 36 = 0$ and $\alpha^2 + \beta^2 = 9$, then the value of p are **[AMU 1991]**

(a) ± 3 (b) ± 6 (c) ± 8 (d) ± 9

209. If α, β, γ are the roots of $2x^3 - 2x - 1 = 0$, then $(\sum \alpha\beta)^2 =$ **[EAMCET 2002]**

(a) – 1 (b) 3 (c) 2 (d) 1

210. If α, β be the roots of $x^2 + px + q = 0$ and $\alpha + h, \beta + h$ are the roots of $x^2 + rx + s = 0$, then **[AMU 2001]**

(a) $\frac{p}{r} = \frac{q}{s}$ (b) $2h = \left[\frac{p}{q} + \frac{r}{s}\right]$ (c) $p^2 - 4q = r^2 - 4s$ (d) $pr^2 = qs^2$

211. The quadratic equation with real coefficients whose one root is $7 + 5i$ will be**[Kerala (Engg.) 2001, 02; Rajasthan PET 199**

(a) $x^2 - 14x - 74 = 0$ (b) $x^2 + 14x + 74 = 0$ (c) $x^2 + 14x - 74 = 0$ (d) $x^2 - 14x + 74 = 0$

212. The quadratic equation with one root as the square root of $-47 + 8\sqrt{-3}$ is **[IIT 1995]**

(a) $x^2 + 2x + 49 = 0$ (b) $x^2 - 2x + 49 = 0$ (c) $x^2 \pm 2x + 49 = 0$ (d) $x^2 \pm 2x - 49 = 0$

213. The quadratic equation whose one root is $\frac{1}{2 + \sqrt{5}}$ will be **[Rajasthan PET 1987]**

(a) $x^2 + 4x - 1 = 0$ (b) $x^2 - 4x - 1 = 0$ (c) $x^2 + 4x + 1 = 0$ (d) None of these

214. The quadratic equation with one root $2 - \sqrt{3}$ is **[Rajasthan PET 1985]**

(a) $x^2 - 4x + 1 = 0$ (b) $x^2 - 4x - 1 = 0$ (c) $x^2 + 4x + 1 = 0$ (d) $x^2 + 4x - 1 = 0$

215. The quadratic equation whose roots are three times the roots of the equation $3ax^2 + 3bx + c = 0$ is **[AMU 1990]**

(a) $ax^2 + bx + c = 0$ (b) $ax^2 + 3bx + c = 0$ (c) $ax^2 + bx + 3c = 0$ (d) $ax^2 + 3bx + 3c = 0$

216. If α, β are the roots of $x^2 + px + q = 0$ then $-\frac{1}{\alpha}, -\frac{1}{\beta}$ are the roots of the equation **[TS Rajendra 1991]**

(a) $qx^2 - px + 1 = 0$ (b) $qx^2 + px + 1 = 0$ (c) $x^2 + px + q = 0$ (d) $x^2 - px + q = 0$

217. If a root of the equation $ax^2 + bx + c = 0$ be reciprocal of a root of the equation $a'x^2 + b'x + c' = 0$, then **[IIT 1968]**

(a) $(cc' - aa')^2 = (ba' - cb')(ab' - bc')$ (b) $(bb' - aa')^2 = (ca' - bc')(ab' - bc')$

(c) $(cc' - aa')^2 = (ba' + cb')(ab' + bc')$ (d) None of these

218. One root of $ax^2 + bx + c = 0$ is reciprocal of other root if **[Rajasthan PET 1985]**

(a) $a + c = 0$ (b) $b + c = 0$ (c) $b - c = 0$ (d) $a - c = 0$

219. If the roots of the equation $5x^2 + 13x + k = 0$ be reciprocals of each other, then k is equal to **[MNR 1980; Rajasthan PET 19**

(a) 0 (b) 5 (c) 1/6 (d) 6

220. If one root of the equation $x^2 = px + q$ is reciprocal of the other, then the correct relationship is **[AMU 1987, 89]**

(a) $q = -1$ (b) $q = 1$ (c) $pq = -1$ (d) $pq = 1$

221. If the roots of the quadratic equation $\frac{x - m}{mx + 1} = \frac{x + n}{nx + 1}$ are reciprocal to each other, then **[MP PET 2001]**

(a) $n = 0$ (b) $m = n$ (c) $m + n = 1$ (d) $m^2 + n^2 = 1$

222. The roots of the quadratic equation $ax^2 + bx + c = 0$ will be reciprocal to each other if

(a) $a = \frac{1}{c}$ (b) $a = c$ (c) $b = ac$ (d) $a = b$

223. If the absolute difference between two roots of the equation $x^2 + px + 3 = 0$ is $\sqrt{p}$, then p equals **[Bihar CEE 1998]**

(a) – 3, 4 (b) 4 (c) – 3 (d) None of these

224. If the roots of equation $x^2 - px + q = 0$ differ by 1, then **[MP PET 1999]**

(a) $p^2 = 4q$ (b) $p^2 = 4q + 1$ (c) $p^2 = 4q - 1$ (d) None of these

225. The numerical difference of the roots of $x^2 - 7x - 9 = 0$ is

(a) 5 (b) $2\sqrt{85}$ (c) $9\sqrt{7}$ (d) $\sqrt{85}$

226. If the difference of the roots of $x^2 - px + 8 = 0$ be 2, then the value of p is **[Roorkee 1992]**

(a) ± 2 (b) ± 4 (c) ± 6 (d) ± 8

227. If the difference of the roots of the equation $x^2 - bx + c = 0$ be 1, then **[Rajasthan PET 1991]**

(a) $b^2 - 4c - 1 = 0$ (b) $b^2 - 4c = 0$ (c) $b^2 - 4c + 1 = 0$ (d) $b^2 + 4c - 1 = 0$

228. If the roots of the equations $x^2 - bx + c = 0$ and $x^2 - cx + b = 0$ differ by the same quantity, then $b + c$ is equal to **[BIT Ranchi 1969; MP PET 1993]**

(a) 4 (b) 1 (c) 0 (d) – 4

229. If the roots of $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4c$ is **[Kurukshetra CEE 1998]**

(a) 1 (b) 2 (c) 3 (d) 4

230. If α, β are the roots of $x^2 - 3x + a = 0, a \in R$ and $\alpha < 1 < \beta$ then

(a) $a \in (-\infty, 2)$ (b) $a \in \left(-\infty, \frac{9}{4}\right)$ (c) $a \in \left(2, \frac{9}{4}\right)$ (d) None of these

231. If α, β be the roots of $4x^2 - 16x + \lambda = 0, \lambda \in R$ such that $1 < \alpha < 2$ and $2 < \beta < 3$ then the number of integral solutions of λ is

(a) 5 (b) 6 (c) 2 (d) 3

232. If X denotes the set of real numbers p for which the equation $x^2 = p(x + p)$ has its roots greater than p then X is equal to

(a) $\left(-2, -\frac{1}{2}\right)$ (b) $\left(-\frac{1}{2}, \frac{1}{4}\right)$ (c) Null set (d) $(-\infty, 0)$

233. If one root of the quadratic equation $ax^2 + bx + c = 0$ is equal to the n^{th} power of the other root, then the value of $(ac^n)^{\frac{1}{n+1}} + (a^n c)^{\frac{1}{n+1}} =$ **[IIT 1983]**

(a) b (b) $-b$ (c) $b^{\frac{1}{n+1}}$ (d) $-b^{\frac{1}{n+1}}$

234. If one root of the equation $ax^2 - bx + c = 0$ is square of the other, then **[Rajasthan PET 1998]**

(a) $a^2c + ac^2 + 3abc - b^3 = 0$ (b) $a^2c + ac^2 - 3abc + b^3 = 0$ (c) $a^3 + b^3 = 3abc$ (d) $(a + b)^3 = 3abc$

235. For the equation $3x^2 + px + 3,\ p > 0$ if one of the root is square of the other, then p is equal to **[IIT Screening 2000]**

(a) $\frac{1}{3}$ (b) 1 (c) 3 (d) $\frac{2}{3}$

236. If one root of equation $px^2 - qx + r = 0$ is double of the other, then

(a) $9q^2 = 2pr$ (b) $2q^2 = 9pr$ (c) $3q^2 = 4pr$ (d) $4q^2 = 3pr$

237. The value of k for which one of the roots of $x^2 - x + 3k = 0$ is double of one of the roots of $x^2 - x + k = 0$ is **[UPSEAT 200**

(a) 1 (b) – 2 (c) 2 (d) None of these

238. The function $f(x) = ax^2 + 2x + 1$ has one double root if **[AMU 1989]**

(a) $a = 0$ (b) $a = -1$ (c) $a = 1$ (d) $a = 2$

239. If $\sin\alpha, \cos\alpha$ are the roots of the equation $ax^2 + bx + c = 0$, then **[MP PET 1993]**

(a) $a^2 - b^2 + 2ac = 0$ (b) $(a - c)^2 = b^2 + c^2$ (c) $a^2 + b^2 - 2ac = 0$ (d) $a^2 + b^2 + 2ac = 0$

240. If the roots of $ax^2 + bx + c = 0$ are α, β and root of $Ax^2 + Bx + c = 0$ are $\alpha - k, \beta - k$, then $\frac{B^2 - 4AC}{b^2 - 4ac}$ is equal to **[Rajasthan PET 1999]**

(a) $\frac{a}{A}$ (b) $\frac{A}{a}$ (c) $\left(\frac{a}{A}\right)^2$ (d) $\left(\frac{A}{a}\right)^2$

241. If the product of roots of the equation $x^2 - 3k\,x + 2e^{2\log k} - 1 = 0$ is 7, then its roots will real when **[Pb. CET 1990; IIT 1984]**

