and $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{6}$

According to addition theorem of probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{3}{6} + \frac{2}{6} - \frac{1}{6} = \frac{4}{6}$$

Verification :

$\therefore$ $(A \cup B) = \{2, 3, 4, 6\}$ and $P((A \cup B) = \frac{n(A \cup B)}{n(S)} = \frac{4}{6}$

Example : 15.

A die is thrown once. What is the probability of getting an odd number or an even number?

Solution :

Let O be an event of getting an odd number, E be an event of getting an even number and $(O \cup E)$ is an event of getting an odd number or an even number.

Here, $S = \{1, 2, 3, 4, 5, 6\}$, $O = \{1, 3, 5\}$, $E = \{2, 4, 6\}$

and $(O \cup E)$ (i.e., O and E are mutually exclusive)

$\therefore P(O) = \frac{3}{6}$, $P(E) = \frac{3}{6}$ and $P(O \cup E) = P(O) + P(E) = \frac{3}{6} + \frac{3}{6} = \frac{6}{6} = \mathbf{1}$

Note: Here, $(O \cup E) = \{1, 2, 3, 4, 5, 6\} = S$.

Verification, $P(S) = 1$

Example : 16.

A card is drawn from a pack of 52 playing cards. What is the probability that it is a Club or a King ?

Solution :

Let C be an event of getting a club(♣), K be an event of getting a king and CK is an event of getting a club king (♣K) .

Here, $P(C) = \frac{13}{52}$, $P(K) = \frac{4}{52}$ and $P(CK) = \frac{n(CK)}{n(S)} = \frac{1}{52}$

We know that, $P(C \cup K) = P(C) + P(K) - P(CK)$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \mathbf{0.3077}$$

Example : 17.

In a town 15% of the population read newspaper A, 13% of the population read newspaper B and 8% of the population read newspapers A and B. Find the probability that a person selected at random shall read either paper A or paper B.

Solution :

Here, $P(A) = 15\% = 0.15$, $P(B) = 13\% = 0.13$ and $P(A \cap B) = 8\% = 0.08$

According to addition theorem of probability,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.15 + 0.13 - 0.08 = \mathbf{0.2}$$

Example : 18

A box contains 200 bolts and 300 nuts. 20% of bolts and half of the nuts are rusted. If one item is selected at random, what is the probability that it is a rusted item or a bolt ?

Solution : Here, total number of itmes = 200 + 300 = 500

Number of rusted bolts = 20% of 200 = $\frac{20}{100} \times 200 = 40$

Number of rusted nuts = $\frac{1}{2} \times 300 = 150$

$\therefore$ Number of rusted items = 40 + 150 = 190

Let R be an event of getting rusted item, B be an event of getting bolt, and RB be an event of getting rusted bolt.

$\therefore P(R) = \frac{190}{500}$, $P(B) = \frac{200}{500}$ and $P(R \cap B) = \frac{40}{500}$

We know that, $P(R \cup B) = P(R) + P(B) - P(R \cap B) = \frac{190}{500} + \frac{200}{500} - \frac{40}{500} = \frac{350}{500} = \mathbf{0.7}$

Example :19. If $P(A) = \frac{1}{10}, P(B) = \frac{1}{5}$ and $P(A \cap B) = \frac{1}{20}$ then, find $P(A \cup B)$.

Solution : We know that, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{10} + \frac{1}{5} - \frac{1}{20} = \frac{5}{20} \text{ or } \mathbf{0.25}$$

Example : 20.

If $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{6}$ then, find $P(A \cup B)$.

Solution : We know that, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{2}{3} \text{ or } \mathbf{0.6667}$$

Example : 21.

If $P(A) = \frac{1}{8}, P(B) = \frac{1}{6}$ and $P(A \cup B) = \frac{1}{4}$ then, find $P(A \cap B)$.

Solution : We know that, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{8} + \frac{1}{6} - \frac{1}{4} = \frac{1}{24} \text{ or } \mathbf{0.0417}$$

Example : 22.

If $P(A \cup B) = \frac{1}{3}, P(A) = \frac{1}{6}$ and $P(A \cap B) = \frac{1}{12}$ then, find $P(B)$.

Solution : We know that, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\frac{1}{3} = \frac{1}{6} + P(B) - \frac{1}{12}$$

$$\Rightarrow P(B) = \frac{1}{3} - \frac{1}{6} + \frac{1}{12} = \frac{1}{4} \text{ or } \mathbf{0.25}$$

Addition theorem of probability for two mutually exclusive events : (MEE)

Statement :

Let A and B be two mutually exclusive events with respective probabilities P (A) and P(B). Then, the probability of occurrence of at least one of these two events is –

$P(A \cup B) = P(A) + P(B)$ or $P(A+B) = P(A) + P(B)$

Proof: Let S be the sample space, A and B be two mutually exclusive events.

By definition, $P(A) = \frac{n(A)}{n(S)}$, $P(B) = \frac{n(B)}{n(S)}$ and $P(A \cup B) = \frac{n(A \cup B)}{n(S)}$

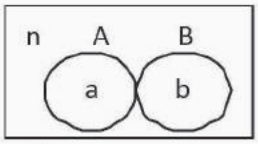
Consider $P(A \cup B) = \frac{n(A \cup B)}{n(S)} = \frac{n(A) + n(B)}{n(S)}$ [$\because$ for MEE $n(A \cup B) = n(A) + n(B)$]

$$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)}$$

$$= P(A) + P(B)$$

Alternative Proof

Out of 'n' exhaustive outcomes of a random experiment, if 'a' outcomes are favourable to event A, 'b' outcomes are favourable to event B and there are no common outcomes.



Then, $P(A) = \frac{a}{n}$ and $P(B) = \frac{b}{n}$

Here, favourable outcomes to event (A or B) are a + b

$$\therefore P(A \cup B) = \frac{a+b}{n} = \frac{a}{n} + \frac{b}{n} = P(A) + P(B)$$

i.e., For mutually exclusive events, the probability of sum of events is equal to sum of probabilities of events.

Note:

If $A_1, A_2, \ldots, A_k$ are ' k ' MEE. Then, $P(A_1 \cup A_2 \cup \ldots \cup A_k) = P(A_1) + P(A_2) + \ldots + P(A_k)$

Example : 23.

When two different coins are tossed, find the probability of getting one or more heads (at least one head).

Solution :

Let A be an event of getting 2 heads, B be an event of getting 1 head and $(A \cup B)$ be an event of getting one or more heads.

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{1}{4}$, $P(B) = \frac{n(B)}{n(S)} = \frac{2}{4}$ and $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{0}{4} = 0 = P(\Phi)$

According to addition theorem of MEE; $P(A \cup B) = P(A) + P(B) = \frac{1}{4} + \frac{2}{4} = \frac{3}{4} = \mathbf{0.75}$

Verification: Here, $(A \cup B) = \{HH, HT, TH\}$ Thus, $P(A \cup B) = \frac{n(A \cup B)}{n(S)} = \frac{3}{4} = \mathbf{0.75}$

Example : 24.

A die is thrown once. What is the probability of getting an odd number or a multiple of 4 ?

Solution :

Let A be an event of getting an odd number, B be an event of getting a multiple of 4 and $(A \cup B)$ is an event of getting an odd number or a multiple of 4.

Here, $S = \{1, 2, 3, 4, 5, 6\}$, $A = \{1, 3, 5\}$, $B = \{4\}$ and $(A \cap B) = \Phi$ i.e., A and B are MEE

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{3}{6}$, $P(B) = \frac{n(B)}{n(S)} = \frac{1}{6}$ and $P(A \cup B) = P(A) + P(B) = \frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3} = \mathbf{0.67}$

Example : 25.

A card is drawn from a pack of 52 playing cards. What is the probability that it is a: (i) Red or Club ? (ii) King or Queen ?

Solution :

Let, R is an event of getting red, C is an event of getting club (♣), K is an event of getting king and Q is an event of getting queen.

(i) Here, R and C are mutually exclusive (i.e., $R \cap C = \Phi$). $P(R) = \frac{26}{52}$, $P(C) = \frac{13}{52}$

$$\therefore P(R \cup C) = P(R) + P(C) = \frac{26}{52} + \frac{13}{52} = \frac{39}{52} = \mathbf{0.75}$$

(ii) Here, K and Q are mutually exclusive (i.e., $K \cap Q = \Phi$). $P(K) = \frac{4}{52}$, $P(Q) = \frac{4}{52}$

$$\therefore P(K \cup Q) = P(K) + P(Q) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \mathbf{0.1538}$$

Example : 26.

A bag contains 6 red and 4 white balls. Two balls are drawn from the bag. What is the probability that they are of the same colour ?

Solution :

Let R be an event of getting two red balls and W be an event of getting two white balls.

Here, $P(R) = \frac{6_{C_2}}{10_{C_2}} = \frac{15}{45}$, $P(W) = \frac{4_{C_2}}{10_{C_2}} = \frac{6}{45}$

and R and W are mutually exclusive (i.e., $R \cap W = \Phi$).

$$\therefore P(R \cup W) = P(R) + P(W) = \frac{15}{45} + \frac{6}{45} = \frac{21}{45} = \mathbf{0.47}$$

Example : 27.

The probabilities of two mutually exclusive events are 1/4 and 1/6. Find the probability of occurrence of at least one of the events.

Solution:

Let, A and B be the two mutually exclusive events (i.e., $A \cap B = \Phi$).

Here, $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{6}$

$$\therefore P(A \cup B) = P(A) + P(B) = \frac{1}{4} + \frac{1}{6} = \frac{5}{12} = \mathbf{0.4167}$$

Independent events :

If the occurrence or non-occurrence of one event does not influence the occurrence of another event then those events are said to be independent.

Ex: If two cards are drawn from a pack of 52 playing cards one after the other **with replacement,** then the event of drawing a king in the first draw and the event of drawing a king in the second draw are independent events.

Dependent events :

If the occurrence or non-occurrence of one event influences the occurrence of another event then those events are said to be dependent.

Ex : If two cards are drawn from a pack of 52 playing cards one after the other **without replacement,** then the event of drawing a king in the first draw and the event of drawing a king in the second draw are dependent events.

Conditional probability (Probability of dependent events) :

The probability of occurrence of one event under the condition that another event has already occurred is known as conditional probability.

The probability that B will occur under the condition that A has already occurred is denoted by P(B | A).

$$P(B|A) = \frac{\text{Total number of favourable cases of B}}{\text{Total number of favourable cases of A}}$$

The probability that A will occur under the condition that B has already occurred is denoted by P(A | B).

Multiplication theorem of probability for two dependent events :

Statement :

In a random experiment, if the event B is dependent on another event A, then probability of occurrence of both these events is given by

$P(A \cap B) = P(A)\, P(B|A)$

Proof : Let S be the sample space, A and B be two events.

By definition, $P(A) = \frac{n(A)}{n(S)}$, $P(B) = \frac{n(B)}{n(S)}$ and $P(A \cap B) = \frac{n(A \cap B)}{n(S)}$

Suppose that event A has occurred. Then there are n(A) possible cases. Out of these n(A) possible cases, $n(A \cap B)$ cases are favourable to event B.

By definition of conditional probability,

$$P(B|A) = \frac{\text{Total number of favourable cases of B}}{\text{Total number of favourable cases of A}} = \frac{n(A \cap B)}{n(A)}$$

Consider, $P(A \cap B) = \frac{n(A \cap B)}{n(S)}$ [Multiply and divide by n(A)]

$$= \frac{n(A)}{n(S)} \times \frac{n(A \cap B)}{n(A)}$$

$$= P(A)\ P(B|A)$$

Similarly, it can be proved that $P(A \cap B) = P(B)\ P(A | B)$ when event A depends on event B. Where $P(A | B)$ is the conditional probability of A given B.

Alternative Proof.

Out of 'n' exhaustive outcomes of a random experiment, let 'a' outcomes be favourable to event A, 'b' outcomes be favourable to event B and 'c' outcomes are favourable to both A and B.

Then, $P(A) = \frac{a}{n}$, $P(B) = \frac{b}{n}$ and $P(A \cap B) = \frac{c}{n}$

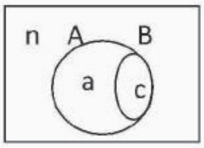
Suppose that event A has occurred. Then there are 'a' possible cases. Out of these 'a' possible cases, 'c' cases are favourable to event B .

By definition of conditional probability

$$P(B|A) = \frac{\text{Total number of favourable cases of B}}{\text{Total number of favourable cases of A}}$$

$P(B|A) = \frac{c}{a}$ [divide both numerator and denominator by ' n ']

$= \frac{c/n}{a/n} = \frac{P(A \cap B)}{P(A)}$



$\Rightarrow P(A \cap B) = P(A)\, P(B|A)$

Note : If A, B and C are three dependent events.

Then, $P(A \cap B \cap C) = P(A).\ P(B|A).\ P(C|AB)$

Example : 28.

A card is drawn from a pack of 52 playing cards. What is the probability that it is a king known that drawn card is black ?

Solution :

Let (K|B) be an event of getting a king known that drawn card is black, (BK) is an event of getting black-king card and B is an event of getting black card.

Here, $P(B) = \frac{26}{52}$ and $P(BK) = \frac{2}{52}$ $\therefore P(K|B) = \frac{P(BK)}{P(B)} = \frac{2/52}{26/52} = \frac{2}{26} = \mathbf{0.0769}$

Example : 29.

A card is drawn from a pack of 52 playing cards. What is the probability that it is a diamond card known that drawn card is red ?

Solution :

Let (D|R) be an event of getting diamond card known that the drawn card is red, (RD) is an event of getting a red-diamond card and R is an event of getting a red card.

Here, $P(R) = \frac{26}{52}$ and $P(RD) = \frac{13}{52}$ $\therefore P(D/R) = \frac{P(RD)}{P(R)} = \frac{13/52}{26/52} = \frac{1}{2} = \mathbf{0.5}$

Example : 30.

A box contains twelve cards numbered from 1 to 12. A card is drawn randomly from this box. What is the probability that it is an even number card, known that the number on the drawn card is of two digits ?

Solution :

Let A be an event of getting two-digit number card, (B | A) is an event of getting even number card when it is known that the drawn card is of two digits and (AB) is an event of getting two-digit even number card.

$\therefore P(A) = \frac{3}{12}$, $P(A \cap B) = \frac{2}{12}$ and we know that, $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{2/12}{3/12} = \frac{2}{3} = \mathbf{0.6667}$

Example : 31. If $P(AB) = \frac{1}{4}$ and $P(A) = \frac{3}{4}$ then, find $P(B|A)$.

Solution : We know that, $P(B|A) = \frac{P(AB)}{P(A)} = \frac{1/4}{3/4} = \frac{1}{3} = \mathbf{0.3333}$

Example : 32. If $P(AnB) = \frac{1}{3}$ and $P(B) = \frac{2}{3}$ then, find $P(A|B)$.

Solution : We know that, $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/3}{2/3} = \frac{1}{2} = \mathbf{0.5}$

Example : 33. If $P(A) = \frac{5}{6}$ and $P(B|A) = \frac{2}{5}$ then, find $P(A \cap B)$.

Solution : We know that, $P(A \cap B) = P(A)\,P(B|A) = \frac{5}{6} \times \frac{2}{5} = \frac{1}{3} = \mathbf{0.3333}$

Multiplication theorem of probability for two independent events :

[Theorem of compound probability]

Statement :

Let A and B be two independent events with respective probabilities P(A) and P(B). Then, the probability of simultaneous occurrence of A and B is

$P(A \cap B) = P(A) \times P(B)$.

Proof : If a random experiment has n_1 outcomes of which 'a' outcomes are favourable to event A. Another random experiment has n_2 outcomes of which 'b' outcomes are favourable to event B. Thus, out of $n_1 \times n_2$ outcomes ' $a \times b$ ' outcomes are favourable to event A and B.

By definition,

$$P(A) = \frac{a}{n_1},\ P(B) = \frac{b}{n_2},\ P(A \cap B) = \frac{a \times b}{n_1 \times n_2} = \frac{a}{n_1} \times \frac{b}{n_2} = P(A)\ P(B)$$

i.e., For independent events, the probability of product of events is equal to product of probabilities of events.

Thus, Two events A and B are independent if and only if P(AB) = P(A) P(B).

Note : If $A_1, A_2, \ldots, A_k$ are ' k ' independent events,then,

$$P(A_1 \cap A_2 \cap \ldots \cap A_k) = P(A_1) \times P(A_2) \times \ldots \times P(A_k)$$

Example : 34.

The first bag contains 3 red and 2 green balls. The second bag contains 4 red and 5 green balls. A ball is selected from each bag. What is the probability that they are red ?

Solution :

Let A be an event of getting a red ball from the first bag and B be an event of getting a red ball from the second bag.

$\therefore P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{9}$. Here, events A and B are independent

$$\therefore P(A \cap B) = P(A)\ P(B) = \frac{3}{5} \times \frac{4}{9} = \frac{4}{15} = \mathbf{0.2667}$$

Example : 35.

A fair coin and a fair die are thrown. Find the probability that coin shows a head and die shows a multiple of 3.

Solution :

Let A be an event of getting head on the coin and B be an event of getting a multiple of 3 on the die.

$\therefore P(A) = \frac{1}{2}$ and $P(B) = \frac{2}{6}$. Here, events A and B are independent

$$\therefore P(AB) = P(A)\ P(B) = \frac{1}{2} \times \frac{2}{6} = \frac{1}{6} = \mathbf{0.1667}$$

Example : 36.

The Probabilities of three drivers A, B and C driving home safely after consuming liquor are 3/4, 4/5 and 5/6 respectively. One day, the three drivers drive home after consuming liquor in a party. Find the probability that all the three drive home safely.

Solution :

Here, $P(A) = \frac{3}{4}$, $P(B) = \frac{4}{5}$ and $P(C) = \frac{5}{6}$ Here, the events A, B and C are independent.

$\therefore$ P(Probability that all the three drive home safely) = P(ABC)

$$= P(A)\ P(B)\ P(C)$$

$$= \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6}$$

$$= \frac{1}{2} \text{ or } \mathbf{0.5}$$

Example : 37.

A fair coin is tossed thrice. What is the probability that all the three tosses result in heads ?

Solution :

Let A be an event that the first toss results in head, B be an event that the second toss results in head and C be an event that the third toss results in head. Then, A, B and C are results of three different tosses, they are independent. Therefore that all the three tosses result in heads is –

$$P(3\text{ heads}) = P(ABC) = P(A)P(B)P(C) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8} = \mathbf{0.125}$$

Example : 38.

