8. Factorisation

Let us Work Out 8.1

1. Question

Let us factorise the following polynomials:

$x^3 - 3x + 2$

Answer

Given, $f(x) = x^3 - 3x + 2$

In f(x) putting $x = \pm 1, \pm 2, \pm 3$, we see for which value of x, $f(x) = 0$

$f(1) = (1)^3 - 3.(1) + 2 = 0$

We observe that $f(1) = 0$

From factor theorem, we can say, $(x-1)$ is a factor of f(x)

$x^3 - 3x + 2 = x^3 - x^2 + x^2 - x - 2x + 2$

$= x^2(x-1) + x(x-1) - 2(x-1)$

$= (x-1)(x^2 + x - 2)$

$= (x-1)(x^2 + 2x - x - 2)$

$= (x-1)(\, x(x+2) - (x+2)\,)$

$= (x-1)(x-1)(x+2)$

$= (x-1)^2(x+2)$

2. Question

Let us factorise the following polynomials:

$x^3 + 2x + 3$

Answer

Given, $f(x) = x^3 + 2x + 3$

In f(x) putting $x = \pm 1, \pm 2, \pm 3$, we see for which value of x, $f(x) = 0$

Since, each term of f(x) is positive here; so for positive value of x we shall not get the value of f(x) as zero.

Hence, for the negative value of x, the value of f(x) can be zero.

$f(-1)=(-1)^3+2.(-1)+3=0$

We observe that f(−1) = 0

From factor theorem, we can say, (x+1) is a factor of f(x)

$x^3 + 2x + 3 = x^3 + x^2 - x^2 - x + 3x + 3$

$= x^2(x+1)-x(x+1)+3(x+1)$

$= (x+1)(x^2-x+3)$

3. Question

Let us factorise the following polynomials:

$a^3 - 12a - 16$

Answer

Given, $f(a) = a^3 - 12a - 16$

In f(a) putting a=±1, ±2, ±3, we see for which value of a, f(a)=0

$f(-2)=(-2)^3-12.(-2)-16=0$

We observe that f(−2) = 0

From factor theorem, we can say, (a+2) is a factor of f(a)

$a^3 - 12a - 16 = a^3 - 12a - 16$

$= a^3+2a^2-2a^2- 4a - 8a - 16$

$= a^2(a+2)-2a(a+2)-8(a+2)$

$= (a+2)(a^2-2a-8)$

$= (a+2)(a^2-4a+2a-8)$

$= (a+2)(a(a-4) + 2(a-4))$

$= (a+2)(a-4)(a+2)$

$= (a+2)^2(a-4)$

4. Question

Let us factorise the following polynomials:

$x^3 - 6x + 4$

Answer

Given, $f(x) = x^3 - 6x + 4$

In f(x) putting $x=\pm1, \pm2, \pm3$, we see for which value of x, $f(x)=0$

$f(2)=(2)^3-6.2+4=0$

We observe that $f(2) = 0$

From factor theorem, we can say, $(x-2)$ is a factor of f(x)

$x^3 - 6x + 4 = x^3 - 6x + 4$

$= x^3 - 2x^2 + 2x^2 - 4x - 2x + 4$

$= x^2(x-2)+2x(x-2)-2(x-2)$

$= (x-2)(x^2+2x-2)$

5. Question

Let us factorise the following polynomials:

$x^3 - 19x - 30$

Answer

Given, $f(x)= x^3 - 19x - 30$

In f(x) putting $x=\pm1, \pm2, \pm3$, we see for which value of x, $f(x)=0$

$f(-2)=(-2)^3-19.(-2)+30=0$

We observe that $f(-2) = 0$

From factor theorem, we can say, $(x+2)$ is a factor of f(x)

$x^3 - 19x - 30 = x^3 - 19x - 30$

$= x^3+2x^2-2x^2-4x-15x-30$

$= x^2(x+2)-2x(x+2)-15(x+2)$

$= (x+2)(x^2-2x-15)$

$= (x+2)(x^2-5x+3x-15)$

$= (x+2)(x(x-5) + 3(x-5))$

$= (x+2)(x-5)(x+3)$

6. Question

Let us factorise the following polynomials:

$4a^3 - 9a^2 + 3a + 2$

Answer

Given, $f(a) = 4a^3 - 9a^2 + 3a + 2$

In f(a) putting a=±1, ±2, ±3, we see for which value of a, f(a)=0

$f(1)=4.(1)^3 -9.(1)^2 +3.(1)+2=0$

We observe that f(1) = 0

From factor theorem, we can say, (a−1) is a factor of f(a)

$4a^3-9a^2+3a+2= 4a^3-4a^2-5a^2+5a-2a+ 2$

$= 4a^2(a-1)-5a(a-1)-2(a-1)$

$= (a-1)(4a^2-5a-2)$

7. Question

Let us factorise the following polynomials:

$x^3 - 9x^2 + 23x - 15$

Answer

Given, $f(x)= x^3 - 9x^2 + 23x - 15$

In f(x) putting x=±1, ±2, ±3, we see for which value of x, f(x)=0

$f(1)=4.(1)^3 -9.(1)^2 +3.(1)+2=0$

We observe that f(1) = 0

From factor theorem, we can say, (x−1) is a factor of f(x)

$x^3 - 9x^2 + 23x - 15= x^3 - 9x^2 + 23x - 15$

$= x^3-x^2-8x^2+8x+15x-15$

$= x^2(x-1)-8x(x-1)+15(x-1)$

$= (x-1)(x^2-8x+15)$

$= (x-1)(x^2-5x-3x+15)$

$= (x-1)(x(x-5) -3(x-5))$

$= (x-1)(x-5)(x-3)$

8. Question

Let us factorise the following polynomials:

$5a^3 + 11a^2 + 4a - 2$

Answer

Given, $f(a)= 5a^3 + 11a^2 + 4a - 2$

In f(a) putting a=±1, ±2, ±3, we see for which value of a, f(a)=0

$f(-1)=5.(-1)^3+11.(-1)^2+4.(-1)-2=0$

We observe that $f(-1) = 0$

From factor theorem, we can say, (a+1) is a factor of f(a)

$5a^3 + 11a^2 + 4a - 2 = 5a^3 + 11a^2 + 4a - 2$

$= 5a^3+5a^2+6a^2+6a-2a-2$

$= 5a^2(a+1)+6a(a+1)-2(a+1)$

$= (a+1)(5a^2+6a-2)$

9. Question

Let us factorise the following polynomials:

$2x^3 - x^2 + 9x + 5$

Answer

Given, $f(x)= 2x^3 - x^2 + 9x + 5$

In f(x) putting x=±1, $\pm\frac{1}{2}$, ±2, ±3, we see for which value of x, f(x)=0

$f(-\frac{1}{2})=2.(-\frac{1}{2})^3-(-\frac{1}{2})^2-9.(-\frac{1}{2})+5=0$

We observe that $f(-\frac{1}{2}) = 0$

From factor theorem, we can say, for $(x=-\frac{1}{2})$, (2x+1) is a factor of f(x)

$2x^3 - x^2 + 9x + 5 = 2x^3 - x^2 + 9x + 5$

$= 2x^3+x^2-2x^2- x + 10x +5= x^2(2x+1)-x(2x+1)+5(2x+1)$

$= (2x+1)(x^2-x+5)$

10. Question

Let us factorise the following polynomials:

$2y^3 - 5y^2 - 19y + 42$

Answer

Given, $f(y) = 2y^3 - 5y^2 - 19y + 42$

In f(y) putting $y = \pm 1, \pm 2, \pm 3$, we see for which value of y, $f(y) = 0$

$f(2) = 2.(2)^3 - 5.(2)^2 - 19.2 + 42 = 0$

We observe that $f(2) = 0$

From factor theorem, we can say, $(y-2)$ is a factor of f(y)

$2y^3 - 5y^2 - 19y + 42 = 2y^3 - 5y^2 - 19y + 42$

$= 2y^3 - 4y^2 - y^2 + 2y - 21y + 42$

$= 2y^2(y-2) - y(y-2) - 21(y-2)$

$= (y-2)(2y^2 - y - 21)$

Let us Work Out 8.2

1. Question

Let us factorise the following algebraic expressions:

$$\frac{x^4}{16} - \frac{y^4}{81}$$

Answer

Given, $\frac{x^4}{16} - \frac{y^4}{81}$

$\Rightarrow$ This can be written as $\left(\frac{x}{2}\right)^4 - \left(\frac{y}{3}\right)^4$

$$\Rightarrow \left(\frac{x^2}{2^2}\right)^2 - \left(\frac{y^2}{3^2}\right)^2$$

[Since, from the identity III we know that, $(x^2 - y^2) = (x - y)(x + y)$]

$$\Rightarrow \left(\left(\frac{x^2}{2^2}\right) - \left(\frac{y^2}{3^2}\right)\right)\left(\left(\frac{x^2}{2^2}\right) + \left(\frac{y^2}{3^2}\right)\right)$$

$$\Rightarrow \left(\left(\frac{x}{2}\right)^2 - \left(\frac{y}{3}\right)^2\right)\left(\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2\right)$$

$$\Rightarrow \left(\frac{x}{2} - \frac{y}{3}\right)\left(\frac{x}{2} + \frac{y}{3}\right)\left(\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2\right)$$

2. Question

Let us factorise the following algebraic expressions:

$$m^2 + \frac{1}{m^2} + 2 - 2m - \frac{2}{m}$$

Answer

Given, $m^2 + \frac{1}{m^2} + 2 - 2m - \frac{2}{m}$

$$\Rightarrow (m^2 - 2m + 1) + \left(\frac{1}{m^2} - \frac{2}{m} + 1\right)$$

$$\Rightarrow (m - 1)^2 + \left(\frac{1}{m} - 1\right)^2$$

$$\Rightarrow (m - 1)^2 + \left(\frac{1 - m}{m}\right)^2$$

$$\Rightarrow (m - 1)^2 - \left(\frac{1}{m}(m - 1)\right)^2$$

$$\Rightarrow (m - 1)^2\left(1 - \frac{1}{m^2}\right)$$

$$\Rightarrow (m - 1)(m - 1)\left(1 - \frac{1}{m^2}\right)$$

3. Question

Let us factorise the following algebraic expressions:

$9p^2 - 24pq + 16q^2 + 3ap - 4aq$

Answer

Given, $9p^2 - 24pq + 16q^2 + 3ap - 4aq$

$\Rightarrow (3p)^2 - 24pq + (4q)^2 + 3ap - 4aq$

$\Rightarrow (3p)^2 - 2(3p)(4q) + (4q)^2 + 3ap - 4aq$

[from Identity I, $(x - y)^2 = x^2 - 2xy + y^2$]

$\Rightarrow (3p - 4q)^2 + a(3p - 4q)$

$\Rightarrow (3p - 4q)[(3p - 4q) + a]$

$\Rightarrow (3p - 4q)(a + (3p - 4q))$

4. Question

Let us factorise the following algebraic expressions:

$4x^4 + 81$

Answer

Given, $4x^4 + 81$

$\Rightarrow 4x^4 + 3^4$

$\Rightarrow (2x^2)^2 + 9^2$

$\Rightarrow (2x^2 + 9)^2 - 2(2x^2)(9)$

$\Rightarrow (2x^2 + 9)^2 - 36x^2$

$\Rightarrow (2x^2 + 9)^2 - (6x)^2$

[Since, from the identity III we know that, $(x^2 - y^2) = (x - y)(x + y)$]

$\Rightarrow ((2x^2 + 9) - (6x))\ ((2x^2 + 9) + (6x))$

$\Rightarrow (2x^2 + 9 - 6x)(2x^2 + 9 + 6x)$

5. Question

Let us factorise the following algebraic expressions:

$x^4 - 7x^2 + 1$

Answer

Given, $x^4 - 7x^2 + 1$

Can be written as

$\Rightarrow (x^2 + 1)^2 - 9x^2$

$\Rightarrow (x^2 + 1)^2 - (3x)^2$

[Since, from the identity III we know that, $(x^2 - y^2) = (x - y)(x + y)$]

$\Rightarrow ((x^2 + 1) - 3x)((x^2 + 1) + 3x)$

$\Rightarrow (x^2 - 3x + 1)(x^2 + 3x + 1)$

6. Question

Let us factorise the following algebraic expressions:

$p^4 - 11p^2q^2 + q^4$

Answer

Given, $p^4 - 11p^2q^2 + q^4$

$\Rightarrow (p^2)^2 - 11p^2q^2 + (q^2)^2$

$\Rightarrow (p^2)^2 - 2p^2q^2 - 9\,p^2q^2 + (q^2)^2$

$\Rightarrow (p^2)^2 - 2p^2q^2 + (q^2)^2 - 9\,p^2q^2$

$\Rightarrow (p^2 - q^2)^2 - (3pq)^2$

$\Rightarrow ((p^2 - q^2) - 3pq)((p^2 - q^2) + 3pq)$

$\Rightarrow (p^2 - q^2 - 3pq)(p^2 - q^2 + 3pq)$

7. Question

Let us factorise the following algebraic expressions:

$a^2 + b^2 - c^2 - 2ab$

Answer

Given, $a^2 + b^2 - c^2 - 2ab$

$\Rightarrow$ This can be written as $a^2 - 2ab + b^2 - c^2$

$\Rightarrow$ From the identity II, $a^2 - 2ab + b^2 = (a - b)^2$

$\therefore (a - b)^2 - c^2$

$\Rightarrow ((a - b) - c)((a - b) + c)$

[Since, from the identity III we know that, $(x^2 - y^2) = (x - y)(x + y)$]

$\Rightarrow (a - b - c)(a - b + c)$

8. Question

Let us factorise the following algebraic expressions:

$3a\,(3a + 2c) - 4b\,(b + c)$

Answer

Given, $3a(3a + 2c) - 4b(b + c)$

$\Rightarrow 9a^2 + 6ac - 4b^2 - 4bc$

$\Rightarrow (3a)^2 + 2(3)(ac) - (2b)^2 - 4bc + c^2 - c^2$

$\Rightarrow (3a)^2 + 2(3)(ac) + c^2 - ((2b)^2 + 2(2b)(c) + c^2)$

$\Rightarrow (3a + c)^2 - (2b + c)^2$

[Since, from the identity III we know that, $(x^2 - y^2) = (x - y)(x + y)$]

$\Rightarrow (3a + c - 2b - c)(3a + c + 2b + c)$

$\Rightarrow (3a - 2b)(3a + 2b + 2c)$

9. Question

Let us factorise the following algebraic expressions:

$a^2 - 6ab + 12bc - 4c^2$

Answer

Given, $a^2 - 6ab + 12bc - 4c^2$

$\Rightarrow a^2 - 4c^2 - 6ab + 12bc$

$\Rightarrow (a^2 - (2c)^2) - 6b(a - 2c)$

[Since, from the identity III we know that, $(x^2 - y^2) = (x - y)(x + y)$]

$\Rightarrow (a + 2c)(a - 2c) - 6b(a - 2c)$

$\Rightarrow (a - 2c)(a + 2c - 6b)$

10. Question

$3a^2 + 4ab + b^2 - 2ac - c^2$

Answer

Given,$3a^2 + 4ab + b^2 - 2ac - c^2$

$\Rightarrow 3a^2 + 4ab + b^2 - 2ac - c^2$

$\Rightarrow 4a^2 + 4ab + b^2 - a^2 - 2ac - c^2$

$\Rightarrow$ From the identity I, $a^2 + 2ab + b^2 = (a + b)^2$

$\Rightarrow 4a^2 + 4ab + b^2 - (a^2 + 2ac + c^2)$

$\Rightarrow (2a + b)^2 - (a + c)^2$

[Since, from the identity III we know that, $(x^2 - y^2) = (x - y)(x + y)$]

$\Rightarrow ((2a + b) - (a + c))$

$((2a + b) + (a + c)$

$\Rightarrow (2a + b - a - c)(2a + b + a + c)$

11. Question

Let us factorise the following algebraic expressions:

$x^2 - y^2 - 6ax + 2ay + 8a^2$

Answer

Given,$x^2 - y^2 - 2(3ax) + 2ay + 9a^2 - a^2$

$\Rightarrow x^2 - 2(3a)(x) + (3a)^2 - (a^2 - 2ay + y^2)$

$\Rightarrow (x - 3a)^2 - (a - y)^2$

$\Rightarrow (x - 3a - a + y)(x - 3a + a - y)$

[Since, from the identity III we know that, $(x^2 - y^2) = (x - y)(x + y)$]

$\Rightarrow (x - 4a + y)(x - 2a - y)$

12. Question

Let us factorise the following algebraic expressions:

$a^2 - 9b^2 + 4c^2 - 25d^2 - 4ac + 30bd$

Answer

Given, $a^2 - 9b^2 + 4c^2 - 25d^2 - 4ac + 30bd$

$\Rightarrow a^2 - 9b^2 + 4c^2 - 25d^2 - 4ac + 30bd$

$\Rightarrow a^2 - 4ac + 4c^2 - 9b^2 - 25d^2 + 30bd$

$\Rightarrow a^2 - 2(2ac) + (2c)^2 - (3b)^2 - (5d)^2 + 2(3b)(5d)$

$\Rightarrow (a - 2c)^2 - (3b - 5d)^2$

[Since, from the identity III we know that, $(x^2 - y^2) = (x - y)(x + y)$]

$\Rightarrow ((a - 2c) - (3b - 5d))(a - 2c - 3b + 5d)$

$\Rightarrow (a - 2c - 3b - 5d)(a - 2c - 3b + 5d)$

13. Question

Let us factorise the following algebraic expressions:

$3a^2 - b^2 - c^2 + 2ab - 2bc + 2ca$

Answer

Given, $3a^2 - b^2 - c^2 + 2ab - 2bc + 2ca$

$\Rightarrow 4a^2 - a^2 - b^2 - c^2 + 2ab - 2bc + 2ca$

$\Rightarrow (2a)^2 - (a^2 + b^2 + c^2 - 2ab + 2bc - 2ca)$

$\Rightarrow (2a)^2 - (a - b - c)^2$

[Since, from the identity III we know that, $(x^2 - y^2) = (x - y)(x + y)$]

$\Rightarrow (2a - (a - b - c))(2a + a + b + c)$

$\Rightarrow (a + b + c)(3a + b + c)$

14. Question

Let us factorise the following algebraic expressions:

$x^2 - 2x - 22499$

Answer

Given, $x^2 - 2x - 22499$

$\Rightarrow x^2 - 151x + 149x - 22499$

$\Rightarrow x(x - 151) + 149(x - 151)$

$\Rightarrow (x + 149)(x - 151)$

15. Question

Let us factorise the following algebraic expressions:

$(x^2 - y^2)(a^2 - b^2) + 4abxy$

Answer

Given, $(x^2 - y^2)(a^2 - b^2) + 4abxy$

$\Rightarrow ((x - y)(x + y)(a - b)(a + b)) + 4abxy$

$\Rightarrow (ax)^2 - (bx)^2 - (ay)^2 + (by)^2 + 4abxy$

$\Rightarrow (ax)^2 + (by)^2 + 4abxy - (bx)^2 - (ay)^2$

$\Rightarrow (ax)^2 + (by)^2 + 2abxy + 2abxy - (bx)^2 - (ay)^2$

[from Identity I, $(x + y)^2 = x^2 + 2xy + y^2$ and from identity II, $(x - y)^2 = x^2 - 2xy + y^2$]

$\Rightarrow (ax + by)^2 - (ay - bx)^2$

$\Rightarrow ((ax + by) + (ay - bx))((ax + by) - (ay - bx))$

Let us Work Out 8.3

1. Question

Let us factorize the following algebraic expressions:

$t^9 - 512$

Answer

We know, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Given,

$t^9 - 512$

$= (t^3)^3 - (8)^3$

$= (t^3 - 8)\ (t^6 + 8t^3 + 64)$

$= [(t)^3 - (2)^3]\ (t^6 + 8t^3 + 64)$

$= (t - 2)\ (t^2 + 2t + 4)\ (t^6 + 8t^3 + 64)$

2. Question

Let us factorize the following algebraic expressions:

$729p^6 - q^6$

Answer

We know, $a^2 - b^2 = (a + b)\ (a - b)$

Given,

$729p^6 - q^6$

$= (27p^3)^2 - (q^3)^2$

$= (27p^3 + q^3)\ (27p^3 - q^3)$

Now, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ and $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$= [(3p)^3 + (q)^3]\ [(3p)^3 - (q)^3]$

$= (3p + q)(3p^2 - 3pq + q^2)\ (3p - q)(3p^2 + 3pq + q^2)$

3. Question

Let us factorize the following algebraic expressions:

$8(p - 3)^3 + 343$

Answer

We know, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Given,

$8(p - 3)^3 + 343$

$= [8(p - 3)]^3 + (7)^3$

$= [8(p - 3) + 7]\ [\{8(p - 3)\}^2 - 8(p - 3).7 + 7^2]$

$= (8p - 24 + 7)\ [64(p - 3)^2 - 56(p - 3) + 49]$

$= (8p - 17)\ [64(p^2 - 6p + 9) - 56p + 168 + 49]$

$= (8p - 17)\ [64p^2 - 384p + 576 - 56p + 168 + 49]$

$= (8p - 17)\ [64p^2 - 440p + 793]$

4. Question

Let us factorize the following algebraic expressions:

$$\frac{1}{8a^3} + \frac{8}{b^3}$$

Answer

We know, $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Given,

$$\frac{1}{8a^3} + \frac{8}{b^3}$$

$$= \left(\frac{1}{2a}\right)^3 + \left(\frac{2}{b}\right)^3$$

$$= \left(\frac{1}{2a} + \frac{2}{b}\right)\left(\frac{1}{4a^2} - \frac{1}{ab} + \frac{4}{b^2}\right)$$

5. Question

Let us factorize the following algebraic expressions:

$(2a^3 - b^3)^3 - b^9$

Answer

Given,

$(2a^3 - b^3)^3 - b^9$

We know, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

Therefore,

$= (2a^3)^3 - (b^3)^3 - 3(2a^3)(b^3)(2a^3 - b^3) - b^9$

$= 8a^9 - b^9 - 6a^3b^3(2a^3 - b^3) - b^9$

$= 8a^9 - 12a^6b^3 + 6a^3b^6 - 2b^9$

$= 2a^9 - 2b^9 + 6a^9 - 12a^6b^3 + 6a^3b^6$

$= 2(a^9 - b^9) + 6a^3(a^6 - 2a^3b^3 + b^6)$

Now, we know: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ and

$(x - y)^2 = x^2 - 2xy + y^2$

$= 2(a^3 - b^3)(a^6 + a^3b^3 + b^3) + 6a^3(a^3 - b^3)^2$

$= 2(a - b)(a^2 + ab + b^2)(a^6 + a^3b^3 + b^6) + 6a^3(a - b)^2(a^2 + ab + b^2)^2$

$= 2(a - b)(a^2 + ab + b^2)(a^6 + a^3b^3 + b^6 + 3a^3(a - b)(a^2 + ab + b^2))$

$= 2(a - b)(a^2 + ab + b^2)(a^6 + a^3b^3 + b^6 + 3a^3(a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3)$

$= 2(a - b)(a^2 + ab + b^2)(a^6 + a^3b^3 + b^6 + 3a^3(a^3 - b^3))$

$= 2(a - b)(a^2 + ab + b^2)(a^6 + a^3b^3 + b^6 + 3a^6 - 3a^3b^3)$

$= 2(a - b)(a^2 + ab + b^2)(4a^6 - 2a^3b^3 + b^6)$

6. Question

Let us factorize the following algebraic expressions:

$AR^3 - Ar^3 + AR^2h - Ar^2h$

Answer

Given,

$AR^3 - Ar^3 + AR^2h - Ar^2h$

$= A(R^3 - r^3 + R^2h - r^2h)$

$= A[(R^3 - r^3) + h(R^2 - r^2)]$

We know, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ and $a^2 - b^2 = (a + b)(a - b)$

$= A\ [(R - r)\ (R^2 + Rr + r^2) + h\ (R + r)(R - r)]$

$= A(R - r)\ [R^2 + Rr + r^2 + h\ (R + r)]$

$= A(R - r)\ [R(R + r) + r^2 + h(R + r)]$

$= A(R - r)\ (R + r)\ (R + r^2 + h)$

7. Question

Let us factorize the following algebraic expressions:

$a^3 + 3a^2b + 3ab^2 + b^3 - 8$

Answer

Given,

$a^3 + 3a^2b + 3ab^2 + b^3 - 8$

$= (a + b)^3 - (2)^3$

We know, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$= (a + b - 2)\ [(a + b)^2 + 2(a + b) + 2^2]$

We know, $(a + b)^2 = a^2 + 2ab + b^2$

$= (a + b - 2)\ [a^2 + 2ab + b^2 + 2a + 2b + 4]$

8. Question

Let us factorize the following algebraic expressions:

$32x^4 - 500x$

Answer

$32x^4 - 500x$

$= 4x\ (8x^3 - 125)$

$= 4x\ [(2x)^3 - (5)^3]$

We know, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$= 4x\ (2x - 5)\ [(2x)^2 + 2x.5 + (5)^2]$