(a) $k = 1$ (b) $k = 2$ (c) $k = 3$ (d) None of these

242. If a and b are rational and b is not a perfect square then the quadratic equation with rational coefficients whose one root is $\frac{1}{a+\sqrt{b}}$ is

(a) $x^2-2ax+(a^2-b)=0$ (b) $(a^2-b)x^2-2ax+1=0$ (c) $(a^2-b)x^2-2bx+1=0$ (d) None of these

243. If $\frac{1}{4-3i}$ is a root of $ax^2+bx+1=0$, where *a, b* are real, then

(a) $a=25, b=-8$ (b) $a=25, b=8$ (c) $a=5, b=4$ (d) None of these

244. If α, β, γ be the roots of the equation $x(1+x^2)+x^2(6+x)+2=0$ then the value of $\alpha^{-1}+\beta^{-1}+\gamma^{-1}$ is

(a) – 3 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) None of these

245. If the roots of $x^3-12x^2+39x-28=0$ are in A.P. then their common difference is

(a) ±1 (b) ±2 (c) ±3 (d) ±4

246. The roots of the equation $x^3+14x^2-84x-216=0$ are in

(a) A.P. (b) G.P. (c) H.P. (d) None of these

247. If 3 and $1+\sqrt{2}$ are two roots of a cubic equation with rational coefficients, then the equation is

(a) $x^3-5x^2+9x-9=0$ (b) $x^3-3x^2-4x+12=0$ (c) $x^3-5x^2+7x+3=0$ (d) None of these

248. What is the sum of the squares of roots of $x^2-3x+1=0$ **[Karnataka CET 1993]**

(a) 5 (b) 7 (c) 9 (d) 10

249. If $\alpha+\beta=3$ and $\alpha^3+\beta^3=27$, then α and β are the roots of

(a) $3x^2+9x+7=0$ (b) $9x^2-27x+20=0$ (c) $2x^2-6x+15=0$ (d) None of these

250. For what value of λ the sum of the squares of the roots of $x^2+(2+\lambda)x-\frac{1}{2}(1+\lambda)=0$ is minimum **[AMU 1999]**

(a) 3/2 (b) 1 (c) 1/2 (d) 11/4

251. The value of $a(a\geq3)$ for which the sum of the cubes of the roots of $x^2-(a-2)x+(a-3)=0$, assumes the least value is **[Orissa JEE 2002]**

(a) 3 (b) 4 (c) 5 (d) None of these

252. Let α, β be the roots of $x^2+(3-\lambda)x-\lambda=0$. The value of λ for which $\alpha^2+\beta^2$ is minimum, is **[AMU 2000]**

(a) 0 (b) 1 (c) 2 (d) 3

253. If the sum of squares of the roots of the equation $x^2-(a-2)x-(a+1)=0$ is least, then the value of *a* is **[Rajasthan PET 2000. Pb. CET 2002]**

(a) 0 (b) 2 (c) – 1 (d) 1

254. If α, β are roots of $Ax^2+Bx+C=0$ and α^2, β^2 are roots of $x^2+px+q=0$, then *p* is equal to **[Rajasthan PET 1986]**

(a) $(B^2-2AC)/A^2$ (b) $(2AC-B^2)/A^2$ (c) $(B^2-4AC)/A^2$ (d) $(4AC-B^2)/A^2$

255. If α, β are roots of the equation $x^2+x+1=0$ and $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ are roots of the equation $x^2+px+q=0$, then *p* equals **[Rajasthan PET 1987, 93]**

(a) -1 (b) 1 (c) -2 (d) 2

256. If α, β are real and α^2, β^2 are the roots of the equation $a^2x^2+x+1-a^2=0 (a>1)$, then $\beta^2=$ **[EAMCET 1999]**

(a) a^2 (b) $1-\frac{1}{a^2}$ (c) $1-a^2$ (d) $1+a^2$

257. The H.M. of the roots of the equation $x^2-8x+4=0$ is **[Rajasthan PET 1988]**

(a) 1 (b) 2 (c) 3 (d) None of these

258. If α, β are the roots of the equation $x^2+x\sqrt{\alpha}+\beta=0$, then the value of α and β are **[AMU 1990, 92]**

(a) $\alpha=1$ and $\beta=-1$ (b) $\alpha=1$ and $\beta=-2$ (c) $\alpha=2$ and $\beta=1$ (d) $\alpha=2$ and $\beta=-2$

259. If *p* and *q* are the roots of $x^2+px+q=0$, then **[IIT 1995, AIEEE 2002]**

(a) $p=1$ (b) $p=-2$ (c) $p=1$ or 0 (d) $p=-2$ or 0

260. If roots of the equation $2x^2-(a^2+8a+1)x+a^2-4a=0$ are in opposite sign, then **[AMU 1998]**

(a) $0<a<4$ (b) $a>0$ (c) $a<8$ (d) $-4<a<0$

261. Which of the following equation has 1 and –2 as the roots **[SCRA 1999]**

(a) $x^2-x-2=0$ (b) $x^2+x-2=0$ (c) $x^2-x+2=0$ (d) $x^2+x+2=0$

262. If the roots of the equation $x^2+x+1=0$ are in the ratio $m:n$ then **[Rajasthan PET 1994]**

(a) $\sqrt{\frac{m}{n}}+\sqrt{\frac{n}{m}}+1=0$ (b) $\sqrt{m}+\sqrt{n}+1=0$ (c) $\frac{m}{n}+\frac{n}{m}+1=0$ (d) $m+n+1=0$

263. If the roots of the equation $lx^2+nx+n=0$ are in the ratio $p:q$ then $\sqrt{\frac{p}{q}}+\sqrt{\frac{q}{p}}$ is equal to **[Rajasthan PET 1997; BIT Ranc**

(a) $\sqrt{n/l}$ (b) $\sqrt{l/n}$ (c) $\pm\sqrt{n/l}$ (d) $-\sqrt{l/n}$

264. If the roots of the equation $12x^2-mx+5=0$ are in the ratio $2:3$, then $m=$ **[Rajasthan PET 2002]**

(a) $5\sqrt{10}$ (b) $3\sqrt{10}$ (c) $2\sqrt{10}$ (d) None of these

265. If the ratio of the roots of the equation $ax^2+bx+c=0$ be $p:q$, then **[Pb. CET 1994]**

(a) $pqb^2+(P+q)^2ac=0$ (b) $pqb^2-(P+q)^2ac=0$ (c) $pqa^2-(P+q)^2bc=0$ (d) None of these

266. The two roots of an equation $x^3-9x^2+14x+24=0$ are in the ratio $3:2$. The roots will be **[UPSEAT 1999]**

(a) 6, 4, –1 (b) 6, 4, 1 (c) –6, 4, 1 (d) –6, –4, 1

267. The condition that one root of the equation $ax^2+bx+c=0$ is three times the other is **[DCE 2002]**

(a) $b^2=8ac$ (b) $3b^2+16ac=0$ (c) $3b^2=16ac$ (d) $b^2+3ac=0$

268. If the roots of the equation $\frac{x^2-bx}{ax-c}=\frac{\lambda-1}{\lambda+1}$ are such that $\alpha+\beta=0$, then the value of λ is

[Kurukhestra CEE 1995; MP PET 1996, 2002; Rajasthan PET 2001]

(a) $\frac{a-b}{a+b}$ (b) c (c) $\frac{1}{c}$ (d) $\frac{a+b}{a-b}$

269. For the equation $\frac{1}{x+a}-\frac{1}{x+b}=\frac{1}{x+c}$, if the product of the roots is zero, then the sum of the roots is **[AMU 1992]**

(a) 0 (b) $\frac{2ab}{b+c}$ (c) $\frac{2bc}{b+c}$ (d) $-\frac{2bc}{b+c}$

270. If the sum of two of the roots of $x^3+px^2+qx+r=0$ is zero, then $pq=$ **[EAMCET 2003]**

(a) $-r$ (b) r (c) $2r$ (d) $-2r$

271. If the roots of the equation $\frac{1}{x+p}+\frac{1}{x+q}=\frac{1}{r}$ are equal in magnitude but opposite in sign, then the product of the roots will be

[IIT 1967]

(a) $\frac{p^2+q^2}{2}$ (b) $-\frac{(p^2+q^2)}{2}$ (c) $\frac{p^2-q^2}{2}$ (d) $-\frac{(p^2-q^2)}{2}$

272. The value of m for which the equation $x^3-mx^2+3x-2=0$ has two roots equal in magnitute but opposite in sign, is **[Kurukhestra CEE 1996]**

(a) 1/2 (b) 2/3 (c) 3/4 (d) 4/5

273. If $ax^2+bx+c=a(x-\alpha)(x-\beta)$, then $a(\alpha x+1)(\beta x+1)$ is equal to **[AMU 1986]**

(a) ax^2+bx+c (b) cx^2-bx+a (c) cx^2-bx-a (d) cx^2+bx+a

274. If α,β are the roots of the equation $ax^2+bx+c=0$ $(a\neq 0)$ and $\alpha+\delta,\beta+\delta$ are the roots of $Ax^2+Bx+C=0$ $(A\neq 0)$ for some constant, then **[IIT 2000]**

(a) $\frac{b^2-4ac}{a^2}=\frac{B^2-4AC}{A^2}$ (b) $\frac{b^2-2ac}{a^2}=\frac{B^2-2AC}{A^2}$ (c) $\frac{b^2-8ac}{a^2}=\frac{B^2-8AC}{A^2}$ (d) None of these

