CHAPTER

6 Cube and Cube Roots

Learning Objectives

In this Chapter you will learn

- *To find cube of a number.*
- *To learn about different properties of cubes.*
- *To find cube root of a number using different methods.*
- *To use concept of cube and cube roots in solving practical life problems.*

6.1 Introduction :-

The greatest Indian Mathematician **S. Ramanujan** experimented with numbers throughout his life. He loved the numbers. Once another famous mathematician Prof. G.H. Hardy came to visit him in a taxi, whose number was 1729. While talking to Ramanujan, he described the taxi number as "a dull number". But Ramanujan quickly pointed out that the number 1729 is a very interesting number. He said it is the smallest number that can be expressed as a sum of two cubes in two different ways, as :

$1729 = 1728 + 1 = 12^3 + 1^3$ and $1729 = 1000 + 729 = 10^3 + 9^3$

1729 has since been known as Hardy-Ramanujan number. There are infinitaly many such numbers. Few of them are 4104 {(2, 16); (9, 15)}, 13832 {(18, 20); (2, 24)} etc.

6.2 Cube :-

Cube of any number is when its exponent is 3 i.e. when some number is raised to power three.

For example cube of 2 is written as 2^3 which is equal to $2 \times 2 \times 2$.

We can say that cube of any number is a number when that number is mutliplied three times by itself.

For Example: $8^3 = 8\times8\times8 = 512$

$12^3 = 12\times12\times12 = 1728$

$7^3 = 7\times7\times7 = 343$

In Geometry, we use the word cube. A cube is a solid figure whose all sides are equal. (Fig. 6.1). It has six faces, all are squares, for example, dice is a form of cube.

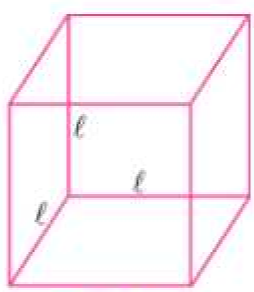

Fig. 6.1

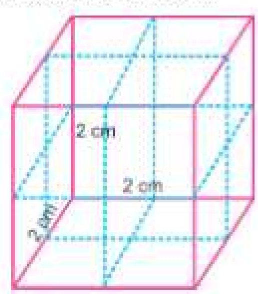

Fig. 6.2

How many cubes of side 1 cm will make a cube of side 2 cm ? Observe fig. 6.2, 8 cubes of side 1cm will make a cube of side 2cm.

Now consider the numbers 1, 8, 27, 64, 125, ...

Now observe :-

$1 = 1 \times 1 \times 1$; $8 = 2 \times 2 \times 2$; $27 = 3 \times 3 \times 3$

$64 = 4 \times 4 \times 4$; $125 = 5 \times 5 \times 5$; and so on.

Each of these is obtained when a number is multiplied three times by itself.

These types of numbers are known as **Perfect Cube or Cube numbers.**

Is 9 a perfect cube ?

As $9 = 3 \times 3$ and there is no natural number which when multiplied three times by itself gives 9 So 9 is not a Perfect cube number.

The following are the cubes of numbers form 1 to 20 (the table for cube of a number.)

Number	1	2	3	4	5	6	7	8	9	10	...	20
Cube	1^3 $=1\times1\times1$ $=1$	2^3 $=2\times2\times2$ $=8$	3^3 $=3\times3\times3$ $=27$	4^3 $=4\times4\times4$ $=64$	5^3 $=5\times5\times5$ $=125$	6^3 $=6\times6\times6$ $=216$	7^3 $=7\times7\times7$ $=343$	8^3 $=8\times8\times8$ $=512$	9^3 $=9\times9\times9$ $=729$	10^3 $=10\times10\times10$ $=1000$	...	20^3 $=20\times20\times20$ $=8000$

(Table 6.1)

Now observe, whether cube of even numbers are even ? and cube of odd numbers are odd ? From table you can observe that cube of even number is even and odd number is odd. Also there are only few perfect cube numbers from 1 to 1000. How many Perfect cubes are there from 1 to 200 ?

Consider few numbers having 1 as the digit at Unit's (ones) place. (Find the cube of them for example 1, 11, 21, 31, 41, ..., 111, etc.)