$= 4x (2x – 5) (4x^2 + 10x + 25)$

9. Question

Let us factorize the following algebraic expressions:

$8a^3 – b^3 – 4ax + 2bx$

Answer

Given,

$8a^3 – b^3 – 4ax + 2bx$

$= (2a)^3 – (b)^3 – 2x(2a – b)$

We know, $a^3 – b^3 = (a – b)(a^2 + ab + b^2)$

$= (2a – b) ((2a)^2 + 2a.b + (b)^2) – 2x(2a – b)$

$= (2a – b) (4a^2 + 2ab + b^2) – 2x(2a – b)$

$= (2a – b) (4a^2 + 2ab + b^2 – 2x)$

10. Question

Let us factorize the following algebraic expressions:

$x^3 – 6x^2 + 12x – 35$

Answer

Given,

$x^3 – 6x^2 + 12x – 35$

$= x^3 – 35 – 6x^2 + 12x$

$= x^3 – 35 – 6x(x – 2)$

Let us Work Out 8.4

1. Question

Let us factorise the following algebraic expressions:

$x^3 + y^3 – 12xy + 64$

Answer

$x^3 + y^3 – 12xy + 64$

$\Rightarrow x^3 + y^3 + 4^3 – 12xy$

$\Rightarrow x^3 + y^3 + 4^3 - 3\times4xy$ ...Equation (i)

We use the identity

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Using the above identity in Equation (i) we get

$x^3 + y^3 + 4^3 - 3\times4xy = (x + y + 4)(x^2 + y^2 + 16 - xy - 4y - 4x)$

$(x + y + 4)(x^2 + y^2 + 16 - xy - 4y - 4x)$

2. Question

Let us factorise the following algebraic expressions:

$8x^3 - y^3 + 1 + 6xy$

Answer

$8x^3 - y^3 + 1 + 6xy$

$\Rightarrow (2x)^3 + (-y)^3 + 1^3 + 6xy$

$\Rightarrow (2x)^3 + (-y)^3 + 1^3 - 3\times(2x)\times(-y)\times1$ ...Equation(i)

We use the identity

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Using the above identity in Equation (i) we get

$\Rightarrow (2x)^3 + (-y)^3 + 1^3 - 3\times(2x)\times(-y) = (2x - y + 1)((2x)^2 + y^2 + 1^2 + 2xy + y - 2x)$

$\Rightarrow (2x)^3 + (-y)^3 + 1^3 - 3\times(2x)\times(-y) = (2x - y + 1)(4x^2 + y^2 + 1 + 2xy + y - 2x)$

$(2x - y + 1)(4x^2 + y^2 + 1 + 2xy + y - 2x)$

3. Question

Let us factorise the following algebraic expressions:

$8a^3 - 27b^3 - 1 - 18ab$

Answer

$8a^3 - 27b^3 - 1 - 18ab$

$\Rightarrow (2a)^3 + (-3b)^3 + (-1)^3 - 18ab$

$\Rightarrow (2a)^3 + (-3b)^3 + (-1)^3 - 3\times(2a)\times(-3b)\times(-1)$ ...Equation(i)

We use the identity

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Using the above identity in Equation (i) we get

$\Rightarrow (2a)^3 + (-3b)^3 + (-1)^3 - 3\times(2a)\times(-3b)\times(-1) = (2a - 3b - + 1)(4a^2 + 9b^2 + 1 + 6ab - 3b - 2a)$

$(2a - 3b - 1)(4a^2 + 9b^2 + 1 - 6ab + 3b + 2a)$

4. Question

Let us factorise the following algebraic expressions:

$1 + 8x^3 + 18xy - 27y^3$

Answer

$1 + 8x^3 + 18xy - 27y^3$

$\Rightarrow 1^3 + (2x)^3 + (-3y)^3 + 18xy$

$\Rightarrow 1^3 + (2x)^3 + (-3y)^3 + 3\times(1)\times(2x)(-3y)$ ...Equation(i)

We use the identity

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Using the above identity in Equation (i) we get

$\Rightarrow 1^3 + (2x)^3 + (-3y)^3 + 3\times(1)\times(2x)(-3y) = (1 + 2x - 3y)(1 + 4x^2 + 9y^2 - 2x + 6xy + 3y)$

$(1 + 2x - 3y)(1 + 4x^2 + 9y^2 - 2x + 6xy + 3y)$

5. Question

Let us factorise the following algebraic expressions:

$(3a - 2b)^3 + (2b - 5c)^3 + (5c - 3a)^3$

Answer

Let us consider $(3a - 2b) = x$, $(2b - 5c) = y$, $(5c - 3a) = z$

So $x + y + z = 3a - 2b + 2b - 5c + 5c - 3a = 0$

$(3a - 2b)^3 + (2b - 5c)^3 + (5c - 3a)^3 = x^3 + y^3 + z^3$

Since $x + y + z = 0$, hence

$x^3 + y^3 + z^3 = 3xyz$

$\Rightarrow (3a - 2b)^3 + (2b - 5c)^3 + (5c - 3a)^3 = 3\times(3a - 2b)\times(2b - 5c)\times (5c - 3a)$

$3(3a - 2b)(2b - 5c)(5c - 3a)$

6. Question

Let us factorise the following algebraic expressions:

$(2x - y)^3 - (x + y)^3 + (2y - x)^3$

Answer

Let us consider $(2x - y) = a$, $(2y - x) = b$

$a + b = (2x - y) + (2y - x) = x + y$

So we can say that

$(2x - y)^3 - (x + y)^3 + (2y - x)^3 = a^3 + b^3 - (a + b)^3$ ...Equation (i)

Using the identity of $(a + b)^3$ we get

$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$\Rightarrow - 3ab(a + b) = a^3 + b^3 - (a + b)^3$

Using the above identity in Equation (i)

$(2x - y)^3 - (x + y)^3 + (2y - x)^3 = - 3ab(a + b)$

$\Rightarrow (2x - y)^3 - (x + y)^3 + (2y - x)^3 = - 3\times(2x - y)\times (2y - x)\times (x + y)$

$\Rightarrow (2x - y)^3 - (x + y)^3 + (2y - x)^3 = 3\times(2x - y)\times (x - 2y)\times (x + y)$

$3(2x - y)(x - 2y)(x + y)$

7. Question

Let us factorise the following algebraic expressions:

$a^6 + 32a^3 - 64$

Answer

$a^6 + 32a^3 - 64$

$\Rightarrow a^6 + 8a^3 - 64 + 24a^3$

$\Rightarrow (a^2)^3 + (2a)^3 + (- 4)^3 - 3\times(a^2)\times(2a)\times(- 4)$ ...Equation (i)

We use the identity

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Using the above identity in Equation (i) we get

$\Rightarrow (a^2)^3 + (2a)^3 + (-4)^3 - 3\times(a^2)\times(2a)\times(-4) = (a^2 + 2a - 4)(a^4 + 4a^2 + 16 - 2a^3 + 8a + 4a^2)$

$\Rightarrow (a^2 + 2a - 4)(a^4 + 8a^2 + 16 - 2a^3 + 8a)$

$(a^2 + 2a - 4)(a^4 - 2a^3 + 8a^2 + 8a + 16)$

8. Question

Let us factorise the following algebraic expressions:

$a^6 - 18a^3 + 125$

Answer

$a^6 - 18a^3 + 125$

$\Rightarrow a^6 + 27a^3 + 125 - 45a^3$

$\Rightarrow (a^2)^3 + (3a)^3 + 5^3 - 45a^3$

$\Rightarrow (a^2)^3 + (3a)^3 + 5^3 - 3\times a^2\times 3a\times 5$ ...Equation (i)

We use the identity

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Using the above identity in Equation (i) we get

$\Rightarrow (a^2)^3 + (3a)^3 + 5^3 - 3\times a^2\times 3a\times 5 = (a^2 + 3a + 5)(a^4 + 9a^2 + 25 - 3a^3 - 15a - 5a^2)$

$\Rightarrow (a^2)^3 + (3a)^3 + 5^3 - 3\times a^2\times 3a\times 5 = (a^2 + 3a + 5)(a^4 + 4a^2 + 25 - 3a^3 - 15a)$

$(a^2 + 3a + 5)(a^4 - 3a^3 + 4a^2 - 15a + 25)$

9. Question

Let us factorise the following algebraic expressions:

$p^3 (q - r)^3 + q^3 (r - p)^3 + r^3 (p - q)^3$

Answer

$p^3 (q - r)^3 + q^3 (r - p)^3 + r^3 (p - q)^3$

$\Rightarrow (pq - pr)^3 + (qr - pq)^3 + (pr - qr)^3$

Let us consider $pq - pr = a$, $qr - pq = b$, $pr - qr = c$

$a + b + c = pq - pr + qr - pq + pr - qr = 0$

Since a + b + c = 0, hence

$a^3 + b^3 + c^3 = 3abc$

$\Rightarrow p^3 (q - r)^3 + q^3 (r - p)^3 + r^3 (p - q)^3 = 3pqr(q - r)(r - p)(p - q)$

$3pqr (q - r)(r - p)(p - q)$

10. Question

Let us factorise the following algebraic expressions:

$$p^3 + \frac{1}{p^3} + \frac{26}{27}$$

Answer

$$p^3 + \frac{1}{p^3} + \frac{26}{27}$$

$$\Rightarrow (p)^3 + \left(\frac{1}{p}\right)^3 - \frac{1}{27} + 1$$

$$\Rightarrow (p)^3 + \left(\frac{1}{p}\right)^3 + \left(\frac{1}{-3}\right)^3 - 3 \times p \times \frac{1}{p} \times \frac{1}{-3} \text{ ...Equation (i)}$$

We use the identity

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Using the above identity in Equation (i) we get

$$\Rightarrow (p)^3 + \left(\frac{1}{p}\right)^3 + \left(\frac{1}{-3}\right)^3 - 3 \times p \times \frac{1}{p} \times \frac{1}{-3} = \left(p^2 + \frac{1}{p^2} + \frac{1}{9} - 1 + \frac{1}{3p} + \frac{p}{3}\right)\left(p + \frac{1}{p} - \frac{1}{3}\right)$$

$$\Rightarrow (p)^3 + \left(\frac{1}{p}\right)^3 + \left(\frac{1}{-3}\right)^3 - 3 \times p \times \frac{1}{p} \times \frac{1}{-3} = \left(p^2 + \frac{1}{p^2} - \frac{8}{9} + \frac{1}{3p} + \frac{p}{3}\right)\left(p + \frac{1}{p} - \frac{1}{3}\right)$$

$$\left(p + \frac{1}{p} - \frac{1}{3}\right)\left(p^2 + \frac{1}{p^2} - \frac{8}{9} + \frac{1}{3p} + \frac{p}{3}\right)$$

Let us Work Out 8.5

1 A. Question

Let us factorise the following algebraic expressions:

$(a + b)^2 – 5a – 5b – 6$

Answer

The given expression can be rewritten as:

$(a + b)^2 – 5(a + b) + 6$

Assume $(a + b) = p$,

$\Rightarrow p^2 – 5p+ 6$

$\Rightarrow p^2 – 2p - 3p + 6$

$\Rightarrow p(p - 2) -3(p - 2)$

$\Rightarrow (p – 3)(p – 2)$

On substituting the value of p, we get,

$(a +b - 3)(a + b – 2)$

1 B. Question

Let us factorise the following algebraic expressions:

$(x + 1) (x + 2) (3x – 1) (3x – 4) + 12$

Answer

The given expression can be rewritten as:

$(x + 1)(3x – 1)(x + 2)(3x – 4) + 12$

$\Rightarrow (3x^2 – x + 3x - 1)(3x^2 - 4x + 6x - 8) + 12$

$\Rightarrow (3x^2 + 2x - 1)(3x^2 + 2x - 8) + 12$

Let $3x^2 + 2x = p$

$\Rightarrow (p - 1)(p – 8) + 12$

$\Rightarrow p^2 – 9x + 8 + 12$

$\Rightarrow p^2 – 9p + 20$

$\Rightarrow p^2 – 5p – 4p + 20$

$\Rightarrow(p – 5)(p – 4)$

On substituting the value of p, we get,

$\Rightarrow (3x^2 + 2x – 5)(3x^2 + 2x – 4)$

$\Rightarrow(3x^2 + 5x - 3x - 5)(3x^2 + 2x - 4)$

$\Rightarrow (x(3x + 5) - 1(3x + 5))(3x^2 + 2x - 4)$

$\Rightarrow (x - 1)(3x + 5) (3x^2 + 2x - 4)$

1 C. Question

Let us factorise the following algebraic expressions:

$x(x^2 - 1) (x + 2) - 8$

Answer

As we know that,

$a^2 - b^2 = (a - b)(a + b)$

The given expression can be rewritten as:

$x(x + 1)(x - 1)(x + 2) - 8$

$\Rightarrow (x^2 + x)(x^2 + x - 2) - 8$

Let $x^2 + x = p$

$\Rightarrow p(p - 2) - 8$

$\Rightarrow p^2 - 2p - 8$

$\Rightarrow p^2 - 4p + 2p - 8$

$\Rightarrow p(p - 4) + 2(p - 4)$

$\Rightarrow (p + 2)(p - 4)$

On substituting the value of p, we get,

$\Rightarrow (x^2 + x + 2)(x^2 + x - 4)$

1 D. Question

Let us factorise the following algebraic expressions:

$7(a^2 + b^2)^2 - 15(a^4 - b^4) + 8 (a^2 - b^2)^2$

Answer

As we know that,

$x^2 - y^2 = (x - y)(x + y)$

The given expression can be rewritten as:

$7(a^2 + b^2)^2 - 15(a^2 + b^2)(a^2 - b^2) + 8\,(a^2 - b^2)^2$

Let $(a^2 + b^2) = p$ and $(a^2 - b^2) = q$

$\Rightarrow 7p^2 - 15pq + 8q^2$

$\Rightarrow 7p^2 - 7pq - 8pq + 8q^2$

$\Rightarrow 7p(p - q) - 8q(p - q)$

$\Rightarrow (7p - 8q)(p - q)$

On substituting the value of p and q, we get,

$\Rightarrow (7(a^2 + b^2) - 8(a^2 - b^2))(\,a^2 + b^2 - a^2 + b^2)$

$\Rightarrow (7a^2 + 7b^2 - 8a^2 + 8b^2)\,(2b^2)$

$\Rightarrow 2b^2(15b^2 - a^2)$

1 E. Question

Let us factorise the following algebraic expressions:

$(x^2 - 1)^2 + 8x(x^2 + 1) + 19x^2$

Answer

As we know that,

$a^2 - b^2 = (a - b)(a + b)$

The given expression can be rewritten as:

$(x + 1)^2(x - 1)^2 + 8x(x^2 + 1) + 19x^2$

Using $(a - b)^2 = a^2 + b^2 - 2ab$, and

$(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow (x^2 + 1 + 2x)(x^2 + 1 - 2x) + 8x(x^2 + 1) + 19x^2$

Let $x^2 + 1 = p$

$\Rightarrow (p + 2x)(p - 2x) + 8xp + 19x^2$

$\Rightarrow p^2 - 4x^2 + 8xp + 19x^2$

$\Rightarrow p^2 + 8xp + 15x^2$

$\Rightarrow p^2 + 3xp + 5xp + 15x^2$

$\Rightarrow p(p+3x) + 5x(p + 3x)$

$\Rightarrow (p + 5x)(p + 3x)$

On substituting the value of p, we get,

$\Rightarrow (x^2 + 5x + 1)(x^2 + 3x + 1)$

1 F. Question

Let us factorise the following algebraic expressions:

$(a - 1)x^2 - x - (a - 2)$

Answer

The given expression can be rewritten as:

$(a - 1)x^2 - x((a - 1) - (a - 2)) - (a - 2)$

$\Rightarrow (a - 1)x(x - 1) + (a - 2)(x - 1)$

$\Rightarrow (x - 1)(ax - x + a - 2)$

1 G. Question

Let us factorise the following algebraic expressions:

$(a - 1)x^2 + a^2xy + (a + 1)y^2$

Answer

Let $p = (a - 1)$ and $q = (a+1)$

As we know that, $a^2 - b^2 = (a - b)(a + b)$

$\Rightarrow pq = a^2 + 1$

The given expression can be rewritten as:

$px^2 + pqxy + pq + qy^2$

$\Rightarrow px(x + qy) + y(x + qy)$

$\Rightarrow (px + y)(x + qy)$

On substituting the value of p and q, we get,

$(ax - x + y)(x + ay + y)$

1 H. Question

Let us factorise the following algebraic expressions:

$x^2 - qx - p^2 + 5pq - 6q^2$

Answer

The given expression can be rewritten as:

$X^2 - qx - (p^2 - 5pq + 6q^2)$

$\Rightarrow x(x - q) - (p^2 - 2pq - 3pq + 6q^2)$

$\Rightarrow x^2 - qx - (p - 2q)(p - 3q)$

$\Rightarrow x^2 + (p - 2q)x - (p - 3q)x - (p - 2q)(p - 3q)$

$\Rightarrow (x - p + 2q)(x + p - 3q)$

1 I. Question

Let us factorise the following algebraic expressions:

$$2\left(a^2 + \frac{1}{a^2}\right) - \left(a - \frac{1}{a}\right) - 7$$

Answer

The given expression can be rewritten as:

$$\Rightarrow 2(a^2 + \frac{1}{a^2} - 2 + 2) - \left(a - \frac{1}{a}\right) - 7$$

$$\Rightarrow 2\left(a^2 + \frac{1}{a^2} - 2(a)\left(\frac{1}{a}\right) + 2\right) - \left(a - \frac{1}{a}\right) - 7$$

As we know that, $x^2 + y^2 - 2xy = (x - y)^2$

$$\Rightarrow 2\left(a - \frac{1}{a}\right)^2 + 4 - \left(a - \frac{1}{a}\right) - 7$$

Let $a + \frac{1}{a} = p$

$\Rightarrow 2p^2 - p - 3$

$\Rightarrow 2p^2 - 3p + 2p - 3$

$\Rightarrow p(2p - 3) + (2p - 3)$

$\Rightarrow (p + 1)(2p - 3)$

On substituting the value of p, we get,

$$\Rightarrow \left(a + \frac{1}{a} + 1\right)\left(2a + \frac{2}{a} - 3\right)$$

1 J. Question

Let us factorise the following algebraic expressions:

$(x^2 - x) y^2 + y - (x^2 + x)$

Answer

The given expression can be rewritten as:

$x(x - 1)y^2 + x^2y - (x^2 - 1)y - x(x+1)$

$\Rightarrow x^2 - xy^2 + x^2y - x^2y + y - x^2 - x$

As we know that $a^2 - b^2 = (a + b)(a - b)$

$\Rightarrow xy[(x - 1)y + x] - (x + 1)[(x - 1)y + x]$

$\Rightarrow (xy - x - 1)(xy + x - y)$

2 A. Question

If $a^2 - b^2 = 11 \times 9$ and a & b are positive integers (a>b) then

A. a = 11, b =9

B. a = 33, b = 3

C. a = 10, b =1

D. a = 100, b =1

Answer

Using the identity $x^2 - y^2 = (x + y)(x - y)$

$\Rightarrow (a + b)(a - b) = 11 \times 9$

Since 11 is a prime number therefore the factors can be equated,

$\Rightarrow a + b = 11$

And $a - b = 9$

On solving above two equations we get,

a = 10 and b = 1

2 B. Question

If $\frac{a}{b} + \frac{b}{a} = 1$, then the value of $a^3 + b^3$ is

A. 1

B. a

C. b

D. 0

Answer

The given equation on LCM reduces to:

$a^2 + b^2 = ab$

Also we know that,

$a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

On substituting the value of $a^2 + b^2$ in above equation, we get,

$\Rightarrow (a + b)(ab - ab)$

$\Rightarrow 0$

2 C. Question

The value of $25^3 - 75^3 + 50^3 + 3 \times 25 \times 75 \times 50$ is

A. 150

B. 0

C. 25

D. 50

Answer

The given expression can be rewritten as:

$25^3 - 75^3 - (25 - 75)^3 - 3(25)(75)(25 - 75)$

As we know that,

$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$\Rightarrow a^3 - b^3 - (a - b)^3 - 3ab(a - b) = 0$

Since we can easily compare the above equation with the given expression by putting a = 25 and b = 75, the value of given expression is 0.

2 D. Question

If a + b + c = 0, then the value of $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$ is

A. 0

B. 1

C. -1

D. 3

Answer

$$\frac{a^2}{bc}+\frac{b^2}{ac}+\frac{c^2}{ab}=\frac{a^3+\ b^3+\ c^3}{abc}$$

We know the identity that, if a + b+ c = 0, then,

$a^3 + b^3 + c^3 = 3abc$

$$\Rightarrow \frac{a^3+\ b^3+\ c^3}{abc}=\frac{3abc}{abc}=3$$

2 E. Question

If $x^2 - px + 12 = (x - 3)(x - a)$ is an identity, then the values of a and p are respectively.

A. a = 4, p = 7

B. a = 7, p = 4

C. a = 4, p = -7

D. a = -4, p = 7

Answer

$(x - 3)(x - a) = x^2 - (3 + a)x + 3a$

$\Rightarrow 3a = 12$

$\Rightarrow a = 4$

Also, p = 3 + a

$\Rightarrow p = 3 + 4 = 7$

3 A. Question

Short answer type questions:

Let us write the simplest value of $\frac{\left(b^2-c^2\right)^3+\left(c^2-a^2\right)^3+\left(a^2-b^2\right)^3}{\left(b-c\right)^3+\left(c-a\right)^3+\left(a-b\right)^3}$

Answer

Let $p = (b^2 - c^2)$, $q = (c^2 - a^2)$ and $r = (a^2 - b^2)$

Since p + q+ r = 0 and we have an identity that if,

$x + y + z = 0$, the $x^3 + y^3 + z^3 = 3xyz$

$\Rightarrow p^3 + q^3 + r^3 = 3pqr$

Similarly in denominator of given fraction, we see that sum of all the individual terms is equal to 0, so same identity can be applied in denominator as well.

The fractions reduces to:

$$\Rightarrow \frac{3(b^2-c^2)(c^2-a^2)(a^2-b^2)}{3(b-c)(c-a)(a-b)}$$

Also as, $x^2 - y^2 = (x - y)(x + y)$

$$\Rightarrow \frac{3(b^2-c^2)(c^2-a^2)(a^2-b^2)}{3(b-c)(c-a)(a-b)} = (a+b)(b+c)(c+a)$$

3 B. Question

Short answer type questions:

Let us write the relation of a, b and c if $a^3 + b^3 + c^3 - 3abc = 0$ and $a + b + c \neq 0$.

Answer

As we know from the identity that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Since $a + b + c \neq 0$, that means,

$a^2 + b^2 + c^2 - ab - bc - ca = 0$

Multiplying both sides by 2, we get,

$a^2 + b^2 - 2ab + b^2 + c^2 - 2bc + a^2 + c^2 - 2ac = 0$

$\Rightarrow (a - b)^2 + (b - c)^2 + (a - c)^2 = 0$

Now this is only possible if:

$a - b = 0$, $b - c = 0$ and $a - c = 0$

$\Rightarrow a = b = c$

3 C. Question

Short answer type questions:

If $a^2 - b^2 = 224$ and a and b are negative integers ($a < b$), then let us write the values of a and b.

Answer

224 can be written as 16×14

$224 = (-16) \times (-14)$

Also using the identity, $x^2 - y^2 = (x + y)(x - y)$

$\Rightarrow a^2 - b^2 = (a + b)(a - b) = (-16) \times (-14)$

On equate the factors, as

$a + b = -16$ and $(a - b) = -14$ and solving these two equations, we get,

$a = -15$ and $b = -1$

3 D. Question

Short answer type questions:

Let us write the value of $(x - a)^3 + (x - b)^3 + (x - c)^3 - 3(x - a)(x - b)(x - c)$ if $3x = a + b + c$.

Answer

As we know the identity that,

If $l + m + n = 0$, then $l^3 + m^3 + n^3 - 3mnl = 0$

Let $p = (x - a)$, $q = (x - b)$, $r = (x - c)$

$p + q + r = (x - a + x - b + x - c) = 3x - a - b - c$

also It is given $3x = a + b + c$,

$\Rightarrow p + q + r = 0$

$\Rightarrow p^3 + q^3 + r^3 - 3pqr = 0$

On substituting the values of p, q and r in above equation, we get,

$(x - a)^3 + (x - b)^3 + (x - c)^3 - 3(x - a)(x - b)(x - c) = 0$

$\therefore$ the required value is 0.

3 E. Question

Short answer type questions:

Let us write the values of a and p if $2x^2 + px + 6 = (2x - a)(x - 2)$ is an identity.

Answer

$(2x - a)(x - 2) = 2x^2 - 4x - ax + 2a$

$\Rightarrow 2x^2 - x(a + 4) + 2a$

Since the above expression is identical to $2x^2 + px + 6$, therefore their coefficients can be equated.

$\Rightarrow 2a = 6$

$\Rightarrow a = 3$

Also, $p = -(a + 4) = -7$

$\therefore$ The values of a and p are 3 and -7 respectively.