275. In a triangle PQR, $\angle R=\frac{\pi}{2}$. If $\tan\left(\frac{p}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of the equation $ax^2+bx+c=0$ $(a\neq 0)$, then **[IIT 1999]**

(a) $a+b=c$ (b) $b+c=0$ (c) $a+c=b$ (d) $b=c$

276. The product of all real roots of the equation $x^2 - |x| - 6 = 0$ is **[Roorkee 2000]**

(a) –9 (b) 6 (c) 9 (d) 36

277. If the sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals then bc^2, ca^2, ab^2 are in **[IIT 1976]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

278. The roots of the equation $x^2 - 2x + A = 0$ are p, q and the roots of the equation $x^2 - 18x + B = 0$ are r, s. If $p < q < r < s$ are in A.P., then **[IIT 1997]**

(a) $A = 3, B = 77$ (b) $A = -3, B = 77$ (c) $A = 3, B = -77$ (d) $A = -3, B = -77$

279. If the roots of the equation $x^2 + bx + c = 0$ and $x^2 + qx + r = 0$ are in the same ratio, then **[EAMCET 1994]**

(a) $r^2c = qb^2$ (b) $r^2b = qc^2$ (c) $c^2r = q^2b$ (d) $b^2r = q^2c$

280. If one root of the equation $x^2 + px + q = 0$ is $2 + \sqrt{3}$, then values of p and q are **[UPSEAT 2002]**

(a) –4, 1 (b) 4, –1 (c) $2, \sqrt{3}$ (d) $-2, -\sqrt{3}$

281. If $1 - i$ is a root of the equation $x^2 - ax + b = 0$, then $b =$ **[EAMCET 2002]**

(a) –2 (b) –1 (c) 1 (d) 2

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282. If α, β are the roots of $x^2 + px + 1 = 0$ and γ, δ are the roots of $x^2 + qx + 1 = 0$, then $q^2 - p^2 =$ **[IIT 1978; DCE 2000]**

(a) $(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta)$ (b) $(\alpha + \gamma)(\beta + \gamma)(\alpha - \delta)(\beta + \delta)$

(c) $(\alpha + \gamma)(\beta + \gamma)(\alpha + \delta)(\beta + \delta)$ (d) None of these

283. If α, β be the roots of $x^2 - px + q = 0$ and α', β' be the roots of $x^2 - p'x + q' = 0$, then the value of $(\alpha - \alpha')^2 + (\beta - \alpha')^2 + (\alpha - \beta')^2 + (\beta - \beta')^2$ is

(a) $2\{p^2 - 2q + p'^2 - 2q' - pp'\}$ (b) $2\{p^2 - 2q + p'^2 - 2q' - qq'\}$

(c) $2\{p^2 - 2q - p'^2 - 2q' - pp'\}$ (d) $2\{p^2 - 2q - p'^2 - 2q' - qq'\}$

284. If α and β are the roots of the equation $x^2 - ax + b = 0$ and $A_n = \alpha^n + \beta^n$, then which of the following is true **[Karnataka CET 2000]**

(a) $A_{n+1} = aA_n + bA_{n-1}$ (b) $A_{n+1} = bA_n + aA_{n-1}$ (c) $A_{n+1} = aA_n - bA_{n-1}$ (d) $A_{n+1} = bA_n - aA_{n-1}$

285. If roots of an equation $x^n - 1 = 0$ are $1, a_1, a_2 ....., a_{n-1}$, then the value of $(1 - a_1)(1 - a_2)(1 - a_3).....(1 - a_{n-1})$ will be **[UPSEAT 1999]**

(a) n (b) n^2 (c) n^n (d) 0

286. If α and β are the roots of $6x^2 - 6x + 1 = 0$, then the value of $\frac{1}{2}[a + b\alpha + c\alpha^2 + d\alpha^3] + \frac{1}{2}[a + b\beta + c\beta^2 + d\beta^3]$ is **[Rajasthan PET 2000]**

(a) $\frac{1}{4}(a + b + c + d)$ (b) $\frac{a}{1} + \frac{b}{2} + \frac{c}{3} + \frac{d}{4}$ (c) $\frac{a}{1} - \frac{b}{2} + \frac{c}{3} - \frac{d}{4}$ (d) None of these

287. If α_1, α_2 are the roots of equation $x^2 - px + 1 = 0$ and β_1, β_2 be those of equation $x^2 - qx + 1 = 0$ and vector $\alpha_1\hat{i} + \beta_1\hat{j}$ is parallel to $\alpha_2\hat{i} + \beta_2\hat{j}$, then

(a) $p = \pm q$ (b) $p = \pm 2q$ (c) $p = 2q$ (d) None of these

288. If the roots of $a_1x^2 + b_1x + c_1 = 0$ are α_1 and β_1 and those of $a_2x^2 + b_2x + c_2 = 0$ are α_2 and β_2 such that $\alpha_1\alpha_2 = \beta_1\beta_2 = 1$, then

(a) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (b) $\frac{a_1}{c_2} = \frac{b_1}{b_2} = \frac{c_1}{a_2}$ (c) $a_1a_2 = b_1b_2 = c_1c_2$ (d) None of these

289. If the sum of the roots of the equation $qx^2 + 2x + 3q = 0$ is equal to their product, then the value of q is equal to

(a) $-\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 3 (d) –6

290. If $x=(\beta-\gamma)(\alpha-\delta),\ y=(\gamma-\alpha)(\beta-\delta),\ z=(\alpha-\beta)(\gamma-\delta)$, then the value of $x^3+y^3+z^3-3xyz$ is

(a) 0 (b) $\alpha^6+\beta^6+\gamma^6+\delta^6$ (c) $\alpha^6\beta^6\gamma^6\delta^6$ (d) None of these

291. If α, β, γ are the roots of the equation $x^3+px^2+qx+r=0$, then $(1-\alpha^2)(1-\beta^2)(1-\gamma^2)$ is equal to

(a) $(1+q)^2-(p+r)^2$ (b) $(1+q)^2+(p+r)^2$ (c) $(1-q)^2+(p-r)^2$ (d) None of these

292. If α, β, γ are the roots of the equation $x^3+ax+b=0$, then $\frac{\alpha^3+\beta^3+\gamma^3}{\alpha^2+\beta^2+\gamma^2}=$

(a) $\frac{3b}{2a}$ (b) $\frac{-3b}{2a}$ (c) $3b$ (d) $2a$

293. If α, β are the roots of $6x^2-2x+1=0$ and $s_x=\alpha^n+\beta^n$, then $\lim_{n\to\infty}\sum_{r=1}^{n} S_r$ is

(a) $\frac{5}{17}$ (b) 0 (c) $\frac{3}{37}$ (d) None of these

294. Let α, β be the roots of the equation $ax^2+bx+c=0$ and let $\alpha^n+\beta^n=S_n$ for $n\ge 1$. Then the value of the determinant

$$\begin{vmatrix} 3 & 1+S_1 & 1+S_2 \\ 1+S_1 & 1+S_2 & 1+S_3 \\ 1+S_2 & 1+S_3 & 1+S_4 \end{vmatrix}$$ is

(a) $\frac{b^2-4ac}{a^4}$ (b) $\frac{(a+b+c)(b^2+4ac)}{a^4}$ (c) $\frac{(a+b+c)(b^2-4ac)}{a^4}$ (d) $\frac{(a+b+c)^2(b^2-4ac)}{a^4}$

295. If α, β are roots of the equation $2x^2+6x+b=0\,(b<0)$, then $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ is less then

(a) 2 (b) –2 (c) 18 (d) None of these

296. If α, β are roots of the equation $ax^2+3x+2=0\ (a<0)$, then $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$ is greater then

(a) 0 (b) 1 (c) 2 (d) None of these

297. If $\alpha, \beta, \gamma, \sigma$ are the roots of the equation $x^4+4x^3-6x^2+7x-9=0$, then the value of $(1+\alpha^2)(1+\beta^2)(1+\gamma^2)(1+\sigma^2)$ is

(a) 5 (b) 9 (c) 11 (d) 13

298. If α and β are the roots of the equation $x^2-p(x+1)-q=0$, then the value of $\frac{\alpha^2+2\alpha+1}{\alpha^2+2\alpha+q}+\frac{\beta^2+2\beta+1}{\beta^2+2\beta+q}$ is

(a) 2 (b) 3 (c) 0 (d) 1

299. If A, G, H be respectively, the A.M., G.M. and H.M. of three positive number a, b, c then the equation whose roots are these number is given by

(a) $x^3-3Ax^2+G^3(3x-1)=0$ (b) $x^3-3Ax^2+3(G^3/H)x-G^3=0$

(c) $x^3+3Ax^2+3(G^3/H)x-G^3=0$ (d) $x^3-3Ax^2-3(G^3/H)x+G^3=0$

300. Let $a=\cos\frac{2\pi}{7}+i\sin\frac{2\pi}{7}$, $A=a+a^2+a^4$ and $B=a^3+a^5+a^6$ then A and B are roots of the equation **[Rajasthan PET 2000]**

(a) $x^2-x+2=0$ (b) $x^2-x-2=0$ (c) $x^2+x+2=0$ (d) None of these

301. If α, β are the roots of the equation $x^2-px+q=0$, then the quadratic equation whose roots are $(\alpha^2-\beta^2)(\alpha^3-\beta^3)$ and $\alpha^3\beta^2+\alpha^2\beta^3$ is **[Roorkee 1994]**

(a) $x^2-Sx+P=0$ (b) $x^2+Sx+P=0$ (c) $x^2+Sx-P=0$ (d) None of these

[Where $S=p(p^4-5p^2q+5q^2)$ and $P=p^2q^2(p^4-5p^2q+4q^2)$]