Number	1	11	21	31	41	...	111	...
Cube	1^3 $=1\times1\times1$ $=1$	11^3 $=11\times11\times11$ $=1331$	21^3 $=21\times21\times21$ $=9261$	31^3 $=31\times31\times31$ $=29791$	41^3 $=41\times41\times41$ $=68921$	...	111^3 $=111\times111\times111$ $=1367631$	...

Table 6.2

What can you say about the ones digit of the cube of a number having 1 as the Unit's (ones) place number ? The unit's place digits of cube of a number having 1 as ones digit is 1.

Similarly, explore the ones digit of cubes of numbers ending in 2, 3, 4, ... etc. You will observe that the ones place digit of the cube of number having ones place 2 is 8; having ones place 3 is 7 and having ones place 4 is 4.

Example 6.1 : What should be the ones digit of the cube of each of the following numbers, tell without actual calculation?

(i) 2561 **(ii) 342** **(iii) 463** **(iv) 1264**

Sol. As we observe that, the ones digit of the cube of number :

(i) 2561 will be 1

(ii) 342 will be 8

(iii) 463 will be 7

(iv) 1264 will be 4

Example 6.2: Find cube of (i) 13 (ii) –4 (iii) $\frac{3}{8}$ (iv) 2.1

Sol. (i) Cube of 13 = $13^3 = 13 \times 13 \times 13 = 2197$

(ii) Cube of –4 = $(-4)^3 = (-4) \times (-4) \times (-4) = -64$

(iii) Cube of $\frac{3}{8} = \left(\frac{3}{8}\right)^3 = \frac{3}{8} \times \frac{3}{8} \times \frac{3}{8} = \frac{27}{512}$

(iv) Cube of 2.1 = $(2.1)^3 = 2.1 \times 2.1 \times 2.1 = 9.261$

Example 6.3: Find the volume of cube having side 3 cm.

Sol. Here, side of cube = 3 cm; As volume of cube = $(\text{side})^3$

So volume of cube having side 3cm = $(3\text{ cm})^3 = 27\text{ cm}^3$

Exercise 6.1

1. What should be the ones digit of the cube of the each of the following numbers tell without actual calculation?

   (i) 231 (ii) 4584 (iii) 6259 (iv) 105 (v) 17 (vi) 120

2. Find the cube of following numbers.

   (i) –9 (ii) 16 (iii) –14 (iv) $\frac{1}{13}$ (v) $\frac{8}{7}$

   (vi) 2.4 (vii) 0.002 (viii) 9.9 (ix) 1.01

3. Find volume of cube having side :

   (i) 4 cm (ii) 15cm (iii) 17cm (iv) 2.3 cm (v) 7.2m

4. **Multiple Choice Questions :**

   (i) Ones digit of cube of 7 is :

   (a) 7 (b) 3 (c) 5 (d) 6

   (ii) Ones digit of cube of a number having 2 at ones place is :

   (a) 2 (b) 4 (c) 6 (d) 8

   (iii) Volume of a cube of side 5cm is:

   (a) 15cm (b) 125cm^3 (c) 45cm^3 (d) 50cm

   (iv) Ones digit of 1823^3 is :

   (a) 3 (b) 9 (c) 7 (d) 6

(v) How many cubes of side 1cm will form a cube of side 2cm.

(a) 2 (b) 4 (c) 6 (d) 8

(vi) What is ones place digit in 626^3.

(a) 2 (b) 3 (c) 4 (d) 6

6.2.1 Some Interesting Patterns :

1. (a) Adding consecutive odd numbers

Observe the following patterns of sums of odd number :

$$1 = 1 = 1^3$$
$$3 + 5 = 8 = 2^3$$
$$7 + 9 + 11 = 27 = 3^3$$
$$13 + 15 + 17 + 19 = 64 = 4^3$$
$$21 + 23 + 25 + 27 + 29 = 125 = 5^3$$
$$31 + 33 + 35 + 37 + 39 + 41 = 216 = 6^3$$

Is it not interesting ? What do you observe?

(b) Subtracting cubes of consecutive numbers.

