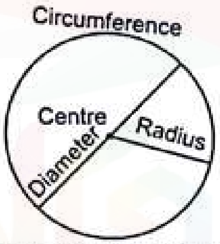
CHAPTER
27

Circle

Introduction

A circle is a simple shape consisting of those points in a plane that are a given distance from a given point. That given point is known as the centre of circle and the distance between any of the points and the centre is called the radius.

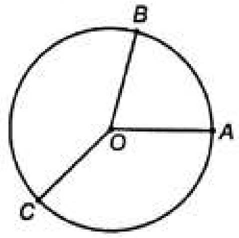
- Circles are simple closed curves which divide the plane into two regions : an interior and an exterior.
- A circle can be defined as the curve traced out by a point that moves so that its distance from a given point is constant.



Circle illustration showing a radius, a diameter, the centre and the

Components of a Circle

Centre



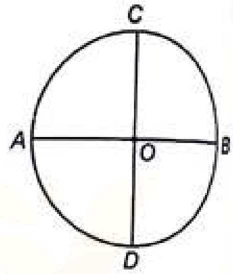
The centre is an essential component in order to construct a circle as without this the existance of circle is not possible. It is the fixed point in the interior of the circle through which all the points lying on the circle maintain a fixed distance. Here, O is the fixed point that is known as centre of the circle and the distances OA, OB and OC all are equal.

Radius

Radius is the fixed distance from the centre of the circle and the points lying on the circle.

Here, in the above figure OA, OB and OC all are having fixed distance, so they are known as radius of the circle.

Diameter



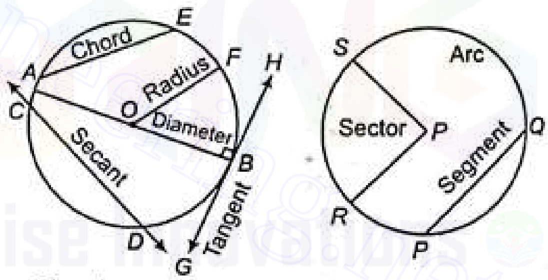
Diameter is the line segment that passes through the centre of the circle and touches both the ends of the circle. The diameter of a circle is twice the radius. Here, AB and CD are the diameters of the circle.

$$AB = CD = 2(OA)$$
$$= 2(OB) = 2(OC) = 2(OD)$$

- A circle can have an infinite number of diameters.

Chord

A chord is a line segment whose end points lie on the circle. A diameter is the longest chord in a circle.



Chord, secant, tangent, and diameter Arc, sector and segment

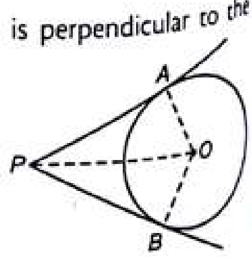
Here, in the above figure AE is a chord and AB is the diameter of the circle which is the longest chord of that circle.

Tangent

A tangent to a circle is a straight line that touches the circle at a single point.

Here, in the above figure GBH is a tangent passing through a single point B lying on the circle.

- A tangent at any point of a circle is perpendicular to the radius through the point of contact.
- If two tangents are drawn to a circle from an external point, then they subtend equal angles at the centre.



- They are equally inclined to the segment joining the centre to the point.

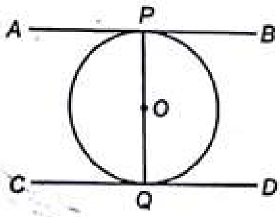
Here, $\angle POA = \angle POB, \angle APO = \angle OPB$

and $\angle AOB + \angle APB = 180°$

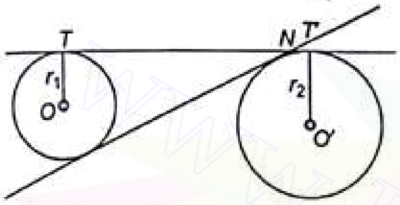
- The length of two tangents drawn from an external point to a circle are equal.

Here, $PA = PB$

- Tangents at the end points of a diameter of a circle are always parallel.



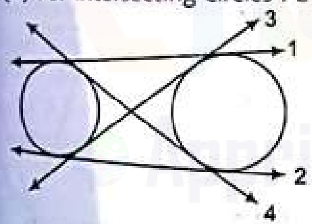
Here, PQ is the diameter, thus $AB \parallel CD$.

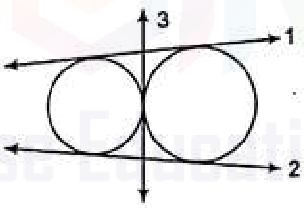


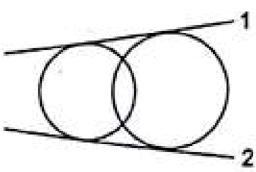
- Length of the direct common tangent is $TT' = \sqrt{(OO')^2 - (r_1 - r_2)^2}$ and length of transverse common tangent is

$$MN = \sqrt{(OO')^2 - (r_1 + r_2)^2}$$

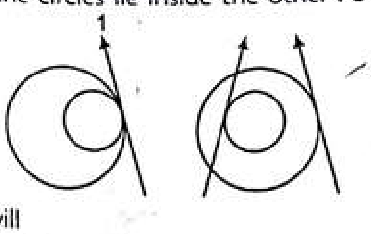
- Number of tangents
  (i) For disjoint circles : 4
  (ii) For the circles touching externally : 3
  (iii) For intersecting circles : 2



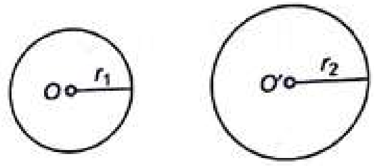




  (iv) For the circles touching internally : 1
  (v) If one of the circles lie inside the other : 0



- Two circles will
  (i) be disjoint when $OO' > r_1 + r_2$.
  (ii) be touching externally when $OO' = r_1 + r_2$.
  (iii) be intersecting when $OO' < r_1 + r_2$.
  (iv) be touching internally when $OO' = |r_2 - r_1|$
  (v) one of the circle will lie inside the other when $OO' < |r_2 - r_1|$.



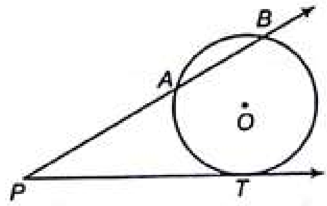
Secant

A secant is an extended chord : a straight line cutting the circle at two points.

Here, in the above figure CD is a secant cutting the circle at two distinct points as C and D.

- If PAB is a secant to a circle intersecting the circle at A and B and PT is a tangent, then

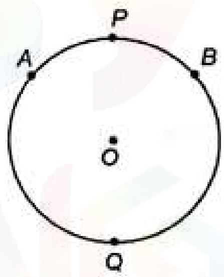
$$PA \times PB = PT^2$$



Arc

Any part of a circle is called an arc of the circle. Here, AB is an arc of circle and written as AB.

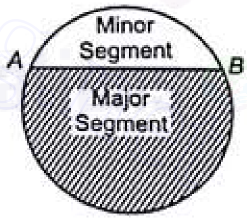
- If arc is smaller than semi-circle it is called a minor arc, otherwise it is called as a major arc.
  Here, APB is a minor arc and AQB is a major arc.



Segment

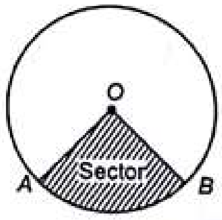
The area enclosed by an arc and its corresponding chord is called a segment of the circle.

- A segment is minor if its arc is minor.
- A segment is major if its arc is major.



Sector

The area enclosed by any two radii and the arc determined by the end points of the radii is called a sector of the circle.



Circumference

Circumference is the distance travelled in going once around a circle.

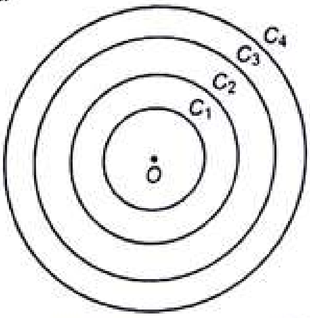
Circumference $= 2 \times \pi \times$ radius of the circle

or

$= \pi \times$ diameter of the circle

Concentric Circles

In a plane two or more circles are called concentric, if they have a common centre.



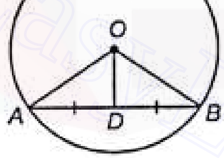
Here, in the adjoining figure C_1, C_2, C_3 and C_4 are known as concentric circles as they have a common centre 'O'.

- An infinite number of circles can be drawn with same centre.

Important Theorems on Circles

Theorem 1 If two arcs of a circle arc congruent, then the corresponding chords are equal.

Theorem 2 The perpendicular from the centre of a circle to a chord bisects the chord.

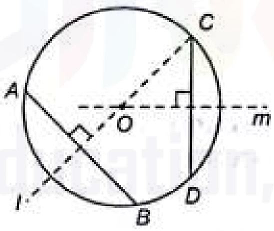


So, here $OD \perp AB$, then $AD = DB$

Theorem 3 The line joining the centre to the mid-point of a chord is perpendicular to the chord.

Here, if $AD = DB$, then $\angle ADO = \angle ODB = 90°$.

Theorem 4 The perpendicular bisectors of two chords of a circle intersect at its centre.

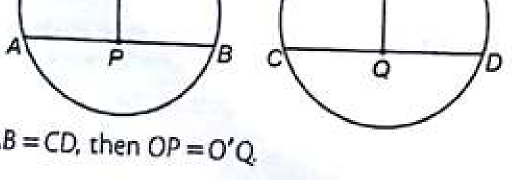


Here, AB, CD are the chords and l, m are perpendicular bisector of AB and CD. So, l and m meet at 'O'.

Theorem 5 There is one and only one circle passing through three non-collinear points.

- An infinite number of circles can be drawn to pass through a single point.
- An infinite number of circles can be drawn to pass through two given points.
- A unique circle can be drawn to pass through three given non-collinear points.

Theorem 6 Equal chords of congruent circles are equidistant from the corresponding centres.

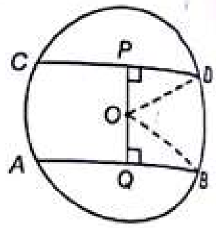


Here, if $AB = CD$, then $OP = O'Q$.

