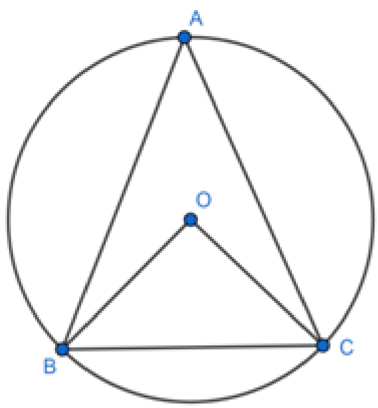
7. Theorems Related to Angles in a Circle

Let us Work Out 7.1

1. Question

O is the circumcentre of the isosceles triangle ABC, whose AB = AC, the point A and BC are on opposite us write by calculating, the values of ∠ABC and ∠ABO.

Answer



Since it is given that ∠BOC = 100° and by the theorem:-

The angle formed at the centre of a circle by an arc, is double of the angle formed by the same arc at any point on circle.

$$\Rightarrow \angle BAC = \frac{1}{2}\angle BOC = 50°$$

In ΔABC, as AB = BC, and we know that angle opposite to equal sides are equal.

$\Rightarrow \angle ACB = \angle ABC = t$

As the sum of all angles in a triangle is equal to 180°

$\Rightarrow 2t + \angle BAC = 180$

$\Rightarrow 2t = 180 - 50$

$\Rightarrow t = 65°$

$\Rightarrow \angle ABC = 65°$

Similarly, in ΔBOC, BO = OC, applying the same principle as above, we get,

$\Rightarrow 2\angle OBC + \angle BOC = 180$

$\Rightarrow 2\angle OBC = 180 - 100$

$\Rightarrow \angle OBC = 40°$

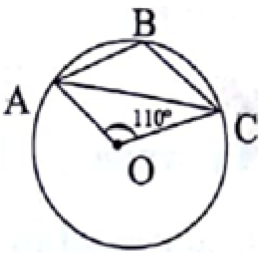
Also, $\angle ABO = \angle ABC - \angle OBC$

$\Rightarrow \angle ABO = 65 - 40 = 15°$

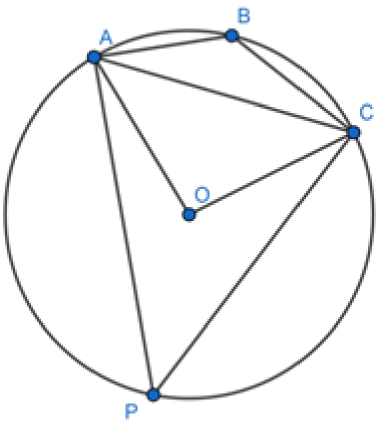
The values of $\angle ABC$ and $\angle ABO$ are 65° and 15°.

2. Question

In the adjoining figure, if O is the centre of circumcircle of ΔABC. And $\angle AOC$ = 110°, let us write by calculating, the value of $\angle ABC$.



Answer



Since it is given that $\angle AOC = 110°$ and by the theorem:-

The angle formed at the centre of a circle by an arc, is double of the angle formed by the same arc at any point on circle.

$$\Rightarrow \angle APC = \frac{1}{2}\angle AOC = 55°$$

Since APCB forms a cyclic quadrilateral and we know that sum of opposite sides of a cyclic quadrilateral is equal to 180°

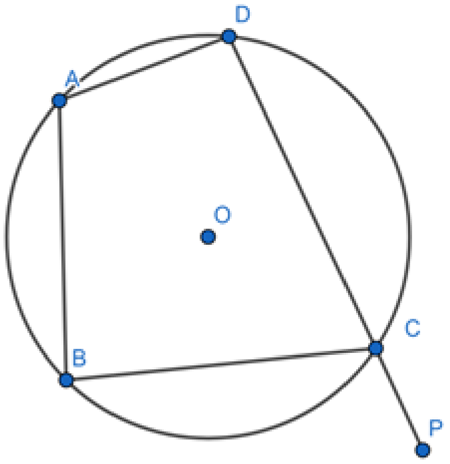
$\Rightarrow \angle APC + \angle ABC = 180$

$\Rightarrow \angle ABC = 180 - 55 = 125°$

3. Question

ABCD is a cyclic quadrilateral of a circle with centre O; DC is extended to the point P. If ∠BCP = 180°, let us write by calculating, the value of ∠BOD.

