16
Simple Harmonic Motion

Periodic and Oscillatory (Vibratory) Motion

(1) A motion, which repeat itself over and over again after a regular interval of time is called a periodic motion

Revolution of earth around the sun (period one year), Rotation of earth about its polar axis (period one day), Motion of hour's hand of a clock (period 12-hour) *etc* are common example of periodic motion.

(2) Oscillatory or vibratory motion is that motion in which a body moves to and fro or back and forth repeatedly about a fixed point in a definite interval of time. In such a motion, the body is confined with in well-defined limits on either side of mean position. Oscillatory motion is also called as harmonic motion.

(i) Common examples are

(a) The motion of the pendulum of a wall clock

(b) The motion of a load attached to a spring, when it is pulled and then released.

(c) The motion of liquid contained in U-tube when it is compressed once in one limb and left to itself.

(d) A loaded piece of wood floating over the surface of a liquid when pressed down and then released executes oscillatory motion.

(ii) Harmonic oscillation is that oscillation which can be expressed in terms of single harmonic function (*i.e.* sine or cosine function). *Example* : $y = a\sin\omega t$ or $y = a\cos\omega t$

(iii) Non-harmonic oscillation is that oscillation which can not be expressed in terms of single harmonic function. It is a combination of two or more than two harmonic oscillations. *Example* : $y = a\sin\omega t + b\sin 2\omega t$.

Simple Harmonic Motion

(1) Simple harmonic motion is a special type of periodic motion, in which a particle moves to and fro repeatedly about a mean position.

(2) In linear S.H.M. a restoring force which is always directed towards the mean position and whose magnitude at any instant is directly proportional to the displacement of the particle from the mean position at that instant *i.e.* Restoring force $\propto$ Displacement of the particle from mean position.

$$F \propto -x \Rightarrow F = -kx$$

Where k is known as force constant. Its S.I. unit is Newton/meter and dimension is $[MT^{-2}]$.

(3) In stead of straight line motion, if particle or centre of mass of body is oscillating on a small arc of circular path, then for angular S.H.M.

Restoring torque $(\tau) \propto -$ Angular displacement (θ)

Some Important Definitions

(1) **Time period** (T) **:** It is the least interval of time after which the periodic motion of a body repeats itself.

S.I. unit of time period is second.

(2) **Frequency** (n) **:** It is defined as the number of oscillations executed by body per second. S.I unit of frequency is hertz (Hz).

(3) **Angular Frequency** (ω) **:** Angular frequency of a body executing periodic motion is equal to product of frequency of the body with factor 2π. Angular frequency $\omega = 2\pi n$

Its unit is *rad/sec.*

(4) **Phase** (ϕ) **:** Phase of a vibrating particle at any instant is a physical quantity, which completely express the position and direction of motion, of the particle at that instant with respect to its mean position.

In oscillatory motion the phase of a vibrating particle is the argument of *sine* or *cosine* function involved to represent the generalised equation of motion of the vibrating particle.

$$y = a\sin\theta = a\sin(\omega t + \phi_0)$$

here, $\theta = \omega t + \phi_0$ = phase of vibrating particle.

ϕ = Initial phase or epoch. It is the phase of a vibrating particle at t = 0.

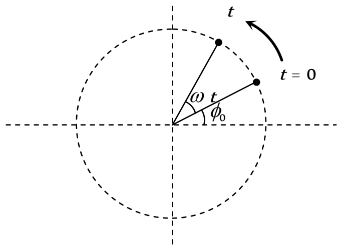


Fig. 16.2

(1) **Same phase :** Two vibrating particle are said to be in same phase, if the phase difference between them is an even multiple of π or path difference is an even multiple of $(\lambda / 2)$ or time interval is an even multiple of $(T/2)$ because 1 time period is equivalent to 2π *rad* or 1 wave length (λ).

(2) **Opposite phase :** When the two vibrating particles cross their respective mean positions at the same time moving in opposite directions, then the phase difference between the two vibrating particles is 180° .

Opposite phase means the phase difference between the particle is an odd multiple of π (say π, 3π, 5π, 7π.....) or the path difference is an odd multiple of λ (say $\frac{\lambda}{2}, \frac{3\lambda}{2}$,........) or the time interval is an odd multiple of $(T/2)$.

(3) **Phase difference :** If two particles performs S.H.M and their equation are

$$y_1 = a\sin(\omega t + \phi_1) \text{ and } y_2 = a\sin(\omega t + \phi_2)$$

then phase difference $\Delta\phi = (\omega t + \phi_2) - (\omega t + \phi_1) = \phi_2 - \phi_1$

Displacement in S.H.M.

(1) The displacement of a particle executing S.H.M. at an instant is defined as the distance of particle from the mean position at that instant.

(2) Simple harmonic motion is also defined as the projection of uniform circular motion on any diameter of circle of reference.

(3) If the projection is taken on *y*-axis. then from the figure

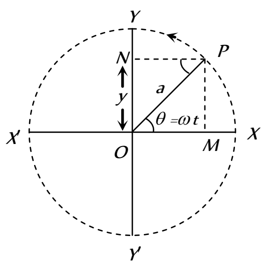


Fig. 16.3

$$y = a\sin\omega t = a\sin\frac{2\pi}{T}t = a\sin 2\pi n t = a\sin(\omega t \pm \phi)$$

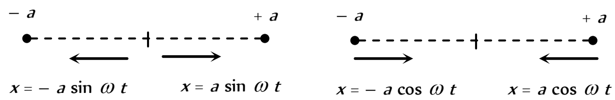
(i) $y = a\sin\omega t$ when the time is noted from the instant when the vibrating particle is at mean position.

(ii) $y = a\cos\omega t$ when the time is noted from the instant when the vibrating particle is at extreme position.

(iii) $y = a\sin(\omega t \pm \phi)$ when the vibrating particle is ϕ phase leading or lagging from the mean position.

(4) If the projection of P is taken on *X*-axis then equations of S.H.M. can be given as

$$x = a\cos(\omega t \pm \phi) = a\cos\left(\frac{2\pi}{T}t \pm \phi\right) = a\cos(2\pi n t \pm \phi)$$



(A) (B)

Fig. 16.4

(5) Direction of displacement is always away from the equilibrium position, particle either is moving away from or is coming towards the equilibrium position.

Velocity in S.H.M.

(1) Velocity of the particle executing S.H.M. at any instant, is defined as the time rate of change of its displacement at that instant.

(2) In case of S.H.M. when motion is considered from the equilibrium position, displacement $y = a\sin\omega t$

So $v = \frac{dy}{dt} = a\omega\cos\omega t = a\omega\sqrt{1 - \sin^2\omega t} = \omega\sqrt{a^2 - y^2}$

[As $\sin\omega t = y/a$]

(3) At mean position or equilibrium position (y = 0 and $\theta = \omega t$ = 0), velocity of particle is maximum and it is $v_{max} = a\omega$.

(4) At extreme position ($y = \pm a$ and $\theta = \omega t = \pi/2$), velocity of oscillating particle is zero *i.e.* v = 0.

(5) From $v = \omega\sqrt{a^2 - y^2} \Rightarrow v^2 = \omega^2(a^2 - y^2) \Rightarrow \frac{v^2}{\omega^2} = a^2 - y^2$

$$\Rightarrow \frac{v^2}{a^2\omega^2} + \frac{y^2}{a^2} = 1$$

This is the equation of ellipse. Hence the graph between v and y is an ellipse.

For ω = 1, graph between v and y is a circle.

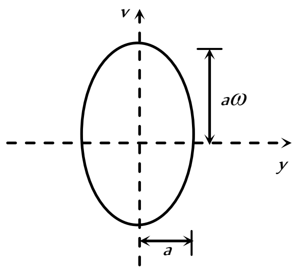


Fig. 16.5

(6) Direction of velocity is either towards or away from mean position depending on the position of particle.

Acceleration in S.H.M.

(1) The acceleration of the particle executing S.H.M. at any instant, is defined as the rate of change of its velocity at that instant. So acceleration

$$A = \frac{dv}{dt} = \frac{d}{dt}(a\omega\cos\omega t) = -\omega^2 a\sin\omega t = -\omega^2 y$$

[As $y = a\sin\omega t$]

(2) In S.H.M. as $|\text{Acceleration}| = \omega^2 y$ is not constant. So equations of translatory motion can not be applied.

(3) In S.H.M. acceleration is maximum at extreme position (at $y = \pm a$). Hence $|A_{max}| = \omega^2 a$ when $|\sin\omega t| = \text{maximum} = 1$ *i.e.* at $t = \frac{T}{4}$ or $\omega t = \frac{\pi}{2}$. From equation (ii) $|A_{max}| = \omega^2 a$ when $y = a$.

(i) In S.H.M. acceleration is minimum at mean position

From equation (i) $A_{min} = 0$ when $\sin\omega t = 0$ *i.e.* at $t = 0$ or $t = \frac{T}{2}$ or $\omega t = \pi$. From equation (ii) $A_{min} = 0$ when $y = 0$

(ii) Acceleration is always directed towards the mean position and so is always opposite to displacement

i.e., $A \propto -y$

Graph between acceleration (A) and displacement (y) is a straight line as shown

Slope of the line $= -\omega$

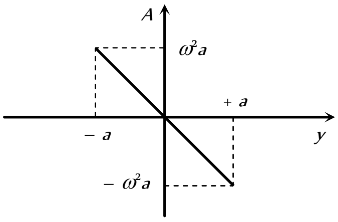


Fig. 16.6

Comparative Study of Displacement Velocity and Acceleration

(1) All the three quantities displacement, velocity and acceleration show harmonic variation with time having same period.

(2) The velocity amplitude is ω times the displacement amplitude

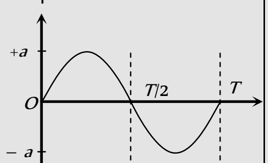
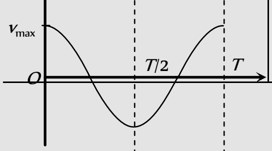
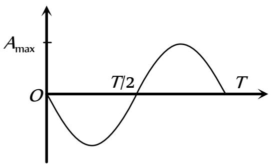
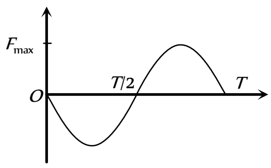
(3) The acceleration amplitude is ω^2 times the displacement amplitude

(4) In S.H.M. the velocity is ahead of displacement by a phase angle $\pi / 2$

(5) In S.H.M. the acceleration is ahead of velocity by a phase angle $\pi / 2$

(6) The acceleration is ahead of displacement by a phase angle of π

Table 16.1 : Various physical quantities in S.H.M. at different position :

Graph	Formula	At mean position	At extreme position
Displacement 	$y = a\sin\omega t$	$y = 0$	$y = \pm a$
Velocity 	$v = a\omega\cos\omega t$ $= a\omega\sin(\omega t + \frac{\pi}{2})$ or $v = \omega\sqrt{a^2 - y^2}$	$v_{max} = a\omega$	$v_{min} = 0$
Acceleration 	$A = -a\omega^2\sin\omega t$ $= a\omega^2\sin(\omega t + \pi)$ or $\|A\| = \omega^2 y$	$A_{min} = 0$	$\|A_{max}\| = \omega^2 a$
Force 	$F = -m\omega^2 a\sin\omega t$ or $F = m\omega^2 y$	$F_{min} = 0$	$F_{max} = m\omega^2 a$

Energy in S.H.M.

(1) **Potential energy :** This is an account of the displacement of the particle from its mean position.

(i) The restoring force $F = -ky$ against which work has to be done. Hence potential energy U is given by

$$U = \int dU = -\int dW = -\int_0^x F dx = \int_0^y ky\, dy = \frac{1}{2}ky^2 + U_0$$

where U_0 = Potential energy at equilibrium position.

If $U_0 = 0$ then $U = \frac{1}{2}m\omega^2 y^2$ [As $\omega^2 = k/m$]

(ii) Also $U = \frac{1}{2}m\omega^2 a^2\sin^2\omega t = \frac{1}{4}m\omega^2 a^2(1 - \cos 2\omega t)$

[As $y = a\sin\omega t$]

Hence potential energy varies periodically with double the frequency of S.H.M.

(iii) Potential energy maximum and equal to total energy at extreme positions

$$U_{max} = \frac{1}{2}ka^2 = \frac{1}{2}m\omega^2 a^2 \text{ when } y = \pm a;\ \omega t = \pi/2;\ t = \frac{T}{4}$$

(iv) Potential energy is minimum at mean position

$U_{min} = 0$ when $y = 0$; $\omega t = 0$; $t = 0$

(2) **Kinetic energy :** This is because of the velocity of the particle

Kinetic Energy $K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(a^2 - y^2)$

[As $v = \omega\sqrt{a^2 - y^2}$]

(i) Also $K = \frac{1}{2}m\omega^2 a^2\cos^2\omega t = \frac{1}{4}m\omega^2 a^2(1 + \cos 2\omega t)$

[As $v = a\omega\cos\omega t$]

Hence kinetic energy varies periodically with double the frequency of S.H.M.

(ii) Kinetic energy is maximum at mean position and equal to total energy at mean position.

$$K_{max} = \frac{1}{2} m\omega^2 a^2 \text{ when } y = 0\,;\; t = 0\,;\; \omega t = 0$$

(iii) Kinetic energy is minimum at extreme position.

$$K_{min} = 0 \qquad \text{when } y = a\,;\; t = T/4\,,\; \omega t = \pi/2$$

(3) **Total mechanical energy :** Total mechanical energy always remains constant and it is equal to sum of potential energy and kinetic energy *i.e.* $E = U + K$

$$E = \frac{1}{2} m\omega^2 (a^2 - y^2) + \frac{1}{2} m\omega^2 y^2 = \frac{1}{2} m\omega^2 a^2$$

Total energy is not a position function.

(4) **Energy position graph**

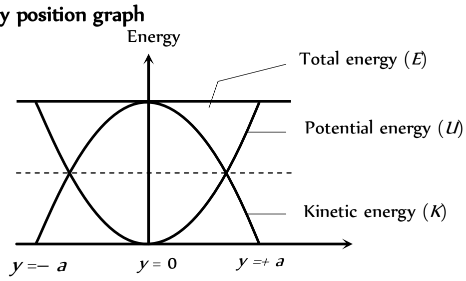


Fig. 16.7

(i) At $y = 0$; $U = 0$ and $K = E$

(ii) At $y = \pm a$; $U = E$ and $K = 0$

(iii) At $y = \pm \frac{a}{2}$; $U = \frac{E}{4}$ and $K = \frac{3E}{4}$

(iv) At $y = \pm \frac{a}{\sqrt{2}}$; $U = K = \frac{E}{2}$

Average Value of P.E. and K.E.

The average value of potential energy for complete cycle is given by

$$U_{average} = \frac{1}{T}\int_0^T U\,dt = \frac{1}{T}\int_0^T \frac{1}{2} m\,\omega^2 a^2 \sin^2(\omega t + \phi) = \frac{1}{4} m\omega^2 a^2$$

The average value of kinetic energy for complete cycle

$$K_{average} = \frac{1}{T}\int_0^T K\,dt = \frac{1}{T}\int_0^T \frac{1}{2} m\,\omega^2 a^2 \cos^2 \omega t\,dt = \frac{1}{4} m\omega^2 a^2$$

Thus average values of kinetic energy and potential energy of harmonic oscillator are equal and each equal to half of the total energy $K_{average} = U_{average} = \frac{1}{2}E = \frac{1}{4} m\omega^2 a^2$.

Differential Equation of S.H.M.

For S.H.M. (linear) Acceleration ∝ – (Displacement)

$$A \propto -y \text{ or } A = -\omega^2 y \text{ or } \frac{d^2y}{dt^2} = -\omega^2 y$$

or $m\frac{d^2y}{dt^2} + ky = 0$ [As $\omega = \sqrt{\frac{k}{m}}$]

For angular S.H.M. $\tau = -c\theta$ and $\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$

where $\omega^2 = \frac{c}{I}$ [As c = Restoring torque constant and I = Moment of inertia]

How to Find Frequency and Time Period of S.H.M.

Step 1 : When particle is in its equilibrium position, balance all forces acting on it and locate the equilibrium position mathematically.

Step 2 : From the equilibrium position, displace the particle slightly by a displacement y and find the expression of net restoring force on it.

Step 3 : Try to express the net restoring force acting on particle as a proportional function of its displacement from mean position. The final expression should be obtained in the form.

$$F = -ky$$

Here we put – *ve* sign as direction of F is opposite to the displacement y. If a be the acceleration of particle at this displacement, we have $a = -\left(\frac{k}{m}\right)y$

Step 4 : Comparing this equation with the basic differential equation of S.H.M. we get $\omega^2 = \frac{k}{m} \Rightarrow \omega = \sqrt{\frac{k}{m}}$ or $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

As ω is the angular frequency of the particle in S.H.M., its time period of oscillation can be given as $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

(i) In different types of S.H.M. the quantities m and k will go on taking different forms and names. In general m is called inertia factor and k is called spring factor.

Thus $T = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}}$ or $n = \frac{1}{2\pi}\sqrt{\frac{\text{Spring factor}}{\text{Inertia factor}}}$

(ii) In linear S.H.M. the spring factor stands for force per unit displacement and inertia factor for mass of the body executing S.H.M. and in Angular S.H.M. k stands for restoring torque per unit angular displacement and inertial factor for moment of inertia of the body executing S.H.M.

For linear S.H.M.

$$T = 2\pi\sqrt{\frac{m}{k}} = \sqrt{\frac{m}{\text{Force/Displacement}}} = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$

Simple Pendulum

(1) An ideal simple pendulum consists of a heavy point mass body (bob) suspended by a weightless, inextensible and perfectly flexible string from a rigid support about which it is free to oscillate.

(2) But in reality neither point mass nor weightless string exist, so we can never construct a simple pendulum strictly according to the definition.

(3) Suppose simple pendulum of length l is displaced through a small angle θ from it's mean (vertical) position. Consider mass of the bob is m and linear displacement from mean position is x

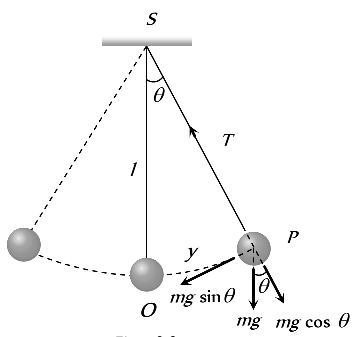


Fig. 16.8

Restoring force acting on the bob

$$F = -mg\sin\theta \quad \text{or} \quad F = -mg\theta = -mg\frac{x}{l}$$

(When θ is small $\sin\theta \simeq \theta = \dfrac{\text{Arc}}{\text{Length}} = \dfrac{OP}{l} = \dfrac{x}{l}$)

$\therefore \quad \dfrac{F}{x} = \dfrac{-mg}{l} = k$ (Spring factor)

So $T = 2\pi\sqrt{\dfrac{\text{Inertia factor}}{\text{Spring factor}}} = 2\pi\sqrt{\dfrac{m}{mg/l}} = 2\pi\sqrt{\dfrac{l}{g}}$

Factor Affecting Time Period of Simple Pendulum

(1) **Amplitude :** The period of simple pendulum is independent of amplitude as long as its motion is simple harmonic. But if θ is not small, $\sin\theta \neq \theta$ then motion will not remain simple harmonic but will become oscillatory. In this situation if θ is the amplitude of motion. Time period

$$T = 2\pi\sqrt{\frac{l}{g}}\left[1 + \frac{1}{2^2}\sin^2\left(\frac{\theta_0}{2}\right) + ...\right] \approx T_0\left[1 + \frac{\theta_0{}^2}{16}\right]$$

(2) **Mass of the bob :** Time period of simple pendulum is also independent of mass of the bob. This is why

(i) If the solid bob is replaced by a hollow sphere of same radius but different mass, time period remains unchanged.

(ii) If a girl is swinging in a swing and another sits with her, the time period remains unchanged.

(3) **Length of the pendulum :** Time period $T \propto \sqrt{l}$ where l is the distance between point of suspension and center of mass of bob and is called effective length.

(i) When a sitting girl on a swinging swing stands up, her center of mass will go up and so l and hence T will decrease.

(ii) If a hole is made at the bottom of a hollow sphere full of water and water comes out slowly through the hole and time period is recorded till the sphere is empty, initially and finally the center of mass will be at the center of the sphere. However, as water drains off the sphere, the center of mass of the system will first move down and then will come up. Due to this l and hence T first increase, reaches a maximum and then decreases till it becomes equal to its initial value.

(iii) Different graphs

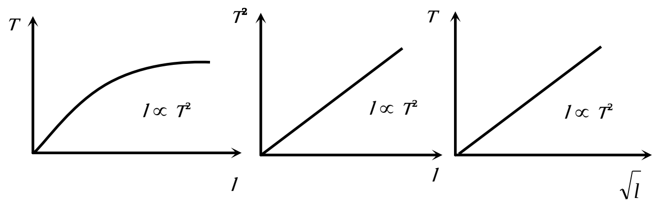


Fig. 16.9

(4) **Effect of g :** $T \propto \dfrac{1}{\sqrt{g}}$ *i.e.* as g increase T decreases.

(i) As we go high above the earth surface or we go deep inside the mines the value of g decrease, hence time period of pendulum (T) increases.

(ii) If a clock, based on simple pendulum is taken to hill (or on any other planet), g will decrease so T will increases and clock will become slower.

(iii) Different graphs

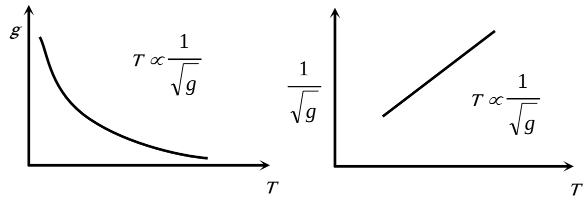


Fig. 16.10

(5) **Effect of temperature on time period :** If the bob of simple pendulum is suspended by a wire then effective length of pendulum will increase with the rise of temperature due to which the time period will increase.

$l = l_0(1 + \alpha\,\Delta\theta)$ (If $\Delta\theta$ is the rise in temperature, l_0 = initial length of wire, l = final length of wire)

$$\frac{T}{T_0} = \sqrt{\frac{l}{l_0}} = (1 + \alpha\,\Delta\theta)^{1/2} \approx 1 + \frac{1}{2}\alpha\,\Delta\theta$$

So $\dfrac{T}{T_0} - 1 = \dfrac{1}{2}\alpha\,\Delta\theta$ *i.e.* $\dfrac{\Delta T}{T} \approx \dfrac{1}{2}\alpha\,\Delta\theta$

Oscillation of Pendulum in Different Situations

(1) **Oscillation in liquid :** If bob a simple pendulum of density ρ is made to oscillate in some fluid of density σ (where $\sigma < \rho$) then time period of simple pendulum gets increased.

As thrust will oppose its weight hence $mg_{eff.} = mg -$ Thrust

or $g_{eff.} = g - \dfrac{V\sigma g}{V\rho}$ *i.e.* $g_{eff.} = g\left[1 - \dfrac{\sigma}{\rho}\right]$

$\Rightarrow \dfrac{g_{eff.}}{g} = \dfrac{\rho - \sigma}{\rho}$

$\Rightarrow \dfrac{T'}{T} = \sqrt{\dfrac{g}{g_{eff.}}} = \sqrt{\dfrac{\rho}{\rho - \sigma}} > 1$

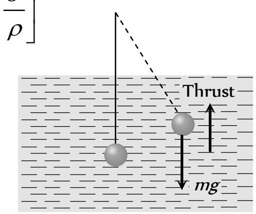


Fig. 16.11

(2) **Oscillation under the influence of electric field :** If a bob of mass m carries a positive charge q and pendulum is placed in a uniform electric field of strength E

(i) If electric field directed vertically upwards.

Effective acceleration

$$g_{eff.} = g - \frac{qE}{m}$$

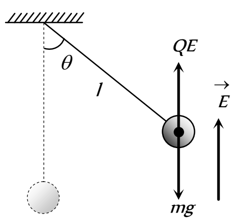


Fig. 16.12

So $T = 2\pi\sqrt{\dfrac{l}{g - \dfrac{qE}{m}}}$

(ii) If electric field is vertically downward then

$g_{eff.} = g + \dfrac{qE}{m}$

$T = 2\pi\sqrt{\dfrac{l}{g + \dfrac{qE}{m}}}$

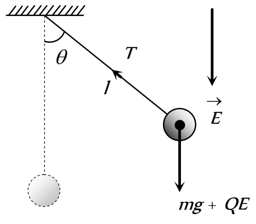


Fig. 16.13

(3) **Pendulum in a lift :** If the pendulum is suspended from the ceiling of the lift.

(i) If the lift is at rest or moving down ward /up ward with constant velocity.

$T = 2\pi\sqrt{\dfrac{l}{g}}$

and $n = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{l}}$

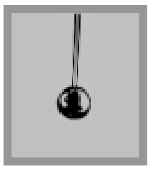


Fig. 16.14

(ii) If the lift is moving up ward with constant acceleration *a*

$T = 2\pi\sqrt{\dfrac{l}{g+a}}$

and $n = \dfrac{1}{2\pi}\sqrt{\dfrac{g+a}{l}}$

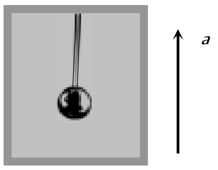


Fig. 16.15

Time period decreases and frequency increases

(iii) If the lift is moving down ward with constant acceleration *a*

$T = 2\pi\sqrt{\dfrac{l}{g-a}}$

and $n = \dfrac{1}{2\pi}\sqrt{\dfrac{g-a}{l}}$

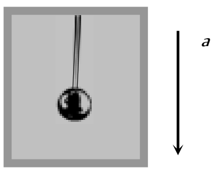


Fig. 16.16

Time period increase and frequency decreases

(iv) If the lift is moving down ward with acceleration $a = g$

$T = 2\pi\sqrt{\dfrac{l}{g-g}} = \infty$

and $n = \dfrac{1}{2\pi}\sqrt{\dfrac{g-g}{l}} = 0$

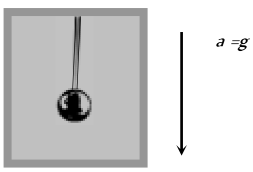


Fig. 16.17

It means there will be no oscillation in a pendulum.

Similar is the case in a satellite and at the centre of earth where effective acceleration becomes zero and pendulum will stop.

(4) **Pendulum in an accelerated vehicle :** The time period of simple pendulum whose point of suspension moving horizontally with acceleration *a*

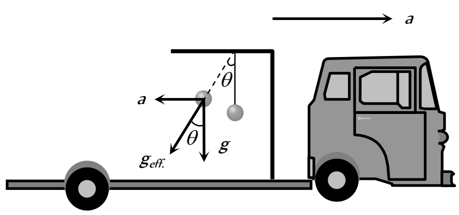


Fig. 16.18

In this case effective acceleration $g_{eff.} = \sqrt{g^2 + a^2}$

$$T = 2\pi\sqrt{\frac{l}{(g^2 + a^2)^{1/2}}} \quad \text{and} \quad \theta = \tan^{-1}(a/g)$$

If simple pendulum suspended in a car that is moving with constant speed *v* around a circle of radius *r*.

$$T = 2\pi\frac{\sqrt{l}}{\sqrt{g^2 + \left(\dfrac{v^2}{r}\right)^2}}$$

Some Other Types of Pendulum

(1) **Infinite length pendulum :** If the length of the pendulum is comparable to the radius of earth then

$$T = 2\pi\sqrt{\frac{1}{g\left[\dfrac{1}{l} + \dfrac{1}{R}\right]}}$$

(i) If $l << R$, then $\dfrac{1}{l} >> \dfrac{1}{R}$ so $T = 2\pi\sqrt{\dfrac{l}{g}}$

(ii) If $l >> R (\to \infty)$ then $\dfrac{1}{l} < \dfrac{1}{R}$

so $T = 2\pi\sqrt{\dfrac{R}{g}} = 2\pi\sqrt{\dfrac{6.4\times10^6}{10}} \cong 84.6$ *minutes*

and it is the maximum time period which an oscillating simple pendulum can have

(iii) If $l = R$ so $T = 2\pi\sqrt{\dfrac{R}{2g}} \cong 1 hour$

(2) **Second's Pendulum :** It is that simple pendulum whose time period of vibrations is two seconds.

Putting T = 2 *sec* and $g = 9.8 m/\sec^2$ in $T = 2\pi\sqrt{\dfrac{l}{g}}$ we get

$l = \dfrac{4\times 9.8}{4\pi^2} = 0.993$ *m* = 99.3 *cm*

Hence length of second's pendulum is 99.3 *cm* or nearly 1 meter on earth surface.

For the moon the length of the second's pendulum will be 1/6 meter

[As $g_{moon} = \dfrac{g_{\text{Earth}}}{6}$]

(3) **Compound pendulum :** Any rigid body suspended from a fixed support constitutes a physical pendulum. Consider the situation when the body is displaced through a small angle θ. Torque on the body about *O* is given by

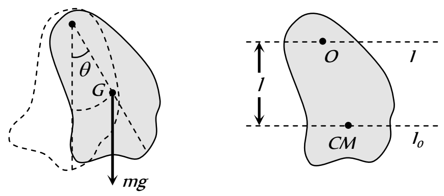


Fig. 16.19

$$\tau = mgl \sin\theta \quad ...(i)$$

where l = distance between point of suspension and centre of mass of the body.

If I be the M.I. of the body about O. Then $\tau = I\alpha$ ...(ii)

From (i) and (ii), we get $I\frac{d^2\theta}{dt^2} = -mgl\sin\theta$ as θ and $\frac{d^2\theta}{dt^2}$ are oppositely directed $\Rightarrow \frac{d^2\theta}{dt^2} = -\frac{mgl}{I}\theta$ since θ is very small

Comparing with the equation $\frac{d^2\theta}{dt^2} = -\omega^2\theta$, we get

$$\omega = \sqrt{\frac{mgl}{I}} \Rightarrow T = 2\pi\sqrt{\frac{I}{mgl}}$$

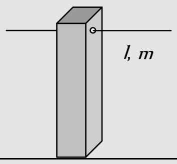
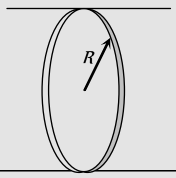
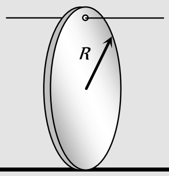
Also $I = I_{cm} + ml^2$ (Parallel axis theorem)

$= mk^2 + ml^2$ (where k = radius of gyration)

$$\therefore\ T = 2\pi\sqrt{\frac{mK^2 + ml^2}{mgl}} = 2\pi\sqrt{\frac{\frac{K^2}{l} + l}{g}} = 2\pi\sqrt{\frac{l_{\text{eff}}}{g}}$$

l_e = Effective length of pendulum = Distance between point of suspension and centre of mass.

Table 16. 2: Some common physical pendulum

Body	Time period
Bar 	$T = 2\pi\sqrt{\frac{2l}{3g}}$
Ring 	$T = 2\pi\sqrt{\frac{2R}{g}}$
Disc 	$T = 2\pi\sqrt{\frac{3R}{2g}}$

Spring System

When a spring is stretched or compressed from its normal position (x = 0) by a small distance x, then a restoring force is produced in the spring because it obeys Hook's law

i.e. $F \propto -x \Rightarrow F = -k\,x$

where k is called spring constant.

(i) It's S.I. unit *Newton/metre*, C.G.S unit *Dyne/cm* and dimension is $[MT^{-2}]$

(ii) Actually k is a measure of the stiffness/softness of the spring.

(iii) For massless spring constant restoring elastic force is same every where

(iv) When a spring compressed or stretched then work done is stored in the form of elastic potential energy in it.

(v) Spring constant depend upon radius and length of the wire used in spring.

(vi) The spring constant k is inversely proportional to the spring length.

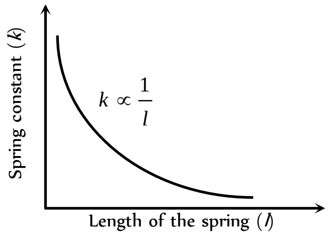


Fig. 16.20

$$k \propto \frac{1}{\text{Extension}} \propto \frac{1}{\text{Length of spring}}$$

That means if the length of spring is halved then its force constant becomes double.

(vii) When a spring of length l is cut in two pieces of length l_1 and l_2 such that $l_1 = nl_2$.

If the constant of a spring is k then spring constant of first part

$$k_1 = \frac{k(n+1)}{n}$$

Spring constant of second part $k_2 = (n+1)k$

and ratio of spring constant $\frac{k_1}{k_2} = \frac{1}{n}$

Spring Pendulum

A point mass suspended from a mass less spring or placed on a frictionless horizontal plane attached with spring (fig.) constitutes a linear harmonic spring pendulum

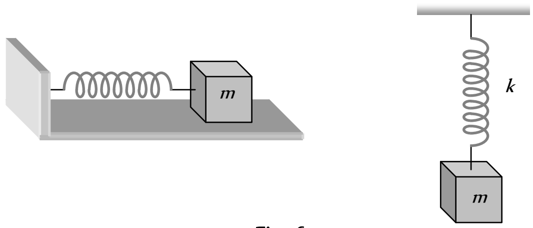


Fig. 16.21

Time period $T = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}} = 2\pi\sqrt{\frac{m}{k}}$

and Frequency $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

(1) Time period of a spring pendulum depends on the mass suspended

$\Rightarrow T \propto \sqrt{m}$ or $n \propto \frac{1}{\sqrt{m}}$ *i.e.* greater the mass greater will be the inertia and so lesser will be the frequency of oscillation and greater will be the time period.

(2) The time period depends on the force constant k of the spring *i.e.*

$T \propto \frac{1}{\sqrt{k}}$ or $n \propto \sqrt{k}$

(3) Time of a spring pendulum is independent of acceleration due to gravity. That is why a clock based on spring pendulum will keep proper time every where on a hill or moon or in a satellite and time period of a spring pendulum will not change inside a liquid if damping effects are neglected.

(4) **Massive spring :** If the spring has a mass M and mass m is suspended from it, effective mass is given by $m_{eff} = m + \frac{M}{3}$. Hence

$$T = 2\pi\sqrt{\frac{m_{eff}}{k}}$$

(5) **Reduced mass :** If two masses of mass m_1 and m_2 are connected by a spring and made to oscillate on horizontal surface, the reduced mass m_r is given by $\frac{1}{m_r} = \frac{1}{m_1} + \frac{1}{m_2}$ so that

$$T = 2\pi\sqrt{\frac{m_r}{k}}$$

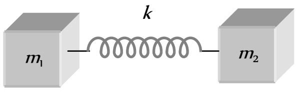


Fig. 16.22

(6) If a spring pendulum, oscillating in a vertical plane is made to oscillate on a horizontal surface, (or on inclined plane) time period will remain unchanged.

(7) Equilibrium position for a spring in a horizontal plain is the position of natural length of spring as weight is balanced by reaction. While in case of vertical motion equilibrium position will be $l + y_0$ with $ky_0 = mg$

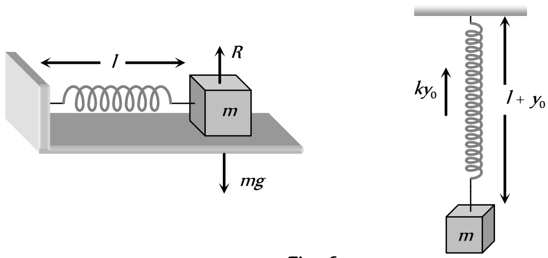


Fig. 16.23

If the stretch in a vertically loaded spring is y_0 then for equilibrium of mass m, $ky_0 = mg$ *i.e.* $\frac{m}{k} = \frac{y_0}{g}$

So that $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{y_0}{g}}$

Time period does not depends on 'g' because along with g, y_0 will also change in such a way that $\frac{y_0}{g} = \frac{m}{k}$ remains constant

Oscillation of Spring Combination

(1) **Series combination :** If two springs of spring constants K_1 and K_2 are joined in series as shown then

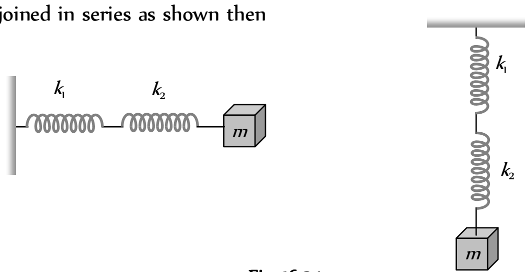


Fig. 16.24

(i) In series combination equal forces acts on spring but extension in springs are different.

