

Ex 2.1

Functions Ex 2.1 Q1(i)

Example of a function which is one-one but not onto.

let $f : N \rightarrow N$ given by $f(x) = x^2$

Check for injectivity:

let $x, y \in N$ such that

$$f(x) = f(y)$$

$$\Rightarrow x^2 = y^2$$

$$\Rightarrow (x - y)(x + y) = 0 \quad [\because x, y \in N \Rightarrow x + y > 0]$$

$$\Rightarrow x - y = 0$$

$$\Rightarrow x = y$$

$\therefore f$ is one-one

Surjectivity: let $y \in N$ be arbitrary, then

$$f(x) = y$$

$$\Rightarrow x^2 = y$$

$$\Rightarrow x = \sqrt{y} \notin N \text{ for non-perfect square value of } y.$$

$\therefore$ No non-perfect square value of y has a pre image in domain N .

$\therefore f : N \rightarrow N$ given by $f(x) = x^2$ is one-one but not onto.

Functions Ex 2.1 Q1(ii)

Example of a function which is onto but not one-one.

let $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^3 - x$

Check for injectivity:

let $x, y \in \mathbb{R}$ such that

$$f(x) = f(y)$$

$$\Rightarrow x^3 - x = y^3 - y$$

$$\Rightarrow x^3 - y^3 - (x - y) = 0$$

$$\Rightarrow (x - y)(x^2 + xy + y^2 - 1) = 0$$

$$\because x^2 + xy + y^2 \geq 0 \Rightarrow x^2 + xy + y^2 - 1 \geq -1$$

$$\therefore x \neq y \text{ for some } x, y \in \mathbb{R}$$

$$\therefore f \text{ is not one-one.}$$

Surjectivity: let $y \in \mathbb{R}$ be arbitrary

then, $f(x) = y$

$$\Rightarrow x^3 - x = y$$

$$\Rightarrow x^3 - x - y = 0$$

we know that a degree 3 equation has a real root.

let $x = \alpha$ be that root

$$\therefore \alpha^3 - \alpha = y$$

$$\Rightarrow f(\alpha) = y$$

Thus for clearly $y \in \mathbb{R}$, there exist $\alpha \in \mathbb{R}$ such that $f(x) = y$

$$\therefore f \text{ is onto}$$

$\therefore$ Hence $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^3 - x$ is not one-one but onto.

Functions Ex 2.1 Q1(iii)

Example of a function which is neither one-one nor onto.

let $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 2$

We know that a constant function is neither one-one nor onto

Here $f(x) = 2$ is a constant function

$\therefore f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 2$ is neither one-one nor onto.

Functions Ex 2.1 Q2

$$\begin{aligned} \text{i)} \quad f_1 &= \{(1, 3), (2, 5), (3, 7)\} \\ A &= \{1, 2, 3\}, \quad B = \{3, 5, 7\} \end{aligned}$$

We can easily observe that in f_1 every element of A has different image from B .

$\therefore f_1$ is one-one

also, each element of B is the image of some element of A .

$\therefore f_1$ is onto.

ii)

$$\begin{aligned} f_2 &= \{(2, a), (3, b), (4, c)\} \\ A &= \{2, 3, 4\} \quad B = \{a, b, c\} \end{aligned}$$

It is clear that different elements of A have different images in B

$\therefore f_2$ is one-one

Again, each element of B is the image of some element of A .

$\therefore f_2$ is onto

$$\begin{aligned} \text{iii)} \quad f_3 &= \{(a, x), (b, x), (c, z), (d, z)\} \\ A &= \{a, b, c, d\} \quad B = \{x, y, z\} \end{aligned}$$

Since, $f_3(a) = x = f_3(b)$ and $f_3(c) = z = f_3(d)$

$\therefore f_3$ is not one-one

Again, $y \in B$ is not the image of any of the elements of A

$\therefore f_3$ is not onto

Functions Ex 2.1 Q3

We have, $f: N \rightarrow N$ defined by $f(x) = x^2 + x + 1$

Check for injectivity:

Let $x, y \in N$ such that

$$\begin{aligned} f(x) &= f(y) \\ \Rightarrow x^2 + x + 1 &= y^2 + y + 1 \\ \Rightarrow x^2 - y^2 + x - y &= 0 \\ \Rightarrow (x - y)(x + y + 1) &= 0 \\ \Rightarrow x - y = 0 & \quad [\because x, y \in N \Rightarrow x + y + 1 > 0] \\ \Rightarrow x &= y \end{aligned}$$

$\therefore f$ is one-one.

Surjectivity:

Let $y \in N$, then

$$\begin{aligned} f(x) &= y \\ \Rightarrow x^2 + x + 1 - y &= 0 \end{aligned}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1 - 4(1 - y)}}{2} \notin N \quad \text{for } y > 1$$

$\therefore$ for $y > 1$, we do not have any pre-image in domain N .

$\therefore f$ is not onto.

Functions Ex 2.1 Q4.

We have, $A = \{-1, 0, 1\}$ and $f: A \rightarrow A$

defined by $f = \{(x, x^2) : x \in A\}$

clearly $f(1) = 1$ and $f(-1) = 1$

$\therefore f(1) = f(-1)$

$\therefore f$ is not one-one

Again $y = -1 \in A$ in the co-domain does not have any pre image in domain A .

$\therefore f$ is not onto.

Functions Ex 2.1 Q5(i)

$f: N \rightarrow N$ given by $f(x) = x^2$

let $x_1 = x_2$ for $x_1, x_2 \in N$

$\Rightarrow x_1^2 = x_2^2 \Rightarrow f(x_1) = f(x_2)$

$\therefore f$ is one-one.

Surjectivity: Since f takes only square value like 1, 4, 9, 16,

so, non-perfect square values in N (co-domain) do not have pre image in domain N .

Thus, f is not onto.

Functions Ex 2.1 Q5(ii)

$f: Z \rightarrow Z$ given by $f(x) = x^2$

Injectivity: let $x_1 \neq -x_1 \in Z$

$\Rightarrow x_1 \neq -x_1$

$\Rightarrow x_1^2 = (-x_1)^2 \Rightarrow f(x_1) = f(-x_1)$

$\Rightarrow f$ is not one-one.

Surjective: Again, f takes only square values 1, 4, 9, 16, ...

So, no non-perfect square values in Z have a pre image in domain Z .

$\therefore f$ is not onto.

Functions Ex 2.1 Q5(iii)

$f: N \rightarrow N$, given by $f(x) = x^3$

Injectivity: let $y, x \in N$ such that

$x = y$

$\Rightarrow x^3 = y^3$

$\Rightarrow f(x) = f(y)$

$\therefore f$ is one-one

Surjective:

$\therefore f$ attains only cubic number like 1, 8, 27, 64, ...

So, no non-cubic values of N (co-domain) have pre image in N (Domain)

$\therefore f$ is not onto.

Functions Ex 2.1 Q5(iv)

$f: Z \rightarrow Z$ given by $f(x) = x^3$

Injectivity: let $x, y \in Z$ such that

$x = y$

$\Rightarrow x^3 = y^3$

$\Rightarrow f(x) = f(y)$

$\Rightarrow f(x) = f(y)$

$\Rightarrow f$ is one-one.

Surjective: Since f attains only cubic values like $\pm 1, \pm 8, \pm 27, \dots$

so, no non-cubic values of Z (co-domain) have pre image in Z (domain)

$\therefore f$ is not onto.

Functions Ex 2.1 Q5(v)

$f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = |x|$

Injectivity: let $x, y \in \mathbb{R}$ such that

$$x = y \text{ but if } y = -x$$

$$\Rightarrow |x| = |y| \Rightarrow |y| = |-x| = x$$

$\therefore f$ is not one-one.

Surjective: Since f attains only positive values, for negative real numbers in $\mathbb{R}$, there is no pre-image in domain $\mathbb{R}$.

$\therefore f$ is not onto.

Functions Ex 2.1 Q5(vi)

$f: \mathbb{Z} \rightarrow \mathbb{Z}$ given by $f(x) = x^2 + x$

Injective: let $x, y \in \mathbb{Z}$ such that

$$f(x) = f(y)$$

$$\Rightarrow x^2 + x = y^2 + y$$

$$\Rightarrow x^2 - y^2 + x - y = 0$$

$$\Rightarrow (x - y)(x + y + 1) = 0$$

$$\Rightarrow \text{either } x - y = 0 \text{ or } x + y + 1 = 0$$

Case I: if $x - y = 0$

$$\Rightarrow x = y$$

$\therefore f$ is injective

Case II if $x + y + 1 = 0$

$$\Rightarrow x + y = -1$$

$$\Rightarrow x \neq y$$

$\therefore f$ is not one to one

Thus, in general, f is not one-one

Surjective:

Since $1 \in \mathbb{Z}$ (co-domain)

Now, we wish to find if there is any pre-image in domain $\mathbb{Z}$.

let $x \in \mathbb{Z}$ such that $f(x) = 1$

$$\Rightarrow x^2 + x = 1 \Rightarrow x^2 + x - 1 = 0$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1+4}}{2} \notin \mathbb{Z}.$$

So, f is not onto.

Functions Ex 2.1 Q5(vii)

$f: \mathbb{Z} \rightarrow \mathbb{Z}$ given by $f(x) = x - 5$

Injective: let $x, y \in \mathbb{Z}$ such that

$$f(x) = f(y)$$

$$\Rightarrow x - 5 = y - 5$$

$$\Rightarrow x = y$$

$\therefore f$ is one-one.

Surjective: let $y \in \mathbb{Z}$ be an arbitrary element

$$\text{then } f(x) = y$$

$$\Rightarrow x - 5 = y$$

$$\Rightarrow x = y + 5 \in \mathbb{Z} \text{ (domain)}$$

Thus, for each element in co-domain $\mathbb{Z}$ there exists an element in domain $\mathbb{Z}$ such that $f(x) = y$

$\therefore f$ is onto.

Since, f is one-one and onto,

$\therefore f$ is bijective.

Functions Ex 2.1 Q5(viii)

$f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = \sin x$

Injective: let $x, y \in \mathbb{R}$ such that

$$f(x) = f(y)$$

$$\Rightarrow \sin x = \sin y$$

$$\Rightarrow x = n\pi + (-1)^n y$$

$$\Rightarrow x \neq y$$

$\therefore f$ is not one-one.

Surjective: let $y \in \mathbb{R}$ be arbitrary such that

$$f(x) = y$$

$$\Rightarrow \sin x = y$$

$$\Rightarrow x = \sin^{-1} y$$

Now, for $y > 1$ $x \notin \mathbb{R}$ (domain)

$\therefore f$ is not onto.

Functions Ex 2.1 Q5(ix)

$f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^3 + 1$

Injective: let $x, y \in \mathbb{R}$ such that

$$f(x) = f(y)$$

$$\Rightarrow x^3 + 1 = y^3 + 1$$

$$\Rightarrow x^3 = y^3$$

$$\Rightarrow x = y$$

$\therefore f$ is one-one.

Surjective:

let $y \in \mathbb{R}$, then

$$f(x) = y$$

$$\Rightarrow x^3 + 1 = y \Rightarrow x^3 + 1 - y = 0$$

We know that degree 3 equation has atleast one real root.

$\therefore$ let $x = \alpha$ be the real root.

$$\therefore \alpha^3 + 1 = y$$

$$\Rightarrow f(\alpha) = y$$

Thus, for each $y \in \mathbb{R}$, there exist $\alpha \in \mathbb{R}$ such that $f(\alpha) = y$

$\therefore f$ is onto.

Since f is one-one and onto, f is bijective.

Functions Ex 2.1 Q5(x)

$f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^3 - x$

Injective: let $x, y \in \mathbb{R}$ such that

$$f(x) = f(y)$$

$$\Rightarrow x^3 - x = y^3 - y$$

$$\Rightarrow x^3 - y^3 - (x - y) = 0$$

$$\Rightarrow (x - y)(x^2 + xy + y^2 - 1) = 0$$

$$\because x^2 + xy + y^2 \geq 0 \Rightarrow x^2 + xy + y^2 - 1 \geq -1$$

$$\therefore x^2 + xy + y^2 - 1 \neq 0$$

$$\Rightarrow x - y = 0 \Rightarrow x = y$$

$\therefore f$ is one-one.