302. Let A, G and H are the A.M., G.M. and H.M. respectively of two unequal positive integers. Then the equation $Ax^2-|G|\,x-H=0$ has

(a) Both roots as fractions (b) At least one root which is a negative fraction

(c) Exactly one positive root (d) At least one root which is an integer

303. Let $x^2 - px + q = 0$, where $p \in R, q \in R$, have the roots α, β such that $\alpha + 2\beta = 0$ then

(a) $2p^2 + q = 0$ (b) $2q^2 + p = 0$ (c) $q < o$ (d) None of these

304. The cubic equation whose roots are the A.M., G.M. and H.M. of the roots of $x^2 - 2px + q^2 = 0$ is

(a) $(x-p)(x-q)(x-p-q) = 0$ (b) $(x-p)(x-|q|)(px-q^2) = 0$

(c) $x^3 - \left(p + |q| + \frac{q^2}{p}\right)x^2 + \left(p|q| + q^2 + \frac{|q|^3}{p}\right)x - |q|^3 = 0$ (d) None of these

305. If α, β are the roots of $x^2 + px + q = 0$ and also of $x^{2n} + p^n x^n + q^n = 0$ and if $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ are the roots of $x^n + 1 + (x+1)^n = 0$, then n is

(a) An odd integer (b) An even integer (c) Any integer (d) None of these

306. If $\cos^4 x + \sin^2 x - p = 0, p \in R$ has real solutions then

(a) $p \le 1$ (b) $\frac{3}{4} \le p \le 1$ (c) $p \ge \frac{3}{4}$ (d) None of these

307. If the ratio of the roots of $\lambda x^2 + \mu x + v = 0$ is equal to the ratio of the roots of $x^2 + x + 1 = 0$ then λ, μ, v are in

(a) A.P. (b) G.P. (c) H.P. (d) None of these

308. P, q, r and s are integers. If the A.M. of the roots of $x^2 - px + q^2 = 0$ and G.M. of the roots of $x^2 - rx + s^2 = 0$ are equal then

(a) q is an odd integer (b) r is an even integer (c) p is an even integer (d) s is an odd integer

309. If the roots of $4x^2 + 5k = (5k+1)x$ differ by unity then the negative value of k is

(a) –3 (b) $-\frac{1}{5}$ (c) $-\frac{3}{5}$ (d) None of these

310. The harmonic mean of the roots of the equation $(5+\sqrt{2})x^2 - (4+\sqrt{5})x + 8 + 2\sqrt{5} = 0$ is

(a) 2 (b) 4 (c) 6 (d) 8

311. If α, β are the roots of $ax^2 + c = bx$ then the equation $(a+cy)^2 = b^2 y$ in y has the roots

(a) α^{-1}, β^{-1} (b) α^2, β^2 (c) $\alpha\beta^{-1}, \alpha^{-1}\beta$ (d) α^{-2}, β^{-2}

312. If the roots of $ax^2 - bx - c = 0$ change by the same quantity then the expression in a, b, c that does not change is

(a) $\frac{b^2 - 4ac}{a^2}$ (b) $\frac{b-4c}{a}$ (c) $\frac{b^2 + 4ac}{a^2}$ (d) None of these

313. If α, β are the roots of $x^2 - px + q = 0$ then the product of the roots of the quadratic equation whose roots are $\alpha^2 - \beta^2$ and $\alpha^3 - \beta^3$ is

(a) $p(p^2 - q)^2$ (b) $p(p^2 - q)(p^2 - 4q)$ (c) $p(p^2 - 4q)(p^2 + q)$ (d) None of these

314. The quadratic equation whose roots are the A.M. and H.M. of the roots of the equation $x^2 + 7x - 1 = 0$ is

(a) $14x^2 + 14x - 45 = 0$ (b) $45x^2 - 14x + 14 = 0$ (c) $14x^2 + 45x - 14 = 0$ (d) None of these

315. If $z_0 = \alpha + i\beta, i = \sqrt{-1}$, then the roots of the cubic equation $x^3 - 2(1+\alpha)x^2 + (4\alpha + \alpha^2 + \beta^2)x + 2(\alpha^2 + \beta^2) = 0$ are

(a) $2, z_0, \bar{z}_0$ (b) $1, z_0, -z_0$ (c) $2, z_0, -\bar{z}_0$ (d) $2, -z_0, \bar{z}_0$

316. Let a, b, c be real numbers and $a \ne 0$. If α is a root of $a^2x^2 + bx + c = 0$, β is a root of $a^2x^2 - bx - c = 0$, and $0 < \alpha < \beta$ then the equation $a^2x^2 + 2bx + 2c = 0$ has a root γ that always satisfies

(a) $\gamma = \frac{1}{2}(\alpha + \beta)$ (b) $\gamma = \alpha + \frac{\beta}{2}$ (c) $\gamma = \alpha$ (d) $\alpha < \gamma < \beta$

317. If $(\lambda^2 + \lambda - 2)x^2 + (\lambda + 2)x < 1$ for all $x \in R$, then λ belongs to the interval

(a) $(-2, 1)$ (b) $\left(-2, \frac{2}{5}\right)$ (c) $\left(\frac{2}{5}, 1\right)$ (d) None of these

318. The least integral value of k for which $(k-2)x^2 + 8x + k + 4 > 0$ for all $x \in R$, is

(a) 5 (b) 4 (c) 3 (d) None of these

319. The set of possible values of λ for which $x^2-(\lambda^2-5\lambda+5)x+(2\lambda^2-3\lambda-4)=0$ has roots whose sum and product are both less then 1 is

(a) $\left(-1, \frac{5}{2}\right)$ (b) $(1, 4)$ (c) $\left[1, \frac{5}{2}\right]$ (d) $\left(1, \frac{5}{2}\right)$

320. The set of the possible values of x such that $5^x+(2\sqrt{3})^{2x}-169$ is always positive is

(a) $[3, +\infty)$ (b) $[2, +\infty)$ (c) $(2, +\infty)$ (d) None of these

321. If all real value of x obtained from the equation $4^x-(a-3)2^x+a-4=0$ are nonpositive then

(a) $a\in(4, 5]$ (b) $a\in(0, 4)$ (c) $a\in(4, +\infty)$ (d) None of these

322. If $ax^2+bx+6=0$ does not have two distinct real roots $a\in R, b\in R$, then the least value of $3a+b$ is

(a) 4 (b) –1 (c) 1 (d) –2

323. If $ab=2a+3b, a>0, b>0$ then the minimum value of ab is

(a) 12 (b) 24 (c) $\frac{1}{4}$ (d) None of these

324. The number of values of k for which $\{x^2-(k-2)x+k^2\}\{x^2+kx+(2k-1)\}$ is a perfect square is

(a) 1 (b) 2 (c) 0 (d) None of these

325. If $x^2-bx+c=0$ has equal integral roots then

(a) b and c are integers

(b) b and c are even integers

(c) b is an even integer and c is a perfect square of a positive integer

(d) None of these

326. Let A, G and H be the A.M., G.M. and H.M. of two positive number a and b. The quadratic equation whose roots are A and H is

(a) $Ax^2-(A^2+G^2)x+AG^2=0$ (b) $Ax^2-(A^2+H^2)x+AH^2=0$

(c) $Hx^2-(H^2+G^2)x+HG^2=0$ (d) None of these

327. If $x^2+y^2+z^2=1$, then the value of $xy+yz+zx$ lies in the interval

(a) $\left[\frac{1}{2}, 2\right]$ (b) $[-1, 2]$ (c) $\left[-\frac{1}{2}, 1\right]$ (d) $\left[-1, \frac{1}{2}\right]$

328. If $px^2+qx+r=0$ has no real roots and p, q, r are real such that $p+r>0$, then

(a) $p-q+r<0$ (b) $p-q+r>0$ (c) $p+r=q$ (d) All of these

329. The quadratic equation $x^2-2x-\lambda=0, \lambda\neq 0$

(a) Cannot have a real root if $\lambda<-1$

(b) Can have a rational root if λ is a perfect square

(c) Cannot have an integral root if $n^2-1<\lambda<n^2+2n$ where $n=0, 1, 2, 3...$

(d) None of these

330. A quadratic equation whose roots are $\left(\frac{\gamma}{\alpha}\right)^2$ and $\left(\frac{\beta}{\alpha}\right)^2$, where α, β, γ are the roots of $x^3+27=0$, is

(a) $x^2-x+1=0$ (b) $x^2+3x+9=0$ (c) $x^2+x+1=0$ (d) $x^2-3x+9=0$

331. If a, b are the real roots of $x^2+px+1=0$ and c, d are the real roots of $x^2+qx+1=0$, then $(a-c)(b-c)(a+d)(b+d)$ is divisible by

(a) $a+b+c+d$ (b) $a+b-c-d$ (c) $a-b+c-d$ (d) $a-b-c-d$

332. If $0<a<5, 0<b<5$ and $\frac{x^2+5}{2}=x-2\cos(a+bx)$ is satisfied for at least one real x then the greatest value of $a+b$ is

(a) π (b) $\frac{\pi}{2}$ (c) 3π (d) 4π

333. $a(x^2-y^2)+\lambda\{x(y+1)+1\}$ can be resolved into linear rational factors. Then

(a) $\lambda = 1$ (b) $\lambda = \frac{4a^2}{a-1}, a \ne 1$ (c) $\lambda = 0,\ a = 1$ (d) None of these

334. If α, β are the roots of the equation $x^2 + x + 3 = 0$ then equation $3x^2 + 5x + 3 = 0$ has a root

(a) $\frac{\alpha}{\beta}$ (b) $\frac{\beta}{\alpha}$ (c) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ (d) None f these

335. If α, β are the roots of $x^2 - 2ax + b^2 = 0$ and γ, δ are the roots of $x^2 - 2bx + a^2 = 0$, then

(a) A.M. of α, β = G.M. of γ, δ (b) G.M. of α, β = A.M. of γ, δ

(c) $\alpha, \beta, \gamma, \delta$ are in A.P. (d) $\alpha, \beta, \gamma, \delta$ are in G.P.