(ii) Spring constants of combination

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2} \Rightarrow k_s = \frac{k_1 k_2}{k_1 + k_2}$$

(iii) If n springs of different force constant are connected in series having force constant k_1, k_2, k_3....... respectively then

$$\frac{1}{k_S} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + ...$$

If all spring have same spring constant then $k_S = \frac{k}{n}$

(iv) Time period of combination $T = 2\pi\sqrt{\frac{m}{k_S}} = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$

(2) **Parallel combination :** If the springs are connected in parallel as shown

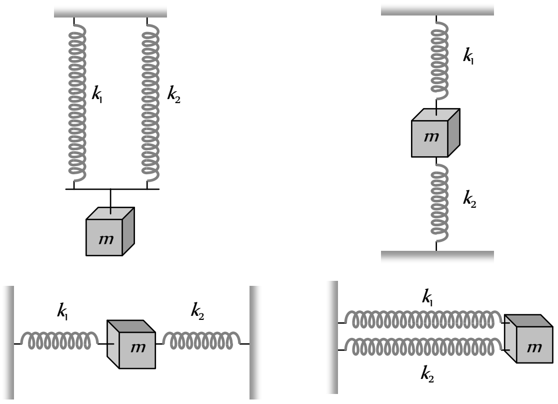


Fig. 16.25

(i) In parallel combination different forces acts on different springs but extension in springs are same

(ii) Spring constants of combination $k_P = k_1 + k_2$

(iii) If n springs of different force constant are connected in parallel having force constant k_1, k_2, k_3....... respectively then $k_P = k_1 + k_2 + k_3 + ...$

If all spring have same spring constant then $k_P = nk$

(iv) Time period of combination $T_P = 2\pi\sqrt{\frac{m}{k_P}} = 2\pi\sqrt{\frac{m}{(k_1 + k_2)}}$

Various Formulae of S.H.M.

(1) **S.H.M. of a liquid in U tube :** If a liquid of density ρ contained in a vertical U tube performs S.H.M. in its two limbs. Then time period

$$T = 2\pi\sqrt{\frac{L}{2g}} = 2\pi\sqrt{\frac{h}{g}}$$

where L = Total length of liquid column,

h = Height of undisturbed liquid in each limb ($L=2h$)

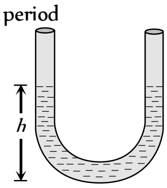


Fig. 16.26

(2) **S.H.M. of a floating cylinder :** If l is the length of cylinder dipping in liquid then

Time period $T = 2\pi\sqrt{\frac{l}{g}}$

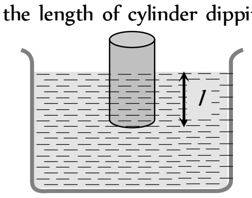


Fig. 16.27

(3) **S.H.M. of a small ball rolling down in hemi-spherical bowl**

$$T = 2\pi\sqrt{\frac{R - r}{g}}$$

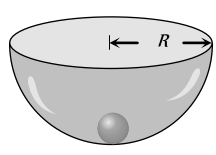


Fig. 16.28

R = Radius of the bowl

r =Radius of the ball

(4) **S.H.M. of a piston in a cylinder**

$$T = 2\pi\sqrt{\frac{Mh}{PA}}$$

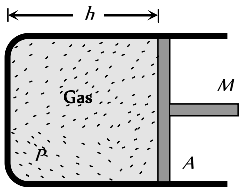


Fig. 16.29

M = mass of the piston

A = area of cross section

h = height of cylinder

P = pressure in a cylinder

(5) **S.H.M. of a body in a tunnel dug along any chord of earth**

$$T = 2\pi\sqrt{\frac{R}{g}} = 84.6 \text{ minutes}$$

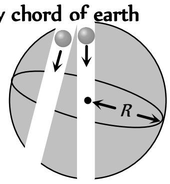


Fig. 16.30

(6) **Torsional pendulum :** In a torsional pendulum an object is suspended from a wire. If such a wire is twisted, due to elasticity it exert a restoring toque $\tau = C\theta$.

In this case time period is given by

$$T = 2\pi\sqrt{\frac{I}{C}}$$

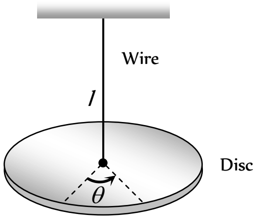


Fig. 16.31

where I = Moment of inertia a disc

C = Torsional constant of wire = $\dfrac{\pi\eta r^4}{2l}$

η = Modulus of elasticity of wire and r = Radius of wire

(7) **Longitudinal oscillations of an elastic wire :** Wire/string pulled a distance Δl and left. It executes longitudinal oscillations. Restoring force

$$F = -AY\left(\frac{\Delta l}{l}\right)$$

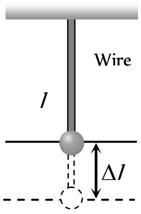


Fig. 16.32

Y = Young's modulus

A = Area of cross-section

Hence $T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{ml}{AY}}$

Free, Damped, Forced and Maintained Oscillations

(1) **Free oscillation**

(i) The oscillation of a particle with fundamental frequency under the influence of restoring force are defined as free oscillations

(ii) The amplitude, frequency and energy of oscillation remains constant

(iii) Frequency of free oscillation is called natural frequency because it depends upon the nature and structure of the body.

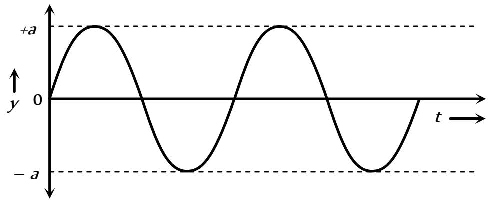


Fig. 16.33

(2) **Damped oscillation**

(i) The oscillation of a body whose amplitude goes on decreasing with time are defined as damped oscillation

(ii) In these oscillation the amplitude of oscillation decreases exponentially due to damping forces like frictional force, viscous force, hystersis *etc*.

(iii) Due to decrease in amplitude the energy of the oscillator also goes on decreasing exponentially

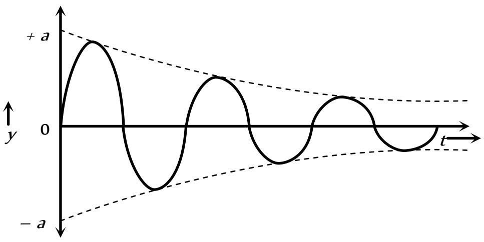


Fig. 16.34

(iv) The force produces a resistance to the oscillation is called damping force.

If the velocity of oscillator is v then

Dumping force $F_d = -bv$, b = damping constant

(v) Resultant force on a damped oscillator is given by

$$F = F_R + F_d = -Kx - Kv \Rightarrow \frac{md^2x}{dt^2} + b\frac{dx}{dt} + Kx = 0$$

(vi) Displacement of damped oscillator is given by

$x = x_m e^{-bt/2m}\sin(\omega' t + \phi)$ where ω' = angular frequency of the damped oscillator = $\sqrt{\omega_0^2 - (b/2m)^2}$

The amplitude decreases continuously with time according to

$$x = x_m e^{-(b/2m)t}$$

(vii) For a damped oscillator if the damping is small then the mechanical energy decreases exponentially with time as

$$E = \frac{1}{2}Kx_m^2 e^{-bt/m}$$

(3) **Forced oscillation**

(i) The oscillation in which a body oscillates under the influence of an external periodic force are known as forced oscillation

(ii) The amplitude of oscillator decrease due to damping forces but on account of the energy gained from the external source it remains constant.

(iii) Resonance : When the frequency of external force is equal to the natural frequency of the oscillator. Then this state is known as the state of resonance. And this frequency is known as resonant frequency.

(iv) While swinging in a swing if you apply a push periodically by pressing your feet against the ground, you find that not only the oscillations can now be maintained but the amplitude can also be increased. Under this condition the swing has forced or driven oscillation.

(v) In forced oscillation, frequency of damped oscillator is equal to the frequency of external force.

(vi) Suppose an external driving force is represented by

$$F(t) = F_0 \cos \omega_d t$$

The motion of a particle under combined action of

(a) Restoring force $(-Kx)$

(b) Damping force $(-bv)$ and

(c) Driving force $F(t)$ is given by $ma = -Kx - bv + F_0 \cos \omega_d t$

$$\Rightarrow m^2 \frac{d^2x}{d^2} + Kx + b\frac{dx}{dt} = F_0 \cos \omega_d t$$

The solution of this equation gives $x = x_0 \sin(\omega_d t + \phi)$ with amplitude $x_0 = \dfrac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2) + (b\omega/m)^2}}$ and $\tan\theta = \dfrac{(\omega^2 - \omega_0^2)}{b\omega/m}$

where $\omega_0 = \sqrt{\dfrac{K}{m}}$ = Natural frequency of oscillator.

(vii) **Amplitude resonance :** The amplitude of forced oscillator depends upon the frequency ω_d of external force.

When $\omega = \omega_d$, the amplitude is maximum but not infinite because of presence of damping force. The corresponds frequency is called resonant frequency (ω_0).

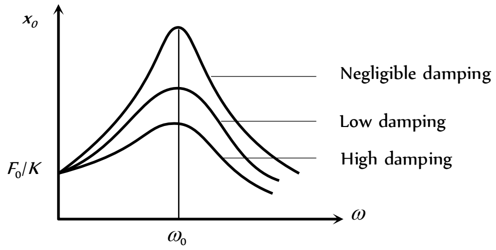


Fig. 16.35

(viii) **Energy resonance :** At $\omega = \omega_0$, oscillator absorbs maximum kinetic energy from the driving force system this state is called energy resonance.

At resonance the velocity of a driven oscillator is in phase with the driving term.

The sharpness of the resonance of a driven oscillator depends on the damping.

In the driven oscillator, the power input of the driving term in maximum at resonance.

(4) **Maintained oscillation :** The oscillation in which the loss of oscillator is compensated by the supplying energy from an external source are known as maintained oscillation.

Super Position of S.H.M's (Lissajous Figures)

If two S.H.M's act in perpendicular directions, then their resultant motion is in the form of a straight line or a circle or a parabola etc. depending on the frequency ratio of the two S.H.M. and initial phase difference. These figures are called Lissajous figures.

Let the equations of two mutually perpendicular S.H.M's of same frequency be

$$x = a_1 \sin\omega t \text{ and } y = a_2 \sin(\omega t + \phi)$$

then the general equation of Lissajou's figure can be obtained as

$$\frac{x^2}{a_1^2} + \frac{y^2}{a_2^2} - \frac{2xy}{a_1 a_2}\cos\phi = \sin^2\phi$$

For ϕ = 0° : $\dfrac{x^2}{a_1^2} + \dfrac{y^2}{a_2^2} - \dfrac{2xy}{a_1 a_2} = 0 \Rightarrow \left(\dfrac{x}{a_1} - \dfrac{y}{a_2}\right)^2 = 0$

$$\Rightarrow \frac{x}{a_1} = \frac{y}{a_2} \Rightarrow y = \frac{a_2}{a_1}x$$

This is a straight line passes through origin and it's slope is $\dfrac{a_2}{a_1}$.

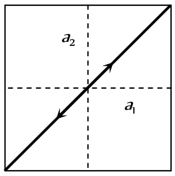
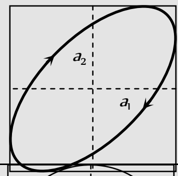
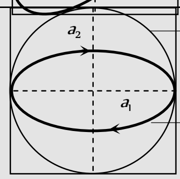
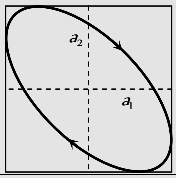
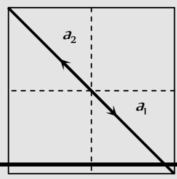


Fig. 16.36

Table 16.3 : Lissajou's figures in other conditions

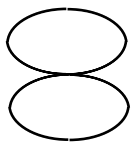
(with $\dfrac{\omega_1}{\omega_2} = 1$**)**

Phase diff.(ϕ)	Equation	Figure
$\dfrac{\pi}{4}$	$\dfrac{x^2}{a_1^2} + \dfrac{y^2}{a_2^2} - \dfrac{\sqrt{2}xy}{a_1 a_2} = \dfrac{1}{2}$	Oblique ellipse 
$\dfrac{\pi}{2}$	$\dfrac{x^2}{a_1^2} + \dfrac{y^2}{a_2^2} = 1$	 $a_1 = a_2$ (Circle) $a_1 \neq a_2$ (Ellipse)
$\dfrac{3\pi}{4}$	$\dfrac{x^2}{a_1^2} + \dfrac{y^2}{a_2^2} - \dfrac{\sqrt{2}xy}{a_1 a_2} = \dfrac{1}{2}$	Oblique ellipse 
π	$\dfrac{x}{a_1} + \dfrac{y}{a_2} = 0$ $\Rightarrow y = -\dfrac{a_2}{a_1}x$	Straight line 

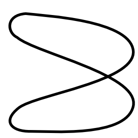
For the frequency ratio $\omega_1 : \omega_2 = 2 : 1$ the two perpendicular S.H.M's are

$$x = a_1 \sin(\omega t + \phi) \text{ and } y = a_2 \sin\omega t$$

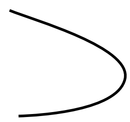
Different Lissajou's figures as follows



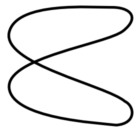
ϕ = 0, π, 2π
Figure of eight



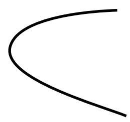
ϕ = $\pi/4$, $3\pi/4$
Double parabola



ϕ = $\pi/2$
Parabola



$\phi = 5\pi/4,\ 7\pi/4$
Double parabola



$\phi = 3\pi/2$
Parabola

Fig. 16.37

Tips & Tricks

✍ Suppose a body of mass m vibrate separately with two different springs (of spring constants k_1 and k_2) with time period T_1 and T_2 respectively. $T_1 = 2\pi\sqrt{\frac{m}{k_1}}$ and $T_2 = 2\pi\sqrt{\frac{m}{k_2}}$

If the same body vibrates with series combination of these two springs then for the system time period $T = \sqrt{T_1^2 + T_2^2}$

If the same body vibrates with parallel combination of these two springs then time period of the system $T = \frac{T_1 T_2}{\sqrt{T_1^2 + T_2^2}}$

✍ The pendulum clock runs slow due to increase in its time period whereas it becomes fast due to decrease in time period.

✍ If infinite spring with force constant $k, 2k, 4k, 8k$ .... respectively are connected in series. The effective force constant of the spring will be $k/2$.

✍ Percentage change in time period with l and g.

If g is constant and length varies by $n\%$. Then % change in time period $\frac{\Delta T}{T}\times 100 = \frac{n}{2}\times 100$

If l is constant and g varies by $n\%$. Then % change in time period $\frac{\Delta T}{T}\times 100 = -\frac{n}{2}\times 100$

(Valid only for small percentage change say 5%).

✍ Suppose a spring of force constant k oscillates with time period T. If it is divided in to n equal parts then spring constant of each part will become nk and time period of oscillation of each part will become $\frac{T}{\sqrt{n}}$.

If these n parts connected in parallel then $k_{eff} = n^2 k$. So time period of the system becomes $T' = \frac{T}{n}$

✍ If a particle performs S.H.M. whose velocity is v_1 at a x_1 distance from mean position and velocity v_2 at distance x_2

$$\omega = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}}\ ;\ T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$$

$$a = \sqrt{\frac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}}\ ;\ v_{\max} = \sqrt{\frac{v_1^2 x_2^2 - v_2^2 x_1^2}{x_2^2 - x_1^2}}$$

✍ If $y_1 = a\sin\omega t$ and $y_2 = b\cos\omega t$ are two S.H.M. then by the superimposition of these two S.H.M. we get $\vec{y} = \vec{y_1} + \vec{y_2}$

$\Rightarrow\ y = a\sin\omega t + b\cos\omega t\ \Rightarrow y = A\sin(\omega t + \phi)$ this is also the equation of S.H.M.; where $A = \sqrt{a^2 + b^2}$ and $\phi = \tan^{-1}(b/a)$

✍ In the absence of resistive force the work done by a simple pendulum in one complete oscillation is zero

✍ If θ is the angular amplitude of pendulum then

Height rises by the bob $h = l(1 - \cos\theta)$

Velocity at mean position

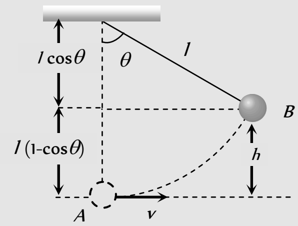
$v = \sqrt{2gl(1 - \cos\theta)}$

Work done in displacement

$W = U = mgl(1 - \cos\theta)$

K.E. at mean position

$KE_{mean} = mgl(1 - \cos\theta)$



Tension in the string of pendulum

At mean position : $T_A(\max) = mg + \frac{mv^2}{l} = (3mg - 2mg\cos\theta)$

At extream position : $T = mg\cos\theta$

Ordinary Thinking

Objective Questions

Displacement of S.H.M. and Phase

1. The phase of a particle executing simple harmonic motion is $\frac{\pi}{2}$ when it has **[MP PET 1985]**
   (a) Maximum velocity (b) Maximum acceleration
   (c) Maximum energy (d) Maximum displacement

2. A particle starts S.H.M. from the mean position. Its amplitude is A and time period is T. At the time when its speed is half of the maximum speed, its displacement y is **[Haryana CEE 1996; CBSE PMT 1996; MH CET 2002]**
   (a) $\frac{A}{2}$ (b) $\frac{A}{\sqrt{2}}$
   (c) $\frac{A\sqrt{3}}{2}$ (d) $\frac{2A}{\sqrt{3}}$

3. The amplitude and the periodic time of a S.H.M. are 5 cm and 6 sec respectively. At a distance of 2.5 cm away from the mean position, the phase will be
   (a) $5\pi/12$ (b) $\pi/4$
   (c) $\pi/3$ (d) $\pi/6$

4. Two equations of two S.H.M. are $y = a\sin(\omega t - \alpha)$ and $y = b\cos(\omega t - \alpha)$. The phase difference between the two is

[MP PMT 1985]

(a) 0° (b) $\alpha°$

(c) 90° (d) 180°

5. The amplitude and the time period in a S.H.M. is 0.5 *cm* and 0.4 *sec* respectively. If the initial phase is $\pi / 2$ radian, then the equation of S.H.M. will be

(a) $y = 0.5 \sin 5\pi t$ (b) $y = 0.5 \sin 4\pi t$

(c) $y = 0.5 \sin 2.5\pi t$ (d) $y = 0.5 \cos 5\pi t$

6. The equation of S.H.M. is $y = a\sin(2\pi nt + \alpha)$, then its phase at time *t* is [DPMT 2001]

(a) $2\pi nt$ (b) α

(c) $2\pi nt + \alpha$ (d) $2\pi t$

7. A particle is oscillating according to the equation $X = 7\cos 0.5\pi t$, where *t* is in second. The point moves from the position of equilibrium to maximum displacement in time

(a) 4.0 *sec* (b) 2.0 *sec*

(c) 1.0 *sec* (d) 0.5 *sec*

8. A simple harmonic oscillator has an amplitude *a* and time period *T*. The time required by it to travel from *x* = *a* to *x* = *a* / 2 is[CBSE PMT 1992; SCRA 1996; BHU 1997]

(a) $T/6$ (b) $T/4$

(c) $T/3$ (d) $T/2$

9. Which of the following expressions represent simple harmonic motion [Roorkee 1999]

(a) $x = A\sin(\omega t + \delta)$ (b) $x = B\cos(\omega t + \phi)$

(c) $x = A\tan(\omega t + \phi)$ (d) $x = A\sin\omega t\cos\omega t$

10. A $1.00 \times 10^{-20} kg$ particle is vibrating with simple harmonic motion with a period of $1.00 \times 10^{-5} sec$ and a maximum speed of $1.00 \times 10^{3} m/s$. The maximum displacement of the particle is

(a) 1.59 *mm* (b) 1.00 *m*

(c) 10 *m* (d) None of these

11. The phase (at a time *t*) of a particle in simple harmonic motion tells [AMU (Engg.) 1999]

(a) Only the position of the particle at time *t*

(b) Only the direction of motion of the particle at time *t*

(c) Both the position and direction of motion of the particle at time *t*

(d) Neither the position of the particle nor its direction of motion at time *t*

12. A particle is moving with constant angular velocity along the circumference of a circle. Which of the following statements is true

(a) The particle so moving executes S.H.M.

(b) The projection of the particle on any one of the diameters executes S.H.M.

(c) The projection of the particle on any of the diameters executes S.H.M.

(d) None of the above

13. A particle is executing simple harmonic motion with a period of *T seconds* and amplitude *a metre*. The shortest time it takes to reach a point $\frac{a}{\sqrt{2}} m$ from its mean position in *seconds* is [EAMCET (Med.) 2000]

(a) *T* (b) *T*/4

(c) *T*/8 (d) *T*/16

14. A simple harmonic motion is represented by $F(t) = 10\sin(20t + 0.5)$. The amplitude of the S.H.M. is

[DPMT 1998; CBSE PMT 2000; MH CET 2001]

(a) *a* = 30 (b) *a* = 20

(c) *a* = 10 (d) *a* = 5

15. Which of the following equation does not represent a simple harmonic motion [Kerala (Med.) 2002]

(a) $y = a\sin\omega t$ (b) $y = a\cos\omega t$

(c) $y = a\sin\omega t + b\cos\omega t$ (d) $y = a\tan\omega t$

16. A particle in S.H.M. is described by the displacement function $x(t) = a\cos(\omega t + \theta)$. If the initial $(t = 0)$ position of the particle is 1 *cm* and its initial velocity is $\pi\, cm/s$. The angular frequency of the particle is $\pi\, rad/s$, then it's amplitude is [CPMT 1989]

(a) 1 *cm* (b) $\sqrt{2}\, cm$

(c) 2 *cm* (d) 2.5 *cm*

17. A particle executes a simple harmonic motion of time period *T*. Find the time taken by the particle to go directly from its mean position to half the amplitude [UPSEAT 2002]

(a) $T/2$ (b) $T/4$

(c) $T/8$ (d) $T/12$

18. A particle executing simple harmonic motion along *y*-axis has its motion described by the equation $y = A\sin(\omega t) + B$. The amplitude of the simple harmonic motion is

[Orissa JEE 2003]

(a) *A* [AMU (Med.) 1999] (b) *B*

(c) *A* + *B* (d) $\sqrt{A + B}$

19. A particle executing S.H.M. of amplitude 4 *cm* and *T* = 4 *sec*. The time taken by it to move from positive extreme position to half the amplitude is [BHU 1995]

(a) 1 *sec* (b) 1/3 *sec*

(c) 2/3 *sec* (d) $\sqrt{3/2}$ *sec*

20. Which one of the following is a simple harmonic motion

[CBSE PMT 1994]

(a) Wave moving through a string fixed at both ends

(b) Earth spinning about its own axis [AMU (Engg.) 1999]

(c) Ball bouncing between two rigid vertical walls

(d) Particle moving in a circle with uniform speed

21. A particle is moving in a circle with uniform speed. Its motion is[CPMT 1978; CI

(a) Periodic and simple harmonic

(b) Periodic but not simple harmonic

(c) A periodic

(d) None of the above

22. Two simple harmonic motions are represented by the equations $y_1 = 0.1\sin\left(100\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1\cos\pi t$. The phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is [AIEEE 2005]

(a) $\frac{-\pi}{3}$ (b) $\frac{\pi}{6}$

(c) $\frac{-\pi}{6}$ (d) $\frac{\pi}{3}$

23. Two particles are executing S.H.M. The equation of their motion are $y_1 = 10\sin\left(\omega t + \frac{\pi T}{4}\right)$, $y_2 = 25\sin\left(\omega t + \frac{\sqrt{3}\pi T}{4}\right)$. What is the ratio of their amplitude [DCE 1996]

(a) 1 : 1 (b) 2 : 5

(c) 1 : 2 (d) None of these

24. The periodic time of a body executing simple harmonic motion is 3 *sec*. After how much interval from time $t = 0$, its displacement will be half of its amplitude [BHU 1998]

(a) $\frac{1}{8}$ *sec* (b) $\frac{1}{6}$ *sec*

(c) $\frac{1}{4}$ *sec* (d) $\frac{1}{3}$ *sec*

25. A system exhibiting S.H.M. must possess [KCET 1994]

(a) Inertia only

(b) Elasticity as well as inertia

(c) Elasticity, inertia and an external force

(d) Elasticity only

26. If $x = a\sin\left(\omega t + \frac{\pi}{6}\right)$ and $x' = a\cos\omega t$, then what is the phase difference between the two waves [RPET 1996]

(a) $\pi / 3$ (b) $\pi / 6$

(c) $\pi / 2$ (d) π

Velocity of Simple Harmonic Motion

1. A simple pendulum performs simple harmonic motion about $X = 0$ with an amplitude A and time period T. The speed of the pendulum at $X = \frac{A}{2}$ will be [MP PMT 1987]

(a) $\frac{\pi A\sqrt{3}}{T}$ (b) $\frac{\pi A}{T}$

(c) $\frac{\pi A\sqrt{3}}{2T}$ (d) $\frac{3\pi^2 A}{T}$

2. A body is executing simple harmonic motion with an angular frequency $2 rad/s$. The velocity of the body at 20 *mm* displacement, when the amplitude of motion is 60 *mm*, is

CPMT 1999]

(a) 40 *mm /s* (b) $60 mm / s$

(c) $113 mm / s$ (d) $120 mm / s$

3. A body of mass 5 *gm* is executing S.H.M. about a point with amplitude 10 *cm*. Its maximum velocity is 100 *cm/sec*. Its velocity will be 50 *cm/sec* at a distance [CPMT 1976]

(a) 5 (b) $5\sqrt{2}$

(c) $5\sqrt{3}$ (d) $10\sqrt{2}$

4. A simple harmonic oscillator has a period of 0.01 *sec* and an amplitude of 0.2 *m*. The magnitude of the velocity in $m\,\sec^{-1}$ at the centre of oscillation is [JIPMER 1997]

(a) 20π (b) 100

(c) 40π (d) 100π

5. A particle executes S.H.M. with a period of 6 second and amplitude of 3 *cm*. Its maximum speed in *cm/sec* is

[AIIMS 1982]

(a) $\pi / 2$ (b) π

(c) 2π (d) 3π

6. A particle is executing S.H.M. If its amplitude is 2 *m* and periodic time 2 *seconds*, then the maximum velocity of the particle will be

(a) $\pi\, m/s$ (b) $\sqrt{2\pi}\, m/s$

(c) $2\pi\, m/s$ (d) $4\pi\, m/s$

7. A S.H.M. has amplitude '*a*' and time period *T*. The maximum velocity will be

[MP PMT 1985; CPMT 1997; UPSEAT 1999]

(a) $\frac{4a}{T}$ (b) $\frac{2a}{T}$

(c) $2\pi\sqrt{\frac{a}{T}}$ (d) $\frac{2\pi a}{T}$

8. A body is executing S.H.M. When its displacement from the mean position is 4 *cm* and 5 *cm*, the corresponding velocity of the body is 10 *cm/sec* and 8 *cm/sec*. Then the time period of the body is [CPMT 1991; MP PE

(a) $2\pi\, sec$ (b) $\pi / 2\, sec$

(c) $\pi\, sec$ (d) $3\pi / 2\, sec$

9. A particle has simple harmonic motion. The equation of its motion is $x = 5\sin\left(4t - \frac{\pi}{6}\right)$, where x is its displacement. If the displacement of the particle is 3 units, then it velocity is

[MP PMT 1994]

(a) $\frac{2\pi}{3}$ (b) $\frac{5\pi}{6}$

(c) 20 (d) 16

10. [Pb. CET 1996; Pb. PMT 1997; AFMC 1998] If a simple pendulum oscillates with an amplitude of 50 *mm* and time period of 2 *sec*, then its maximum velocity is

[AIIMS 1998; MH CET 2000; DPMT 2000]

(a) 0.10 m/s (b) 0.15 m/s

(c) 0.8 m/s (d) 0.26 m/s

11. If the displacement of a particle executing SHM is given by $y = 0.30\sin(220t + 0.64)$ in *metre*, then the frequency and maximum velocity of the particle is [AFMC 1998]

(a) 35 *Hz*, 66 m/s (b) 45 *Hz*, 66 m/s

(c) 58 *Hz*, 113 m/s (d) 35 *Hz*, 132 m/s

12. The maximum velocity and the maximum acceleration of a body moving in a simple harmonic oscillator are $2\,m/s$ and $4\,m/s^2$. Then angular velocity will be

[Pb. PMT 1998; MH CET 1999, 2003]

(a) 3 *rad/sec* (b) 0.5 *rad/sec*

(c) 1 *rad/sec* (d) 2 *rad/sec*

13. If a particle under S.H.M. has time period 0.1 *sec* and amplitude $2\times10^{-3}\,m$. It has maximum velocity

[RPET 2000]

(a) $\frac{\pi}{25}\,m/s$ (b) $\frac{\pi}{26}\,m/s$

(c) $\frac{\pi}{30}\,m/s$ (d) None of these

14. A particle executing simple harmonic motion has an amplitude of 6 *cm*. Its acceleration at a distance of 2 *cm* from the mean position is $8\,cm/s^2$. The maximum speed of the particle is [EAMCET (Engg.) 2000]

(a) 8 *cm/s* (b) 12 *cm/s*

(c) 16 *cm/s* (d) 24 *cm/s*

15. A particle executes simple harmonic motion with an amplitude of 4 *cm*. At the mean position the velocity of the particle is 10 *cm/s*. The distance of the particle from the mean position when its speed becomes 5 *cm/s* is

[EAMCET (Med.) 2000]

(a) $\sqrt{3}\,cm$ (b) $\sqrt{5}\,cm$

(c) $2(\sqrt{3})\,cm$ (d) $2(\sqrt{5})\,cm$

16. Two particles *P* and *Q* start from origin and execute Simple Harmonic Motion along *X*-axis with same amplitude but with periods 3 *seconds* and 6 *seconds* respectively. The ratio of the velocities of *P* and *Q* when they meet is

[EAMCET 2001]

(a) 1 : 2 (b) 2 : 1

(c) 2 : 3 (d) 3 : 2

17. A particle is performing simple harmonic motion with amplitude *A* and angular velocity ω. The ratio of maximum velocity to maximum acceleration is [Kerala (Med.) 2002]

(a) ω (b) $1/\omega$

(c) ω (d) $A\omega$

18. The angular velocities of three bodies in simple harmonic motion are $\omega_1, \omega_2, \omega_3$ with their respective amplitudes as A_1, A_2, A_3. If all the three bodies have same mass and velocity, then

(a) $A_1\omega_1 = A_2\omega_2 = A_3\omega_3$ (b) $A_1\omega_1^2 = A_2\omega_2^2 = A_3\omega_3^2$

(c) $A_1^2\omega_1 = A_2^2\omega_2 = A_3^2\omega_3$ (d) $A_1^2\omega_1^2 = A_2^2\omega_2^2 = A^2$

19. The velocity of a particle performing simple harmonic motion, when it passes through its mean position is

[MH CET (Med.) 2002; BCECE 2004]

(a) Infinity (b) Zero

(c) Minimum (d) Maximum

20. The velocity of a particle in simple harmonic motion at displacement *y* from mean position is

[BCECE 2003; RPMT 2003]

(a) $\omega\sqrt{a^2+y^2}$ (b) $\omega\sqrt{a^2-y^2}$

(c) ωy (d) $\omega^2\sqrt{a^2-y^2}$

21. A particle is executing the motion $x = A\cos(\omega t - \theta)$. The maximum velocity of the particle is

[BHU 2003; CPMT 2004]

(a) $A\omega\cos\theta$ (b) $A\omega$

(c) $A\omega\sin\theta$ (d) None of these

22. A particle executing simple harmonic motion with amplitude of 0.1 *m*. At a certain instant when its displacement is 0.02 *m*, its acceleration is 0.5 *m/s*. The maximum velocity of the particle is (in *m/s*)

(a) 0.01 (b) 0.05

(c) 0.5 (d) 0.25

23. The amplitude of a particle executing SHM is 4 *cm*. At the mean position the speed of the particle is 16 *cm/sec*. The distance of the particle from the mean position at which the speed of the particle becomes $8\sqrt{3}\,cm/s$, will be

[Pb. PET 2003]

(a) $2\sqrt{3}\,cm$ (b) $\sqrt{3}\,cm$

(c) 1 *cm* (d) 2 *cm*

24. The maximum velocity of a simple harmonic motion represented by $y = 3\sin\left(100\,t + \frac{\pi}{6}\right)$ is given by

[BCECE 2005]

(a) 300 (b) $\frac{3\pi}{6}$

(c) 100 (d) $\frac{\pi}{6}$

25. The displacement equation of a particle is $x = 3\sin 2t + 4\cos 2t$. The amplitude and maximum velocity will be respectively

(a) 5, 10 (b) 3, 2

(c) 4, 2 (d) 3, 4

26. Velocity at mean position of a particle executing S.H.M. is *v*, they velocity of the particle at a distance equal to half of the amplitude

(a) 4*v* (b) 2*v*

(c) $\frac{\sqrt{3}}{2}v$ (d) $\frac{\sqrt{3}}{4}v$

27. The instantaneous displacement of a simple pendulum oscillator is given by $x = A\cos\left(\omega t + \frac{\pi}{4}\right)$. Its speed will be maximum at time [BHU 2002]

(a) $\frac{\pi}{4\omega}$ (b) $\frac{\pi}{2\omega}$

(c) $\frac{\pi}{\omega}$ (d) $\frac{2\pi}{\omega}$

Acceleration of Simple Harmonic Motion

1. Which of the following is a necessary and sufficient condition for S.H.M. [NCERT 1974]

(a) Constant period

(b) Constant acceleration

(c) Proportionality between acceleration and displacement from equilibrium position

(d) Proportionality between restoring force and displacement from equilibrium position

2. If a hole is bored along the diameter of the earth and a stone is dropped into hole [CPMT 1984]

(a) The stone reaches the centre of the earth and stops there

(b) The stone reaches the other side of the earth and stops there

(c) The stone executes simple harmonic motion about the centre of the earth

(d) The stone reaches the other side of the earth and escapes into space

3. The acceleration of a particle in S.H.M. is [MP PMT 1993]

(a) Always zero

(b) Always constant

(c) Maximum at the extreme position

(d) Maximum at the equilibrium position

4. The displacement of a particle moving in S.H.M. at any instant is given by $y = a\sin\omega t$. The acceleration after time $t = \frac{T}{4}$ is (where *T* is the time period) [MP PET 1984]

(a) $a\omega$ (b) $-a\omega$

(c) $a\omega^2$ (d) $-a\omega^2$

5. The amplitude of a particle executing S.H.M. with frequency of 60 *Hz* is 0.01 *m*. The maximum value of the acceleration of the particle is

[DPMT 1998; CBSE PMT 1999; AFMC 2001; Pb. PMT 2001; Pb. PET 2001, 02; CPMT 1993, 95, 04; RPMT 2005; MP PMT 2005]

(a) $144\pi^2 m/sec^2$ (b) $144 m/sec^2$

(c) $\frac{144}{\pi^2} m/sec^2$ (d) $288\pi^2 m/sec^2$

6. A small body of mass 0.10 *kg* is executing S.H.M. of amplitude 1.0 *m* and period 0.20 *sec*. The maximum force acting on it is

(a) 98.596 *N* (b) 985.96 *N*

(c) 100.2 *N* (d) 76.23 *N*

7. A body executing simple harmonic motion has a maximum acceleration equal to $24 metres/sec^2$ and maximum velocity equal to $16\ metres/sec$. The amplitude of the simple harmonic motion is