Surjective:

let $y \in \mathbb{R}$, then

$$f(x) = y$$

$$\Rightarrow x^3 - x - y = 0$$

We know that a degree 3 equation has atleast one real solution.

let $x = \alpha$ be that real solution

$$\therefore \alpha^3 - \alpha = y$$

$$\Rightarrow f(\alpha) = y$$

$\therefore$ For each $y \in \mathbb{R}$, there exist $x = \alpha \in \mathbb{R}$

such that $f(\alpha) = y$

$\therefore f$ is onto.

Functions Ex 2.1 Q5(xi)

$f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \sin^2 x + \cos^2 x$.

Injective: since $f(x) = \sin^2 x + \cos^2 x = 1$

$\Rightarrow f(x) = 1$ which is a constant function we know that a constant function is neither injective nor surjective

$\therefore f$ is not one-one and not onto.

Functions Ex 2.1 Q5(xii)

$$f: \mathbb{Q} - [3] \rightarrow \mathbb{Q} \quad \text{defined by } f(x) = \frac{2x+3}{x-3}$$

Injective: let $x, y \in \mathbb{Q} - [3]$ such that

$$f(x) = f(y)$$

$$\Rightarrow \frac{2x+3}{x-3} = \frac{2y+3}{y-3}$$

$$\Rightarrow 2xy - 6x + 3y - 9 = 2xy + 3x - 6y - 9$$

$$\Rightarrow -6x + 3y - 3x + 6y = 0$$

$$\Rightarrow -9(x - y) = 0$$

$$\Rightarrow x = y$$

$$\Rightarrow f \text{ is one-one.}$$

Surjective:

let $y \in \mathbb{Q}$ be arbitrary, then

$$f(x) = y$$

$$\Rightarrow \frac{2x+3}{x-3} = y$$

$$\Rightarrow 2x + 3 = xy - 3y$$

$$\Rightarrow x(2 - y) = -3(y + 1)$$

$$\therefore x = \frac{-3(y+1)}{2-y} \notin \mathbb{Q} - [3] \text{ for } y = 2$$

$$\therefore f \text{ is not onto}$$

Functions Ex 2.1 Q5(xiii)

$$f: \mathbb{Q} \rightarrow \mathbb{Q} \quad \text{defined by } f(x) = x^3 + 1$$

Injective: let $x, y \in \mathbb{Q}$ such that

$$f(x) = f(y)$$

$$\Rightarrow x^3 + 1 = y^3 + 1$$

$$\Rightarrow (x^3 - y^3) = 0$$

$$\Rightarrow (x - y)(x^2 + xy + y^2) = 0$$

$$\text{but } x^2 + xy + y^2 \geq 0$$

$$\therefore x - y = 0$$

$$\Rightarrow x = y$$

$$\therefore f \text{ is injective.}$$

Surjective: let $y \in \mathbb{Q}$ be arbitrary, then

$$f(x) = y$$

$$\Rightarrow x^3 + 1 - y = 0$$

we know that a degree 3 equation has atleast one real solution.

let $x = \alpha$ be that solution

$$\therefore \alpha^3 + 1 = y$$

$$\therefore f(\alpha) = y$$

$$\therefore f \text{ is onto.}$$

Functions Ex 2.1 Q5(xiv)

$f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 5x^3 + 4$

Injective: let $x, y \in \mathbb{R}$ such that

$$f(x) = f(y)$$

$$\Rightarrow 5x^3 + 4 = 5y^3 + 4$$

$$\Rightarrow 5(x^3 - y^3) = 0$$

$$\Rightarrow 5(x - y)(x^2 + xy + y^2) = 0$$

$$\text{but } 5(x^2 + xy + y^2) \geq 0$$

$$\Rightarrow x - y = 0 \Rightarrow x = y$$

$\therefore f$ is one-one

Surjective: let $y \in \mathbb{R}$ be arbitrary, then

$$f(x) = y$$

$$\Rightarrow 5x^3 + 4 = y$$

$$\Rightarrow 5x^3 + 4 - y = 0$$

we know that a degree 3 equation has atleast one real solution.

let $x = \alpha$ be that real solution

$$\therefore 5\alpha^3 + 4 = y$$

$$\therefore f(\alpha) = y$$

$\therefore$ For each $y \in \mathbb{Q}$, there $\alpha \in \mathbb{R}$ such that $f(\alpha) = y$

$\therefore f$ is onto

Since f is one-one and onto

$\therefore f$ is bijective.

Functions Ex 2.1 Q5(xv)

$f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 3 - 4x$

Injective: let $x, y \in \mathbb{R}$ such that

$$f(x) = f(y)$$

$$\Rightarrow 3 - 4x = 3 - 4y$$

$$\Rightarrow -4(x - y) = 0$$

$$\Rightarrow x = y$$

$\therefore f$ is one-one.

Surjective: let $y \in \mathbb{R}$ be arbitrary, such that

$$f(x) = y$$

$$\Rightarrow 3 - 4x = y$$

$$\Rightarrow x = \frac{3 - y}{4} \in \mathbb{R}$$

Thus for each $y \in \mathbb{R}$, there exist $x \in \mathbb{R}$ such that

$$f(x) = y$$

$\therefore f$ is onto.

Hence, f is one-one and onto and therefore bijective.

Functions Ex 2.1 Q5(xvi)

$f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 1 + x^2$

Injective: let $x, y \in \mathbb{R}$ such that

$$f(x) = f(y)$$

$$\Rightarrow 1 + x^2 = 1 + y^2$$

$$\Rightarrow x^2 - y^2 = 0$$

$$\Rightarrow (x - y)(x + y) = 0$$

either $x = y$ or $x = -y$ or $x \neq y$

$\therefore f$ is not one-one.

Surjective: let $y \in \mathbb{R}$ be arbitrary, then

$$f(x) = y$$

$$\Rightarrow 1 + x^2 = y$$

$$\Rightarrow x^2 + 1 - y = 0$$

$$\therefore x = \pm\sqrt{y-1} \notin \mathbb{R} \text{ for } y < 1$$

$\therefore f$ is not onto.

Functions Ex 2.1 Q6

Given, $f: A \rightarrow B$ is injective such that $\text{range}(f) = \{a\}$

We know that in injective map different elements have different images.

$\therefore A$ has only one element.

Functions Ex 2.1 Q7

$A = \mathbb{R} - \{3\}$, $B = \mathbb{R} - \{1\}$

$f: A \rightarrow B$ is defined as $f(x) = \left(\frac{x-2}{x-3}\right)$.

Let $x, y \in A$ such that $f(x) = f(y)$.

$$\Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3}$$

$$\Rightarrow (x-2)(y-3) = (y-2)(x-3)$$

$$\Rightarrow xy - 3x - 2y + 6 = xy - 3y - 2x + 6$$

$$\Rightarrow -3x - 2y = -3y - 2x$$

$$\Rightarrow 3x - 2x = 3y - 2y$$

$$\Rightarrow x = y$$

Therefore, f is one-one.

Let $y \in B = \mathbb{R} - \{1\}$.

Then, $y \neq 1$.

The function f is onto if there exists $x \in A$ such that $f(x) = y$.

Now,

$$f(x) = y$$

$$\Rightarrow \frac{x-2}{x-3} = y$$

$$\Rightarrow x - 2 = xy - 3y$$

$$\Rightarrow x(1 - y) = -3y + 2$$

$$\Rightarrow x = \frac{2-3y}{1-y} \in A \quad [y \neq 1]$$

Thus, for any $y \in B$, there exists $\frac{2-3y}{1-y} \in A$ such that

$$f\left(\frac{2-3y}{1-y}\right) = \frac{\left(\frac{2-3y}{1-y}\right) - 2}{\left(\frac{2-3y}{1-y}\right) - 3} = \frac{2-3y-2+2y}{2-3y-3+3y} = \frac{-y}{-1} = y.$$

$\therefore f$ is onto.

Hence, function f is one-one and onto.

Functions Ex 2.1 Q8

We have $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x - [x]$

Now,

check for injectivity:

$$\because f(x) = x - [x] \Rightarrow f(x) = 0 \text{ for } x \in \mathbb{Z}$$

$$\therefore \text{Range of } f = [0, 1] \neq \mathbb{R}$$

$\therefore f$ is not one-one, whereas many-one

Again, Range of $f = [0, 1] \neq \mathbb{R}$

$\therefore f$ is an into function

Functions Ex 2.1 Q9

Suppose $f(n_1) = f(n_2)$

If n_1 is odd and n_2 is even, then we have

$$n_1 + 1 = n_2 - 1 \Rightarrow n_2 - n_1 = 2, \text{ not possible}$$

If n_1 is even and n_2 is odd, then we have

$$n_1 - 1 = n_2 + 1 \Rightarrow n_1 - n_2 = 2, \text{ not possible}$$

Therefore, both n_1 and n_2 must be either odd or even.

Suppose both n_1 and n_2 are odd.

$$\text{Then, } f(n_1) = f(n_2) \Rightarrow n_1 + 1 = n_2 + 1 \Rightarrow n_1 = n_2$$

Suppose both n_1 and n_2 are even.

$$\text{Then, } f(n_1) = f(n_2) \Rightarrow n_1 - 1 = n_2 - 1 \Rightarrow n_1 = n_2$$

Thus, f is one-one.

Also, any odd number $2r + 1$ in the co-domain $\mathbb{N}$ will have an even number

as image in domain $\mathbb{N}$ which is

$$f(n) = 2r + 1 \Rightarrow n - 1 = 2r + 1 \Rightarrow n = 2r + 2$$

any even number $2r$ in the co-domain $\mathbb{N}$ will have an odd number

as image in domain $\mathbb{N}$ which is

$$f(n) = 2r \Rightarrow n + 1 = 2r \Rightarrow n = 2r - 1$$

Thus, f is onto.

Functions Ex 2.1 Q10

We have $A = \{1, 2, 3\}$

All one-one functions from $A = \{1, 2, 3\}$ to itself are obtained by re-arranging elements of A .

Thus all possible one-one functions are:

$$\text{i)} f(1) = 1, f(2) = 2, f(3) = 3$$

$$\text{ii)} f(1) = 2, f(2) = 3, f(3) = 1$$

$$\text{iii)} f(1) = 3, f(2) = 1, f(3) = 2$$

$$\text{iv)} f(1) = 1, f(2) = 3, f(3) = 2$$

$$\text{v)} f(1) = 3, f(2) = 2, f(3) = 1$$

$$\text{vi)} f(1) = 2, f(2) = 1, f(3) = 3$$

Functions Ex 2.1 Q11

We have $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = 4x^3 + 7$

Let $x, y \in \mathbb{R}$ such that

$$f(a) = f(b)$$

$$4a^3 + 7 = 4b^3 + 7$$

$$a = b$$

f is one-one.

Now let $y \in \mathbb{R}$ be arbitrary, then

$$f(x) = y$$

$$4x^3 + 7 = y$$

$$x = (y - 7)^{\frac{1}{3}} \in \mathbb{R}$$

f is onto.

Hence the function is a bijection

Functions Ex 2.1 Q12

We have $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = e^x$

let $x, y \in \mathbb{R}$, such that

$$f(x) = f(y)$$

$$\Rightarrow e^x = e^y$$

$$\Rightarrow e^{x-y} = 1 = e^0$$

$$\Rightarrow x - y = 0$$

$$\Rightarrow x = y$$

$\therefore f$ is one-one

clearly range of $f = (0, \infty) \neq \mathbb{R}$

$\therefore f$ is not onto

When co-domain is replaced by $\mathbb{R}_0^+$ i.e., $(0, \infty)$ then f becomes an onto function.

Functions Ex 2.1 Q13

We have $f: \mathbb{R}_0^+ \rightarrow \mathbb{R}$ given by $f(x) = \log_a x : a > 0$

let $x, y \in \mathbb{R}_0^+$, such that

$$f(x) = f(y)$$

$$\Rightarrow \log_a x = \log_a y$$

$$\Rightarrow \log_a \left(\frac{x}{y} \right) = 0$$

$$\Rightarrow \frac{x}{y} = 1$$

$$\Rightarrow x = y$$

$\therefore f$ is one-one

Now, let $y \in \mathbb{R}$ be arbitrary, then

$$f(x) = y$$

$$\Rightarrow \log_a x = y \quad \Rightarrow x = a^y \in \mathbb{R}_0^+ \quad \left[\because a > 0 \Rightarrow a^y > 0 \right]$$

Thus, for all $y \in \mathbb{R}$, there exist $x = a^y$ such that $f(x) = y$

$\therefore f$ is onto

$\therefore f$ is one-one and onto $\therefore f$ is bijective

Functions Ex 2.1 Q14

Since f is one-one, three elements of $\{1, 2, 3\}$ must be taken to 3 different elements of the co-domain $\{1, 2, 3\}$ under f .