Answer



Since ∠BCP = 108° and DCP is a straight line,

⇒ ∠BCD = 180 – 108

∠BCD = 72°

By the theorem:-

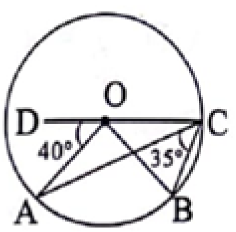
The angle formed at the centre of a circle by an arc, is double of the angle formed by the same arc at any point on circle.

$\Rightarrow \angle BOD = 2\angle BCD = 144°$

∴ value of ∠BOD is 144°.

4. Question

In the adjacent figure O is the centre of the circle; ∠AOD = 40° and ∠ACB = 35°; let us write by calculating the value of ∠BCO and ∠BOD, and answer with reason.



Answer

As ∠ACB = 35° and by the theorem:-

The angle formed at the centre of a circle by an arc, is double of the angle formed by the same arc at any point on circle.

$\Rightarrow \angle AOB = 2\angle ACB = 70°$

As ∠BOD = ∠AOD + ∠AOB

⇒ ∠BOD = 40 + 70 = 110°

Also since DOC forms a straight line,

⇒ ∠BOC = 180 - ∠BOD = 180 – 110 = 70°

In ΔBOC, OB = OC and as we know that angle opposite to equal sides are equal.

⇒ ∠OCB = ∠OBC

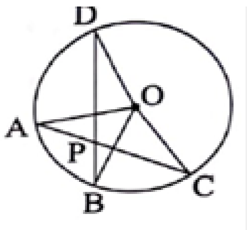
Sum of angles in a triangle is equal to 180.

⇒ 2∠BCO = 180 – 70

⇒ ∠BCO = 55°

5. Question

O is the centre of circle in the picture beside, if ∠APB = 80°, let us find the sum of the measures of ∠AOB and ∠COD and answer with reason.



Answer

In ΔAPB, sum of all angle is equal to 180°

⇒ ∠PAB + ∠ABP = 180 - ∠APB

⇒ ∠PAB + ∠ABP = 180 – 80 = 100 --------- (1)

Also by the theorem:-

The angle formed at the centre of a circle by an arc, is double of the angle formed by the same arc at any point on circle.

⇒ $\angle ABP = \frac{1}{2}\angle AOD$ and $\angle BAC = \frac{1}{2}\angle BOC$

Substituting above values in eq. (1)

$$\Rightarrow \frac{1}{2}(\angle AOD + \angle BOC) = 100$$

⇒ ∠AOD + ∠BOC = 200

Also, as the sum of angles around the point is equal to 360°

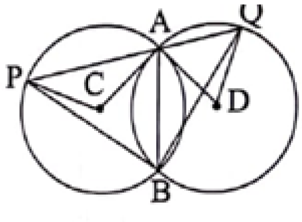
$\Rightarrow \angle AOD + \angle BOC + \angle AOB + \angle COD = 360$

$\Rightarrow \angle AOB + \angle COD = 360 - 200 = 160°$

6. Question

Like the adjoining figure, we draw two circles with centres C and D which intersect each other at the points A and B. We draw a straight line through the point A which intersects the circle with centre C at the point P and the circle with centre D at the point Q.

Let us prove that (i) $\angle PBQ = \angle CAD$ (ii) $\angle BPC = \angle BQD$



Answer

(i) In ΔACB and ΔADB,

By the theorem:-

The angle formed at the centre of a circle by an arc, is double of the angle formed by the same arc at any point on circle.