A coach wants to select a player for a cricket match with following qualities.

(i) A good batsman : probability of getting this is 3/10. (ii) A good bowler : probability of getting this is 2/10. (iii) A good fielder :

probability of getting this is 5/10.

Find the probability of getting such a player, when these three attributes are independent.

Solution :

Let A be an event of getting a good batsman, B be an event of getting a good bowler and C be an event of getting a good fielder.

Here, $P(A) = \frac{3}{10}$, $P(B) = \frac{2}{10}$ and $P(C) = \frac{5}{10}$ Here, the events A, B and C are independent.

$\therefore$ P(a player has qualities A, B and C) = P(A) P(B) P(C) = $\frac{3}{10} \times \frac{2}{10} \times \frac{5}{10} = \frac{3}{100}$ or **0.03**

Miscellaneous Examples :

Example : 39.

A card is drawn from a pack of 52 playing cards. What is the probability that it is a : (i) Face card (ii) Honour card ?

Solution :

(i) F = { Jacks, Queens, Kings } $\therefore P(F) = \frac{12}{52}$

(ii) H = { Jacks, Queens, Kings, Aces } $\therefore P(H) = \frac{16}{52}$

Example : 40.

What is the probability that there will be 53 Mondays in a randomly selected i) Non-Leap year ii) Leap year ?

Solution :

i) As we know a non-leap year contains 365 days. Out of them, 364 days make 52 complete weeks. The remaining one day may be Mon, Tue, Wed, Thu, Fri, Sat, Sun therefore the sample space is-

S = { Mon, Tue, Wed, Thu, Fri, Sat, Sun } and M = { Mon }

$\therefore$ By definition, P(53 Mondays in non-leap year) = $\frac{n(M)}{n(S)} = \frac{1}{7}$

ii) As we know a leap year contains 366 days. Out of them, 364 days make 52 complete weeks For remaining two days, the sample space is-

S = { (Mon, Tue), (Tue, Wed), (Wed, Thu) (Thu, Fri), (Fri, Sat), (Sat, Sun), (Sun, Mon) } and M = { (Mon, Tue), (Sun, Mon)}

$\therefore$ By definition, P(53 Mondays in leap year) = $\frac{n(M)}{n(S)} = \mathbf{\frac{2}{7}}$

Example : 41.

If S = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 } and A = { 3, 6, 9 } then, find P(A^{I}).

Solution :

Here, $P(A) = \frac{3}{10}$ and P(A) + P(A′) = 1 ($\because$ A and A′ are complimentary events)

$$\therefore\ P(A') = 1 - P(A) = 1 - \frac{3}{10} = \frac{7}{10} \text{ or } \mathbf{0.7}$$

Example : 42. If S = { E_1, E_2 } and $P(E_1) = \frac{2}{5}$ then, find $P(E_2)$.

Solution :

Here, $P(E_1) + P(E_2) = 1$ ($\because$ E_1 and E_2 are complimentary events)

$$P(E_2) = 1 - \frac{2}{5} = \frac{3}{5} = \mathbf{0.6}$$

Example : 43.

A husband and his wife attended an interview. The probability of selecting husband is $\frac{2}{5}$ and that of wife is $\frac{1}{2}$. Find the probability of selecting at least one of them.

Solution :

Here, $P(H) = \frac{2}{5}$, $P(W) = \frac{1}{2}$

We know that, $P(H\cup W) = P(H) + P(W) - P(H\cap W)$

$P(H\cup W) = P(H) + P(W) - P(H)\,P(W)$ ($\because$ H and W are independent events)

$$= \frac{2}{5} + \frac{1}{2} - \frac{2}{5} \times \frac{1}{2}$$

$= \frac{2}{5} + \frac{1}{2} - \frac{1}{5}$

$= \frac{7}{10} = \mathbf{0.7}$

It is simplified using calculator as:
2 ÷ 5 [M +] 1 ÷ 2 [M +]
2 ÷ 5 × 1 ÷ 2 [M -]
[MR] [MC]

Example : 44.

The probability that a boy will pass an examination is $\frac{3}{5}$ and that of a girl is $\frac{2}{5}$. What is the probability that at least one of them passes the examination ?

Solution : Here , $P(B) = \frac{3}{5}$, $P(G) = \frac{2}{5}$

We know that, $P(B \cup G) = P(B) + P(G) - P(B \cap G)$

$P(B \cup G) = P(B) + P(G) - P(B)\,P(G)$ ($\because$ B and G are independent events)

$= \frac{3}{5} + \frac{2}{5} - \frac{3}{5} \times \frac{2}{5}$

$= \frac{3}{5} + \frac{2}{5} - \frac{6}{25}$

$= \frac{19}{25}$ or **0.76**

It is simplified using calculator as:
3 ÷ 5 |M +| 2 ÷ 5 M +
3 ÷ 5 × 2 ÷ 5 [M -]
[MR] [MC]

Example: 45.

A machine has two parts A and B. In a given period the probability of failure of part A is 0.07 and that of B is 0.05. What is the probability that the machine fails in that period ?

Solution :

Here, P(A) = 0.07 and P(B) = 0.05

Now, P(Failure of both part) = $P(A \cup B) = P(A)\,P(B)$

($\because$ parts work independently)

$= 0.07 \times 0.05 = \mathbf{0.0035}$

P(The machine fails)=$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.07 + 0.05 - 0.0035 = \mathbf{0.1165}$

Example : 46.

A red ball is drawn from a bag containing 2 red and 3 white balls. Find (i) odds in favour of the event ? (ii) Odds against the event ?

Solution :

Here, number of total balls is 2 + 3 = 5 and R is the event of drawing a red ball.

$\therefore$ P(in favour of the event) = P(R) = $\frac{2}{5}$ $\Rightarrow$ P(against the event) = $\frac{3}{5}$

Hence, the odds in favour of the event are 2 : 3 and the odds against the event are 3 : 2

Example : 47.

The odds in favour of A solving the problem are 14 to16 and odds against B solving a problem are 8 to 6. If both of them try individually, what is the probability that the problem is solved ?

Solution :

Here, $P(A) = \frac{14}{30}$ and $P(B') = \frac{8}{14} \Rightarrow P(B) = \frac{6}{14}$

We know that, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = P(A) + P(A) - P(A)\, P(B)$ ($\because$ A and B are independent events)

$$= \frac{14}{30} + \frac{6}{14} - \frac{14}{30} \times \frac{6}{14}$$

$$= \frac{292}{420} \text{ or } 0.6952$$

Example : 48.

If P(A) = 0.8, P(B) = 0.5, $P(A \cup B) = 0.9$ then, find P(A|B). Are events A and B independent?

Solution :

We know that, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$0.9 = 0.8 + 0.5 - P(A \cap B)$$

i.e., $P(A \cap B) = 0.8 + 0.5 - 0.9 = 0.4$

$$\therefore \; P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.4}{0.5} = 0.8$$

Here, $P(AB) = P(A)\, P(B)$

$0.4 = 0.8 \times 0.5$ $\therefore$ **events A and B are independent.**

Example : 49.

If A and B are two independent events and $P(A) = 0.6$, $P(B) = 0.5$ then, find $P(A \cup B)$.

Solution:

We know that, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 0.6 + 0.5 - 0.6 \times 0.5$ ($\because$ A and B are ind)

i.e., $P(A \cup B) = 0.6 + 0.5 - 0.3 =$ **0.8**

Example : 50.

If $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(B|A) = \frac{2}{5}$ then, find $P(A \cup B)$.

Solution :

We know that, $P(A \cap B) = P(A)\, P(B|A) = \frac{1}{2} \times \frac{2}{5} = \frac{1}{5}$

and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{5}$$

$$= \frac{19}{30} = \mathbf{0.6333}$$

It is simplified using calculator as:
1 ÷ 2 [M +] 1 ÷ 3 [M +]
1 ÷ 5 [M −] [MR] [MC]

Example : 51.

A problem in Statistics is given to solve to 2 students A and B. Their chances of solving it are $\frac{3}{4}$ and $\frac{3}{5}$ respectively. If they try independently, what is the probability that it is solved ?

Solution :

$P(A \cup B) = P(A) + P(B) - P(AB) = \frac{3}{4} + \frac{3}{5} - \frac{3}{4} \times \frac{3}{5} = \frac{18}{20}$ or **0.9**

OR

$P(A\cup B) = P(A)P(B') + P(A')P(B) + P(A)P(B)$

$$= \frac{3}{4} \times \frac{2}{5} + \frac{1}{4} \times \frac{3}{5} + \frac{3}{4} \times \frac{3}{5} = \frac{6}{20} + \frac{3}{20} + \frac{9}{20} = \mathbf{0.9}$$

OR

$P(A\cup B) = 1 - P(A\cup B)' = 1 - P(A'\cap B')$

$= 1 - P(A')\cap P(B')$ ($\because$ A and B independent events)

$$= 1 - \frac{1}{4} \times \frac{2}{5} = 1 - \frac{2}{20} \text{ or } \mathbf{0.9}$$

Example : 52.

One card is drawn from a well shuffled pack of 52 cards. What is the probability that it is neither a spade nor an ace ?

Solution :

Let A denote the event that the card drawn is spade and B denotes an ace. The event that the card drawn is neither a spade nor an ace is $A'\cap B'$.

Here, $P(A) = \frac{13}{52}$, $P(B) = \frac{4}{52}$ and $P(A\cap B) = \frac{1}{52}$

$P(A'\cap B') = 1 - P(A\cup B) = 1 - [\ P(A) + P(B) - P(A\cap B)\ ]$

$$= 1 - \left[\frac{13}{52} + \frac{4}{52} - \frac{1}{52} \right] = 1 - \frac{16}{52} = \frac{36}{52} \text{ or } \mathbf{0.6923}$$

Example : 53.

A fair coin is tossed four times. Find the probability of obtaining : (i) Head in all the tosses, (ii) Tail in at least one of the tosses.

Solution :

(i) P(Head in all tosses) = $P(A\cap B\cap C\cap D) = P(A)P(B)P(C)P(D)$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \mathbf{\frac{1}{16}}$$

(ii) P(Tail in at least one of the tosses) = 1 - P(head in all tosses)

$$= 1 - \frac{1}{16} = \mathbf{\frac{15}{16}}$$

Example : 54. Out of 5 persons, 2 are graduates. 2 persons are selected from these 5 persons. Find the probability that at least one of the selected persons is a graduate.

Solution:

P(at least one graduate) = 1 – P(none is graduate)

$$= 1 - \frac{3c_2}{5c_2} = 1 - \frac{3}{10} = \frac{7}{10} \text{ or } \mathbf{0.7}$$

OR

P(at least one graduate) = P(One graduate) + P(Two graduates)

$$= \frac{2c_1 \times 3c_1}{5c_2} + \frac{2c_2}{5c_2} = \frac{2 \times 3}{10} + \frac{1}{10} = \frac{7}{10} \text{ or } \mathbf{0.7}$$

Example : 55. A, B and C hit a target with probabilities 0.6, 0.5 and 0.4 respectively. If they hit at the target independently, find the probability that : (i) none of them hit the target (ii) the target is hit.

Solution : Here,

$P(A) = 0.6$, $P(A') = 0.4$, $P(B) = 0.5$, $P(B') = 0.5$, $P(C) = 0.4$, $P(C') = 0.6$

(i) P(none of them hit the target) = $P(A' \cap B' \cap C')$

$= P(A') \cap P(B') \cap P(C')$ (∵ they independent events)

$= 0.4 \times 0.5 \times 0.6 = \mathbf{0.12}$

(ii) P(the target is hit) = $P(A \cup B \cup C) = 1 - P(A \cup B \cup C)' = 1 - P(A' \cap B' \cap C')$

$= 1 - P(A') \cap P(B') \cap P(C')$

$= 1 - 0.12 = \mathbf{0.88}$

Example: 56. The probability that India wins a cricket match is 0.6. If India plays three matches, find the probability that it wins : (i) at least one match (ii) all the three matches.

Solution :

Let A : India wins the first match, B : India wins the second match and C : India wins the third match.

Here, P(A) = P(B) = P(C) = 0.6 $\therefore$ P(A′) = P(B′) = P(C′) = 0.4

i) P(Wins at least one match) = 1 – P (Wins none)

$= 1 - (0.4 \times 0.4 \times 0.4)$

$= 1 - 0.064$

$= \mathbf{0.946}$

ii) P(Wins all matches) = P(A)P(B)P(C) = $0.6 \times 0.6 \times 0.6 = \mathbf{0.216}$

Example : 57.

A box contains 5 white and 3 black balls. Two balls are drawn one after the other. Find the probability of getting a white ball in the first draw and a black ball in the second draw when (i) the first drawn ball is replaced (ii) the first drawn ball is not replaced.

Solution :

Let A be an event of getting white ball in the first draw and B be an event of getting black ball in the second draw. Here, P(A)= $\frac{5}{8}$ and when the ball is replaced, getting black ball in the second draw becomes unconditional i.e., P(B)=$\frac{3}{8}$, when the ball is not replaced, getting black ball in second draw becomes conditional

i.e., $P(B\,|\,A) = \frac{3}{7}$ ($\because$ Remaining total balls are 7)

$\therefore$ (i) $P(A \cap B) = P(A)\,P(B) = \frac{5}{8} \times \frac{3}{8} = \mathbf{\frac{15}{64}}$

(ii) $P(A \cap B) = P(A)\,P(B|A) = \frac{5}{8} \times \frac{3}{7} = \mathbf{\frac{15}{56}}$

Example : 58.

A box has 3 red and 2 green balls. Two balls are drawn one after the other. Find the probability that the balls drawn would be red if the ball drawn first is-

(a) Returned to the box before the second draw is made. (Draw with replacement)

(b) Not returned to the box before the second draw is made. (Draw without replacement)

Solution :

Let A be an event of getting red ball in the first draw and B be an event of getting red ball in the second draw. Here, $P(A)= \frac{3}{5}$

When the ball is replaced getting red ball in the second draw becomes unconditional. i.e., $P(B) = \frac{3}{5}$

When the ball is not replaced getting red ball in the second draw becomes conditional.

i.e., $P(B|A) = \frac{2}{4}$ ($\because$ Remaining total balls are 4)

$\therefore$ (a) $P(A \cap B) = P(A)\, P(B) = \frac{3}{5} \times \frac{3}{5} = \frac{9}{25} = \mathbf{0.36}$

(b) $P(A \cap B) = P(A)\, P(B|A) = \frac{3}{5} \times \frac{2}{4} = \frac{3}{10} = \mathbf{0.3}$

Example : 59.

The first box contains 2 red and 3 green marbles. The second box contains 4 red and 5 green marbles. One marble is transferred from the first box to the second and then one marble is drawn from the second box. Find the chance that it is green.

Solution:

Let A be an event of transferring red marble from the first to the second, B be an event of transferring green marble from the first to the second, (C | A) be an event of drawing green marble from the second box given that the transferred marble is red and (C | B) be an event of drawing green marble from the second box given that the transferred marble is green

$\therefore P(A) = \frac{2}{5}, P(B) = \frac{3}{5}, P(C|A) = \frac{5}{10}$ and $P(C|B) = \frac{6}{10}$

$P(A \cap C) = P(A)\ P(C|A) = \frac{2}{5} \times \frac{5}{10}$ and $P(B \cap C) = P(B)\ P(C|B) = \frac{3}{5} \times \frac{6}{10}$

$$P(\text{Green marble}) = P(A)\ P(C|A) + P(B)\ P(C|B) = \frac{2}{5} \times \frac{5}{10} + \frac{3}{5} \times \frac{6}{10}$$

$$= \frac{10}{50} + \frac{18}{50} = \frac{28}{50} \text{ or } \mathbf{0.56}$$

Example : 60.

The first box contains 3 white and 5 black marbles. The second box contains 6 white and 4 black marbles. A box is selected at random and then one marble is drawn from it. Find the probability that it is white.

Solution :

Let A be an event of selecting a box, B be an event of drawing one white marble from the first box and C be an event of drawing one white marble from the second box.

Here, number of boxes is two $\therefore P(A) = \frac{1}{2}, P(B) = \frac{3}{8}$ and $P(C) = \frac{6}{10}$

$P(A \cap B) = P(A)\ P(B) = \frac{1}{2} \times \frac{3}{8}$ and $P(A \cap C) = P(A)\ P(C) = \frac{1}{2} \times \frac{6}{10}$

$P(\text{White marble}) = P(A)\ P(B) + P(A)\ P(C)$

$$= \frac{1}{2} \times \frac{3}{8} + \frac{1}{2} \times \frac{6}{10}$$

$$= \frac{3}{16} + \frac{3}{10}$$

$$= \frac{39}{80} \text{ or } \mathbf{0.4875}$$

> It is simplified using calculator as;
> 1 ÷ 2 × 3 ÷ 8 [M +]
> 1 ÷ 2 × 6 ÷ 10 [M +]
> [MR] [MC]

Example: 61.

A card is drawn at random from a pack of 52 playing cards.

(i) What is the probability that it is a heart ?

(ii) If it is known that the card drawn is red, what is the probability that it is a heart ?

Solution :

There are 52 equally likely, mutually exclusive and exhaustive outcomes. Let events A and B be A : Card drawn is red. B : Card drawn is heart.

There are 26 red cards and 13 hearts in a pack of cards. Therefore, event A has 26 favourable outcomes, event B has 13 favourable outcomes and event $(A \cap B)$ has 13 favourable outcomes.

Therefore, $P(A) = \frac{26}{52}$, $P(B) = \frac{13}{52}$ and $P(A \cap B) = \frac{13}{52}$

(i) The unconditional probability of drawing a heart is $P(B) = \frac{13}{52}$ = **0.25**

(ii) The conditional probability of drawing a heart given that it is red card is–

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{13/52}{26/52} = \frac{13}{26} = \mathbf{0.5}$$

Example : 62.

There are 7 PU, 3 Degree and 2 PG students in a training camp. What is the probability of selecting a team of four leaders two from PU, one from Degree and one from PG ?

Solution :

Here, number of total students in training camp is 12. Out of which, selection of four leaders is in ${}^{12}c_4$ ways.

$$\therefore P(\text{Team contains 2 PU, 1 Degree and 1 PG leaders}) = \frac{7c_2 \times 3c_1 \times 2c_1}{12c_4}$$

$$= \frac{21 \times 3 \times 2}{495} = \mathbf{0.2545}$$

Example : 63.