Hence, f has to be onto.

Functions Ex 2.1 Q15

Suppose f is not one-one.

Then, there exists two elements, say 1 and 2 in the domain whose image in the co-domain is same.

Also, the image of 3 under f can be only one element.

Therefore, the range set can have at most two elements of the co-domain $\{1, 2, 3\}$

i.e f is not an onto function, a contradiction.

Hence, f must be one-one.

Functions Ex 2.1 Q16

Onto functions from the set $\{1, 2, 3, \dots, n\}$ to itself is simply a permutation on n symbols $1, 2, \dots, n$.

Thus, the total number of onto maps from $\{1, 2, \dots, n\}$ to itself is the same as the total number of permutations on n symbols $1, 2, \dots, n$, which is $n!$.

Functions Ex 2.1 Q17

Let $f_1 : R \rightarrow R$ and $f_2 : R \rightarrow R$ be two functions given by:

$$f_1(x) = x$$

$$f_2(x) = -x$$

We can easily verify that f_1 and f_2 are one-one functions.

Now,

$$(f_1 + f_2)(x) = f_1(x) + f_2(x) = x - x = 0$$

$\therefore f_1 + f_2 : R \rightarrow R$ is a function given by

$$(f_1 + f_2)(x) = 0$$

Since $f_1 + f_2$ is a constant function, it is not one-one.

Functions Ex 2.1 Q18

Let $f_1 : Z \rightarrow Z$ defined by $f_1(x) = x$ and

$f_2 : Z \rightarrow Z$ defined by $f_2(x) = -x$

Then f_1 and f_2 are surjective functions.

Now,

$f_1 + f_2 : Z \rightarrow Z$ is given by

$$(f_1 + f_2)(x) = f_1(x) + f_2(x) = x - x = 0$$

Since $f_1 + f_2$ is a constant function, it is not surjective.

Functions Ex 2.1 Q19

Let $f_1 : R \rightarrow R$ be defined by $f_1(x) = x$

and $f_2 : R \rightarrow R$ be defined by $f_2(x) = x$

clearly f_1 and f_2 are one-one functions.

Now,

$F = f_1 \times f_2 : R \rightarrow R$ is defined by

$$F(x) = (f_1 \times f_2)(x) = f_1(x) \times f_2(x) = x^2 \dots\dots\dots (i)$$

Clearly, $F(-1) = 1 = F(1)$

$\therefore F$ is not one-one

Hence, $f_1 \times f_2 : R \rightarrow R$ is not one-one.

Functions Ex 2.1 Q20

Let $f_1 : R \rightarrow R$ and $f_2 : R \rightarrow R$ are two functions defined by $f_1(x) = x^3$ and $f_2(x) = x$ clearly f_1 & f_2 are one-one functions.

Now,

$\frac{f_1}{f_2} : R \rightarrow R$ given by

$$\left(\frac{f_1}{f_2}\right)(x) = \frac{f_1(x)}{f_2(x)} = x^2 \text{ for all } x \in R.$$

let $\frac{f_1}{f_2} = f$

$\therefore F = R \rightarrow R$ defined by $f(x) = x^2$

now, $F(1) = 1 = F(-1)$

$\therefore F$ is not one-one

$\therefore \frac{f_1}{f_2} = R \rightarrow R$ is not one-one.

Functions Ex 2.1 Q22

We have $f : R \rightarrow R$ given by $f(x) = x - [x]$

Now,

check for injectivity:

$$\because f(x) = x - [x] \Rightarrow f(x) = 0 \text{ for } x \in Z$$

$$\therefore \text{Range of } f = [0, 1] \neq R$$

$\therefore f$ is not one-one, where as many-one

Again, Range of $f = [0, 1] \neq R$

$\therefore f$ is an into function

Functions Ex 2.1 23

Suppose $f(n_1) = f(n_2)$

If n_1 is odd and n_2 is even, then we have

$$n_1 + 1 = n_2 - 1 \Rightarrow n_2 - n_1 = 2, \text{ not possible}$$

If n_1 is even and n_2 is odd, then we have

$$n_1 - 1 = n_2 + 1 \Rightarrow n_1 - n_2 = 2, \text{ not possible}$$

Therefore, both n_1 and n_2 must be either odd or even.

Suppose both n_1 and n_2 are odd.

$$\text{Then, } f(n_1) = f(n_2) \Rightarrow n_1 + 1 = n_2 + 1 \Rightarrow n_1 = n_2$$

Suppose both n_1 and n_2 are even.

$$\text{Then, } f(n_1) = f(n_2) \Rightarrow n_1 - 1 = n_2 - 1 \Rightarrow n_1 = n_2$$

Thus, f is one - one.

Also, any odd number $2r + 1$ in the co - domain N will have an even number as image in domain N which is

$$f(n) = 2r + 1 \Rightarrow n - 1 = 2r + 1 \Rightarrow n = 2r + 2$$

any even number $2r$ in the co - domain N will have an odd number as image in domain N which is

$$f(n) = 2r \Rightarrow n + 1 = 2r \Rightarrow n = 2r - 1$$

Thus, f is onto.

Ex 2.2

Functions Ex2.2 Q1(i)

Since, $f: R \rightarrow R$ and $g: R \rightarrow R$

$\therefore f \circ g: R \rightarrow R$ and $g \circ f: R \rightarrow R$

Now, $f(x) = 2x + 3$ and $g(x) = x^2 + 5$

$$g \circ f(x) = g(2x + 3) = (2x + 3)^2 + 5$$

$$\Rightarrow g \circ f(x) = 4x^2 + 12x + 14$$

$$f \circ g(x) = f(g(x)) = f(x^2 + 5) = 2(x^2 + 5) + 3$$

$$\Rightarrow f \circ g(x) = 2x^2 + 13$$

Functions Ex2.2 Q1(ii)

$$f(x) = 2x + x^2 \quad \text{and} \quad g(x) = x^3$$

$$g \circ f(x) = g(f(x)) = g(2x + x^2)$$

$$g \circ f(x) = (2x + x^2)^3$$

$$f \circ g(x) = f(g(x)) = f(x^3)$$

$$\therefore f \circ g(x) = 2x^3 + x^6$$

Functions Ex2.2 Q1(iii)

$$f(x) = x^2 + 8 \text{ and } g(x) = 3x^3 + 1$$

$$\text{Thus, } g \circ f(x) = g[f(x)]$$

$$\Rightarrow g \circ f(x) = g[x^2 + 8]$$

$$\Rightarrow g \circ f(x) = 3[x^2 + 8]^3 + 1$$

$$\text{Similarly, } f \circ g(x) = f[g(x)]$$

$$\Rightarrow f \circ g(x) = f[3x^3 + 1]$$

$$\Rightarrow f \circ g(x) = [3x^3 + 1]^2 + 8$$

$$\Rightarrow f \circ g(x) = [9x^6 + 1 + 6x^3] + 8$$

$$\Rightarrow f \circ g(x) = 9x^6 + 6x^3 + 9$$

Functions Ex2.2 Q1(iv)

$$f(x) = x \text{ and } g(x) = |x|$$

$$\text{Now, } g \circ f(x) = g(f(x)) = g(x)$$

$$\therefore g \circ f(x) = |x|$$

$$\text{and, } f \circ g(x) = f(g(x)) = f(|x|)$$

$$\therefore f \circ g(x) = |x|$$

Functions Ex2.2 Q1(v)

$$f(x) = x^2 + 2x - 3 \text{ and } g(x) = 3x - 4$$

$$\text{Now, } g \circ f(x) = g(f(x)) = g(x^2 + 2x - 3)$$

$$\therefore g \circ f(x) = 3(x^2 + 2x - 3) - 4$$

$$\Rightarrow g \circ f(x) = 3x^2 + 6x - 13$$

$$\text{and, } f \circ g(x) = f(g(x)) = f(3x - 4)$$

$$\therefore f \circ g(x) = (3x - 4)^2 + 2(3x - 4) - 3$$

$$= 9x^2 + 16 - 24x + 6x - 8 - 3$$

$$\therefore f \circ g(x) = 9x^2 - 18x + 5$$

Functions Ex2.2 Q1(vi)

$$f(x) = 8x^3 \text{ and } g(x) = x^{1/3}$$

$$\text{Now, } g \circ f(x) = g(f(x)) = g(8x^3)$$

$$= (8x^3)^{1/3}$$

$$\therefore g \circ f(x) = 2x$$

$$\text{and, } f \circ g(x) = f(g(x)) = f(x^{1/3})$$

$$= 8(x^{1/3})^3$$

$$\therefore f \circ g(x) = 8x$$

Functions Ex2.2 Q2

Let $f = \{(3, 1), (9, 3), (12, 4)\}$ and
 $g = \{(1, 3), (3, 3), (4, 9), (5, 9)\}$

Now,

$$\text{range of } f = \{1, 3, 4\}$$

$$\text{domain of } f = \{3, 9, 12\}$$

$$\text{range of } g = \{3, 9\}$$

$$\text{domain of } g = \{1, 3, 4, 5\}$$

since, range of $f \subset$ domain of g

$\therefore g \circ f$ is well defined.

Again, range of $g \subseteq$ domain of f

$\therefore f \circ g$ is well defined.

$$\text{Now } g \circ f = \{(3, 3), (9, 3), (12, 9)\}$$

$$f \circ g = \{(1, 1), (3, 1), (4, 3), (5, 3)\}$$

Functions Ex2.2 Q3

We have,

$$f = \{(1, -1), (4, -2), (9, -3), (16, 4)\} \text{ and}$$

$$g = \{(-1, -2), (-2, -4), (-3, -6), (4, 8)\}$$

Now,

$$\text{Domain of } f = \{1, 4, 9, 16\}$$

$$\text{Range of } f = \{-1, -2, -3, 4\}$$

$$\text{Domain of } g = \{-1, -2, -3, 4\}$$

$$\text{Range of } g = \{-2, -4, -6, 8\}$$

Clearly range of $f =$ domain of g

$\therefore g \circ f$ is defined.

but, range of $g \neq$ domain of f

$\therefore f \circ g$ is not defined.

Now,

$$g \circ f(1) = g(-1) = -2$$

$$g \circ f(4) = g(-2) = -4$$

$$g \circ f(9) = g(-3) = -6$$

$$g \circ f(16) = g(4) = 8$$

$$\therefore g \circ f = \{(1, -2), (4, -4), (9, -6), (16, 8)\}$$

Functions Ex2.2 Q4

$A = \{a, b, c\}$, $B = \{u, v, w\}$ and
 $f = A \rightarrow B$ and $g : B \rightarrow A$ defined by
 $f = \{(a, v), (b, u), (c, w)\}$ and
 $g = \{(u, b), (v, a), (w, c)\}$

For both f and g , different elements of domain have different images
 $\therefore f$ and g are one-one

Again for each element in co-domain of f and g , there is a pre image in domain
 $\therefore f$ and g are onto

Thus, f and g are bijectives.