Theorem 7 Chords which are equidistant from the corresponding centres are equal.

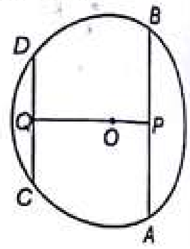
So, in above figure, if $OP = O'Q$, then $AB = CD$.

Theorem 8 Equal chords of a circle are equidistant from the centre.



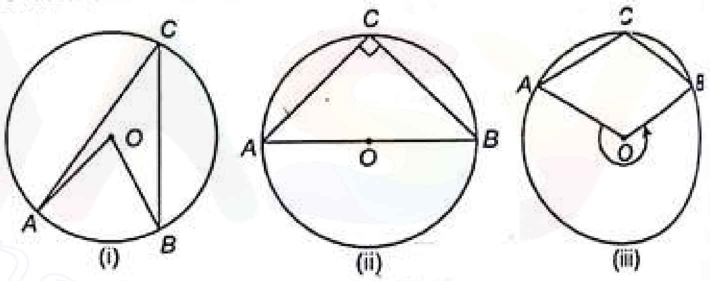
Here, if AB and CD are equal chords of circle, then $OP = OQ$.
(in conversely)
If $OP = OQ$, then also $AB = CD$
i.e., chords at equal distance for the centre are equal.

Theorem 9 Of any two chords of a circle, the greater chord is nearer to the centre.



Here, if $OQ > OP$, then $AB > CD$, where AB and CD are chords.

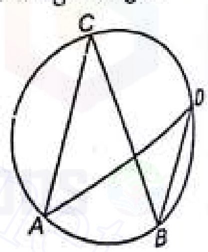
Theorem 10 The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the circumference of circle. Here, three case arises.



Here, in all the three cases $\angle AOB = 2\angle ACB$.

Theorem 11 The angle in a semi-circle is a right angle.

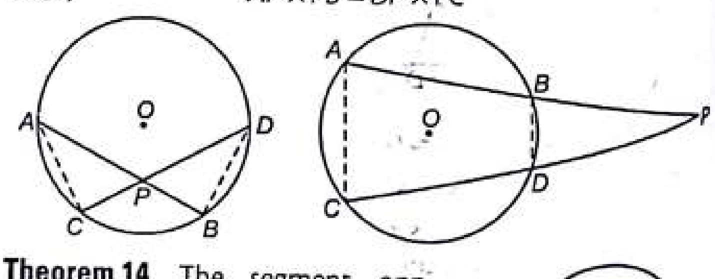
Theorem 12 Angle in the same segment of the circle are equal.



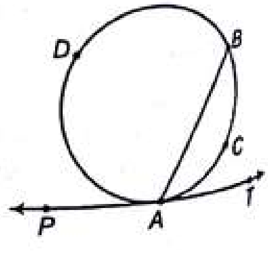
Here, $\angle ACB = \angle ADB$

Theorem 13 If two chords AB and CD of a circle intersect inside or outside the circle when produced at a point P.

Then, $AP \times PB = DP \times PC$

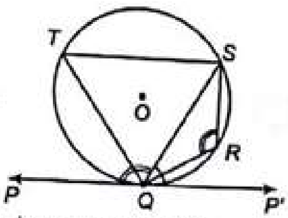


Theorem 14 The segment opposite to the angle formed by the chord of a circle with the tangent to a point is called the alternate segment for that angle.



Here, for $\angle BAT$, the alternate segment is ADB and for $\angle PAB$ the alternate segment is ACB.

If from the point of contact, a chord is drawn, then the angle which this chord makes with the given line, are equal, respectively to the angles formed in the corresponding alternate segments.



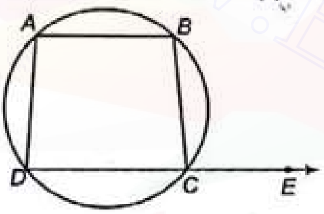
Here, in the adjoining figure, PP' is the tangent at Q to the circle.

$$\angle QTS = \angle SQP \text{ and } \angle PQS = \angle QRS$$

Cyclic Quadrilateral

A quadrilateral whose all vertices lie on a circle is called a cyclic quadrilateral.

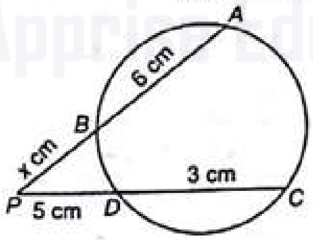
- $\angle A, \angle B, \angle C$ and $\angle D$ are interior angles.
- $\angle A$ and $\angle C$ are opposite angles also $\angle B$ and $\angle D$.
- $\angle A + \angle C = 180° = \angle B + \angle D$.
- The exterior angle, formed by producing a side of a cyclic quadrilateral is equal to the interior opposite angles.



Here, $\angle BAD = \angle BCE$

Example 1. In the given figure, chords AB and CD of a circle intersect externally at P. If $AB = 6$ cm, $CD = 3$ cm and $PD = 5$ cm, then the measurement of PB is

(a) 2.5 cm (b) 4 cm
(c) 10 cm (d) None of these



Sol. (b) $PA \times PB = PC \times PD \Rightarrow (x+6) \times x = 8 \times 5$

$\Rightarrow \quad x^2 + 6x - 40 = 0 \Rightarrow (x+10)(x-4) = 0$

$\Rightarrow \quad x = 4$, as $x \neq 10$

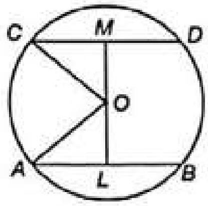
$\therefore \quad PB = 4$ cm

Example 2. AB and CD are two parallel chords on the opposite sides of the centre of the circle. If $AB = 10$ cm, $CD = 24$ cm and the radius of the circle is 13 cm, then what is the distance between the chords?

(a) 10 cm (b) 17 cm
(c) 24 cm (d) None of these

Sol. (b) From O draw $OL \perp AB$ and $OM \perp CD$. Join OA and OC.

$$AL = \frac{1}{2} AB = 5 \text{ cm}, OA = 13 \text{ cm}$$



$$OL^2 = OA^2 - AL^2 = (13)^2 - (5)^2 = (169 - 25) = 144$$

$\Rightarrow \quad OL = \sqrt{144} = 12$ cm

Now, $\quad CM = \frac{1}{2} \times CD = 12$ cm and $OC = 13$ cm

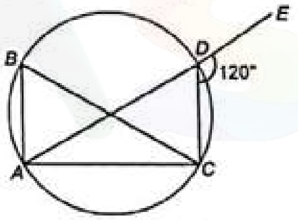
$\therefore \quad OM^2 = OC^2 - CM^2 = (13)^2 - (12)^2 = (169 - 144) = 25$

$\Rightarrow \quad OM = \sqrt{25} = 5$ cm

$\Rightarrow \quad ML = OM + OL = (5 + 12)$ cm $= 17$ cm

Example 3. In the given figure, what is the measure of $\angle ABC$?

(a) 30° (b) 45° (c) 60° (d) 75°



Sol. (c) $\angle ADC + \angle EDC = 180°$ (linear angle)

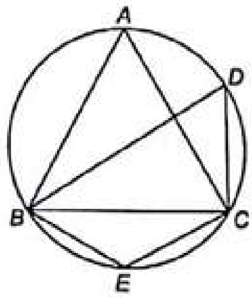
$\Rightarrow \quad \angle ADC + 120° = 180° \Rightarrow \angle ADC = 60°$

$\therefore \quad \angle ABC = \angle ADC = 60°$

(angles lying in the same segment of the circle)

Example 4. In the adjoining figure, ΔABC is an isosceles triangle with $AB = AC$ and $\angle ABC = 50°$. Then, the measure of $\angle BDC$ is

(a) 80° (b) 100° (c) 40° (d) 160°



Sol. (a) As, $\quad AB = AC$

$\Rightarrow \quad \angle ACB = \angle ABC = 50°$

$\therefore \quad \angle BAC = 180° - (50° + 50°) = 80°$

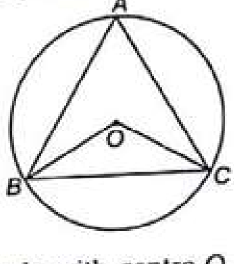
$\therefore \quad \angle BDC = \angle BAC = 80°$

(angle lying in the same segment of the circle)

Exercise

1. In a circle with centre O and radius 5 cm, AB is a chord of length 8 cm. If $OM \perp AB$, what is the length of OM?
   (a) 4 cm (b) 5 cm
   (c) 3 cm (d) None of these

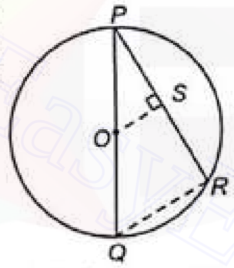
2. An equilateral ΔABC is inscribed in a circle with centre O. Then, $\angle BOC$ is equal to



   (a) 120°
   (b) 75°
   (c) 180°
   (d) 60°

3. A square $ABCD$ is inscribed in a circle with centre O. Then, the angle subtended by each side of the square at the centre O is
   (a) 120° (b) 180° (c) 45° (d) 90°

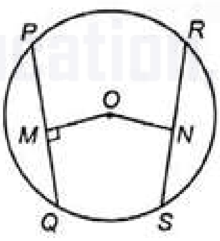
4. In the given figure, PQ is the diameter of a circle with centre at O. OS is perpendicular to PR. Then, OS is equal to



   (a) $\frac{1}{4}QR$ (b) $\frac{1}{3}QR$
   (c) $\frac{1}{2}QR$ (d) QR

5. If two circles C_1 and C_2 have three points in common, then
   (a) C_1 and C_2 are the same circle
   (b) C_1 and C_2 are concentric
   (c) C_1 and C_2 have different centres
   (d) None of the above

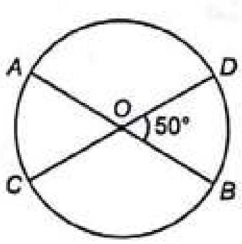
6. In the given figure, OM and ON are the perpendiculars drawn on the chords PQ and RS. If $OM = ON = 6$ cm. Then,



   (a) $PQ \geq RS$
   (b) $PQ < RS$
   (c) $PQ \leq RS$
   (d) $PQ = RS$

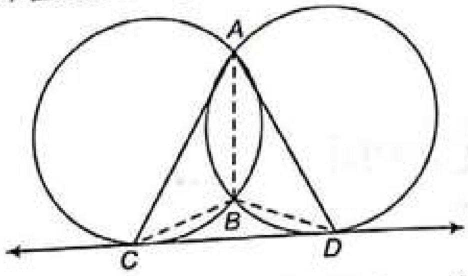
7. PQ and RS are two chords of a circle intersecting at O. Then,
   (a) $\Delta POS \cong \Delta QOR$
   (b) ar $(\Delta POS) =$ ar (ΔQOR)
   (c) $\Delta POS \sim \Delta QOR$
   (d) Both (a) and (b)

8. Diameter AB and CD of a circle intersect at O. If $m\angle BOD = 50°$, then mOD is



   (a) 50°
   (b) 180°
   (c) 130°
   (d) 310°

9. CD is a direct common tangent to two circles intersecting each other at A and B. Then, $\angle CAD + \angle CBD$ is equal to?