Observe the following Pattern :

$2^3 - 1^3 = 1 + 2 \times 1 \times 3 = 1 + 6 = 7$

$3^3 - 2^3 = 1 + 3 \times 2 \times 3 = 1 + 18 = 19$

$4^3 - 3^3 = 1 + 4 \times 3 \times 3 = 1 + 36 = 37$

$5^3 - 4^3 = 1 + 5 \times 4 \times 3 = 1 + 60 = 61$

Is it not interesting? What do you observe?

2. Cubes and their Prime factors

Consider the following Prime factorisation of some numbers and their cubes.

Prime factorisation of a number	Prime factorisation of its cube
$10 = 2 \times 5$	$10^3 = 1000 = 2\times2\times2\times5\times5\times5 = 2^3 \times 5^3$
$18 = 2 \times 3 \times 3$	$18^3 = 5832 = 2\times2\times2\times3\times3\times3\times3\times3\times3 = 2^3\times3^3\times3^3$
$14 = 2 \times 7$	$14^3 = 2744 = 2\times2\times2\times7\times7\times7 = 2^3 \times 7^3$
$15 = 3 \times 5$	$15^3 = 3375 = 3\times3\times3\times5\times5\times5 = 3^3 \times 5^3$
$20 = 2\times2\times5$	$20^3 = 8000 = 2\times2\times2\times2\times2\times2\times5\times5\times5 = 2^3\times2^3\times5^3$

Observe that each prime factor of a number appears three times in the prime factorisation of its cube.

In the prime factorisation of any number, if each factor appears three times, then the number is perfect cube.

Example 6.4: By Prime Factorisation check whether following are perfect cube or not ?

(i) 512 (ii) 5000 (iii) 1372 (iv) 1331

Sol. (i) 512

Prime factorisation of 512 is =

$\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$

As we can group prime factors in triplets

$\therefore$ It is a perfect cube.

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

(ii) 5000

Prime factorisation of 5000 = $\underline{2\times2\times2}\times\underline{5\times5\times5}\times5$

As prime factors of 5000 cannot be grouped in triplets.

$\therefore$ It is not a perfect cube.

2	5000
2	2500
2	1250
5	625
5	125
5	25
5	5
	1

(iii) 1372

Prime factorisation of 1372 = $2 \times 2 \times \underline{7 \times 7 \times 7}$

As prime factors of 1372 cannot be grouped in triplets.

$\therefore$ It is not a perfect cube.

2	1372
2	686
7	343
7	49
7	7
	1

(iv) 1331

Prime Factorisation of 1331 = $\underline{11 \times 11 \times 11}$

As prime factors of 1331 can be grouped in triplets.

$\therefore$ it is a perfect cube.

11	1331
11	121
11	11
	1

Exercise 6.2

1. Express the following numbers as the sum of consecutive odd natural numbers.

   (i) 7^3 (ii) 8^3 (iii) 9^3

2. Find the value of following by using suitable pattern?

(i) $12^3 - 11^3$ (ii) $20^3 - 19^3$ (iii) $51^3 - 50^3$

3. Which of the following are perfect cubes ?

(i) 225 (ii) 10648 (iii) 1125 (iv) 2744

6.2.2 Smallest multiple that is Perfect Cube :-

In last section we have observed that every number is not a perfect cube. If a number is not a perfect cube, we can find the smallest natural number by which the given number must be multiplied, so that the product is a perfect cube. We can also find the smallest number to divide the given number, so that quotient is a perfect cube.

Example 6.5 : Is 243 a perfect cube ? If not, find the smallest number by which 243 must be multiplied so that the product is a perfect cube. Also find the number.

Sol. The prime factorisation of 243 is

$$243 = \underbrace{3 \times 3 \times 3} \times 3 \times 3 = 3^3 \times 3^2$$

3	243
3	81
3	27
3	9
3	3
	1

The prime factor 3 does not appear in a group of three. So 243 is not a perfect cube. To make it perfect cube, we need one more 3, so to make 243 a perfect cube. we have to multiply it by 3.

$$243 \times 3 = 729 = \underbrace{3 \times 3 \times 3} \times \underbrace{3 \times 3 \times 3} = 729$$

Now, it is a perfect cube number. Hence, the smallest number to be multiplied is 3 and 729 is the number which is perfect cube.