[MP PMT 1995; DPMT 2002; RPET 2003; Pb. PET 2004]

(a) $\frac{32}{3} metres$ (b) $\frac{3}{32} metres$

(c) $\frac{1024}{9} metres$ (d) $\frac{64}{9} metres$

8. For a particle executing simple harmonic motion, which of the following statements is not correct

[MP PMT 1997; AIIMS 1999; Kerala PMT 2005]

(a) The total energy of the particle always remains the same

(b) The restoring force of always directed towards a fixed point

(c) The restoring force is maximum at the extreme positions

(d) The acceleration of the particle is maximum at the equilibrium position

9. A particle of mass 10 *grams* is executing simple harmonic motion with an amplitude of 0.5 *m* and periodic time of $(\pi/5)$ *seconds*. The maximum value of the force acting on the particle is[MP PET 1999; MP PMT

(a) 25 *N* (b) 5 *N*

(c) 2.5 *N* (d) 0.5 *N*

10. The displacement of an oscillating particle varies with time (in *seconds*) according to the equation $y\ (cm) = \sin\frac{\pi}{2}\left(\frac{t}{2}+\frac{1}{3}\right)$. The maximum acceleration of the particle is approximately

(a) $5.21 cm/s^2$ (b) $3.62 cm/s^2$

(c) $1.81 cm/s^2$ (d) $0.62 cm/s^2$

11. A particle moving along the *x*-axis executes simple harmonic motion, then the force acting on it is given by

[CBSE PMT 1994]

(a) – *A Kx* (b) *A* cos (*Kx*)

(c) *A exp* (– *Kx*) (d) *A Kx*

Where *A* and *K* are positive constants

12. A body is vibrating in simple harmonic motion with an amplitude of 0.06 *m* and frequency of 15 *Hz*. The velocity and acceleration of body is [AFMC 1999]

(a) $5.65\, m/s$ and $5.32\times 10^2 m/s^2$

(b) $6.82\, m/s$ and $7.62\times 10^2 m/s^2$

(c) $8.91\, m/s$ and $8.21\times 10^2 m/s^2$

(d) $9.82\, m/s$ and $9.03\times 10^2 m/s^2$

13. A particle executes harmonic motion with an angular velocity and maximum acceleration of 3.5 *rad/sec* and 7.5 *m/s* respectively. The amplitude of oscillation is

[AIIMS 1999; Pb. PET 1999]

(a) 0.28 *m* (b) 0.36 *m*

(c) 0.53 *m* (d) 0.61 *m*

14. A 0.10 *kg* block oscillates back and forth along a horizontal surface. Its displacement from the origin is given by: $x = (10\,cm)\cos[(10\,rad/s)t + \pi/2\,rad]$. What is the maximum acceleration experienced by the block [AMU (Engg.) 2000]

(a) $10\ m/s^2$ (b) $10\,\pi\ m/s^2$

(c) $\frac{10\pi}{2} m/s^2$ (d) $\frac{10\pi}{3} m/s^2$

15. In S.H.M. maximum acceleration is at

[RPET 2001; BVP 2003]

(a) Amplitude (b) Equilibrium

(c) Acceleration is constant (d) None of these

16. A particle is executing simple harmonic motion with an amplitude of 0.02 *metre* and frequency 50 *Hz*. The maximum acceleration of the particle is [MP PET 2001]

(a) $100\ m/s^2$ (b) $100\pi^2\ m/s^2$

(c) $100\ m/s^2$ (d) $200\pi^2\ m/s^2$

17. Acceleration of a particle, executing SHM, at it's mean position is [MH CET (Med.) 2002]

(a) Infinity (b) Varies

(c) Maximum (d) Zero

18. Which one of the following statements is true for the speed v and the acceleration a of a particle executing simple harmonic motion

(a) When v is maximum, a is maximum

(b) Value of a is zero, whatever may be the value of v

(c) When v is zero, a is zero

(d) When v is maximum, a is zero

19. What is the maximum acceleration of the particle doing the SHM $y = 2\sin\left[\frac{\pi t}{2} + \phi\right]$ where 2 is in *cm* [DCE 2003]

(a) $\frac{\pi}{2} cm/s^2$ (b) $\frac{\pi^2}{2} cm/s^2$

(c) $\frac{\pi}{4} cm/s^2$ (d) $\frac{\pi}{4} cm/s^2$

20. A particle executes linear simple harmonic motion with an amplitude of 2 *cm*. When the particle is at 1 *cm* from the mean position the magnitude of its velocity is equal to that of its acceleration. Then its time period in seconds is

[Kerala PET 2005]

(a) $\frac{1}{2\pi\sqrt{3}}$ (b) $2\pi\sqrt{3}$

(c) $\frac{2\pi}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2\pi}$

21. In simple harmonic motion, the ratio of acceleration of the particle to its displacement at any time is a measure of

[UPSEAT 2001]

(a) Spring constant (b) Angular frequency

(c) (Angular frequency)· (d) Restoring force

Energy of Simple Harmonic Motion

1. The total energy of a particle executing S.H.M. is proportional to

[CPMT 1974, 78; EAMCET 1994; RPET 1999; MP PMT 2001; Pb. PMT 2002; MH CET 2002]

(a) Displacement from equilibrium position

(b) Frequency of oscillation

(c) Velocity in equilibrium position

(d) Square of amplitude of motion

2. A particle executes simple harmonic motion along a straight line with an amplitude A. The potential energy is maximum when the displacement is [CPMT 1982]

(a) $\pm A$ (b) Zero

(c) $\pm\frac{A}{2}$ (d) $\pm\frac{A}{\sqrt{2}}$

3. A particle is vibrating in a simple harmonic motion with an amplitude of 4 *cm*. At what displacement from the equilibrium position, is its energy half potential and half kinetic [NCERT 1984; MNR 1995; RPMT 1995; DCE 2000; UPSEAT 2000]

(a) 1 *cm* (b) $\sqrt{2}$ *cm*

(c) 3 *cm* (d) $2\sqrt{2}$ *cm*

4. For a particle executing simple harmonic motion, the kinetic energy K is given by $K = K_o \cos^2 \omega t$. The maximum value of potential energy is [CPMT 1981]

(a) K_0 [CBSE PMT 2004] (b) Zero

(c) $\frac{K_0}{2}$ (d) Not obtainable

5. The potential energy of a particle with displacement X is $U(X)$. The motion is simple harmonic, when (K is a positive constant)

(a) $U = -\frac{KX^2}{2}$ (b) $U = KX^2$

(c) $U = K$ (d) $U = KX$

6. The kinetic energy and potential energy of a particle executing simple harmonic motion will be equal, when displacement (amplitude = a) is

[MP PMT 1987; CPMT 1990; DPMT 1996; MH CET 1997, 99; AFMC 1999; CPMT 2000]

(a) $\frac{a}{2}$ (b) $a\sqrt{2}$

(c) $\frac{a}{\sqrt{2}}$ (d) $\frac{a\sqrt{2}}{3}$

7. The total energy of the body executing S.H.M. is E. Then the kinetic energy when the displacement is half of the amplitude, is

[RPMT 1994, 96; CBSE PMT 1995; JIPMER 2002]

(a) $\frac{E}{2}$ (b) $\frac{E}{4}$

(c) $\frac{3E}{4}$ (d) $\frac{\sqrt{3}}{4}E$

8. The potential energy of a particle executing S.H.M. is 2.5 *J*, when its displacement is half of amplitude. The total energy of the particle be

(a) 18 *J* (b) 10 *J*

(c) 12 *J* (d) 2.5 *J*

9. The angular velocity and the amplitude of a simple pendulum is ω and a respectively. At a displacement X from the mean position if its kinetic energy is T and potential energy is V, then the ratio of T to V is [CBSE PMT 1991]

(a) $X^2\omega^2/(a^2 - X^2\omega^2)$ (b) $X^2/(a^2 - X^2)$

(c) $(a^2 - X^2\omega^2)/X^2\omega^2$ (d) $(a^2 - X^2)/X^2$

10. When the potential energy of a particle executing simple harmonic motion is one-fourth of its maximum value during the oscillation, the displacement of the particle from the equilibrium position in terms of its amplitude a is

[CBSE PMT 1993; EAMCET (Engg.) 1995;

MP PMT 1994, 2000; MP PET 1995, 96, 2002]

(a) $a/4$ (b) $a/3$

(c) $a/2$ (d) $2a/3$

11. A particle of mass 10 *gm* is describing S.H.M. along a straight line with period of 2 *sec* and amplitude of 10 *cm*. Its kinetic energy when it is at 5 *cm* from its equilibrium position is

(a) $37.5\pi^2 ergs$ (b) $3.75\pi^2 ergs$

(c) $375\pi^2 ergs$ (d) $0.375\pi^2 ergs$

12. When the displacement is half the amplitude, the ratio of potential energy to the total energy is

[CPMT 1999; JIPMER 2000; Kerala PET 2002]

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) 1 (d) $\frac{1}{8}$

13. The P.E. of a particle executing SHM at a distance *x* from its equilibrium position is

[Roorkee 1992; CPMT 1997; RPMT 1999]

(a) $\frac{1}{2}m\omega^2 x^2$ (b) $\frac{1}{2}m\omega^2 a^2$

(c) $\frac{1}{2}m\omega^2(a^2 - x^2)$ (d) Zero

14. A vertical mass-spring system executes simple harmonic oscillations with a period of 2 *s*. A quantity of this system which exhibits simple harmonic variation with a period of 1 *s* is

(a) Velocity

(b) Potential energy

(c) Phase difference between acceleration and displacement

(d) Difference between kinetic energy and potential energy

15. For any S.H.M., amplitude is 6 *cm*. If instantaneous potential energy is half the total energy then distance of particle from its mean position is [RPET 2000]

(a) 3 *cm* (b) 4.2 *cm*

(c) 5.8 *cm* (d) 6 *cm*

16. A body of mass $1\,kg$ is executing simple harmonic motion. Its displacement $y(cm)$ at t seconds is given by $y = 6\sin(100t + \pi/4)$. Its maximum kinetic energy is

[EAMCET (Engg.) 2000]

(a) 6 *J* (b) 18 *J*

(c) 24 *J* (d) 36 *J*

17. A particle is executing simple harmonic motion with frequency *f*. The frequency at which its kinetic energy change into potential energy is [MP PET 2000]

(a) *f*/2 (b) *f*

(c) 2 *f* (d) 4 *f*

18. There is a body having mass *m* and performing S.H.M. with amplitude *a*. There is a restoring force $F = -Kx$, where *x* is the displacement. The total energy of body depends upon

[CBSE PMT 2001]

(a) *K, x* (b) *K, a*

(c) *K, a, x* (d) *K, a, v*

[MP PMT 1996]

19. The total energy of a particle executing S.H.M. is 80 *J*. What is the potential energy when the particle is at a distance of 3/4 of amplitude from the mean position

[Kerala (Engg.) 2001]

(a) 60 *J* (b) 10 *J*

(c) 40 *J* (d) 45 *J*

20. In a simple harmonic oscillator, at the mean position

[AIEEE 2002]

(a) Kinetic energy is minimum, potential energy is maximum

(b) Both kinetic and potential energies are maximum

(c) Kinetic energy is maximum, potential energy is minimum

(d) Both kinetic and potential energies are minimum

21. Displacement between maximum potential energy position and maximum kinetic energy position for a particle executing S.H.M. is

(a) $-a$ (b) $+a$

(c) $\pm a$ (d) $\pm\frac{a}{4}$

22. When a mass *M* is attached to the spring of force constant *k*, then the spring stretches by *l*. If the mass oscillates with amplitude *l*, what will be maximum potential energy stored in the spring

(a) $\frac{kl}{2}$ (b) $2kl$

(c) $\frac{1}{2}Mgl$ [SCRA 1998] (d) *Mgl*

23. The potential energy of a simple harmonic oscillator when the particle is half way to its end point is (where *E* is the total energy)

(a) $\frac{1}{8}E$ (b) $\frac{1}{4}E$

(c) $\frac{1}{2}E$ (d) $\frac{2}{3}E$

24. A body executes simple harmonic motion. The potential energy (P.E.), the kinetic energy (K.E.) and total energy (T.E.) are measured as a function of displacement *x*. Which of the following statements is true [AIEEE 2003]

(a) P.E. is maximum when $x = 0$

(b) K.E. is maximum when $x = 0$

(c) T.E. is zero when $x = 0$

(d) K.E. is maximum when *x* is maximum

25. If $\langle E \rangle$ and $\langle U \rangle$ denote the average kinetic and the average potential energies respectively of mass describing a simple harmonic motion, over one period, then the correct relation is

(a) $\langle E \rangle = \langle U \rangle$ (b) $\langle E \rangle = 2\langle U \rangle$

(c) $\langle E \rangle = -2\langle U \rangle$ (d) $\langle E \rangle = -\langle U \rangle$

26. The total energy of a particle, executing simple harmonic motion is

(a) $\propto x$ (b) $\propto x^2$

(c) Independent of x (d) $\propto x^{1/2}$

27. The kinetic energy of a particle executing S.H.M. is 16 *J* when it is at its mean position. If the mass of the particle is 0.32 *kg*, then what is the maximum velocity of the particle

[MH CET 2004]

(a) $5m/s$ (b) $15m/s$

(c) $10m/s$ (d) $20m/s$

28. Consider the following statements. The total energy of a particle executing simple harmonic motion depends on its

(1) Amplitude (2) Period (3) Displacement

Of these statements **[RPMT 2001; BCECE 2005]**

(a) (1) and (2) are correct

(b) (2) and (3) are correct

(c) (1) and (3) are correct

(d) (1), (2) and (3) are correct

29. A particle starts simple harmonic motion from the mean position. Its amplitude is *a* and total energy *E*. At one instant its kinetic energy is $3E/4$. Its displacement at that instant is

[Kerala PET 2005]

(a) $a/\sqrt{2}$ (b) $a/2$

(c) $\frac{a}{\sqrt{3/2}}$ (d) $a/\sqrt{3}$

30. A particle executes simple harmonic motion with a frequency f. The frequency with which its kinetic energy oscillates is **[IIT JEE 1973, 87; Manipal MEE 1995; MP PET 1997; DCE 1997; DCE 1999; UPSEAT 2000; RPET 2002; RPMT 2004; BHU 2005]**

(a) $f/2$ (b) f

(c) $2f$ (d) $4f$

31. The amplitude of a particle executing SHM is made three-fourth keeping its time period constant. Its total energy will be

(a) $\frac{E}{2}$ (b) $\frac{3}{4}E$

(c) $\frac{9}{16}E$ (d) None of these

32. A particle of mass *m* is hanging vertically by an ideal spring of force constant *K*. If the mass is made to oscillate vertically, its total energy is **[CPMT 1978; RPET 1999]**

(a) Maximum at extreme position

(b) Maximum at mean position

(c) Minimum at mean position

(d) Same at all position

33. A body is moving in a room with a velocity of 20 *m* / s perpendicular to the two walls separated by 5 *meters*. There is no friction and the collisions with the walls are elastic. The motion of the body is **[MP PMT 1999]**

(a) Not periodic

(b) Periodic but not simple harmonic

(c) Periodic and simple harmonic

(d) Periodic with variable time period

34. A body is executing Simple Harmonic Motion. At a displacement x its potential energy is E_1 and at a displacement y its potential energy is E_2. The potential energy E at displacement $(x+y)$ is **[EAMCET 20**

(a) $\sqrt{E} = \sqrt{E_1} - \sqrt{E_2}$ (b) $\sqrt{E} = \sqrt{E_1} + \sqrt{E_2}$

(c) $E = E_1 + E_2$ (d) $E = E_1 + E_2$

Time Period and Frequency

1. A particle moves such that its acceleration a is given by $a = -bx$, where x is the displacement from equilibrium position and b is a constant. The period of oscillation is

[NCERT 1984; CPMT 1991; MP PMT 1994; MNR 1995; UPSEAT 2000]

(a) $2\pi\sqrt{b}$ (b) $\frac{2\pi}{\sqrt{b}}$

(c) $\frac{2\pi}{b}$ (d) $2\sqrt{\frac{\pi}{b}}$

2. The equation of motion of a particle is $\frac{d^2y}{dt^2} + Ky = 0$, where K is positive constant. The time period of the motion is given by

(a) $\frac{2\pi}{K}$ (b) $2\pi K$

(c) $\frac{2\pi}{\sqrt{K}}$ (d) $2\pi\sqrt{K}$

3. A tunnel has been dug through the centre of the earth and a ball is released in it. It will reach the other end of the tunnel after

(a) 84.6 *minutes*

(b) 42.3 *minutes*

(c) 1 day

(d) Will not reach the other end

[RPMT 2004]

4. The maximum speed of a particle executing S.H.M. is $1m/s$ and its maximum acceleration is $1.57 m/sec^2$. The time period of the particle will be **[DPMT 2002]**

(a) $\frac{1}{1.57}sec$ (b) 1.57 *sec*

(c) 2 *sec* (d) 4 *sec*

5. The motion of a particle executing S.H.M. is given by $x = 0.01\sin 100\pi(t + .05)$, where x is in *metres* and time is in seconds. The time period is **[CPMT 1990]**

(a) 0.01 *sec* (b) 0.02 *sec*

(c) 0.1 *sec* (b) 0.2 *sec*

6. The kinetic energy of a particle executing S.H.M. is 16 *J* when it is in its mean position. If the amplitude of oscillations is 25 *cm* and the mass of the particle is 5.12 *kg*, the time period of its oscillation is

[Haryana CEE 1996; AFMC 1998]

(a) $\frac{\pi}{5}$ *sec* (b) 2π *sec*

(c) 20π *sec* (d) 5π *sec*

7. The acceleration of a particle performing S.H.M. is $12 cm/sec^2$ at a distance of 3 *cm* from the mean position. Its time period is [MP PET 1996; MP PMT 1997]

(a) 0.5 *sec* (b) 1.0 *sec*

(c) 2.0 *sec* (d) 3.14 *sec*

8. To make the frequency double of an oscillator, we have to [CPMT 1999]

(a) Double the mass

(b) Half the mass

(c) Quadruple the mass

(d) Reduce the mass to one-fourth

9. What is constant in S.H.M. [UPSEAT 1999]

(a) Restoring force (b) Kinetic energy

(c) Potential energy (d) Periodic time

10. If a simple harmonic oscillator has got a displacement of 0.02 *m* and acceleration equal to $2.0 ms^{-2}$ at any time, the angular frequency of the oscillator is equal to [CBSE PMT 1992; RPMT 1996]

(a) $10\, rad s^{-1}$ (b) $0.1\, rad s^{-1}$

(c) $100\, rad s^{-1}$ (d) $1\, rad s^{-1}$

11. The equation of a simple harmonic motion is $X = 0.34\cos(3000t + 0.74)$ where *X* and *t* are in *mm* and *sec*. The frequency of motion is [Kerala (Engg.) 2002]

(a) 3000 (b) $3000/2\pi$

(c) $0.74/2\pi$ (d) $3000/\pi$

12. Mark the wrong statement [MP PMT 2003]

(a) All S.H.M.'s have fixed time period

(b) All motion having same time period are S.H.M.

(c) In S.H.M. total energy is proportional to square of amplitude

(d) Phase constant of S.H.M. depends upon initial conditions

13. A particle in SHM is described by the displacement equation $x(t) = A\cos(\omega t + \theta)$. If the initial ($t = 0$) position of the particle is 1 *cm* and its initial velocity is π *cm/s*, what is its amplitude? The angular frequency of the particle is πs^{-1} [DPMT 2004]

(a) 1 *cm* (b) $\sqrt{2}$ *cm*

(c) 2 *cm* (d) 2.5 *cm*

14. A particle executes SHM in a line 4 *cm* long. Its velocity when passing through the centre of line is 12 *cm/s*. The period will be

(a) 2.047 *s* (b) 1.047 *s*

(c) 3.047 *s* (d) 0.047 *s*

15. The displacement *x* (in *metre*) of a particle in, simple harmonic motion is related to time *t* (in seconds) as

$$x = 0.01\cos\left(\pi t + \frac{\pi}{4}\right)$$

The frequency of the motion will be [UPSEAT 2004]

(a) 0.5 *Hz* (b) 1.0 *Hz*

(c) $\frac{\pi}{2} Hz$ (d) $\pi\, Hz$

16. A simple harmonic wave having an amplitude *a* and time period *T* is represented by the equation $y = 5\sin\pi(t+4)m$. Then the value of amplitude (*a*) in (*m*) and time period (*T*) in second are [Pb. PET 2004]

(a) $a = 10, T = 2$ (b) $a = 5, T = 1$

(c) $a = 10, T = 1$ (d) $a = 5, T = 2$

17. A particle executing simple harmonic motion of amplitude 5 *cm* has maximum speed of 31.4 *cm/s*. The frequency of its oscillation is

(a) 3 *Hz* (b) 2 *Hz*

(c) 4 *Hz* (d) 1 *Hz*

18. The displacement *x* (in *metres*) of a particle performing simple harmonic motion is related to time *t* (in *seconds*) as $x = 0.05\cos\left(4\pi t + \frac{\pi}{4}\right)$. The frequency of the motion will be

(a) 0.5 *Hz* (b) 1.0 *Hz*

(c) 1.5 *Hz* (d) 2.0 *Hz*

Simple Pendulum

1. The period of a simple pendulum is doubled, when [CPMT 1974; MNR 1980; AFMC 1995; Pb. PET/PMT 2002]

(a) Its length is doubled

(b) The mass of the bob is doubled

(c) Its length is made four times

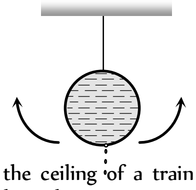
(d) The mass of the bob and the length of the pendulum are doubled

2. The period of oscillation of a simple pendulum of constant length at earth surface is *T*. Its period inside a mine is [CPMT 1973; DPMT 2001]

(a) Greater than *T* (b) Less than *T*

(c) Equal to *T* (d) Cannot be compared

3. A simple pendulum is made of a body which is a hollow sphere containing mercury suspended by means of a wire. If a little mercury is drained off, the period of pendulum will [NCERT 1972; BHU 1979]



(a) Remains unchanged

(b) Increase

(c) Decrease

(d) Become erratic

4. A pendulum suspended from the ceiling of a train has a period *T*, when the train is at rest. When the train is accelerating with a uniform acceleration *a*, the period of oscillation will [NCERT 1980; CPMT 1997] [Pb. PET 2000]

(a) Increase (b) Decrease

(c) Remain unaffected (d) Become infinite

5. The mass and diameter of a planet are twice those of earth. The period of oscillation of pendulum on this planet will be (If it is a second's pendulum on earth) [IIT 1973; DCE 2002]

(a) $\frac{1}{\sqrt{2}}$ *sec* (b) $2\sqrt{2}$ *sec*

(c) 2 *sec* (d) $\frac{1}{2}$ *sec*

6. A simple pendulum is set up in a trolley which moves to the right with an acceleration a on a horizontal plane. Then the thread of the pendulum in the mean position makes an angle θ with the vertical

(a) $\tan^{-1}\frac{a}{g}$ in the forward direction

(b) $\tan^{-1}\frac{a}{g}$ in the backward direction

(c) $\tan^{-1}\frac{g}{a}$ in the backward direction

(d) $\tan^{-1}\frac{g}{a}$ in the forward direction

7. Which of the following statements is not true ? In the case of a simple pendulum for small amplitudes the period of oscillation is

(a) Directly proportional to square root of the length of the pendulum

(b) Inversely proportional to the square root of the acceleration due to gravity

(c) Dependent on the mass, size and material of the bob

(d) Independent of the amplitude

8. The time period of a second's pendulum is 2 *sec*. The spherical bob which is empty from inside has a mass of 50 *gm*. This is now replaced by another solid bob of same radius but having different mass of 100 *gm*. The new time period will be

(a) 4 *sec* (b) 1 *sec*

(c) 2 *sec* (d) 8 *sec*

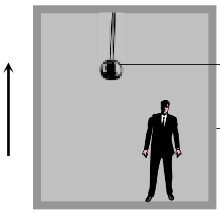
9. A man measures the period of a simple pendulum inside a stationary lift and finds it to be T sec. If the lift accelerates upwards with an acceleration $g/4$, then the period of the pendulum will be

(a) T

(b) $\frac{T}{4}$

(c) $\frac{2T}{\sqrt{5}}$

(d) $2T\sqrt{5}$



10. A simple pendulum is suspended from the roof of a trolley which moves in a horizontal direction with an acceleration a, then the time period is given by $T = 2\pi\sqrt{\frac{l}{g'}}$, where g' is equal to

(a) g (b) $g-a$

(c) $g+a$ (d) $\sqrt{g^2+a^2}$

11. A second's pendulum is placed in a space laboratory orbiting around the earth at a height $3R$, where R is the radius of the earth. The time period of the pendulum is

[CPMT 1989; RPMT 1995]

(a) Zero (b) $2\sqrt{3}$ *sec*

(c) 4 *sec* (d) Infinite

12. The bob of a simple pendulum of mass m and total energy E will have maximum linear momentum equal to

[CPMT 1983] [MP PMT 1986]

(a) $\sqrt{\frac{2E}{m}}$ (b) $\sqrt{2mE}$

(c) $2mE$ (d) mE^2

13. The length of the second pendulum on the surface of earth is 1 *m*. The length of seconds pendulum on the surface of moon, where g is 1/6th value of g on the surface of earth, is

[CPMT 1971]

(a) 1 / 6 *m* (b) 6 *m*

(c) 1 / 36 *m* (d) 36 *m*

[NCERT 1982]

14. If the length of second's pendulum is decreased by 2%, how many seconds it will lose per day [CPMT 1992]

(a) 3927 *sec* (b) 3727 *sec*

(c) 3427 *sec* (d) 864 sec

15. The period of simple pendulum is measured as T in a stationary lift. If the lift moves upwards with an acceleration of 5 g, the period will be [MNR 1979]

(a) The same (b) Increased by 3/5

(c) Decreased by 2/3 times (d) None of the above

[NCERT 1972]

16. The length of a simple pendulum is increased by 1%. Its time period will [MP PET 1994; RPET 2001]

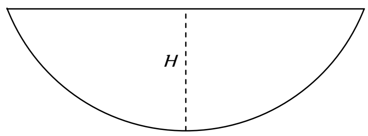
(a) Increase by 1% (b) Increase by 0.5%

(c) Decrease by 0.5% (d) Increase by 2%

17. A simple pendulum with a bob of mass 'm' oscillates from A to C and back to A such that PB is H. If the acceleration due to gravity is 'g', then the velocity of the bob as it passes through B is

[NCERT 1990; BHU 2001]

[CBSE PMT 1995; DPMT 1995; Pb. PMT 1996]



(a) mgH (b) $\sqrt{2gH}$

(c) $2gH$ (d) Zero

18. Identify correct statement among the following

[BHU 1997]

[Manipal MEE 1995]

(a) The greater the mass of a pendulum bob, the shorter is its frequency of oscillation

(b) A simple pendulum with a bob of mass M swings with an angular amplitude of 40^o. When its angular amplitude is 20^o, the tension in the string is less than $Mg\cos 20^o$.

(c) As the length of a simple pendulum is increased, the maximum velocity of its bob during its oscillation will also decreases

(d) The fractional change in the time period of a pendulum on changing the temperature is independent of the length of the pendulum

19. The bob of a pendulum of length l is pulled aside from its equilibrium position through an angle θ and then released. The bob will then pass through its equilibrium position with a speed v, where v equals [Haryana CEE 1996]

(a) $\sqrt{2gl(1-\sin\theta)}$ (b) $\sqrt{2gl(1+\cos\theta)}$

(c) $\sqrt{2gl(1-\cos\theta)}$ (d) $\sqrt{2gl(1+\sin\theta)}$

20. A simple pendulum executing S.H.M. is falling freely along with the support. Then

(a) Its periodic time decreases

(b) Its periodic time increases

(c) It does not oscillate at all

(d) None of these

21. A pendulum bob has a speed of 3 m/s at its lowest position. The pendulum is 0.5 m long. The speed of the bob, when the length makes an angle of 60^o to the vertical, will be (If $g = 10 m/s^2$)

(a) $3 m/s$ (b) $\frac{1}{3} m/s$

(c) $\frac{1}{2} m/s$ (d) $2 m/s$

22. The time period of a simple pendulum is 2 *sec*. If its length is increased 4 times, then its period becomes

[CBSE PMT 1999; DPMT 1999]

(a) 16 *sec* (b) 12 *sec*

(c) 8 *sec* (d) 4 *sec*

23. If the metal bob of a simple pendulum is replaced by a wooden bob, then its time period will [AIIMS 1998, 99]

(a) Increase

(b) Decrease

(c) Remain the same

(d) First increase then decrease

24. In a simple pendulum, the period of oscillation T is related to length of the pendulum l as [EAMCET (Med.) 1995]

(a) $\frac{l}{T}$ = constant (b) $\frac{l^2}{T}$ = constant

(c) $\frac{l}{T^2}$ = constant (d) $\frac{l^2}{T^2}$ = constant

25. A pendulum has time period T. If it is taken on to another planet having acceleration due to gravity half and mass 9 times that of the earth then its time period on the other planet will be

(a) $\sqrt{T}$ (b) T

(c) $T^{1/3}$ (d) $\sqrt{2}$ T

26. A simple pendulum is executing simple harmonic motion with a time period T. If the length of the pendulum is increased by 21%, the percentage increase in the time period of the pendulum of increased length is

[BHU 1994, 96; Pb. PMT 1995; AFMC 2001;

AIIMS 2001; AIEEE 2003]

(a) 10% (b) 21%

(c) 30% (d) 50%

27. If the length of simple pendulum is increased by 300%, then the time period will be increased by [RPMT 1999]

(a) 100% (b) 200%

(c) 300% (d) 400%

28. The length of a seconds pendulum is [RPET 2000]

(a) 99.8 *cm* (b) 99 *cm*

(c) 100 *cm* (d) None of these

29. The time period of a simple pendulum in a lift descending with constant acceleration g is [DCE 1998; MP PMT 2001]

(a) $T = 2\pi\sqrt{\frac{l}{g}}$ (b) $T = 2\pi\sqrt{\frac{l}{2g}}$

(c) Zero (d) Infinite

[MP PET 1996]

30. A chimpanzee swinging on a swing in a sitting position, stands up suddenly, the time period will

[KCET (Engg./Med.) 2000; AIEEE 2002; DPMT 2004]

(a) Become infinite (b) Remain same

(c) Increase (d) Decrease

31. The acceleration due to gravity at a place is $\pi^2 m/sec^2$. Then the time period of a simple pendulum of length one *metre* is

(a) $\frac{2}{\pi} sec$ (b) $2\pi\, sec$

(c) $2\, sec$ (d) $\pi\, sec$

32. A plate oscillated with time period 'T'. Suddenly, another plate put on the first plate, then time period [AIEEE 2002]

(a) Will decrease (b) Will increase

(c) Will be same (d) None of these

33. A simple pendulum of length l has a brass bob attached at its lower end. Its period is T. If a steel bob of same size, having density x times that of brass, replaces the brass bob and its length is changed so that period becomes $2T$, then new length is

(a) 2 l (b) 4 l

(c) 4 l x (d) $\frac{4l}{x}$

34. In a seconds pendulum, mass of bob is 30 *gm*. If it is replaced by 90 *gm* mass. Then its time period will

[CMEET Bihar 1995]

[Orissa PMT 2001]

(a) 1 *sec* (b) 2 *sec*

(c) 4 *sec* (d) 3 *sec*

35. The time period of a simple pendulum when it is made to oscillate on the surface of moon [J & K CET 2004]

(a) Increases (b) Decreases

(c) Remains unchanged (d) Becomes infinite

36. A simple pendulum is attached to the roof of a lift. If time period of oscillation, when the lift is stationary is *T*. Then frequency of oscillation, when the lift falls freely, will be [DCE 2002]

(a) Zero (b) *T*

(c) 1/*T* (d) None of these

37. A simple pendulum, suspended from the ceiling of a stationary van, has time period *T*. If the van starts moving with a uniform velocity the period of the pendulum will be [RPMT 2003]

(a) Less than *T* (b) Equal to 2*T*

(c) Greater than *T* (d) Unchanged

38. If the length of the simple pendulum is increased by 44%, then what is the change in time period of pendulum [MH CET 2004; UPSEAT 2005]

(a) 22% (b) 20%

(c) 33% (d) 44%

39. To show that a simple pendulum executes simple harmonic motion, it is necessary to assume that [CPMT 2001]

(a) Length of the pendulum is small

(b) Mass of the pendulum is small

(c) Amplitude of oscillation is small

(d) Acceleration due to gravity is small

40. The height of a swing changes during its motion from 0.1 *m* to 2.5 *m*. The minimum velocity of a boy who swings in this swing is

(a) 5.4 m/s (b) 4.95 m/s

(c) 3.14 m/s (d) Zero

41. The amplitude of an oscillating simple pendulum is 10*cm* and its period is 4 *sec*. Its speed after 1 *sec* after it passes its equilibrium position, is

(a) Zero (b) $0.57 m/s$

(c) $0.212 m/s$ (d) $0.32 m/s$

42. A simple pendulum consisting of a ball of mass *m* tied to a thread of length *l* is made to swing on a circular arc of angle θ in a vertical plane. At the end of this arc, another ball of mass *m* is placed at rest. The momentum transferred to this ball at rest by the swinging ball is [NCERT 1977]

(a) Zero (b) $m\theta\sqrt{\frac{g}{l}}$

(c) $\frac{m\theta}{l}\sqrt{\frac{l}{g}}$ (d) $\frac{m}{l}2\pi\sqrt{\frac{l}{g}}$

43. A simple pendulum hangs from the ceiling of a car. If the car accelerates with a uniform acceleration, the frequency of the simple pendulum will [Pb. PMT 2000]

(a) Increase (b) Decrease

(c) Become infinite (d) Remain constant

44. The periodic time of a simple pendulum of length 1 *m* and amplitude 2 *cm* is 5 seconds. If the amplitude is made 4 *cm*, its periodic time in seconds will be [MP PMT 1985]

(a) 2.5 (b) 5

(c) 10 (d) $5\sqrt{2}$

45. The ratio of frequencies of two pendulums are 2 : 3, then their length are in ratio [DCE 2005]

(a) $\sqrt{2/3}$ (b) $\sqrt{3/2}$

(c) $4/9$ (d) $9/4$

46. Two pendulums begin to swing simultaneously. If the ratio of the frequency of oscillations of the two is 7 : 8, then the ratio of lengths of the two pendulums will be [J & K CET 2005]

(a) 7 : 8 (b) 8 : 7

(c) 49 : 64 (d) 64 : 49

47. A simple pendulum hanging from the ceiling of a stationary lift has a time period T_1. When the lift moves downward with constant velocity, the time period is T_2, then [Orissa JEE 2005]

(a) T_2 is infinity (b) $T_2 > T_1$

(c) $T_2 < T_1$ (d) $T_2 = T_1$

48. If the length of a pendulum is made 9 times and mass of the bob is made 4 times then the value of time period becomes [BHU 2005]

(a) 3*T* (b) 3/2*T*

(c) 4*T* (d) 2*T*

49. A simple pendulum is taken from the equator to the pole. Its period [CPMT 1997]

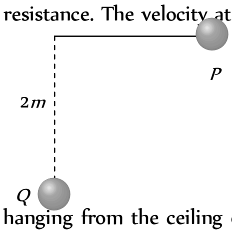
(a) Decreases

(b) Increases

(c) Remains the same

(d) Decreases and then increases

50. A pendulum of length 2*m* lift at *P*. When it reaches *Q*, it losses 10% of its total energy due to air resistance. The velocity at *Q* is



(a) 6 *m/sec*

(b) 1 *m/sec*

(c) 2 *m/sec*

(d) 8 *m/sec*

51. There is a simple pendulum hanging from the ceiling of a lift. When the lift is stand still, the time period of the pendulum is *T*. If the resultant acceleration becomes $g/4$, then the new time period of the pendulum is [DCE 2004]