Now,

$$\begin{aligned}
 g \circ f &= \{(a, a), (b, b), (c, c)\} \text{ and} \\
 f \circ g &= \{(u, u), (v, v), (w, w)\}
 \end{aligned}$$

Functions Ex2.2 Q5

We have, $f : R \rightarrow R$ given by $f(x) = x^2 + 8$ and
 $g : R \rightarrow R$ given by $g(x) = 3x^3 + 1$

$$\begin{aligned}
 \therefore f \circ g(x) &= f(g(x)) = f(3x^3 + 1) \\
 &= (3x^3 + 1)^2 + 8
 \end{aligned}$$

$$\therefore f \circ g(2) = (3 \times 8 + 1)^2 + 8 = 625 + 8 = 633$$

Again

$$\begin{aligned}
 g \circ f(x) &= g(f(x)) = g(x^2 + 8) \\
 &= 3(x^2 + 8)^3 + 1
 \end{aligned}$$

$$\therefore g \circ f(1) = 3(1 + 8)^3 + 1 = 2188$$

Functions Ex2.2 Q6

We have, $f : R^+ \rightarrow R^+$ given by
 $f(x) = x^2$
 $g : R^+ \rightarrow R^+$ given by
 $g(x) = \sqrt{x}$

$$\therefore f \circ g(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x$$

Also,

$$g \circ f(x) = g(f(x)) = g(x^2) = \sqrt{x^2} = x$$

Thus,

$$f \circ g(x) = g \circ f(x)$$

Functions Ex2.2 Q7

We have, $f : R \rightarrow R$ and $g : R \rightarrow R$ are two functions defined by
 $f(x) = x^2$ and $g(x) = x + 1$

Now,

$$f \circ g(x) = f(g(x)) = f(x + 1) = (x + 1)^2$$

$$\therefore f \circ g(x) = x^2 + 2x + 1 \dots\dots\dots (i)$$

$$g \circ f(x) = g(f(x)) = g(x^2) = x^2 + 1 \dots\dots\dots (ii)$$

from (i) & (ii)

$$f \circ g \neq g \circ f$$

Functions Ex2.2 Q8

Let $f: R \rightarrow R$ and $g: R \rightarrow R$ are defined as

$$f(x) = x + 1 \text{ and } g(x) = x - 1$$

Now,

$$\begin{aligned} f \circ g(x) &= f(g(x)) = f(x - 1) = x - 1 + 1 \\ &= x = I_R \dots\dots\dots (i) \end{aligned}$$

Again,

$$\begin{aligned} f \circ g(x) &= f(g(x)) = g(x + 1) = x + 1 - 1 \\ &= x = I_R \dots\dots\dots (ii) \end{aligned}$$

from (i) & (ii)

$$f \circ g = g \circ f = I_R$$

Functions Ex2.2 Q9

We have, $f: N \rightarrow Z_0$, $g: Z_0 \rightarrow Q$ and

$$h: Q \rightarrow R$$

$$\text{Also, } f(x) = 2x, \quad g(x) = \frac{1}{x} \text{ and } h(x) = e^x$$

Now, $f: N \rightarrow Z_0$ and $h \circ g: Z_0 \rightarrow R$

$$\therefore (h \circ g) \circ f: N \rightarrow R$$

also, $g \circ f: N \rightarrow Q$ and $h: Q \rightarrow R$

$$\therefore h \circ (g \circ f): N \rightarrow R$$

Thus, $(h \circ g) \circ f$ and $h \circ (g \circ f)$ exist and are function from N to set R .

$$\begin{aligned} \text{Finally, } (h \circ g) \circ f(x) &= (h \circ g)(f(x)) = (h \circ g)(2x) \\ &= h\left(\frac{1}{2x}\right) \\ &= e^{1/2x} \end{aligned}$$

$$\begin{aligned} \text{now, } h \circ (g \circ f)(x) &= h \circ (g(2x)) = h\left(\frac{1}{2x}\right) \\ &= e^{1/2x} \end{aligned}$$

Hence, associativity verified.

Functions Ex2.2 Q10

We have,

$$\begin{aligned} h \circ (g \circ f)(x) &= h(g \circ f(x)) = h(g(f(x))) \\ &= h(g(2x)) = h(3(2x) + 4) \\ &= h(6x + 4) = \sin(6x + 4) \quad \forall x \in \mathbf{N} \\ ((h \circ g) \circ f)(x) &= (h \circ g)(f(x)) = (h \circ g)(2x) \\ &= h(g(2x)) = h(3(2x) + 4) \\ &= h(6x + 4) = \sin(6x + 4) \quad \forall x \in \mathbf{N} \end{aligned}$$

This shows, $h \circ (g \circ f) = (h \circ g) \circ f$

Functions Ex2.2 Q11

Define $f: \mathbf{N} \rightarrow \mathbf{N}$ by, $f(x) = x + 1$

And, $g: \mathbf{N} \rightarrow \mathbf{N}$ by,

$$g(x) = \begin{cases} x - 1 & \text{if } x > 1 \\ 1 & \text{if } x = 1 \end{cases}$$

We first show that f is not onto.

For this, consider element 1 in co-domain $\mathbf{N}$. It is clear that this element is not an image of any of the elements in domain $\mathbf{N}$.

Therefore, f is not onto.

Now, $g \circ f: \mathbf{N} \rightarrow \mathbf{N}$ is defined by,

Functions Ex2.2 Q12

Define $f: \mathbf{N} \rightarrow \mathbf{Z}$ as $f(x) = x$ and $g: \mathbf{Z} \rightarrow \mathbf{Z}$ as $g(x) = |x|$.

We first show that g is not injective.

It can be observed that:

$$g(-1) = |-1| = 1$$

$$g(1) = |1| = 1$$

Therefore, $g(-1) = g(1)$, but $-1 \neq 1$.

Therefore, g is not injective.

Now, $g \circ f: \mathbf{N} \rightarrow \mathbf{Z}$ is defined as $g \circ f(x) = g(f(x)) = g(x) = |x|$.

Let $x, y \in \mathbf{N}$ such that $g \circ f(x) = g \circ f(y)$.

$$\Rightarrow |x| = |y|$$

Since x and $y \in \mathbf{N}$, both are positive.

$$\therefore |x| = |y| \Rightarrow x = y$$

Hence, $g \circ f$ is injective

Functions Ex2.2 Q13

We have, $f: A \rightarrow B$ and $g: B \rightarrow C$ are one-one functions

Now we have to prove $g \circ f: A \rightarrow C$ is one-one

let $x, y \in A$ such that

$$g \circ f(x) = g \circ f(y)$$

$$\Rightarrow g(f(x)) = g(f(y))$$

$$\Rightarrow f(x) = f(y) \quad [\because g \text{ is one-one}]$$

$$\Rightarrow x = y \quad [\because f \text{ is one-one}]$$

$\therefore g \circ f$ is one-one function

Functions Ex2.2 Q14

We have, $f: A \rightarrow B$ and $g: B \rightarrow C$ are onto functions.

Now, we need to prove: $g \circ f: A \rightarrow C$ is onto.

let $y \in C$, then

$$g \circ f(x) = y$$

$$\Rightarrow g(f(x)) = y \dots\dots\dots (i)$$

Since g is onto, for each element in C , then exists a preimage in B .

$$\therefore g(x) = y \dots\dots\dots (ii)$$

From (i) & (ii)

$$f(x) = \alpha.$$

Since f is onto, for each element in B there exists a preimage in A

$$\therefore f(x) = \alpha \dots\dots\dots (iii)$$

From (ii) and (iii) we can conclude that for each $y \in C$, there exists a preimage in A such that $g \circ f(x) = y$

$\therefore g \circ f$ is onto

Ex 2.3

Functions Ex 2.3 Q 1(i)

$$f(x) = e^x \quad \text{and} \quad g(x) = \log_e x$$

$$\text{Now, } f \circ g(x) = f(g(x)) = f(\log_e x) = e^{\log_e x} = x$$

$$f \circ g(x) = x$$

$$g \circ f(x) = g(f(x)) = g(e^x) = \log_e e^x = x$$

$$\Rightarrow g \circ f(x) = x$$

Functions Ex 2.3 Q 1(ii)

$$f(x) = x^2, \quad g(x) = \cos x$$

Domain of f and Domain of $g = \mathbb{R}$

Range of $f = [0, \infty)$

Range of $g = (-1, 1)$

$\therefore$ Range of $f \subset$ domain of $g \Rightarrow g \circ f$ exist

Range of $g \subset$ domain of $f \Rightarrow f \circ g$ exist

Now,

$$g \circ f(x) = g(f(x)) = g(x^2) = \cos x^2$$

And

$$f \circ g(x) = f(g(x)) = f(\cos x) = \cos^2 x$$

Functions Ex 2.3 Q1(iii)

$$f(x) = |x| \text{ and } g(x) = \sin x$$

$$\text{Range of } f = [0, \infty) \subset \text{Domain of } g = \mathbb{R} \Rightarrow g \circ f \text{ exist}$$

$$\text{Range of } g = [-1, 1] \subset \text{Domain of } f = \mathbb{R} \Rightarrow f \circ g \text{ exist}$$

Now,

$$f \circ g(x) = f(g(x)) = f(\sin x) = |\sin x|$$

And

$$g \circ f(x) = g(f(x)) = g(|x|) = \sin|x|$$

Functions Ex 2.3 Q1(iv)

$$f(x) = x + 1 \text{ and } g(x) = e^x$$

$$\text{Range of } f = \mathbb{R} \subset \text{Domain of } g = \mathbb{R} \Rightarrow g \circ f \text{ exist}$$

$$\text{Range of } g = (0, \infty) \subset \text{Domain of } f = \mathbb{R} \Rightarrow f \circ g \text{ exist}$$

Now,

$$g \circ f(x) = g(f(x)) = g(x + 1) = e^{x+1}$$

And

$$f \circ g(x) = f(g(x)) = f(e^x) = e^x + 1$$

Functions Ex 2.3 Q1(v)

$$f(x) = \sin^{-1} x \text{ and } g(x) = x^2$$

$$\text{Range of } f = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \subset \text{Domain of } g = \mathbb{R} \Rightarrow g \circ f \text{ exist}$$

$$\text{Range of } g = [0, \infty) \subseteq \text{Domain of } f = \mathbb{R} \Rightarrow f \circ g \text{ exist}$$

Now,

$$f \circ g(x) = f(g(x)) = f(x^2) = \sin^{-1} x^2$$

And

$$g \circ f(x) = g(f(x)) = g(\sin^{-1} x) = (\sin^{-1} x)^2$$

Functions Ex 2.3 Q 1(vi)

$$f(x) = x + 1 \text{ and } g(x) = \sin x$$

$$\text{Range of } f = \mathbb{R} \subset \text{Domain of } g = \mathbb{R} \Rightarrow g \circ f \text{ exists}$$

$$\text{Range of } g = [-1, 1] \subset \text{Domain of } f = \mathbb{R} \Rightarrow f \circ g \text{ exists}$$

Now,

$$f \circ g(x) = f(g(x)) = f(\sin x) = \sin x + 1$$

And

$$g \circ f(x) = g(f(x)) = g(x + 1) = \sin(x + 1)$$

Functions Ex 2.3 Q1(vii)

$$f(x) = x + 1 \text{ and } g(x) = 2x + 3$$

$$\text{Range of } f = \mathbb{R} \subseteq \text{Domain of } g = \mathbb{R} \Rightarrow g \circ f \text{ exist}$$

$$\text{Range of } g = \mathbb{R} \subseteq \text{Domain of } f = \mathbb{R} \Rightarrow f \circ g \text{ exist}$$

Now,

$$f \circ g(x) = f(g(x)) = f(2x + 3) = (2x + 3) + 1 = 2x + 4$$

And

$$g \circ f(x) = g(f(x)) = g(x + 1) = 2(x + 1) + 3$$

$$\Rightarrow g \circ f(x) = 2x + 5$$

Functions Ex 2.3 Q1(viii)

$$f(x) = c, \quad c \in \mathbb{R} \text{ and}$$

$$g(x) = \sin x^2$$

Range of $f = \mathbb{R} \subset \text{Domain of } g = \mathbb{R} \Rightarrow g \circ f \text{ exist}$
 Range of $g = [-1, 1] \subset \text{Domain of } f = \mathbb{R} \Rightarrow f \circ g \text{ exist}$

Now,

$$g \circ f(x) = g(f(x)) = g(c) = \sin c^2$$

And

$$f \circ g(x) = f(g(x)) = f(\sin x^2) = c$$

Functions Ex 2.3 Q1(ix)

$$f(x) = x^2 + 2 \text{ and } g(x) = 1 - \frac{1}{1-x}$$

Range of $f = (2, \infty) \subset \text{Domain of } g = \mathbb{R} \Rightarrow g \circ f \text{ exist}$

Range of $g = \mathbb{R} - [1] \subset \text{Domain of } f = \mathbb{R} \Rightarrow f \circ g \text{ exist}$

Now,

$$f \circ g(x) = f(g(x)) = f\left(\frac{-x}{1-x}\right) = \frac{x^2}{(1-x)^2} + 2$$

And

$$g \circ f(x) = g(f(x)) = g(x^2 + 2) = \frac{-(x^2 + 2)}{1 - (x^2 + 2)}$$

$$\Rightarrow g \circ f(x) = \frac{x^2 + 2}{x^2 + 1}$$

Functions Ex 2.3 Q2

We have, $f(x) = x^2 + x + 1$ and $g(x) = \sin x$

Now,

$$f \circ g(x) = f(g(x)) = f(\sin x)$$

$$\Rightarrow f \circ g(x) = \sin^2 x + \sin x + 1$$

Again, $g \circ f(x) = g(f(x)) = g(x^2 + x + 1)$

$$\Rightarrow g \circ f(x) = \sin(x^2 + x + 1)$$

Clearly

$$f \circ g \neq g \circ f$$

Functions Ex 2.3 Q3

We have $f(x) = |x|$

We assume the domain of $f = \mathbb{R}$

Range of $f = [0, \infty)$

$\therefore$ Range of $f \subset \text{domain of } f$

$\therefore f \circ f$ exists.