Example 6.6 : Is 675 a perfect cube ? If not, find the smallest natural number by which 675 must be multiplied so that the product is a perfect cube.

Sol. Let us find the prime factorisation of

$$675 = \underbrace{3 \times 3 \times 3} \times 5 \times 5$$

3	675
3	225
3	75
5	25
5	5
	1

The prime factor 5 does not appear in group of three.

So, 675 is not a perfect cube. To make it a perfect cube, we need one more 5. In that case

$$675 \times 5 = \underbrace{3 \times 3 \times 3} \times \underbrace{5 \times 5 \times 5} = 3375$$

which is a perfect cube. Hence, the smallest natural number by which 675 should be multiplied to make it a perfect cube is 5.

Example 6.7 : Is 31944 a perfect cube ? If not, then by which smallest natural number should 31944 be divided so that quotient is a perfect cube ?

Sol. Let us first find the prime factorisation of 31944.

Now $31944 = \underbrace{2 \times 2 \times 2} \times 3 \times \underbrace{11 \times 11 \times 11}$

2	31944
2	15972
2	7986
3	3993
11	1331
11	121
11	11
	1

The prime factor 3 does not appear in a group of three, so 31994 is not a perfect cube. In the factorisation 3 appears only once.

So, if we divide the number by 3, then the prime factorisation of quotient will not contain 3.

So, $31944 \div 3 = 10648$

Hence the smallest number by which 31944 should be divided to make it a perfect cube is 3.

The perfect cube in that case is 10648.

Exercise 6.3

1. Find the smallest number by which each of the following numbers must be multiplied to obtain a perfect cube.

   (i) 81 (ii) 100 (iii) 72 (iv) 625 (v 2916 (vi) 41503

2. Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube.

   (i) 2187 (ii) 78125 (iii) 16384 (iv) 19773 (v) 36501 (vi) 23625

3. Check which of the following are perfect cubes :

   (i) 2700 (ii) 16000 (iii) 8000 (iv) 27000 (v) 125000 (vi) 15125

   Which pattern do you observe in these Perfect cubes ?

4. **Multiple Choice Questions :**

   (i) By what number 108 be multiplied to make it a perfect cube.

   (a) 2 (b) 3 (c) 4 (d) 6

   (ii) By what number 625 be divided so as to make it a perfect cube?

   (a) 5 (b) 8 (c) 6 (d) 9

   (iii) Which of following is not a perfect cube ?

   (a) 16 (b) 27 (c) 64 (d) 125

   (iv) Find the number which when multiplied with 500 makes it a perfect cube?

   (a) 5 (b) 2 (c) 3 (d) 6

   (v) Find $7^3 - 6^3$:

   (a) 127 (b) 397 (c) 1141 (d) 200

6.3 Cube Roots:-

Sometime, we have to find the number whose cube is given. For example, if the volume of a cube is 64 cm^3, what should be the length of side of cube ? Now, here we need to find a number whose cube is 64.

In last chapter we have already studied about square and square root. As you know, finding the square root is inverse operation of squaring. Similarly, finding the cube root is inverse operation of finding cube.

We know that $4^3 = 64$. So we say that cube root of 64 is 4. We write $\sqrt[3]{64} = 4$. The symbol

$\sqrt[3]{\ }$ denotes cube root. In terms of power, we write it as $(\ )^{1/3}$. Study the following (table 6.3) :

$1^3 = 1$	$2^3 = 8$	$3^3 = 27$	$4^3 = 64$	$5^3 = 125$	$6^3 = 216$
$\sqrt[3]{1}$ $= 1$	$\sqrt[3]{8} = (2^3)^{1/3}$ $= 2$	$\sqrt[3]{27} = (3^3)^{1/3}$ $= 3$	$\sqrt[3]{64} = (4^3)^{1/3}$ $= 4$	$\sqrt[3]{125} = (5^3)^{1/3}$ $= 5$	$\sqrt[3]{216} = (6^3)^{1/3}$ $= 6$

Table 6.3

6.3.1 Cube Root through Prime factorisation :-

We can find the cube root of a given cube number by its Prime factorisation. Study the following examples :

Example 6.8 : Find the cube root of 42875.