(a) 0.8 *T* (b) 0.25 *T*

(c) 2 *T* (d) 4 *T*

52. The period of a simple pendulum measured inside a stationary lift is found to be *T*. If the lift starts accelerating upwards with acceleration of $g/3$, then the time period of the pendulum is [RPMT 2000; DPM

(a) $\frac{T}{\sqrt{3}}$ (b) $\frac{T}{3}$

(c) $\frac{\sqrt{3}}{2}T$ (d) $\sqrt{3}\,T$

53. Time period of a simple pendulum will be double, if we [MH CET 2003]

(a) Decrease the length 2 times
(b) Decrease the length 4 times
(c) Increase the length 2 times
(d) Increase the length 4 times

54. Length of a simple pendulum is l and its maximum angular displacement is θ, then its maximum K.E. is [RPMT 1995; BHU 2003]

(a) $mgl\sin\theta$ (b) $mgl(1+\sin\theta)$
(c) $mgl(1+\cos\theta)$ (d) $mgl(1-\cos\theta)$

55. The velocity of simple pendulum is maximum at [RPMT 2004]

(a) Extremes (b) Half displacement
(c) Mean position (d) Every where

56. A simple pendulum is vibrating in an evacuated chamber, it will oscillate with [Pb. PMT 2004]

(a) Increasing amplitude (b) Constant amplitude
(c) Decreasing amplitude (d) First (c) then (a)

57. The time period of a simple pendulum of length L as measured in an elevator descending with acceleration $\frac{g}{3}$ is [CPMT 2000]

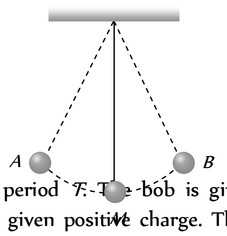
(a) $2\pi\sqrt{\frac{3L}{g}}$ (b) $\pi\sqrt{\left(\frac{3L}{g}\right)}$
(c) $2\pi\sqrt{\left(\frac{3L}{2g}\right)}$ (d) $2\pi\sqrt{\frac{2L}{3g}}$

58. If a body is released into a tunnel dug across the diameter of earth, it executes simple harmonic motion with time period

(a) $T=2\pi\sqrt{\frac{R_e}{g}}$ (b) $T=2\pi\sqrt{\frac{2R_e}{g}}$
(c) $T=2\pi\sqrt{\frac{R_e}{2g}}$ (d) $T=2$ seconds

59. What is the velocity of the bob of a simple pendulum at its mean position, if it is able to rise to vertical height of 10*cm* (g = 9.8 *m/s*)

(a) 2.2 *m/s*
(b) 1.8 *m/s*
(c) 1.4 *m/s*
(d) 0.6 *m/s*



60. A simple pendulum has time period T. The bob is given negative charge and surface below it is given positive charge. The new time period will be [AFMC 2004]

(a) Less than T (b) Greater than T
(c) Equal to T (d) Infinite

61. What effect occurs on the frequency of a pendulum if it is taken from the earth surface to deep into a mine [AFMC 2005]

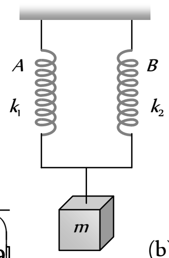
(a) Increases
(b) Decreases
(c) First increases then decrease
(d) None of these

Spring Pendulum

1. Two bodies M and N of equal masses are suspended from two separate massless springs of force constants k_1 and k_2 respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude M to that of N is [IIT-JEE 1988; MP PET 1997, 2001; MP PMT 1997; BHU 1998; Pb. PMT 1998; MH CET 2000, 03; AIEEE 2003]

(a) $\frac{k_1}{k_2}$ (b) $\sqrt{\frac{k_1}{k_2}}$
(c) $\frac{k_2}{k_1}$ (d) $\sqrt{\frac{k_2}{k_1}}$

2. A mass m is suspended by means of two coiled spring which have the same length in unstretched condition as in figure. Their force constant are k_1 and k_2 respectively. When set into vertical vibrations, the period will be [MP PMT 2001]



(a) $2\pi\sqrt{\left(\frac{m}{k_1k_2}\right)}$ [CPMT 1999] (b) $2\pi\sqrt{m\left(\frac{k_1}{k_2}\right)}$
(c) $2\pi\sqrt{\left(\frac{m}{k_1-k_2}\right)}$ (d) $2\pi\sqrt{\left(\frac{m}{k_1+k_2}\right)}$

3. A spring has a certain mass suspended from it and its period for vertical oscillation is T. The spring is now cut into two equal halves and the same mass is suspended from one of the halves. The period of vertical oscillation is now [BHU 2000] [MP PET 1995]

(a) $\frac{T}{2}$ (b) $\frac{T}{\sqrt{2}}$
(c) $\sqrt{2}T$ (d) $2T$

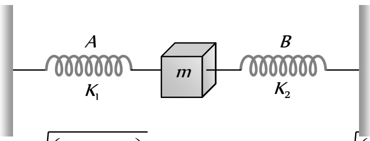
4. Two masses m_1 and m_2 are suspended together by a massless spring of constant k. When the masses are in equilibrium, m_1 is removed without disturbing the system. Then the angular frequency of oscillation of m_2 is

(a) $\sqrt{\frac{k}{m_1}}$ (b) $\sqrt{\frac{k}{m_2}}$

(c) $\sqrt{\frac{k}{m_1+m_2}}$ (d) $\sqrt{\frac{k}{m_1 m_2}}$

5. In arrangement given in figure, if the block of mass m is displaced, the frequency is given by

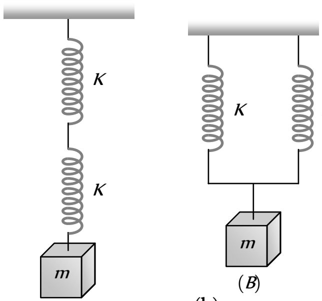
[BHU 1994; Pb. PET 2001]



(a) $n = \frac{1}{2\pi}\sqrt{\left(\frac{k_1-k_2}{m}\right)}$ (b) $n = \frac{1}{2\pi}\sqrt{\left(\frac{k_1+k_2}{m}\right)}$

(c) $n = \frac{1}{2\pi}\sqrt{\left(\frac{m}{k_1+k_2}\right)}$ (d) $n = \frac{1}{2\pi}\sqrt{\left(\frac{m}{k_1-k_2}\right)}$

6. Two identical spring of constant K are connected in series and parallel as shown in figure. A mass m is suspended from them. The ratio of their frequencies of vertical oscillations will be [MP PET 1993; BHU 1997]



(A) (B)

(a) 2 : 1 (b) 1 : 1

(c) 1 : 2 (d) 4 : 1

7. A mass m is suspended from the two coupled springs connected in series. The force constant for springs are K_1 and K_2. The time period of the suspended mass will be

[CBSE PMT 1990; Pb. PET 2002]

(a) $T = 2\pi\sqrt{\left(\frac{m}{K_1+K_2}\right)}$ (b) $T = 2\pi\sqrt{\left(\frac{m}{K_1+K_2}\right)}$

(c) $T = 2\pi\sqrt{\left(\frac{m(K_1+K_2)}{K_1K_2}\right)}$ (d) $T = 2\pi\sqrt{\left(\frac{mK_1K_2}{K_1+K_2}\right)}$

8. A spring is stretched by 0.20 *m*, when a mass of 0.50 *kg* is suspended. When a mass of 0.25 *kg* is suspended, then its period of oscillation will be $(g = 10 m/s^2)$

(a) 0.328 *sec* (b) 0.628 *sec*

(c) 0.137 *sec* (d) 1.00 *sec*

9. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes simple harmonic oscillations with a time period T. If the mass is increased by m then the time period becomes $\left(\frac{5}{4}T\right)$. The ratio of $\frac{m}{M}$ is[CPMT 1991]

(a) 9/16 (b) 25/16

(c) 4/5 (d) 5/4

10. A spring having a spring constant 'K' is loaded with a mass 'm'. The spring is cut into two equal parts and one of these is loaded again with the same mass. The new spring constant is [NCERT 1990; KCET 1999; Kerala PMT 2004; BCECE 2004]

(a) $K/2$ (b) K

(c) $2K$ (d) K^2

11. A weightless spring which has a force constant oscillates with frequency n when a mass m is suspended from it. The spring is cut into two equal halves and a mass $2m$ is suspended from it. The frequency of oscillation will now become

[CPMT 1988]

(a) n (b) $2n$

(c) $n/\sqrt{2}$ (d) $n(2)^{1/2}$

12. A mass M is suspended from a light spring. An additional mass m added displaces the spring further by a distance x. Now the combined mass will oscillate on the spring with period[CPMT 1989, 1998 ; UPSEA

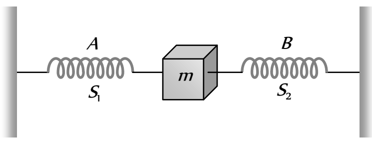
(a) $T = 2\pi\sqrt{(mg/x(M+m))}$

(b) $T = 2\pi\sqrt{((M+m)x/mg)}$

(c) $T = (\pi/2)\sqrt{(mg/x(M+m))}$

(d) $T = 2\pi\sqrt{((M+m)/mgx)}$

13. In the figure, S_1 and S_2 are identical springs. The oscillation frequency of the mass m is f. If one spring is removed, the frequency will become [CPMT 1971]



(a) f (b) $f \times 2$

(c) $f \times \sqrt{2}$ (d) $f/\sqrt{2}$

14. The vertical extension in a light spring by a weight of 1 *kg* suspended from the wire is 9.8 *cm*. The period of oscillation

[CPMT 1981; MP PMT 2003]

(a) 20π sec (b) 2π sec

(c) $2\pi/10$ sec (d) 200π sec

15. A particle of mass 200 *gm* executes S.H.M. The restoring force is provided by a spring of force constant 80 *N / m*. The time period of oscillations is [MP PET 1994]

(a) 0.31 *sec* (b) 0.15 *sec*

(c) 0.05 *sec* (d) 0.02 *sec*

16. The length of a spring is l and its force constant is k. When a weight W is suspended from it, its length increases by x. If the spring is cut into two equal parts and put in parallel and the same weight W is suspended from them, then the extension will be

(a) $2x$ (b) x

(c) $\frac{x}{2}$ (d) $\frac{x}{4}$

17. A block is placed on a frictionless horizontal table. The mass of the block is m and springs are attached on either side with force constants K_1 and K_2. If the block is displaced a little and left to oscillate, then the angular frequency of oscillation will be

(a) $\left(\frac{K_1+K_2}{m}\right)^{1/2}$ (b) $\left[\frac{K_1K_2}{m(K_1+K_2)}\right]^{1/2}$

(c) $\left[\frac{K_1K_2}{(K_1-K_2)m}\right]^{1/2}$ (d) $\left[\frac{K_1^2+K_2^2}{(K_1+K_2)m}\right]^{1/2}$

18. A uniform spring of force constant k is cut into two pieces, the lengths of which are in the ratio 1 : 2. The ratio of the force constants of the shorter and the longer pieces is [Manipal MEE 1995]

(a) 1 : 3 (b) 1 : 2

(c) 2 : 3 (d) 2 : 1

19. A mass m =100 *gms* is attached at the end of a light spring which oscillates on a frictionless horizontal table with an amplitude equal to 0.16 *metre* and time period equal to 2 *sec*. Initially the mass is released from rest at t = 0 and displacement $x=-0.16$ *metre*. The expression for the displacement of the mass at any time t is [MP PMT 1995]

(a) $x=0.16\cos(\pi t)$ (b) $x=-0.16\cos(\pi t)$

(c) $x=0.16\sin(\pi t+\pi)$ (d) $x=-0.16\sin(\pi t+\pi)$

20. A block of mass m, attached to a spring of spring constant k, oscillates on a smooth horizontal table. The other end of the spring is fixed to a wall. The block has a speed v when the spring is at its natural length. Before coming to an instantaneous rest, if the block moves a distance x from the mean position, then

(a) $x=\sqrt{m/k}$ (b) $x=\frac{1}{v}\sqrt{m/k}$

(c) $x=v\sqrt{m/k}$ (d) $x=\sqrt{mv/k}$

21. The force constants of two springs are K_1 and K_2. Both are stretched till their elastic energies are equal. If the stretching forces are F_1 and F_2, then $F_1:F_2$ is [MP PET 2002]

(a) $K_1:K_2$ (b) $K_2:K_1$

(c) $\sqrt{K_1}:\sqrt{K_2}$ (d) $K_1^2:K_2^2$

22. A mass m is vertically suspended from a spring of negligible mass; the system oscillates with a frequency n. What will be the frequency of the system if a mass 4 m is suspended from the same spring

(a) $n/4$ (b) $4n$

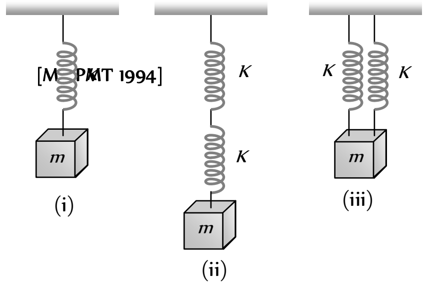
(c) $n/2$ (d) $2n$

23. If the period of oscillation of mass m suspended from a spring is 2 *sec*, then the period of mass $4m$ will be [AIIMS 1998]

(a) 1 *sec* (b) 2 *sec*

(c) 3 *sec* (d) 4 *sec*

24. Five identical springs are used in the following three configurations. The time periods of vertical oscillations in configurations (i), (ii) and (iii) are in the ratio [AMU 1995]

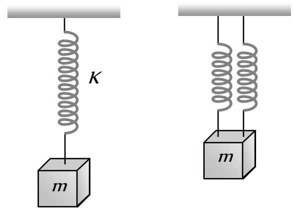


[MP PMT 1994]

(a) $1:\sqrt{2}:\frac{1}{\sqrt{2}}$ (b) $2:\sqrt{2}:\frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}}:2:1$ (d) $2:\frac{1}{\sqrt{2}}:1$

25. A mass m performs oscillations of period T when hanged by spring of force constant K. If spring is cut in two parts and arranged in parallel and same mass is oscillated by them, then the new time period will be [CPMT 1995; RPET 1997; RPMT 2003]



(a) $2T$

(b) T

(c) $\frac{T}{\sqrt{2}}$

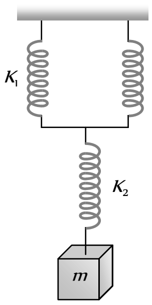
(d) $\frac{T}{2}$ [MP PET 1996]

26. If a watch with a wound spring is taken on to the moon, it [AFMC 1993]

(a) Runs faster (b) Runs slower

(c) Does not work (d) Shows no change

27. What will be the force constant of the spring system shown in the figure [RPET 1996; Kerala (Med./ Engg.) 2005]



(a) $\frac{K_1}{2}+K_2$

(b) $\left[\frac{1}{2K_1}+\frac{1}{K_2}\right]^{-1}$ [CBSE PMT 1998]

(c) $\frac{1}{2K_1}+\frac{1}{K_2}$

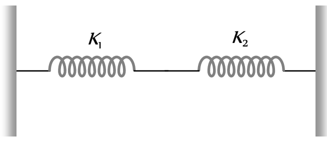
(d) $\left[\frac{2}{K_1}+\frac{1}{K_1}\right]^{-1}$

28. Two springs have spring constants K_A and K_B and $K_A>K_B$. The work required to stretch them by same extension will be

(a) More in spring A (b) More in spring B

(c) Equal in both (d) Noting can be said

29. The effective spring constant of two spring system as shown in figure will be [RPMT 1999]



(a) $K_1 + K_2$ (b) $K_1K_2 / K_1 + K_2$

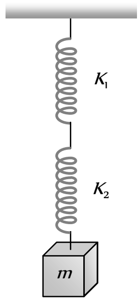
(c) $K_1 - K_2$ (d) $K_1K_2 / K_1 - K_2$

30. A mass m attached to a spring oscillates every 2 *sec*. If the mass is increased by 2 *kg*, then time-period increases by 1 *sec*. The initial mass is [CBSE PMT 2000; AIIMS 2000; MP PET 2000; DPMT 2001; Pb. PMT 2003]

(a) 1.6 *kg* (b) 3.9 *kg*

(c) 9.6 *kg* (d) 12.6 *kg*

31. A mass M is suspended by two springs of force constants K_1 and K_2 respectively as shown in the diagram. The total elongation (stretch) of the two springs is [MP PMT 2000; RPET 2001]



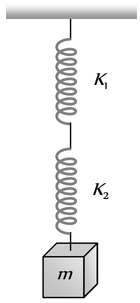
(a) $\dfrac{Mg}{K_1 + K_2}$

(b) $\dfrac{Mg(K_1 + K_2)}{K_1K_2}$

(c) $\dfrac{Mg\, K_1K_2}{K_1 + K_2}$

(d) $\dfrac{K_1 + K_2}{K_1K_2Mg}$

32. The frequency of oscillation of the springs shown in the figure will be [AIIMS 2001; Pb. PET 2002]



(a) $\dfrac{1}{2\pi}\sqrt{\dfrac{K}{m}}$

(b) $\dfrac{1}{2\pi}\sqrt{\dfrac{(K_1 + K_2)m}{K_1K_2}}$

(c) $2\pi\sqrt{\dfrac{K}{m}}$

(d) $\dfrac{1}{2\pi}\sqrt{\dfrac{K_1K_2}{m(K_1 + K_2)}}$

33. The scale of a spring balance reading from 0 to 10 *kg* is 0.25 m long. A body suspended from the balance oscillates vertically with a period of $\pi/10$ second. The mass suspended is (neglect the mass of the spring) [Kerala (Engg.) 2001]

(a) 10 *kg* (b) 0.98 *kg*

(c) 5 *kg* (d) 20 *kg*

34. If a spring has time period T, and is cut into n equal parts, then the time period of each part will be [AIEEE 2002]

(a) $T\sqrt{n}$ (b) $T/\sqrt{n}$

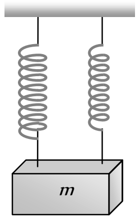
(c) nT (d) T

35. One-forth length of a spring of force constant K is cut away. The force constant of the remaining spring will be [MP PET 2002]

(a) $\dfrac{3}{4}K$ (b) $\dfrac{4}{3}K$

(c) K (d) 4 K

36. A mass m is suspended separately by two different springs of spring constant K_1 and K_2 gives the time-period t_1 and t_2 respectively. If same mass m is connected by both springs as shown in figure then time-period t is given by the relation [CBSE PMT 2002]



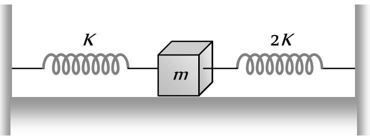
(a) $t = t_1 + t_2$

(b) $t = \dfrac{t_1 . t_2}{t_1 + t_2}$

(c) $t^2 = {t_1}^2 + {t_2}^2$

(d) $t^{-2} = {t_1}^{-2} + {t_2}^{-2}$

37. Two springs of force constants K and $2K$ are connected to a mass as shown below. The frequency of oscillation of the mass is [RPMT 1996; DCE 200



(a) $(1/2$ ... $/m)$

(c) $(1/2\pi)\sqrt{(3K/m)}$ (d) $(1/2\pi)\sqrt{(m/K)}$

38. Two springs of constant k_1 and k_2 are joined in series. The effective spring constant of the combination is given by [CBSE PMT 2004]

(a) $\sqrt{k_1k_2}$ (b) $(k_1 + k_2)/2$

(c) $k_1 + k_2$ (d) $k_1k_2 / (k_1 + k_2)$

39. A particle at the end of a spring executes simple harmonic motion with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T, then [AIEEE 2004]

(a) $T = t_1 + t_2$ (b) $T^2 = t_1^2 + t_2^2$

(c) $T^{-1} = t_1^{-1} + t_2^{-1}$ (d) $T^{-2} = t_1^{-2} + t_2^{-2}$

40. Infinite springs with force constant k, $2k$, $4k$ and $8k$.... respectively are connected in series. The effective force constant of the spring will be [J & K CET 2004]

(a) 2K (b) k

(c) k/2 (d) 2048

41. To make the frequency double of a spring oscillator, we have to

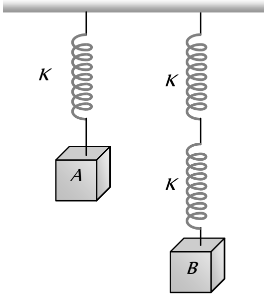
(a) Reduce the mass to one fourth

(b) Quardruple the mass

(c) Double of mass

(d) Half of the mass

42. The springs shown are identical. When $A = 4kg$, the elongation of spring is 1 *cm*. If $B = 6kg$, the elongation produced by it is



(a) 4 *cm* (b) 3 *cm*

(c) 2 *cm* (d) 1 *cm*

43. When a body of mass 1.0 *kg* is suspended from a certain light spring hanging vertically, its length increases by 5 *cm*. By suspending 2.0 *kg* block to the spring and if the block is pulled through 10 *cm* and released the maximum velocity in it in *m/s* is : (Acceleration due to gravity $= 10 m/s^2$) [EAMCET 2003]

(a) 0.5 (b) 1

(c) 2 (d) 4

44. Two springs with spring constants $K_1 = 1500\, N/m$ and $K_2 = 3000\, N/m$ are stretched by the same force. The ratio of potential energy stored in spring will be [RPET 2001]

(a) 2 : 1 (b) 1 : 2

(c) 4 : 1 (d) 1 : 4

45. If a spring extends by *x* on loading, then energy stored by the spring is (if *T* is the tension in the spring and *K* is the spring constant)

(a) $\frac{T^2}{2x}$ (b) $\frac{T^2}{2K}$

(c) $\frac{2K}{T^2}$ (d) $\frac{2T^2}{K}$

46. A weightless spring of length 60 *cm* and force constant 200 *N/m* is kept straight and unstretched on a smooth horizontal table and its ends are rigidly fixed. A mass of 0.25 *kg* is attached at the middle of the spring and is slightly displaced along the length. The time period of the oscillation of the mass is [MP PET 2003]

(a) $\frac{\pi}{20} s$ (b) $\frac{\pi}{10} s$

(c) $\frac{\pi}{5} s$ (d) $\frac{\pi}{\sqrt{200}} s$

47. The time period of a mass suspended from a spring is *T*. If the spring is cut into four equal parts and the same mass is suspended from one of the parts, then the new time period will be [MP PMT 2002; CBSE PMT 2003]

(a) *T* (b) $\frac{T}{2}$

(c) 2 *T* (d) $\frac{T}{4}$

48. A mass *M* is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes S.H.M. of time period *T*. If the mass is increased by m, the time period becomes 5*T*/3. Then the ratio of *m*/*M* is [Pb. PET 2002]

(a) $\frac{5}{3}$ (b) $\frac{3}{5}$

(c) $\frac{25}{9}$ (d) $\frac{16}{9}$

49. An object is attached to the bottom of a light vertical spring and set vibrating. The maximum speed of the object is 15 *cm/sec* and the period is 628 *milli-seconds*. The amplitude of the motion in centimeters is [EAMCET 2003]

(a) 3.0 (b) 2.0

(c) 1.5 (d) 1.0

50. When a mass *m* is attached to a spring, it normally extends by 0.2 *m*. The mass *m* is given a slight addition extension and released, then its time period will be [MH CET 2001]

(a) $\frac{1}{7}$ *sec* (b) 1 *sec*

(c) $\frac{2\pi}{7}$ *sec* (d) $\frac{2}{3\pi}$ *sec*

51. If a body of mass 0.98 *kg* is made to oscillate on a spring of force constant 4.84 *N/m*, the angular frequency of the body is

(a) 1.22 *rad/s* (b) 2.22 *rad/s*

(c) 3.22 *rad/s* (d) 4.22 *rad/s*

52. A mass *m* is suspended from a spring of length *l* and force constant *K*. The frequency of vibration of the mass is f_1. The spring is cut into two equal parts and the same mass is suspended from one of the parts. The new frequency of vibration of mass is f_2. Which of the following relations between the frequencies is correct [AFMC 2000]

[NCERT 1983; CPMT 1986; MP PMT 1991; DCE 2002]

(a) $f_1 = \sqrt{2} f_2$ (b) $f_1 = f_2$

(c) $f_1 = 2 f_2$ (d) $f_2 = \sqrt{2} f_1$

53. A mass *m* oscillates with simple harmonic motion with frequency $f = \frac{\omega}{2\pi}$ and amplitude *A* on a spring with constant *K*, therefore

(a) The total energy of the system is $\frac{1}{2} KA^2$

(b) The frequency is $\frac{1}{2\pi}\sqrt{\frac{K}{M}}$

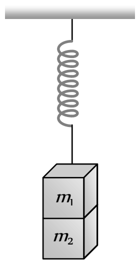
(c) The maximum velocity occurs, when $x = 0$

(d) All the above are correct

54. Two masses m_1 and m_2 are suspended together by a massless spring of constant *K*. When the masses are in equilibrium, m_1 is removed without disturbing the system. The amplitude of oscillations is

(a) $\frac{m_1 g}{K}$

(b) $\frac{m_2 g}{K}$



(c) $\frac{(m_1+m_2)g}{K}$

(d) $\frac{(m_1-m_2)g}{K}$

55. A spring executes SHM with mass of 10*kg* attached to it. The force constant of spring is 10*N/m*.If at any instant its velocity is 40*cm/sec*, the displacement will be (where amplitude is 0.5*m*)

(a) 0.09 *m* (b) 0.3 *m*

(c) 0.03 *m* (d) 0.9 *m*

Superposition of S.H.M's and Resonance

1. The S.H.M. of a particle is given by the equation $y = 3\sin\omega t + 4\cos\omega t$. The amplitude is [MP PET 1993]

(a) 7 (b) 1

(c) 5 (d) 12

2. If the displacement equation of a particle be represented by $y = A\sin PT + B\cos PT$, the particle executes [MP PET 1986]

(a) A uniform circular motion

(b) A uniform elliptical motion

(c) A S.H.M.

(d) A rectilinear motion

3. The motion of a particle varies with time according to the relation $y = a(\sin\omega t + \cos\omega t)$, then

(a) The motion is oscillatory but not S.H.M.

(b) The motion is S.H.M. with amplitude *a*

(c) The motion is S.H.M. with amplitude $a\sqrt{2}$

(d) The motion is S.H.M. with amplitude $2a$

4. The resultant of two rectangular simple harmonic motions of the same frequency and unequal amplitudes but differing in phase by $\frac{\pi}{2}$ is [BHU 2003; CPMT 2004; MP PMT 1989, 2005; BCECE 2005]

(a) Simple harmonic (b) Circular

(c) Elliptical (d) Parabolic

5. The composition of two simple harmonic motions of equal periods at right angle to each other and with a phase difference of π results in the displacement of the particle along

(a) Straight line (b) Circle

(c) Ellipse (d) Figure of eight

6. Two mutually perpendicular simple harmonic vibrations have same amplitude, frequency and phase. When they superimpose, the resultant form of vibration will be [MP PMT 1992]

(a) A circle (b) An ellipse

(c) A straight line (d) A parabola

7. The displacement of a particle varies according to the relation $x = 4(\cos\pi t + \sin\pi t)$. The amplitude of the particle is [AIEEE 2003]

(a) 8 (b) – 4

(c) 4 (d) $4\sqrt{2}$

8. A S.H.M. is represented by $x = 5\sqrt{2}(\sin 2\pi t + \cos 2\pi t)$. The amplitude of the S.H.M. is [MH CET 2004] [RPMT 2004]

(a) 10 *cm* (b) 20 *cm*

(c) $5\sqrt{2}$ *cm* (d) 50 *cm*

9. Resonance is an example of [CBSE PMT 1999; BHU 1999; 2005]

(a) Tuning fork (b) Forced vibration

(c) Free vibration (d) Damped vibration

10. In case of a forced vibration, the resonance wave becomes very sharp when the [CBSE PMT 2003]

(a) Restoring force is small

(b) Applied periodic force is small

(c) Quality factor is small

(d) Damping force is small

11. Amplitude of a wave is represented by

$$A = \frac{c}{a+b-c}$$

Then resonance will occur when [CPMT 1984]

(a) $b = -c/2$ (b) $b = 0$ and $a = -c$

(c) $b = -a/2$ (d) None of these

12. A particle with restoring force proportional to displacement and resisting force proportional to velocity is subjected to a force $F\sin\omega t$. If the amplitude of the particle is maximum for $\omega = \omega_1$ and the energy of the particle is maximum for $\omega = \omega_2$, then (where ω_0 natural frequency of oscillation of particle)

(a) $\omega_1 = \omega_0$ and $\omega_2 \neq \omega_o$ (b) $\omega_1 = \omega_0$ and $\omega_2 = \omega_o$

(c) $\omega_1 \neq \omega_0$ and $\omega_2 = \omega_o$ (d) $\omega_1 \neq \omega_0$ and $\omega_2 \neq \omega_o$

13. A simple pendulum is set into vibrations. The bob of the pendulum comes to rest after some time due to [AFMC 2003; JIPMER 1999]

(a) Air friction

(b) Moment of inertia [CBSE PMT 1990]

(c) Weight of the bob

(d) Combination of all the above

14. A simple pendulum oscillates in air with time period *T* and amplitude *A*. As the time passes [CPMT 2005]

(a) *T* and *A* both decrease

(b) *T* increases and *A* is constant

(c) *T* increases and *A* decreases

(d) *T* decreases and *A* is constant

Critical Thinking

Objective Questions

1. Two particles executes S.H.M. of same amplitude and frequency along the same straight line. They pass one another when going in opposite directions, and each time their displacement is half of their amplitude. The phase difference between them is

(a) 30° (b) 60°

(c) 90° (d) 120°

2. The displacement of a particle varies with time as $x = 12\sin\omega t - 16\sin^3\omega t$ (in *cm*). If its motion is S.H.M., then its maximum acceleration is

(a) $12\,\omega^2$ (b) $36\,\omega^2$

(c) $144\,\omega^2$ (d) $\sqrt{192}\,\omega^2$

3. A linear harmonic oscillator of force constant $2\times10^6\,N/m$ times and amplitude 0.01 *m* has a total mechanical energy of 160 *joules*. Its

[IIT JEE 1989; CPMT 1995; CBSE PMT 1996; KECT (Med.) 1999; AMU (Engg.) 2000; UPSEAT 2001]

(a) Maximum potential energy is 100 *J*

(b) Maximum K.E. is 100 *J*

(c) Maximum P.E. is 160 *J*

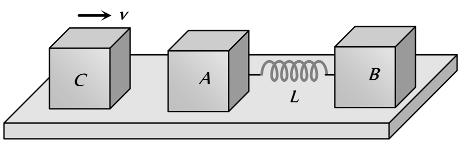
(d) Minimum P.E. is zero

4. A particle of mass *m* is executing oscillations about the origin on the *x*-axis. Its potential energy is $U(x) = k[x]^3$, where *k* is a positive constant. If the amplitude of oscillation is *a*, then its time period *T* is

(a) Proportional to $\frac{1}{\sqrt{a}}$ (b) Independent of *a*

(c) Proportional to $\sqrt{a}$ (d) Proportional to $a^{3/2}$

5. Two blocks *A* and *B* each of mass *m* are connected by a massless spring of natural length *L* and spring constant *K*. The blocks are initially resting on a smooth horizontal floor with the spring at its natural length as shown in figure. A third identical block *C* also of mass *m* moves on the floor with a speed *v* along the line joining *A* and *B* and collides with *A*. Then [IIT-JEE 1993]



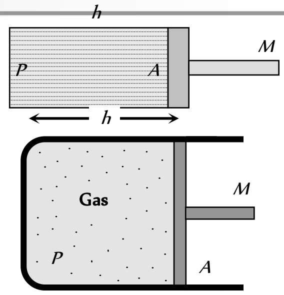
(a) The kinetic energy of the *A*-*B* system at maximum compression of the spring is zero

(b) The kinetic energy of the *A*-*B* system at maximum compression of the spring is $mv^2/4$

(c) The maximum compression of the spring is $v\sqrt{m/K}$

(d) The maximum compression of the spring is $v\sqrt{m/2K}$

6. A cylindrical piston of mass *M* slides smoothly inside a long cylinder closed at one end, enclosing a certain mass of gas. The cylinder is kept with its axis horizontal. If the piston is disturbed from its equilibrium position, it oscillates simple harmonically. The period of oscillation will be [IIT-JEE 1981]



(a) $T = 2\pi\sqrt{\left(\frac{Mh}{PA}\right)}$

(b) $T = 2\pi\sqrt{\left(\frac{MA}{Ph}\right)}$

(c) $T = 2\pi\sqrt{\left(\frac{M}{PAh}\right)}$
[MP PMT 1999]

(d) $T = 2\pi\sqrt{MPhA}$

7. A sphere of radius *r* is kept on a concave mirror of radius of curvature *R*. The arrangement is kept on a horizontal table (the surface of concave mirror is frictionless and sliding not rolling). If the sphere is displaced from its equilibrium position and left, then it executes S.H.M. The period of oscillation will be

(a) $2\pi\sqrt{\left(\frac{(R-r)1.4}{g}\right)}$ (b) $2\pi\sqrt{\left(\frac{R-r}{g}\right)}$

(c) $2\pi\sqrt{\left(\frac{rR}{a}\right)}$ (d) $2\pi\sqrt{\left(\frac{R}{gr}\right)}$

8. The amplitude of vibration of a particle is given by $a_m = (a_0)/(a\omega^2 - b\omega + c)$; where a_0, a, b and *c* are positive. The condition for a single resonant frequency is

[CPMT 1982]

(a) $b^2 = 4ac$ (b) $b^2 > 4ac$
[IIT-JEE 1998]
(c) $b^2 = 5ac$ (d) $b^2 = 7ac$

9. A *U* tube of uniform bore of cross-sectional area *A* has been set up vertically with open ends facing up. Now *m* gm of a liquid of density *d* is poured into it. The column of liquid in this tube will oscillate with a period *T* such that

(a) $T = 2\pi\sqrt{\frac{M}{g}}$ (b) $T = 2\pi\sqrt{\frac{MA}{gd}}$

(c) $T = 2\pi\sqrt{\frac{M}{gdA}}$ (d) $T = 2\pi\sqrt{\frac{M}{2Adg}}$

10. A particle is performing simple harmonic motion along *x*-axis with amplitude 4 *cm* and time period 1.2 *sec*. The minimum time taken by the particle to move from *x* =2 *cm* to *x* = + 4 *cm* and back again is given by [AIIMS 1995]

(a) 0.6 *sec* (b) 0.4 *sec*

(c) 0.3 *sec* (d) 0.2 *sec*

11. A large horizontal surface moves up and down in SHM with an amplitude of 1 *cm*. If a mass of 10 *kg* (which is placed on the surface) is to remain continually in contact with it, the maximum frequency of S.H.M. will be

[SCRA 1994; AIIMS 1995]

(a) 0.5 *Hz* (b) 1.5 *Hz*

(c) 5 *Hz* (d) 10 *Hz*

12. Due to some force F_1 a body oscillates with period 4/5 *sec* and due to other force F_2 oscillates with period 3/5 *sec*. If both forces act simultaneously, the new period will be

[RPET 1997]

(a) 0.72 *sec* (b) 0.64 *sec*

(c) 0.48 *sec* (d) 0.36 *sec*

13. A horizontal platform with an object placed on it is executing S.H.M. in the vertical direction. The amplitude of oscillation is $3.92\times10^{-3}m$. What must be the least period of these oscillations, so that the object is not detached from the platform

(a) 0.1256 *sec* (b) 0.1356 *sec*

(c) 0.1456 *sec* (d) 0.1556 *sec*

14. A particle executes simple harmonic motion (amplitude = *A*) between $x=-A$ and $x=+A$. The time taken for it to go from 0 to *A*/2 is T_1 and to go from *A*/2 to *A* is T_2. Then

[IIT-JEE (Screening) 2001]

(a) $T_1 < T_2$ (b) $T_1 > T_2$

(c) $T_1 = T_2$ (d) $T_1 = 2T_2$

15. A simple pendulum of length *L* and mass (bob) *M* is oscillating in a plane about a vertical line between angular limits $-\phi$ and $+\phi$. For an angular displacement $\theta(|\theta|<\phi)$, the tension in the string and the velocity of the bob are *T* and *v* respectively. The following relations hold good under the above conditions

(a) $T\cos\theta = Mg$

(b) $T - Mg\cos\theta = \frac{Mv^2}{L}$

(c) The magnitude of the tangential acceleration of the bob $|a_T| = g\sin\theta$

(d) $T = Mg\cos\theta$

16. Two simple pendulums of length 5 *m* and 20 *m* respectively are given small linear displacement in one direction at the same time. They will again be in the phase when the pendulum of shorter length has completed .... oscillations.