Four cards are drawn at random from a well shuffled pack of 52 playing cards. What is the probability that they belong to different suits ?

Solution :

$$P(\text{Cards belong to different suits}) = \frac{13c_1 \times 13c_1 \times 13c_1 \times 13c_1}{52c_4}$$

$$= \frac{13 \times 13 \times 13 \times 13}{270725} = \mathbf{0.1055}$$

Example : 64.

Two fair dice are rolled once. Find the probability that the sum of the numbers-obtained is: i) 5 ii) 10 iii) 5 or 10 iv) at least 10 v) at most 5 vi) odd vii) even.

Solution :

When we rolled two fair dice once, the different sums obtained are given below.

Different sums	Different possibilities	Total number of possibilities	Corresponding probabilities
2	(1,1)	1	1/36
3	(1,2)(2,1)	2	2/36
4	(1,3)(2,2)(3,1)	3	3/36
5	(1,4)(2,3)(3,2)(4,1)	4	4/36
6	(1,5)(2,4)(3,3)(4,2)(5,1)	5	5/36
7	(1,6)(2,5)(3,4)(4,3)(5,2)(6,2)	6	6/36
8	(2,6)(3,5)(4,4)(5,3)(6,2)	5	5/36
9	(3,6)(4,5)(5,4)(6,3)	4	4/36
10	(4,6)(5,5)(6,4)	3	3/36
11	(5,6)(6,5)	2	2/36
12	(6,6)	1	1/36

Let A be an event that the sum of numbers obtained is 5, B be an event that the sum of numbers obtained is 10.

i) $P(A) = \frac{4}{36}$

ii) $P(B) = \frac{3}{36}$

iii) $P(A+B) = P(A) + P(B) = \frac{4}{36} + \frac{3}{36} = \frac{7}{36}$ ($\because$ A and B are MEE)

iv) $P(\text{sum is at least } 10) = P(\text{sum is } 10 \text{ or } 11 \text{ or } 12) = \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{6}{36}$

v) $P(\text{sum is at most } 5) = P(\text{sum is } 2 \text{ or } 3 \text{ or } 4 \text{ or } 5) = \frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36} = \frac{10}{36}$

vi) P(sum is an odd number) = P(sum is 3 or 5 or 7 or 9 or 11)

$$= \frac{2}{36} + \frac{4}{36} + \frac{6}{36} + \frac{4}{36} + \frac{2}{36} = \mathbf{\frac{18}{36}}$$

vii) P(sum is an even number) = 1 - P(sum is an odd number) = $1 - \frac{18}{36} = \mathbf{\frac{18}{36}}$

Example : 65.

Two fair dice are rolled. If the sum of the numbers obtained is 4. Find the probability that the numbers obtained on both the dice are even.

Solution :

Let A be the event that sum of the numbers is 4 and B be the event that the numbers on both the dice are even. Here, we have to find $P(B\,|\,A) = \frac{P(A\cap B)}{P(A)}$

Event A has 3 favourable outcomes, namely (1,3) **(2,2)** and (3,1)

$\therefore P(A) = \frac{3}{36}$ and P (Sum is 4 and numbers are even) = $P(A\cap B) = \frac{1}{36}$

Thus, P(Numbers are even given Sum is 4) = $P(B|A) = \frac{P(A\cap B)}{P(A)} = \frac{1/36}{3/36} = \frac{1}{3} = \mathbf{0.3333}$

Example : 66.

A bag contains 4 red balls and some blue balls. If the probability of drawing a blue ball is double to that of a red ball, then find the number of blue balls in the bag.

Solution :

Here, number of blue balls = n(B), P(B) = 2P(R) and $P(R) = \frac{4}{4 + n(B)}$

We know that, P(R) + P(B) = 1 $\Rightarrow$ P(R) + 2P(R) = 1

i.e., 3P(R) = 1 $\Rightarrow P(R) = \frac{1}{3} = \frac{4}{4 + n(B)} \Rightarrow 4 + n(B) = 12 \Rightarrow n(B) = \mathbf{8}$

Example : 67.

A bag contains 5 white balls and some black balls. If the probability of drawing a black ball is triple to that of a white ball, determine the number of black balls in the bag.

Solution :

Here, number of black balls $= n(B)$, $P(B) = 2P(W)$ and $P(W) = \frac{5}{5+n(B)}$

We know that, $P(W) + P(B) = 1 \Rightarrow P(W) + 3P(W) = 1$

i.e., $4P(W) = 1 \Rightarrow P(W) = \frac{1}{4} = \frac{5}{5+n(B)} \Rightarrow 5 + n(B) = 20 \Rightarrow n(B) = \mathbf{15}$

Example : 68.

A jar contains 24 marbles, in which some are green and others are blue. If the probability of a green marble is $\frac{2}{3}$ then find the number of blue marbles in the jar.

Solution :

Here, $P(G) = \frac{2}{3} \Rightarrow P(B) = \frac{1}{3} = \frac{n(B)}{24} \Rightarrow n(B) = \frac{24}{3} = 8$

Examples on statistical definition :

Example : 69.

Out of 500 babies born in a community in a year, 260 were females. Find the probability that a new–born baby is a female.

Solution:

Here, n = 500 births were observed and among them m = 260 have resulted in births to female children. Therefore, P(new–born baby is female) $\frac{m}{n} = \frac{260}{500} = \mathbf{0.52}$

Example : 70.

Team A played 96 cricket matches with team B. Among them, 36 matches were won by team A. Find the probability that the 97th match that team A played with team B would be won by team A.

Solution : Here, n = 96 and m = 36.

Therefore, P(A wins the 97th match is) $= \frac{m}{n} = \frac{36}{96} = \mathbf{0.375}$

Questions

1. Define an Experiment.
2. Define an outcome.
3. What is a random experiment ? Give an example.
4. Define a Sample space.
5. Write the sample space, when i) two coins are tossed once. ii) a die thrown once.
6. What is an event ?
7. Define null event. Give an example.
8. Define simple event. Give an example.
9. Define compound event. Give an example.
10. Define favourable outcomes with an example.
11. Define exhaustive outcomes with an example.
12. Define equally likely events with an example.
13. What is union of events ?
14. What is intersection of events ?
15. Define mutually exclusive events with an example.
16. What is complement of an event ? Give an example.
17. Mention the methods of assigning probabilities.
18. What is the base for classical (mathematical) method of assigning probabilities ?
19. What is the base for statistical (empirical) method of assigning probabilities?
20. Give the classical (mathematical) definition of probability.
21. Write the limitations of classical (mathematical) definition of probability.
22. Give the statistical (empirical) definition of probability.
23. Give the axiomatic definition of probability.
24. What is the probability of null event ?
25. What is the probability of sure event ?

26. Show that, $0 \leq P(A) \leq 1$.
27. If A^I is the complementary event of A, then show that $P(A)+P(A^I)=1$.
28. If $P(A) = 0.4$ then, find $P(A^I)$.
29. State and prove addition theorem of probability for two non-mutually exclusive events.
30. State and prove addition theorem of probability for two mutually exclusive events.
31. Define independent events with an example.
32. Define dependent events with an example.
33. Define conditional probability.
34. State and prove multiplication theorem of probability for two dependent events.
35. State and prove multiplication theorem of probability for two independent events.

Exercise Problems

1. A coin is tossed once. Find the probability of getting a (a) head (b) tail (c) head or tail.
2. A die is thrown once. What is the probability of getting a/an (a) 2 or 5 (b) odd or multiple of 3 ?
3. A card is drawn randomly from a pack of 52 playing cards. Find the probability that it is : (i) a King (ii) Not a king (iii) a Spade (iv) a Red (v) a King or a Spade (vi) a Spade or a Red.
4. A box contains cards numbered from 1 to 20. A card is drawn randomly from it. Find the probability of getting a card with: (i) an odd number (ii) a multiple of 4 (iii) a perfect square.
5. When three coins are tossed at a time. Find the probability of getting: (i) only heads (ii) at least two heads.
6. From a group of 6 boys and 4 girls, two are selected at random.

Find the probability that: (a) both are boys (b) both are girls (c) one is boy and other is a girl.

7. A bag contains 6 white and 4 black balls. What is the probability that : (i) A ball drawn at random is white ? (ii) Two balls drawn at random are one of each colour ?
8. A box contains 5 red and 4 green balls. Two balls are drawn at random from this box. Find the probability that they are : (a) red (b) green (c) of the same colour (d) of different colours.
9. Two cards are drawn successively from a pack of 52 playing cards. What is the probability that drawn cards are (a) Kings (b) Spades (c) Reds (d) Spades or Reds ?
10. A box contains 5 red, 4 green and 3 blue marbles. Three marbles are drawn at random from this box. Find the probability that they are of : (i) different colours (ii) the same colour.
11. A bag contains 5 tickets numbered from 1 to 5. Two tickets are drawn at random. What is the probability that the sum of obtained numbers is (i) odd (ii) even ?
12. Two red balls are drawn from a bag containing 3 red and 4 white balls. Find odds against the event.
13. For a university cricket team 2 players are to be selected from a certain college among 5 batsmen, 3 bowlers and 2 wicket-keepers. Find the probability of selecting- (i) a batsman and a wicket-keeper (ii) bowlers only.
14. A firm wants to select three clerks among 3 graduates, 5 undergraduates and 8 matriculates. What is the probability of selecting : (a) one graduate and two matriculates, (b) two undergraduates and one matriculate ?
15. In a hostel 60% of residents drink tea, 50% of residents drink coffee and 20% of residents drink both tea and coffee. Find the probability that a randomly selected resident drinks either tea or coffee.

16. The probability that a contractor will get a plumbing contract is 1/2 and the probability that he will not get an electrical contract is 2/3. If the probability of getting at least one of these contracts is 2/3. What is the probability that he will get both ?
17. A box contains 40 nails and 20 screws. ¼th of nails and 20% of the screws are rusted. If one item is selected at random, what is the probability that it is a rusted item or a screw ?
18. If $P(A) = \frac{1}{13}$, $P(B) = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{52}$ then, find the value of $P(A \cup B)$.
19. A card is drawn from a pack of 52 playing cards, what is the probability that it is king card known that the drawn card is spade?
20. If $P(A \cap B) = \frac{1}{3}$ and $P(B) = \frac{2}{3}$ then, find $P(A|B)$.
21. If $P(A) = \frac{2}{3}$ and $P(B|A) = \frac{3}{5}$ then, find $P(A \cap B)$.
22. A bag has 5 balls of which 3 are white. Another bag has 6 balls of which 2 are white. From each of the bags one ball is drawn at random. Find the probability that both are white.
23. A purse contains 4 silver and 2 gold coins. Another purse contains 3 silver and 4 gold coins. If a coin is selected at random from one of the two purses, what is the probability that it is a silver coin ?
24. Probability that A solves a problem is 2/3 and that B solves it is 3/5. Find the Probability that : a) both of them solve, b) none of them solves.
25. Three persons A, B and C are able to hit a target 6, 5 and 8 times respectively with 10 shots. If each of them fires once at the target, what is the probability that the target is hit ?
26. The probability of a bomb hitting a target is 3/5. If 3 are aimed at a bridge what is the probability that bridge is hit?
27. The probabilities of a husband and his wife surviving for 20 more years are 0.8 and 0.9 respectively. Find the probability that after 20 years: (i) both of them are alive. (ii) at least one of them is alive.

28. An aircraft is equipped with two engines that operate independently. The probability of an engine failure is 0.01. What is the probability of successful flight, if one engine is needed for successful operation of the aircraft ?

29. An urn contains 3 bags. Contents of the bags are- I bag : 3 red and 2 green, II bag : 4 red and 3 green, III bag : 2 red and 2 green. One bag is selected at random and then a ball is drawn from it. Find the probability that it is red.

30. The first box has 6 red and 4 white balls. The second box has 3 red and 5 white balls. A coin is tossed. If head turns up, a ball is randomly drawn from the first box. If tail turns up, a ball is randomly drawn from the second box. Find the probability that the drawn ball is red.

31. In a college, there are 5 lady lecturers, among them 3 are doctorates. If a committee consisting 3 of them is formed, what is the probability that at least two are doctors ?

32. Two fair dice are rolled. Find the probability that : (i) both the dice show number 5, (ii) the sum of numbers is 7 or 11, (iii) the sum is divisible by 3 (iv) product of numbers obtained is 36.

33. A bag has 5 red and 4 blue marbles. Another bag has 3 red and 7 blue marbles. A ball is drawn from the first bag and is placed in the second. Then, a ball is drawn from the second bag, what is the probability that it is red ?

34. The first box contains 4 white, 3 black balls. The second box contains 4 white, 5 black balls. One of the boxes is selected at random and from the selected box two balls are randomly drawn. Find the probability that they are of different colours.

35. A bag contains 5 white balls and some black balls. If the probability of drawing a black ball is double to that of a white ball, then find the number of black balls in the bag.

36. A box contains 20 balls, in which some are blue and others are red. If the probability a blue ball is $\frac{1}{4}$, then find the number of red balls in the box.

Answers

(1) (a) $\frac{1}{2}$, (b) $\frac{1}{2}$, (c) 1

(2) (a) $\frac{1}{3}$, (b) $\frac{2}{3}$

(3) (i) $\frac{4}{52} = 0.0769$, (ii) $\frac{48}{52} = 0.9231$, (iii) $\frac{13}{52} = 0.25$, (iv) $\frac{26}{52} = 0.5$, (v) $\frac{16}{52} = 0.3071$, (vi) $\frac{39}{52} = 0.75$

(4) (i) $\frac{1}{2}$, (ii) $\frac{1}{4}$, (iii) $\frac{1}{5}$

(5) (i) $\frac{1}{8}$, ii) $\frac{1}{2}$

(6) (a) $\frac{15}{45} = \frac{1}{3}$, (b) $\frac{6}{45} = \frac{2}{15}$, (c) $\frac{24}{45} = \frac{8}{15}$

(7) (i) $\frac{6}{10} = \frac{3}{5}$, (ii) $\frac{24}{45} = \frac{8}{15}$

(8) (a) $\frac{10}{36} = \frac{5}{8}$, (b) $\frac{6}{36} = \frac{1}{6}$, (c) $\frac{16}{36} = \frac{4}{9}$, (d) $\frac{20}{36} = \frac{5}{9}$

(9) (a) $\frac{6}{1326} = \frac{1}{221}$, (b) $\frac{78}{1326} = \frac{1}{17}$, (c) $\frac{325}{1326} = \frac{25}{102}$, (d) $\frac{403}{1326} = \frac{31}{102}$

(10) (i) $\frac{60}{220} = \frac{3}{11}$, (ii) $1 - \frac{3}{11} = \frac{8}{11}$

(11) (i) $\frac{3c_1 \times 2c_1}{5c_2} = \frac{6}{10}$, (ii) $\frac{3c_2 + 2c_2}{5c_2} = \frac{4}{10}$

(12) 6 : 1

(13) (i) $\frac{10}{45} = \frac{2}{9}$, (ii) $\frac{1}{15}$

(14) (a) $\frac{84}{560} = \frac{3}{20}$ or 0.15, (b) $\frac{80}{560} = \frac{1}{7}$

(15) 0.9

(16) $\frac{1}{6}$

(17) $\frac{1}{2}$

(18) $\frac{16}{52}$

(19) $\frac{1}{13}$

(20) $\frac{1}{2}$

(21) $\frac{2}{5}$

(22) $\frac{3}{5} \times \frac{2}{6} = \frac{1}{5}$

(23) $\frac{4}{6} \times \frac{3}{7} = \frac{2}{7}$

(24) (a) $\frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$, (b) $1 - (\frac{1}{3} \times \frac{2}{5}) = \frac{13}{15}$

(25) $1 - (\frac{4}{10} \times \frac{5}{10} \times \frac{2}{10}) = \frac{48}{50}$

(26) $\frac{117}{125}$

(27) (i) 0.72, (ii) 0.98

(28) 0.9999

(29) $\frac{117}{210}$

(30) $\frac{3}{10} + \frac{3}{16} = \frac{39}{80}$

(31) $\frac{6}{10} + \frac{1}{10} = \frac{7}{10}$

(32) (i) $\frac{1}{36}$, (ii) $\frac{8}{36} = \frac{2}{9}$, (iii) $\frac{1}{3}$, (iv) $\frac{1}{36}$

(33) $\frac{32}{99}$

(34) $\frac{71}{126}$

(35) 15

(36) 15.

Unit-X

RANDOM VARIABLE

Introduction :

Logically by a random variable we mean a real number "X" associated with the outcomes of a random experiment. It can take any one of the various possible values each with definite probability.

Suppose two fair coins are tossed, the sample space is S ={HH, HT, TH, TT}.

If to every sample points in the sample space, a number is assigned as below :

Sample Point	TT	TH	HT	HH
Number	0	1	1	2

Here, the assigned numbers indicate the number of heads obtained in each case. Let, The number of heads be denoted by 'X', then X is a function on the sample space. It takes the values 0, 1 and 2 with respective probabilities $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{1}{4}$. i.e., P(X=0) = P (no head) = $\frac{1}{4}$

$$P(X=1) = P\text{ (one head)} = \frac{2}{4}$$

$$P(X=2) = P\text{ (two heads)} = \frac{1}{4}$$

Here, 'X' is called a **random variable** or a **variate.**

Definition : Random variable is a function which assigns a real number to every sample point in the sample space.

The set of such real values is the range of the random variable.

Types of Random Variables : Broadly random variables are classified as below:

i) Discrete random variable.

ii) Continuous random variable.

Discrete random variable:

A random variable 'X' which takes the specified values $x_1, x_2, \ldots, x_n$ with a respective probabilities $p_1, p_2, \ldots, p_n$ is a **discrete random variable.**

Here the values $x_1, x_2, \ldots, x_n$ form the range of the random variable.

Ex : 1.

Let 'X' denotes the number of heads obtained while tossing two fair coins. Then 'X' is a discrete random variable which takes the specified values 0, 1 and 2 with respective probabilities $\frac{1}{4}, \frac{2}{4}$ and $\frac{1}{4}$.

Ex : 2.

Let 'X' denotes the number obtained while throwing a fair die. Then 'X' is a discrete random variable taking the specified values 1, 2, 3, 4, 5 and 6 with respective probabilities $\frac{1}{6}$ each.

Continuous random variable :

A random variable which assumes all the possible values in its range is called a **continuous random variable.**

Ex : 1. Let 'X' denotes the marks obtained by a group of students in an examination. Then 'X' is a continuous random variable.