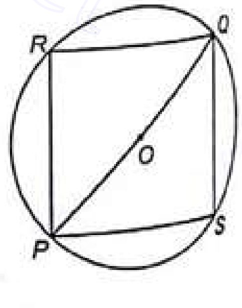
   (a) 180° (b) 90° (c) 360° (d) 120°

10. In a circle of radius 17 cm, two parallel chords are drawn on opposite side of a diameter. The distance between the chords is 23 cm. If the length of one chord is 16 cm, then the length of the other is
    (a) 34 cm (b) 15 cm
    (c) 23 cm (d) 30 cm

11. It is not possible to draw a circle having its centre on a fixed straight line l and passing through two points A and B not on l, if___
    (a) l is parallel to $\overline{AB}$
    (b) l is the perpendicular bisector of $\overline{AB}$
    (c) l is perpendicular to $\overline{AB}$ but does not bisect it
    (d) l is not perpendicular to $\overline{AB}$ but bisects it

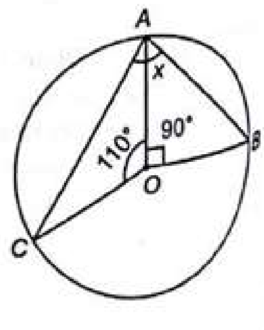
12. If AB is a chord of a circle, P and Q are the two points on the circle different from A and B, then
    (a) the angles subtended at P and Q by AB are always equal
    (b) the sum of the angles subtended by AB at P and Q is always equal to two right angles
    (c) the angles subtended by AB at P and Q are either equal or supplementary
    (d) the sum of the angles subtended at P and Q is to four right angles

13. In the adjoining figure, POQ is the diameter of the circle, R and S are any two points on the circles. Then,



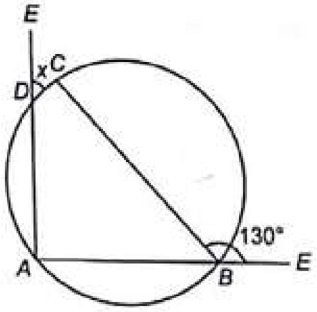
    (a) $\angle PRQ > \angle PSQ$
    (b) $\angle PRQ < \angle PSQ$
    (c) $\angle PRQ = \angle PSQ$
    (d) $\angle PRQ = \frac{1}{2}\angle PSQ$

14. If O is the centre of the circle, the value of 'x' in the adjoining figure is



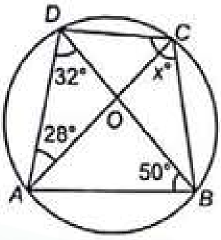
    (a) 80°
    (b) 70°
    (c) 60°
    (d) 50°

15. In the given figure A, B, C, D are the concyclic points. The value of 'x' is



(a) 50°
(b) 60°
(c) 70°
(d) 90°

16. If O is the centre of the circle, then 'x' is



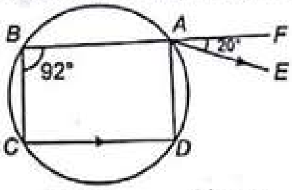
(a) 72°
(b) 62°
(c) 82°
(d) 52°

17. Two circles touch each other internally. Their radii are 2 cm and 3 cm. The biggest chord of the outer circle which is outside the inner circle is of length
(a) $2\sqrt{2}$ cm (b) $3\sqrt{2}$ cm (c) $2\sqrt{3}$ cm (d) $4\sqrt{2}$ cm

18. If two circles are such that the centre of one lies on the circumference of the other, then the ratio of the common chord of the two circles to the radius of any one of the circles is
(a) 2 : 1 (b) $\sqrt{3}$: 1 (c) $\sqrt{5}$: 1 (d) 4 : 1

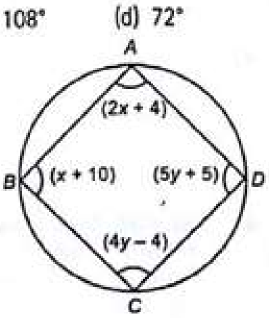
19. The incircle of a ΔABC touches the sides AB, BC and AC at the points P, Q, R, respectively, then which of the following statements is/are correct?
I. $AP + BQ + CR = PB + QC + RA$
II. $AP + BQ + CR = \frac{1}{2}$ (perimeter of ΔABC)
III. $AP + BQ + CR = 3(AB + BC + CA)$
(a) I, II and III (b) Only I (c) II and III (d) I and II

20. In the given figure, $ABCD$ is a cyclic quadrilateral. AE is drawn parallel to CD and BA is produced. If $\angle ABC = 92°$ and $\angle FAE = 20°$, then $\angle BCD$ is equal to



(a) 88° (b) 98° (c) 108° (d) 72°

21. The values of x and y in the figure are measure of angles, then x + y is equal to

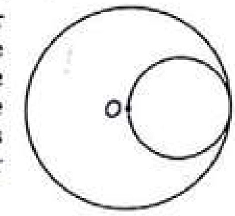


(a) 90°
(b) 85°
(c) 75°
(d) 65°

22. Which of the following statements is incorrect?
(a) A circle is symmetrical about the diameter
(b) Two circles are divided symmetrically by the link passing through their centres
(c) More than one circle can be drawn through three non-collinear points
(d) Two circles cannot cut each other in more than two points without coinciding entirely

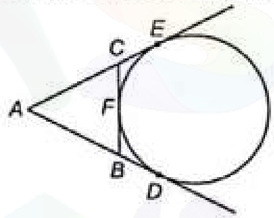
23. Three points A, B, C are on the same line. A circle passes through B and C. Then, the locus of the tangent drawn from A to the circle, if the diameter of the circle is $2a$, is
(a) $x^2 + y^2 = a^2$ (b) $xx_1 + yy_1 = a^2$
(c) $xy = 0$ (d) $x + y = 0$

24. In the adjoining figure, a smaller circle touches a larger circle internally and passes through the centre O of the larger circle. If the area of the smaller circle is 200 cm^2, the area of the larger circle in sq cm is



(a) 400 (b) 600 (c) 800 (d) 1000

25. In the adjoining figure AD, AE and BC are tangent to the circle at D, E, F, respectively, then

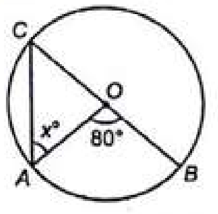


(a) $AD = AB + BC + AC$ (b) $2AD = AB + BC + AC$
(c) $AD = \frac{1}{2}(AB + BC + AC)$ (d) $3AD = AB + BC + AC$

26. S_1 and S_2 are two circles on a plane with radii 4 cm, and 2 cm, respectively and the distance between their centres is 3 cm. Which one of the following statements is true?
(a) S_2 lies entirely within the circle S_1
(b) S_1 and S_2 touch each other internally
(c) S_1 and S_2 touch each other externally
(d) S_1 and S_2 intersect in two distinct points

27. ACB is a tangent to a circle at C, CD and CE are chords such that $\angle ACE > \angle ACD$. If $\angle ACD = \angle BCE = 50°$, then
(a) $CD = CE$
(b) ED is not parallel to AB
(c) ED passes through the centre of the circle
(d) ΔCDE is a right angled triangle

28. If 'O' is the centre of circle, then x is equal to



(a) 80° (b) 60° (c) 40° (d) 20°

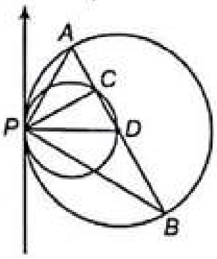
29. If two equal circles touch each other externally, the common tangent divides the line of centres in the ratio

(a) 1 : 1 (b) 2 : 1 (c) 1 : 2 (d) 3 : 2

30. M and N are the centres of two circles whose radii are 7 cm and 4 cm, respectively. The direct common tangents to the circles meet MN in P. Then, P divides MN in the ratio

(a) 7 : 4 internally (b) 4 : 7 internally
(c) 7 : 4 externally (d) 4 : 7 externally

31. Two circle touch internally at a point P and a chord AB of the larger circle intersects the other circle at C and D. Then, which statement is true?

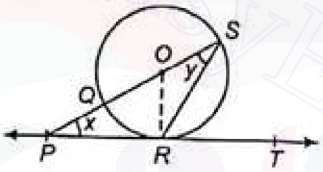


(a) $\angle CPA = \angle DPB$
(b) $\angle CPA = \angle CPD$
(c) $\angle DPC = \angle DPY$
(d) $\angle APC = \angle DPB$

32. With the vertices of a ΔABC as centre three circles are described, each touching the other two circle externally. If the sides of the triangle are 9 cm, 7 cm and 6 cm. Then, the radius of the circle are

(a) 4, 5, 2 (b) 4, 5, 6
(c) 3, 2, 3 (d) All equal to 3 cm

33. In the given figure PT touches the circle with centre O at R. Diameter SQ when produced meet PT at P. If $\angle SPR = x$ and $\angle QSR = y$, then $x + 2y$ is equal to



(a) 180° (b) 90°
(c) 135° (d) None of these

34. Consider the following statements

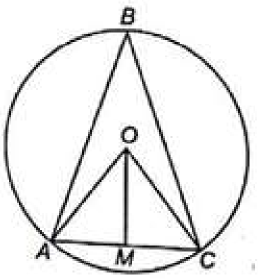
I. The opposite angles of a cyclic quadrilateral are supplementary.

II. Angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Which one of the following is correct in respect of the above statements? **(CDS 2011 II)**

(a) Statement I ⇒ Statement II
(b) Statement II ⇒ Statement I
(c) Statement I ⇔ Statement II
(d) Neither Statement I ⇒ Statement II nor Statement II ⇒ Statement I

35. In the given figure, O is the centre of the circle, $OA = 3$ cm, $AC = 3$ cm and OM is perpendicular to AC. What is $\angle ABC$ equal to? **(CDS 2011 II)**



(a) 60°
(b) 45°
(c) 30°
(d) None of the above

36. Consider the following statements

I. Let P be a point on a straight line L. Let Q, R and S be the points on the same plane containing the line L such that PQ, PR and PS are perpendicular to L. Then, there exists no triangle with vertices Q, R, S.

II. Let C be a circle passing through three distinct points D, E and F such that the tangent at D to the circle C is parallel to EF. Then, DEF is an isosceles triangle.

Which of the statement(s) given above is/are correct? **(CDS 2011 II)**

(a) I only (b) II only
(c) Both I and II (d) Neither I nor II

37. Two circles touch each other internally. Their radii are 4 cm and 6 cm. what is the length of the longest chord of the outer circle which is outside the inner circle? **(CDS 2011 II)**

(a) $4\sqrt{2}$ cm (b) $4\sqrt{3}$ cm (c) $6\sqrt{3}$ cm (d) $8\sqrt{2}$ cm

38. The distance between the centres of two circles having radii 4.5 cm and 3.5 cm, respectively is 10 cm. What is the length of the transverse common tangent of these circles? **(CDS 2011 II)**

(a) 8 cm (b) 7 cm
(c) 6 cm (d) None of these

39. ABC is an equilateral triangle inscribed in a circle with $AB = 5$ cm. Let the bisector of the angle A meet BC in X and the circle in Y. What is the value of $AX \cdot AY$? **(CDS 2011 II)**

(a) 16 cm^2 (b) 20 cm^2 (c) 25 cm^2 (d) 30 cm^2

40. Two unequal circles are touching each other externally at P, APB and CPD are two secants cutting the circles at A, B, C and D. Which one of the following is correct? **(CDS 2011 II)**

(a) $ACBD$ is parallelogram (b) $ACBD$ is a trapezium
(c) $ACBD$ is a rhombus (d) None of these

41. A bicycle is running straight towards North. What is the locus of the centre of the front wheel of the bicycle whose diameter is d? **(CDS 2011 II)**

(a) A line parallel to the path of the wheel of the bicycle at a height d cm
(b) A line parallel to the path of the wheel of the bicycle at a height $d/2$ cm
(c) A circle of radius $d/2$ cm
(d) A circle of radius d cm

42. $ABCD$ is a quadrilateral, the sides of which touch a circle. Which one of the following is correct? **(CDS 2010 II)**

(a) $AB + AD = CB + CD$ (b) $AB : CD = AD : BC$
(c) $AB + CD = AD + BC$ (d) $AB : AD = CB : CD$

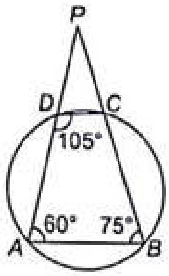
43. Let PAB be a secant to a circle intersecting at points A and B and PC is a tangent. Which one of the following is correct? **(CDS 2010 II)**

(a) The area of rectangle with PA, PB as sides is equal to the area of square with PC as sides
(b) The area of rectangle with PA, PC as sides is equal to the area of square with PB as sides

(c) The area of rectangular with PC, PB as sides is equal to the area of square with PA as side

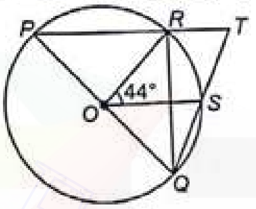
(d) The perimeter of rectangle with PA, PB as sides is equal to the perimeter of square with PC as side

44. In the given figure, if $\angle BAD = 60°$, $\angle ADC = 105°$, then what is $\angle DPC$ equal to? **(CDS 2010 II)**



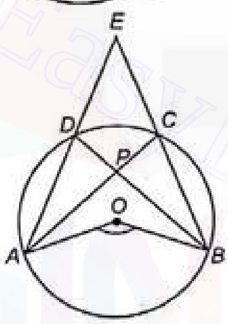
(a) 40°
(b) 45°
(c) 50°
(d) 60°

45. In the given figure, PQ is a diameter of the circle whose centre is at O. If $\angle ROS = 44°$ and OR is a bisector of $\angle PQR$, then what is the value of $\angle RTS$? **(CDS 2010 II)**



(a) 46°
(b) 64°
(c) 69°
(d) None of the above

46. In the given figure, O is the centre of the circle. AC and BD intersect at P. If $\angle AOB = 100°$ and $\angle DAP = 30°$, what is $\angle APB$? **(CDS 2010 II)**

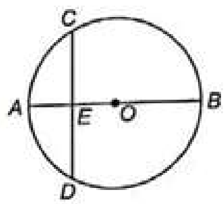


(a) 77°
(b) 80°
(c) 85°
(d) 90°

47. Two circles touch each other externally at P. Two secants APB and CPD are drawn through P to meet the circle at A, C and B, D, respectively. Then, which one of the following is correct? **(CDS 2010 I)**

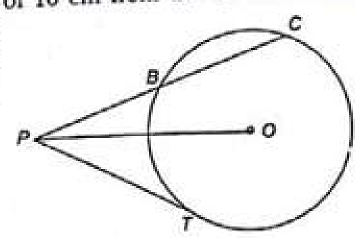
(a) AC is perpendicular to BD
(b) AC intersects BD
(c) AC is parallel to BD
(d) None of the above

48. In the given figure, AB is a diameter of a circle and CD is perpendicular to AB, if $AB = 10$ cm and $AE = 2$ cm, then what is the length of ED? **(CDS 2010 I)**



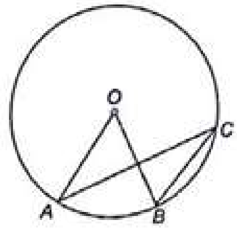
(a) 5 cm (b) 4 cm
(c) $\sqrt{10}$ cm (d) $\sqrt{20}$ cm

49. In the given figure, PT is a tangent to a circle of radius 6 cm. If P is at a distance of 10 cm from the centre O and $PB = 5$ cm, then what is the length of the chord BC? **(CDS 2009 II)**



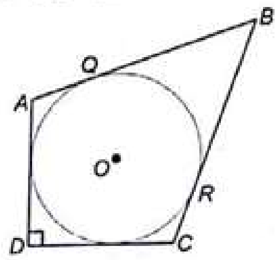
(a) 7.8 cm
(b) 8 cm
(c) 8.4 cm
(d) 9 cm

50. In the given figure, $\angle AOB = 46°$, AC and OB intersect each other at right angles. What is the measure of $\angle OBC$? (O is the centre of the circle.) **(CDS 2009 II)**



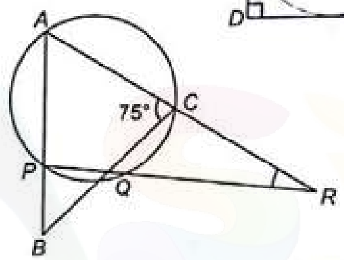
(a) 44° (b) 46°
(c) 67° (d) 78.5°

51. In the given figure, a circle is inscribed in a quadrilateral $ABCD$. Given that, $BC = 38$ cm, $QB = 27$ cm, $DC = 25$ cm and AD is perpendicular to DC. What is the radius of the circle? **(CDS 2009 I)**



(a) 11 cm
(b) 14 cm
(c) 15 cm
(d) 16 cm

52.



In the figure given above, what is $\angle CBA$? **(CDS 2009 I)**

(a) 30° (b) 45° (c) 50° (d) 60°

53. A, B, C and D are four distinct points on a circle whose centre is at O.
If $\angle OBD - \angle CDB = \angle CBD - \angle ODB$, then what is $\angle A$ equal to? **(CDS 2009 I)**

(a) 45° (b) 60° (c) 120° (d) 135°

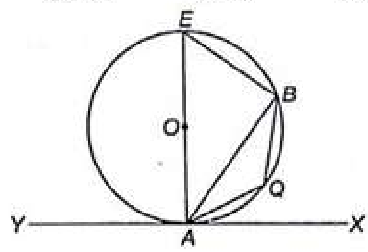
54. PQ is a common chord of two circles. APB is a secant line joining points A and B on the two circles. Two tangents AC and BC are drawn. If $\angle ACB = 45°$, then what is $\angle AQB$ equal to? **(CDS 2009 I)**

(a) 75° (b) 90° (c) 120° (d) 135°

55. $ABCD$ is concyclic quadrilateral. The tangents at A and C intersect each other at P. If $\angle ABC = 100°$, then what is $\angle APC$ equal to? **(CDS 2009 I)**

(a) 10° (b) 20° (c) 30° (d) 40°

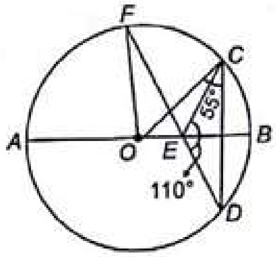
56.



In the figure given above, YAX is a tangent to the circle with centre O. If $\angle BAX = 70°$ and $\angle BAQ = 40°$, then what is $\angle ABQ$ equal to? **(CDS 2009 I)**

(a) 20° (b) 30° (c) 35° (d) 40°

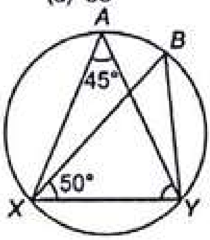
57.



In the figure given above, AB is a diameter of the circle with centre O and $EC = ED$. What is $\angle EFO$? **(CDS 2008 II)**

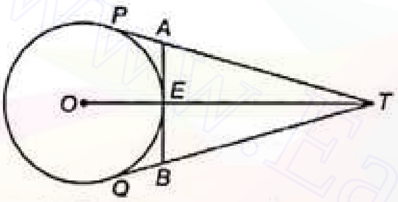
(a) 15° (b) 20° (c) 25° (d) 30°

58. In the given figure, what is $\angle BYX$ equal to? **(CDS 2008 I)**



(a) 85°
(b) 50°
(c) 45°
(d) 90°

59.