336. If the roots of the equation $ax^2 - 4x + a^2 = 0$ are imaginary and the sum of the roots is equal to their product then a is

(a) –2 (b) 4 (c) 2 (d) None of these

Condition for common roots

Basic Level

337. If equations $x^2 + bx + a = 0$ and $x^2 + ax + b = 0$ have one root common and $a \ne b$, then **[Rajasthan PET 1992; IIT 1986]**

(a) $a + b = 1$ (b) $a - b = 1$ (c) $a + b = -1$ (d) $a + b = 0$

338. If equations $x^2 + 2x + 3\lambda = 0$ and $2x^2 + 3x + 5\lambda = 0$ have one non-zero root common, then λ is equal to **[Rajasthan PET 19**

(a) 2 (b) –1 (c) 1 (d) 3

339. If $x^2 + ax + 10 = 0$ and $x^2 + bx - 10 = 0$ have a common root, then $a^2 - b^2$ is equal to **[Kerala (Engg.) 2002]**

(a) 10 (b) 20 (c) 30 (d) 40

340. If two equations $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ have a common root, then the value of $(a_1b_2 - a_2b_1).(b_1c_2 - c_1b_2)$ is **[Roorkee 1992]**

(a) $-(a_1c_2 - a_2c_1)^2$ (b) $(a_1a_2 - c_1c_2)^2$ (c) $(a_1c_1 - a_2c_2)^2$ (d) $(a_1c_2 - c_1a_2)^2$

341. If the roots of $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ are the same, then **[Kurukshetra CEE 1995]**

(a) $a_1 = a_2, b_1 = b_2, c_1 = c_2$ (b) $c_1 = c_2 = 0$

(c) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (d) $a_1 = b_1 = c_1; a_2 = b_2 = c_2$

342. If one root of the equation $(k^2 + 1)x^2 + 13x + 4k = 0$ is reciprocal of the other then k has the value

(a) $-2 + \sqrt{3}$ (b) $2 - \sqrt{3}$ (c) 1 (d) None of these

343. If the product of the roots of the equation $x^2 - 5x + 4^{\log_2 \lambda} = 0$ is 8 then λ is

(a) $\pm 2\sqrt{2}$ (b) $2\sqrt{2}$ (c) 3 (d) None of these

344. If the absolute value of the difference of roots of the equation $x^2 + px + 1 = 0$ exceeds $\sqrt{3p}$ then

(a) $p < -1$ or $p > 4$ (b) $p > 4$ (c) $-1 < p < 4$ (d) $0 \le p < 4$

345. If α, β are roots of $x^2 + px + q = 0$ and γ, δ are the roots of $x^2 + px - r = 0$, then $(\alpha - \gamma)(\alpha - \delta)$ is equal to

(a) $q + r$ (b) $q - r$ (c) $-(q + r)$ (d) $-(p + q + r)$

346. If the equation $2x^2 + 3x + 5\lambda = 0$ and $x^2 + 2x + 3\lambda = 0$ have a common root, then $\lambda =$ **[Rajasthan PET 1989]**

(a) 0 (b) –1 (c) 0, –1 (d) 2, –1

347. If a root of the equations $x^2 + px + q = 0$ and $x^2 + \alpha x + \beta = 0$ is common, then its value will be (where $p \ne \alpha$ and $q \ne \beta$) **[IIT 1974, 76; Rajasthan PET 1997]**

(a) $\frac{q - \beta}{\alpha - p}$ (b) $\frac{p\beta - \alpha q}{q - \beta}$ (c) $\frac{q - \beta}{\alpha - p}$ or $\frac{p\beta - \alpha q}{q - \beta}$ (d) None of these

348. If $ax^2+bx+c=0$ and $bx^2+cx+a=0$ have a common root and $a \neq 0$, then $\frac{a^3+b^3+c^3}{abc}=$ **[IIT 1982; MNR 1983]**

(a) 1 (b) 2 (c) 3 (d) None of these

349. If the equation $x^2+px+q=0$ and $x^2+qx+p=0$, have a common root, then $p+q+1=$ **[Orissa JEE 2002]**

(a) 0 (b) 1 (c) 2 (d) –1

Advance Level

350. If every pair from among the equation $x^2+px+qr=0$, $x^2+qx+rp=0$ and $x^2+rx+pq=0$ has a common root, then the product of three common roots is

(a) pqr (b) $2pqr$ (c) $p^2q^2r^2$ (d) None of these

351. If the equation $x^2+px+qr=0$ and $x^2+qx+pr=0$ have a common root, then the sum and product of their other roots are respectively

(a) r, pq (b) $-r, pq$ (c) pq, r (d) $-pq, r$

352. The value of 'a' for which the equations $x^3+ax+1=0$ and $x^4+ax^2+1=0$ have a common root is

(a) 2 (b) –2 (c) 0 (d) None of these

353. If the equations $ax^2+bx+c=0$ and $cx^2+bx+a=0,\ a \neq c$ have a negative common root then the value of $a-b+c$ is

(a) 0 (b) 2 (c) 1 (d) None of these

354. If $x^2+ax+b=0$ and $x^2+bx+a=0,\ a \neq b$, have a common root α then

(a) $a+b=1$ (b) $\alpha+1=0$ (c) $\alpha=1$ (d) $a+b+1=0$

355. If α is a root of the equation $2x(2x+1)=1$ then the other root is

(a) $3\alpha^3-4\alpha$ (b) $-2\alpha(\alpha+1)$ (c) $4\alpha^3-3\alpha$ (d) None of these

356. The common roots of the equations $x^3+2x^2+2x+1=0$ and $1+x^{130}+x^{1988}=0$ are (where ω is a nonreal cube root of unity)

(a) ω (b) ω^2 (c) –1 (d) $\omega-\omega^2$

357. If a, b, c are rational and no two of them are equal then the equations $(b-c)x^2+(c-a)x+a-b=0$ and $a(b-c)x^2+b(c-a)x+c(a-b)=0$

(a) Have rational roots (b) Will be such at least one has rational roots

(c) Have exactly one root common (d) Have at least one root common

358. If the equations $ax^2+bx+c=0$ and $x^3+3x^2+3x+2=0$ have two common roots, then

(a) $a=b\neq c$ (b) $a=-b=c$ (c) $a=b=c$ (d) None of these

359. The equations $ax^2+bx+a=0$ and $x^3-2x^2+2x-1=0$ have 2 roots in common. Then $a+b$ must be equal to

(a) 1 (b) –1 (c) 0 (d) None of these

360. If a, b, c are in G.P. then the equations $ax^2+2bx+c=0$ and $dx^2+2ex+f=0$ have a common root if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in **[IIT 1985; Pb. CET 2000; DCE 2000]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

361. If the equations $x^2+ix+a=0$, $x^2-2x+ia=0,\ a\neq 0$ have a common root then

(a) a is real (b) $a=\frac{1}{2}+i$

(c) $a=\frac{1}{2}-i$ (d) The other root is also common

362. If $x^2-2r.p_r x+r=0;\ r=1, 2, 3$ are three quadratic equations of which each pair has exactly one root common then the number of solutions of the triplet (p_1, p_2, p_3) is

(a) 2 (b) 1 (c) 9 (d) 27

363. If x, y, z are three consecutive terms of a G.P., where $x>0$ and the common ratio is r, then the inequality $z+3x>4y$ holds for

(a) $r \in (-\infty, 1)$ (b) $r = \frac{24}{5}$ (c) $r \in (3, +\infty)$ (d) $r = \frac{1}{2}$

364. If x is real, then the value of $x^2 - 6x + 13$ will not be less then **[Rajasthan PET 1986]**
(a) 4 (b) 6 (c) 7 (d) 8

365. If x be real, the least value of $x^2 - 6x + 10$ is **[Kurukshetra CEE 1998]**
(a) 1 (b) 2 (c) 3 (d) 10

366. The smallest value of $x^2 - 3x + 3$ in the interval $(-3, 3/2)$ is **[EAMCET 1991]**
(a) $3/4$ (b) 5 (c) –15 (d) –20

367. If $x = 2 + 2^{1/3} + 2^{2/3}$, then $x^3 - 6x^2 + 6x$ equals **[Rajasthan PET 1995; MNR 1985]**
(a) 2 (b) –2 (c) 0 (d) 1

368. If x be real, then the minimum value of $x^2 - 8x + 17$ is **[MNR 1980]**
(a) –1 (b) 0 (c) 1 (d) 2

369. If x be real, then the maximum value of $5 + 4x - 4x^2$ will be equal to **[MNR 1979]**
(a) 5 (b) 6 (c) 1 (d) 2

370. The expression $ax^2 + bx + c$ has the same sign as of 'a' of **[Kurukshetra CEE 1995]**
(a) $b^2 - 4ac > 0$ (b) $b^2 - 4ac = 0$
(c) $b^2 - 4ac \le 0$ (d) b and c have the same sign as a.