Ex : 2. Let 'X' denote the height of a group of students. Then 'X' is a continuous random variable.

Generally, random variables are denoted by capital letters like X, Y, Z etc.. If 'X' is a random variable and the values assumed by random variable 'X' are denoted by small letter x.

Probability Mass Function (pmf) :

Let 'X' be a discrete random variable. And let p(x) be a function such that $p(x) = p(X=x)$. Then p(x) is called the probability mass function of 'X'. If the following conditions are satisfied,

i) $p(x) \geq 0$ for all x, and

ii) $\Sigma\, p(x) = 1$.

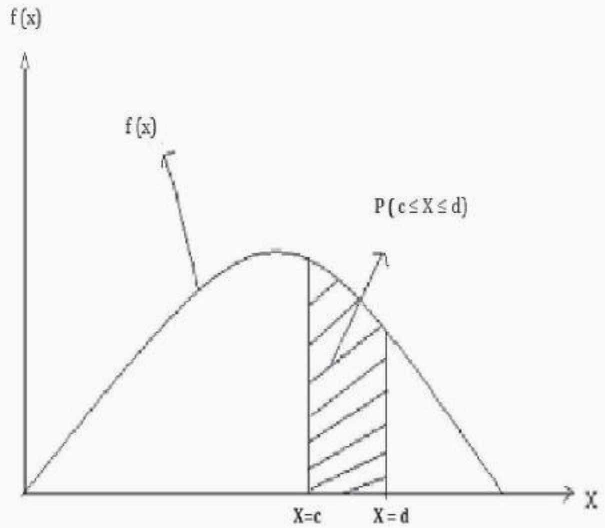
Probability density function (pdf) : Let 'X' be a continuous random variable taking values in the interval [a, b].

A function f(x) is said to be the probability density function of the continuous random variable 'X', if it satisfies the following conditions:

i) $f(x) \geq 0$ for all ' x ' in the interval [a, b].

ii) For two distinct numbers c & d in the interval [a, b]

$P(c \leq X \leq d)$=(Area under the probability curve between ordinates at X =c and X=d).



iii) Total area under the curve is 1. i.e., $P(-\infty < x < \infty) = 1$.

Probability Distribution :

The probability distribution is analogous to that of the frequency distribution. Just as frequency distribution tells how the total frequency is distributed among the different values of the variable. Similarly, a probability distribution tells us how total probability is distributed among the various values of the random variable.

Definition:

A systematic presentation of the values taken by a random variable with respective probabilities is called the **probability distribution** of a random variable.

i.e., probability distribution of a random variable 'X':

X	x_1	x_2	x_3	....	x_n
p(x)	p_1	p_2	p_3	....	p_n

Where, $\sum p(x) = p_1 + p_2 + p_3 + \ldots + p_n = 1$

Example :

Three fair coins are tossed once. Then, its probability distribution is

X	0	1	2	3
p(x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Where, ' X ' denotes the number of heads.

Mathematical Expectation :

1) Mathematical expectation of a discrete random variable :

Let 'X' be a discrete random variable which can takes any one of the values $x_1, x_2, x_3, \ldots, x_n$ with respective probabilities $p_1, p_2, p_3, \cdots, p_n$ then, the mathematical expectation of 'X' is usually denoted by E(X) and is defined as:

$$E(X) = x_1p_1 + x_2p_2 + x_3p_3 + \ldots + x_np_n = \sum x\,p(x)$$

2) **Mathematical expectation of a function h(X) of X :** Let 'X' be a discrete random variable with probability mass function p(x). Then, mathematical expectation of any function h(X) defined on X is

$$E[h(X)] = \sum h(x)\,p(x)$$

Results: 1. **Mean =** $E(X) = \sum x\,p(x)$

2. For a random variable X, the variance is

$$Var(X) = E[X - E(X)]^2$$

On simplification, $Var(X) = E(X^2) - [E(X)]$

The square root of variance is the standard deviation.

i.e., **S.D(X) =** $\sqrt{\mathbf{Var(X)}}$

3. If X is a random variable and a, b are constants then;

i) E(a) = a

ii) E(aX) = a E(X)

iii) E(aX + b) = a E(X) + b

iv) Var (a) = 0

v) Var (aX) = $\mathbf{a^2}$ **Var(X)**

vi) Var (aX + b) = $\mathbf{a^2}$ **Var(X)**

Proof : Let ' X ' be a discrete random variable with probability mass function (pmf) p(x) then,

$E(X) = \sum x\, p(x)$

i) $E[h(X)] = \sum h(x)\, p(x)$

$E(a) = \sum a.\, p(x)$

$= a \sum p(x)$

$= a \times 1 \quad \because \sum p(x)=1$

$= a$

ii) $E(aX) = \sum ax\, p(x)$

$= a \sum x\, p(x)$

$= a\, E(X)$

i) $E(aX + b) = \sum (ax + b)\, p(x)$

$= \sum ax\, p(x) + \sum b\, p(x)$

$= a \sum x\, P(x) + b \sum p(x)$

$= a\, E(X) + b \quad \because \sum p(x) = 1$

iv) $Var(a) = E[a - E(a)]^2$

$= E[a - a]^2 \quad \because E\,(a) = a$

$= E(0)^2 = 0$

v) $Var(aX) = E[aX - E(aX)]^2$

$= E[aX - aE(X)]^2 \quad \because E(aX) = a\,E(X)$

$= a^2 E[X - E(X)]^2$

$= a^2 Var(X)$

vi) $Var(aX+b) = E[(aX + b) - E(aX + b)]^2$

$= E[aX + b - aE(X) - b]^2$

$= E[aX - aE(X)]^2$

$= E[a\{X - E(X)\}]^2$

$= a^2E[X - E(X)]^2 = a^2 Var(X)$

Note: i) $Var(aX) = Var(aX + b)$

ii) We know that, $Var(X) > 0$

i.e., $E(X^2) - [E(X)]^2 > 0$

i.e., $E(X^2) > [E(X)]^2$

Bivariate Probability Function :

Let X and Y be two discrete random variables which are defined on a sample space of a random experiment. Let p(x, y) be a function such that p(x, y) = p[X=x, Y=y] then, p(x, y) is called **bivariate probability function (joint probability function)** of X and Y.

Marginal Probability Function:

Let $p_1(x)$ be the marginal probability function of X and $p_2(y)$ be the marginal probability function of Y, obtained from the bivariate probability function of p(x, y)

i.e., $p_1(x) = \sum_y p(X = x, Y = y) = \sum_y p(x, y)$

and $p_2(y) = \sum_x p(X = x, Y = y) = \sum_x p(x, y)$

Independent Random Variables :

Two random variables X and Y are said to be independent if and only if $p(x, y) = p_1(x).\ p_2(y)$, for all x and y.

Addition theorem :

Statement : Let X and Y be two random variables with respective expectations E(X) and E(Y). Then, expectation of the sum of these random variables is:

E(X+Y)=E(X) + E(Y)

Proof : Let X and Y be two discrete random variables, then their joint probability distribution is,

$p(x,y) = p[X=x, Y=y]$ and

The marginal distributions of X and Y are respectively,

$P_1(x) = \sum_y P(x,y)$ .... (1)

$P_2(y) = \sum_x P(x,y)$ .... (2)

Then by definition of expectation,

$$E(X+Y) = \sum_x \sum_y (x+y)\,p(x,y)$$

$$= \sum_x \sum_y x p(x,y) + \sum_x \sum_y y p(x,y)$$

$$= \sum_x x \sum_y p(x,y) + \sum_y y \sum_x p(x,y)$$; From equations (1) and (2)

$$= \sum x\, p_1(x) + \sum y\, p_2(y)$$

$\therefore E(X+Y) = E(X) + E(Y)$

Hence the theorem.

In general, $E(X_1 + X_2 + ...+X_n) = E(X_1) + E(X_2) +...+E(X_n)$

Multiplication Theorem :

Statement: Let X and Y be two **independent** random variables with respective expectations E(X) and E(Y). Then expectation of the product of these random variables is ;

$E(XY) = E(X)\, E(Y)$

Proof: Let X and Y be two Independent random variables then

$P(x, y) = p_1(x)\, p_2(y)$ .... (1)

Then by the definition of expectation we have

$E(XY) = \Sigma \Sigma\; x\, y\, p(x, y)$

$= \Sigma \Sigma\; x\, y\, p_1(x)\, p_2(y)$ from equation (1)

$= \Sigma x\, p_1(x)\; \Sigma\, y\, p_2(y)$

$\therefore$ **$E(XY) = E(X)\,E(Y)$**

Hence the theorem.

In general if $x_1, x_2, x_3, \ldots, x_n$ are ' n ' independent random variables then,

$E[X_1\, X_2\, X_3 \ldots X_n] = E(X_1).\, E(X_2).\, E(X_3) \ldots E(X_n)$

Covariance :

Let X and Y be two random variables. Then covariance between X and Y is :

$Cov(X, Y) = E\,[(X - E(X))\,(Y - E(Y))]$

$= E\,(XY) - E(X)\,E(Y)$

Coefficient of Correlation :

The coefficient of correlation between two random variables X and Y is defined as :

$$\gamma = \frac{Cov\,(X, Y)}{\sqrt{Var(X)}\;\sqrt{Var\,(Y)}}$$

$$\gamma = \frac{E(XY) - E(X)\,E(Y)}{\sqrt{E(X^2) - [E(X)]^2}\;\sqrt{E(Y^2) - [E(Y)]^2}}$$

Results :

For two independent random variables X and Y

i) $Cov\,(X, Y) = 0$

ii) Coefficient of correlation $= \gamma = 0$

Proof :

We know that, when X and Y are independent random variables then,

$E\,(XY) = E(X)\,E(Y)$ ——(1)

i) By definition of covariance

$$\text{Cov}(X, Y) = E(XY) - E(X)\,E(Y)$$
$$= E(X)\,E(Y) - E(X)\,E(Y), \text{ by equation (1)}$$
$$= 0$$

$\therefore$ Cov (X, Y) = 0

Hence the Result.

ii) When X and Y are independent, Cov (X,Y) = 0, then

$$\Upsilon = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X)}\,\sqrt{\text{Var}(Y)}} = \frac{0}{\sqrt{\text{Var}(X)}\sqrt{\text{Var}(Y)}} = 0$$

$\therefore\ \boldsymbol{\Upsilon = 0}$

Hence the result.

Probability Distributions:

Example : 1.

A die is tossed once, getting an odd number is termed as a success. Find the probability distribution of the number of successes.

Solution :

Since the cases favourable to getting an odd number (success) in a throw of a die are {1, 3, 5}

Probability of success = $P(s) = \frac{3}{6} = \frac{1}{2}$ and

Probability of failure = $P(f) = \frac{3}{6} = \frac{1}{2}$

Let X denotes the number of success in throw of a die, and then X is a random variable which takes the values 0 and 1, with respective probabilities, $\frac{1}{2}$ each.

Hence the probability distribution of X is:

x	0	1
p(x)	$\frac{1}{2}$	$\frac{1}{2}$

Example : 2.

Obtain the probability distribution of the number of heads in three tosses of a coin.

Solution :

Let X denotes the number of heads obtained, then X is a random variable which takes the values 0, 1, 2 and 3.

The sample space is S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

Hence, the probability distribution of X is :

X	0	1	2	3
p(x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Example : 3.

An urn contains 6 red and 4 white balls. 3 balls are drawn at random from the urn. Obtain the probability distribution of the number of white balls drawn.

Solution :

Let X denotes the number of white balls in the draw of 3 balls, then X is a random variable which takes the values 0, 1, 2 and 3.

$$P(X=0)= \frac{6C_3 \times 4C_0}{10C_3} = \frac{5}{30}$$

$$P(X=1)= \frac{6C_2 \times 4C_1}{10C_3} = \frac{15}{30}$$

$$P(X=2)= \frac{6C_1 \times 4C_2}{10C_3} = \frac{9}{30}$$

$$P(X=3)= \frac{6C_0 \times 4C_3}{10C_3} = \frac{1}{30}$$

Hence, the probability distribution of 'X' is:

X	0	1	2	3
p(x)	$\frac{5}{30}$	$\frac{15}{30}$	$\frac{9}{30}$	$\frac{1}{30}$

Mathematical Expectation :

Example : 4.

A die is thrown. What is the expectation of the number on it ?

Solution:

Let X denotes the number obtained on the die. Then X is a random variable which can take any one of the values 1, 2, 3, ..., 6 each with equal probability $\frac{1}{6}$.

That is,

X	1	2	3	4	5	6
p(x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$E(X) = \sum x\, p(x)$$

$$= 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6}$$

$$= \frac{1}{6}(1 + 2 + 3 + 4 + 5 + 6) = \frac{21}{6} = \frac{7}{2}$$

$$\therefore E(X) = 3.5$$

Example : 5.

Find the mathematical expectation of the number of heads obtained when two fair coins are tossed once (or one coin is tossed twice).

Solution :

Let X denotes the number of heads obtained. Then X is a random variable which takes the values 0, 1 and 2 with respective probabilities $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{1}{4}$. Thus, the probability distribution of X is:

X	0	1	2
p(x)	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

The mathematical expectation of the number of heads is:

$$E(X) = \sum x\, p(x)$$

$$= 0 \times \frac{1}{4} + 1 \times \frac{1}{2} + 2 \times \frac{1}{4}$$

$$= 0 + \frac{1}{2} + \frac{1}{2} = 1 \text{ (head)}$$

Example : 6.

For the following probability distribution, find E(X), Var(X), S.D(X), E(2X+3), Var(2X+3) and S.D(2X+3).

X	0	1	2	3
p(x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Solution:

x	0	1	2	3	
p(x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	$\sum p(x) = 1$
x^2	0	1	4	9	-
2x+3	3	5	7	9	-
xp(x)	0	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{3}{8}$	$\sum xp(x) = \frac{12}{8}$
$x^2p(x)$	0	$\frac{3}{8}$	$\frac{12}{8}$	$\frac{9}{8}$	$\sum x^2p(x) = \frac{24}{8}$
(2x+3)p(x)	$\frac{3}{8}$	$\frac{15}{8}$	$\frac{21}{8}$	$\frac{9}{8}$	$\sum(2x+3)p(x) = \frac{48}{8}$

$$E(X) = \sum xp(x) = \frac{12}{8}$$

$$E(X^2)=\sum x^2\, p(x)= \frac{24}{8}$$

$$\therefore Var(X) = E(X^2) - [E(X)]^2 = \frac{24}{8} - \left(\frac{12}{8}\right)^2 = \frac{24}{8} - \frac{144}{64}$$

$$Var(X) = \frac{192-144}{64} = \frac{48}{64} = \frac{3}{4} = 0.75$$

$$S.D(X) = \sqrt{Var(x)} = \sqrt{0.75} = 0.8660$$

$$E(2X+3) = \sum(2X+3)p(x) = \frac{48}{8} = 6$$

We know that $Var(ax+b) = a^2 Var(X)$

Therefore $Var(2X+3) = 2^2\ Var(X) = 4 \times \frac{3}{4} = 3$

$$S.D(2X+3) = \sqrt{Var(2X+3)} = \sqrt{3} = 1.7321$$

Example : 7. The probability distribution of a random variable is given in the following table.

X	-2	3	1
p(x)	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

Find E(X), E (X^2+2), $E\left(\frac{X}{6}\right)$, Var (X), $Var\left(\frac{X}{6}\right)$, $Var\left(\frac{X}{6}+\frac{1}{6}\right)$ and $S.D\left(\frac{X}{6}+\frac{1}{6}\right)$ Var(-X).

Solution :

X	-2	3	1	Total
p(x)	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$	$\sum p(x) = 1$
x^2	4	9	1	-
$\sum x\ p(x)$	$\frac{-2}{3}$	$\frac{3}{2}$	$\frac{1}{6}$	$\sum x\ p(x) = 1$
$x^2\ p(x)$	$\frac{4}{3}$	$\frac{9}{2}$	$\frac{1}{6}$	$\sum x^2\ p(x) = 6$

Therefore, $E(X) = \sum x\, p(x) = 1$

$E(X^2) = \sum x^2 p(x) = 6$

$Var(X) = E(X^2) - [E(X)]^2$

$= 6 - 1^2 = 6 - 1 = 5$

By using the results,

$E(X^2 + 2) = E(X^2) + 2 = 6 + 2 = 8$

$E\left(\frac{X}{6}\right) = \frac{1}{6} E(X) = \frac{1}{6}(1) = \frac{1}{6}$

$Var\left(\frac{X}{6}\right) = \frac{1}{6^2} Var(X) = \frac{1}{36} \times 5 = \frac{5}{36}$

$Var\left(\frac{X}{6} + \frac{1}{6}\right) = \frac{1}{6^2} Var(X) = \frac{1}{36} \times 5 = \frac{5}{36}$

$S.D\left(\frac{X}{6} + \frac{1}{6}\right) = \sqrt{Var\left(\frac{X}{6} + \frac{1}{6}\right)} = \sqrt{\frac{5}{36}} = 0.3727$

$Var(-X) = (-1^2)\, Var(X) = 1 \times 5 = 5$

Note: $Var(X) = Var(-X)$

Example : 8.

A random variable X assumes the values 1 and 0 with respective probabilities p and $q = 1 - p$. Find its mean, variance and standard deviation.

Solution: The probability distribution of X is

x	1	0
p(x)	p	q

Mean $= E(X) = \Sigma\, xp(x)$

$= 1 \times p + 0 \times q = p$

$$E(X^2) = \sum x^2 p(x)$$

$$= 1^2 \times p + 0^2 \times q = p$$

$$Var(X) = E(X^2) - [E(X)]^2$$

$$= p - p^2 = p(1-p) = pq \qquad [\text{where, } q = 1-p]$$

$$S.D(X) = \sqrt{Var(X)} = \sqrt{pq}$$

Example : 9.

If a random variable X assumes the values 0 and 1 with P(X=0) =3P(X=1), then find E(X) and V(X).

Solution: $P(x=1) = p$ .... (1)

Given, $P(x=0) = 3P(x=1) = 3p$.