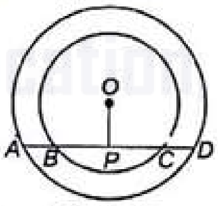
From a point T, 13 cm away from the centre O of a circle of radius 5 cm, tangents PT and QT are drawn. What is the length of AB? **(CDS 2008 I)**

(a) $\frac{19}{3}$ cm (b) $\frac{20}{3}$ cm (c) $\frac{40}{13}$ cm (d) $\frac{22}{3}$ cm

60. With A, B and C as centres, three circles are drawn such that they touch each other externally. If the sides of the ΔABC are 4 cm, 6 cm and 8 cm, then what is the sum of the radii of the circles? **(CDS 2008 I)**

(a) 9 cm (b) 10 cm (c) 12 cm (d) 14 cm

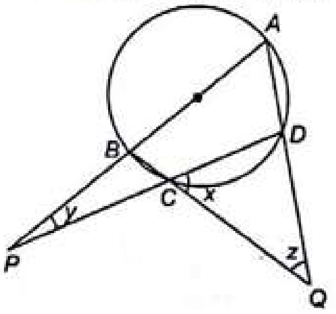
61. In the given figure, AD is a straight line, OP perpendicular to AD and O is the centre of both circles. If $OA = 20$ cm, $OB = 15$ cm and $OP = 12$ cm, what is AB equal to? **(CDS 2008 I)**



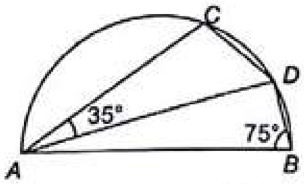
(a) 7 cm (b) 8 cm (c) 10 cm (d) 12 cm

62. In the given figure, if $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$, where $\angle DCQ = x$, $\angle BPC = y$ and $\angle DQC = z$, then what are the values of x, y and z, respectively? **(CDS 2007 II)**

(a) 33°, 44° and 55°
(b) 36°, 48° and 60°
(c) 39°, 52° and 65°
(d) 42°, 56° and 70°



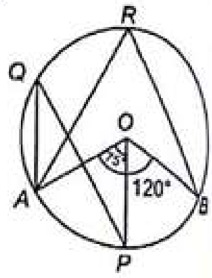
63.



In the figure given above, C and D are points on the semi-circle described on AB as diameter. $\angle ABD = 75°$ and $\angle DAC = 35°$. What is the $\angle BDC$? **(CDS 2007 II)**

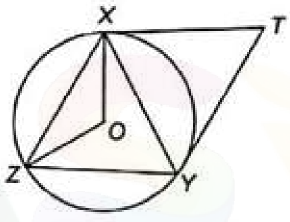
(a) 130° (b) 110°
(c) 90° (d) 100°

64. In the given figure, if $\angle AOP = 75°$ and $\angle ACB = 120°$, then what is $\angle AQP$? **(CDS 2007 II)**



(a) 45°
(b) 37.5°
(c) 30°
(d) 22.5°

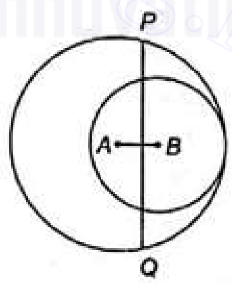
65.



In the figure given above, O is the centre of the circumcircle of the ΔXYZ. Tangents at X and Y intersect at T. If $\angle XTY = 80°$, what is the value of $\angle ZXY$? **(CDS 2007 II)**

(a) 20° (b) 40°
(c) 60° (d) 80°

66. Two circles with centres A and B touch each other internally, as shown in the given figure. Their radii are 5 and 3 units, respectively. Perpendicular bisector of AB meets the bigger circle in P and Q. What is the length of PQ? **(CDS 2007 II)**



(a) $2\sqrt{6}$ (b) $\sqrt{34}$
(c) $4\sqrt{6}$ (d) $6\sqrt{2}$

67. In a ΔABC, $AB = AC$. A circle through B touches AC at D and intersects AB at P. If D is the mid-point of AC, which one of the following is correct? **(CDS 2007 II)**

(a) $AB = 2AP$ (b) $AB = 3AP$
(c) $AB = 4AP$ (d) $2AB = 5AP$

Answers

1. (c)	2. (a)	3. (d)	4. (c)	5. (a)	6. (d)	7. (d)	8. (c)	9. (a)	10. (d)
11. (c)	12. (c)	13. (c)	14. (a)	15. (a)	16. (c)	17. (d)	18. (b)	19. (d)	20. (c)
21. (d)	22. (c)	23. (b)	24. (c)	25. (b)	26. (d)	27. (a)	28. (c)	29. (a)	30. (c)
31. (a)	32. (a)	33. (b)	34. (d)	35. (c)	36. (c)	37. (a)	38. (c)	39. (c)	40. (d)
41. (b)	42. (c)	43. (a)	44. (b)	45. (d)	46. (b)	47. (d)	48. (b)	49. (a)	50. (c)
51. (b)	52. (d)	53. (b)	54. (d)	55. (b)	56. (b)	57. (b)	58. (a)	59. (b)	60. (a)
61. (a)	62. (b)	63. (a)	64. (b)	65. (d)	66. (c)	67. (c)			

Hints and Solutions

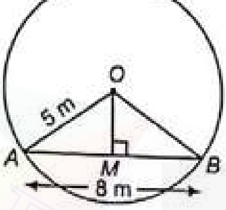
1. $OA = 5$ cm,

$AM = \frac{1}{2}AB, AM = 4$ cm

$OM^2 = OA^2 - AM^2$

$= 5^2 - 4^2$

$= 9\ OM = 3$ cm



2. Method I. $\angle BOC = 90° + \frac{1}{2}\angle A$

$= 90° + \frac{1}{2}60°$

$\Rightarrow \quad \angle BOC = 120°$

Method II. $\angle BOC = 2\angle A = 2 \times 60°$ (by theorem)

$\Rightarrow \quad \angle BOC = 120°$

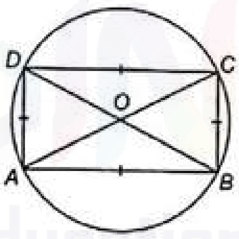
3. A square has four equal side.

$\therefore$ Each side subtends the same angle at the centre O.

Let angle subtended be $x°$.

So, $4x° = 360°$

$\Rightarrow \quad x° = \frac{360°}{4} = 90°$



4. As, O is mid-point of PQ and $\angle PRQ = 90°$ (angle is semi-circle)

So, $OS \parallel QR$ as both $\perp PR$

$\therefore \quad \frac{PO}{PQ} = \frac{OS}{QR}$

$\frac{OS}{QR} = \frac{1}{2} \Rightarrow OS = \frac{1}{2}QR$

5. Clearly, two circle can intersect at two points only.

6. $PQ = RS$ as equal chords of circle are equidistant from the distance.

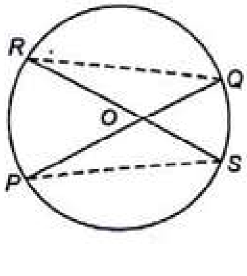
7. As, in ΔPOS and ΔROQ

$OR = OQ = OP = OS$

and $\angle ROQ = \angle POS$

(Vertically opposite angle)

Also, arc are equal.



8. $m\angle BOD = 50°$

$\Rightarrow \quad \angle BOD = 50°$

$\Rightarrow \quad \angle AOD = 180° - \angle BOD = 130°$

$\Rightarrow \quad mOD = 130°$

9. Here, $\angle CAB = \angle BCD$ (angles in alternate segments)

and $\angle DAB = \angle CDB$ (angles in alternate segments)

$\angle CAD = \angle CAB + \angle DAB = \angle BCD + \angle CDB$

$\Rightarrow \quad \angle CAD + \angle CBD = \angle BCD + \angle CDB + \angle CBD$

$= 180°$ (angles of a Δ)

10. Here, $BE = \frac{1}{2}AB = 8$ cm

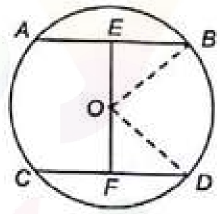
$OB = OD = 17$ cm

$OE = \sqrt{OB^2 - BE^2} = \sqrt{17^2 - 8^2}$

$= \sqrt{225} = 15$ cm

$\therefore \quad OF = EF - OE$

$= (23 - 15) = 8$ cm



$FD = \sqrt{OD^2 - OF^2} = \sqrt{17^2 - 8^2} = 15$ cm

$\therefore \quad CD = 2FD = 30$ cm

11. Let P be a fixed point on l. Then, a circle can be drawn through the points P, A, B only when $PA = PB$. When $l \perp AB$ and l does not bisect AB, then $PA \neq PB$, so in this case, the circle cannot be drawn to pass through P, A, B.

12. There are two possibilities

Case I When P and Q are on the same side of AB. In this case $\angle APB = \angle AQB$ (angle in the same segment).

Case II When P and Q are on the opposite of AB. In this case PAQB is a cyclic quadrilateral.

So, $\angle APB + \angle AQB = 180°$

Here, (c) is correct option.

13. $\angle PRQ = \angle PSQ = 90°$ (each angle in semi-circle)

14. $\angle COB = 360° - (110° + 90°) = 160°$

$\therefore \quad x = \angle CAB = \frac{1}{2}\angle COB = \frac{1}{2} \times 160° = 80°$

15. $\angle CBF = \angle CDA \Rightarrow \angle CDA = 130°$

$\angle CDA + x = 180°$ (linear pair)

$x = 180° - 130° = 50°$

16. $\angle ACB = \angle ADB = 32°$

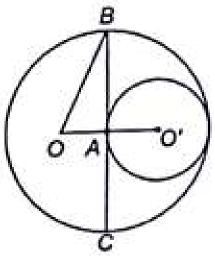
$\angle ACD = \angle ABD = 50°$

$\therefore \quad x = \angle BCD = \angle ACB + \angle ACD = 82°$

17. Here, $OA = 10$ cm, $OB = 3$ cm

$\therefore \quad AB = \sqrt{3^2 - 1^2} = \sqrt{8}$ cm

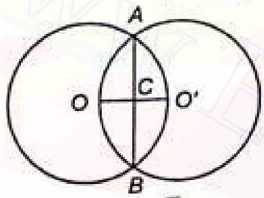
$\therefore$ Required length $= BC = 2AB = 2\sqrt{8}$ cm $= 4\sqrt{2}$ cm



18. Here, let O, O' be the centres of the circle.
As, the centre of each lies on the circumference of the other, the two circles will have the same radius. Let it be r.