371. The value of $x^2 + 2bx + c$ is positive if **[Roorkee 1995]**
(a) $b^2 - 4c > 0$ (b) $b^2 - 4c < 0$ (c) $c^2 < b$ (d) $b^2 < c$

372. The values of 'a' for which $(a^2 - 1)x^2 + 2(a-1)x + 2$ is positive for any x are **[UPSEAT 2001]**
(a) $a \ge 1$ (b) $a \le 1$ (c) $a > -3$ (d) $a < -3$ or $a > 1$

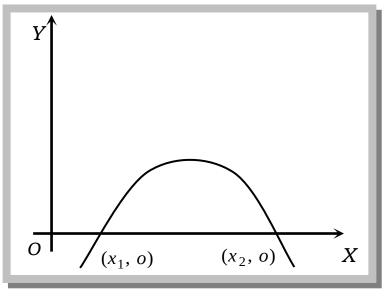
Quadratic Expressions

Basic Level

373. If x is real, then the maximum and minimum values of the expression $\frac{x^2 - 3x + 4}{x^2 + 3x + 4}$ will be **[IIT 1984]**
(a) 2, 1 (b) $5, \frac{1}{5}$ (c) $7, \frac{1}{7}$ (d) None of these

374. If x is real, then the value of $\frac{x^2 + 34x - 71}{x^2 + 2x - 7}$ does not lie between **[Roorkee 1983, 89]**
(a) –9 and –5 (b) –5 and 9 (c) 0 and 9 (d) 5 and 9

375. The adjoining figure shows the graph of $y = ax^2 + bx + c$. Then



(a) $a < 0$ (b) $b^2 < 4ac$
(c) $c > 0$ (d) a and b are of opposite signs

376. If $x + 2$ is a common factor of $px^2 + qx + r$ and $qx^2 + px + r$, then
(a) $p = q = r$ (b) $p = q$ or $p + q + r = 0$ (c) $p = r$ or $p + q + r = 0$ (d) $q = r$ or $p + q + r = 0$

377. $x^2 - 11x + a$ and $x^2 - 14x + 2a$ will have a common factor, if $a =$ **[Roorkee 1981]**

(a) 24 (b) 0, 24 (c) 3, 24 (d) 0, 3

378. If $x^2 - 3x + 2$ is a factor of $x^4 - px^2 + q$, then **[IIT 1974; MP PET 1995]**

(a) $p = 4, q = 5$ (b) $p = 5, q = 4$ (c) $p = -5, q = -4$ (d) None of these

379. If $x + 1$ is a factor of $x^4 - (p-3)x^3 - (3p-5)x^2 + (2p-7)x + 6$, then p is equal to **[IIT 1975]**

(a) – 4 (b) 4 (c) –1 (d) 1

380. If $x^2 + px + 1$ is a factor of the expression $ax^3 + bx + c$, then **[IIT 1980]**

(a) $a^2 + c^2 = -ab$ (b) $a^2 - c^2 = -ab$ (c) $a^2 - c^2 = ab$ (d) None of these

381. The condition that $x^3 - 3px + 2q$ may be divisible by a factor of the form $x^2 + 2ax + a^2$ is **[AMU 2002]**

(a) $3p = 2q$ (b) $3p + 2q = 0$ (c) $p^3 = q^2$ (d) $27p^3 = 4q^2$

382. If x be real then $\frac{(x-a)(x-b)}{x-c}$ will take all real values when **[IIT 1984; Karnataka CET 2002]**

(a) $a < b < c$ (b) $a > b > c$ (c) $a < c < b$ (d) Always

383. Let $y = \sqrt{\frac{(x+1)(x-3)}{(x-2)}}$, then all real values of x for which y takes real values, are **[IIT 1980]**

(a) $-1 \le x < 2$ or $x \ge 3$ (b) $-1 \le x < 3$ or $x > 2$ (c) $1 \le x < 2$ or $x \ge 3$ (d) None of these

384. The graph of the curve $x^2 = 3x - y - 2$ is

(a) Between the lines $x = 1$ and $x = \frac{3}{2}$ (b) Between the lines $x = 1$ and $x = 2$

(c) Strictly below the line $4y = 1$ (d) None of these

385. If $x^2 + px + 1$ is a factor of the expression $ax^3 + bx + c$ then

(a) $a^2 + c^2 = -ab$ (b) $a^2 - c^2 = -ab$ (c) $a^2 - c^2 = ab$ (d) None of these

386. If $x + \lambda y - 2$ and $x - \mu y + 1$ are factors of the expression $6x^2 - xy - y^2 - 6x + 8y - 12$, then

(a) $\lambda = \frac{1}{3}, \mu = \frac{1}{2}$ (b) $\lambda = 2, \mu = 3$ (c) $\lambda = \frac{1}{3}, \mu = -\frac{1}{2}$ (d) None of these

Advance Level

387. Given that, for all real x, the expression $\frac{x^2 - 2x + 4}{x^2 + 2x + 4}$ lies between $\frac{1}{3}$ and 3. The values between which the expression $\frac{9.3^{2x} + 6.3^x + 4}{9.3^{2x} - 6.3^x + 4}$ lies are **[Karanataka CET 1998]**

(a) $\frac{1}{3}$ and 3 (b) –2 and 0 (c) –1 and 1 (d) 0 and 2

388. If x, y, z are real and distinct, then $u = x^2 + 4y^2 + 9z^2 - 6yz - 3zx - 2xy$ is always **[IIT 1979]**

(a) Non-negative (b) Non-positive (c) Zero (d) None of these

389. If $x + y$ and $y + 3x$ are two factors of the expression $\lambda x^3 - \mu x^2 y + xy^2 + y^3$, then the third factor is

(a) $y + 3x$ (b) $y - 3x$ (c) $y - x$ (d) None of these

390. If $\log_{10} x + \log_{10} y \ge 2$ then the smallest possible value of $x + y$ is

(a) 10 (b) 30 (c) 20 (d) None of these

391. If α be the number of solutions of equation $[\sin x] = |x|$, where $[x]$ denote the integral part of x and m be the greatest value of $\cos(x^2 + xe^x - [x])$ on the interval $[-1, 1]$, then

(a) $\alpha = m$ (b) $\alpha < m$ (c) $\alpha > m$ (d) $\alpha \ne m$

392. If $f(x) = 3^x + 4^x + 5^x - 6^x$, then $f(x) < f(3)$ for

(a) Only one value of x (b) No value of x (c) Only two values of x (d) Infinitely many values of x

393. If $f(x)=\sum_{r=0}^{100} a_r x^r$ and $f(0)$ and $f(1)$ are odd numbers, then for any integer x

(a) $f(x)$ is odd or even according as x is odd or even (b) $f(x)$ is even or odd according as x is odd or even

(c) $f(x)$ is even for all integral values of x (d) $f(x)$ is odd for all integral values of x

394. If $x \in [2, 4]$ then for the expression $x^2-6x+5=0$

(a) The least value $=-4$ (b) The greatest value $=4$ (c) The least value $=3$ (d) The greatest value $=-3$

395. The value of 'a' for which $(a^2-1)x^2+2(a-1)x+2$ is positive for any x are

(a) $a \geq 1$ (b) $a \leq 1$ (c) $a \geq -3$ (d) $a \leq -3$ or $a \geq 1$

396. Let $f(x)$ be a quadratic expression which is positive for all real values of x, then for all real $x, 10[f(x)+f(-x)]$ *is*

(a) > 0 (b) ≥ 0 (c) < 0 (d) ≤ 0

397. The constant term of the quadratic expression $\sum_{k=1}^{n}\left(x-\frac{1}{k+1}\right)\left(x-\frac{1}{k}\right)$ as $n \to \infty$ is

(a) –1 (b) 0 (c) 1 (d) None of these

398. Let $f(x)=(1+b^2)x^2+2bx+1$ and let $m(b)$ be the minimum value of $f(x)$. As b varies, the range of $m(b)$ is

(a) $[0, 1]$ (b) $\left[0, \frac{1}{2}\right]$ (c) $\left[\frac{1}{2}, 1\right]$ (d) $(0, 1]$

399. If $p(x)$ be a polynomial satisfying the identity $p(x^2)+2x^2+10x=2xp(x+1)+3$, then $p(x)$ is given by

(a) $2x+3$ (b) $3x-4$ (c) $3x+2$ (d) $2x-3$

400. Let $y=\frac{\sin x \cos 3x}{\cos x \sin 3x}$, then

(a) y may be equal to $\frac{1}{3}$ (b) y may be equal to 3

(c) Set of possible value of y is $\left(-\infty, \frac{1}{3}\right) \cup (3, \infty)$ (d) Set of possible values of y is $\left(-\infty, \frac{1}{3}\right] \cup (3, \infty)$

401. If $a=\frac{x^2-2x+4}{x^2+2x+4}$, and equation of lines AB and CD be $3y=x$ and $y=3x$ respectively, then for all real x, point $P(a, a^2)$

(a) Lies in the acute angle between lines AB and CD (b) Lies in the obtuse angle between lines AB and CD

(c) Cannot be in the acute angle between lines AB and CD (d) Cannot lie in the obtuse angle between lines AB and CD

Position of roots

Basic Level

402. If a, b, c are real numbers such that $a+b+c=0$, then the quadratic equation $3ax^2+2bx+c=0$ has **[MNR 1992; DCE 1995**

(a) At least one root in [0, 1] (b) At least one root in [1, 2]

(c) At least one root in [–1, 0] (d) None of these

403. The number of values of k for which the equation $x^2-3x+k=0$ has two real and distinct roots lying in the interval (0, 1), are **[UPSEAT 2001; Kurukshetra CEET 2002]**

(a) 0 (b) 2 (c) 3 (d) Infinitely many

404. The value of k for which the equation $(k-2)x^2+8x+k+4=0$ has both real, distinct and negative is **[Orissa JEE 2002]**