[CBSE PMT 1998; JIPMER 2001, 02]

(a) 5 (b) 1

(c) 2 (d) 3

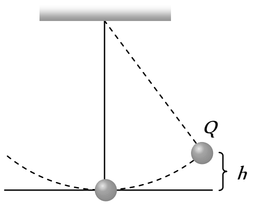
17. The bob of a simple pendulum is displaced from its equilibrium position *O* to a position *Q* which is at height *h* above *O* and the bob is then released. Assuming the mass of the bob to be *m* and time period of oscillations to be 2.0 *sec*, the tension in the string when the bob passes through *O* is

(a) $m(g+\pi\sqrt{2gh})$

(b) $m(g+\sqrt{\pi^2 gh})$

(c) $m\left(g+\sqrt{\frac{\pi^2}{2}gh}\right)$

(d) $m\left(g+\sqrt{\frac{\pi^2}{3}gh}\right)$



18. The metallic bob of a simple pendulum has the relative density ρ. The time period of this pendulum is *T*. If the metallic bob is immersed in water, then the new time period is given by

(a) $T\frac{\rho-1}{\rho}$ (b) $T\frac{\rho}{\rho-1}$

(c) $T\sqrt{\frac{\rho-1}{\rho}}$ [AIIMS 1999] (d) $T\sqrt{\frac{\rho}{\rho-1}}$

19. A clock which keeps correct time at 20^oC, is subjected to 40^oC. If coefficient of linear expansion of the pendulum is $12\times10^{-6}/^\circ C$. How much will it gain or loose in time

[BHU 1998]

(a) 10.3 *seconds / day* (b) 20.6 *seconds / day*

(c) 5 *seconds / day* (d) 20 *minutes / day*

20. The period of oscillation of a simple pendulum of length *L* suspended from the roof of a vehicle which moves without friction down an inclined plane of inclination α, is given by

[IIT-JEE (Screening) 2000]

(a) $2\pi\sqrt{\frac{L}{g\cos\alpha}}$ (b) $2\pi\sqrt{\frac{L}{g\sin\alpha}}$

(c) $2\pi\sqrt{\frac{L}{g}}$ (d) $2\pi\sqrt{\frac{L}{g\tan\alpha}}$

[IIT 1986; UPSEAT 1998]

21. The bob of a simple pendulum executes simple harmonic motion in water with a period *t*, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is (4/3) ×1000 *kg/m*. What relationship between *t* and t_0 is true [AIEEE 2004]

(a) $t = t_0$ (b) $t = t_0/2$

(c) $t = 2t_0$ (d) $t = 4t_0$

22. A spring of force constant *k* is cut into two pieces such that one piece is double the length of the other. Then the long piece will have a force constant of

[IIT-JEE (Screening) 1999]

(a) $(2/3)k$ (b) $(3/2)k$

(c) $3k$ (d) $6k$

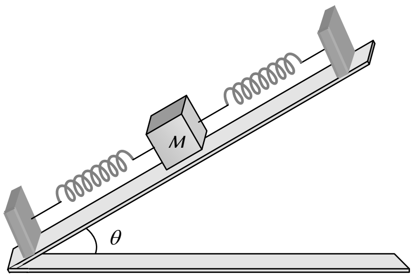
23. One end of a long metallic wire of length *L* is tied to the ceiling. The other end is tied to massless spring of spring constant *K*. *A* mass *m* hangs freely from the free end of the spring. The area of cross-section and Young's modulus of the wire are *A* and *Y* respectively. If the mass is slightly pulled down and released, it will oscillate with a time period *T* equal to [IIT 1993]

[AMU 1995]

(a) $2\pi\left(\frac{m}{K}\right)$ (b) $2\pi\left\{\frac{(YA+KL)m}{YAK}\right\}^{1/2}$

(c) $2\pi\frac{mYA}{KL}$ (d) $2\pi\frac{mL}{YA}$

24. On a smooth inclined plane, a body of mass *M* is attached between two springs. The other ends of the springs are fixed to firm supports. If each spring has force constant *K*, the period of oscillation of the body (assuming the springs as massless) is



(a) $2\pi\left(\frac{m}{2K}\right)^{1/2}$

(b) $2\pi\left(\frac{2M}{K}\right)^{1/2}$

(c) $2\pi\frac{Mg\sin\theta}{2K}$

(d) $2\pi\left(\frac{2Mg}{K}\right)^{1/2}$

25. A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0 -An external force $F(t)$ proportional to $\cos\omega t((\omega \neq \omega_0)$ is applied to the oscillator. The time displacement of the oscillator will be proportional to

(a) $\frac{m}{\omega_0^2 - \omega^2}$ (b) $\frac{1}{m(\omega_0^2 - \omega^2)}$

(c) $\frac{1}{m(\omega_1^2 + \omega^2)}$ (d) $\frac{m}{\omega_1^2 + \omega^2}$

26. A 15 g ball is shot from a spring gun whose spring has a force constant of 600 N/m. The spring is compressed by 5 cm. The greatest possible horizontal range of the ball for this compression is (g = 10 m/s) [DPMT 2004]

(a) 6.0 m (b) 10.0 m

(c) 12.0 m (d) 8.0 m

27. An ideal spring with spring-constant K is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension in the spring is

[IIT-JEE (Screening) 2002]

(a) 4 Mg/K (b) 2 Mg/K

(c) Mg/K (d) $Mg/2K$

28. The displacement y of a particle executing periodic motion is given by $y = 4\cos^2(t/2)\sin(1000t)$. This expression may be considered to be a result of the superposition of .... independent harmonic motions [IIT 1992]

(a) Two (b) Three

(c) Four (d) Five

29. Three simple harmonic motions in the same direction having the same amplitude a and same period are superposed. If each differs in phase from the next by 45^o, then

(a) The resultant amplitude is $(1+\sqrt{2})a$

(b) The phase of the resultant motion relative to the first is 90°

(c) The energy associated with the resulting motion is $(3+2\sqrt{2})$ times the energy associated with any single motion

(d) The resulting motion is not simple harmonic

30. The function $\sin^2(\omega t)$ represents [AIEEE 2005]

(a) A simple harmonic motion with a period $2\pi/\omega$

(b) A simple harmonic motion with a period π/ω

(c) A periodic but not simple harmonic motion with a period $2\pi/\omega$

(d) A periodic but not simple harmonic, motion with a period π/ω

31. A simple pendulum has time period T_1. The point of suspension is now moved upward according to equation $y = kt^2$ where $k = 1\,m/sec^2$. If new time period is T_2 then ratio $\frac{T_1^2}{T_2^2}$ will be

(a) 2/3 (b) 5/6

(c) 6/5 (d) 3/2

32. A simple pendulum is hanging from a peg inserted in a vertical wall. Its bob is stretched in horizontal position from the wall and is left free to move. The bob hits on the wall the coefficient of restitution $\frac{2}{\sqrt{5}}$ [AIEEE 2004] After how many collisions the amplitude of vibration will become less than 60°

[UPSEAT 1999]

(a) 6 (b) 3

(c) 5 (d) 4

33. A brass cube of side a and density σ is floating in mercury of density ρ. If the cube is displaced a bit vertically, it executes S.H.M. Its time period will be

(a) $2\pi\sqrt{\frac{\sigma a}{\rho g}}$ (b) $2\pi\sqrt{\frac{\rho a}{\sigma g}}$

(c) $2\pi\sqrt{\frac{\rho g}{\sigma a}}$ (d) $2\pi\sqrt{\frac{\sigma g}{\rho a}}$

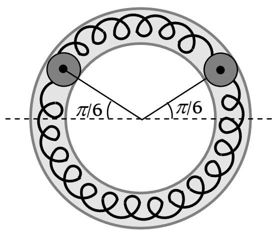
34. Two identical balls A and B each of mass 0.1 kg are attached to two identical massless springs. The spring mass system is constrained to move inside a rigid smooth pipe bent in the form of a circle as shown in the figure. The pipe is fixed in a horizontal plane. The centres of the balls can move in a circle of radius 0.06 m. Each spring has a natural length of 0.06π m and force constant 0.1N/m. Initially both the balls are displaced by an angle $\theta = \pi/6$ radian with respect to the diameter PQ of the circle and released from rest. The frequency of oscillation of the ball B is

(a) $\pi\,Hz$

(b) $\frac{1}{\pi}Hz$

(c) $2\pi\,Hz$ [IIT JEE 1999]

(d) $\frac{1}{2\pi}Hz$

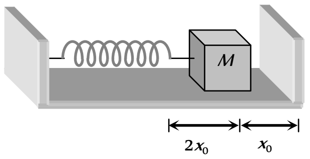


35. A disc of radius R and mass M is pivoted at the rim and is set for small oscillations. If simple pendulum has to have the same period as that of the disc, the length of the simple pendulum should be

(a) $\frac{5}{4}R$ (b) $\frac{2}{3}R$

(c) $\frac{3}{4}R$ (d) $\frac{3}{2}R$

36. One end of a spring of force constant k is fixed to a vertical wall and the other to a block of mass m resting on a smooth horizontal surface. There is another wall at a distance x_0 from the black. The spring is then compressed by $2x_0$ and released. The time taken to strike the wall is



(a) $\frac{1}{6}\pi\sqrt{\frac{k}{m}}$ (b) $\sqrt{\frac{k}{m}}$

(c) $\frac{2\pi}{3}\sqrt{\frac{m}{k}}$ (d) $\frac{\pi}{4}\sqrt{\frac{k}{m}}$

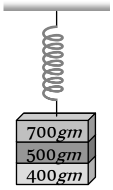
37. Three masses 700*g*, 500*g*, and 400*g* are suspended at the end of a spring a shown and are in equilibrium. When the 700*g* mass is removed, the system oscillates with a period of 3 seconds, when the 500 *gm* mass is also removed, it will oscillate with a period of

(a) 1 *s*

(b) 2 *s*

(c) 3 *s*

(d) $\sqrt{\frac{12}{5}}\ s$



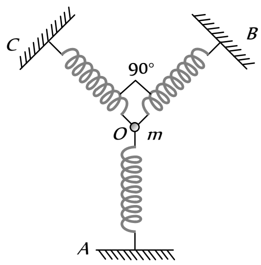
38. A particle of mass *m* is attached to three identical springs *A, B* and *C* each of force constant *k* a shown in figure. If the particle of mass *m* is pushed slightly against the spring *A* and released then the time period of oscillations is

(a) $2\pi\sqrt{\frac{2m}{k}}$

(b) $2\pi\sqrt{\frac{m}{2k}}$

(c) $2\pi\sqrt{\frac{m}{k}}$

(d) $2\pi\sqrt{\frac{m}{3k}}$



39. A hollow sphere is filled with water through a small hole in it. It is then hung by a long thread and made to oscillate. As the water slowly flows out of the hole at the bottom, the period of oscillation will

[MP PMT 1994; KCET 1994; RPET 1996; AFMC 2000; CBSE PMT 2000; CPMT 2001; AIEEE 2005]

(a) Continuously decrease

(b) Continuously increase

(c) First decrease and then increase to original value

(d) First increase and then decrease to original value

40. Two simple pendulums whose lengths are 100 *cm* and 121 *cm* are suspended side by side. Their bobs are pulled together and then released. After how many minimum oscillations of the longer pendulum, will the two be in phase again

(a) 11 (b) 10

(c) 21 (d) 20

41. The amplitude of a damped oscillator becomes half in one minute. The amplitude after 3 *minute* will be $\frac{1}{X}$ times the original, where *X* is [CPMT 1989; DPMT 2002]

(a) 2×3 (b) 2^3

(c) 3^2 (d) 3×2^2

42. Which of the following function represents a simple harmonic oscillation [AIIMS 2005]

(a) $\sin\omega t - \cos\omega t$ (b) $\sin^2\omega t$

(c) $\sin\omega t + \sin 2\omega t$ (d) $\sin\omega t - \sin 2\omega t$

43. A uniform rod of length 2.0 *m* is suspended through an end and is set into oscillation with small amplitude under gravity. The time period of oscillation is approximately

[AMU (Med.) 2000]

(a) 1.60 *sec* (b) 1.80 *sec*

(c) 2.0 *sec* (d) 2.40 *sec*

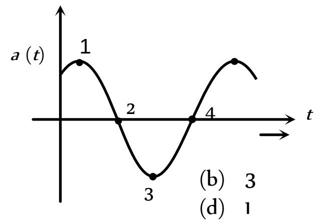
GQ Graphical Questions

1. A particle is executing S.H.M. Then the graph of acceleration as a function of displacement is

(a) A straight line (b) A circle

(c) An ellipse (d) A hyperbola

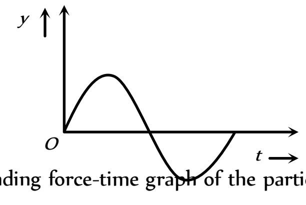
2. The acceleration *a* of a particle undergoing S.H.M. is shown in the figure. Which of the labelled points corresponds to the particle being at $-x$ [AMU (Med.) 2000]



(a) 4 (b) 3

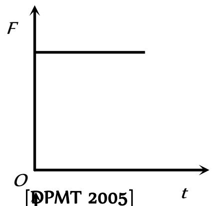
(c) 2 (d) 1

3. The displacement time graph of a particle executing S.H.M. is as shown in the figure [KCET 2003]

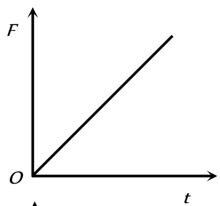


The corresponding force-time graph of the particle is

(a)

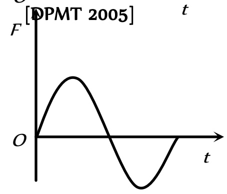


(b)

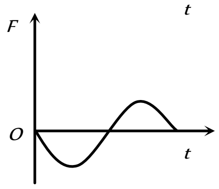


[DPMT 2005]

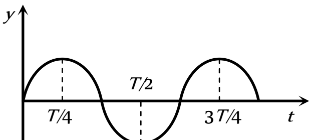
(c)



(d)

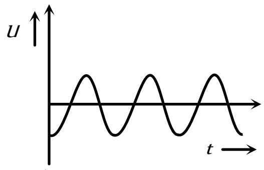
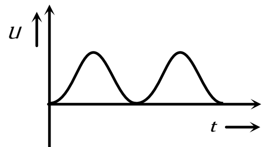
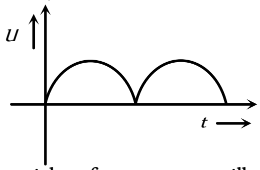
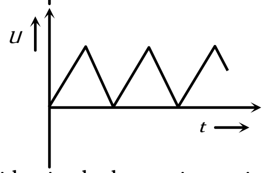


4. The graph shows the variation of displacement of a particle executing S.H.M. with time. We infer from this graph that

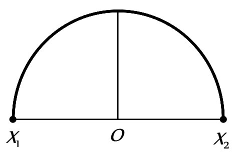
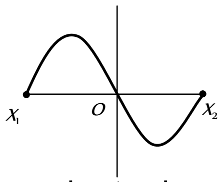
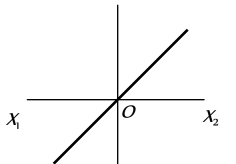
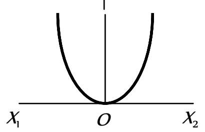


(a) The force is zero at time $3T/4$
(b) The velocity is maximum at time $T/2$
(c) The acceleration is maximum at time T
(d) The P.E. is equal to total energy at time $T/2$

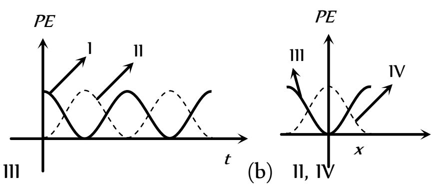
5. As a body performs S.H.M., its potential energy U varies with time as indicated in [AMU (Med.) 2001]

(a) 
(b) 
(c) 
(d) 

6. A particle of mass m oscillates with simple harmonic motion between points x_1 and x_2, the equilibrium position being O. Its potential energy is plotted. It will be as given below in the graph

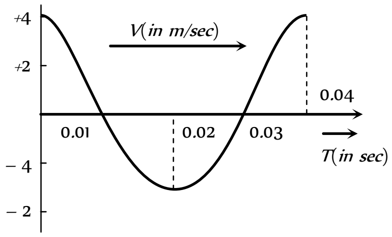
(a) 
(b) 
(c) 
(d) 

7. For a particle executing S.H.M. the displacement x is given by $x = A\cos\omega t$. Identify the graph which represents the variation of potential energy (P.E.) as a function of time t and displacement x



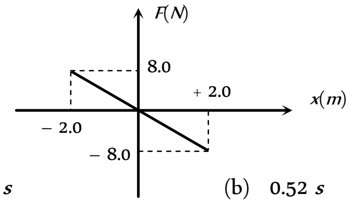
(a) I, III
(b) II, IV
(c) II, III
(d) I, IV

8. The velocity-time diagram of a harmonic oscillator is shown in the adjoining figure. The frequency of oscillation is [CPMT 1989]



(a) 25 Hz
(b) 50 Hz
(c) 12.25 Hz
(d) 33.3 Hz

9. A body of mass 0.01 kg executes simple harmonic motion (S.H.M.) about $x = 0$ under the influence of a force shown below : The period of the S.H.M. is [AMU (Med.) 2002]

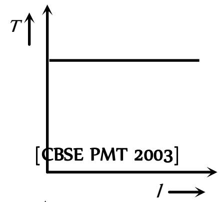
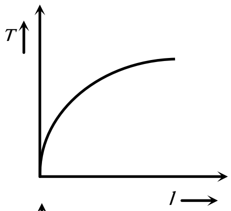
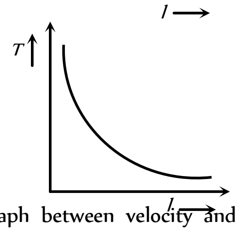
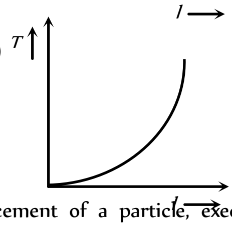


(a) 1.05 s
(b) 0.52 s
(c) 0.25 s
(d) 0.30 s

10. For a simple pendulum the graph between L and T will be. [CPMT 1992]

(a) Hyperbola
(b) Parabola
(c) A curved line
(d) A straight line

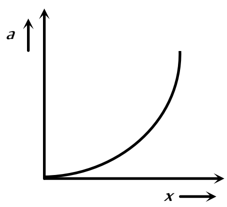
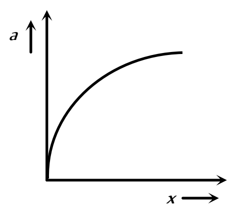
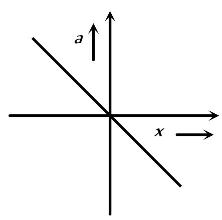
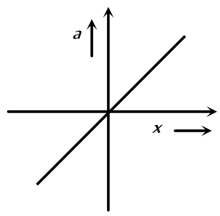
11. In case of a simple pendulum, time period versus length is depicted by [DCE 1999, 2001]

(a) 
[CBSE PMT 2003]
(b) 
(c) 
(d) 

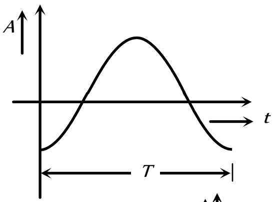
12. Graph between velocity and displacement of a particle, executing S.H.M. is [DPMT 2005]

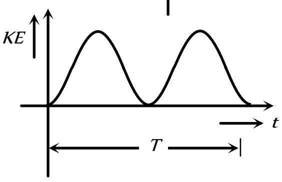
(a) A straight line
(b) A parabola
(c) A hyperbola
(d) An ellipse

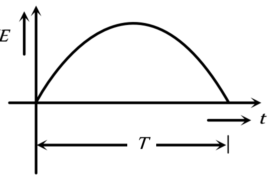
13. The variation of the acceleration a of the particle executing S.H.M. with displacement x is as shown in the figure [IIT JEE (Screening) 2003]

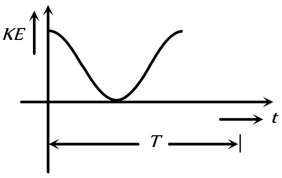
(a) 
(b) 
(c) 
(d) 

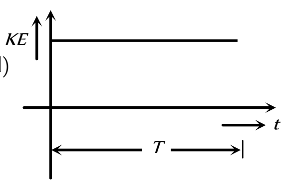
14. Acceleration A and time period T of a body in S.H.M. is given by a curve shown below. Then corresponding graph, between kinetic energy (K.E.) and time t is correctly represented by



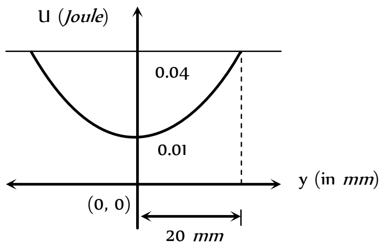
(a)


(b)


(c)


(d)


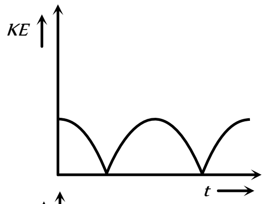
15. The variation of potential energy of harmonic oscillator is as shown in figure. The spring constant is

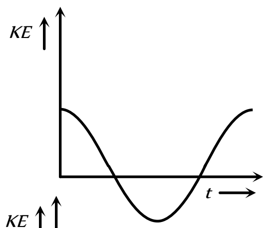


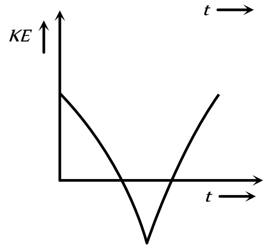
(a) 1×10^2 N/m (b) 150 N/m

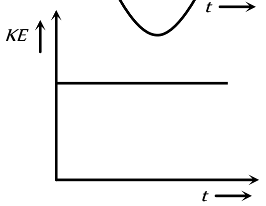
(c) 0.667×10^2 N/m (d) 3×10^2 N/m

16. A body performs S.H.M. Its kinetic energy K varies with time t as indicated by graph

(a)


(b)


(c)


(d)


AR Assertion & Reason

For AIIMS Aspirants

Read the assertion and reason carefully to mark the correct option out of the options given below:

(a) If both assertion and reason are true and the reason is the correct explanation of the assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of the assertion.
(c) If assertion is true but reason is false.
(d) If the assertion and reason both are false.
(e) If assertion is false but reason is true.

1. Assertion : All oscillatory motions are necessarily periodic motion but all periodic motion are not oscillatory.
Reason : Simple pendulum is an example of oscillatory motion.

2. Assertion : Simple harmonic motion is a uniform motion.
Reason : Simple harmonic motion is the projection of uniform circular motion.

3. Assertion : Acceleration is proportional to the displacement. This condition is not sufficient for motion in simple harmonic.
Reason : In simple harmonic motion direction of displacement is also considered.

4. Assertion : Sine and cosine functions are periodic functions.
Reason : Sinusoidal functions repeats it values after a definite interval of time.

5. Assertion : The graph between velocity and displacement for a harmonic oscillator is a parabola.
Reason : Velocity does not change uniformly with displacement in harmonic motion.

6. Assertion : When a simple pendulum is made to oscillate on the surface of moon, its time period increases.
Reason : Moon is much smaller as compared to earth.

7. Assertion : Resonance is special case of forced vibration in which the natural frequency of vibration of the body is the same as the impressed frequency of external periodic force and the amplitude of forced vibration is maximum.
Reason : The amplitude of forced vibrations of a body increases with an increase in the frequency of the externally impressed periodic force.

[AIIMS 1994]

8. Assertion : The graph of total energy of a particle in SHM *w.r.t.*, position is a straight line with zero slope.
Reason : Total energy of particle in SHM remains constant throughout its motion.

9. Assertion : The percentage change in time period is 1.5%, if the length of simple pendulum increases by 3%.
Reason : Time period is directly proportional to length of pendulum.

10. Assertion : The frequency of a second pendulum in an elevator moving up with an acceleration half the acceleration due to gravity is 0.612 s^{-}.
Reason : The frequency of a second pendulum does not depend upon acceleration due to gravity.

11. Assertion : Damped oscillation indicates loss of energy.
Reason : The energy loss in damped oscillation may be due to friction, air resistance etc.

12. Assertion : In a S.H.M., kinetic and potential energies become equal when the displacement is $1/\sqrt{2}$ times the amplitude.

Reason : In SHM, kinetic energy is zero when potential energy is maximum.

13. Assertion : If the amplitude of a simple harmonic oscillator is doubled, its total energy becomes four times.

Reason : The total energy is directly proportional to the square of amplitude of vibration of the harmonic oscillator.

14. Assertion : For an oscillating simple pendulum, the tension in the string is maximum at the mean position and minimum at the extreme position.

Reason : The velocity of oscillating bob in simple harmonic motion is maximum at the mean position.

15. Assertion : The spring constant of a spring is *k*. When it is divided into *n* equal parts, then spring constant of one piece is *k/n*.

Reason : The spring constant is independent of material used for the spring.

16. Assertion : The periodic time of a hard spring is less as compared to that of a soft spring.

Reason : The periodic time depends upon the spring constant, and spring constant is large for hard spring.

17. Assertion : In extreme position of a particle executing S.H.M., both velocity and acceleration are zero.

Reason : In S.H.M., acceleration always acts towards mean position.

18. Assertion : Soldiers are asked to break steps while crossing the bridge.

Reason : The frequency of marching may be equal to the natural frequency of bridge and may lead to resonance which can break the bridge.

[AIIMS 2001]

19. Assertion : The amplitude of oscillation can never be infinite.

Reason : The energy of oscillator is continuously dissipated.

20. Assertion : In S.H.M., the motion is 'to and fro' and periodic.

Reason : Velocity of the particle $(v) = \omega\sqrt{k^2 - x^2}$ (where x is the displacement and k is amplitude)

[AIIMS 2002]

21. Assertion : The amplitude of an oscillating pendulum decreases gradually with time

Reason : The frequency of the pendulum decreases with time [AIIMS 2003]

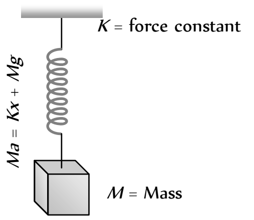
22. Assertion : In simple harmonic motion, the velocity is maximum when acceleration is minimum

Reason : Displacement and velocity of S.H.M. differ is phase by $\pi/2$ [AIIMS 1999]

23. Assertion : Consider motion for a mass spring system under gravity, motion of M is not a simple harmonic motion unless Mg is negligibly small.

Reason : For simple harmonic motion acceleration must be proportional to displacement and is directed towards the mean position

[SCRA 1994]



Answers

Displacement of S.H.M. and Phase

1	b,d	2	c	3	d	4	c	5	d
6	c	7	c	8	a	9	a,b,d	10	a
11	c	12	c	13	c	14	c	15	d
16	b	17	d	18	a	19	c	20	a
21	b	22	c	23	b	24	c	25	b
26	a								

Velocity of Simple Harmonic Motion

1	a	2	c	3	c	4	c	5	b
6	c	7	d	8	c	9	d	10	b
11	a	12	d	13	a	14	b	15	c
16	b	17	b	18	a	19	d	20	b
21	b	22	c	23	d	24	a	25	a
26	c	27	a						

Acceleration of Simple Harmonic Motion

1	d	2	c	3	c	4	d	5	a
6	a	7	a	8	d	9	d	10	d
11	a	12	a	13	d	14	a	15	a
16	d	17	d	18	d	19	b	20	c
21	c								

Energy of Simple Harmonic Motion

1	d	2	a	3	d	4	a	5	a
6	c	7	c	8	b	9	d	10	c
11	c	12	b	13	a	14	a	15	b
16	b	17	c	18	b	19	d	20	c
21	c	22	c	23	b	24	b	25	a
26	c	27	c	28	a	29	b	30	c
31	c	32	d	33	b	34	b		

Time Period and Frequency

1	b	2	c	3	b	4	d	5	b
6	a	7	d	8	d	9	d	10	a
11	b	12	b	13	b	14	b	15	a
16	d	17	d	18	d				

Simple Pendulum

1	c	2	a	3	b	4	b	5	b
6	b	7	c	8	c	9	c	10	d
11	d	12	b	13	a	14	d	15	d

16	b	17	b	18	c	19	c	20	c
21	d	22	d	23	c	24	c	25	d
26	a	27	a	28	b	29	d	30	d
31	c	32	c	33	b	34	b	35	a
36	a	37	d	38	b	39	c	40	d
41	a	42	a	43	a	44	b	45	d
46	d	47	b	48	a	49	a	50	a
51	c	52	c	53	c	54	d	55	c
56	b	57	c	58	a	59	c	60	a
61	b								

Spring Pendulum

1	d	2	d	3	b	4	b	5	b
6	c	7	c	8	b	9	a	10	c
11	a	12	b	13	d	14	c	15	a
16	d	17	a	18	d	19	b	20	c
21	c	22	c	23	d	24	a	25	d
26	d	27	b	28	a	29	a	30	a
31	b	32	d	33	b	34	b	35	b
36	d	37	c	38	d	39	b	40	c
41	a	42	b	43	b	44	a	45	b
46	a	47	b	48	d	49	c	50	c
51	b	52	d	53	d	54	a	55	b

Superposition of S.H.M's and Resonance

1	c	2	c	3	c	4	c	5	a
6	c	7	d	8	a	9	b	10	d
11	b	12	c	13	a	14	c		

Critical Thinking Questions

1	d	2	b	3	b,c	4	a	5	b,d
6	a	7	b	8	a	9	d	10	b
11	c	12	c	13	a	14	a	15	b,c
16	c	17	a	18	d	19	a	20	a
21	c	22	b	23	b	24	a	25	b
26	b	27	b	28	b	29	a,c	30	d
31	c	32	b	33	a	34	b	35	d
36	c	37	b	38	b	39	d	40	b
41	b	42	a	43	d				

Graphical Questions

1	a	2	d	3	d	4	d	5	b
6	d	7	a	8	a	9	d	10	b
11	b	12	d	13	c	14	a	15	b
16	a								

Assertion and Reason

1	b	2	e	3	a	4	a	5	e
6	b	7	c	8	a	9	c	10	c
11	b	12	b	13	a	14	b	15	e
16	a	17	e	18	a	19	a	20	b
21	c	22	b	23	e				

Answers and Solutions

Displacement of S.H.M. and Phase

1. (b,d) For S.H.M. displacement $y = a\sin\omega t$ and acceleration

$A = -\omega^2 y\sin\omega t$ these are maximum at $\omega t = \frac{\pi}{2}$.

2. (c) $v_{max} = \omega A \Rightarrow v = \frac{\omega A}{2} = \omega\sqrt{A^2 - y^2}$

$\Rightarrow A^2 - y^2 = \frac{A^2}{4} \Rightarrow y^2 = \frac{3A^2}{4} \Rightarrow y = \frac{\sqrt{3}A}{2}$

3. (d) Equation of motion is $y = 5\sin\frac{2\pi t}{6}$. For $y = 2.5\,cm$

$2.5 = 5\sin\frac{2\pi t}{6} \Rightarrow \frac{2\pi t}{6} = \frac{\pi}{6} \Rightarrow t = \frac{1}{2}sec$

and phase $= \frac{2\pi t}{6} = \frac{\pi}{6}$.

4. (c) $y = a\sin(\omega t - \alpha) = a\cos\left(\omega t - \alpha - \frac{\pi}{2}\right)$

Another equation is given $y = \cos(\omega t - \alpha)$

So, there exists a phase difference of $\frac{\pi}{2} = 90°$

5. (d) $y = a\sin(\omega t + \phi)$

$= a\sin\left(\frac{2\pi}{T}t + \phi\right) \Rightarrow y = 0.5\sin\left(\frac{2\pi}{0.4}t + \frac{\pi}{2}\right)$

$y = 0.5\sin\left(5\pi t + \frac{\pi}{2}\right) = 0.5\cos 5\pi t$

6. (c) $y = a\sin(2\pi nt + \alpha)$. Its phase at time $t = 2\pi nt + \alpha$

7. (c) From given equation $\omega = \frac{2\pi}{T} = 0.5\pi \Rightarrow T = 4\ sec$

Time taken from mean position to the maximum displacement $= \frac{1}{4}T = 1\ sec.$

8. (a) It is required to calculate the time from extreme position. Hence, in this case equation for displacement of particle can be written as $x = a\sin(\omega t + \frac{\pi}{2}) = a\cos\omega t$

$\Rightarrow \frac{a}{2} = a\cos\omega t \Rightarrow \omega t = \frac{\pi}{3} \Rightarrow \frac{2\pi}{T}.t = \frac{\pi}{3} \Rightarrow t = \frac{T}{6}$

9. (a,b,d) $x = a\sin\omega t\cos\omega t = \frac{a}{2}\sin 2\omega t$

10. (a) $v_{\max} = a\omega = a \times \frac{2\pi}{T} \Rightarrow a = \frac{v_{\max} \times T}{2\pi}$

$a = \frac{1.00 \times 10^3 \times (1 \times 10^{-5})}{2\pi} = 1.59\ mm$

11. (c)

12. (c)

13. (c) $y = a\sin\frac{2\pi}{T}t \Rightarrow \frac{a}{\sqrt{2}} = a\sin\frac{2\pi}{T}\cdot t$

$\Rightarrow \sin\frac{2\pi}{T}t = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4} \Rightarrow \frac{2\pi}{T}t = \frac{\pi}{4} \Rightarrow t = \frac{T}{8}$

14. (c)

15. (d) Standard equation of S.H.M. $\frac{d^2y}{dt^2} = -\omega^2 y$, is not satisfied by $y = a\tan\omega t$.

16. (b) $x = a\cos(\omega t + \theta)$ ....(i)

and $v = \frac{dx}{dt} = -a\omega\sin(\omega t + \theta)$ ....(ii)

Given at $t = 0$, $x = 1\,cm$ and $v = \pi$ and $\omega = \pi$

Putting these values in equation (i) and (ii) we will get $\sin\theta = \frac{-1}{a}$ and $\cos\theta = \frac{1}{a}$

$\Rightarrow \sin^2\theta + \cos^2\theta = \left(-\frac{1}{a}\right)^2 + \left(\frac{1}{a}\right)^2 \Rightarrow a = \sqrt{2}\,cm$

17. (d) $y = A\sin\omega t = \frac{A\sin 2\pi}{T}t \Rightarrow \frac{A}{2} = A\sin\frac{2\pi t}{T} \Rightarrow t = \frac{T}{12}$.

18. (a) The amplitude is a maximum displacement from the mean position.

19. (c) Equation of motion $y = a\cos\omega t$

$\Rightarrow \frac{a}{2} = a\cos\omega t \Rightarrow \cos\omega t = \frac{1}{2} \Rightarrow \omega t = \frac{\pi}{3}$

$\Rightarrow \frac{2\pi t}{T} = \frac{\pi}{3} \Rightarrow t = \frac{\frac{\pi}{3}\times T}{2\pi} = \frac{4}{3\times 2} = \frac{2}{3}\sec$

20. (a) Simple harmonic waves are set up in a string fixed at the, two ends.

21. (b)

22. (c) $v_1 = \frac{dy_1}{dt} = 0.1\times 100\pi\cos\left(100\pi t + \frac{\pi}{3}\right)$

$v_2 = \frac{dy_2}{dt} = -0.1\pi\sin\pi t = 0.1\pi\cos\left(\pi t + \frac{\pi}{2}\right)$

Phase difference of velocity of first particle with respect to the velocity of 2nd particle at $t = 0$ is

$\Delta\phi = \phi_1 - \phi_2 = \frac{\pi}{3} - \frac{\pi}{2} = -\frac{\pi}{6}$.