Now,

$$f \circ f(x) = f(f(x)) = f(|x|) = \|x\| = f(x)$$

$\therefore f \circ f = f$

Functions Ex 2.3 Q4

$$f(x) = 2x + 5 \text{ and } g(x) = x^2 + 1$$

- ∴ Range of $f = R$ and range of $g = [1, \infty]$
- ∴ Range of $f \subseteq \text{Domain of } g(R)$ and range of $g \subseteq \text{domain of } f(R)$
- ∴ both $fo g$ and $go f$ exist.

$$\begin{aligned} \text{i)} \quad f \circ g(x) &= f(g(x)) = f(x^2 + 1) \\ &= 2(x^2 + 1) + 5 \end{aligned}$$

$$\Rightarrow f \circ g(x) = 2x^2 + 7$$

$$\begin{aligned} \text{ii)} \quad g \circ f(x) &= g(f(x)) = g(2x + 5) \\ &= (2x + 5)^2 + 1 \end{aligned}$$

$$\Rightarrow g \circ f(x) = 4x^2 + 20x + 26$$

$$\begin{aligned} \text{iii)} \quad f \circ f(x) &= f(f(x)) = f(2x + 5) \\ &= 2(2x + 5) + 5 \\ f \circ f(x) &= 4x + 15 \end{aligned}$$

$$\begin{aligned} \text{iv)} \quad f^2(x) &= [f(x)]^2 = (2x + 5)^2 \\ &= 4x^2 + 20x + 25 \end{aligned}$$

$$\begin{aligned} \therefore \text{ from (iii) \& (iv)} \\ f \circ f &\neq f^2 \end{aligned}$$

Functions Ex 2.3 Q5

We have, $f(x) = \sin x$ and $g(x) = 2x$.

Domain of f and g is R

$$\text{Range of } f = [-1, 1]$$

$$\text{Range of } g = R$$

- ∴ Range of $f \subseteq \text{Domain } g$ and
- Range of $g \subseteq \text{Domain } f$

∴ $fo g$ and $go f$ both exist.

$$\text{i)} \quad g \circ f(x) = g(f(x)) = g(\sin x) = g \circ f(x) = 2 \sin x$$

$$\text{ii)} \quad f \circ g(x) = f(g(x)) = f(2x) = \sin 2x$$

$$\therefore g \circ f \neq f \circ g$$

Functions Ex 2.3 Q6

f, g , and h are real functions given by $f(x) = \sin x$, $g(x) = 2x$ and $h(x) = \cos x$

To prove: $f \circ g = g \circ (fh)$

L.H.S

$$\begin{aligned} f \circ g(x) &= f(g(x)) \\ &= f(2x) = \sin 2x \\ &\Rightarrow f \circ g(x) = 2 \sin x \cos x \dots\dots\dots (A) \end{aligned}$$

R.H.S

$$\begin{aligned} g \circ (fh)(x) &= g \circ (f(x) \cdot h(x)) \\ &= g(\sin x \cos x) \\ g \circ (fh)(x) &= 2 \sin x \cos x \dots\dots\dots (B) \end{aligned}$$

from A & B

$$f \circ g(x) = g \circ (fh)(x)$$

Functions Ex 2.3 Q7

We are given that f is a real function and g is a function given by $g(x) = 2x$
 To prove; $g \circ f = f + f$.

L.H.S

$$\begin{aligned} g \circ f(x) &= g(f(x)) = 2f(x) \\ &= f(x) + f(x) = \text{R.H.S} \end{aligned}$$

$$\Rightarrow g \circ f = f + f$$

Functions Ex 2.3 Q8

$$f(x) = \sqrt{1-x}, \quad g(x) = \log_e^x$$

Domain of f and g are R .

$$\text{Range of } f = (-\infty, 1)$$

$$\text{Range of } g = (0, e)$$

Clearly $\text{Range } f \subset \text{Domain } g \Rightarrow g \circ f$ exists

$\text{Range } g \subset \text{Domain } f \Rightarrow f \circ g$ exists

$$\begin{aligned} \therefore g \circ f(x) &= g(f(x)) = g(\sqrt{1-x}) \\ g \circ f(x) &= \log_e^{\sqrt{1-x}} \end{aligned}$$

Again

$$\begin{aligned} f \circ g(x) &= f(g(x)) = f(\log_e^x) \\ f \circ g(x) &= \sqrt{1 - \log_e^x} \end{aligned}$$

Functions Ex 2.3 Q9

$$f: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \rightarrow R \text{ and } g: [-1, 1] \rightarrow R \text{ defined as } f(x) = \tan x \text{ and } g(x) = \sqrt{1-x^2}$$

$$\begin{aligned} \text{Range of } f: \text{let } y &= f(x) \Rightarrow y = \tan x \\ &\Rightarrow x = \tan^{-1} y \end{aligned}$$

$$\text{Since } x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), y \in (-\infty, \infty)$$

$$\therefore \text{Range of } f \subset \text{domain of } g = [-1, 1]$$

$$\therefore g \circ f \text{ exists.}$$

By similar argument $f \circ g$ exists.

Now,

$$f \circ g(x) = f(g(x)) = f(\sqrt{1-x^2})$$

$$f \circ g(x) = \tan \sqrt{1-x^2}$$

Again

$$\begin{aligned} g \circ f(x) &= g(f(x)) \\ &= g(\tan x) \\ g \circ f(x) &= \sqrt{1 - \tan^2 x} \end{aligned}$$

Functions Ex 2.3 Q10

$$f(x) = \sqrt{x+3} \text{ and } g(x) = x^2 + 1$$

Now,

$$\text{Range of } f = [-3, \infty] \text{ and}$$

$$\text{Range of } g = (1, \infty)$$

Then, Range of $f \subset \text{Domain } g$ and

Range of $g \subset \text{Domain } f$

$\therefore f \circ g$ and $g \circ f$ exist.

Now,

$$f \circ g(x) = f(g(x)) = f(x^2 + 1)$$

$$f \circ g(x) = \sqrt{x^2 + 4}$$

Again,

$$g \circ f(x) = g(f(x)) = g(\sqrt{x+3})$$

$$= (\sqrt{x+3})^2 + 1$$

$$g \circ f(x) = x + 4$$

Functions Ex 2.3 Q11(i)

We have, $f(x) = \sqrt{x-2}$

Clearly, $\text{Domain}(f) = [2, \infty)$ and $\text{Range}(f) = [0, \infty)$.

We observe that $\text{range}(f)$ is not a subset of $\text{domain of } f$.

$$\begin{aligned} \therefore \text{Domain of } (f \circ f) &= \{x : x \in \text{Domain}(f) \text{ and } f(x) \in \text{Domain}(f)\} \\ &= \{x : x \in [2, \infty) \text{ and } \sqrt{x-2} \in [2, \infty)\} \\ &= \{x : x \in [2, \infty) \text{ and } \sqrt{x-2} \geq 2\} \\ &= \{x : x \in [2, \infty) \text{ and } x-2 \geq 4\} \\ &= \{x : x \in [2, \infty) \text{ and } x \geq 6\} \\ &= [6, \infty) \end{aligned}$$

Now,

$$(f \circ f)(x) = f(f(x)) = f(\sqrt{x-2}) = \sqrt{\sqrt{x-2}-2}$$

$\therefore f \circ f : [6, \infty) \rightarrow \mathbb{R}$ defined as

$$(f \circ f)(x) = \sqrt{\sqrt{x-2}-2}$$

Functions Ex 2.3 Q11(ii)

We have, $f(x) = \sqrt{x-2}$

Clearly, $\text{Domain}(f) = [2, \infty)$ and $\text{Range}(f) = [0, \infty)$.

We observe that $\text{range}(f)$ is not a subset of domain of f .

$$\begin{aligned}\therefore \text{Domain of } (f \circ f) &= \{x : x \in \text{Domain}(f) \text{ and } f(x) \in \text{Domain}(f)\} \\ &= \{x : x \in [2, \infty) \text{ and } \sqrt{x-2} \in [2, \infty)\} \\ &= \{x : x \in [2, \infty) \text{ and } \sqrt{x-2} \geq 2\} \\ &= \{x : x \in [2, \infty) \text{ and } x-2 \geq 4\} \\ &= \{x : x \in [2, \infty) \text{ and } x \geq 6\} \\ &= [6, \infty)\end{aligned}$$

Clearly, $\text{range of } f = [0, \infty) \not\subset \text{Domain of } (f \circ f)$.

$$\begin{aligned}\therefore \text{Domain of } ((f \circ f) \circ f) &= \{x : x \in \text{Domain}(f) \text{ and } f(x) \in \text{Domain}(f \circ f)\} \\ &= \{x : x \in [2, \infty) \text{ and } \sqrt{x-2} \in [6, \infty)\} \\ &= \{x : x \in [2, \infty) \text{ and } \sqrt{x-2} \geq 6\} \\ &= \{x : x \in [2, \infty) \text{ and } x-2 \geq 36\} \\ &= \{x : x \in [2, \infty) \text{ and } x \geq 38\} \\ &= [38, \infty)\end{aligned}$$

Now,

$$(f \circ f)(x) = f(f(x)) = f(\sqrt{x-2}) = \sqrt{\sqrt{x-2}-2}$$

$$(f \circ f \circ f)(x) = (f \circ f)(f(x)) = (f \circ f)(\sqrt{x-2}) = \sqrt{\sqrt{\sqrt{x-2}-2}-2}$$

$\therefore f \circ f \circ f : [38, \infty) \rightarrow \mathbb{R}$ defined as

$$(f \circ f \circ f)(x) = \sqrt{\sqrt{\sqrt{x-2}-2}-2}$$

Functions Ex 2.3 Q11(iii)

We have, $f(x) = \sqrt{x-2}$

Clearly, $\text{Domain}(f) = [2, \infty)$ and $\text{Range}(f) = [0, \infty)$.

We observe that $\text{range}(f)$ is not a subset of domain of f .

$$\begin{aligned}\therefore \text{Domain of } (f \circ f) &= \{x: x \in \text{Domain}(f) \text{ and } f(x) \in \text{Domain}(f)\} \\ &= \{x: x \in [2, \infty) \text{ and } \sqrt{x-2} \in [2, \infty)\} \\ &= \{x: x \in [2, \infty) \text{ and } \sqrt{x-2} \geq 2\} \\ &= \{x: x \in [2, \infty) \text{ and } x-2 \geq 4\} \\ &= \{x: x \in [2, \infty) \text{ and } x \geq 6\} \\ &= [6, \infty)\end{aligned}$$

Clearly, $\text{range of } f = [0, \infty) \not\subset \text{Domain of } (f \circ f)$.

$$\begin{aligned}\therefore \text{Domain of } ((f \circ f) \circ f) &= \{x: x \in \text{Domain}(f) \text{ and } f(x) \in \text{Domain}(f \circ f)\} \\ &= \{x: x \in [2, \infty) \text{ and } \sqrt{x-2} \in [6, \infty)\} \\ &= \{x: x \in [2, \infty) \text{ and } \sqrt{x-2} \geq 6\} \\ &= \{x: x \in [2, \infty) \text{ and } x-2 \geq 36\} \\ &= \{x: x \in [2, \infty) \text{ and } x \geq 38\} \\ &= [38, \infty)\end{aligned}$$

Now,

$$(f \circ f)(x) = f(f(x)) = f(\sqrt{x-2}) = \sqrt{\sqrt{x-2}-2}$$

$$(f \circ f \circ f)(x) = (f \circ f)(f(x)) = (f \circ f)(\sqrt{x-2}) = \sqrt{\sqrt{\sqrt{x-2}-2}-2}$$

$\therefore f \circ f \circ f : [38, \infty) \rightarrow \mathbb{R}$ defined as

$$(f \circ f \circ f)(x) = \sqrt{\sqrt{\sqrt{x-2}-2}-2}$$

$$(f \circ f \circ f)(38) = \sqrt{\sqrt{\sqrt{38-2}-2}-2} = \sqrt{\sqrt{\sqrt{36}-2}-2} = \sqrt{\sqrt{6-2}-2} = \sqrt{\sqrt{4}-2} = \sqrt{2-2} = 0$$

Functions Ex 2.3 Q11(iv)

We have, $f(x) = \sqrt{x-2}$

Clearly, $\text{Domain}(f) = [2, \infty)$ and $\text{Range}(f) = [0, \infty)$.