(a) 0 (b) 2 (c) 3 (d) –4

Advance Level

405. Let *a, b, c* be real number $a \neq 0$. If α is a root of $a^2x^2 + bx + c = 0$; β is a root of $a^2x^2 - bx - c = 0$ and $0 < \alpha < \beta$; then the equation $a^2x^2 + 2bx + 2c = 0$ has a root γ which always satisfies **[IIT 1989]**

(a) $\gamma = \frac{\alpha + \beta}{2}$ (b) $\gamma = \alpha + \frac{\beta}{2}$ (c) $\gamma = \alpha$ (d) $\alpha < \gamma < \beta$

406. Let *a, b, c* be non-zero real numbers such that $\int_0^1 (1 + \cos^8 x)(ax^2 + bx + c)\,dx = \int_0^2 (1 + \cos^8 x)(ax^2 + bx + c)\,dx$, then the quadratic equation $ax^2 + bx + c = 0$ has

(a) No root in (0, 2) (b) At least one root in (0, 1) (c) A double root in (0, 2) (d) Two imaginary roots

407. For the equation $2x^2 + 6\sqrt{2}x + 1 = 0$

(a) Roots are rational (b) If one root is $p + \sqrt{q}$ then the other is $-p + \sqrt{q}$

(c) Roots are irrational (d) If one root is $p + \sqrt{q}$ then the other is $p - \sqrt{q}$

408. The values of *a* for which both roots of the equation $(1 - a^2)x^2 + 2ax - 1 = 0$ lie between 0 and 1 are given by

(a) $a > 2$ (b) $1 < a < 2$ (c) $-\infty < a < \infty$ (d) None of these

409. If *p, q* be non-zero real numbers and $f(x) \neq 0$ in [0, 2] and $\int_0^1 f(x).(x^2 + px + q)\,dx = \int_0^2 f(x).(x^2 + px + q)\,dx = 0$ then equation $x^2 + px + q = 0$ has

(a) Two imaginary roots (b) No root in (0, 2)

(c) One root in (0, 1) and other in (1, 2) (d) One root in $(-\infty, 0)$ and other in $(2, \infty)$

410. If $a, b, c \in R, a \neq 0$ and $(b-1)^2 < 4ac$, then the number of real roots of the system of equation (in three unknowns x_1, x_2, x_3)

$ax_1^2 + bx_1 + c = x_2$, $ax_2^2 + bx_2 + c = x_3$, $ax_3^2 + bx_3 + c = x_1$ is

(a) 0 (b) 1 (c) 2 (d) 3

411. If $0 < \alpha < \frac{\pi}{4}$, equation $(x - \sin\alpha)(x - \cos\alpha) - 2 = 0$ has

(a) Both roots in $(\sin\alpha, \cos\alpha)$ (b) Both roots in $(\cos\alpha, \sin\alpha)$

(c) One root in $(-\infty, \cos\alpha)$ and other in $(\sin\alpha, \infty)$ (d) One root in $(-\infty, \sin\alpha)$ and other in $(\cos\alpha, \infty)$

412. For equation $x^3 - 6x^2 + 9x + k = 0$ to have exactly one root in (1, 3), the set of values of *k* is

(a) (– 4, 0) (b) (1, 3) (c) (0, 4) (d) None of these

413. Let $f(x) = x^3 - 6x^2 + 3(1+\pi)x + 7$, $p > q > r$, then $\frac{\{x - f(p)\}\{x - f(r)\}}{x - f(q)}$ has no value in

(a) (p, q) (b) (q, r) (c) (r, ∞) (d) None of these

414. If $a + b + 2c = 0,\ c \neq 0$, then equation $ax^2 + bx + c = 0$ has

(a) At least one root in (0, 1) (b) At least one root in (0, 2)

(c) At least one root in (–1, 1) (d) None of these

415. If $ax^2 - bx + c = 0$ has two distinct real roots in (0, 1), where $a, b, c \in N$, then $16c(a - b + c)$

(a) $= a^2$ (b) $< a^2$ (c) $> a^2$ (d) $\geq a^2$

Solution of Quadratic inequations

Basic Level

416. If $a < b$, then the solution of $x^2 + (a+b)x + ab < 0$, is given by

(a) $a < x < b$ (b) $x < a$ or $x > b$ (c) $-b < x < -a$ (d) $x < -b$ or $x < -a$

417. The solution of $6 + x - x^2 > 0$ is **[DCE 2000; Kurakshetra CEE 1999]**

(a) $-1 < x < 2$ (b) $-2 < x < 3$ (c) $-2 < x < -1$ (d) None of these

418. For all $x \in R$, if $mx^2 - 9mx + 5m + 1 > 0$, then m lies in the interval **[AMU 1989]**

(a) $\left(-\frac{4}{61}, 0\right)$ (b) $\left[0, \frac{4}{61}\right)$ (c) $\left(\frac{4}{61}, \frac{61}{4}\right)$ (d) $\left(-\frac{61}{4}, 0\right]$

419. If $x^2 - 1$ is a factor of $x^4 + ax^3 + 3x - b$, then

(a) $a = 3, b = -1$ (b) $a = -3, b = 1$ (c) $a = 3, b = 1$ (d) None of these

420. If $(x-1)^3$ is factor of $x^4 + ax^3 + bx^2 + cx - 1$ then the other factor is

(a) $x - 3$ (b) $x + 1$ (c) $x + 2$ (d) None of these

421. The set of values of x which satisfy $5x + 2 < 3x + 8$ and $\frac{x+2}{x-1} < 4$, is **[EAMCET 1989]**

(a) (2, 3) (b) $(-\infty, 1) \cup (2, 3)$ (c) $(-\infty, 1)$ (d) (1, 3)

422. The solution of the equation $2x^2 + 3x - 9 \le 0$ is given by **[Kurukshetra CEE 1998]**

(a) $\frac{3}{2} \le x \le 3$ (b) $-3 \le x \le \frac{3}{2}$ (c) $-3 \le x \le 3$ (d) $\frac{3}{2} \le x \le 2$

423. The complete solution of the inequation $x^2 - 4x < 12$ is **[AMU 1999]**

(a) $x < -2$ or $x > 6$ (b) $-6 < x < 2$ (c) $2 < x < 6$ (d) $-2 < x < 6$

424. If x is real and satisfies $x + 2 > \sqrt{x+4}$, then **[AMU 1999]**

(a) $x < -2$ (b) $x > 0$ (c) $-3 < x < 0$ (d) $-3 < x < 4$

425. If $a < 0$ then the inequality $ax^2 - 2x + 4 > 0$ has the solution represented by **[AMU 2001]**

(a) $\frac{1+\sqrt{1-4a}}{a} > x > \frac{1-\sqrt{1-4a}}{a}$ (b) $x < \frac{1-\sqrt{1-4a}}{a}$

(c) $x < 2$ (d) $2 > x > \frac{1+\sqrt{1-4a}}{a}$

Advance Level

426. If x satisfies $|x-1| + |x-2| + |x-3| \ge 6$, then

(a) $0 \le x \le 4$ (b) $x \le -2$ or $x \ge 4$ (c) $x \le 0$ (d) None of these

427. The number of positive integral solutions of $\frac{x^2(3x-4)^3(x-2)^4}{(x-5)^5(2x-7)^6} \le 0$ is

(a) 4 (b) 3 (c) 2 (d) 1

428. If $5^x + (2\sqrt{3})2^x \ge 13^x$, then the solution set for x is

(a) $[2, \infty)$ (b) {2} (c) $(-\infty, 2]$ (d) [0, 2]

429. The inequality $|2x - 3| < 1$ is valid when x lies in **[IIT 1993]**

(a) (3, 4) (b) (1, 2) (c) (–1, 2) (d) (–4, 3)

430. The graph of the function $y = 16x^2 + 8(a+5)x - 7a - 5$ is strictly above the x-axis, then 'a' must satisfy the inequality

(a) $-15 < a < -2$ (b) $-2 < a < -1$ (c) $5 < a < 7$ (d) None of these

431. If x is a real number such that $x(x^2+1), (-1/2)x^2, 6$ are three consecutive terms of an A.P. then the next two consecutive term of the A.P. are

(a) 14, 6 (b) –2, –10 (c) 14, 22 (d) None of these

432. If x, y are rational numbers such that $x + y + (x - 2y)\sqrt{2} = 2x - y + (x - y - 1)\sqrt{6}$, then

(a) x and y connot be determined (b) $x = 2, y = 1$

(c) $x = 5, y = 1$ (d) None of these

433. If $[x]=$ the greatest interger less than or equal to x, and (x)= the least integer greatest than or equal to x and $[x]^2+(x)^2>25$ then x belongs to

(a) [3, 4] (b) $(-\infty, -4]$ (c) $[4, +\infty)$ (d) $(-\infty, -4]\cup[4, +\infty)$

434. The set of real values of x satisfying $|x-1|\le 3$ and $|x-1|\ge 1$ is

(a) [2, 4] (b) $(-\infty, 2]\cup[4, +\infty)$ (c) $[-2, 0]\cup[2, 4]$ (d) None of these

435. The set of real values of x satisfying $\big||x-1|-1\big|\le 1$ is

(a) $[-1, 3]$ (b) $[0, 2]$ (c) $[-1, 1]$ (d) None of these

436. If $x\in Z$ (the set of integers) such that $x^2-3x<4$ then the number of possible values of x is

(a) 3 (b) 4 (c) 6 (d) None of these

437. If x is an interger satisfying $x^2-6x+5\le 0$ and $x^2-2x>0$ then the number of possible values of x is

(a) 3 (b) 4 (c) 2 (d) Infinite

438. The solution set of the ineuation $\log_{1/3}(x^2+x+1)+1>0$ is

(a) $(-\infty, -2)\cup(1, +\infty)$ (b) $[-1, 2]$ (c) $(-2, 1)$ (d) $(-\infty, +\infty)$