Hence the probability distribution of 'X' is

x	1	0
p(x)	p	3p

Since, $p+3p = 1, \Rightarrow 4p = 1, \Rightarrow p = \frac{1}{4}$

$$E(X) = \sum x\, p(x) = 1 \times p + 0 \times 3p = p = \frac{1}{4}$$

$$E(X^2)=\sum x^2 p(x) = 1^2 \times p + 0^2 \times 3p = p = \frac{1}{4}$$

$$Var(X) = E(X^2) - [E(X)]^2 = \frac{1}{4} - \left(\frac{1}{4}\right)^2 = \frac{1}{4} - \frac{1}{16} = \frac{4-1}{16} = \frac{3}{16}$$

Example : 10. If $E(X^2) = 74$ and $Var(X) = 49$. Find the E(X) and S.D(X).

Solution :

$$Var(X) = E(X^2) - [E(X)]^2$$

$$49 = 74 - [E(X)]^2$$

$$\Rightarrow [E(X)]^2 = 74 - 49 = 25$$

$\Rightarrow E(X) = \pm 5$

$S.D\ (X) = \sqrt{Var(X)} = \sqrt{49} = 7$

Example : 11. If Var(X) = 9 and E(X) =4, find $E(x^2)$

Solution:

$$Var(X) = E(X^2) - [E(X)]^2$$

$$\Rightarrow E(X^2) = Var(X) + [E(X)]^2$$

$$= 9 + 16 = 25$$

i.e., $E(X^2) = 25$

Example : 12.

If E(X) = 3, find E(3X), E(-X), E(-3X), E(3X-2), E(-2X+4) and E(8).

Solution : By using results,

$E(3X) = 3E(X) = 3 \times 3 = 9$

$E(-X) = -E(X) = -3$

$E(-3X) = -3E(X) = -3 \times 3 = -9$

$E(3X - 2) = 3E(X) - 2 = 3 \times 3 - 2 = 9 - 2 = 7$

$E(-2X + 4) = -2E(X) + 4 = -2 \times 3 + 4 = -6 + 4 = -2$

$E(8) = 8$

Example : 13. If Var (X) =4, find i) Var(-X), ii) Var (2X) iii) Var(3X – 5), iv) S.D (X) and v) Var(7).

Solution: Given, Var(X) = 4

i) $Var(-X) = Var[(-1)X] = (-1)^2 Var(X) = 1 \times 4 = 4$

ii) $Var(2X) = (2)^2 Var(X) = 4 \times 4 = 16$

iii) $Var(3X - 5) = (3)^2 Var(X) = 9 \times 4 = 36$

iv) $S.D(X) = \sqrt{Var(X)} = \sqrt{4} = 2$

v) $Var(7) = 0$

Example : 14. If If $E(X) = 7$ and $E(X^2) = 64$. Find $Var(8X + 8)$.

Solution :

$$Var(X) = E(X^2) - [E(X)]^2 = 64 - 49 = 15$$

$$\therefore Var\ (8X + 8) = 8^2 Var(X)$$

$$= 64 \times 15 = 960$$

Example : 15.

For the following probability distribution, find $E(2X + 4)$, $E(3x^2 - 5)$, $Var(X)$, $Var(3X)$, $Var(2X+4)$, $Var\left(-\frac{x}{2}\right)$, $Var(-X+2)$, $S.D(X)$ and $S.D(-5X+2)$.

x	5	1
p(x)	$\frac{1}{3}$	$\frac{2}{3}$

Solution :

$$E(X) = \sum x\ p(x) = 5 \times \frac{1}{3} + 1 \times \frac{2}{3} = \frac{5}{3} + \frac{2}{3} = \frac{7}{3}$$

$$E(X^2) = \sum x^2 p(x) = 5^2 \times \frac{1}{3} + 1^2 \times \frac{2}{3} = \frac{25}{3} + \frac{2}{3} = \frac{27}{3} = 9$$

$$E(2X + 4) = 2E(X) + 4 = 2 \times \frac{7}{3} + 4 = \frac{14}{3} + 4 = \frac{14+12}{3} = \frac{26}{3}$$

$$E(3X^2 - 5) = 3E(X^2) - 5 = 3 \times 9 - 5 = 27 - 5 = 22$$

$$Var(X) = E(X^2) - [E(X)]^2 = 9 - \frac{49}{9} = \frac{81-49}{9} = \frac{32}{9} = 3.5556$$

$$Var\ (3X) = 3^2 Var(X) = 9 \times \frac{32}{9} = 32$$

$$Var(2X+4) = 2^2 Var(X) = 4 \times \frac{32}{9} = \frac{128}{9}$$

$$Var(-\frac{X}{2}) = Var[(-\frac{1}{2})\ X] = (-\frac{1}{2})^2\ Var\ (X) = \frac{1}{4} \times \frac{32}{9} = \frac{8}{9}$$

$Var(-X+2) = (-1)^2 Var(X) = 1 \times \frac{32}{9} = \frac{32}{9}$

$S.D(X) = \sqrt{Var(X)} = \sqrt{3.5556} = 1.8856$

$S.D(-5X + 2) = \sqrt{Var(-5X + 2)} = \sqrt{(-5)^2\, Var(X)} = \sqrt{25 \times 3.5556} = 9.4281$

Example :16.

A random variable X has the following probability distribution.

x	-2	-1	0	1	2	3
p(x)	0.1	0.1	0.2	2k	0.3	0.1

Find the value of k and calculate mean and variance of X.

Solution :

Here we know that, $\sum p(x) = 1$

$0.1 + 0.1 + 0.2 + 2k + 0.3 + 0.1 = 1$

$0.8 + 2k = 1$

$\Rightarrow 2k = 1 - 0.8$

$2k = 0.2$

$\Rightarrow k = \frac{0.2}{2} = 0.1$

$E(X) = \sum x\, p(x)$

$= (-2) x (0.1) + (-1) x (0.1) + 0 x (0.2) + 1 x (2k) + 2 x (0.3) + 3 x (0.1)$

$= -0.2 - 0.1 + 0 + 2(0.1) + 0.6 + 0.6 + 0.3$

$= -0.3 + 1.1$

$= 0.8$

$E(X^2) = \sum x^2 p(x)$

$= (-2^2)(0.1) + (-1^2)(0.1) + 0^2 (0.2) + 1^2 (2k) + 2^2 (0.3) + 3^2 (0.1)$

$= 0.4 + 0.1 + 0 + 0.2 + 1.2 + 0.9 = 2.8$

$Var(X) = E(X^2) - [E(X)]^2$

$= 2.8 - (0.8)^2 = 2.8 - 0.64 = 2.16$

Example : 17.

Find k, mean and variance of the following distribution.

X	-3	-2	0	2	3
p(x)	$\frac{k}{6}$	$\frac{k}{2}$	$\frac{2k}{3}$	$\frac{k}{2}$	$\frac{k}{6}$

Solution : For the probability distribution,

$$\sum p(x) = 1$$

$$\Rightarrow \frac{k}{6} + \frac{k}{2} + \frac{2k}{3} + \frac{k}{2} + \frac{k}{6} = 1$$

$$\Rightarrow \frac{k+3k+4k+3k+k}{6} = 1$$

$$\Rightarrow \frac{12k}{6} = 1$$

$$\Rightarrow k = \frac{1}{2}$$

$$E(X) = \sum x\, p(x)$$

$$= (-3)\frac{k}{6} + (-2)\frac{k}{2} + (0)\frac{2k}{3} + (2)\frac{k}{2} + (3)\frac{k}{6}$$

$$= \frac{k}{2} - k + 0 + k + \frac{k}{2} = 0$$

$$E(X^2) = \sum x^2\, p(x)$$

$$= (-3)^2\frac{k}{6} + (-2)^2\frac{k}{2} + (0)^2\frac{2k}{3} + (2)^2\frac{k}{2} + (3)^2\frac{k}{6}$$

$$= \frac{9k}{6} + \frac{4k}{2} + \frac{4k}{2} + \frac{9k}{6}$$

$$= k\left(\frac{9+12+12+9}{6}\right) = k\left(\frac{42}{6}\right) = k\,(7) = \frac{1}{2}(7) = \frac{7}{2}$$

$$\therefore \text{Var}(X) = E(X^2) - [E(X)]^2 = \frac{7}{2} - 0 = \frac{7}{2}$$

Example : 18.

From a bag containing 4 white and 6 red balls, three balls are drawn at random. If each white ball drawn carries a reward of Rs.4 and each red ball Rs.6. Find the expected reward of the draw.

Solution :

Let X be the reward. Then X is a random variable which takes the values 18, 16, 14 and 12, with respective probabilities

$$\frac{6C_3 \times 4C_0}{10C_3}, \frac{6C_2 \times 4C_1}{10C_3}, \frac{6C_1 \times 4C_2}{6C_3} \text{ and } \frac{6C_0 \times 4C_3}{10C_3}$$

Thus, the probabilities distribution of X is

X	18	16	14	12
p(x)	$\frac{5}{30}$	$\frac{15}{30}$	$\frac{9}{30}$	$\frac{1}{30}$

Expected reward of the draw is

$$E(X) = \sum xp(x)$$

$$= 18 \times \frac{5}{30} + 16 \times \frac{15}{30} + 14 \times \frac{9}{30} + 12 \times \frac{1}{30}$$

$$= \frac{90}{30} + \frac{240}{30} + \frac{126}{30} + \frac{12}{30}$$

$$= \frac{468}{30}$$

$$= \text{Rs. } 15.6$$

Example : 19.

A player tosses two fair coins. He wins Rs.5, if two heads occurs, Rs.2 if one head occurs and Re.1 if no head occurs.

i) Find his expected gain.

ii) How much he should pay to play the game if it is to be fair ?

Solution :

Let X be the amount he gets. Then X is a random variable which takes the values 1, 2 and 5 with respective probabilities $\frac{1}{4}, \frac{2}{4}$ and $\frac{1}{4}$. Then the probability distribution of X is

X	1	2	5
p(x)	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

i) Expected gain of a player is

$$E(X) = \sum xp(x) = 1 \times \frac{1}{4} + 2 \times \frac{2}{4} + 5 \times \frac{1}{4}$$

$$= \frac{10}{4} = \text{Rs. } 2.50$$

ii) The game is said to be fair if the mathematical expectation of the gain of the player is zero. Hence, the player should pay Rs.2.50 to play the game, if this is to be fair.

Example : 20.

A box contains 12 items of which 3 are defective. A sample of 3 items is selected at random from the box, what is the expected number of the defective items ?

Solution :

Let X be the number of defective items in the draw. Then, X is a random variable which takes the values 0, 1, 2 and 3 with respective probabilities,

$\frac{9C_3 \times 3C_0}{12C_3}, \frac{9C_2 \times 3C_1}{12C_3}, \frac{9C_1 \times 3C_2}{12C_3}$ and $\frac{9C_0 \times 3C_3}{12C_3}$

Thus, the probability distribution of X is

X	0	1	2	3
p(x)	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

$$E(X)=\sum x\, p(x)$$

$$=0 \times \frac{27}{64} + 1 \times \frac{27}{64} + 2 \times \frac{9}{64} + 3 \times \frac{1}{64}$$

$$= \frac{27+18+3}{64}$$

$$= \frac{48}{64} = \mathbf{0.75}$$

Example : 21.

A boy is asked to throw a fair die once. He is assured of an amount (Rupees) equal to the number occurring in the throw. Find his expectation.

Solution:

Let X denotes the amount that the boy gets. Then X is a random variable which takes the values 1, 2, 3, 4, 5 and 6 with respective probabilities $\frac{1}{6}$ each. Then the probability distribution of X is

X	1	2	3	4	5	6
p(x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$E(X) = \sum xp(x)$$

$$=1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6}$$

$$= \frac{1}{6}[1 + 2 + 3 + 4 + 5 + 6]$$

$$= \frac{1}{6}[21]$$

$$= \frac{7}{2}$$

$$\therefore\ E(X) = \text{Rs.}3.5$$

Example : 22.

A box contains 6 tickets. Two of the tickets carry a prize of Rs.5 each and the other four tickets carry a prize of Re.1 each. If two tickets are drawn at random, then what is the expected value of the prize ?

Solution:

Let X denotes the prize amount of the tickets drawn. Then X is a random variable which takes the values 10, 6 and 2 with respective probabilities,

$$\frac{2C_2}{6C_2}, \frac{2C_1 \times 4C_1}{6C_2} \text{ and } \frac{4C_2}{6C_2}.$$

Hence the probability distribution of X is

X	10	6	2
p(x)	$\frac{2}{30}$	$\frac{16}{30}$	$\frac{12}{30}$

$$E(X) = \sum x\, p(x)$$

$$= 10 \times \frac{2}{30} + 6 \times \frac{16}{30} + 2 \times \frac{12}{30}$$

$$= \frac{20}{30} + \frac{96}{10} + \frac{24}{30}$$

$$= \frac{140}{30}$$

= Rs. 4.67

Example : 23.

A person enters into a competition of hitting a target. If he hits the target, he gets Rs. 10. Otherwise, he loses Rs.5. If the probability of hitting the target is $\frac{3}{10}$. Find his expectation.

Solution :

Let X be the amount that a person gets. Then X is a random variable which takes the values 10 and -5 with respective probabilities $\frac{3}{10}$ and $\frac{7}{10}$

Then the probability distribution is

X	10	-5
p(x)	$\frac{3}{10}$	$\frac{7}{10}$

$$E(X) = \sum x\, p(x) = 10 \times \frac{3}{10} + (-5) \times \frac{7}{10} = 3 - 3.5 = -0.5 = \textbf{Rs. 0.5 (Loss).}$$

Example : 24. A purse has 10 one rupee coins, 6 fifty paise coins and 4 ten paise coins. A coin is randomly drawn from the purse. Find the expectation of the amount drawn.

Solution:

Let X be the amount in paise. Then, X is a random variable which takes the values 100, 50 and 10 with respective probabilities $\frac{10}{20}, \frac{6}{20}$ and $\frac{4}{20}$.

Then the probability distribution of X is

X	100	50	10
p(x)	$\frac{10}{20}$	$\frac{6}{20}$	$\frac{4}{20}$

$$E(X) = \sum x\, p(x)$$

$$= (100) \times \frac{10}{20} + (50) \times \frac{6}{20} + (10) \times \frac{4}{20}$$

$$= \frac{1000 + 300 + 40}{20}$$

$$= \frac{1340}{20}$$

= 67 paise.

Example : 25. There are 100 tickets in a lottery. There is one first prize worth Rs.25/- and two second prizes worth Rs.10/- each. What is the expected prize amount that a particular lottery ticket fetches ? If a lottery ticket is bought for Re. 1, what is the expected loss?

Solution :

Let X be the prize amount that a particular lottery ticket fetches. Then, X is a random variable which takes the values 25, 10 and 0 with respective probabilities, $\frac{1}{100}$, $\frac{2}{100}$ and $\frac{97}{100}$.

Then the probability distribution of X is

X	25	10	0
p(x)	$\frac{1}{100}$	$\frac{2}{100}$	$\frac{97}{100}$

$$E(X) = \sum x\, p(x)$$

$$= 25 \times \frac{1}{100} + 10 \times \frac{2}{100} + 0 \times \frac{97}{100}$$

$$= \frac{25 + 20}{100}$$

$$= \frac{45}{100}$$

= 45 paise.

The ticket is bought for 1 rupee.

∴ Expected loss = 100 – 45 = 55 paise

Example : 26.

A person, by paying Rs. 5 enters into a game of shooting a target. With one shot, if he hits the target, he gets Rs. 25. Otherwise, he gets nothing. If his probability of hitting the target is $\frac{1}{7}$, then find his expected loss.

Solution :

Let X denote the amount that the person receives. Then, X is a random variable which takes the values -5 and 20 with respective probabilities, $\frac{6}{7}$ and $\frac{1}{7}$.

Thus, the probability distribution of X is

X	-5	20
p(x)	$\frac{6}{7}$	$\frac{1}{7}$

$$E(X) = \sum x\, p(x)$$

$$= (-5) \times \frac{6}{7} + 20 \times \frac{1}{7}$$

$$= -\frac{30}{7} + \frac{20}{7} = -\frac{10}{7} = 1.43 \text{ Rs (loss)}$$

Example : 27.

Akshay gets Rs.5 if the sum of the numbers on two dice is 7 and he gets Rs. 3 if both the dice show even numbers. Otherwise, he has to pay Rs.4. Find Akshay's expected gain.

Solution:

Let, X denotes the amount that Akshay gets. Then, X is a random variable which takes the values 5, 3 and -4 with respective probabilities, $\frac{6}{36}$, $\frac{9}{36}$ and $\frac{21}{36}$. Hence the probability distribution of X is

X	-4	3	5
p(x)	$\frac{21}{36}$	$\frac{9}{36}$	$\frac{6}{36}$

$$E(X) = \sum x\, p(x)$$

$$= (-4) \times \frac{21}{36} + 3 \times \frac{9}{36} + 5 \times \frac{6}{36}$$

$$= -\frac{84}{36} + \frac{27}{36} + \frac{30}{36}$$

$$= -\frac{27}{36}$$

= -0.75 i.e., **Rs. 0.75 (loss)**

Example : 28.

Find the expectation of the sum of the numbers obtained in the throw of (i) 2 dice and (ii) n dice.

Solution :

(i) Let X and Y denotes the number obtained on the first and the second die respectively. Then each of them is a random variable which takes the values 1, 2, 3, 4, 5 and 6 with probability $\frac{1}{6}$ each.

Then, $E(X) = \sum x\, p(x)$

$$= 1\times\frac{1}{6} + 2\times\frac{1}{6} + 3\times\frac{1}{6} + 4\times\frac{1}{6} + 5\times\frac{1}{6} + 6\times\frac{1}{6}$$

$$= \frac{1}{6}[1 + 2 + 3 + 4 + 5 + 6]$$

$$= \frac{7}{2}$$

Similarly, $E(Y) = \frac{7}{2}$

Thus, expectation of the numbers obtained on 2 dice is

$$E(X + Y) = E(X) + E(Y)$$

$$= \frac{7}{2} + \frac{7}{2} = 7$$

(ii) Let $x_1, x_2, x_3, \ldots, x_n$ denote the numbers obtained on the first, second,.....,n^{th} die respectively. Then,

$$E[X_1+X_2+X_3+\ldots+X_n] = E(X_1) + E(X_2) + E(X_3) + \ldots + E(X_n)$$

$$= \frac{7}{2} + \frac{7}{2} + \frac{7}{2} + \ldots + \frac{7}{2}$$

$$= \frac{7}{2}n$$

Example : 29.

There are 10 tickets in a bag which are numbered 1, 2, 3,.....,10. Two tickets are drawn randomly one after the other with replacement. Find the expectation of (i) the sum of the numbers drawn (ii) product of the numbers drawn.