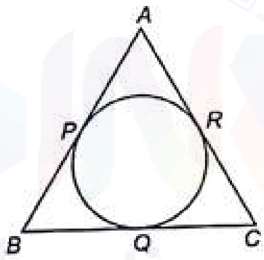
$\therefore \quad OC = O'C = \frac{r}{2}$

$\therefore \quad AC = \sqrt{OA^2 - OC^2} = \sqrt{r^2 - \frac{r^2}{4}} = \frac{\sqrt{3}}{2}r$



Hence, $\frac{\text{Common chord}}{\text{Radius}} = \frac{\sqrt{3}r}{2} = \sqrt{3}:1$

19. As, the tangents drawn from an external point to a circle are equal.



$\therefore \quad AP = AR, BQ = BP$

and $\quad CR = QC$

$\therefore \quad AP + BQ + CR = BP + QC + RA$

and perimeter of

$\Delta ABC = AB + BC + CA$

$= (AP + PB) + (BQ + QC) + (OR + RA)$

$= (AP + BQ) + (BQ + CR) + (CR + AP)$

$= 2(AP + BQ + CR)$

$\therefore \quad AP + BQ + CR = \frac{1}{2}$ (perimeter of ΔABC)

20. $\angle B + \angle D = 180°$

$\Rightarrow \quad \angle D = 180° - \angle B = 180° - 92° = 88°$

$\angle DAE = \angle D = 88°$

$\angle FAD = 88° + 20° = 108°$

$\angle BCD = \angle FAD = 108°$

$\Rightarrow \quad \angle BCD = 108°$

21. As, $\angle B + \angle D = 180°$ and $\angle A + \angle C = 180°$

$x + 10 + 5y + 5 = 180°$

$x + 5y = 165° \quad$ ...(i)

$2x + 4 + 4y - 4 = 180°$

$2x + 4y = 180° \quad$ ...(ii)

Solving, x and y are 40° and 25°

$x + y = 40° + 25° = 65°$

22. Clearly, not more than one circle can be drawn through three non-collinear points.

23. The focus of the tangent from a point A.
Let having coordinate (x_1, y_1) is equation of the tangent, which is $xx_1 + yy_1 = a^2$.

24. Let the radius of the larger circle be R, then radius of smaller circle $= \frac{R}{2}$

$\therefore \quad \pi\frac{R^2}{4} = 200 \Rightarrow \pi R^2 = 4 \times 200$

$\pi R^2 = 800$

Here, area of larger circle $= 800\text{ cm}^2$

25. As, the tangents drawn to a circle from a point outside it are equal.

We have, $\quad AD = AE, BD = BF$ and $CE = CF$

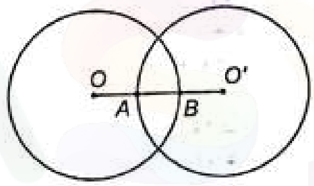
$AD = AB + BD = AB + BF$

$AD = AC + EC = AC + CF$

$\therefore \quad 2AD = AB + AC + (BF + CF)$

$2AD = AB + AC + BC$

26. $OB = 4$ cm



$O'A = 2$ cm

$OO' = 3$ cm

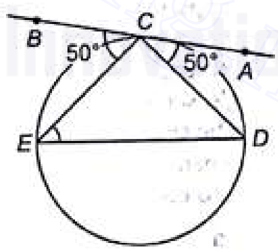
As, $\quad OO' \neq OB + O'A$

So, circle does not touch each other externally.

Also, $\quad OO' \neq OB - O'A$

So, circle does not touch internally, hence they cut each other at two distinct points.

27. Join ED, then



$\angle DEC = \angle ACD = 50°$

(angle in alternate segment)

$\angle EDC = \angle BCE = 50°$

(cyclic in alternate segment)

$\therefore \quad \angle DEC = \angle EDC$

So, $\quad CD = CE$

28. $\angle OAC = \angle OCA$

$2\angle OAC = 80° \quad$ (External angle of ΔOAC)

$\angle OAC = \frac{80°}{2} = 40°$

$\Rightarrow \quad x = 40°$

29. Since, the direct common tangents to two circles divides the line joining their centres externally in the ratio of their radii. Here, both the circles being of equal radii. This ratio is 1 : 1.

30. The direct common tangent divides the line joining the centres of two circles externally in the ratio of their sides.

$\therefore$ Required ratio $= \frac{7}{4}$ or $7:4$

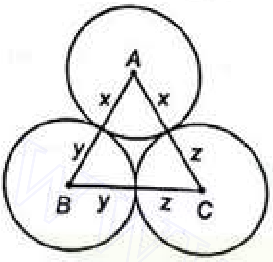
31. $\angle BPY = \angle BAP$ (angles in alternate segments)

$\angle DPY = \angle DCP$ (angles in alternate segments)

$\therefore$ $\angle DPY - \angle BPY = \angle DCP - \angle BAP$

or $\angle DPB = \angle CPA$

32. Let $AB = 9$ cm, $BC = 7$ cm, $AC = 6$ cm



Let x, y, z be radii of circles with centre A, B, C.

$x + y = 9$, $y + z = 7$ and $z + x = 6$

$\therefore$ $2(x + y + z) = 22$

or $(x + y + z) = 11$

$\therefore$ $z = 11 - 9 = 2$ cm

$x = 11 - 7 = 4$ cm

$y = 11 - 6 = 5$ cm

So, radii are 4 cm, 5 cm and 2 cm.

33. $\angle SRQ = 90°$ angle in semi-circle

$\angle QRP = \angle QSR = y°$ (angle in alternate segments)

Also, $\angle PRS = 90 + y°$

In ΔPRS,

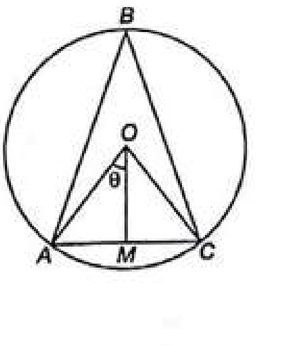
$$\angle SRP + \angle RPS + \angle PSR = 180°$$
$$(90 + y°) + x° + y° = 180°$$
$$x + 2y° = 90°$$

34. I. It is true that the opposite angles of a cyclic quadrilateral are supplementary.

II. It is also true that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Hence, both statements are individually true, but neither statements implies to each other.

35. Given, $OA = 3$ cm and



$$AC = 3 \text{ cm} \Rightarrow AM = \frac{3}{2} \text{ cm}$$

In ΔOAM,

$$\sin\theta = \frac{AM}{OA} = \frac{3/2}{3}$$

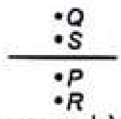
$\Rightarrow$ $\sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$

$\therefore$ $\angle AOC = 2\theta = 60°$

By using the property of circle,

$$\angle ABC = \frac{1}{2}\angle AOC = \frac{60°}{2} = 30°$$

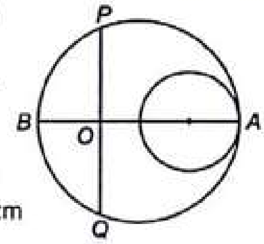
36. I. It is clear from the figure that points Q, S and R in a straight line.



II. Since, PQ is parallel to EF.

$\therefore$ $\angle PDE = \angle DEF$ (alternate angle)

37. OA = Diameter of inner circle = 4 cm



and $OB = AB - OA = 6 - 4 = 2$ cm

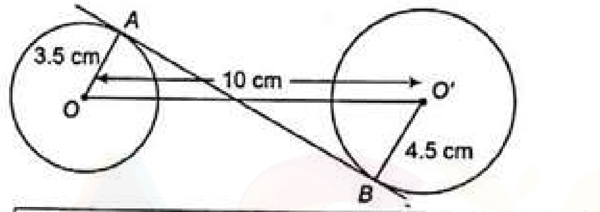
PQ and AB are two chord of outer circle.

$\therefore$ $OA \times OB = OP \times OQ$ $(\because OP = OQ)$

$\Rightarrow$ $4 \times 2 = OP^2 \Rightarrow OP = 2\sqrt{2}$ cm

$\therefore$ $PQ = 2 \times 2\sqrt{2} = 4\sqrt{2}$ cm

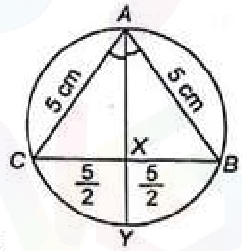
38. From figure, length of the transverse common tangent of these circles



$= \sqrt{(\text{Distance between the centres of circles})^2 - (\text{Sum of radius})^2}$

$= \sqrt{10^2 - (4.5 + 3.5)^2} = \sqrt{10^2 - 8^2} = \sqrt{36} = 6$ cm

39. In ΔABC,



$$BX = \frac{5}{2} \text{ cm}, CX = \frac{5}{2} \text{ cm}$$

and $AX = \frac{\sqrt{3}}{2} \times 5 = \frac{5\sqrt{3}}{2}$ cm

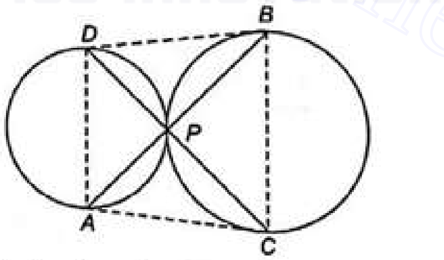
AY and BC are the chord of circle

$\therefore$ $AX \cdot XY = BX \cdot XC$

$\Rightarrow$ $\frac{5\sqrt{3}}{2} \cdot XY = \frac{5}{2} \cdot \frac{5}{2} \Rightarrow XY = \frac{5}{2\sqrt{3}}$

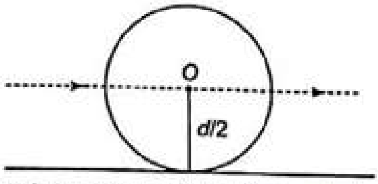
$\therefore$ $AX \cdot AY = \left(\frac{5\sqrt{3}}{2} + \frac{5}{2\sqrt{3}}\right) \times \frac{5}{2\sqrt{3}} = 25 \text{ cm}^2$

40.