$\Rightarrow \angle PCA = 2\angle PBA$ and $\angle ADQ = 2\angle ABQ$

$\Rightarrow \angle PBQ = \angle ABQ + \angle PBA = \frac{1}{2}(\angle PCA + \angle ADQ)$ --------- (1)

In ΔPCA, $\angle PAC + \angle CPA = 180 - \angle PCA$

$\Rightarrow 2\angle CAP = 180 - \angle PCA$ [As $\angle PAC = \angle CPA$]

Similarly, in ΔADQ,

$\Rightarrow 2\angle QAD = 180 - \angle ADQ$ [As $\angle QAD = \angle AQD$]

Since, PAQ is a straight line,

$\Rightarrow \angle CAD = 180 - (\angle CAP + \angle QAD)$

On substituting values in above equation, we get,

$\Rightarrow \angle CAD = \frac{1}{2}(\angle PCA + \angle ADQ)$ ------------ (2)

From (1) and (2), we get, $\angle CAD = \angle PBQ$ ---------- (3)

Hence, proved.

(ii) Also, as $\angle CBA = \angle CAB$ and $\angle ABD = \angle BAD$

⇒ ∠CBA + ∠ABD = ∠CAB + ∠BAD

⇒ ∠CAD = ∠CBD ------------- (4)

From equations (3) and (4), we get,

∠PBQ = ∠CBD

⇒ ∠PBQ - ∠CBD = 0

⇒ ∠PBC - ∠QBD = 0

⇒ ∠PBC = ∠QBD ---------------- (5)

As PC = CB and we know that angles opposite to equal sides are equal

⇒ ∠BPC = ∠PBC ----------------- (6)

As BD = DQ and we know that angles opposite to equal sides are equal.

⇒ ∠BQD = ∠QBD ----------------- (7)

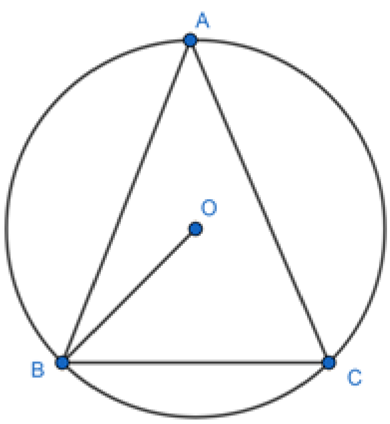
From equations (5), (6) and (7)

∠BPC = ∠BQD

7. Question

If the circumcentre of triangle ABC is O; let us prove that ∠OBC + ∠BAC = 90°.

Answer



By the theorem:-

The angle formed at the centre of a circle by an arc, is double of the angle formed by the same arc at any point on circle.

$\Rightarrow \angle BOC = 2\angle BAC$

As OB = OC and we know that angles opposite to equal sides are equal.

⇒ ∠OBC = ∠OCB

In ΔOBC, as sum of all sides of a triangle is equal to 180°.

⇒ ∠OBC + ∠OCB + ∠BOC = 180°

⇒ 2∠OBC + ∠BOC = 180°

⇒ 2∠OBC + 2∠BAC = 180°

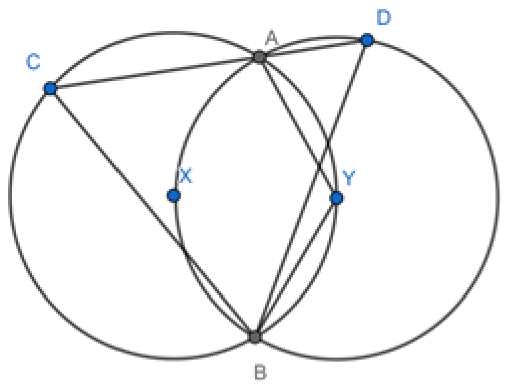
⇒ ∠OBC + ∠BAC = 90°

Hence, Proved.