Solution:

Let X and Y denote the numbers on the first and the second tickets respectively. Then each of them is a random variable which takes the values 1, 2, 3, ..., 10 with probability $\frac{1}{10}$ each. Thus

$$E(X) = \sum x\, p(x)$$

$$= 1 \times \frac{1}{10} + 2 \times \frac{1}{10} + 3 \times \frac{1}{10} + 4 \times \frac{1}{10} + 5 \times \frac{1}{10} + 6 \times \frac{1}{10} + 7 \times \frac{1}{10} + 8 \times \frac{1}{10} + 9 \times \frac{1}{10} + 10 \times \frac{1}{10}$$

$$= \frac{1}{10}[1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10]$$

$$= \frac{55}{10} = 5.5$$

Similarly, E(Y) = 5.5

(i) $E(X+Y) = E(X) + E(Y) = 5.5 + 5.5 = 11$

(ii) $E(X\,Y) = E(X)\,E(Y) = (5.5)\,(5.5) = 30.25$

Example : 30.

Find the expectation of the product of the numbers obtained in the throw of (i) 2 dice (ii) n dice.

Solution :

(i) Let X and Y denote the numbers obtained on the first and the second die respectively.

Then, $E(X) = E(Y) = \frac{7}{2}$

Here, X and Y are two independent random variables as the two dice are independently thrown. $\therefore$ $E(X\,Y) = E(X)\,E(Y) = \frac{7}{2} \times \frac{7}{2} = \frac{49}{4}$

(ii) Let $X_1, X_2, \ldots, X_n$ denote the numbers obtained on the first, second, ..., n^{th} die respectively. Then $E(X_1) = E(X_2) = \ldots = E(X_n) = \frac{7}{2}$

Here also $X_1, X_2, \ldots, X_n$ are independent of each other as all the dice are thrown independently.

$$\therefore E(X_1\,X_2 \ldots\ldots X_n) = E(X_1)\,E(X_2) \ldots E(X_n)$$

$$= \frac{7}{2} \times \frac{7}{2} \times \ldots \times \frac{7}{2} = \left(\frac{7}{2}\right)^n$$

Example : 31.

In a bivariate data, $E(X) = 4$, $E(Y) = 10$, $E(X^2) = 25$, $E(Y^2) = 136$ and $E(XY) = 28$. Find Var (X), Var (Y), S.D(X), S.D (Y), Cov (X,Y), γ and Var(4x).

Solution:

$$Var(X) = E(X^2) - [E(X)]^2 = 25 - 4^2 = 25 - 16 = 9$$

$$Var(Y) = E(Y^2) - [E(Y)]^2 = 136 - 10^2 = 136 - 100 = 36$$

$$S.D\,(X) = \sqrt{Var(X)} = \sqrt{9} = 3$$

$$S.D\,(Y) = \sqrt{Var(Y)} = \sqrt{36} = 6$$

$$Cov\,(X, Y) = E(XY) - E(X)\,E(Y) = 28 - 4 \times 10 = 28 - 40 = -12$$

$$\gamma = \frac{Cov\,(X,Y)}{S.D(X)\,S.D(Y)} = \frac{-12}{3 \times 6} = -0.67$$

$$Var\,(4X) = 16\,[Var(X)] = 16 \times 9 = 144$$

Example: 32. From the following joint probability distribution, find the coefficient of correlation. Also find E (X+Y).

Y \ X	-1	0	1	Total
-1	0.1	0	0.1	0.2
0	0.1	0.3	0.2	0.6
1	0	0.1	0.1	0.2
Total	0.2	0.4	0.4	1

Solution : The marginal probability distributions of X and Y are:

X	-1	0	1
p(x)	0.2	0.4	0.4

and

Y	-1	0	1
p(y)	0.2	0.6	0.2

$E(X) = \sum x\, p(x)$

$= -1 \times 0.2 + 0 \times 0.4 + 1 \times 0.4$

$= -0.2 + 0 + 0.4 \quad = 0.2$

$E(X^2) = \sum x^2\, p(x)$

$= (-1)^2(0.2) + 0^2\,(0.4) + 1^2(0.4)$

$= 0.2 + 0 + 0.4 \; = 0.6$

$\text{Var}\,(X) = E(X^2) - [E(X)]^2$

$= 0.6 - (0.2)^2$

$= 0.6 - 0.04 = 0.56$

$\text{S.D}\,(X) = \sqrt{\text{Var}(X)}$

$= \sqrt{0.56} \; = 0.7483$

$E(Y) = \sum y\, p(y)$

$= -1 \times 0.2 + 0 \times 0.6 + 1 \times 0.2$

$= -0.2 + 0 + 0.2 = 0$

$E(Y^2) = \sum y^2\, p(y)$

$= (-1)^2(0.2) + 0^2\,(0.6) + 1^2(0.2)$

$= 0.2 + 0 + 0.2 = 0.4$

$$\text{Var }(Y) = E(Y^2) - [E(Y)]^2$$
$$= 0.4 - (0)^2$$
$$= 0.4 - 0 = 0.4$$

$$\text{S.D }(Y) = \sqrt{\text{Var}(Y)}$$
$$= \sqrt{0.4} = 0.6325$$

$$E(X\,Y) = \Sigma\Sigma\, xy\, p(x,y)$$

$$= (-1)(-1)(0.1) + (-1)(0)(0) + (-1)(1)(0.1) + (0)(-1)(0.1) + (0)(0)(0.3) + (0)(1)(0.2) + (1)(-1)(0) + (1)(0)(0.1) + (1)(1)(0.1)$$

$$= 0.1 + 0 - 0.1 + 0 + 0 + 0 + 0 + 0 + 0.1$$
$$= 0.1$$

$$\text{Cov}(X,Y) = E(XY) - E(X)\,E(Y) = 0.1 - (0.2)(0) = 0.1$$

$$\gamma = \frac{\text{Cov}(X,Y)}{\text{S.D.}(X)\ \text{S.D.}(Y)} = \frac{0.1}{(0.7483)(0.6325)} = \frac{0.1}{0.4733} = 0.2113$$

$$E(X+Y) - E(X) + E(Y) = (0.2) + (0) = 0.2$$

Example : 33.

From the following joint probability distribution of X and Y find (a) k (b) E(3X + 2Y) and (c) γ.

X \ Y	1	3	9
2	0.1	0.1	0.05
4	0.2	K	0.1
6	0.1	0.15	0.2

Solution :

The marginal probability distribution of X and Y are

x	2	4	6
p(x)	0.25	0.3+k	0.45

and

y	1	3	9
p(y)	0.4	0.25+k	0.35

Since, $\sum p(x) = 1$

i.e., $0.25 + 0.3 + k + 0.45 = 1$

$\Rightarrow k = 1 - 1 = 0$

$E(X) = \sum x\, p(x)$

$= 2 \times 0.25 + 4 \times 0.3 + 6 \times 0.45$

$= 0.5 + 1.2 + 2.7$

$= 4.4$

$E(X^2) = \sum x^2\, p(x)$

$= 2^2(0.25) + 4^2(0.3) + 6^2(0.45)$

$= 1.0 + 4.8 + 16.2$

$= 22$

$Var(X) = E(X^2) - [E(X)]^2$

$= 22 - (4.4)^2$

$= 22 - 19.36$

$= 2.64$

$E(Y) = \sum y\, p(y)$

$= 1 \times 0.4 + 3 \times 0.25 + 9 \times 0.35$

$= 0.4 + 0.75 + 3.15$

$= 4.3$

$E(Y^2) = \sum y^2\, p(y)$

$= 1^2(0.4) + 3^2(0.25) + 9^2(0.35)$

$= 0.4 + 2.25 + 28.35$

$= 31$

$Var(Y) = E(Y^2) - [E(Y)]^2$

$= 31 - (4.3)^2$

$= 31 - 18.49$

$= 12.51$

$S.D\ (X) = \sqrt{Var\ (X)} = \sqrt{2.64} = 1.62$

$S.D\ (Y) = \sqrt{Var\ (Y)} = \sqrt{12.51} = 3.54$

$E(XY) = \Sigma\Sigma xy\ P(x,y)$

$= (2)\ (1)\ (0.1) + (2)\ (3)\ (0.1) + (2)\ (9)\ (0.05) + (4)\ (1)\ (0.2) + (4)\ (3)\ (0) + (4)\ (9)\ (0.1) + (6)\ (1)\ (0.1) + (6)\ (3)\ (0.15) + (6)\ (9)\ (0.2)$

$= 0.2 + 0.6 + 0.9 + 0.8 + 0 + 3.6 + 0.6 + 2.7 + 10.8$

$= 20.2$

$Cov\ (X, Y) = E(XY) - E(X)\ E(Y)$

$= 20.2 - (4.4)\ (4.3)$

$= 20.2 - 18.92$

$= 1.28$

$E(3X + 2Y) = 3E(X) + 2\ E(Y)$

$= 3 \times 4.4 + 2 \times 4.3$

$= 13.2 + 8.6$

$= 21.8$

$$\gamma = \frac{Cov\ (X,Y)}{S.D(X)\ S.D(Y)} = \frac{1.28}{(1.62)(3.54)} = \frac{1.28}{5.7348} = \mathbf{0.2231}$$

Questions

1. Define a 'Random Variable', 'Discrete Random Variable' and 'Continuous Random Variable'.
2. What is meant by 'Probability Distribution' and 'Probability Mass Function' ?
3. Define 'Mathematical Expectation'.
4. Express variance in terms of expectation.
5. If X is a random variable a and b are any two constants, then prove that;

   (i) $E(a)=a$ (ii) $E(aX) = a\,E(X)$ (iii) $E(aX+b) = a\,E(X)+b$

   (iv) $V(a) = 0$ (v) $V(aX) = a^2\,V(X)$ (vi) $V(aX+b) = a^2\,V(X)$
6. Define a 'Joint Probability Mass Function' and 'Marginal Probability Distribution'.
7. When two random variables 'X' and 'Y' are said to be independent ?
8. State the addition theorem of expectation for two random variables X and Y.
9. State the multiplication theorem of expectation for two independent random variables X and Y.
10. Prove addition theorem of expectation for two discrete random variables X and Y.
11. Prove multiplication theorem of expectation for two independent random variables X and Y.
12. Express covariance in terms of expectation.
13. Write the formula for correlation coefficient in terms of expectation.
14. For two independent random variables X and Y, what are the values of (i) Cov(X,Y) and (ii) γ .

Exercise Problems

1. If ' X ' is a discrete random variate, show that (i) E(4) = 4 (ii) E(5X) = 5E(X) and (iii) E(3X+5) = 3E(X)+5.
2. If ' X ' is a discrete random variable, show that (i) Var(6) = 0 (ii) Var (-6X) = 36Var(X) (iii) Var (6X+7) = 36Var(X).
3. If V(X) = 3, then find the values of (i) Var(-X) (ii) Var(3X) (iii) Var(3X+3) (iv) $Var\left(\frac{X}{3}\right)$ and (v) Var(3 – X). Ans: (i) 3 (ii) 27 (iii) 27 (iv) $\frac{1}{3}$ (v) 3.
4. If E(X) = 5 and $E(X^2)$ = 34, find S.D(X). Ans:3.
5. If $E(X^2)$ = 25 and Var (X) = 16, find E(X). Ans: 3.
6. If E(X) = 10 and S.D(X) = 12, find $E(X^2)$. Ans: 244
7. If E(X) = 4, find E(4X), E(-X), E(-4X), E(-4X+4) and E(10).
   Ans: 16, -4, -16, -12 and 10.
8. If E(X) = 4 and E(Y) = 5, find E(4X+5Y), E(-4X+4Y) and E(6X-6Y).
   Ans: 41 4 and -6.
9. For the following probability distribution, find E(X), Var(X), S.D(X) and E(2X-4).

X	-1	0	1	2
p(x)	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{2}{5}$

Ans: 0.9, 1.29, 1.135 and -0.22.

10. Find the mean and variance of the following distribution.

x	0	1	2	3	4
p(x)	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{16}$

Ans: 1.312, 0.965

11. From the following probability distribution, find the missing probability, mean and standard deviation of ' X '

x	-2	-1	0	1	2
p(x)	0.2	0.3	0.2	?	0.1

Ans: 0.2, -0.3 and 1.039

12. Find the mean, variance and the value of 'k' of the following probability distribution.

X	-3	-2	0	2	3
p(x)	$\frac{k}{6}$	$\frac{k}{2}$	$\frac{2k}{3}$	$\frac{k}{2}$	$\frac{k}{6}$

Ans: 0, 7k and k = 0.5

13. A random variable 'X' assumes the values 10 and 20 with respective probabilities $\frac{1}{3}$ and $\frac{2}{3}$. Find its mean and variance.

Ans: 16.67 and 22.22

14. A bag has 4 white and 6 red balls. Two balls are randomly drawn from the bag, find the expected number of (i) white balls and (ii) red balls. Ans: (i) 0.8□ 1 ball and (ii) 1.2□ 1 ball.

15. A bag contains 4 green and 3 red balls. A man draws 3 balls at random from the bag. If he is to receive Rs.20 for every green ball he draws and Rs.10 for every red one. What is his expectation ?

Ans: Rs. 47.14

16. A person throws a biased coin. He gets Rs.8 if head appears otherwise he gets Rs.4. If the probability of occurrence of head is $\frac{1}{3}$, find his expectation and variance. Ans: 5.33 and 3.56

17. A man throws a fair die. If the throw results in an even number, he gets Rs.5, otherwise he loses Rs.10, find his expectation.

Ans: Loss of Rs.2.5

18. A man throws a fair die once. If the number obtained is divisible by 3, he gets Rs.9, otherwise he losses Rs.5, find his expectation.

Ans: Loss of Rs.0.33

19. A person, by paying Rs.5 enters into a game of shooting a target. With one shot, if he hits the target, he gets Rs.25, otherwise he gets nothing. If his probability of hitting the target is $\frac{1}{7}$. Find his expected net loss. Ans: Loss of Rs.1.43

20. In a lottery, there are 1000 tickets costing Re.1 each. There is one first prize worth Rs.100, two second prizes worth Rs.20 each and ten third prizes worth Rs.10 each. Find the expected loss in buying one ticket. Ans: Loss of Rs.0.76

21. A bag has 3 one-rupee, 4 two rupees and 2 five rupees coins. A boy picks two coins at random from the bag. What is the expectation of the amount he has picked ? Ans: Rs.4.67

22. If ' X ' is a random variable, show that $E(X^2) > [E(X)]^2$

23. Using addition theorem of expectation, find the expectation of the number of heads obtained in tossing of ten coins. Ans: 5

24. From the following joint probability distribution of X and Y. Find the value of k, E(X+Y) and γ

X \ Y	1	3	9
2	0.1	0.1	0.05
4	0.2	0	0.1
6	0.1	0.15	k

Ans: 0.2, 8.7 and -0.45

25. In a bivatiate data, E(X) = 0, E(Y) = 12, $E(X^2) = 49$, $E(Y^2) = 145$ and E(XY) = 3.5. Find Cov(X, Y) and γ. Ans: 3.5 and 0.5

26. For the following joint probability distribution of X and Y, find γ and E(3X+4Y).

X \ Y	1	2	3
-5	0	0.1	0.1
0	0.1	0.2	0.2
5	0.2	0.1	0

Ans: -0.267 and 9.5

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LIST OF PRACTICALS

Practical 1: Classification - A

1. A review of the first 40 pages of a text book reveals the following printing mistakes:

   5 1 2 4 0 2 2 5 1 4 3 2 4 2 3 3 4 5 1 2

   3 4 5 3 0 4 2 6 1 2 4 2 3 6 2 3 2 2 2 4

   Prepare a frequency distribution of printing mistakes.

2. Following is the data regarding the number of children per couple in a certain locality.

   0, 1, 2, 4, 3, 4, 4, 1, 0, 2, 1, 2, 3, 4, 2, 3, 2, 3, 1, 3, 4, 2, 3, 1, 0, 0, 2, 3, 2, 3.

   Tabulate above information.

3. The ages (in years) of 60 persons living in an apartment are given below:

   30 45 48 55 39 25 31 12 18 21 54 59 51 33 43

   44 10 38 19 26 47 35 37 41 46 33 51 37 58 58

   17 19 23 26 29 38 57 36 35 44 43 27 31 43 22

   34 15 36 20 18 33 12 20 44 25 31 19 26 34 40

   Prepare frequency table by taking a class interval of width 10 each.

4. Following is the marks obtained by 32 students in an examination, prepare a frequency distribution table.

   67 60 69 70 62 63 69 70 58 56 57 54 55 70 60 70

   60 65 70 56 67 58 60 59 61 63 69 67 61 60 59 57

Practical 2: Classification - B

1. The following are the marks secured by 48 students out of a maximum of 50, in an entrance examination. Form a frequency distribution with class intervals of 5 each.

15	27	21	18	21	10	07	0	08	02	10	29	01	04	11	20
12	16	18	28	24	23	32	20	24	16	15	14	25	34	15	05
30	22	17	13	03	17	19	14	11	16	19	15	08	15	19	06

2. Following are the marks obtained by a class of 60 students in an examination. Construct a frequency table with class intervals as 0-9, 10-19, 20-29 etc.

34	48	30	40	14	16	54	22	30	51	18	22	27	19	55
30	39	10	21	26	60	32	40	38	51	34	59	11	45	61
52	19	15	22	04	19	42	40	07	30	17	23	43	14	18
35	45	50	48	47	40	53	25	35	36	38	49	26	43	28

3. The data given below relate to the marks obtained by 20 students in two subjects. Represent the data by a bivariate frequency table with class intervals 15-25, 25-35... and so on for subject A and 62-64, 64-66... and so on for subject B.