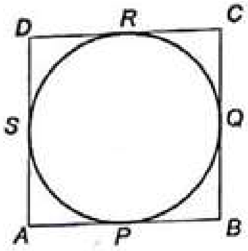
Hence, it is clear from the figure that ACBD is a quac ateral.

41.



The locus of the centre of the front wheel of the cycle is a line parallel to the path of the wheel of the bicycle a height $d/2$ cm.

42. We know that, two tangents drawn from an external point to a circle are of equal lengths.



$\therefore$ $AP = AS$

$BP = BQ$

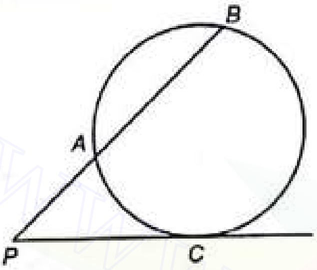
$CR = CQ$

$DR = DS$

$\Rightarrow$ $AP + BP + CR + DR = AS + BQ + CQ + DS$

$\Rightarrow$ $AB + CD = AD + BC$

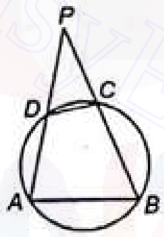
43. If a secant to a circle intersect circle at points A and B and PC is a tangent to circle, then



$PC^2 = PA \times PB$

which is equivalent to area of rectangle with PA and PB as sides is equal to the area of square with PC as side.

44. Given, $\angle BAD = 60°$, $\angle ADC = 105°$



In cyclic quadrilateral $ABCD$,

$\angle BAD + \angle BCD = 180°$

$\Rightarrow$ $\angle BAD = 180° - 60° = 120°$

Now, $\angle BCD + \angle DCP = 180°$ (straight line)

$\Rightarrow$ $\angle DCP = 180° - 120° = 60°$

and $\angle ADC + \angle CDP = 180°$ (straight line)

$\Rightarrow$ $105° + \angle CDP + 180°$

$\Rightarrow$ $\angle CDP = 75°$

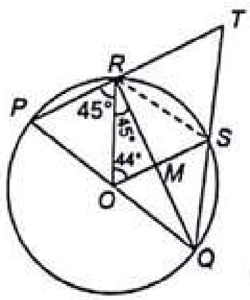
Hence, in ΔCPD,

$\angle DCP + \angle CDP + \angle DPC = 180°$

$\Rightarrow$ $60° + 75° + \angle DPC = 180°$

$\Rightarrow$ $\angle DPC = 180° - 135° = 45°$

45. Since, OR is a bisector of $\angle PRQ$.



$\therefore$ $\angle PRO = \angle ORQ = 45°$

Also, $OP = OR$ (radius)

$\therefore$ $\angle OPR = 45°$

In ΔORS,

$OR = OS$

$\Rightarrow$ $\angle ORS = \angle OSR = \dfrac{180° - 44°}{2} = 68°$

$\therefore$ $\angle MRS = 68° - 45° = 23°$

$\Rightarrow$ $\angle PRS = 90° + 23° = 113°$

By properties of cyclic quadrilateral,

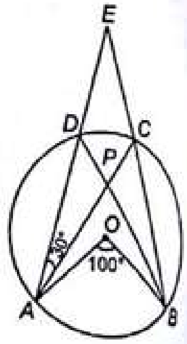
$\angle PRS + \angle PQS = 180°$

$\Rightarrow$ $\angle PQS = 180° - 113° = 67°$

In ΔPTQ, $\angle QPT + \angle PQT + \angle PTQ = 180°$

$\Rightarrow$ $\angle PTQ = 180° - 45° - 67° = 68°$

46. Since, $\angle ADB = \dfrac{1}{2} \angle AOB = 50°$



In ΔDPA,

$\angle DAP + \angle ADP + \angle DPA = 180°$

$\Rightarrow$ $30° + 50° + \angle DPA = 180°$

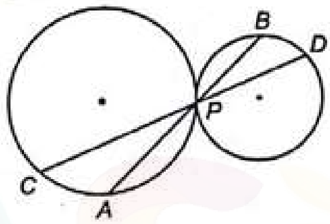
$\Rightarrow$ $\angle DPA = 100°$

Also, DPB be a straight line,

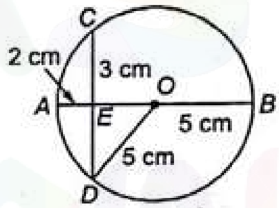
$\therefore$ $\angle DPA + \angle APB = 180°$

$\Rightarrow$ $\angle APB = 180° - 100° = 80°$

47. It is clear from the figure that none of the option is correct.



48. In ΔOED,



$(OD)^2 = (DE)^2 + (EO)^2$

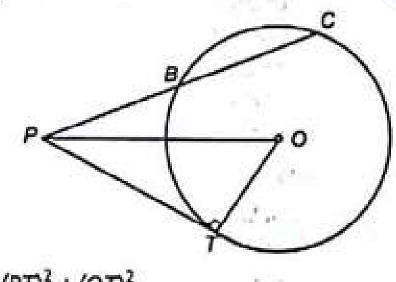
$\Rightarrow$ $(5)^2 = (DE)^2 + (3)^2$

$\Rightarrow$ $(DE)^2 = 25 - 9 = 16 \Rightarrow DE = 4$ cm

49. Given, $PO = 10$ cm, radius $OT = 6$ cm

and $PB = 5$ cm

In ΔOTP,



$(OP)^2 = (PT)^2 + (OT)^2$

$\Rightarrow$ $(10)^2 = (PT)^2 + 6^2 \Rightarrow PT = 8$ cm

By using theorem of circle,

$(PT)^2 = PB \times PC$

$\Rightarrow$ $8^2 = 5 \times (BC + PB)$

$\Rightarrow$ $64 = 5(BC + 5)$

$\Rightarrow$ $5BC = 39$

$\Rightarrow$ $BC = 7.8$ cm

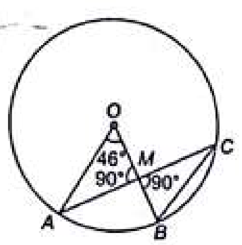
50. Since, angle subtend on the circumference is half of the angle subtend on centre.

$\therefore \quad \angle ACB = \frac{1}{2}\angle AOB = \frac{1}{2} \times 46° = 23°$

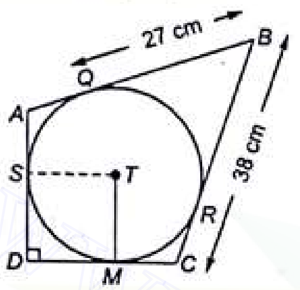
In ΔMCB,

$\angle C + \angle B + \angle M = 180°$

$\Rightarrow 23° + \angle B + 90° = 180° \Rightarrow \angle B = 67°$



51. We know that, the tangents drawn to the circle from a point outside the circle are always equal.

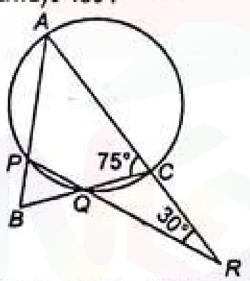


$\therefore \quad BQ = BR = 27 \text{ cm} \Rightarrow RC = 38 - 27 = 11 \text{ cm}$

$\therefore \quad RC = CM = 11 \text{cm}$

Now, $\quad DM = 25 - 11 = 14 \text{ cm}$

52. We know that, the sum of opposite angles in a cyclic quadrilateral is always 180°.



$\angle ACQ + \angle APQ = 180° \Rightarrow 75° + \angle APQ = 180°$

$\Rightarrow \quad \angle APQ = 180° - 75° = 105°$

Since, $\quad \angle APQ + \angle BPQ = 180°$ (straight line)

$\therefore \quad 105° + \angle BPQ = 180°$

$\Rightarrow \quad \angle BPQ = 180° - 105° = 75°$

Since, $\angle ACQ$ is exterior angle of ΔRCQ.

$\therefore \quad \angle ACQ = \angle CRQ + \angle CQR$

$\Rightarrow \quad 75° = 30° + \angle CQR \Rightarrow \angle CQR = 45°$

In ΔBPQ, $\quad \angle B + \angle P + \angle Q = 180°$

$\Rightarrow \quad \angle B + 75° + 45° = 180° \Rightarrow \angle B = 60°$

53. Given, $\angle OBD + \angle ODB = \angle CBD + \angle CDB$

Let $\quad \angle OBD = \angle ODB = \theta$

and $\quad \angle DBC = \theta_1, \angle BDC = \theta_2$

$\therefore \quad \theta + \theta = \theta_1 + \theta_2 \quad$ ...(i)

$\Rightarrow \quad 2\theta = \theta_1 + \theta_2$

$\therefore \quad \angle BOD = 180° - 2\theta$

$\Rightarrow \quad \angle BCD = \frac{360° - (180° - 2\theta)}{2}$

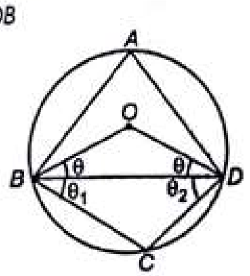
(by properties of circle)

$\Rightarrow \quad 180° - (\theta_1 + \theta_2) = 90° + \theta$

$\Rightarrow \quad 180° - 2\theta = 90° + \theta \Rightarrow 90° = 3\theta$

$\Rightarrow \quad \theta = 30°$

$\therefore \quad \angle BOD = 120° \Rightarrow \angle BAD = 60°$

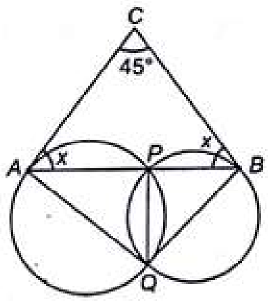


54. We know that the tangents drawn from an outer point on a circle are always equal. So, in ΔCAB, two angles $\angle CAB$ and $\angle CBA$ are equal.