23. (b) $\frac{a_1}{a_2} = \frac{10}{25} = \frac{2}{5}$

24. (c) $y = a\sin\frac{2\pi}{T}t \Rightarrow \frac{a}{2} = a\sin\frac{2\pi t}{3} \Rightarrow \frac{1}{2} = \sin\frac{2\pi t}{3}$

$\Rightarrow \sin\frac{2\pi t}{3} = \sin\frac{\pi}{6} \Rightarrow \frac{2\pi t}{3} = \frac{\pi}{6} \Rightarrow t = \frac{1}{4}\sec$

25. (b)

26. (a) $x = a\sin\left(\omega t + \frac{\pi}{6}\right)$ and $x' = a\cos\omega t = a\sin\left(\omega t + \frac{\pi}{2}\right)$

$\therefore \Delta\phi = \left(\omega t + \frac{\pi}{2}\right) - \left(\omega t + \frac{\pi}{6}\right) = \frac{\pi}{3}$

Velocity of Simple Harmonic Motion

1. (a) Velocity of a particle executing S.H.M. is given by

$v = \omega\sqrt{a^2 - x^2} = \frac{2\pi}{T}\sqrt{A^2 - \frac{A^2}{4}} = \frac{2\pi}{T}\sqrt{\frac{3A^2}{4}} = \frac{\pi A\sqrt{3}}{T}$.

2. (c) $v = \omega\sqrt{(a^2 - y^2)} = 2\sqrt{60^2 - 20^2} = 113\,mm/s$.

3. (c) It is given $v_{max} = 100\,cm/sec$, $a = 10\,cm$.

$\Rightarrow v_{max} = a\omega \Rightarrow \omega = \frac{100}{10} = 10\,rad/sec$

Hence $v = \omega\sqrt{a^2 - y^2} \Rightarrow 50 = 10\sqrt{(10)^2 - y^2}$

$\Rightarrow y = 5\sqrt{3}\,cm$

4. (c) At centre $v_{max} = a\omega = a.\frac{2\pi}{T} = \frac{0.2\times 2\pi}{0.01} = 40\pi$

5. (b) $v_{max} = a\omega = a\frac{2\pi}{T} = 3\times\frac{2\pi}{6} = \pi\,cm/s$

6. (c) $v_{max} = \omega a = \frac{2\pi}{T}\times a \Rightarrow v_{max} = \frac{2\times\pi\times 2}{2} = 2\pi\ m/s$

7. (d) $v_{max} = a\omega = \frac{a.2\pi}{T} = \frac{2\pi a}{T}$

8. (c) $v = \omega\sqrt{a^2 - y^2} \Rightarrow 10 = \omega\sqrt{a^2 - (4)^2}$ and $8 = \omega\sqrt{a^2 - (5)^2}$

On solving $\omega = 2 \Rightarrow \omega = \frac{2\pi}{T} = 2 \Rightarrow T = \pi\ sec$

9. (d) From the given equation, $a = 5$ and $\omega = 4$

$\therefore v = \omega\sqrt{a^2 - y^2} = 4\sqrt{(5)^2 - (3)^2} = 16$

10. (b) $v_{max} = a\omega = a\times\frac{2\pi}{T} = (50\times 10^{-3})\times\frac{2\pi}{2} = 0.15\,m/s$

11. (a) $n = \frac{\omega}{2\pi} = \frac{220}{2\pi} = 35\,Hz$

$v_{max} = \omega a = 220\times 0.30\,m/s = 66\,m/s$

12. (d) $v_{max} = a\omega$ and $A_{max} = a\omega^2 \Rightarrow \omega = \frac{A_{max}}{v_{max}} = \frac{4}{2} = 2\,rad/sec$

13. (a) $v_{max} = a\omega = \frac{a\times 2\pi}{T} = \frac{2\times 10^{-3}\times 2\pi}{0.1} = \frac{\pi}{25}m/s$

14. (b) $A = \omega^2 y \Rightarrow \omega = \sqrt{A/y} = \sqrt{\frac{8}{2}} = 2\,rad/sec$

Now $v_{max} = a\omega = 6\times 2 = 12\,cm/sec$

15. (c) $v_{max} = a\omega \Rightarrow \omega = \frac{v_{max}}{a} = \frac{10}{4}$

Now, $v = \omega\sqrt{a^2 - y^2} \Rightarrow v^2 = \omega^2(a^2 - y^2) \Rightarrow y^2 = a^2 - \frac{v^2}{\omega^2}$

$\Rightarrow y = \sqrt{a^2 - \frac{v^2}{\omega^2}} = \sqrt{4^2 - \frac{5^2}{(10/4)^2}} = 2\sqrt{3}\ cm$

16. (b) The particles will meet at the mean position when P completes one oscillation and Q completes half an oscillation

So $\frac{v_P}{v_Q} = \frac{a\omega_P}{a\omega_Q} = \frac{T_Q}{T_P} = \frac{6}{3} = \frac{2}{1}$

17. (b) $\frac{v_{max}}{A_{max}} = \frac{a\omega}{a\omega^2} = \frac{1}{\omega}$

18. (a) Velocity is same. So by using $v = a\omega$

$\Rightarrow A_1\omega_1 = A_2\omega_2 = A_3\omega_3$

19. (d) In S.H.M. at mean position velocity is maximum

So $v = a\omega$ (maximum)

20. (b)

21. (b)

22. (c) Acceleration $A = \omega^2 y \Rightarrow \omega = \sqrt{\frac{A}{y}} = \sqrt{\frac{0.5}{0.02}} = 5$

Maximum velocity $v_{max} = a\omega = 0.1 \times 5 = 0.5$

23. (d) At mean position velocity is maximum

i.e., $v_{max} = \omega a \Rightarrow \omega = \frac{v_{max}}{a} = \frac{16}{4} = 4$

$\therefore v = \omega\sqrt{a^2 - y^2} \Rightarrow 8\sqrt{3} = 4\sqrt{4^2 - y^2}$

$\Rightarrow 192 = 16(16 - y^2) \Rightarrow 12 = 16 - y^2 \Rightarrow y = 2\,cm.$.

24. (a) $v_{max} = a\omega = 3 \times 100 = 300$

25. (a) $x = 3\sin 2t + 4\cos 2t$. From given equation

$a_1 = 3, a_2 = 4,$ and $\phi = \frac{\pi}{2}$

$\therefore a = \sqrt{a_1^2 + a_2^2} = \sqrt{3^2 + 4^2} = 5 \Rightarrow v_{max} = a\omega = 5 \times 2 = 10$

26. (c) Velocity in mean position $v = a\omega$, velocity at a distance of half amplitude.

$v' = \omega\sqrt{a^2 - y^2} = \omega\sqrt{a^2 - \frac{a^2}{4}} = \sqrt{\frac{3}{2}}\,a\omega = \sqrt{\frac{3}{2}}\,v$

27. (a) $x = A\cos\left(\omega t + \frac{\pi}{4}\right)$ and $v = \frac{dx}{dt} = -A\omega\sin\left(\omega t + \frac{\pi}{4}\right)$

For maximum speed,

$\sin\left(\omega t + \frac{\pi}{4}\right) = 1 \Rightarrow \omega t + \frac{\pi}{4} = \frac{\pi}{2}$ or $\omega t = \frac{\pi}{2} - \frac{\pi}{4} \Rightarrow t = \frac{\pi}{4\omega}$

Acceleration of Simple Harmonic Motion

1. (d) $F = -kx$

2. (c) The stone execute S.H.M. about centre of earth with time period $T = 2\pi\sqrt{\frac{R}{g}}$; where R = Radius of earth.

3. (c) Acceleration $= \omega^2 a$ at extreme position is maximum.

4. (d) $-a\omega^2$ when it is at one extreme point.

5. (a) Maximum acceleration $= a\omega^2 = a \times 4\pi^2 n^2$

$= 0.01 \times 4 \times (\pi)^2 \times (60)^2 = 144\pi^2 m/\sec$

6. (a) Maximum acceleration

$A_{max} = a\omega^2 = \frac{a \times 4\pi^2}{T^2} = \frac{1 \times 4 \times (3.14)^2}{0.2 \times 0.2}$

$F_{max} = m \times A_{max} = \frac{0.1 \times 4 \times (3.14)^2}{0.2 \times 0.2} = 98.596\ N$

7. (a) Maximum velocity $= a\omega = 16$

Maximum acceleration $= \omega^2 a = 24$

$\Rightarrow a = \frac{(a\omega)^2}{\omega^2 a} = \frac{16 \times 16}{24} = \frac{32}{3} m$

8. (d) Acceleration $\propto$ – displacement, and direction of acceleration is always directed towards the equilibrium position.

9. (d) Maximum force $= m(a\omega^2) = ma\left(\frac{4\pi^2}{T^2}\right)$

$= 0.5\left(\frac{4\pi^2}{\pi^2/25}\right) \times 0.01 = 0.5 N$

10. (d) $a_{max} = \omega^2 a = \left(\frac{\pi}{4}\right)^2 a = 0.62\ cm/sec^2$ $[\because a = 1]$

11. (a) For S.H.M. $F = -kx$.

$\therefore$ Force = Mass × Acceleration $\propto -x$

$\Rightarrow F = -Akx$; where A and k are positive constants.

12. (a) Velocity $v = a\omega = a \times 2\pi n$

$= 0.06 \times 2\pi \times 15 = 5.65\ m/s$

Acceleration $A = \omega^2 a = 4\pi^2 n^2 a = 5.32 \times 10^2 m/s^2$

13. (d) $A_{max} = a\omega^2 \Rightarrow a = \frac{A_{max}}{\omega^2} = \frac{7.5}{(3.5)^2} = 0.61\ m$

14. (a) $a = 10 \times 10^{-2} m$ and $\omega = 10\ rad/sec$

$A_{max} = \omega^2 a = 10 \times 10^{-2} \times 10^2 = 10\ m/\sec^2$

15. (a) $A_{max} = \omega^2 a$

16. (d) $A_{max} = 4\pi^2 n^2 a = 4\pi^2 \times (50)^2 \times 0.02 = 200\pi^2 m/s$

17. (d) $A = -\omega^2 y$ at mean position $y = 0$

So acceleration is minimum (zero).

18. (d) In S.H.M. $v = \sqrt{a^2 - y^2}$ and $a = -\omega^2 y$ when $y = a$

$\Rightarrow v_{min} = 0$ and $a_{max} = -\omega^2 a$

19. (b) Comparing given equation with standard equation,

$y = a\sin(\omega t + \phi)$, we get, $a = 2\,cm$, $\omega = \frac{\pi}{2}$

$\therefore A_{max} = \omega^2 A = \left(\frac{\pi}{2}\right)^2 \times 2 = \frac{\pi^2}{2} cm/s^2$.

20. (c) Velocity $v = \omega\sqrt{A^2 - x^2}$ and acceleration $= \omega^2 x$

Now given, $\omega^2 x = \omega\sqrt{A^2 - x^2} \Rightarrow \omega^2.1 = \omega\sqrt{2^2 - 1^2}$

$\Rightarrow \omega = \sqrt{3}\ \therefore\ T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{3}}$

21. (c) $a = -\omega^2 x \Rightarrow \left|\frac{a}{x}\right| = \omega^2$

Energy of Simple Harmonic Motion

1. (d) $E = \frac{1}{2} m\omega^2 a^2 \Rightarrow E \propto a^2$

2. (a) P.E. $= \frac{1}{2}m\omega^2 x^2$

It is clear P.E. will be maximum when x will be maximum *i.e.*, at $x = \pm A$

3. (d) Let x be the point where K.E. = P.E.

Hence $\frac{1}{2}m\omega^2(a^2 - x^2) = \frac{1}{2}m\omega^2 x^2$

$\Rightarrow 2x^2 = a^2 \Rightarrow x = \frac{a}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}\ cm$

4. (a) Since maximum value of $\cos^2 \omega t$ is 1.

$\therefore K_{\max} = K_o \cos^2 \omega t = K_o$

Also $K_{\max} = PE_{\max} = K_o$

5. (a) $F = -kx \Rightarrow dW = Fdx = -kxdx$

So $\int_0^W dW = \int_0^x -kx\, dx \Rightarrow W = U = -\frac{1}{2}kx^2$

6. (c) Suppose at displacement y from mean position potential energy = kinetic energy

$\Rightarrow \frac{1}{2}m(a^2 - y^2)\omega^2 = \frac{1}{2}m\omega^2 y^2$

$\Rightarrow a^2 = 2y^2 \Rightarrow y = \frac{a}{\sqrt{2}}$

7. (c) Total energy in SHM $E = \frac{1}{2}m\omega^2 a^2$; (where a = amplitude)

Potential energy $U = \frac{1}{2}m\omega^2(a^2 - y^2) = E - \frac{1}{2}m\omega^2 y^2$

When $y = \frac{a}{2} \Rightarrow U = E - \frac{1}{2}m\omega^2\left(\frac{a^2}{4}\right) = E - \frac{E}{4} = \frac{3E}{4}$

8. (b) $\frac{\text{Potential energy } (U)}{\text{Total energy } (E)} = \frac{\frac{1}{2}m\omega^2 y^2}{\frac{1}{2}m\omega^2 a^2} = \frac{y^2}{a^2}$

So $\frac{2.5}{E} = \frac{\left(\frac{a}{2}\right)^2}{a^2} \Rightarrow E = 10 J$

9. (d) Kinetic energy $T = \frac{1}{2}m\omega^2(a^2 - x^2)$

and potential energy, $V = \frac{1}{2}m\omega^2 x^2 \quad \therefore \frac{T}{V} = \frac{a^2 - x^2}{x^2}$

10. (c) $\frac{U}{U_{\max}} = \frac{\frac{1}{2}m\omega^2 y^2}{\frac{1}{2}m\omega^2 a^2} \Rightarrow \frac{1}{4} = \frac{y^2}{a^2} \Rightarrow y = \frac{a}{2}$

11. (c) Kinetic energy $K = \frac{1}{2}m\omega^2(a^2 - y^2)$

$= \frac{1}{2} \times 10 \times \left(\frac{2\pi}{2}\right)^2 [10^2 - 5^2] = 375\,\pi^2 ergs$

12. (b) $\frac{U}{E} = \frac{\frac{1}{2}m\omega^2 y^2}{\frac{1}{2}m\omega^2 a^2} = \frac{y^2}{a^2} = \frac{\left(\frac{a}{2}\right)^2}{a} = \frac{1}{4}$

13. (a)

14. (a) The time period of potential energy and kinetic energy is half that of SHM.

15. (b) If at any instant displacement is y then it is given that $U = \frac{1}{2} \times E \Rightarrow \frac{1}{2}m\omega^2 y^2 = \frac{1}{2} \times \left(\frac{1}{2}m\omega^2 a^2\right)$

$\Rightarrow y = \frac{a}{\sqrt{2}} = \frac{6}{\sqrt{2}} = 4.2\,cm$

16. (b) So $a = 6cm,\ \omega = 100 rad/\sec$

$K_{\max} = \frac{1}{2}m\omega^2 a^2 = \frac{1}{2} \times 1 \times (100)^2 \times (6 \times 10^{-2})^2 = 18\ J$

17. (c) In S.H.M., frequency of K.E. and P.E.

= 2 × (Frequency of oscillating particle)

18. (b) Total energy $U = \frac{1}{2}Ka^2$

19. (d) $\frac{U}{E} = \frac{\frac{1}{2}m\omega^2 y^2}{\frac{1}{2}m\omega^2 a^2} = \frac{y^2}{a^2} \Rightarrow \frac{U}{80} = \frac{\left(\frac{3}{4}a\right)^2}{a^2} = \frac{9}{16} \Rightarrow U = 45\ J$

20. (c)

21. (c) Maximum potential energy position is $y = \pm a$

and maximum kinetic energy position is $y = 0$

22. (c) $Mg = Kl \Rightarrow U_{\max} = \frac{1}{2}Kl^2 = \frac{1}{2}mgl$

23. (b) $\frac{U}{E} = \frac{\frac{1}{2}m\omega^2 y^2}{\frac{1}{2}m\omega^2 a^2} = \frac{y^2}{a^2} = \frac{\left(\frac{a}{2}\right)^2}{a^2} = \frac{1}{4} \Rightarrow U = \frac{E}{4}$

24. (b) In S.H.M., at mean position *i.e.* at $x = 0$ kinetic energy will be maximum and *pE* will be minimum. Total energy is always constant.

25. (a) In SHM for a complete cycle average value of kinetic energy and potential energy are equal *i.e.* $\langle E \rangle = \langle U \rangle = \frac{1}{4}m\omega^2 a^2$

26. (c) Total energy $= \frac{1}{2}m\omega^2 a^2$ = constant

27. (c) Kinetic energy at mean position,

$K_{\max} = \frac{1}{2}mv_{\max}^2. \Rightarrow v_{\max} = \sqrt{\frac{2K_{\max}}{m}}$

$= \sqrt{\frac{2 \times 16}{0.32}} = \sqrt{100} = 10\,m/s.$

28. (a) $E = \frac{1}{2}ma^2\omega^2 = \frac{1}{2}ma^2\left(\frac{4\pi^2}{T^2}\right) \Rightarrow E \propto \frac{a^2}{T^2}$

29. (b) $\frac{K}{E} = \frac{\frac{1}{2}m\omega^2(a^2 - y^2)}{\frac{1}{2}m\omega^2 a^2} = \frac{a^2 - y^2}{a^2} = 1 - \frac{y^2}{a^2}$

So, $\frac{\left(\frac{3E}{4}\right)}{E} = 1 - \frac{y^2}{a^2} \Rightarrow \frac{y^2}{a^2} = 1 - \frac{3}{4} = \frac{1}{4} \Rightarrow y = \frac{a}{2}$.

30. (c) Kinetic energy $K = \frac{1}{2}mv^2 = \frac{1}{2}ma^2\omega^2\cos^2\omega t$

$= \frac{1}{2}m\omega^2a^2(1+\cos 2\omega t)$ hence kinetic energy varies periodically with double the frequency of S.H.M. *i.e.* 2ω.

31. (c) $E = \frac{1}{2}m\omega^2a^2 \Rightarrow \frac{E'}{E} = \frac{a'^2}{a^2} \Rightarrow \frac{E'}{E} = \frac{\left(\frac{3}{4}a\right)^2}{a^2}$ $\left(\because a' = \frac{3}{4}a\right)$

$\Rightarrow E' = \frac{9}{16}E$

32. (d) In simple harmonic motion, energy changes from kinetic to potential and potential to kinetic but the sum of two always remains constant.

33. (b) Body collides elastically with walls of room. So, there will be no loss in its energy and it will remain colliding with walls of room, so it's motion will be periodic.

There is no change in energy of the body, hence there is no acceleration, so it's motion is not SHM.

34. (b) $E_1 = \frac{1}{2}Kx^2 \Rightarrow x = \sqrt{\frac{2E_1}{K}}$, $E_2 = \frac{1}{2}Ky^2 \Rightarrow y = \sqrt{\frac{2E_2}{K}}$

and $E = \frac{1}{2}K(x+y)^2 \Rightarrow x+y = \sqrt{\frac{2E}{K}}$

$\Rightarrow \sqrt{\frac{2E_1}{K}} + \sqrt{\frac{2E_2}{K}} = \sqrt{\frac{2E}{K}} \Rightarrow \sqrt{E_1} + \sqrt{E_2} = \sqrt{E}$

Time Period and Frequency

1. (b) In the given case, $\frac{\text{Displacement}}{\text{Acceleration}} = \frac{1}{b}$

$\therefore$ Time period $T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}} = \frac{2\pi}{\sqrt{b}}$

2. (c) On comparing with standard equation $\frac{d^2y}{dt^2} + \omega^2 y = 0$ we get $\omega^2 = K \Rightarrow \omega = \frac{2\pi}{T} = \sqrt{K} \Rightarrow T = \frac{2\pi}{\sqrt{K}}$.

3. (b) Ball execute S.H.M. inside the tunnel with time period $T = 2\pi\sqrt{\frac{R}{g}} = 84.63$ min

Hence time to reach the ball from one end to the other end of the tunnel $t = \frac{84.63}{2} = 42.3$ min.

4. (d) Given max velocity $\omega a = 1$ and maximum acceleration $\omega^2 a = 1.57$

$\therefore \frac{\omega^2 a}{\omega a} = 1.57 \Rightarrow \omega = 1.57 \Rightarrow \frac{2\pi}{T} = 1.57 \Rightarrow T = 4$

5. (b) $\omega = \frac{2\pi}{T} = 100\pi \Rightarrow T = 0.02$ sec

6. (a) At mean position, the kinetic energy is maximum.

Hence $\frac{1}{2}ma^2\omega^2 = 16$

On putting the values we get $\omega = 10 \Rightarrow T = \frac{2\pi}{\omega} = \frac{\pi}{5}$ sec

7. (d) $T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\frac{3}{12}} = \pi = 3.14$ sec

8. (d) $\omega = \sqrt{\frac{k}{m}} \Rightarrow \frac{\omega_2}{\omega_1} = \sqrt{\frac{m_1}{m_2}} \Rightarrow 2 = \sqrt{\frac{m_1}{m_2}} \Rightarrow m_2 = \frac{m_1}{4}$

9. (d)

10. (a) $\omega = \sqrt{\frac{\text{Acceleration}}{\text{Displacement}}} = \sqrt{\frac{2.0}{0.02}} = 10\, rad\, s^{-1}$

11. (b) From given equation $\omega = 3000$, $\Rightarrow n = \frac{\omega}{2\pi} = \frac{3000}{2\pi}$

12. (b)

13. (b) Given, $v = \pi\, cm/\sec$, $x = 1\, cm$ and $\omega = \pi s^{-1}$

using $v = \omega\sqrt{a^2 - x^2} \Rightarrow \pi = \pi\sqrt{a^2 - 1}$

$\Rightarrow 1 = a^2 - 1 \Rightarrow a = \sqrt{2}\, cm$.

14. (b) Length of the line = Distance between extreme positions of oscillation = 4 *cm*

So, Amplitude $a = 2\, cm$.

also $v_{\max} = 12\, cm/s$.

$\because v_{\max} = \omega a = \frac{2\pi}{T}a$

$\Rightarrow T = \frac{2\pi a}{v_{\max}} = \frac{2 \times 3.14 \times 2}{12} = 1.047$ sec

15. (a) Comparing given equation with standard equation,

$x = a\cos(\omega t + \phi)$ we get, $a = 0.01$ and $\omega = \pi$

$\Rightarrow 2\pi n = \pi \Rightarrow n = 0.5\, Hz$

16. (d) $y = 5\sin(\pi t + 4\pi)$, comparing it with standard equation

$y = a\sin(\omega t + \phi) = a\sin\left(\frac{2\pi t}{T} + \phi\right)$

$a = 5\, m$ and $\frac{2\pi t}{T} = \pi t \Rightarrow T = 2$ *sec*.

17. (d) $v_{\max} = a\omega = a \times 2\pi n \Rightarrow n = \frac{v_{\max}}{2\pi a} = \frac{31.4}{2 \times 3.14 \times 5} = 1\, Hz$

18. (d) From the given equitation $\omega = 2\pi n = 4\pi \Rightarrow n = 2Hz$

Simple Pendulum

1. (c) $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \sqrt{l}$

2. (a) Inside the mine g decreases

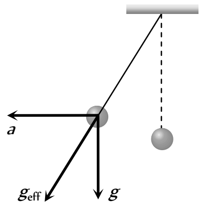
hence from $T = 2\pi\sqrt{\frac{l}{g}}$; T increase

3. (b) When a little mercury is drained off, the position of *c.g.* of ball falls (*w.r.t.* fixed and) so that effective length of pendulum increases hence T increase.

4. (b) Initially time period was $T = 2\pi\sqrt{\frac{l}{g}}$.

When train accelerates, the effective value of g becomes $\sqrt{(g^2 + a^2)}$ which is greater than g

Hence, new time period, becomes less than the initial time period.



5. (b) As we know $g = \frac{GM}{R^2}$

$$\Rightarrow \frac{g_{\text{earth}}}{g_{\text{planet}}} = \frac{M_e}{M_p} \times \frac{R_\rho^2}{R_e^2} \Rightarrow \frac{g_e}{g_p} = \frac{2}{1}$$

Also $T \propto \frac{1}{\sqrt{g}} \Rightarrow \frac{T_e}{T_p} = \sqrt{\frac{g_p}{g_e}} \Rightarrow \frac{2}{T_p} = \sqrt{\frac{1}{2}}$

$$\Rightarrow T_p = 2\sqrt{2}\ \text{sec}.$$

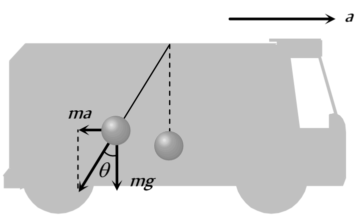
6. (b) In accelerated frame of reference, a fictitious force (pseudo force) ma acts on the bob of pendulum as shown in figure.

Hence,

$\tan\theta = \frac{ma}{mg} = \frac{a}{g}$

$\Rightarrow$

$\theta = \tan^{-1}\left(\frac{a}{g}\right)$ in the backward direction.



7. (c)

8. (c) $T = 2\pi\sqrt{\frac{l}{g}}$ (Independent of mass)

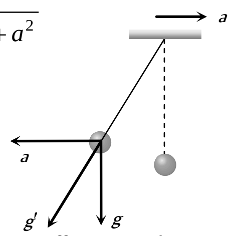
9. (c) In stationary lift $T = 2\pi\sqrt{\frac{l}{g}}$

In upward moving lift $T' = 2\pi\sqrt{\frac{l}{(g+a)}}$

(a = Acceleration of lift)

$$\Rightarrow \frac{T'}{T} = \sqrt{\frac{g}{g+a}} = \sqrt{\frac{g}{\left(g + \frac{g}{4}\right)}} = \sqrt{\frac{4}{5}} \Rightarrow T' = \frac{2T}{\sqrt{5}}$$

10. (d) $g' = \sqrt{g^2 + a^2}$



11. (d) In the given case effective acceleration $g_{eff} = 0 \Rightarrow T = \infty$

12. (b) $p_{\max} = \sqrt{2m\,E_{\max}}$

13. (a) $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow \sqrt{\frac{l}{g}}$ = constant

$$\Rightarrow l \propto g; \Rightarrow \frac{l_m}{1} = \frac{1}{6}\frac{g}{g} \Rightarrow l_m = \frac{1}{6}m$$

14. (d) $T \propto \sqrt{l} \Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l} = \frac{0.02}{2} = 0.01 \Rightarrow \Delta T = 0.01T$

Loss of time per day $= 0.01 \times 24 \times 60 \times 60 = 864\ \text{sec}$

15. (d) $\frac{T'}{T} = \sqrt{\frac{g}{g'+a}} = \sqrt{\frac{g}{g+5g}} = \sqrt{\frac{1}{6}} \Rightarrow T' = \frac{T}{\sqrt{6}}$

16. (b) $T \propto \sqrt{l} \Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l} = \frac{1}{2} \times 1\% = 0.5\%$

17. (b) At B, the velocity is maximum using conservation of mechanical energy

$$\Delta PE = \Delta KE \Rightarrow mgH = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gH}$$

18. (c)

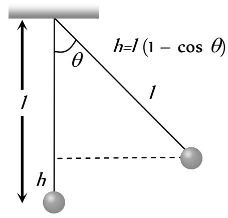
19. (c) If suppose bob rises up to a height h as shown then after releasing potential energy at extreme position becomes kinetic energy of mean position

$$\Rightarrow mgh = \frac{1}{2}mv_{\max}^2 \Rightarrow v_{\max} = \sqrt{2gh}$$

Also, from figure $\cos\theta = \frac{l-h}{l}$

$\Rightarrow h = l(1 - \cos\theta)$

So, $v_{\max} = \sqrt{2gl(1-\cos\theta)}$

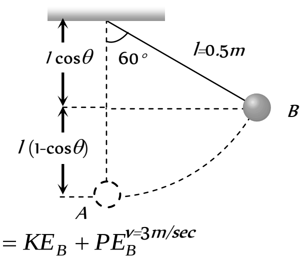


20. (c) $T = 2\pi\sqrt{\frac{l}{g}}$; for freely falling system effective $g = 0$

so $T = \infty$ or $n = 0$

It means that pendulum does not oscillate at all.

21. (d) Let bob velocity be v at point B where it makes an angle of 60° with the vertical, then using conservation of mechanical energy



$$KE_A + PE_A = KE_B + PE_B$$

$$\Rightarrow \frac{1}{2}m \times 3^2 = \frac{1}{2}mv^2 + mgl(1 - \cos\theta)$$

$$\Rightarrow 9 = v^2 + 2 \times 10 \times 0.5 \times \frac{1}{2} \Rightarrow v = 2\ m/s$$

22. (d) $T \propto \sqrt{l} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}} \Rightarrow \frac{2}{T_2} = \sqrt{\frac{l}{4l}} \Rightarrow T_2 = 4\,sec$

23. (c) Remains the same because time period of simple pendulum T is independent of mass of the bob

24. (c) $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow \frac{l}{T^2} = \frac{g}{4\pi^2}$ = constant

25. (d) $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \frac{1}{\sqrt{g}} \Rightarrow \frac{T_P}{T_e} = \sqrt{\frac{g_e}{g_P}} = \sqrt{\frac{2}{1}} \Rightarrow T' = \sqrt{2}T$

26. (a) If initial length $l_1 = 100$ then $l_2 = 121$

By using $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}}$

Hence, $\frac{T_1}{T_2} = \sqrt{\frac{100}{121}} \Rightarrow T_2 = 1.1\,T_1$

% increase = $\frac{T_2 - T_1}{T_1} \times 100 = 10\%$

27. (a) $\frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{100}{400}}$ (If $l_1 = 100$ then $l_2 = 400$)

$\Rightarrow T_2 = 2T_1$

Hence % increase $= \frac{T_2 - T_1}{T_1} \times 100 = 100\%$

28. (b) $T = 2\pi\sqrt{l/g} \Rightarrow l = \frac{gT^2}{4\pi^2} = \frac{9.8 \times 4}{4 \times \pi^2} = 99\ cm$

29. (d) This is the case of freely falling lift and in free fall of lift effective g for pendulum will be zero. So $T = 2\pi\sqrt{\frac{l}{0}} = \infty$

30. (d) After standing centre of mass of the oscillating body will shift upward therefore effective length will decrease and by $T \propto \sqrt{l}$, time period will decrease.

31. (c) $T = 2\pi\sqrt{l/g} = 2\pi\sqrt{\frac{1}{\pi^2}}$ = 2 *sec*

32. (c) Time period is independent of mass of pendulum.

33. (b) $T \propto \sqrt{l}$ Time period depends only on effective length. Density has no effect on time period. If length made 4 times then time period becomes 2 times.

34. (b) Time period is independent of mass of bob of pendulum.

35. (a) At the surface of moon, g decreases hence time period increases $\left(\text{as } T \propto \frac{1}{\sqrt{g}}\right)$

36. (a) When lift falls freely effective acceleration and frequency of oscillations be zero

$g_{eff} = 0 \Rightarrow T' = \infty$, hence a frequency = 0.

37. (d) Effective value of 'g' remains unchanged.

38. (b) $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}} = \sqrt{\frac{144}{100}} = \frac{12}{10}$

$\Rightarrow T_2 = 1.2\ T_1$

Hence % increase $= \frac{T_2 - T_1}{T_1} \times 100 = 20\%$

39. (c) If amplitude is large motion will not remain simple harmonic.

40. (d) Minimum velocity is zero at the extreme positions.

41. (a) At the time $t = \frac{T}{4} = \frac{4}{4} = 1\sec$ after passing from mean position, the body reaches at it's extreme position. At extreme, position velocity of body becomes zero.

42. (a) No momentum will be transferred because, at extreme position the velocity of bob is zero.

43. (a) In this case frequency of oscillation is given by $n = \frac{1}{2\pi}\sqrt{\frac{\sqrt{g^2 + a^2}}{l}}$ where a is the acceleration of car. If a increases then n also increases.

44. (b) As periodic time is independent of amplitude.

45. (d) Frequency $n \propto \frac{1}{\sqrt{l}} \Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{l_2}{l_1}} \Rightarrow \frac{l_1}{l_2} = \frac{n_2^2}{n_1^2} = \frac{3^2}{2^2} = \frac{9}{4}$

46. (d) Suppose at $t = 0$, pendulums begins to swing simultaneously. Hence, they will again swing simultaneously if $n_1T_1 = n_2T_2$

$\Rightarrow \frac{n_1}{n_2} = \frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}} \Rightarrow \frac{l_1}{l_2} = \left(\frac{n_2}{n_1}\right)^2 = \left(\frac{8}{7}\right)^2 = \frac{64}{49}$

47. (b) $T \propto \frac{1}{\sqrt{g}}$ and g is same in both cases so time period remain same.

48. (a) $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \sqrt{l}$, hence if l made 9 times T becomes 3 times.

Also time period of simple pendulum does not depends on the mass of the bob.

49. (a) As we go from equator to pole the value of g increases. Therefore time period of simple pendulum $\left(T \propto \frac{1}{\sqrt{g}}\right)$ decreases. $\left(\because T \propto \frac{1}{\sqrt{g}}\right)$

50. (a) If v is velocity of pendulum at Q and 10% energy is lost while moving from P to Q

Hence, by applying conservation of between P and Q

$\frac{1}{2}mv^2 = 0.9\,(mgh) \Rightarrow v^2 = 2 \times 0.9 \times 10 \times 2 \Rightarrow v = 6\,m/sec$

51. (c) $T \propto \frac{1}{\sqrt{g}} \Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{g_1}{g_2}} = \sqrt{\left(\frac{g}{g/4}\right)} \Rightarrow T_2 = 2T_1 = 2T$

52. (c) For stationary lift $T_1 = 2\pi\sqrt{\frac{l}{g}}$

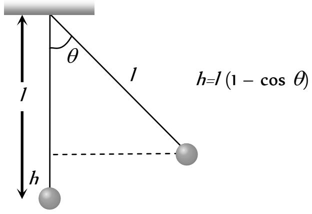
For ascending lift with acceleration a, $T_2 = 2\pi\sqrt{\frac{l}{g + a}}$

$\Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{g + a}{g}} \Rightarrow \frac{T}{T_2} = \sqrt{\frac{g + \frac{g}{3}}{g}} = \sqrt{\frac{4}{3}} \Rightarrow T_2 = \frac{\sqrt{3}}{2}T$

53. (c) $T \propto \sqrt{l}$

54. (d) Kinetic energy will be maximum at mean position.

From law of conservation of energy maximum kinetic energy at mean position = Potential energy at displaced position



$\Rightarrow K_{\max} = mgh = mgl(1 - \cos\theta)$

55. (c)

56. (b) As it is clear that in vacuum, the bob will not experience any frictional force. Hence, there shall be no dissipation therefore, it will oscillate with constant amplitude.

57. (c) The effective acceleration in a lift descending with acceleration $\frac{g}{3}$ is $g_{eff} = g - \frac{g}{3} = \frac{2g}{3}$

$\therefore\ T = 2\pi\sqrt{\left(\frac{L}{g_{eff}}\right)} = 2\pi\sqrt{\left(\frac{L}{2g/3}\right)} = 2\pi\sqrt{\left(\frac{3L}{2g}\right)}$

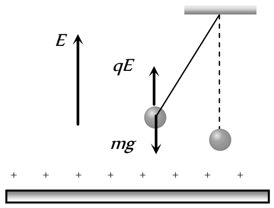
58. (a)

59. (c) According to the principle of conservation of energy, $\frac{1}{2}mv^2 = mgh$ or $v = \sqrt{2gh} = \sqrt{2\times 9.8\times 0.1} = 1.4\, m/s.$

60. (a) In this case time period of pendulum becomes

$T' = 2\pi\sqrt{\frac{l}{\left(g + \frac{qE}{m}\right)}}$

$\Rightarrow T' < T$



61. (b) In deep mine $g' = g\left(1 - \frac{d}{R}\right)$; *i.e., g* decreases so according to $n \propto \sqrt{g}$, frequency also decreases.