8. Question

Each of two equal circles passes through the centre of the other and the two circles intersect each other at the points A and B. If a straight line through the point A intersects the two circles at points C and D, let us prove that ΔABCD is an equilateral triangle.

Answer



As the circles with centre X and Y are equal, the radius of both the circles are equal.

By the theorem:-

The angle formed at the centre of a circle by an arc, is double of the angle formed by the same arc at any point on circle.

⇒ ∠AXB = 2∠ACB ------------ (1)

Since AXBC is a quadrilateral triangle, therefore the sum of opposite sides of a quadrilateral is equal to 180°

⇒ ∠AYB + ∠ACB = 180 --------- (2)

Also, by using above theorem, we get

⇒ ∠AYB = 2∠ADB ------------ (3)

In ΔAXB and ΔAYB,

AX = AY (radius of equal circles)

BX = BY (radius of equal circles)

AB = AB (common)

⇒ $\Delta AXB \cong \Delta AYB$, by SSS congruency

∴ $\angle AXB = \angle AYB$ ------------ (4)

From equation (1), (2) and (4) we get,

⇒ $3\angle ACB = 180$

⇒ $\angle ACB = 60°$

From equation (1), (3) and (4) we get,

$\angle ACB = \angle ADB = 60°$

In ΔBCD, sum of all angles of a triangle is equal to 180°

⇒ $\angle ACB + \angle ADB + \angle CBD = 180°$

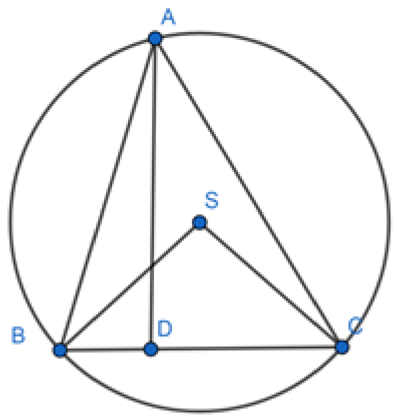
⇒ $\angle CBD = 60°$

As each angle in ΔBCD is equal to 60°, therefore ΔBCD is an equilateral triangle.

9. Question

S is the centre of the circumcircle of ΔABC and if $AD \perp BC$, let us prove that $\angle BAD = \angle SAC$.

Answer



Since AD is perpendicular to BC, $\angle ADB = 90°$

⇒ $\angle ABD + \angle BAD = 90$ ----------- (1)

By the theorem:-

The angle formed at the centre of a circle by an arc, is double of the angle formed by the same arc at any point on circle.

$\Rightarrow \angle ASC = 2\angle ABC = 2\angle ABD$ ------------ (2)

From equation (1) and (2), we get,

$\frac{1}{2}\angle ASC + \angle BAD = 90$ ------------- (3)

In ΔASC, as AS = SC and we know that angles opposite to equal sides are equal.

$\Rightarrow \angle SAC = \angle SCA$

Also, $\angle SAC + \angle SCA + \angle ASC = 180°$

$\Rightarrow 2\angle SAC + \angle ASC = 180°$

$\Rightarrow \angle ASC = 180° - 2\angle SAC$ ------------ (4)

From equation (3) and (4), we get,

$$\frac{1}{2}(180 - 2\angle SAC) + \angle BAD = 90$$

$\Rightarrow \angle BAD = \angle SAC$

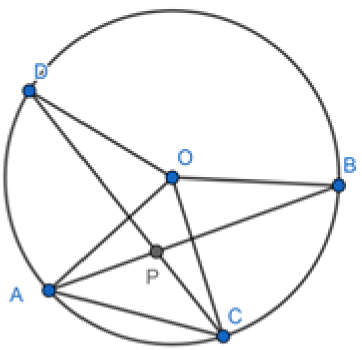
Hence, proved.

10. Question

Two chords AB and CD of a circle with centre O intersect each other at the points P, let us prove that $\angle AOD + \angle BOC = 2\angle BPC$.