$\therefore \quad 45° + x + x = 180°$

$\Rightarrow \quad 2x = 180° - 45°$

$\Rightarrow \quad x = 67\frac{1°}{2}$



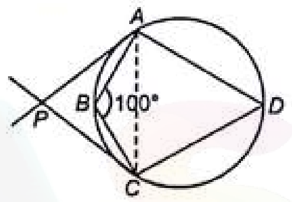
$\angle AQP = \angle x = \angle BQP = 67\frac{1°}{2}$

(alternate interior segment properties)

$\Rightarrow \quad \angle AQB = \angle AQP + \angle BQP$

$= 67\frac{1°}{2} + 67\frac{1°}{2} = 135°$

55. We know that, the sum of opposite angles of a cyclic quadrilateral is always 180°.



$\therefore \quad \angle B + \angle D = 180° \Rightarrow 100 + \angle D = 180° \Rightarrow \angle D = 80°$

$\therefore \quad \angle ACP = \angle PAC = 80°$

(by theorem of alternate interior segment)

In ΔPAC,

$\angle P + \angle PAC + \angle PCA = 180°$

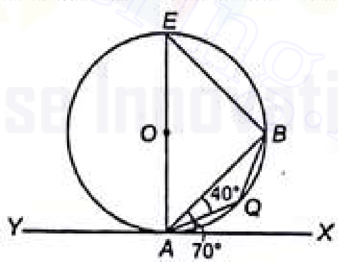
$\Rightarrow \quad \angle P + 80° + 80° = 180°$

$\Rightarrow \quad \angle P = 180° - 160° = 20°$

56. Given, $\angle BAX = 70°$ and $\angle BAQ = 40°$

$\angle QAX = 70° - 40° = 30°$

$\because \quad \angle EAX = 90° \Rightarrow \angle EAB = 90° - 70° = 20°$



Since, $AQBE$ is a cyclic quadrilateral.

$\therefore \quad \angle EAQ + \angle EBQ = 180° \Rightarrow \angle EBQ = 180° - 60° = 120°$

But $\quad \angle EBA = 90°$

$\therefore \quad \angle ABQ = 120° - 90° = 30°$

57. Given, $\quad EC = ED$

$\Rightarrow \quad \angle EDC = \angle ECD = 35°$

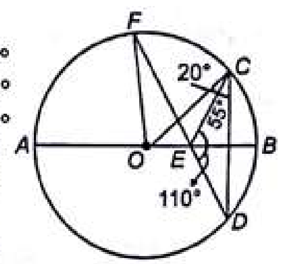
Since, $\quad \angle OCD = 55°$

Then, $\quad \angle OCE = 20°$

By using the theorem that triangle on the same segment of a circle makes an equal angles.

Here, OE is a segment, which makes a ΔOFE and ΔOCE.

Therefore, $\quad \angle OCE = \angle EFO = 20°$



58. We know that, the triangle of same segment of a circle makes an equal angles.

$\therefore \quad \angle XBY = \angle XAY = 45°$

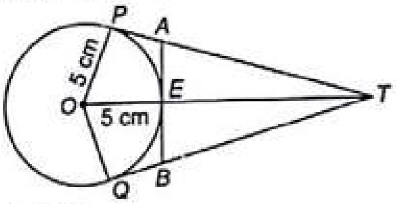
In ΔBXY,

$\angle BXY + \angle XBY + \angle BYX = 180°$

$\Rightarrow \quad 50° + 45° + \angle BYX = 180° \quad (\because \angle BXY = 50°)$

$\Rightarrow \quad \angle BYX = 180° - 95° = 85°$

59. $OT = 13$ cm, $OE = 5$ cm $= OP$ (radius)



$ET = 13 - 5 = 8$ cm

In ΔOPT,

$$13^2 - 5^2 = PT^2 \Rightarrow PT = 12$$

Let $\quad \angle ATE = \theta$

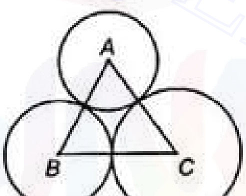
In ΔOPT, $\quad \tan\theta = \frac{OP}{PT} = \frac{5}{12}$ ...(i)

Now, in ΔATE, $\quad \tan\theta = \frac{AE}{ET} = \frac{AE}{8}$

$\Rightarrow \quad AE = 8\tan\theta = 8 \times \frac{5}{12} = \frac{10}{3}$ [from Eq. (i)]

$\therefore \quad AB = 2AE = 2 \times \frac{10}{3} = \frac{20}{3}$ cm

60. Let r_1, r_2 and r_3 be the radii of the circles with centres A, B and C, respectively.



Given that, $AB = 4$ cm, $BC = 6$ cm and $CA = 8$ cm

$\therefore \quad r_1 + r_2 = 4$ ...(i)

$r_2 + r_3 = 6$ ...(ii)

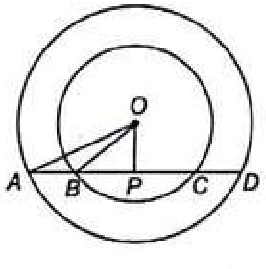
and $\quad r_3 + r_1 = 8$ ...(iii)

On adding Eqs. (i), (ii) and (iii), we get

$$2(r_1 + r_2 + r_3) = 4 + 6 + 8 \Rightarrow r_1 + r_2 + r_3 = 9$$

Thus, the required sum is 9 cm.

61. In ΔOPB,



$OB^2 = OP^2 + BP^2$

$\Rightarrow \quad (15)^2 = (12)^2 + BP^2$

$\Rightarrow \quad 225 - 144 = BP^2$

$(BP)^2 = 81 \Rightarrow BP = 9$ cm

And in ΔAOP,

$(OA)^2 = (OP)^2 + (AP)^2$

$\Rightarrow \quad (20)^2 = (12)^2 + (AP)^2$

$\Rightarrow \quad (AP)^2 = 400 - 144 = 256 \Rightarrow AP = 16$ cm

Hence, $\quad AB = AP - BP = 16 - 9 = 7$ cm

62. Given, $\quad \frac{x}{3} = \frac{y}{4} = \frac{z}{5} = k$ (say)

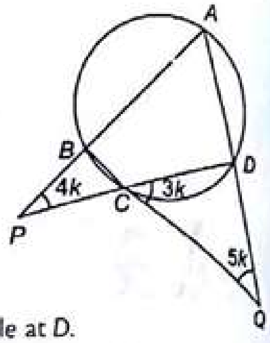
$\therefore \quad x = 3k, y = 4k$ and $z = 5k$

Since, $\quad \angle DCQ = \angle BCP = 3k$ (vertically opposite angle)

In ΔDCQ,

$\angle CDQ = 180° - (3k + 5k) = 180° - 8k$

By properties of cyclic quadrilateral,



$\angle QDC = \angle CBA = 180° - 8k$

$\Rightarrow \quad \angle PBC = 8k$

In ΔPBC,

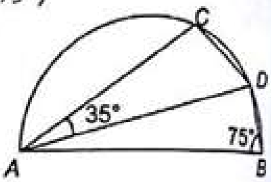
$\angle P + \angle B + \angle C = 180°$

$\therefore \quad 4k + 8k + 3k = 180°$

$\Rightarrow \quad k = \frac{180°}{15}\ k = 12°$

$\therefore \quad x = 36°, y = 48°, z = 60°$

63. Since, ΔADB is a right angled triangle at D.



$\therefore \quad \angle DAB = 180° - (90° + 75°)$

$\Rightarrow \quad \angle DAB = 15°$

Also, $ABCD$ is cyclic quadrilateral.

$\therefore \quad \angle CAB + \angle CDB = 180°$

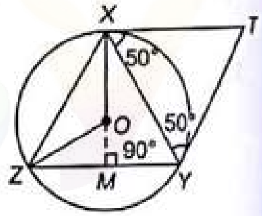
$\Rightarrow \quad \angle CDB = 180° - (35° + 15°)$

$= 130°$

64. We know that,

$$\angle AQP = \frac{1}{2}\angle AOP = \frac{1}{2}75° = 37.5°$$

65. Since, $XT = YT$



$\therefore \quad \angle TXY = \angle TYX = 50°$

Also, OX is perpendicular to XT.

$\therefore \quad \angle OXT = 90°$

$\therefore \quad \angle OXY = 90° - 50° = 40°$

Also, OM is perpendicular to ZY.

In ΔXMY,

$\angle MXY + \angle XYM + \angle XMY = 180°$

$\therefore \quad 40° + \angle XYM + 90° = 180° \Rightarrow \angle XYM = 50°$

Also, by property of circle,

$\angle TXY = \angle XZY = 50°$

In ΔXYZ,

$\angle X + \angle Y + \angle Z = 180°$

$\therefore \quad \angle X = 180° - 50° - 50° = 80°$

66. Here, $\quad AP = AM = 5$

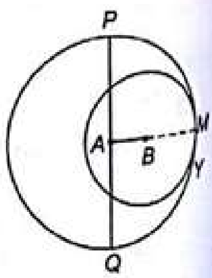
$BM = 3$

$\therefore \quad AB = AM - BM = 5 - 3 = 2$

PQ bisects AB perpendicularly.

$\therefore \quad AO = OB = 1$

Now in right angled ΔAOP,



$\therefore \quad (PA)^2 = (OA)^2 + (OP)^2$

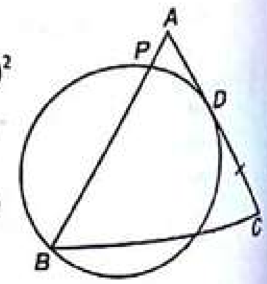
$\Rightarrow \quad (5)^2 = (1)^2 + (OP)^2$

$\Rightarrow \quad 24 = (OP)^2$

$\Rightarrow \quad OP = 2\sqrt{6}$

$\therefore \quad PQ = 2OP = 2 \times 2\sqrt{6} = 4\sqrt{6}$

67. By using theorem,



$$AB \times AP = AD^2 = \left(\frac{AC}{2}\right)^2 = \frac{1}{4}(AC)^2$$

$\Rightarrow AB \times AP = \frac{1}{4}(AB)^2$

$(\because AC = AB$ given$)$

$\Rightarrow \quad AB = 4AP$