Spring Pendulum

1. (d) Maximum velocity $= a\omega = a\sqrt{\frac{k}{m}}$

Given that $a_1\sqrt{\frac{K_1}{m}} = a_2\sqrt{\frac{K_2}{m}} \Rightarrow \frac{a_1}{a_2} = \sqrt{\frac{K_2}{K_1}}$

2. (d) Given spring system has parallel combination, so

$k_{eq} = k_1 + k_2$ and time period $T = 2\pi\sqrt{\frac{m}{(k_1+k_2)}}$

3. (b) $T = 2\pi\sqrt{\frac{m}{k}}$. Also spring constant $(k) \propto \frac{1}{\text{Length}(l)}$, when the spring is half in length, then k becomes twice.

$\therefore T' = 2\pi\sqrt{\frac{m}{2k}} \Rightarrow \frac{T'}{T} = \frac{1}{\sqrt{2}} \Rightarrow T' = \frac{T}{\sqrt{2}}$

4. (b) $\omega = \sqrt{\frac{k}{m}}$

5. (b) With respect to the block the springs are connected in parallel combination.

$\therefore$ Combined stiffness $k = k + k$ and $n = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}$

6. (c) $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow \frac{n_S}{n_P} = \sqrt{\frac{k_S}{k_P}} \Rightarrow \frac{n_s}{n_p} = \sqrt{\frac{\left(\frac{k}{2}\right)}{2k}} = \frac{1}{2}$

7. (c) In series $k_{eq} = \frac{k_1k_2}{k_1+k_2}$ so time period $T = 2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}}$

8. (b) Force constant $k = \frac{F}{x} = \frac{0.5\times 10}{0.2} = 25\ N/m$

Now $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.25}{25}} = 0.628\ sec$

9. (a) $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow m \propto T^2 \Rightarrow \frac{m_2}{m_1} = \frac{T_2^2}{T_1^2}$

$\Rightarrow \frac{M+m}{M} = \left(\frac{\frac{5}{4}T}{T}\right)^2 \Rightarrow \frac{m}{M} = \frac{9}{16}$

10. (c) Spring constant $(k) \propto \frac{1}{\text{Length of the spirng}(l)}$

as length becomes half, k becomes twice is $2k$.

11. (a) $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow \frac{n}{n'} = \sqrt{\frac{k}{m}\times\frac{m'}{K'}} = \sqrt{\frac{k}{m}\times\frac{2m}{2K}} = 1 \Rightarrow n' = n$

12. (b) As mg produces extension x, hence $k = \frac{mg}{x}$

$\therefore\ T = 2\pi\sqrt{\frac{(M+m)}{k}} = 2\pi\sqrt{\frac{(M+m)x}{mg}}$

13. (d) For the given figure $f = \frac{1}{2\pi}\sqrt{\frac{k_{eq}}{m}} = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$ .....(i)

If one spring is removed, then $k = k$ and

$f' = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ ....(ii)

From equation (i) and (ii), $\frac{f}{f'} = \sqrt{2} \Rightarrow f' = \frac{f}{\sqrt{2}}$

14. (c) $\because\ mg = kx \Rightarrow \frac{m}{k} = \frac{x}{g} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{x}{g}}$

$= 2\pi\sqrt{\frac{9.8\times 10^{-2}}{9.8}} = \frac{2\pi}{10}\ sec$

15. (a) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.2}{80}} = 0.31\ sec$

16. (d) Spring is cut into two equal halves so spring constant of each part = $2k$

These parts are in parallel so $K_{eq} = 2K + 2K = 4K$

Extension force (*i.e.* W) is same hence by using $F = kx \Rightarrow$ $4k \times x' = kx \Rightarrow x' = \frac{x}{4}$.

17. (a) In this case springs are in parallel, so $k_{eq} = k_1 + k_2$

and $\omega = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{k_1 + k_2}{m}}$

18. (d) Force constant $(k) \propto \frac{1}{\text{Length of the spring}(l)}$

$\Rightarrow \frac{k_1}{k_2} = \frac{l_2}{l_1} = \frac{2}{1}$

19. (b) Standard equation for given condition

$x = a\cos\frac{2\pi}{T}t \Rightarrow x = -0.16\cos(\pi t)$

[As $a = -0.16$ *meter*, $T = 2$ *sec*]

20. (c) By using conservation of mechanical energy

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \Rightarrow x = v\sqrt{m/k}$

21. (c) Given elastic energies are equal *i.e.*, $\frac{1}{2}k_1x_1^2 = \frac{1}{2}k_2x_2^2$

$\Rightarrow \frac{k_1}{k_2} = \left(\frac{x_2}{x_1}\right)^2$ and using $F = kx$

$\Rightarrow \frac{F_1}{F_2} = \frac{k_1x_1}{k_2x_2} = \frac{k_1}{k_2} \times \sqrt{\frac{k_2}{k_1}} = \sqrt{\frac{k_1}{k_2}}$

22. (c) $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow n \propto \frac{1}{\sqrt{m}} \Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{m_2}{m_1}}$

$\Rightarrow \frac{n}{n_2} = \sqrt{\frac{4m}{m}} \Rightarrow n_2 = \frac{n}{2}$

23. (d) $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{4m}{m}} = 2 \Rightarrow T_2 = 2 \times 2 = 4s$

24. (a) $T \propto \frac{1}{\sqrt{k}} \Rightarrow T_1 : T_2 : T_3 = \frac{1}{\sqrt{k}} : \frac{1}{\sqrt{k/2}} : \frac{1}{\sqrt{2k}} = 1 : \sqrt{2} : \frac{1}{\sqrt{2}}$

25. (d) $T \propto \frac{1}{\sqrt{k}} \Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{k_1}{k_2}} = \sqrt{\frac{k}{4k}} = \frac{1}{2} \Rightarrow T_2 = \frac{T_1}{2}$

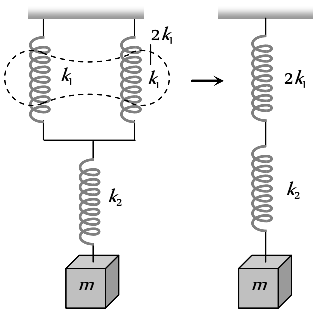
26. (d) The time period of oscillation of a spring does not depend on gravity.

27. (b) In series combination

$\frac{1}{k_S} = \frac{1}{2k_1} + \frac{1}{k_2}$

$\Rightarrow$

$k_S = \left[\frac{1}{2k_1} + \frac{1}{k_2}\right]^{-1}$



28. (a) Work done in stretching $(W) \propto$ Stiffness of spring (*i.e.* k)

$\because k_A > k_B \Rightarrow W_A > W_B$

29. (a) When external force is applied, one spring gets extended and another one gets contracted by the same distance hence force due to two springs act in same direction.

i.e. $F = F_1 + F_2 \Rightarrow -kx = -k_1x - k_2x \Rightarrow k = k_1 + k_2$

30. (a) $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{m_2}{m_1}} \Rightarrow \frac{3}{2} = \sqrt{\frac{m+2}{m}} \Rightarrow \frac{9}{4} = \frac{m+2}{m}$

$\Rightarrow m = \frac{8}{5}kg = 1.6\ kg$

31. (b) For series combination $k_{eq} = \frac{k_1k_2}{k_1 + k_2}$

$F = k_{eq}x \Rightarrow mg = \left(\frac{k_1k_2}{k_1 + k_2}\right)x \Rightarrow x = \frac{mg(k_1 + k_2)}{k_1k_2}$

32. (d) $n = \frac{1}{2\pi}\sqrt{\frac{k_{eq}}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_1k_2}{(k_1 + k_2)m}}$

33. (b) Using $F = kx \Rightarrow 10g = k \times 0.25 \Rightarrow k = \frac{10g}{0.25} = 98 \times 4$

Now $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow m = \frac{T^2}{4\pi^2}k$

$\Rightarrow m = \frac{\pi^2}{100} \times \frac{1}{4\pi^2} \times 98 \times 4 = 0.98\ kg$

34. (b) When spring is cut into n equal parts then spring constant of each part will be nk and so using $T \propto \frac{1}{\sqrt{k}}$, time period will be $T/\sqrt{n}$.

35. (b) By using $K \propto \frac{1}{l}$

Since one fourth length is cut away so remaining length is $\frac{3}{4}th$, hence k becomes $\frac{4}{3}$ times *i.e.*, $k' = \frac{4}{3}x$.

36. (d) $t_1 = 2\pi\sqrt{\frac{m}{K_1}}$ and $t_2 = 2\pi\sqrt{\frac{m}{K_2}}$

Equivalent spring constant for shown combination is $K_1 + K_2$. So time period t is given by $t = 2\pi\sqrt{\frac{m}{K_1 + K_2}}$

By solving these equations we get $t^{-2} = t_1^{-2} + t_2^{-2}$

37. (c) $n = \frac{1}{2\pi}\sqrt{\frac{K_{effective}}{m}} = \frac{1}{2\pi}\sqrt{\frac{(K + 2K)}{m}} = \frac{1}{2\pi}\sqrt{\frac{3K}{m}}$

38. (d) In series combination

$\frac{1}{k_S} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{k_2 + k_1}{k_1k_2} \Rightarrow k_S = \frac{k_1k_2}{k_1 + k_2}$.

39. (b) $t_1 = 2\pi\sqrt{\frac{m}{k_1}}$ and $t_2 = 2\pi\sqrt{\frac{m}{k_2}}$

In series, effective spring constant, $k = \frac{k_1 k_2}{k_1 + k_2}$

$\therefore$ Time period, $T = 2\pi\sqrt{\frac{m(k_1+k_2)}{k_1 k_2}}$ .....(ii)

Now, $t_1^2 + t_2^2 = 4\pi^2 m\left(\frac{1}{k_1}+\frac{1}{k_2}\right) = \frac{4\pi^2 m(k_1+k_2)}{k_1 k_2}$

$t_1^2 + t_2^2 = T^2$. [Using equation (ii)]

40. (c) $\frac{1}{k_{eff}} = \frac{1}{k}+\frac{1}{2k}+\frac{1}{4k}+\frac{1}{8k}+....$

$= \frac{1}{k}\left[1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+.....\right] = \frac{1}{k}\left(\frac{1}{1-1/2}\right) = \frac{2}{k}$

(By using sum of infinite geometrical progression

$a+\frac{a}{r}+\frac{a}{r^2}+...\infty$ sum $(S) = \frac{a}{1-r}$)

$\therefore k_{eff} = \frac{k}{2}$.

41. (a) $n \propto \sqrt{\frac{k}{m}}$

42. (b) $F = kx \Rightarrow mg = kx \Rightarrow m \propto kx$

Hence $\frac{m_1}{m_2} = \frac{k_1}{k_2}\times\frac{x_1}{x_2} \Rightarrow \frac{4}{6} = \frac{k}{k/2}\times\frac{1}{x_2}$

$\Rightarrow x_2 = 3\,cm$.

43. (b) Initially when 1 *kg* mass is suspended then by using $F = kx$

$\Rightarrow mg = kx \Rightarrow k = \frac{mg}{x} = \frac{1\times10}{5\times10^{-2}} = 200\,\frac{N}{m}$

Further, the angular frequency of oscillation of 2 *kg* mass is

$\omega = \sqrt{\frac{k}{M}} = \sqrt{\frac{200}{2}} = 10\,rad/sec$

Hence, $v_{\max} = a\omega = (10\times10^{-2})\times10 = 1\,m/s$

44. (a) $U = \frac{F^2}{2K} \Rightarrow U \propto \frac{1}{K} \Rightarrow \frac{U_1}{U_2} = \frac{K_2}{K_1} = 2$

45. (b) $U = \frac{1}{2}Kx^2$ but $T = Kx$

So energy stored $= \frac{1}{2}\frac{(Kx)^2}{K} = \frac{1}{2}\frac{T^2}{K}$

46. (a) System is equivalent to parallel combination of springs $\therefore K_{eq} = K_1 + K_2 = 400$ and

$T = 2\pi\sqrt{\frac{m}{K_{eq}}} = 2\pi\sqrt{\frac{0.25}{400}} = \frac{\pi}{20}$

47. (b) By cutting spring in four equal parts force constant (K) of each parts becomes four times $\left(\because k \propto \frac{1}{l}\right)$ so by using

$T = 2\pi\sqrt{\frac{m}{K}}$; time period will be half *i.e.* $T' = T/2$

48. (d) $T \propto \sqrt{m} \Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{m_2}{m_1}} \Rightarrow \frac{5}{3} = \sqrt{\frac{M+m}{M}}$

$\Rightarrow \frac{25}{9} = \frac{M+m}{M} \Rightarrow \frac{m}{M} = \frac{16}{9}$

49. (c) $v_{\max} = a\omega = a\frac{2\pi}{T}$

$\Rightarrow a = \frac{v_{\max}T}{2\pi} = \frac{15\times628\times10^{-3}}{2\times3.14} = 1.5\,cm$

50. (c) $Kx = mg \Rightarrow \frac{m}{K} = \frac{x}{g}$

So $T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{x}{g}} = 2\pi\sqrt{\frac{0.2}{9.8}} = \frac{2\pi}{7}$ sec

51. (b) $\omega = \sqrt{k/m} = \sqrt{\frac{4.84}{0.98}} = 2.22\,rad/\sec$

52. (d) When spring is cut into two equal parts then spring constant of each part will be $2K$ and so using $n \propto \sqrt{K}$, new frequency will be $\sqrt{2}$ times *i.e.* $f_2 = \sqrt{2}\,f_1$.

53. (d)

54. (a) With mass m_2 alone, the extension of the spring l is given as

$m_2 g = kl$ ...(i)

With mass (m_1+m_2), the extension l' is given by

$(m_1+m_2)g = k(l+\Delta l)$ ....(ii)

The increase in extension is Δl which is the amplitude of vibration. Subtracting (i) from (ii), we get

$m_1 g = k\Delta l$ or $\Delta l = \frac{m_1 g}{k}$

55. (b) Angular velocity $\omega = \sqrt{\left(\frac{k}{m}\right)} = \sqrt{\left(\frac{10}{10}\right)} = 1$

Now $v = \omega\sqrt{a^2 - y^2} \Rightarrow y^2 = a^2 - \frac{v^2}{\omega^2} = (0.5)^2 - \frac{(0.4)^2}{1^2}$

$\Rightarrow y^2 = 0.9 = y = 0.3\,m$

Superposition of S.H.M.'s and Resonance

1. (c) Resultant amplitude $= \sqrt{3^2+4^2} = 5$

2. (c) $y = A\sin PT + B\cos PT$

Let $A = r\cos\theta,\ B = r\sin\theta$

$\Rightarrow y = r\sin(PT+\theta)$ which is the equation of SHM.

3. (c) $y = a(\cos\omega t + \sin\omega t) = a\sqrt{2}\left[\frac{1}{\sqrt{2}}\cos\omega t + \frac{1}{\sqrt{2}}\sin\omega t\right]$

$= a\sqrt{2}[\sin45°\cos\omega t + \cos45°\sin\omega t]$

$= a\sqrt{2}\sin(\omega t + 45°) \Rightarrow$ Amplitude $= a\sqrt{2}$

4. (c) If first equation is $y_1 = a_1\sin\omega t \Rightarrow \sin\omega t = \frac{y_1}{a_1}$ ... (i)

then second equation will be $y_2 = a_2\sin\left(\omega t + \frac{\pi}{2}\right)$

$= a_2\left[\sin\omega t\cos\frac{\pi}{2} + \cos\omega t\sin\frac{\pi}{2}\right] = a_2\cos\omega t$

$\Rightarrow \cos\omega t = \frac{y_2}{a_2}$ ... (ii)

By squaring and adding equation (i) and (ii)

$$\sin^2\omega t + \cos^2\omega t = \frac{y_1^2}{a_1^2} + \frac{y_2^2}{a_2^2}$$

$\Rightarrow \frac{y_1^2}{a_1^2} + \frac{y_2^2}{a_2^2} = 1$; This is the equation of ellipse.

5. (a) If $y_1 = a_1\sin\omega t$ and $y_2 = a_2\sin(\omega t + \pi)$

$\Rightarrow \frac{y_1}{a_1} + \frac{y_2}{a_2} = 0 \Rightarrow y_2 = -\frac{a_2}{a_1}y_1$

This is the equation of straight line.

6. (c) If $y_1 = a_1\sin\omega t$ and $y_2 = a_2\sin(\omega t + 0) = a_2\sin\omega t$

$\Rightarrow \frac{y_1^2}{a_1^2} + \frac{y_2^2}{a_2^2} - \frac{2y_1y_2}{a_1a_2} = 0 \Rightarrow y_2 = \frac{a_2}{a_1}y_1$

This is the equation of straight line.

7. (d) For given relation

Resultant amplitude $= \sqrt{4^2 + 4^2} = 4\sqrt{2}$

8. (a) $x = 5\sqrt{2}(\sin 2\pi t + \cos 2\pi t)$

$= 5\sqrt{2}\sin 2\pi t + 5\sqrt{2}\cos 2\pi t$

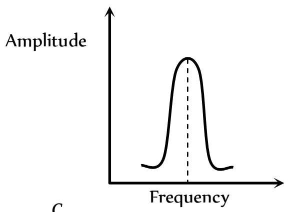
$x = 5\sqrt{2}\sin 2\pi t + 5\sqrt{2}\sin\left(2\pi t + \frac{\pi}{2}\right)$

Phase difference between constituent waves $\phi = \frac{\pi}{2}$

$\therefore$ Resultant amplitude $A = \sqrt{(5\sqrt{2})^2 + (5\sqrt{2})^2}$ =10 *cm.*

9. (b)

10. (d) Less damping force gives a taller and narrower resonance peak



11. (b) $A = \frac{c}{a + b - c}$; when $b = 0$, $a = c$ amplitude

$A \to \infty$. This corresponds to resonance.

12. (c) Energy of particle is maximum at resonant frequency *i.e.,* $\omega_2 = \omega_o$. For amplitude resonance (amplitude maximum) frequency of driver force $\omega = \sqrt{\omega_o^2 - b^2 2m^2} \Rightarrow \omega_1 \neq \omega_o$

13. (a)

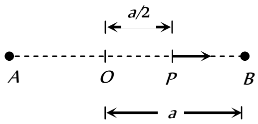
14. (c)

Critical Thinking Questions

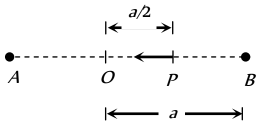
1. (d) $y = a\sin(\omega t + \phi_0)$. According to the question

$y = \frac{a}{2} \Rightarrow \frac{a}{2} = a\sin(\omega t + \phi_0) \Rightarrow (\omega t + \phi_0) = \phi = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

Physical meaning of $\phi = \frac{\pi}{6}$: Particle is at point P and it is going towards B



Physical meaning of $\phi = \frac{5\pi}{6}$: Particle is at point P and it is going towards O



So phase difference $\Delta\phi = \frac{5\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3} = 120°$

2. (b) $x = 12\sin\omega t - 16\sin^3\omega t = 4[3\sin\omega t - 4\sin^3\omega t]$

$= 4[\sin 3\omega t]$ (By using $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$)

$\therefore$ maximum acceleration $A_{max} = (3\omega)^2 \times 4 = 36\omega^2$

3. (b,c) Harmonic oscillator has some initial elastic potential energy and amplitude of harmonic variation of energy is

$\frac{1}{2}Ka^2 = \frac{1}{2} \times 2 \times 10^6 \times (0.01)^2 = 100J$

This is the maximum kinetic energy of the oscillator. Thus $K_{max} = 100J$

This energy is added to initial elastic potential energy may give maximum mechanical energy to have value $160J$.

4. (a) $U = k|x|^3 \Rightarrow F = -\frac{dU}{dx} = -3k|x|^2$ ...(i)

Also, for SHM $x = a\sin\omega t$ and $\frac{d^2x}{dt^2} + \omega^2 x = 0$

$\Rightarrow$ acceleration $a = \frac{d^2x}{dt^2} = -\omega^2 x \Rightarrow F = ma$

$= m\frac{d^2x}{dt^2} = -m\omega^2 x$ ...(ii)

From equation (i) & (ii) we get $\omega = \sqrt{\frac{3kx}{m}}$

$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{3kx}} = 2\pi\sqrt{\frac{m}{3k(a\sin\omega t)}} \Rightarrow T \propto \frac{1}{\sqrt{a}}$.

5. (b, d) Let the velocity acquired by A and B be V, then

$mv = mV + mV \Rightarrow V = \frac{v}{2}$

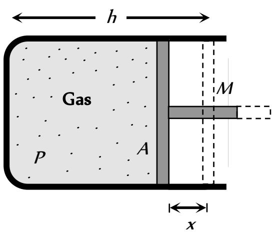
Also $\frac{1}{2}mv^2 = \frac{1}{2}mV^2 + \frac{1}{2}mV^2 + \frac{1}{2}kx^2$

Where x is the maximum compression of the spring. On solving the above equations, we get $x = v\left(\frac{m}{2k}\right)^{1/2}$

At maximum compression, kinetic energy of the

$A - B$ system $= \frac{1}{2}mV^2 + \frac{1}{2}mV^2 = mV^2 = \frac{mv^2}{4}$

6. (a) Let the piston be displaced through distance x towards left, then volume decreases, pressure increases. If ΔP is increase in pressure and ΔV is decrease in volume, then considering the process to take place gradually (i.e. isothermal)



$P_1V_1 = P_2V_2$

$\Rightarrow PV = (P + \Delta P)(V - \Delta V)$

$\Rightarrow PV = PV + \Delta PV - P\Delta V - \Delta P\Delta V$

$\Rightarrow \Delta P.V - P.\Delta V = 0$ (neglecting $\Delta P.\Delta V$)

$\Delta P(Ah) = P(Ax) \Rightarrow \Delta P = \frac{P.x}{h}$

This excess pressure is responsible for providing the restoring force (F) to the piston of mass M.

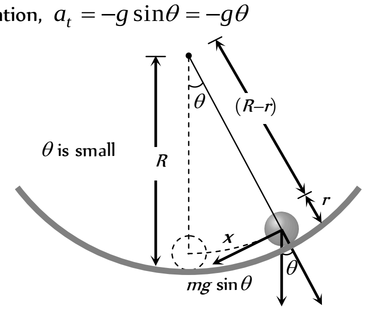
Hence $F = \Delta P.A = \frac{PAx}{h}$

Comparing it with $|F| = kx \Rightarrow k = M\omega^2 = \frac{PA}{h}$

$\Rightarrow \omega = \sqrt{\frac{PA}{Mh}} \Rightarrow T = 2\pi\sqrt{\frac{Mh}{PA}}$

Short trick : by checking the options dimensionally. Option (a) is correct.

7. (b) Tangential acceleration, $a_t = -g\sin\theta = -g\theta$



$a_t = -g\frac{x}{(R-r)}$

Motion is S.H.M., with time period

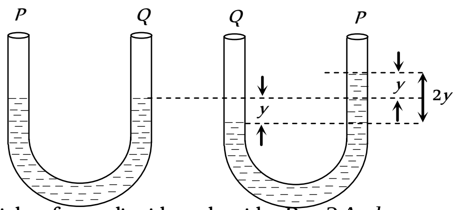
$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}} = 2\pi\sqrt{\frac{x}{\frac{gx}{(R-r)}}} = 2\pi\sqrt{\frac{R-r}{g}}$

8. (a) For resonance amplitude must be maximum which is possible only when the denominator of expression is zero *i.e.*

$a\omega^2 - b\omega + c = 0 \Rightarrow \omega = \frac{+b \pm \sqrt{b^2 - 4ac}}{2a}$

For a single resonant frequency, $b^2 = 4ac$.

9. (d) If the level of liquid is depressed by y *cm* on one side, then the level of liquid in column P is $2y$ *cm* higher than B as shown.



The weight of extra liquid on the side $P = 2Aydg$.

This becomes the restoring force on mass M.

$\therefore$ Restoring acceleration $= \frac{-2Aydg}{M}$

This relation satisfies the condition of SHM i.e. $a \propto -y$.

Hence time period $T = 2\pi\sqrt{\frac{\text{Displacement}}{|\text{Acceleration}|}}$

$= 2\pi\sqrt{\frac{y}{\frac{2Aydg}{M}}} \Rightarrow T = 2\pi\sqrt{\frac{M}{2Adg}}$

10. (b) Time taken by particle to move from $x=0$ (mean position) to $x = 4$ (extreme position) $= \frac{T}{4} = \frac{1.2}{4} = 0.3\ s$

Let t be the time taken by the particle to move from $x=0$ to $x=2$ *cm*

$y = a\sin\omega t \Rightarrow 2 = 4\sin\frac{2\pi}{T}t \Rightarrow \frac{1}{2} = \sin\frac{2\pi}{1.2}t$

$\Rightarrow \frac{\pi}{6} = \frac{2\pi}{1.2}t \Rightarrow t = 0.1\ s$. Hence time to move from $x = 2$ to $x = 4$ will be equal to $0.3 - 0.1 = 0.2$ s

Hence total time to move from $x = 2$ to $x = 4$ and back again $= 2 \times 0.2 = 0.4\,\text{sec}$

11. (c) For body to remain in contact $a_{max} = g$

$\therefore \omega^2 A = g \Rightarrow 4\pi^2 n^2 A = g$

$\Rightarrow n^2 = \frac{g}{4\pi^2 A} = \frac{10}{4(3.14)^2 0.01} = 25 \Rightarrow n = 5\ Hz$

12. (c) Under the influence of one force $F_1 = m\omega_1^2 y$ and under the action of another force, $F_2 = m\omega_2^2 y$.

Under the action of both the forces $F = F_1 + F_2$

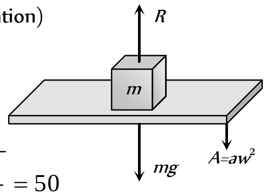
$\Rightarrow m\omega^2 y = m\omega_1^2 y + m\omega^2 y$

$\Rightarrow \omega^2 = \omega_1^2 + \omega_2^2 \Rightarrow \left(\frac{2\pi}{T}\right)^2 = \left(\frac{2\pi}{T_1}\right)^2 + \left(\frac{2\pi}{T_2}\right)^2$

$\Rightarrow T = \sqrt{\frac{T_1^2 T_2^2}{T_1^2 + T_2^2}} = \sqrt{\frac{\left(\frac{4}{5}\right)^2\left(\frac{3}{5}\right)^2}{\left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2}} = 0.48 \text{ sec}$

13. (a) By drawing free body diagram of object during the downward motion at extreme position, for equilibrium of mass

$mg - R = mA$ (A = Acceleration)



For critical condition $R = 0$

so $mg = mA \Rightarrow mg = ma\omega^2$

$\Rightarrow \omega = \sqrt{g/a} = \sqrt{\frac{9.8}{3.92 \times 10^{-3}}} = 50$

$\Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{50} = 0.1256 sec$

14. (a) Using $x = A\sin\omega t$

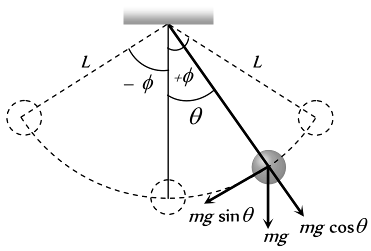
For $x = A/2$, $\sin\omega T_1 = 1/2 \Rightarrow T_1 = \frac{\pi}{6\omega}$

For $x = A$, $\sin\omega(T_1 + T_2) = 1 \Rightarrow T_1 + T_2 = \frac{\pi}{2\omega}$

$\Rightarrow T_2 = \frac{\pi}{2\omega} - T_1 = \frac{\pi}{2\omega} - \frac{\pi}{6\omega} = \frac{\pi}{3\omega}$ i.e. $T_1 < T_2$

Alternate method : In S.H.M., velocity of particle also oscillates simple harmonically. Speed is more near the mean position and less near the extreme position. Therefore the time taken for the particle to go from 0 to $\frac{A}{2}$ will be less than the time taken to go from $\frac{A}{2}$ to A. Hence $T_1 < T_2$.

15. (b, c) From following figure it is clear that



$T - Mg\cos\theta =$ Centripetal force

$\Rightarrow T - Mg\cos\theta = \frac{Mv^2}{L}$

Also tangential acceleration $|a_T| = g\sin\theta$.

16. (c) If t is the time taken by pendulums to come in same phase again first time after $t = 0$.

and N_S = Number of oscillations made by shorter length pendulum with time period T_S.

N_L = Number of oscillations made by longer length pendulum with time period T_L.

Then $t = N_S T_S = N_L T_L$

$\Rightarrow N_S 2\pi\sqrt{\frac{5}{g}} = N_L \times 2\pi\sqrt{\frac{20}{g}}$ $\left(\because T = 2\pi\sqrt{\frac{l}{g}}\right)$

$\Rightarrow N_S = 2N_L$ i.e. if $N_L = 1 \Rightarrow N_S = 2$

17. (a) Tension in the string when bob passes through lowest point

$T = mg + \frac{mv^2}{r} = mg + mv\omega$ $(\because v = r\omega)$

putting $v = \sqrt{2gh}$ and $\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$

we get $T = m(g + \pi\sqrt{2gh})$

18. (d) When the bob is immersed in water its effective weight = $\left(mg - \frac{m}{\rho}g\right) = mg\left(\frac{\rho - 1}{\rho}\right)$ $\therefore g_{eff} = g\left(\frac{\rho - 1}{\rho}\right)$

$\frac{T'}{T} = \sqrt{\frac{g}{g_{eff}}} \Rightarrow T' = T\sqrt{\frac{\rho}{(\rho - 1)}}$

19. (a) Time period $T \propto \sqrt{l} \Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l} = \frac{1}{2}\alpha\Delta\theta$

Also according to thermal expansion $l' = (1 + \alpha\Delta\theta)$

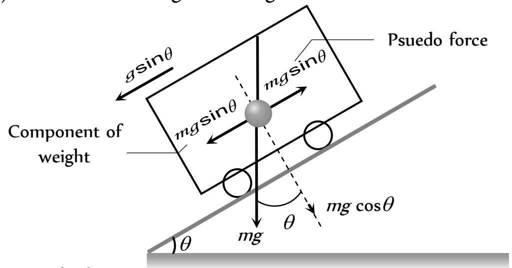
$\frac{\Delta l}{l} = \alpha + \theta$. Hence $\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l} = \frac{1}{2}\alpha\Delta\theta$

$= \frac{1}{2} \times 12 \times 10^{-6} \times (40 - 20) = 12 \times 10^{-5}$

$\Rightarrow \Delta T = 12 \times 10^{-5} \times 86400$ *seconds / day*

$\therefore \Delta T \approx 10.3$ *seconds / day*

20. (a) See the following force diagram.



Vehicle is moving down the frictionless inclined surface so, it's acceleration is $g\sin\theta$. Since vehicle is accelerating, a pseudo force $m(g\sin\theta)$ will act on bob of pendulum which cancel the $\sin\theta$ component of weight of the bob.

Hence net force on the bob is $F_{\_} = mg\cos\theta$ or net acceleration of the bob is $g_{eff} = g\cos\theta$

$\therefore$ Time period $T = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{l}{g\cos\theta}}$

21. (c) $\because t_o = 2\pi\sqrt{\frac{l}{g}}$

Effective weight of bob inside water,

$W' = mg - \text{thrust} = V\rho g - V\rho' g$

$\Rightarrow V\rho g_{eff} = V(\rho - \rho')g$, where, ρ = Density of bob

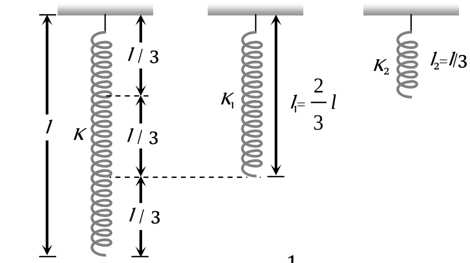
$\Rightarrow g_{eff} = \left(1 - \frac{\rho'}{\rho}\right) g$ and ρ' = Density of water

$\therefore t = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{l}{(1-\rho'/\rho)g}}$

$\therefore \frac{t}{t_0} = \sqrt{\frac{1}{1-\rho'/\rho}} = \sqrt{\frac{1}{1-\frac{3}{4}}}$ $(\because \rho' = 10^3 kg/m^3$, $\rho = \frac{4}{3}\times 10^3 kg/m^3$

$\Rightarrow t = 2t_0$.

22. (b)



Force constant $(k) \propto \frac{1}{\text{Length of spring}}$

$\Rightarrow \frac{K}{K_1} = \frac{l_1}{l} = \frac{\frac{2}{3}l}{l} \Rightarrow K_1 = \frac{3}{2}K$.

23. (b) The wire may be treated as a string for which force constant $k_1 = \frac{\text{Force}}{\text{Extension}} = \frac{YA}{L}$ $\left(\because Y = \frac{F}{A}\times\frac{L}{\Delta L}\right)$

Spring constant of the spring $k_2 = K$

Hence spring constant of the combination (series)

$k_{eq} = \frac{k_1 k_2}{k_1 + k_2} = \frac{(YA/L)K}{(YA/L)+K} = \frac{YAK}{YA+KL}$

$\because$ Time period $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\left[\frac{(YA+KL)m}{YAK}\right]^{1/2}$

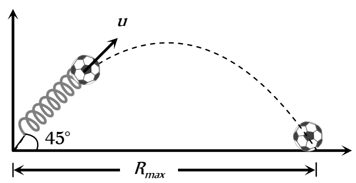
24. (a) Slope is irrelevant hence $T = 2\pi\left(\frac{M}{2K}\right)^{1/2}$

25. (b) For forced oscillation,

$x = x_0 \sin(\omega t + \phi)$ and $F = F_0 \cos\omega t$

where, $x_0 = \frac{F_o}{m(\omega_o^2 - \omega^2)} \propto \frac{1}{m(\omega_o^2 - \omega^2)}$.

26. (b) For getting horizontal range, there must be some inclination of spring with ground to project ball.



$\Rightarrow R_{max} = \frac{u^2}{g}$ .....(i)

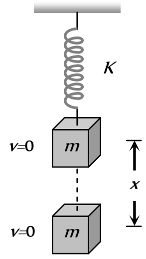
But K.E. acquired by ball = P.E. of spring gun

$\Rightarrow \frac{1}{2}mu^2 = \frac{1}{2}kx^2 \Rightarrow u^2 = \frac{kx^2}{m}$ .....(ii)

From equation (i) and (ii)

$R_{max} = \frac{kx^2}{mg} = \frac{600\times(5\times10^{-2})^2}{15\times10^{-3}\times10} = 10\,m.$

27. (b) Let x be the maximum extension of the spring. From energy conservation



Loss in gravitational potential energy

= Gain in potential energy of spring

$Mgx = \frac{1}{2}Kx^2$

$\Rightarrow x = \frac{2Mg}{K}$

28. (b) $y = 4\cos^2\left(\frac{t}{2}\right)\sin 1000\,t$

$\Rightarrow y = 2(1+\cos t)\sin 1000\,t$

$\Rightarrow y = 2\sin 1000\,t + 2\cos t\,\sin 1000t$

$\Rightarrow y = 2\sin 1000\,t + \sin 999\,t + \sin 1001\,t$

It is a sum of three S.H.M.

29. (a, c) Let simple harmonic motions be represented by

$y_1 = a\sin\left(\omega t - \frac{\pi}{4}\right)$; $y_2 = a\sin\omega t$ and

$y_3 = a\sin\left(\omega t + \frac{\pi}{4}\right)$. On superimposing, resultant SHM will be $y = a\left[\sin\left(\omega t - \frac{\pi}{4}\right) + \sin\omega t + \sin\left(\omega t + \frac{\pi}{4}\right)\right]$

$= a\left[2\sin\omega t\cos\frac{\pi}{4} + \sin\omega t\right]$

$= a[\sqrt{2}\sin\omega t + \sin\omega t] = a(1+\sqrt{2})\sin\omega t$

Resultant amplitude $= (1+\sqrt{2})a$

Energy is S.H.M. $\propto$ (Amplitude)^2

$\therefore \frac{E_{\text{Resultant}}}{E_{\text{Single}}} = \left(\frac{A}{a}\right)^2 = (\sqrt{2}+1)^2 = (3+2\sqrt{2})$

$\Rightarrow E_{\text{Resultant}} = (3+2\sqrt{2})E_{\text{Single}}$

30. (d) $y = \sin^2\omega t = \frac{1-\cos 2\omega t}{2} \Rightarrow$ Period, $T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

The given function is not satisfying the standard differential equation of S.H.M. $\frac{d^2y}{dx^2} = -\omega^2 y$. Hence it represents periodic motion but not S.H.M.