We observe that $\text{range}(f)$ is not a subset of domain of f .

$$\begin{aligned}\therefore \text{Domain of } (f \circ f) &= \{x: x \in \text{Domain}(f) \text{ and } f(x) \in \text{Domain}(f)\} \\ &= \{x: x \in [2, \infty) \text{ and } \sqrt{x-2} \in [2, \infty)\} \\ &= \{x: x \in [2, \infty) \text{ and } \sqrt{x-2} \geq 2\} \\ &= \{x: x \in [2, \infty) \text{ and } x-2 \geq 4\} \\ &= \{x: x \in [2, \infty) \text{ and } x \geq 6\} \\ &= [6, \infty)\end{aligned}$$

Now,

$$(f \circ f)(x) = f(f(x)) = f(\sqrt{x-2}) = \sqrt{\sqrt{x-2}-2}$$

$\therefore f \circ f : [6, \infty) \rightarrow \mathbb{R}$ defined as

$$(f \circ f)(x) = \sqrt{\sqrt{x-2}-2}$$

$$f^2(x) = [f(x)]^2 = [\sqrt{x-2}]^2 = x-2$$

$\therefore f^2 : [2, \infty) \rightarrow \mathbb{R}$ defined as

$$f^2(x) = x-2$$

$\therefore f \circ f \neq f^2$

Functions Ex 2.3 Q12

$$f(x) = \begin{cases} 1+x & 0 \leq x \leq 2 \\ 3-x & 2 \leq x \leq 3 \end{cases}$$

$\therefore$ Range of $f = [0, 3] \subseteq$ Domain of f .

$$\therefore f \circ f(x) = f(f(x)) = f \begin{cases} 1+x & 0 \leq x \leq 2 \\ 3-x & 2 < x \leq 3 \end{cases}$$

$$f \circ f(x) = \begin{cases} 2+x & 0 \leq x \leq 1 \\ 2-x & 1 < x \leq 2 \\ 4-x & 2 < x \leq 3 \end{cases}$$

Ex 2.5

Functions Ex 2.5 Q 1.

$$\begin{aligned} \text{i)} \quad & f : \{1, 2, 3, 4\} \rightarrow \{10\} \text{ given by} \\ & f\{(1, 10), (2, 10), (3, 10), (4, 10)\} \end{aligned}$$

clearly f is many-one function

$\Rightarrow f$ is not bijective

$\Rightarrow f$ is not invertible

$$\begin{aligned} \text{ii)} \quad & g : \{5, 6, 7, 8\} \rightarrow \{1, 2, 3, 4\} \text{ given by} \\ & g\{(5, 4), (6, 3), (7, 4), (8, 2)\} \end{aligned}$$

Since, 5 and 7 have same image 4

$\therefore g$ is not bijective

$\Rightarrow g$ is not bijective

$\Rightarrow g$ is not invertible

$$\begin{aligned} \text{iii)} \quad & h : \{2, 3, 4, 5\} \rightarrow \{7, 9, 11, 13\} \text{ given by} \\ & h\{(2, 7), (3, 9), (4, 11), (5, 13)\} \end{aligned}$$

We can observe that different element of domain have different image in co-domain.

Functions Ex 2.5 Q2

$$A = \{0, -1, -3, 2\}, \quad B = \{-9, -3, 0, 6\}$$

$f: A \rightarrow B$ is defined by $f(x) = 3x$

Since different elements of A have different images in B .

$\therefore f$ is one-one

Again, each element in B has a preimage in A .

$\therefore f$ is onto

$\therefore f$ is one-one bijective

$\Rightarrow f^{-1}: B \rightarrow A$ exists and is given by

$$f^{-1}(x) = \frac{x}{3}$$

$$A = \{1, 3, 5, 7, 9\}, \quad B = \{0, 1, 9, 25, 49, 81\}$$

$f: A \rightarrow B$ be a function defined by $f(x) = x^2$

Since different elements of A have different images in B .

$\therefore f$ is one-one

Again, $0 \in B$ does not have a preimage in A .

$\therefore f$ is not onto

Hence, f^{-1} does not exist.

Functions Ex 2.5 Q3

Given that $f: \{1, 2, 3\} \rightarrow \{a, b, c\}$ and $g: \{a, b, c\} \rightarrow \{\text{apple}, \text{ball}, \text{cat}\}$ such that

$f(1) = a, f(2) = b, f(3) = c, g(a) = \text{apple}, g(b) = \text{ball}$ and $g(c) = \text{cat}$

We need to prove that f, g and $g \circ f$ are invertible.

In order to prove that f is invertible it is sufficient to show that

$f: \{1, 2, 3\} \rightarrow \{a, b, c\}$ is a bijection.

f is one – one:

Each and every element of the set $\{1, 2, 3\}$ is having an image in the set $\{a, b, c\}$

Thus, f is one – one.

Obviously, the number of element of the sets $\{1, 2, 3\}$ and $\{a, b, c\}$ are equal and hence

f is onto.

Thus, the function f is invertible.

Similarly, let us observe for the function g :

g is one – one:

Each and every element of the set $\{a, b, c\}$ is having an image in the set $\{\text{apple}, \text{ball}, \text{cat}\}$

Thus, g is one – one.

Obviously, the number of element of the sets $\{a, b, c\}$ and $\{\text{apple}, \text{ball}, \text{cat}\}$ are equal and hence

g is onto.

Thus, the function g is invertible.

Now let us consider the function $g \circ f = g[f(x)]$

Each and every element of the set $\{1,2,3\}$ is having an image in the set $\{\text{apple}, \text{ball}, \text{cat}\}$.

Therefore, $g \circ f = \{(1, \text{apple}), (2, \text{ball}), (3, \text{cat})\}$

Thus, $g \circ f$ is one – one.

Since the number of elements in the sets $\{1,2,3\}$ and $\{\text{apple}, \text{ball}, \text{cat}\}$ are equal.

Hence $g \circ f$ is onto.

Therefore, function $g \circ f$ is invertible.

Let us now find f^{-1} :

We have $f: \{1,2,3\} \rightarrow \{a,b,c\}$

Thus, $f^{-1}: \{a,b,c\} \rightarrow \{1,2,3\}$

$\Rightarrow f^{-1} = \{(a,1), (b,2), (c,3)\}$

Let us now find g^{-1} :

We have $g: \{a,b,c\} \rightarrow \{\text{apple}, \text{ball}, \text{cat}\}$

Thus, $g^{-1}: \{\text{apple}, \text{ball}, \text{cat}\} \rightarrow \{a,b,c\}$

$\Rightarrow g^{-1} = \{(\text{apple}, a), (\text{ball}, b), (\text{cat}, c)\}$

Let us now find $f^{-1} \circ g^{-1}$:

$\Rightarrow f^{-1} \circ g^{-1} = \{(\text{apple}, 1), (\text{ball}, 2), (\text{cat}, 3)\} \dots (1)$

Also, let us find, $(g \circ f)^{-1}$:

$\Rightarrow (g \circ f)^{-1} = \{(\text{apple}, 1), (\text{ball}, 2), (\text{cat}, 3)\} \dots (2)$

From (1) and (2), we have,

$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

Functions Ex 2.5 Q4

Given that

$$A = \{1, 2, 3, 4\}, \quad B = \{3, 5, 7, 9\}, \quad C = \{7, 23, 47, 79\}$$

$f: A \rightarrow B$ and $g: B \rightarrow C$ are two functions defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2$

Now,

$$g \circ f(x) = g(f(x)) = g(2x + 1) = (2x + 1)^2 - 2$$

$$\Rightarrow g \circ f(x) = 4x^2 + 4x - 1$$

Now,

$$f: A \rightarrow B \text{ given by } f(x) = 2x + 1$$

Clearly f is one-one and onto, $\therefore f$ is bijective

$\Rightarrow f^{-1}$ exists

$$\therefore f^{-1} = \{(3, 1), (5, 2), (7, 3), (9, 4)\}$$

Again, $g: B \rightarrow C$ given by $g(x) = x^2 - 2$

Clearly g is one-one and onto $\Rightarrow g^{-1}$ exists

$$g^{-1} = \{(7, 3), (23, 5), (47, 7), (79, 9)\}$$

$$f \circ^{-1} g^{-1} = \{(7, 1), (23, 2), (47, 3), (79, 4)\} \dots (A)$$

$$\text{Now, } g \circ f(x) = 4x^2 + 4x - 1$$

Clearly $g \circ f$ is one-one and onto $\Rightarrow (g \circ f)^{-1}$ exists.

Hence,

$$(g \circ f)^{-1} = \{(7, 1), (23, 2), (47, 3), (79, 4)\} \dots (B)$$

From (A) & (B) we have $g \circ f^{-1} = f \circ^{-1} g^{-1}$

Functions Ex 2.5 Q5

Given that $f: Q \rightarrow Q$ defined by $f(x) = 3x + 5$.

To prove that f is invertible, we need to prove that f is one – one and onto.

Let $(x, y) \in Q$ be such that, $f(x) = f(y)$

$$\Rightarrow 3x + 5 = 3y + 5$$

$$\Rightarrow x = y$$

So, f is an injection.

Let y be an arbitrary element of Q such that $f(x) = y$.

$$\Rightarrow 3x + 5 = y$$

$$\Rightarrow 3x = y - 5$$

$$\Rightarrow x = \frac{y-5}{3}$$

Thus, for any $y \in Q$ there exists $x = \frac{y-5}{3} \in Q$ such that

$$f(x) = f\left(\frac{y-5}{3}\right) = 3 \frac{y-5}{3} + 5 = y$$

Thus, $f: Q \rightarrow Q$ is a bijection and hence invertible.

Let f^{-1} denotes the inverse of f .

Thus, $f \circ f^{-1}(x) = x$ for all $x \in Q$

$$\Rightarrow f[f^{-1}(x)] = x \text{ for all } x \in Q.$$

$$\Rightarrow 3f^{-1}(x) + 5 = x \text{ for all } x \in Q.$$

$$\Rightarrow f^{-1}(x) = \frac{x-5}{3} \text{ for all } x \in Q$$

Functions Ex 2.5 Q6

$f: \mathbf{R} \rightarrow \mathbf{R}$ is given by, $f(x) = 4x + 3$

One-one:

Let $f(x) = f(y)$.

$$\Rightarrow 4x + 3 = 4y + 3$$

$$\Rightarrow 4x = 4y$$

$$\Rightarrow x = y$$

Therefore f is a one-one function.

Onto:

For $y \in \mathbf{R}$, let $y = 4x + 3$.

$$\Rightarrow x = \frac{y-3}{4} \in \mathbf{R}$$

Therefore, for any $y \in \mathbf{R}$, there exists $x = \frac{y-3}{4} \in \mathbf{R}$ such that

$$f(x) = f\left(\frac{y-3}{4}\right) = 4\left(\frac{y-3}{4}\right) + 3 = y.$$

Therefore, f is onto.

Thus, f is one-one and onto and therefore, f^{-1} exists.

Let us define $g: \mathbf{R} \rightarrow \mathbf{R}$ by $g(x) = \frac{x-3}{4}$

$$\text{Now, } (g \circ f)(x) = g(f(x)) = g(4x+3) = \frac{(4x+3)-3}{4} = x$$

$$(f \circ g)(y) = f(g(y)) = f\left(\frac{y-3}{4}\right) = 4\left(\frac{y-3}{4}\right) + 3 = y - 3 + 3 = y$$

Therefore, $g \circ f = f \circ g = I_{\mathbf{R}}$

Hence, f is invertible and the inverse of f is given by

$$f^{-1}(y) = g(y) = \frac{y-3}{4}.$$

Functions Ex 2.5 Q7

$f: \mathbf{R}_+ \rightarrow [4, \infty)$ is given as $f(x) = x^2 + 4$.