Sol. Let us find Prime factorisation of the number 42875.

Now $42875 = 5 \times 5 \times 5 \times 7 \times 7 \times 7$

$$\sqrt[3]{42875} = \sqrt[3]{5\times5\times5\times7\times7\times7}$$

$$= 5 \times 7 = 35$$

5	42875
5	8575
5	1715
7	343
7	49
7	7
	1

Example 6.9 : Find cube root of 175616 by Prime factorisation.

Sol. Prime factorisation of

$175616 = \underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{7\times7\times7}$

So $$\sqrt[3]{175616} = \sqrt[3]{\underbrace{2\times2\times2}\times\underbrace{2\times2\times2}\times\underbrace{2\times2\times2}\times\underbrace{7\times7\times7}}$$

$$= 2 \times 2 \times 2 \times 7 = 56$$

2	175616
2	87808
2	43904
2	21952
2	10976
2	5488
2	2744
2	1372
2	686
7	343
7	49
7	7
	1

Exercise 6.4

1. Cube of a number is 64. Find the number.
2. Cube of a number is 3375. Find the number.
3. Find the cube root of each of the following numbers by prime factorisation :

   (i) 5832 (ii) 216000 (iii) 456533 (iv) 729000

   (v) 85184 (vi) 328509
4. **Multiple Choice Questions :**

   (i) What is cube root of 512 ?

   (a) 2 (b) 4 (c) 6 (d) 8

   (ii) Find $\sqrt[3]{1728}$.

   (a) 10 (b) 12 (c) 14 (d) 16

   (iii) Find cube root of 1331.

   (a) 11 (b) 21 (c) 31 (d) 23

   (iv) A perfect cube ends with digit 2 what will be ones digit of its cube root.

   (a) 4 (b) 2 (c) 6 (d) 8

Learning Outcomes

After completion of the chapter, the students are now able to:

- *Find cube of a number.*
- *Understand about different properties of cubes.*
- *Find cube root of a number using different methods.*
- *Use concept of cube and cube roots in solving practical life problems.*

Answers

Exercise 6.1

1. (i) 1 (ii) 4 (iii) 9 (iv) 5 (v) 3 (vi) 0
2. (i) –729 (ii) 4096 (iii) –2744 (iv) $\frac{1}{2197}$ (v) $\frac{512}{343}$

   (vi) 13.824 (vii) 0.000000008 (viii) 970.299 (ix) 1.030301

3. (i) 64 cm^3 (ii) 3375cm^3 (iii) 4913cm^3 (iv) 12.167 cm^3 (vi) 373.248 cm^3

4. (i) b (ii) d (iii) b (iv) c (v) d (vi) d

Exercise 6.2

1. (i) $7^3 = 43 + 45 + 47 + 49 + 51 + 53 + 55$

   (ii) $8^3 = 57 + 59 + 61 + 63 + 65 + 67 + 69 + 71$

   (iii) $9^3 = 73 + 75 + 77 + 79 + 81 + 83 + 85 + 87 + 89$

2. (i) $12^3 - 11^3 = 1 + 12 \times 11 \times 3$; (ii) $20^3 - 19^3 = 1 + 20 \times 19 \times 3$

   (iii) $51^3 - 50^3 = 1 + 51 \times 50 \times 3$

3. (ii) and (iv)

Exercise 6.3

1. (i) 9 (ii) 10 (iii) 3 (iv) 25 (v) 2 (vi) 11

2. (i) 3 (ii) 5 (iii) 4 (iv) 9 (v) 3 (vi) 7

3. (i) No (ii) No (iii) Yes (iv) Yes (v) Yes (vi) No

   [**Hints :** Pattern : As $(a \times b)^m = a^m \times b^m$ $\sqrt[3]{ab} = (a)^{1/3} \times b^{1/3}$]

4. (i) a (ii) a (iii) a (iv) b (v) a

Exercise 6.4

1. 4 2. 15

3. (i) 18 (ii) 60 (iii) 77 (iv) 90 (v) 44 (vi) 69

4. (i) d (ii) b (iii) a (iv) d