31. (c) $y = Kt^2 \Rightarrow \frac{d^2y}{dt^2} = a_y = 2K = 2\times 1 = 2\,m/s$ ($\because K = 1\,m/s$)

Now, $T_1 = 2\pi\sqrt{\frac{l}{g}}$ and $T_2 = 2\pi\sqrt{\frac{l}{(g+a_y)}}$

Dividing, $\frac{T_1}{T_2} = \sqrt{\frac{g + a_y}{g}} \Rightarrow \sqrt{\frac{6}{5}} \Rightarrow \frac{T_1^2}{T_2^2} = \frac{6}{5}$

32. (b) From the relation of restitution $\frac{h_n}{h_0} = e^{2n}$ and

$$h_n = h_0(1 - \cos 60°) \Rightarrow \frac{h_n}{h_0} = 1 - \cos 60° = \left(\frac{2}{\sqrt{5}}\right)^{2n}$$

$$\Rightarrow 1 - \frac{1}{2} = \left(\frac{4}{5}\right)^n \Rightarrow \frac{1}{2} = \left(\frac{4}{5}\right)^n$$

Taking log of both sides we get

$$\log 1 - \log 2 = n(\log 4 - \log 5)$$

$$0 - 0.3010 = n(0.6020 - 0.6990)$$

$$-0.3010 = -n \times 0.097 \Rightarrow n = \frac{0.3010}{0.097} = 3.1 \approx 3$$

33. (a) As a is the side of cube σ is its density.

Mass of cube is $a^2\sigma$, its weight $= a^3\sigma g$

Let h be the height of cube immersed in liquid of density ρ in equilibrium then, $F = a^2 h \rho g = Mg = a^3 \sigma g$

If it is pushed down by y then the buoyant force $F' = a^2(h + y)\rho g$

Restoring force is $\Delta F = F' - F = a^2(h + y)\sigma g - a^2 h \sigma g$

$$= a^2 y \rho g$$

Restoring acceleration $= \frac{\Delta F}{M} = -\frac{a^2 y \rho g}{M} = -\frac{a^2 \rho g}{a^2 \sigma} y$

Motion is S.H.M.

$$\Rightarrow T = 2\pi\sqrt{\frac{a^3\sigma}{a^2\rho g}} = 2\pi\sqrt{\frac{a\sigma}{\rho g}}$$

34. (b) As here two masses are connected by two springs, this problem is equivalent to the oscillation of a reduced mass m_r of a spring of effective spring constant.

$$T = 2\pi\sqrt{\frac{m_r}{K_{eff.}}}$$

Here $m_r = \frac{m_1 m_2}{m_1 + m_2} = \frac{m}{2} \Rightarrow K_{eff.} = K_1 + K_2 = 2K$

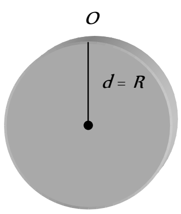
$$\therefore n = \frac{1}{2\pi}\sqrt{\frac{K_{eff.}}{m_r}} = \frac{1}{2\pi}\sqrt{\frac{2K}{m} \times 2} = \frac{1}{\pi}\sqrt{\frac{K}{m}} = \frac{1}{\pi}\sqrt{\frac{0.1}{0.1}} = \frac{1}{\pi} Hz$$

35. (d) Time period of a physical pendulum

$$T = 2\pi\sqrt{\frac{I_0}{mgd}} = 2\pi\sqrt{\frac{\left(\frac{1}{2}mR^2 + mR^2\right)}{mgR}}$$

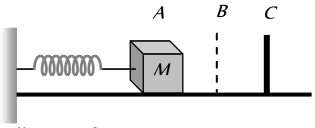
$$= 2\pi\sqrt{\frac{3R}{2g}} \quad .....(i)$$

$$T_{simple\ pendulum} = 2\pi\sqrt{\frac{l}{g}} \quad .....(ii)$$



Equating (i) and (ii), $l = \frac{3}{2}R$.

36. (c) The total time from A to C



$t_{Ac} = t_{AB} + t_{BC}$

$= (T/4) + t_{BC}$

where T = time period of oscillation of spring mass system

t_{BC} can be obtained from, $BC = AB\sin(2\pi/T)t_{BC}$

Putting $\frac{BC}{AB} = \frac{1}{2}$ we obtain $t_{BC} = \frac{T}{12}$

$$\Rightarrow t_{AC} = \frac{T}{4} + \frac{T}{12} = \frac{2\pi}{3}\sqrt{\frac{m}{k}}.$$

37. (b) When mass 700 *gm* is removed, the left out mass (500 + 400) gm oscillates with a period of 3 sec

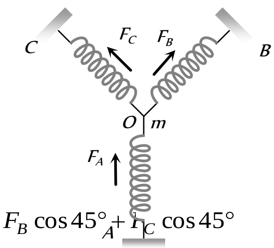
$$\therefore 3 = t = 2\pi\sqrt{\frac{(500 + 400)}{k}} \quad ......(i)$$

When 500 *gm* mass is also removed, the left out mass is 400 *gm*.

$$\therefore t' = 2\pi\sqrt{\frac{400}{k}} \quad .....(ii)$$

$$\Rightarrow \frac{3}{t'} = \sqrt{\frac{900}{400}} \Rightarrow t' = 2\,\text{sec}$$

38. (b) When the particle of mass m at O is pushed by y in the direction of A The spring A will be compressed by y while spring B and C will be stretched by $y' = y\cos 45°$. So that the total restoring force on the mass m along OA.



$$F_{net} = F_A + F_B\cos 45° + F_C\cos 45°$$

$$= ky + 2ky'\cos 45° = ky + 2k(y\cos 45°)\cos 45° = 2ky$$

Also $F_{net} = k'y \Rightarrow k'y = 2ky \Rightarrow k' = 2k$

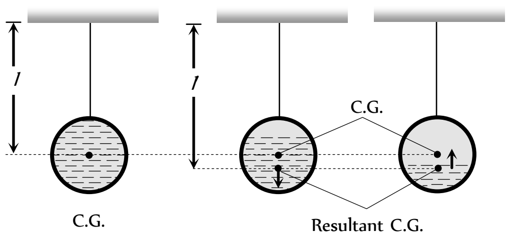
$$T = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{m}{2k}}$$

39. (d) The given system is like a simple pendulum, whose effective length (l) is equal to the distance between point of suspension and C.G. (Centre of Gravity) of the hanging body.

When water slowly flows out the sphere, the C.G. of the system is lowered, and hence l increases, which in turn increases time period (as $T \propto \sqrt{l}$).

After some time weight of water left in sphere become less than the weight of sphere itself, so the resultant C.G. gets clear the C.G. of sphere itself i.e. l decreases and hence T increases.

Finally when the sphere becomes empty, the resulting C.G. is the C.G. of sphere i.e. length becomes equal to the original length and hence the time period becomes equal to the same value as when it was full of water.



40. (b) Let T_1 and T_2 are the time period of the two pendulums $T_1 = 2\pi\sqrt{\frac{100}{g}}$ and $T_2 = 2\pi\sqrt{\frac{121}{g}}$

$(T_1 < T_2$ because $l_1 < l_2)$.

Let at $t = 0$, they start swinging together. Since their time periods are different, the swinging will not be in unision always. Only when number of completed oscillation differs by an integer, the two pendulum will again begin to swing together.

Let longer length pendulum complete n oscillation and shorter length pendulum complete $(n+1)$ oscillation, for the unision swinging, then $(n+1)T_1 = nT_2$

$$(n+1)\times 2\pi\sqrt{\frac{100}{g}} = n\times 2\pi\sqrt{\frac{121}{g}} \Rightarrow n = 10$$

41. (b) Amplitude of damped oscillator

$A = A_0 e^{-\lambda t}$; $\lambda =$ constant, t = time

For t =1 min. $\frac{A_0}{2} = A_0 e^{-\lambda t} \Rightarrow e^{\lambda} = 2$

For t = 3 min. $A = A_0 e^{-\lambda\times 3} = \frac{A_0}{(e^{\lambda})^3} = \frac{A_0}{2^3}$

$\Rightarrow X = 2^3$

42. (a) The standard differential equation is satisfied by only the function $\sin\omega t - \cos\omega t$. Hence it represents S.H.M.

43. (d) This is the special case of physical pendulum and in this case $T = 2\pi\sqrt{\frac{2l}{3g}}$

$$\Rightarrow T = 2\times 3.14\sqrt{\frac{2\times 2}{3\times 9.8}} = 2.31\,\text{sec} \approx 2.4\,\text{sec}$$

Graphical Questions

1. (a) Because acceleration $\propto$ displacement.

2. (d) Using acceleration $A = -\omega^2 x$

$At - x_{max}$ A will be maximum and positive.

3. (d) Acceleration $= -\omega^2 y$. So $F = -m\omega^2 y$

y is sinusoidal function.

So F will be also sinusoidal function with phase difference π

4. (d) At time $\frac{T}{2}; v = 0$ $\therefore$ Total energy = Potential energy.

5. (b) PE varies from zero to maximum. It is always positive sinusoidal function.

6. (d) Potential energy of particle performing SHM is given by: $PE = \frac{1}{2}m\omega^2 y^2$ *i.e.* it varies parabolically such that at mean position it becomes zero and maximum at extreme position.

7. (a) Potential energy is minimum (in this case zero) at mean position $(x = 0)$ and maximum at extreme position $(x = \pm A)$.

At time $t = 0$, $x = A$, hence potential should be maximum. Therefore graph I is correct. Further in graph III. Potential energy is minimum at $x = 0$, hence this is also correct.

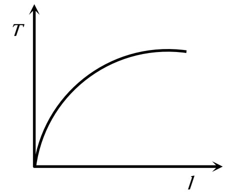
8. (a) $f = \frac{1}{T} = \frac{1}{0.04} = 25\ Hz$

9. (d) From graph, slope $K = \frac{F}{x} = \frac{8}{2} = 4$

$$T = 2\pi\sqrt{\frac{m}{K}} \Rightarrow T = 2\pi\sqrt{\frac{0.01}{4}} = 0.3\,\text{sec}$$

10. (b) $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow l \propto T^2$ (Equation of parabola)

11. (b) $T \propto \sqrt{l} \Rightarrow T^2 \propto l$



12. (d) In simple harmonic motion

$y = a\sin\omega t$ and $v = a\omega\cos\omega t$ from this we have

$\frac{y^2}{a^2} + \frac{v^2}{a^2\omega^2} = 1$, which is a equation of ellipse.

13. (c)

14. (a) In S.H.M. when acceleration is negative maximum or positive maximum, the velocity is zero so kinetic energy is also zero. Similarly for zero acceleration, velocity is maximum so kinetic energy is also maximum.

15. (b) Total potential energy = 0.04 J

Resting potential energy =0.01 J

Maximum kinetic energy =(0.04–0.01)

$$= 0.03J = \frac{1}{2}m\,\omega^2 a^2 = \frac{1}{2}ka^2$$

$$0.03 = \frac{1}{2}\times k\times\left(\frac{20}{1000}\right)^2$$

$k = 0.06\times 2500\ N/m = 150\ N/m$.

16. (a) Kinetic energy varies with time but is never negative.

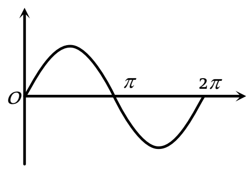
Assertion and Reason

1. (b) Both assertion and reason are correct but reason is not the correct explanation of assertion.

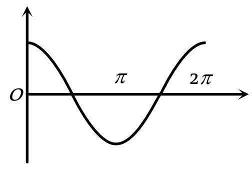
2. (e) In simple harmonic motion, $v = \omega\sqrt{a^2 - y^2}$ as y changes, velocity v will also change. So simple harmonic motion is not uniform motion. But simple harmonic motion may be defined as the projection of uniform circular motion along one of the diameter of the circle.

3. (a) In SHM, the acceleration is always in a direction opposite to that of the displacement *i.e.,* proportional to $(-y)$.

4. (a) A periodic function is one whose value repeats after a definite interval of time. $\sin\theta$ and $\cos\theta$ are periodic functions because they repeat itself after 2π interval of time.



sin curve



cos curve

It is also true that moon is smaller than the earth, but this statement is not explaining the assertion.

5. (e) In SHM, $v = \omega\sqrt{a^2 - y^2}$ or $v^2 = \omega^2 a^2 - \omega^2 y^2$.

Dividing both sides by $\omega^2 a^2$, $\frac{v^2}{\omega^2 a^2} + \frac{y^2}{a^2} = 1$. This is the equation of an ellipse. Hence the graph between v and y is an ellipse not a parabola.

6. (b) $T = 2\pi\sqrt{\frac{l}{g}}$. On moon, g is much smaller compared to g on earth. Therefore, T increases.

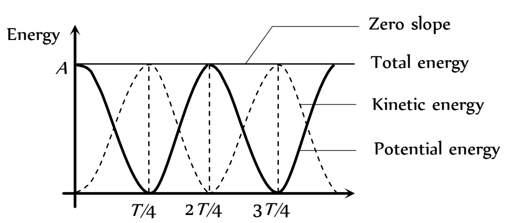
7. (c) Amplitude of oscillation for a forced, damped oscillator is $A = \frac{F_0 / m}{\sqrt{(\omega^2 - \omega_0^2) + (b\omega / m)^2}}$, where b is constant related to the strength of the resistive force, $\omega_0 = \sqrt{k/m}$ is natural frequency of undamped oscillator ($b = 0$).

When the frequency of driving force (ω) $\approx$ ω_0, then amplitude A is very larger.

For $\omega < \omega_0$ or $\omega > \omega_0$ the amplitude decrease.

8. (a) The total energy of S.H.M. = Kinetic energy of particle + potential energy of particle.

The variation of total energy of the particle in SHM with time is shown in a graph.



9. (c) Time period of simple pendulum of length l is,

$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \sqrt{l} \Rightarrow \sqrt{\frac{\Delta T}{T}} = \frac{1}{2}\frac{\Delta l}{l}$$

$$\therefore \frac{\Delta T}{T} = \frac{1}{2} \times 3 = 1.5\%$$

10. (c) Frequency of second pendulum $n = (1/2)s^{-1}$. When elevator is moving upwards with acceleration *g/2*, the effective acceleration due to gravity is

$$g = g + a = g + g/2 = 3g/2.$$

As $n = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$ so $n^2 \propto g$.

$$\therefore \frac{n_1^2}{n_2^2} = \frac{g_1}{g} = \frac{3g/2}{g} = \frac{3}{2} \text{ or, } \frac{n_1}{n} = \sqrt{\frac{3}{2}} = 1.225$$

or, $n_1 = 1.225n = 1.225 \times (1/2) = 0.612\ s^{-1}$.

11. (b) Energy of damped oscillator at an any instant t is given by

$$E = E_0 e^{-bt/m} \text{ [where } E_0 = \frac{1}{2}kx^2 = \text{maximum energy]}$$

Due to damping forces the amplitude of oscillator will go on decreasing with time whose energy is expressed by above equation.

12. (b) In SHM. $K.E. = \frac{1}{2}m\omega^2(a^2 - y^2)$ and $P.E. = \frac{1}{2}m\omega^2 y^2$.

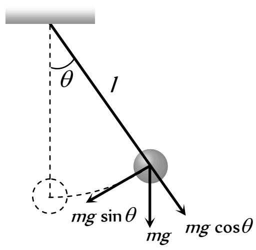
For *K.E.* = *P.E.* $\Rightarrow 2y^2 = a^2 \Rightarrow y = a/\sqrt{2}$. Since total energy remains constant through out the motion, which is $E = K.E. + P.E.$ So, when *P.E.* is maximum then *K.E.* is zero and viceversa.

13. (a) Total energy of the harmonic oscillator,

$$E = \frac{1}{2}m\omega^2 a^2 \text{ i.e., } E \propto a^2.$$

Therefore $\frac{E'}{E} = \left(\frac{2a}{a}\right)^2$ or, $E' = 4E$.

14. (b) In simple pendulum, when bob is in deflection position, the tension in the string is $T = mg\cos\theta + \frac{mv^2}{l}$. Since the value of θ is different at different positions, hence tension in the string is not constant throughout the oscillation. At end points θ is maximum; the value of $\cos\theta$ is least, hence the value of tension in the string is least. At the mean position, the value of $\theta = 0°$ and $\cos 0° = 1$, so the value of tension is largest.



Also velocity is given by $v = \omega\sqrt{a^2 - y^2}$ which is maximum when $y = 0$, at mean position.

15. (e) Spring constant $\propto \frac{1}{\text{Length of spring}}$

$\Rightarrow k' = \frac{k}{n}$

Also, spring constant depends on material properties of the spring.

Hence assertion is false, but reason is true.

16. (a) The time period of a oscillating spring is given by,

$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow T \propto \frac{1}{\sqrt{k}}$. Since the spring constant is large for hard spring, therefore hard spring has a less periodic time as compared to soft spring.

17. (e) In simple harmonic motion the velocity is given by,

$v = \omega\sqrt{a^2 - y^2}$ at extreme position, $y = a$.

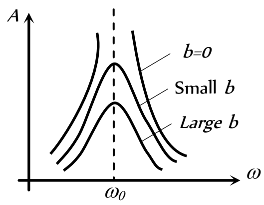
$\therefore v = 0$. But acceleration $A = -\omega^2 a$, which is maximum at extreme position.

18. (a) If the soldiers while crossing a suspended bridge march in steps, the frequency of marching steps of soldiers may match with the natural frequency of oscillations of the suspended bridge. In that situation resonance will take place, then the amplitude of oscillation of the suspended bridge will increase enormously, which may cause the collapsing of the bridge. To avoid situations the soldiers are advised to go out steps on suspended bridge.

19. (a) From equation, amplitude of oscillation

$$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + (b\omega/m)^2}}$$

In absence of damping force (b = 0), that the steady state amplitude approaches infinity as $\omega \to \omega_0$. That is, if there is no resistive force in the system and then it is possible to drive an oscillator with sinusoidal force at the resonance frequency, the amplitude of motion will build up without limit. This does not occur in practice because some damping is always present in real oscillation.



20. (b)

21. (c) The amplitude of an oscillating pendulum decreases with time because of friction due to air. Frequency of pendulum is independent $\left(T = \frac{1}{2\pi}\sqrt{\frac{g}{l}}\right)$ of amplitude.

22. (b) $x = a\sin\omega t$ and $v = \frac{dx}{dt} = a\omega\cos\omega t$

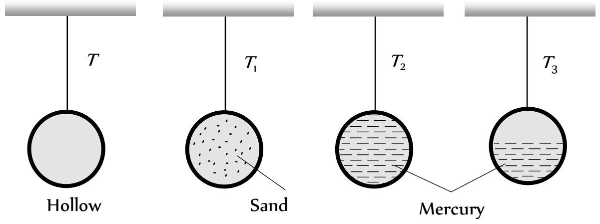
It is clear phase difference between 'x' and 'a' is $\pi/2$.

23. (e)

Simple Harmonic Motion

SET Self Evaluation Test -16

1. The period of a simple pendulum, whose bob is a hollow metallic sphere, is T. The period is T_1 when the bob is filled with sand, T_2 when it is filled with mercury and T_3 when it is half filled with mercury. Which of the following is true



(a) $T = T_1 = T_2 > T_3$ (b) $T_1 = T_1 = T_3 > T$

(c) $T > T_3 > T_1 = T_2$ (d) $T = T_1 = T_2 < T_3$

2. A pendulum clock that keeps correct time on the earth is taken to the moon it will run (it is given that $g_{Moon} = g_{Earth}/6$)

(a) At correct rate (b) 6 time faster

(c) $\sqrt{6}$ times faster (d) $\sqrt{6}$ times slowly

3. A pendulum has time period T in air. When it is made to oscillate in water, it acquired a time period $T' = \sqrt{2}T$. The density of the pendulum bob is equal to (density of water = 1)

(a) $\sqrt{2}$ (b) 2

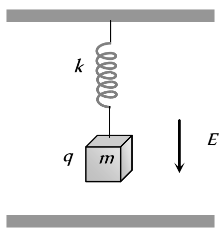
(c) $2\sqrt{2}$ (d) None of these

4. An object of mass 0.2 kg executes simple harmonic along X-axis with frequency of $\frac{25}{\pi} Hz$. At the position $x = 0.04m$, the object has kinetic energy of 0.5 J and potential energy of 0.4 J amplitude of oscillation in meter is equal to

(a) 0.05 (b) 0.06

(c) 0.01 (d) None of these

5. Time period of a block suspended from the upper plate of a parallel plate capacitor by a spring of stiffness k is T. When block is uncharged. If a charge q is given to the block them, the new time period of oscillation will be



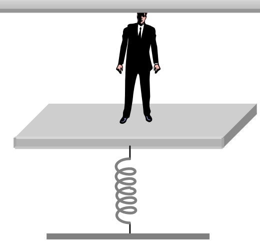
(a) T

(b) $> T$

(c) $< T$

(d) $\geq T$

6. A man weighing 60 kg stands on the horizontal platform of a spring balance. The platform starts executing simple harmonic motion of amplitude 0.1 m and frequency $\frac{2}{\pi} Hz$. Which of the following statement is correct



(a) The spring balance reads the weight of man as 60 kg

(b) The spring balance reading fluctuates between 60 kg. and 70 kg

(c) The spring balance reading fluctuates between 50 kg and 60 kg

(d) The spring balance reading fluctuates between 50 kg and 70 kg

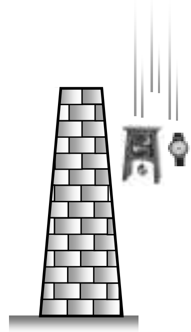
7. A man having a wrist watch and a pendulum clock rises on a TV tower. The wrist watch and pendulum clock per chance fall from the top of the tower. Then

(a) Both will keep correct time during the fall.

(b) Both will keep incorrect time during the fall.

(c) Wrist watch will keep correct time and clock will become fast.

(d) Clock will stop but wrist watch will function normally.



8. A force of 6.4 N stretches a vertical spring by 0.1 m. The mass that must be suspended from the spring so that it oscillates with a period of $\left(\frac{\pi}{4}\right)$ sec. is **[Roorkee 1990]**

(a) $\left(\frac{\pi}{4}\right) kg$ (b) $1 \, kg$

(c) $\left(\frac{1}{\pi}\right) kg$ (d) $10 \, kg$

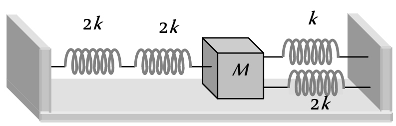
9. A spring with 10 coils has spring constant k. It is exactly cut into two halves, then each of these new springs will have a spring constant **[Kerala PMT 2004]**

(a) $k/2$ (b) $3k/2$

(c) $2k$ (d) $3k$

(e) $4k$

10. Four massless springs whose force constants are $2k$, $2k$, k and $2k$ respectively are attached to a mass M kept on a frictionless plane (as shown in figure). If the mass M is displaced in the horizontal direction, then the frequency of oscillation of the system is



(a) $\frac{1}{2\pi}\sqrt{\frac{k}{4M}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{4k}{M}}$

(c) $\frac{1}{2\pi}\sqrt{\frac{k}{7M}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{7k}{M}}$

11. Values of the acceleration A of a particle moving in simple harmonic motion as a function of its displacement x are given in the table below.

$A\ (mm\ s^{-2})$	16	8	0	– 8	– 16
$x\ (mm)$	– 4	– 2	0	2	4

The period of the motion is

(a) $\frac{1}{\pi}s$ (b) $\frac{2}{\pi}s$

(c) $\frac{\pi}{2}s$ (d) πs

12. Two pendulums have time periods T and $\frac{5T}{4}$. They start S.H.M. at the same time from the mean position. What will be the phase difference between them after the bigger pendulum has complete one oscillation

(a) 45° (b) 90°

(c) 60° (d) 30°

13. The periodic time of a particle doing simple harmonic motion is 4 *second*. The time taken by it to go from its mean position to half the maximum displacement (amplitude) is

(a) 2*s* (b) 1*s*

(c) $\frac{2}{3}s$ (d) $\frac{1}{3}s$

14. The displacement of a particle from its mean position (in metre) is given by $y = 0.2\sin(10\pi t + 1.5\pi)\cos(10\pi t + 1.5\pi)$. The motion of particle is **[CPMT 1998]**

(a) Periodic but not S.H.M.

(b) Non-periodic

(c) Simple harmonic motion with period 0.1 *s*

(d) Simple harmonic motion with period 0.2 *s*

15. The kinetic energy and the potential energy of a particle executing S.H.M. are equal. The ratio of its displacement and amplitude will be **[RPMT 2003; CPMT 2001]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{1}{2}$ (d) $\sqrt{2}$

16. Two simple pendulums of lengths 1.44 *m* and 1 *m* start swinging together. After how many vibrations will they again start swinging together **[J & K CET 2005]**

(a) 5 oscillations of smaller pendulum

(b) 6 oscillations of smaller pendulum

(c) 4 oscillations of bigger pendulum

(d) 6 oscillations of bigger pendulum

17. Equations $y_1 = A\sin\omega t$ and $y_2 = \frac{A}{2}\sin\omega t + \frac{A}{2}\cos\omega t$ represent S.H.M. The ratio of the amplitudes of the two motions is

(a) 1 (b) 2

(c) 0.5 (d) $\sqrt{2}$

18. A particle doing simple harmonic motion, amplitude = 4 *cm*, time period = 12 *sec*. The ratio between time taken by it in going from its mean position to 2 *cm* and from 2 *cm* to extreme position is

(a) 1 (b) 1/3

(c) 1/4 (d) 1/2

19. On a planet a freely falling body takes 2 *sec* when it is dropped from a height of 8 *m*, the time period of simple pendulum of length 1 *m* on that planet is **[Pb. PMT 2004]**

(a) 3.14 *sec* (b) 16.28 *sec*

(c) 1.57 *sec* (d) None of these

20. If a simple pendulum is taken to place where *g* decreases by 2%, then the time period **[Pb. PET 2002]**

(a) Decreases by 1% (b) Increases by 2%

(c) Increases by 2% (d) Increases by 1%

21. Two simple pendulum first of bob mass M_1 and length L_1 second of bob mass M_2 and length L_2. $M_1 = M_2$ and $L_1 = 2L_2$. If these vibrational energy of both is same. Then which is correct

(a) Amplitude of *B* greater than *A*

(b) Amplitude of *B* smaller than *A*

(c) Amplitudes will be same

(d) None of these

Answers and Solutions

(SET -16)

1. (d) Time period of pendulum doesn't depends upon mass but it depends upon length (distance between point of suspension and centre of mass).

In first three cases length are same so $T = T_1 = T_2$ but in last case centre of mass lowers which in turn increases the length. So in this case time period will be more than the other cases.

2. (d) $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow \frac{T_e}{T_m} = \sqrt{\frac{g_m}{g_e}} = \sqrt{\frac{g_e/6}{g_e}} = \frac{1}{\sqrt{6}}$

$\Rightarrow\ T_m = \sqrt{6}T_e$ *i.e.* clock becomes slower.

3. (b) The effective acceleration of a bob in water

$= g' = g\left(1-\frac{\sigma}{\rho}\right)$ where σ and ρ are the density of water and the bob respectively. Since the period of oscillation of the bob in air and water are given as $T = 2\pi\sqrt{\frac{l}{g}}$ and

$T' = 2\pi\sqrt{\frac{l}{g'}}$

$\therefore\ \frac{T}{T'} = \sqrt{\frac{g'}{g}} = \sqrt{\frac{g(1-\sigma/\rho)}{g}} = \sqrt{1-\frac{\sigma}{\rho}} = \sqrt{1-\frac{1}{\rho}}$

Putting $\frac{T}{T'} = \frac{1}{\sqrt{2}}$. We obtain, $\frac{1}{2} = 1-\frac{1}{\rho}\ \Rightarrow \rho = 2$

4. (b) $E = \frac{1}{2}m\omega^2A^2 \Rightarrow E = \frac{1}{2}m(2\pi f)^2A^2 \Rightarrow A = \frac{1}{2\pi f}\sqrt{\frac{2E}{m}}$

Putting $E = K + U$ we obtain,

$A = \frac{1}{2\pi\left(\frac{25}{\pi}\right)}\sqrt{\frac{2\times(0.5+0.4)}{0.2}} \Rightarrow A = 0.06\,m$

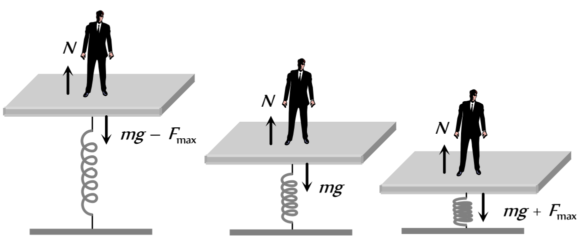
5. (a) The forces that act on the block are qE and mg. Since qE and mg are constant forces, the only variable elastic force changes by kx. Where x is the elongation in the spring $\Rightarrow$ unbalanced (restoring) force $= F = -kx$

$\Rightarrow\ -m\omega^2X = -KX \Rightarrow\ \omega = \sqrt{\frac{k}{M}} = T$.

6. (d) The maximum force acting on the body executing simple harmonic motion is

$m\omega^2a = m\times(2\pi f)^2a = 60\times\left(2\pi\times\frac{2}{\pi}\right)^2\times 0.1\ N$

$= 60\times16\times0.1 = 96N = \frac{96}{9.8} \approx 10kgf$ and this force is towards mean position.



The reaction of the force on the platform away from the mean position. It reduces the weight of man on upper extreme *i.e.* net weight = (60 – 10) *kgf*.

This force adds to the weight at lower extreme position *i.e.* net weight becomes = (60 + 10) *kgf*.

Therefore, the reading the weight recorded by spring balance fluctuates between 50 *kgf* and 70 *kgf*.

7. (d) Function of wrist watch depends upon spring action so it is not effected by gravity but pendulum clock has time period, $T = 2\pi\sqrt{\frac{l}{g}}$. During free fall effective acceleration becomes zero, so time period comes out to be infinity i.e. the clock stops.

8. (b) Force constant of a spring is given by $F = kx$

$6.4 = k(0.1)$ or $k = 64N/m$

$\because\ T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow \frac{\pi}{4} = 2\pi\sqrt{\frac{m}{64}}\ ;\ \frac{m}{64} = \left(\frac{1}{8}\right)^2\ ;\ m = 1kg$

9. (b) $K \propto \frac{1}{l} \Rightarrow Kl = K'\times\frac{l}{2} \Rightarrow K' = 2K$

10. (b) The two springs on left side having spring constant of $2k$ each are in series, equivalent constant is $\frac{1}{\left(\frac{1}{2k}+\frac{1}{2k}\right)} = k$. The two springs on right hand side of mass M are in parallel. Their effective spring constant is $(k+2k) = 3k$.

Equivalent spring constants of value k and $3k$ are in parallel and their net value of spring constant of all the four springs is $k + 3k = 4k$

$\therefore$ Frequency of mass is $n = \frac{1}{2\pi}\sqrt{\frac{4k}{M}}$

11. (d) $|A| = \omega x \Rightarrow \frac{|A|}{x} = \omega^2$

From the given value $\frac{|A|}{x} = \omega^2 = 4\ \Rightarrow \omega = 2$.

Also $\omega = \frac{2\pi}{T} \Rightarrow 2 = \frac{2\pi}{T} \Rightarrow T = \pi\ sec$

12. (b) $\frac{5T}{4} = T + \frac{T}{4}$

By the time, the bigger pendulum makes one full oscillation, the smaller pendulum will make $\left(1+\frac{1}{4}\right)$ oscillation. The bigger pendulum will be in the mean position and the smaller one will be in the positive extreme position. Thus, phase difference = 90°

13. (d) $y = A\sin\left(\frac{2\pi}{T}\right)\cdot t$

$\Rightarrow\ \frac{A}{2} = A\sin\left(\frac{2\pi}{4}\right)t \Rightarrow \frac{\pi t}{2} = \frac{\pi}{6} \Rightarrow t = \frac{1}{3}\ sec$

14. (c) $y = 0.2\sin(10\pi t + 1.5\pi)\cos(10\pi t + 1.5\pi)$

$= 0.1\sin 2(10\pi t + 1.5\pi)$ $[\because \sin 2A = 2\sin A\cos A]$

$= 0.1\sin(20\pi t + 3.0\pi)$

$\therefore$ Time period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{20\pi} = \frac{1}{10} = 0.1\ sec$

15. (a) Given $K.E. = P.E. \Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$\Rightarrow \frac{1}{2}m\omega^2(a^2 - x^2) = \frac{1}{2}m\omega^2 x^2$

$\Rightarrow a^2 - x^2 = x^2 \Rightarrow x^2 = \frac{a^2}{2} \Rightarrow \frac{x}{a} = \frac{1}{\sqrt{2}}$.

16. (b) $n \propto \frac{1}{\sqrt{l}} \Rightarrow \frac{n_2}{n_1} = \sqrt{\frac{1.44}{1}} = \frac{1.2}{1} \Rightarrow n_2 = 1.2n_1$

For n be integer minimum value of n should be 5 and then n = 6 *i.e.*, after 6 oscillations of smaller pendulum both will be in phase.

17. (d) $y_2 = \frac{A}{2}\sin\omega t + \frac{A}{2}\cos\omega t$

$y_2 = \frac{A}{2}(\sin\omega t + \cos\omega t) = \frac{A}{2} \times \sqrt{2}\,[\sin(\omega t + 45^o)]$

$y_2 = \frac{A}{\sqrt{2}}\sin(\omega t + 45^o) \Rightarrow \frac{A_1}{A_2} = \frac{A}{A/\sqrt{2}} = \sqrt{2}$

18. (d) $\omega = \frac{2\pi}{T} = \frac{2\pi}{12} = \frac{\pi}{6}\frac{rad}{sec}$ (For y = 2 *cm*) $2 = 4\left(\sin\frac{\pi}{6}t_1\right)$

By solving $t_1 = 1\ sec$ (For y = 4 *cm*) $t_2 = 3\ sec$

So time taken by particle in going from 2 *cm* to extreme position is $t_2 - t_1 = 2\ sec$. Hence required ratio will be $\frac{1}{2}$.

19. (a) On a planet, if a body dropped initial velocity (u = 0) from a height h and takes time t to reach the ground then $h = \frac{1}{2}g_P t^2 \Rightarrow g_P = \frac{2h}{t^2} = \frac{2\times 8}{4} = 4\ m/s^2$

Using $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T = 2\pi\sqrt{\frac{1}{4}} = \pi = 3.14\ sec.$

20. (d) $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \frac{l}{\sqrt{g}}$

$\Rightarrow \frac{\Delta T}{T}\times 100 = -\frac{1}{2}\left(\frac{\Delta g}{g}\right)\times 100 = -\frac{1}{2}(-2\%) = 1\%.$

21. (b) $n = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \Rightarrow n \propto \frac{1}{\sqrt{l}} \Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{l_2}{l_1}} = \sqrt{\frac{L_2}{2L_2}}$

$\Rightarrow \frac{n_1}{n_2} = \frac{1}{\sqrt{2}} \Rightarrow n_2 = \sqrt{2}\,n_1 \Rightarrow n_2 > n_1$

Energy $E = \frac{1}{2}m\omega^2 a^2 = 2\pi^2 mn^2 a^2$

$\Rightarrow \frac{a_1^2}{a_2^2} = \frac{m_2 n_2^2}{m_1 n_1^2}$ ($\because$ E is same)

Given $n_2 > n_1$ and $m_1 = m_2 \Rightarrow a_1 > a_2$

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