Sl. No.	Marks in Subject A	Marks in subject B
1	52	67
2	70	70
3	35	65
4	36	65
5	37	64
6	48	69
7	24	63
8	17	65
9	28	70
10	43	71

Sl. No.	Marks in Subject A	Marks in Subject B
11	29	62
12	63	70
13	39	67
14	22	63
15	34	68
16	40	67
17	32	69
18	20	66
19	48	68
20	29	67

4. Below are the ages of husbands and wives prepare a bivariate frequency distribution with suitable width:

Sl. No.	1	2	3	4	5	6	7	8	9	10
Age of Husband (in years)	24	25	24	26	21	28	30	24	33	30
Age of wife (in years)	23	22	24	25	20	25	29	21	29	28

Sl. No.	11	12	13	14	15	16	17	18	19	20
Age of Husband (in years)	28	24	27	28	29	30	31	30	25	24
Age of wife (in years)	24	22	26	27	27	30	29	28	24	23

Practical 3: Tabulation - A

1. Draft a blank table, ready to be filled showing the distribution of population in a village classified according to:
   (i) Sex: Men, Women
   (ii) Occupation: Agriculturist and non-agriculturist
   (iii) Marital status: Married, Single.
2. Draft a blank table to show the distribution of workers of a factory according to:
   (i) Sex: male, female (ii) Age groups (in years) : below 30, 30-40, 40 and above (iii) Salary grades: below Rs.5000, Rs.5000-10000 and Rs.10,000 & above.
3. Draft a blank table to show the classification of the population of a town according to:
   (i) Sex-men, women
   (ii) Religion-Hindu, Muslim and others
   (iii) Literacy-literates and illiterates.
4. Draft a blank table to show:
   (i) Sex: men, women
   (ii) Ranks: supervisors, assistants and clerks
   (iii) Age group (in years) : 20-30, 30-50 and 50-60
5. Draft a neat blank table to present the data relating to the college students according to faculty-arts, commerce and science, classes-I PUC and II PUC, sex-boys and girls and for the year 2011-12.

Practical 4: Tabulation - B

1. Tabulate the following information giving in a suitable title
   "In 2005 out of total of 1750 workers of a factory, 200 workers were members to trade union. The number of women employed was 200 of which 175 did not belong to trade union. 2010 the number of union workers increased to 1580 of which 1290 were men. On the other hand, the number of nonunion workers fell down to 208 of which 180 were men."

2. In 2008, out of the total customers visiting the Mrushttanna Darsini, 75 were non vegetarians and 125 were vegetarians. In total there were 55 male non vegetarian customers and 30 female vegetarian customers. In 2010 the total number of customers increased by 25%, while the non vegetarian customers increased by 20%. In all there were 170 male customers among them 65 are non vegetarians. Tabulate the above information.

3. In a sample study about the food habits in two towns, the following information was obtained.

   Town A: 55 % persons were males

   35 % were non vegetarians

   28 % were male non vegetarians

   Town B: 52 % persons were males

   28 % were non vegetarians

   26 % were male non vegetarians.

   Tabulate the above information.

4. In the house of Loksabha there were 543 members were present. During the discussion on a motion of F.D.I. put to vote, 400 voted in favour of resolution. The government members (ruling party) in the house were 380, 65 members belonging to the opposition voted for the resolution. All the members are belonging to either of the two groups and there no absentees.

   Tabulate the above information.

5. In Hubli there were 20 lakh people, out of this, 7 lakh people lived in central Hubli, and the rest in surrounding areas. In central Hubli there were 3 lakh male people, out of which 2 lakh were literate. In central Hubli, 1 lakh ladies were illiterates. In surrounding areas there were 10 lakh male people, out of which 7 lakh were literate. In surrounding areas literate ladies were 2 lakh.

   Tabulate the above information

Practical 5: Diagrammatic representation - A

1. The following table gives the percentages of married women of Indian population under various ages. Represent the data by a simple bar diagram.

Age (in years)	18	20	22	24	26
Married women (in %)	17	17	51	62	66

2. Represent the following data regarding the production of paddy (in 000's tons) by simple bar diagram.

Years	2000	2001	2002	2003
Production	45	49	50	52

3. Following is the information relating to the number of students admitted at a college during the three years. Prepare the multiple bar diagram

Years	Arts	Science	Commerce
2008	120	180	150
2009	130	250	175
2010	150	280	200

4. The production of sugar and rice of a region are given below:

Years		2005	2006	2007	2008	2009	2010
Production (in Metric tons)	Sugar	25	28	30	32	34	35
	Rice	30	35	39	40	42	46

Draw a multiple bar diagram to represent the data.

5. The following table shows the results of C.P.T. students at an examination centre for the last three years. Represent the data in component bar diagram.

Years	I class	II class	III class	Failed
2005	30	80	130	82
2006	35	110	160	75
2007	30	130	180	80

Practical 6: Diagrammatic representation - B

1.Percentage of men and women in India are given below according to occupation. Draw a percentage bar diagram.

Sl. No.	Occupation/Labour	Men (%)	Women (%)
1	Industrial	45	55
2	Agricultural	55	45
3	House hold industries	61	39
4	Miscellaneous	88	12

2. Represent the following data by a percentage bar diagram.

Items of expenditure	Family A (Rs.)	Family B (Rs.)
Food	1500	1500
Clothing	1250	600
Education	250	500
Others	190	700

3. Represent the following data by subdivided bar diagram.

Expenditure (in Rs.)	Family A	Family B
Food	2000	2000
Clothing	600	480
Education	100	180
Fuel	140	60
House rent	600	192
Misc.	100	48

4. Represent the following data regarding the sixth five year plan public sector outlays by a pie diagram.

Heads	Center (Rs. Crores)
Agriculture	4765
Irrigation	6635
Energy	9995
Industry	12770
Transport	12200
Social service	8216
Total	54581

Practical 7: Graphical representation - A

1. Represent the following data by means of Histogram and then find the mode graphically:

Weekly wages (in Rs.)	20-25	25-30	30-35	35-40	40-45	35-50
No. of workers	6	18	25	14	10	8

2. Construct a Histogram for the following frequency distribution:

Variable	35-40	40-45	45-50	50-65	65-75
Frequency	12	30	22	30	28

3. Represent the following data by means of Histogram and locate mode from the histogram:

Age (in years)	0-10	10-20	20-30	30-40	40-60
No. of persons	40	60	150	110	100

4. Monthly profits of 100 shops are distributed as follows:

Profit per shop (in 000's Rs.)	5-10	10-15	15-20	20-25	25-30	30-35
No. of shops	6	18	25	14	10	8

Draw a Histogram and frequency polygon.

5. From the following data draw a frequency polygon.

Marks	20-30	30-40	40-50	40-60	60-70
No. of students	5	12	20	13	8

Practical 8: Graphical representation - B

1. Draw an 'ogive' and from it read the median and quartiles.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No. of students	15	29	60	42	24	18	17

2. Draw two Ogives from following data and locate the median.

Class	100-200	200-300	300-400	400-500	500-600	600-700
Frequency	20	40	80	60	20	10

3. Draw Ogive curves from the following data and measure the median value.

C. I	0 -10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	5	11	21	16	10

4. Using the following table draw a Ogive curve and hence determine the value of median.

Wages in Rs.	Up to 180	Up to 190	Up to 200	Up to 210	Up to 220	Up to 230	Up to 240	Up to 250
No. of workers	6	15	34	53	78	101	111	115

Practical 9: Measures of Central tendency- A

1. Compute arithmetic mean for the following data:

x	11	12	13	14	15
f	2	5	7	4	2

2. Find mean for the following frequency distribution using step deviation method.

Marks	Less than 10	Less than 20	Less than 30	Less than 40	Less than 50	Less than 60	Less than 70	Less than 80
f	5	13	20	32	60	80	90	100

3. If mean is 15.38, find the missing frequency from the following data:

x	10	12	14	16	18	20
f	3	7	-	20	8	5

4. For the following data if $\bar{x} = 8.84$, find the missing frequency.

C.I	4 – 6	6 – 8	8 – 10	10 – 12	12 – 14
f	4	-	17	10	5

5. The mean of 20 values is 45. If one of these values is wrongly taken as 46 instead of 64, find the correct mean.

6. The mean wages of 150 labourers working in a factory running 3 shifts of 60, 40 and 50 labourers is Rs.114. The Mean wages of 60 labourers working in I shift is Rs.121.50 and that of 40 labourers working in II shift is Rs.107.75. Find the Mean wage of 50 labourers working in III Shift.
7. Mean weight of 10 apples is 133gms. When two apples are added, the Mean becomes 137gms. Find the Mean weight of the added apples.
8. A firm of ready-made garments make both men's and women's shirts. Its profit average is 6% of sales. Its profit in men's shirt average is 8% of sales. Women's shirts comprise 60% of the output of the firm. What is the average profit in women's shift made by the firm.
9. The Mean marks of 150 students of a class is 65.6. The mean marks of boys is 60 and mean marks of girls is 72. Find no. of boys & girls in the class.
10. For the following data, show that $\Sigma(x - \bar{x})=0$

    11, 13, 7, 10, 15, 3, 12, 11, 4, 14.
11. Show that the sum of the squared deviations about the mean ($\bar{x}$=7.4) is lesser than the sum of the squared deviations about an assumed value 7 for the data given below,

    4, 6, 7, 9, 11
12. If $\Sigma wx = 150$ and $\Sigma w = 10$, find $\bar{x}_w$.

Practical 10: Measures of Central tendency- B

1. Compute median for the following data.

Income(Rs.)	Below 3000	Below 4000	Below 5000	Below 6000	Below 7000
No. of Persons	10	22	37	45	50

2. For the following distribution M=33, Find the missing frequency.

C.I	0-10	10-20	20-30	30-40	40-50	50-60	60-70
f	14	16	40	-	20	18	12

3. The median for the following distribution is 26. Find the missing frequency.

C.I	0-10	10-20	20-30	30-40	40-50
f	3	5	-	6	4

4. Calculate median and mode for the following data.

Height (cm)	Less than 145	145-150	150-155	155-160	160 & above
No. of persons	5	10	15	10	5

5. Find GM and HM of 9, 15, 25.6, 32.47
6. Calculate GM & HM for the following distribution.

C.I	0-10	10-20	20-30	30-40	40-50
f	4	8	10	6	7

7. The population of a city increased at the rates of 20% and 12% in 2 successive years. In the next 3 years it decreased at the rates of 5%, 7% and 4% respectively. Find the average rate of growth.
8. A boy climbs up a slide at a speed of 16cms per second and comes down at a speed of 45cms per second. Find his average speed.
9. Find Q_1 and P_{20} for the following distribution.

C.I	50-55	55-60	60-65	65-70	70-75	75-80	80-85
f	5	10	22	30	16	12	15

10. Calculate D_2, Q_3 and P_{35} for the following distribution.

C.I	40 - 59	60 - 79	80 - 89	90 - 99	100-109	110-119	120-139	140-159
f	5	16	27	35	38	21	14	4

Practical 11: Measures of Dispersion- A

1. Find range and its relative measure from the following data: 40, 85, 100, 15, 5, 70, 65.
2. Compute coefficient of Q.D for the following distribution.

C.I	0 – 9	10 – 19	20 – 29	30 – 39	40 – 49
f	5	10	22	30	16

3. Calculate Q.D for the data given below:

Mid points	100.5	101.5	102.5	103.5	104.5
f	3	10	15	8	4

4. Calculate mean deviation from median for the following data.

x	5	15	25	35	45	55	65
f	8	12	10	8	3	2	7

5. Calculate mean deviation from mean and median for the following distribution.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
f	7	32	56	106	180	164	86	44

6. Compute mean deviation from mode for the following distribution.

C.I	0-10	10-20	20-30	30-40	40-50
f	5	8	12	8	5

Practical 12: Measures of Dispersion- B

1. Calculate S.D for the following data:

x	600	800	1000	1200	1400
f	5	11	26	10	8

2. Compute S.D for the following distribution.

Marks	0-15	15-30	30-45	45-60	60-75	75-90	90-105
No. of students	20	30	30	35	45	15	5

3. A sample of 35 values has mean 80 and S.D. 4. A second sample of 65 values has mean 70 and S.D. 3. Find the S.D. of the hundred values togather.

4. Two samples of size 10 and 20 have mean 22.5. If their respective standard deviations are 12 and 7. Find the S.D of the combined sample.

5. Compute co-efficient of variation for the following distribution.

Marks	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
No. of students	8	12	30	20	10

6. Scores of two golfers in 12 rounds were as follows:

   Golfer(A): 74, 75, 78, 72, 77, 79, 81, 76, 73, 71, 71, 73

   Golfer(B): 86, 84, 80, 88, 89, 85, 86, 82, 83, 70, 71, 70

   Find out which golfer scores more and who may be considered to be a more consistent player.

Practical 13: Measures of Skewness

1. The sum of lower and upper quartiles is 55 and their difference is 15. If the median is 30 find the co-efficient of skewness.
2. Compute Karl-pearson's co-efficient of skewness for the following distribution.

Wages (Rs.)	70–80	80–90	90–100	100–110	110–120	120–130	130–140	140–150
No. of persons	12	18	35	42	50	45	20	8

3. Calculate Pearson's co-efficient of skewness for the following data.

C.I	10-19	20-29	30-39	40-49	50-59
f	4	5	12	7	4

4. Compute Bowley's co-efficient of skewness for the following distribution.

X	58	59	60	61	62	63	64	65
f	10	18	30	42	35	28	16	8

5. Compute Bowley's co-efficient of skewness for the following distribution.

Age (Years)	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
No. of persons	18	16	15	12	10	7	3	1

6. The first four central moments of a distribution are 0, 12, -3 and176 comment on skewness and kurtosis.

Practical 14: Correlation- A

1. Draw a scatter diagram for the data given below and interpret.

x	3	6	9	12	15	18	21	24	27
y	8	12	16	20	24	28	32	36	40

2. Draw a scatter diagram for the data given below and interpret.

x	25	50	75	100	125	150	175	200	225	250
y	5	8	4	3	7	6	9	10	5	7

3. Calculate Pearson's coefficient of correlation for the age of husband and wife.

Age of husband	23	27	28	29	30	31	33	35	36	39
Age of wife	18	22	23	24	25	26	28	29	30	32

4. Calculate Pearson's coefficient of correlation between price and supply from the following data.

Price (in Rs.)	11	12	13	20	14	15	16	17	18	19
Supply (in kg)	30	29	29	15	25	24	24	24	22	18

5. Family income and its percentage spent on food in the case of 100 families gave the following bivariate frequency distribution. Calculate the coefficient of correlation.

Food expenditure	Family income (in' 000 Rs.)				
%	20 - 30	30 - 40	40 - 50	50 - 60	60 – 70
10 – 15	-	-	-	3	7
15 – 20	-	4	9	4	3
20 – 25	7	6	12	5	-
25 – 30	3	10	19	8	-

Practical 15: Correlation- B

1. Calculate Karl Pearson's Coefficient of correlation between x and y

x \ y	115	120	125	130
10	-	-	6	11
20	-	2	4	10
30	-	3	1	5
40	3	2	3	1
50	10	4	5	-

2. Calculate coefficient of correlation between the marks obtained by the batch of 100 students in Accountancy and Statistics as given in the following table.

Marks in Statistics	Marks in Accountancy				
	20-30	30-40	40-50	50-60	60-70
15-25	5	9	3	-	-
25-35	-	10	25	2	-
35-45	-	1	12	2	-
45-55	-	-	4	16	5
55-65	-	-	-	4	2

3. Two judges have ranked 12 students as follows:

Students	A	B	C	D	E	F	G	H	I	J	K	L
Rank by I judge	5	2	4	1	8	9	10	6	3	11	7	12
Rank by II judge	6	9	7	10	1	2	4	12	3	5	11	8

Calculate rank correlation coefficient.

4. From the following marks obtained by the students in Accountancy and Statistics papers, compute rank coefficient of correlation.

Students	A	B	C	D	E	F	G	H
Marks in Accountancy	15	20	28	12	40	60	20	80
Marks in Statistics	40	30	50	30	20	10	30	60

5. The following are the marks of 9 students in the preparatory and find examinations. Calculate the coefficient of rank correlation and comment.

Students	1	2	3	4	5	6	7	8	9
Preparatory	10	23	18	50	47	61	11	42	49
Final	25	40	38	61	50	88	40	40	49

Practical 16: Regression

1. From the following table showing age of cars of a certain make and maintenance costs, obtain the regression equation for costs related to age.

Age of cars(years)	2	4	6	7	8	10	12
Annual maintenance cost (thousands of rupee)	16	15	18	19	17	21	20

2. From the following data, find the two regression equations.

x	1	2	3	4	5
y	2	3	5	4	6

Also find the most probable value of y when x = 2.5 .

3. You are given the following data

	x	y
A.M	36	85
S.D	11	8

Correlation coefficient between x and y is 0.66

i. Find the two regression equations.

ii. Estimate the value of x when y = 75.

4. Given the following data

Variance of x = 9

Regression equation of y on x: $4x - 5y + 33 = 0$

Regression equation of x on y: $20x - 9y - 107 = 0$

Find, i) the mean values of x and y.

ii) The coefficient of correlation between x and y.

iii) The S.D of y.

5. The following table shows the frequency distribution of 100 couples classified according to their ages.

Wife's age (years)	Husband's age (years)			
	20 – 25	25 – 30	30 – 35	35 – 40
15 – 20	20	10	3	2
20 – 25	4	18	16	4
25 – 30	-	5	11	-
30 – 35	-	-	2	-
35 – 40	-	-	-	5

Estimate the age of husband when wife's age is 28 years.

Practical 17:

Association of attributes, Interpolation and Extrapolation

1. Find Yule's coefficient of association from the following data:

	Smokers	Non-smokers
Drinkers	260	90
Non-drinkers	40	60

2. Prepare a 2×2 contingency table from the following information. Also Calculate Yule's coefficient of association and interpret the result.

$N = 1500$, $(\alpha) = 1117$, $(B) = 360$, $(AB) = 35$.

3. In a co-education institution out of 200 students, 150 were boys. They wrote an examination and it was found that 120 passed. 10 girls failed. Is there any association between sex and success in examination?

4. In order to ascertain, if marriage has any effect on the examination result of students, 1000 students were selected at random. Of the 1000 students, 375 were married. Of the married students 167 passed and on the unmarried students 203 failed. Find Yule's coefficient of association between marriage and failure of students in the examination.

5. Using the binomial expansion method of interpolation find the probable production for the year 2002.

Year	2000	2001	2002	2003	2004	2005
Production ('000 tons)	39	85	-	151	264	388

6. Using the binomial expansion method of interpolation find the probable production for the year 2004.

Year	2000	2002	2004	2006
Value	103	107	-	157

7. Interpolate the missing figure in the following table with the help of suitable formula.

Year	2006	2007	2008	2009	2010	2011
Value	200	225	250	270	-	295

8. Extrapolate the population of a city for the year 2010.

Year	2005	2006	2007	2008	2009	2010
Population (lakhs)	4	4.2	4.3	4.5	4.8	-

Practical 18: Probability

1. A die is thrown once, what is the probability of getting a (i) multiple of 3 (ii) non-multiple of 3.

2. Two cards are drawn from a pack of 52 playing cards. What is the probability that they are: (a) Blacks (b) Queens (c) Blacks or Queens?