439. If $3^{x/2}+2^x>25$ then the solution set is

(a) R (b) $(2, +\infty)$ (c) $(4, +\infty)$ (d) None of these

440. The solution set of $\dfrac{x^2-3x+4}{x+1}>1, x\in R$, is

(a) $(3, +\infty)$ (b) $(-1, 1)\cup(3, +\infty)$ (c) $[-1, 1]\cup[3, +\infty)$ (d) None of these

441. The equation $|x+1||x-1|=a^2-2a-3$ can have real solutions for x if a belongs to

(a) $(-\infty, -1]\cup[3, +\infty)$ (b) $[1-\sqrt{5}, 1+\sqrt{5}]$ (c) $[1-\sqrt{5}, -1]\cup[3, 1+\sqrt{5}]$ (d) None of these

Miscellaneous Problems

Basic Level

442. If $x^2+2x+2xy+my-3$ has two rational factors, then the value of m will be **[Rajasthan PET 1990]**

(a) $-6, -2$ (b) $-6, 2$ (c) $6, -2$ (d) 6, 2

443. If $x^2-hx-21=0$, $x^2-3hx+35=0$ $(h>0)$ has a common root, then the value of h is equal to **[EAMCET 1986]**

(a) 1 (b) 2 (c) 3 (d) 4

444. Minimum value of $(a+b+c+d)\left(\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}+\dfrac{1}{d}\right)$ is

(a) 4 (b) 9 (c) 16 (d) 25

445. Let $f(x)=ax^3+5x^2-bx+1$. If $f(x)$ when divided by $2x+1$ leaves 5 as remainder, and $f'(x)$ is divisible by $3x-1$ then

(a) $a=26, b=10$ (b) $a=24, b=11$ (c) $a=26, b=12$ (d) None of these

446. $x^{3^n}+y^{3^n}$ is divisible by $x+y$ if

(a) n is any integer ≥ 0 (b) n is an odd positive integer

(c) n is an even positive integer (d) n is a rational number

447. The number of solution of the equation $|x|=\cos x$ is

(a) One (b) Two (c) Three (d) Zero

448. The line $y+14=0$ cuts the curve whose equation is $x(x^2+x+1)+y=0$ at

(a) Three real points (b) One real point (c) At least one real point (d) No real point

449. Let R=the set of real numbers, J = the set of integers, N= the set of natural numbers. If S be the solution set of the equation $(x)^2 + [x]^2 = (x-1)^2 + [x+1]^2$, where (x) = the least integer greater then or equal to x and $[x]$ = the greatest integer less than or equal to x, then

(a) $S = R$ (b) $S = R - Z$ (c) $S = R - N$ (d) None of these

450. The number of real roots or $x^8 - x^5 + x^2 - x + 1 = 0$ is equal to

(a) 0 (b) 2 (c) 4 (d) 6

451. The number of positive real roots of $x^4 - 4x - 1 = 0$ is

(a) 3 (b) 2 (c) 1 (d) 0

452. The number of negative real roots of $x^4 - 4x - 1 = 0$ is

(a) 3 (b) 2 (c) 1 (d) 0

453. The number of complex roots of the equation $x^4 - 4x - 1 = 0$ is

(a) 3 (b) 2 (c) 1 (d) 0

454. $x^2 - 4$ is a factor of $f(x) = (a_1x^2 + b_1x + c_1).(a_2x^2 + b_2x + c_2)$ if

(a) $b_1 = 0, c_1 + 4a_1 = 0$ (b) $b_2 = 0, c_2 + 4a_2 = 0$

(c) $4a_1 + 2b_1 + c_1 = 0, 4a_2 + c_2 = 2b_2$ (d) $4a_1 + c_1 = 2b_1, 4a_2 + 2b_2 + c_2 = 0$

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Answer Sheet

Quadratic Equations and Inequations ***Assignment (Basic and Advance Level)***

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
a	c	a	d	d	c	d	d	b	d	d	d	c	d	c	a	c	b	d	c
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
c	d	c	c	b	c	d	c	b	c	b	c	a	a	b	c	b	b	b	c
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
a	c	a	a	d	d	c	a	a	c	b	a	a	a	a	d	b	b	c	b
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
c	a	c	b	a	d	c	a	a	a	d	c	b	d	b	b	b	c	b	a
81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
d	c	b	b	b	a	c	c	b	d	b	c	b	c	b	a	b	a	a	a
101	**102**	**103**	**104**	**105**	**106**	**107**	**108**	**109**	**110**	**111**	**112**	**113**	**114**	**115**	**116**	**117**	**118**	**119**	**120**
c	b	c	b	a	a	a	a	c	d	c	c	b	a	c	d	b	c	b	a
121	**122**	**123**	**124**	**125**	**126**	**127**	**128**	**129**	**130**	**131**	**132**	**133**	**134**	**135**	**136**	**137**	**138**	**139**	**140**
d	a	a	c	c	c	d	d	c	c	b	b	b	c	b	d	d	a	a	b
141	**142**	**143**	**144**	**145**	**146**	**147**	**148**	**149**	**150**	**151**	**152**	**153**	**154**	**155**	**156**	**157**	**158**	**159**	**160**
a	b	a	a,c	c	b	c	a	c	b	a	b	a	d	b	d	a	a	b	b
161	**162**	**163**	**164**	**165**	**166**	**167**	**168**	**169**	**170**	**171**	**172**	**173**	**174**	**175**	**176**	**177**	**178**	**179**	**180**
b	c	b	b	b	a	c	c	b	b	a	b	c	d	b	b	a	a	b	d
181	**182**	**183**	**184**	**185**	**186**	**187**	**188**	**189**	**190**	**191**	**192**	**193**	**194**	**195**	**196**	**197**	**198**	**199**	**200**
a	a	a	c	a	c	d	b	c	d	b	d	a	c	a	a	b	a	d	d
201	**202**	**203**	**204**	**205**	**206**	**207**	**208**	**209**	**210**	**211**	**212**	**213**	**214**	**215**	**216**	**217**	**218**	**219**	**220**
b	c	c	c	a	d	b	d	d	c	d	c	a	a	d	a	a	d	b	a
221	**222**	**223**	**224**	**225**	**226**	**227**	**228**	**229**	**230**	**231**	**232**	**233**	**234**	**235**	**236**	**237**	**238**	**239**	**240**
a	b	b	b	d	c	a	d	a	a	d	c	b	a	c	b	b	c	a	d
241	**242**	**243**	**244**	**245**	**246**	**247**	**248**	**249**	**250**	**251**	**252**	**253**	**254**	**255**	**256**	**257**	**258**	**259**	**260**
b	b	a	c	c	b	d	b	d	c	a	c	d	b	b	b	a	b	a	a
261	**262**	**263**	**264**	**265**	**266**	**267**	**268**	**269**	**270**	**271**	**272**	**273**	**274**	**275**	**276**	**277**	**278**	**279**	**280**
b	a	c	a	b	a	c	a	d	b	b	b	b	a	a	a	a	b	d	a
281	**282**	**283**	**284**	**285**	**286**	**287**	**288**	**289**	**290**	**291**	**292**	**293**	**294**	**295**	**296**	**297**	**298**	**299**	**300**
d	a	a	c	a	b	a	b	a	a	a	a	b	d	b	d	d	d	b	c
301	**302**	**303**	**304**	**305**	**306**	**307**	**308**	**309**	**310**	**311**	**312**	**313**	**314**	**315**	**316**	**317**	**318**	**319**	**320**
a	b,c	a,c	b,c	b	b	b	c	b	b	d	c	b	c	a	d	b	a	d	c
321	**322**	**323**	**324**	**325**	**326**	**327**	**328**	**329**	**330**	**331**	**332**	**333**	**334**	**335**	**336**	**337**	**338**	**339**	**340**
a	d	b	a	c	a,c	c	b	a,c	c	a,b	c	c	a	a,b	c	c	c	d	d
341	**342**	**343**	**344**	**345**	**346**	**347**	**348**	**349**	**350**	**351**	**352**	**353**	**354**	**355**	**356**	**357**	**358**	**359**	**360**
c	b	b	b	c	c	c	c	a	a	b	b	a	c,d	b,c	a,b	a,c	c	c	a
361	**362**	**363**	**364**	**365**	**366**	**367**	**368**	**369**	**370**	**371**	**372**	**373**	**374**	**375**	**376**	**377**	**378**	**379**	**380**
c	a	a,b,c,d	a	a	a	a	c	b	c	d	d	c	d	d	b	b	b	b	c
381	**382**	**383**	**384**	**385**	**386**	**387**	**388**	**389**	**390**	**391**	**392**	**393**	**394**	**395**	**396**	**397**	**398**	**399**	**400**
c	c	a	c	c	a	a	a	b	c	a	d	d	a,d	d	a	c	d	a	c
401	**402**	**403**	**404**	**405**	**406**	**407**	**408**	**409**	**410**	**411**	**412**	**413**	**414**	**415**	**416**	**417**	**418**	**419**	**420**
d	a	a	c	d	b	b,c	a	c	a	d	a	d	a,b,c	b	c	b	b	b	b
421	**422**	**423**	**424**	**425**	**426**	**427**	**428**	**429**	**430**	**431**	**432**	**433**	**434**	**435**	**436**	**437**	**438**	**439**	**440**
b	b	d	b	a	c	b	c	b	a	c	b	d	a	a	a,b	a	c	c	b

441	442	443	444	445	446	447	448	449	450	451	452	453	454
a,c	c	d	c	c	a	b	b	b	a	c	c	b	a,b,c,d