One-one:

Let $f(x) = f(y)$.

$$\Rightarrow x^2 + 4 = y^2 + 4$$

$$\Rightarrow x^2 = y^2$$

$$\Rightarrow x = y \quad [\text{as } x = y \in \mathbf{R}_+]$$

Therefore, f is a one-one function.

Onto:

For $y \in [4, \infty)$, let $y = x^2 + 4$.

$$\Rightarrow x^2 = y - 4 \geq 0 \quad [\text{as } y \geq 4]$$

$$\Rightarrow x = \sqrt{y-4} \geq 0$$

Therefore, for any $y \in \mathbf{R}$, there exists $x = \sqrt{y-4} \in \mathbf{R}$ such that

$$f(x) = f(\sqrt{y-4}) = (\sqrt{y-4})^2 + 4 = y - 4 + 4 = y.$$

Therefore, f is onto.

Thus, f is one-one and onto and therefore, f^{-1} exists.

Let us define $g: [4, \infty) \rightarrow \mathbf{R}_+$ by,

$$g(y) = \sqrt{y-4}$$

$$\text{Now, } g \circ f(x) = g(f(x)) = g(x^2 + 4) = \sqrt{(x^2 + 4) - 4} = \sqrt{x^2} = x$$

$$\text{And, } f \circ g(y) = f(g(y)) = f(\sqrt{y-4}) = (\sqrt{y-4})^2 + 4 = (y-4) + 4 = y$$

Therefore, $g \circ f = f \circ g = I_{\mathbf{R}}$

Hence, f is invertible and the inverse of f is given by

$$f^{-1}(y) = g(y) = \sqrt{y-4}.$$

Functions Ex 2.5 Q8

It is given that $f(x) = \frac{(4x+3)}{(6x-4)}$, $x \neq \frac{2}{3}$.

$$\begin{aligned} (f \circ f)(x) &= f(f(x)) = f\left(\frac{4x+3}{6x-4}\right) \\ &= \frac{4\left(\frac{4x+3}{6x-4}\right) + 3}{6\left(\frac{4x+3}{6x-4}\right) - 4} = \frac{16x+12+18x-12}{24x+18-24x+16} = \frac{34x}{34} = x \end{aligned}$$

Therefore, $f \circ f(x) = x$, for all $x \neq \frac{2}{3}$.

$$\Rightarrow f \circ f = I$$

Hence, the given function f is invertible and the inverse of f is f itself.

Functions Ex 2.5 Q9

$f: \mathbf{R}_+ \rightarrow [-5, \infty)$ is given as $f(x) = 9x^2 + 6x - 5$.

Let y be an arbitrary element of $[-5, \infty)$.

Let $y = 9x^2 + 6x - 5$.

$$\Rightarrow y = (3x+1)^2 - 1 - 5 = (3x+1)^2 - 6$$

$$\Rightarrow (3x+1)^2 = y+6$$

$$\Rightarrow 3x+1 = \sqrt{y+6} \quad [\text{as } y \geq -5 \Rightarrow y+6 > 0]$$

$$\Rightarrow x = \frac{\sqrt{y+6}-1}{3}$$

Therefore, f is onto, thereby range $f = [-5, \infty)$.

Let us define $g: [-5, \infty) \rightarrow \mathbf{R}_+$ as $g(y) = \frac{\sqrt{y+6}-1}{3}$.

We now have:

$$\begin{aligned} (g \circ f)(x) &= g(f(x)) = g(9x^2 + 6x - 5) \\ &= g((3x+1)^2 - 6) \\ &= \frac{\sqrt{(3x+1)^2 - 6 + 6} - 1}{3} \\ &= \frac{3x+1-1}{3} = x \end{aligned}$$

$$\begin{aligned}
 \text{And, } (f \circ g)(y) &= f(g(y)) = f\left(\frac{\sqrt{y+6}-1}{3}\right) \\
 &= \left[3\left(\frac{\sqrt{y+6}-1}{3}\right) + 1\right]^2 - 6 \\
 &= (\sqrt{y+6})^2 - 6 = y + 6 - 6 = y
 \end{aligned}$$

Therefore, $\text{gof} = I_{\mathbb{R}}$ and $\text{fog} = I_{(-5, \infty)}$

Hence, f is invertible and the inverse of f is given by

$$f^{-1}(y) = g(y) = \frac{\sqrt{y+6}-1}{3}.$$

Functions Ex 2.5 Q10

$f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by

$$f(x) = x^3 - 3$$

Injectivity:

$$\begin{aligned}
 &\text{let } f(x_1) = f(x_2) \\
 \Rightarrow &x_1^3 - 3 = x_2^3 - 3 \\
 \Rightarrow &x_1^3 = x_2^3 \\
 \Rightarrow &x_1 = x_2 \\
 \Rightarrow &f \text{ is one-one}
 \end{aligned}$$

Surjectivity: let $y \in \mathbb{R}$ be arbitrary such that

$$\begin{aligned}
 &f(x) = y \\
 \Rightarrow &x^3 - 3 - y = 0
 \end{aligned}$$

We know that an equation of odd degree must have at least one real solution.

let $x = \alpha$ be that solution

$$\begin{aligned}
 \therefore &\alpha^3 - 3 = y \\
 \Rightarrow &f(\alpha) = y
 \end{aligned}$$

so, for each $y \in \mathbb{R}$ in co-domain there exist $\alpha \in \mathbb{R}$ in domain

$$\Rightarrow f \text{ is onto}$$

Thus, f is one-one and onto, so

f^{-1} exists

Now,

$$\begin{aligned}
 \therefore &f(x) = x^3 - 3 = y \\
 \Rightarrow &x^3 = 3 + y \\
 \Rightarrow &x = \sqrt[3]{3 + y} \\
 \Rightarrow &f^{-1}(x) = \sqrt[3]{3 + x}
 \end{aligned}$$

Thus, $f^{-1} : \mathbb{R} \rightarrow \mathbb{R}$ be the inverse function defined by $f^{-1}(x) = (x + 3)^{1/3}$

finally,

$$\begin{aligned}
 f^{-1}(24) &= (24 + 3)^{1/3} = 3 \\
 f^{-1}(5) &= (5 + 3)^{1/3} = 2
 \end{aligned}$$

Functions Ex 2.5 Q11

We have,

$f: \mathbb{R} \rightarrow \mathbb{R}$ is a function defined by

$$f(x) = x^3 + 4$$

Injectivity: let $f(x_1) = f(x_2)$ for $x_1, x_2 \in \mathbb{R}$

$$\Rightarrow x_1^3 + 4 = x_2^3 + 4$$

$$\Rightarrow x_1^3 = x_2^3$$

$$\Rightarrow x_1 = x_2$$

$$\Rightarrow f \text{ is one-one}$$

Surjectivity: let $y \in \mathbb{R}$ be arbitrary such that

$$f(x) = y$$

$$\Rightarrow x^3 + 4 = y$$

$$\Rightarrow x^3 + 4 - y = 0$$

We know that an odd degree equation must have a real root.

$$\Rightarrow x^3 + 4 = y \Rightarrow f(x) = y$$

$$\Rightarrow f \text{ is onto}$$

Since f is one-one and onto

$$\Rightarrow f \text{ is bijective}$$

finally,

$$f(x) = y$$

$$\Rightarrow x^3 + 4 = y$$

$$\Rightarrow x^3 = y - 4$$

$$\Rightarrow x = (y - 4)^{1/3}$$

$$\therefore f^{-1}(x) = (x - 4)^{1/3}$$

$$\therefore f^{-1}(3) = (3 - 4)^{1/3} = -1$$

Functions Ex 2.5 Q12

Given that $f(x) = 2x$ and $g(x) = x + 2$.

We need to prove that f and g are bijective maps.

Let $x, y \in Q$.

Consider $f(x) = f(y)$

$$\Rightarrow 2x = 2y$$

$$\Rightarrow x = y$$

$\Rightarrow f$ is one - one.

Let y be an arbitrary element of Q such that $f(x) = y$

$$\text{Then } f(x) = y = 2x \Rightarrow x = \frac{y}{2}$$

Thus, for any $y \in Q$, there exists $x = \frac{y}{2} \in Q$ such that,

$$f(x) = f\left(\frac{y}{2}\right) = 2 \frac{y}{2} = y$$

So $f: Q \rightarrow Q$ is a bijection and hence invertible.

Let f^{-1} denote the inverse of f .

$$\text{Thus, } f^{-1}(x) = \frac{x}{2} \dots (1)$$

Let $x, y \in Q$.

Consider $g(x) = g(y)$

$$\Rightarrow x + 2 = y + 2$$

$$\Rightarrow x = y$$

$\Rightarrow g$ is one - one.

Let y be an arbitrary element of Q such that $g(x) = y$

$$\text{Then } g(x) = y = x + 2 \Rightarrow x = y - 2$$

Thus, for any $y \in Q$, there exists $x = y - 2, y \in Q$ such that,

$$g(x) = g(y - 2) = y - 2 + 2 = y$$

So $g: Q \rightarrow Q$ is a bijection and hence invertible.

Let g^{-1} denote the inverse of g .

$$\text{Thus, } g^{-1}(x) = x - 2 \dots (2)$$

Now consider $g \circ f = g[f(x)] = g(2x) = 2x + 2$

$$\text{Thus, } (g \circ f)^{-1} = \frac{x - 2}{2} \dots (3)$$

From (1) and (2), we have

$$f^{-1} \circ g^{-1} = f^{-1}[g^{-1}(x)] = f^{-1}[x - 2] = \frac{x - 2}{2} \dots (4)$$

From (3) and (4), it is clear that

$$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$$

Functions Ex 2.5 Q13

Given that $f(x) = \frac{x-2}{x-3}$;

Let $f(x) = y$;

$$\Rightarrow y = \frac{x-2}{x-3}$$

Interchange x and y ;

$$\Rightarrow x = \frac{y-2}{y-3}$$

$$\Rightarrow (y-3)x = y-2$$

$$\Rightarrow xy - 3x = y - 2$$

$$\Rightarrow xy - y = 3x - 2$$

$$\Rightarrow y(x-1) = 3x-2$$

$$\Rightarrow y = \frac{3x-2}{x-1}$$

$$\Rightarrow f^{-1}(x) = \frac{3x-2}{x-1}$$

Functions Ex 2.5 Q14

$f: \mathbb{R}^+ \rightarrow [-9, \infty)$ given by $f(x) = 5x^2 + 6x - 9$

For any $x, y \in \mathbb{R}^+$

$$f(x) = f(y)$$

$$\Rightarrow 5x^2 + 6x - 9 = 5y^2 + 6y - 9$$

$$\Rightarrow 5(x^2 - y^2) + 6(x - y) = 0$$

$$\Rightarrow (x - y)[5(x + y) + 6] = 0$$

$$\Rightarrow x - y = 0 \quad \left[\because 5(x + y) + 6 \neq 0 \text{ as } x, y \in \mathbb{R}^+ \right]$$

$$\Rightarrow x = y$$

So, f is an injection.

Let y be an arbitrary element of $[-9, \infty)$.

$$f(x) = y$$

$$\Rightarrow 5x^2 + 6x - 9 = y$$

$$\Rightarrow 25x^2 + 30x - 45 = 5y$$

$$\Rightarrow 25x^2 + 30x + 9 - 54 = 5y$$

$$\Rightarrow (5x + 3)^2 = 5y + 54$$

$$\Rightarrow (5x + 3) = \sqrt{5y + 54}$$

$$\Rightarrow x = \frac{\sqrt{5y + 54} - 3}{5}$$

Now, $y \in [-9, \infty)$

$$\Rightarrow y \geq -9$$

$$\Rightarrow 5y + 54 \geq 9$$

$$\Rightarrow \sqrt{5y + 54} \geq 3$$

$$\Rightarrow \sqrt{5y + 54} - 3 \geq 0$$

$$\Rightarrow \frac{\sqrt{5y + 54} - 3}{5} \geq 0$$

$$\Rightarrow x \geq 0 \Rightarrow x \in \mathbb{R}^+$$

Thus, for every $y \in [-9, \infty)$ there exist $x = \frac{\sqrt{5y + 54} - 3}{5} \in \mathbb{R}^+$ such that $f(x) = y$.

So, $f: \mathbb{R}^+ \rightarrow [-9, \infty)$ is onto.

Thus, $f: \mathbb{R}^+ \rightarrow [-9, \infty)$ is a bijection and hence invertible.