3. In a firm, 40% of the employees are women and 60% are men. Among them, it is found that $\frac{1}{8}^{th}$ of the women and $\frac{1}{10}^{th}$ of the men wear spectacles. Find the probability that a randomly selected person is: (i) a man or a person wearing spectacles (ii) a woman or a man wearing spectacles.

4. A box contains 5 red and 3 blue balls. Two balls are drawn from this box. What is the probability that they are of the same colour?

5. In a college, there are 800 students, out of which 360 are girls. It is known that out of 360, 10% of the girls study in I P U C Commerce. What is the probability that a randomly chosen student studies in I P U C Commerce given that the chosen student is a girl?

6. A box contains 4 red and 6 blue balls. Two balls are drawn from this box one after the other. What is the probability that they are red, if first drawn ball is (i) not replaced (ii) replaced?

7. The first box contains 3 white, 2 black balls. The second box contains 2 white, 4 black balls. A ball is transferred from the first box to the second box and then a ball is drawn from the second box. What is the probability that it is white in colour?

8. A problem in Statistics is given to solve to 3 students A, B and C. Their probabilities of solving it are 4/5, 3/4 and, 3/5 respectively. If they try individually, what is the probability that it is solved?
   [Hint: P(at least one) = 1 - P(none)]

9. The odds against a person hitting a target are 3 to 2. If he fires twice at a target, what is the probability that the target is hit?

10. Two dice are thrown once. Find the probability of (i) sum of numbers obtained is 5, (ii) sum of numbers obtained is divisible by 5, (iii) product of numbers obtained is 5, (iv) product of numbers obtained is divisible by 5, (v) sum of numbers obtained is 13.

Practical 19: Random Variable-A

1. A person tosses a coin thrice. Find the expected number of heads obtained.
2. Find the mean and variance of a random variable X which assumes the values -1, 0 and 1 with respective probabilities ¼, ½ and ¼.
3. A purse has 4 five rupees coins and 6 two rupees coins. A boy picks a coin at random from the bag, what is the expected amount he gets?
4. A bag contains 6 tickets numbered 1 to 6. A person draws two tickets at random. If the sum of the numbers on the tickets drawn is even, he gets Rs.10, otherwise he loses Rs.5. Show that the expectation of his gain is Re.1.
5. Two fair coins are tossed once. A person receives Rs.10 if both head appears and Rs.5 if both tail appears, otherwise he loses Rs.8, find the expectation of a person.
6. The probability of a person hitting a target is 2/3. If he hits the target he gets Rs.100, otherwise he loses Rs.10. Find his expectation.
7. Find the value of k and then find the mean of the following distribution.

x	1	2	3	4	5	6
p(x)	0.1	0.15	k	0.25	0.18	0.12

8. A box contains 8 items of which 2 are defective. A man selects 3 items. Find the expected number of defective items in the selection.
9. Given the following probability distribution, find E(X).

x	-2	-1	1	2
p(x)	1/5	2/10	3/10	2/5

10. Calculate E(X+4) for the following probability distribution.

x	10	15	20
p(x)	1/6	2/6	3/6

Practical 20: Random Variable-B

1. If E(X) = 2, find the value of E(3X-6).
2. If E(X) = 5 and $E(X^2)$ = 75, find the standard deviation.
3. For a random variable X, V(X) = 4. Find V(2X+4) and V(-3X).
4. In a bivariate data E(X)= 4, E(Y) = 10, $E(X^2)$= 25, $E(Y^2)$ = 136 and E(XY) = 20. Find Var(X), Var(Y) , S.D(X), Cov(X,Y), Var(4X) and γ_{xy}.
5. In a bivariate data E(X) = 4, E(Y) = 6 and E(XY) = 36. Obtain Cov (X,Y).
6. From the following bivariate data of X and Y find (i) 'k' (ii) E(2X+3Y) (iii) Cov (X,Y) and (iv) Coefficient of correlation between X and Y (γ_{xy}).

x \ y	0	10	20
1	0	0.1	0.1
2	0.1	0.2	0.1
3	0.2	k	0.1

7. For the following joint probability distribution of X and Y

x \ y	1	2	3
-5	0	0.1	0.1
0	0.1	0.1	0.2
1	0.1	0.2	0.1

Find (i) E(XY) and (ii) γ_{xy}.

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QUESTION PAPER PATTERN FOR I P.U.C

Sub: STATISTICS (31)

Time: 3 Hours 15 Minutes Maximum Marks : 100

I	Answer any 10 questions out of 12 questions.	[Q No 1 to 12] : (VSA)	10/12 X 1 = 10
II	Answer any 10 questions out of 12 questions.	[Q No 13 to 24] : (SA)	10/12 X 2 = 20
III	Answer any 08 questions out of 12 questions.	[Q No 25 to 36] : (LA)	08/12 X 5 = 40
IV	Answer any 2 questions out of 4 questions.	[Q No 37 to 40] : (ET)	2/4 X 10 = 20
V	Answer any 2 questions out of 4 questions.	[Q No 41 to 44] : (LA)	2/4 X 5 = 10
		Total	32/44 100

Unit No	Unit Name	No of Hours			Marks Allotted	% weightage of Marks	Number of Questions					
		Th	Pr	Tot			VSA	SA	LA	ET	LA	Tot
I	Introduction to Statistics and some basic concepts	7	-	7	8	5	1	1	1			3
II	Organization of data	7	-	7	8	5	1	1	1			3
III	Classification and Tabulation of data	10	8	18	16	10	2	2	1		1	6
IV	Diagrammatic and Graphical representation of data	10	8	18	16	10	2	2	1		1	6
V	Analysis of univariate data	25	10	35	36	23	2	2	1	2	1	8
VI	Analysis of bivariate data	15	6	21	23	15	1	1	2	1		5
VII	Association of attributes	5	1	6	7	4	-	1	1			2
VIII	Interpolation and Extrapolation	5	1	6	6	4	1	-	1			2
IX	Theory of Probability	14	2	16	18	12	1	1	2	a	x	4+
X	Random Variable and Mathematical expectation	14	4	18	18	12	1	1	1	b		4+
Total		112	40	152	156	100	12	12	12	4	4	44

Question Type	Marks
VSA: Very Short Answer	1
SA : Short Answer	2
LA : Long Answer	5
ET : Essay Type	10

Note: 1 One 10 mark question and one 5 mark question (i.e., questions a, b and x) are to set from combination of untis IX and X covering both units.

2 Six problems, two theory and two proof oriented questions should be given in section-C.

QUESTION PAPER PATTERN FOR I P.U.C

Sub: STATISTICS (31)

Time: 3 Hours 15 Minutes		Maximum Marks : 100
I	Answer any 10 questions out of 12 questions. [Q No 1 to 12] : (VSA)	10/12 X 1 = 10
II	Answer any 10 questions out of 12 questions. [Q No 13 to 24] : (SA)	10/12 X 2 = 20
III	Answer any 08 questions out of 12 questions. [Q No 25 to 36] : (LA)	08/12 X 5 = 40
IV	Answer any 2 questions out of 4 questions. [Q No 37 to 40] : (ET)	2/4 X 10 = 20
V	Answer any 2 questions out of 4 questions. [Q No 41 to 44] : (LA)	2/4 X 5 = 10
	Total	32/44 100

Unit No	Unit Name	No of Hours			Marks Allotted	% weightage of Marks	Number of Questions					
		Th	Pr	Tot			VSA	SA	LA	ET	LA	Tot
I	Introduction to Statistics and some basic concepts	7	-	7	8	5	1	1	1			3
II	Organization of data	7	-	7	8	5	1	1	1			3
III	Classification and Tabulation of data	10	8	18	16	10	2	2	1		1	6
IV	Diagrammatic and Graphical representation of data	10	8	18	16	10	2	2	1		1	6
V	Analysis of univariate data	25	10	35	36	23	2	2	1	2	1	8
VI	Analysis of bivariate data	15	6	21	23	15	1	1	2	1		5
VII	Association of attributes	5	1	6	7	4	-	1	1			2
VIII	Interpolation and Extrapolation	5	1	6	6	4	1	-	1			2
IX	Theory of Probability	14	2	16	18	12	1	1	2	a	x	4+
X	Random Variable and Mathematical expectation	14	4	18	18	12	1	1	1	b		4+
Total		112	40	152	156	100	12	12	12	4	4	44

Question Type	Marks
VSA: Very Short Answer	1
SA : Short Answer	2
LA : Long Answer	5
ET : Essay Type	10

Note: 1 One 10 mark question and one 5 mark question (i.e., questions a, b and x) are to set from combination of untis IX and X covering both units.

2 Six problems, two theory and two proof oriented questions should be given in section-C.

BLUE PRINT FOR I P.U.C. MADEL QUESTION PAPER.

Sub: STATISTICS (31)

Time: 3 Hours 15 Minutes | Maximum Marks : 100

I Answer any 10 questions out of 12 questions. [Q No 1 to 12] : (VSA) 10/12 X 1 = 10

II Answer any 10 questions out of 12 questions. [Q No 13 to 24] : (SA) 10/12 X 2 = 20

III Answer any 08 questions out of 12 questions. [Q No 25 to 36] : (LA) 08/10 X 5 = 40

IV Answer any 2 questions out of 4 questions. [Q No 37 to 40] : (ET) 2/4 X 10 = 20

V Answer any 2 questions out of 4 questions. [Q No 41 to 44] : (LA) 2/4 X 5 = 10

Total 32/44 100

Unit No	Unit Name	No of Hours			Marks Allotted	% weightage of Marks	No. of Questions					Knowledge				Understanding				Application				Skill					Total No. Questions
		Th	Pr	Tot			VSA	SA	LA	ET	LA	VSA	SA	LA	ET	VSA	SA	LA	ET	VSA	SA	LA	ET	VSA	SA	LA	ET	LA	
I	Introduction to Statistics and some basic concepts	7	-	7	8	5	1	1	1			1					1					1							3
II	Organization of data	7	-	7	8	5	1	1	1			1	1									1							3
III	Classification and Tabulation of data	10	8	18	16	10	2	2	1		1	2					2					1						1	6
IV	Diagrammatic and Graphical representation of data	10	8	18	16	10	2	2	1		1	2					2									1		1	6
V	Analysis of univariate data	25	10	35	36	23	2	2	1	2	1				2	2	2	1										1	8
VI	Analysis of bivariate data	15	6	21	23	15	1	1	2	1						1	1	2									1		5
VII	Association of attributes	5	1	6	7	4	-	1	1								1					1							2
VIII	Interpolation and Extrapolation	5	1	6	6	4	1	-	1							1						1							2
IX	Theory of Probability	14	2	16	18	12	1	1	2	0.5						1	1	2					0.5						4.5
X	Random Variable and Mathematical expectation	14	4	18	18	12	1	1	1	0.5	1	1					1	1					0.5					1	4.5
	Total	112	40	152	156	100	12	12	12	4	4	7	1	0	2	5	11	6	0	0	0	5	1	0	0	1	1	4	44
					Marks for each Question		1	2	5	10	5	1	2	5	10	1	2	5	10	1	2	5	10	1	2	5	10	5	
					Total Marks		12	24	60	40	20	7	2	0	20	5	22	30	0	0	0	25	10	0	0	5	10	20	
					Grand Total of Marks		156					29				57				35				35					
					Percentage		100					19				37				22				22					

Question Type	Marks
VSA: Very Short Answer	1
SA : Short Answer	2
LA : Long Answer	5
ET : Essay Type	10

I P U C MODEL QUESTION PAPER

Sub: STATISTICS (31)

Time: 3Hours 15 Minutes **Maximum Marks: 100**

Note: 1. Graph sheets and statistical tables will be supplied on request.

2. Scientific calculators may be used.

3. All working steps should be clearly shown.

SECTION - A

I. Answer any TEN of the following questions: **$10 \times 1 = 10$**

1. State Croxton and Cowden's definition of Statistics.
2. Who is an investigator?
3. What is tabulation of the data?
4. Write the formula of mid-point of a class.
5. Which graph is used to locate median?
6. What is class frequency?
7. Find the geometric mean of 2 and 8.
8. For a data if $D_5 = 50$, then what is the value of P_{50}?
9. Mention the type of correlation between 'speed of a vehicle and distance covered by it'.
10. What is interpolation?
11. If $P(A) = \frac{2}{5}$, then find $P(A^1)$.
12. Define a random variable.

SECTION - B

II. Answer any TEN of the following questions: **$10 \times 2 = 20$**

13. What is continuous variable? Give an example.
14. Mention two methods of sampling.
15. What do you mean by inclusive class intervals? Give an example.
16. What are stubs and captions of a table?

17. Mention two objectives of diagrams and graphs.
18. What is Histogram?
19. Find the harmonic mean of 1, 1/2, 1/3, 1/4.
20. For a data, if median is 50 and mean deviation from median is 12, then find its coefficient.
21. What are regression lines? Where do they intersect?
22. In case of two attributes, if N=250, (AB)=30, (A)=100 and (B)=50, then find the remaining frequencies .
23. Two cards are drawn from a pack of 52 playing cards. What is the probability that they are kings?
24. If E(X)=3 and $E(X^2)$=25, then find SD(X).

SECTION - C

III. Answer any EIGHT of the following questions: $8 \times 5 = 40$

25. Write the functions of Statistics.
26. What are the guidelines for the construction of a questionnaire?
27. Prepare a blank table showing the particulars relating to the students of a college classified according to-
    (i) Faculty: Arts, Commerce, Science.
    (ii) Caste: SC/ST, Group I, Others.
    (iii) Sex: Male and Female.
28. Following is data regarding strength of a college. Draw percentage bar diagram.

Academic Year	Number of students		
	Male	Female	Total
2009-10	350	150	500
2010-11	800	200	1000
2011-12	1200	800	2000
2012-13	1000	1000	2000

29. For the following observations, find mean, median and mode.
    12, 42, 25, 35, 67, 25, 56, 5, 75.

30. Explain the types of correlation with examples.

31. Marks obtained by five students in two subjects are as follows. Find Spearman's rank correlation coefficient.

Student	A	B	C	D	E
Marks in Accountancy	56	45	76	89	65
Marks in Statistics	65	54	67	98	56

32 200 candidates appeared for II PUC Examination in a college and 60 of them succeeded. 35 received a special coaching in tutorial class and out of them 20 candidates succeeded. Using Yule's coefficient, discuss whether the special coaching is effective or not.

33. Interpolate the index for 2008 from the following data.

Year	2006	2007	2008	2009	2010
Index No.	278	281	—	313	322

34. State and prove addition theorem of probability for two non-mutually exclusive events.

35. A bag contains 6 red and 4 white balls. Two balls are drawn from the bag randomly. What is the probability that they are of the; (i) same colour (ii) different colour?

36. If X is a random variable and 'a' is any constant, then prove that $E(aX) = a\,E(X)$ and $Var\,(aX) = a^2\,Var\,(X)$.

SECTION - D

IV. Answer any TWO of the following questions: **$2 \times 10 = 20$**

37. Which series is better? and which is more consistent?

X	44	48	50	52	56
Y	37	39	40	41	43

38. Find Karl Pearson's coefficient of skewness for the following data.

C I	10 -14	15 -19	20 - 24	25 - 29	30 - 34
f	5	9	15	13	8

39. For the following bivariate data, find Karl Pearson's coefficient of correlation (γ).

X \ Y	0	1	2	3
20-25	30	-	-	-
25-30	8	22	16	4
30-35	4	-	5	3
35-40	-	2	5	1

40. (a) The probability that a boy will pass an examination is 3/5 and that of a girl is 2/5 . Find the probability that i) both of them pass the examination. ii) at least one of them passes the examination.

(b) Find the mathematical expectation of the number of heads obtained when two fair coins are tossed once.

SECTION – E

(Practical oriented questions)

V. Answer any TWO of the following questions: $2 \times 5 = 10$

41. Following are weights (in Kg) of forty students of a college. Prepare frequency distribution table with suitable class intervals.

Weights (in Kg) 45, 56, 50, 41, 55, 51, 46, 50, 45, 57, 64, 48, 53, 43, 57, 44, 54, 59, 49,52
42, 61, 51, 63, 48, 56, 45, 50, 55,63, 45,55, 60, 50, 46, 50, 42, 52, 62, 50

42. Draw histogram. Hence, find the value of mode.

C I	10 - 20	20-30	30-40	40-50	50-60
f	3	7	10	8	2

43. For the following data find the value of second quartile (Q_2).

Daily Wages (in Rs)	Below 100	Below 200	Below 300	Below 400	Below 500
No. of Workers	10	30	80	105	120

44 There are 10 tickets in a bag which are numbered 1, 2, 3,.....,10. Two tickets are drawn randomly one after the other with replacement. Find the expectation of the sum of the numbers drawn.

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Logarithms

	0	1	2	3	4	5	6	7	8	9	Mean Differences								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0298	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0765	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	3	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	9	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	4	5	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	6	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

Logarithms

	0	1	2	3	4	5	6	7	8	9	Mean Differences								
											1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	7
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	4	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8457	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8527	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

Antilogarithms

	0	1	2	3	4	5	6	7	8	9	Mean Differences								
											1	2	3	4	5	6	7	8	9
.00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
.01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
.02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
.03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
.04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
.05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
.06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
.09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
.16	1445	1449	1452	1455	1459	1460	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
.17	1479	1483	1486	1489	1492	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
.20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
.22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
.28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
.37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2391	1	1	2	2	3	3	4	4	5
.38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
.40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
.44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
.49	3090	3097	3105	3112	3119	3126	3133	3142	3148	3155	1	1	2	3	4	4	5	6	6

Antilogarithms

	0	1	2	3	4	5	6	7	8	9	Mean Differences 1	2	3	4	5	6	7	8	9
.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
.51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
.55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
.56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
.58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
.59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
.60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
.62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
.63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
.64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
.66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
.67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	5	7	8	9	10
.69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
.70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
.71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
.72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
.73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
.74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
.76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
.77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
.78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
.79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
.80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
.81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
.82	6607	6622	6637	6653	6663	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
.83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
.85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
.86	7244	7261	7278	7295	7311	7324	7345	7362	7379	7392	2	3	5	7	8	10	12	13	15
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
.88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
.89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	16	17
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	16	17
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
.95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
.97	9333	9354	9376	9397	9419	9441	9462	9484	9504	9528	2	4	7	9	11	13	15	17	20
.98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
.99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20