Let f^{-1} denote the inverse of f .

Then,

$$(f \circ f^{-1})(y) = y \text{ for all } y \in [-9, \infty)$$

$$f(f^{-1}(y)) = y \text{ for all } y \in [-9, \infty)$$

$$\Rightarrow 5(f^{-1}(y))^2 + 6(f^{-1}(y)) - 9 = y \text{ for all } y \in [-9, \infty)$$

$$\Rightarrow 25(f^{-1}(y))^2 + 30(f^{-1}(y)) - 45 = 5y \text{ for all } y \in [-9, \infty)$$

$$\Rightarrow 25(f^{-1}(y))^2 + 30(f^{-1}(y)) + 9 = 5y + 54 \text{ for all } y \in [-9, \infty)$$

$$\Rightarrow (5f^{-1}(y) + 3)^2 = 5y + 54 \text{ for all } y \in [-9, \infty)$$

$$\Rightarrow 5f^{-1}(y) + 3 = \sqrt{5y + 54} \text{ for all } y \in [-9, \infty)$$

$$\Rightarrow f^{-1}(y) = \frac{\sqrt{5y + 54} - 3}{5}$$

Functions Ex 2.5 Q15

We have given that

$f: \mathbb{R} \rightarrow (-1, 1)$ defined by

$$f(x) = \frac{10^x - 10^{-x}}{10^x + 10^{-x}} \text{ is invertible}$$

let $f(x) = y$

$$\Rightarrow \frac{10^x - 10^{-x}}{10^x + 10^{-x}} = y$$

$$\Rightarrow \frac{10^{2x} - 1}{10^{2x} + 1} = y$$

$$\Rightarrow 10^{2x} - 1 = y(10^{2x} + 1)$$

$$\Rightarrow 10^{2x} - 10^{2x}y = y + 1$$

$$\Rightarrow 10^{2x}(1 - y) = y + 1$$

$$\Rightarrow 10^{2x} = \frac{y + 1}{1 - y}$$

$$\Rightarrow 2x = \log_{10} \left(\frac{1 + y}{1 - y} \right)$$

$$x = \frac{1}{2} \log_{10} \left(\frac{1 + y}{1 - y} \right)$$

$$\therefore f^{-1}(x) = \frac{1}{2} \log_{10} \left(\frac{1 + x}{1 - x} \right)$$

Functions Ex 2.5 Q16

We have given that

$f : \mathbb{R} \rightarrow (0, 2)$ defined by

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} + 1 \text{ is invertible.}$$

$$\text{let } f(x) = y$$

$$\Rightarrow \frac{e^x - e^{-x}}{e^x + e^{-x}} + 1 = y$$

$$\Rightarrow \frac{2e^x}{e^x + e^{-x}} = y$$

$$\Rightarrow \frac{2e^{2x}}{e^{2x} + 1} = y$$

$$\Rightarrow 2e^{2x} = y(e^{2x} + 1)$$

$$\Rightarrow e^{2x}(2 - y) = y$$

$$\Rightarrow e^{2x} = \frac{y}{2 - y} \Rightarrow x = \frac{1}{2} \log_e \left(\frac{y}{2 - y} \right)$$

$$\Rightarrow f^{-1}(x) = \frac{1}{2} \log_e \left(\frac{x}{2 - x} \right)$$

Functions Ex 2.5 Q17

Given: that

$f : [-1, \infty) \rightarrow [-1, \infty)$ is a function

given by $f(x) = (x+1)^2 - 1$

In order to show that f is invertible, we need to prove that f is bijective.

Injective: let $x, y \in [-1, \infty)$, Such that

$$f(x) = f(y)$$

$$\Rightarrow (x+1)^2 - 1 = (y+1)^2 - 1$$

$$\Rightarrow (x+1)^2 = (y+1)^2$$

$$\Rightarrow x+1 = y+1 \quad [x, y \in [-1, \infty)]$$

$$\Rightarrow x = y$$

$$\Rightarrow f \text{ is one-one}$$

Surjectivity: let $y \in [-1, \infty)$ be arbitrary

such that $f(x) = y$

$$\Rightarrow (x+1)^2 - 1 = y$$

$$\Rightarrow (x+1)^2 = y+1$$

$$\Rightarrow x+1 = \sqrt{y+1}$$

$$\Rightarrow x = \sqrt{y+1} - 1 \in [-1, \infty)$$

So, for each $y \in [-1, \infty)$ (co-domain) there exist $x = \sqrt{y+1} - 1 \in [-1, \infty)$ (domain)

$\therefore f$ is onto

Thus, f is bijective $\Rightarrow f$ is invertible.

Now,

$$f(x) = f^{-1}(x)$$

$$\Rightarrow (x+1)^2 - 1 = \sqrt{x+1} - 1$$

$$\Rightarrow (x+1)^2 - \sqrt{x+1} = 0$$

$$\Rightarrow \sqrt{x+1} \left((x+1)^{3/2} - 1 \right) = 0$$

$$\Rightarrow \sqrt{x+1} = 0 \text{ or } (x+1)^{3/2} - 1 = 0$$

$$\Rightarrow x = -1 \text{ or } x = 0$$

$$\therefore x = 0, -1$$

Hence, $S = \{0, -1\}$

Functions Ex 2.5 Q18

$A = \{x \in \mathbb{R} : -1 \leq x \leq 1\}$ and $f: A \rightarrow A$, $g: A \rightarrow A$ are two functions defined by $f(x) = x^2$ and $g(x) = \sin\left(\frac{\pi x}{2}\right)$

Here, $f: A \rightarrow A$ is defined by

$$f(x) = x^2$$

Clearly f is not injective, $\because f(1) = f(-1) = 1$

So, f is not bijective and hence not invertible.

Hence, f^{-1} does not exist

Now, $g: A \rightarrow A$ defined by

$$g(x) = \sin\left(\frac{\pi x}{2}\right)$$

Injectivity: Let $x_1 = x_2$

$$\begin{aligned} \Rightarrow \quad \frac{\pi x_1}{2} &= \frac{\pi x_2}{2} \\ \Rightarrow \quad \sin\left(\frac{\pi x_1}{2}\right) &= \sin\left(\frac{\pi x_2}{2}\right) \quad [\because -1 \leq x \leq 1] \\ \Rightarrow \quad g(x_1) &= g(x_2) \\ \Rightarrow \quad g &\text{ is one-one(i)} \end{aligned}$$

Surjectivity: let y be arbitrary such that

$$\begin{aligned} g(x) &= y \\ \Rightarrow \quad \sin\left(\frac{\pi x}{2}\right) &= y \\ \Rightarrow \quad \frac{\pi x}{2} &= \sin^{-1} y \\ \Rightarrow \quad x &= \frac{2}{\pi} \sin^{-1} y = [-1, 1] \end{aligned}$$

Thus, for each y in codomain, there exists x in domain, such that

$$\begin{aligned} g(x) &= y \\ \Rightarrow \quad g &\text{ is surjective(ii)} \end{aligned}$$

From (i) & (ii)

Functions Ex 2.5 Q19

Given: $f: \mathbb{R} \rightarrow \mathbb{R}$ is a function defined by

$$f(x) = \cos(x+2)$$

Injectivity: let $x, y \in \mathbb{R}$ such that

$$\begin{aligned} f(x) &= f(y) \\ \Rightarrow \quad \cos(x+2) &= \cos(y+2) \\ \Rightarrow \quad x+2 &= 2n\pi \pm y+2 \\ \Rightarrow \quad x &= 2n\pi \pm y \\ \Rightarrow \quad x &\neq y \\ \Rightarrow \quad f &\text{ is not one-one} \end{aligned}$$

Hence, f is not bijective

$$\Rightarrow \quad f \text{ is not invertible}$$

Functions Ex 2.5 Q20

We have, $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c, d\}$

We know that a function from A to B is said to be bijection if it is one-one and onto. This means different elements of A has different image in B . Also each element of B has preimage in A .

Let f_1, f_2, f_3 and f_4 are the functions from A to B .

$$f_1 = \{(1, a), (2, b), (3, c), (4, d)\}$$

$$f_2 = \{(1, b), (2, c), (3, d), (4, a)\}$$

$$f_3 = \{(1, c), (2, d), (3, a), (4, b)\}$$

$$f_4 = \{(1, d), (2, a), (3, b), (4, c)\}$$

we can verify that f_1, f_2, f_3 and f_4 are bijective from A to B .

Now,

$$f_1^{-1} = \{(a, 1), (b, 2), (c, 3), (d, 4)\}$$

$$f_2^{-1} = \{(b, 1), (c, 2), (d, 3), (a, 4)\}$$

$$f_3^{-1} = \{(c, 1), (d, 2), (a, 3), (b, 4)\}$$

$$f_4^{-1} = \{(d, 1), (a, 2), (b, 3), (c, 4)\}$$

Functions Ex 2.5 Q21

Given: A and B are two sets with finite elements.

$f : A \rightarrow B$ and $g : B \rightarrow A$ are injective map.

To prove: f is bijective

Proof: Since, $f : A \rightarrow B$ is injective we need to show f is surjective only.

Now,

$g : B \rightarrow A$ is injective

$\Rightarrow$ each element of B has image in A .

Functions Ex 2.5 Q22

We have,

$f : \mathbb{Q} \rightarrow \mathbb{Q}$ and $g : \mathbb{Q} \rightarrow \mathbb{Q}$ are two function defined by
 $f(x) = 2x$ and $g(x) = x + 2$

Now, $f : \mathbb{Q} \rightarrow \mathbb{Q}$ defined by $f(x) = 2x$

Injectivity: let $x, y \in \mathbb{Q}$ such that

$$f(x) = f(y) \Rightarrow 2x = 2y \Rightarrow x = y \\ \Rightarrow f \text{ is one-one}$$

Surjectivity: let $y \in \mathbb{Q}$ such that

$$f(x) = y \Rightarrow 2x = y \Rightarrow x = \frac{y}{2} \in \mathbb{Q}$$

$\therefore$ For each $y \in \mathbb{Q}$ (co-domain) there exist $x = \frac{y}{2} \in \mathbb{Q}$ (domain) such that $f(x) = y$

$\Rightarrow f$ is onto

$\therefore f$ is bijective

Again for $g : \mathbb{Q} \rightarrow \mathbb{Q}$ defined by

$$g(x) = x + 2$$

Injectivity: let $x, y \in \mathbb{Q}$ such that

$$g(y) = g(x) \Rightarrow y + 2 = x + 2 \Rightarrow y = x \\ \Rightarrow g \text{ is one-one}$$

Surjectivity: let $y \in \mathbb{Q}$ be arbitrary such that

$$g(x) = y \Rightarrow x + 2 = y \Rightarrow x = y - 2 \in \mathbb{Q}$$

Thus, for each $y \in \mathbb{Q}$ (co-domain), there exist $x = y - 2 \in \mathbb{Q}$ such that $g(x) = y$

$\therefore g$ is onto

Hence, g is bijective.

$$g \circ f(x) = g(f(x)) = g(2x) = 2x + 2$$

$$\Rightarrow g \circ f(x) = 2x + 2$$

f and g are bijective $\Rightarrow g \circ f$ is bijective

$$\Rightarrow (g \circ f)^{-1} \text{ exist}$$

$$\text{Now, } (g \circ f)(x) = 2x + 2$$

$$\Rightarrow (g \circ f)^{-1}(2x + 2) = x$$

$$\Rightarrow (g \circ f)^{-1}(2x) = x - 2$$

$$(g \circ f)^{-1}(x) = \frac{1}{2}(x - 2) \quad \dots A$$

Again,

$$f \text{ is bijective} \Rightarrow f^{-1} \text{ exist}$$

$\therefore f^{-1} : \mathbb{Q} \rightarrow \mathbb{Q}$ defined by

$$f^{-1}(x) = \frac{x}{2}$$

Also, g is bijective $\Rightarrow g^{-1}$ exist.

$\therefore g^{-1} : \mathbb{Q} \rightarrow \mathbb{Q}$ defined by

$$g^{-1}(x) = x - 2$$

$$\therefore f^{-1} \circ g^{-1}(x) = f^{-1}(g^{-1}(x))$$

$$= f^{-1}(x - 2)$$

$$(f^{-1} \circ g^{-1})(x) = \frac{1}{2}(x - 2) \dots \dots \dots (B)$$

From (A) & (B)

$$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$$