If AOD and BOC are supplementary to each other, let us prove that the two chords are perpendicular to each other.

Answer



By the theorem:-

The angle formed at the centre of a circle by an arc, is double of the angle formed by the same arc at any point on circle.

$\Rightarrow \angle DOA = 2\angle DCA$

Similarly,

$\angle BOC = 2\angle BAC$

Adding above equations, we get,

$\Rightarrow \angle AOD + \angle BOC = 2(\angle BAC + \angle DCA)$ ----------- (1)

In ΔAPC, $\angle PAC + \angle PCA = \angle BPC$ by exterior angles property.

$\Rightarrow \angle BAC + \angle DCA = \angle BPC$

Hence, proved.

If AOD and BOC are supplementary to each other,

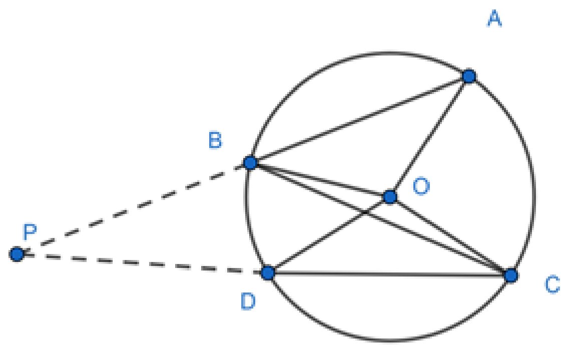
$\Rightarrow \angle BAC + \angle DCA = 90$

And from above theorem, $\angle BPC = 90°$

11. Question

If two chords AB and CD of a circle with centre O, when produced intersect each other at the point P, let us prove that $\angle AOC - \angle BOD = 2\angle BPC$.

Answer



The angle formed at the centre of a circle by an arc, is double of the angle formed by the same arc at any point on circle.

$\Rightarrow \angle AOC = 2\angle ABC$

Similarly, the angle formed at the centre of a circle by an arc, is double of the angle formed by the same arc at any point on circle.

$\Rightarrow \angle BOD = 2\angle BCD$.

In ΔBPC, $\angle ABC = \angle BPC + \angle BCP$

On substituting the values of $\angle ABC$ and $\angle BCP$ in above

equation, we get,

$$\Rightarrow \frac{1}{2}\angle AOC = \angle BPC + \frac{1}{2}\angle BOD$$

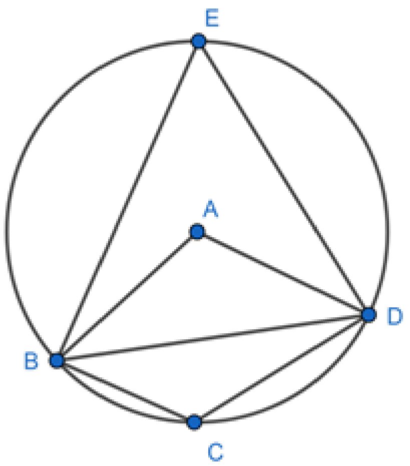
$\Rightarrow \angle AOC - \angle BOD = 2\angle BPC$

Hence, proved.

12. Question

We drew a circle with the point A of quadrilateral ABCD as centre which passes through the points B,C and D. Let us prove that $\angle CBD + \angle CDB = \frac{1}{2}\angle BAD$

Answer



By the theorem:-

The angle formed at the centre of a circle by an arc, is double of the angle formed by the same arc at any point on circle.

$\angle BAD = 2\angle BED$

Since BCDE forms cyclic quadrilateral, sum of opposite angles in a cyclic quadrilateral is equal to 180°

$\Rightarrow \angle BCD + \angle BED = 180$

$$\Rightarrow \angle BCD = 180 - \frac{1}{2}\angle BAD$$

In ΔBCD, sum of all angles is equal to 180°

$\Rightarrow \angle BCD + \angle CBD + \angle CDB = 180$

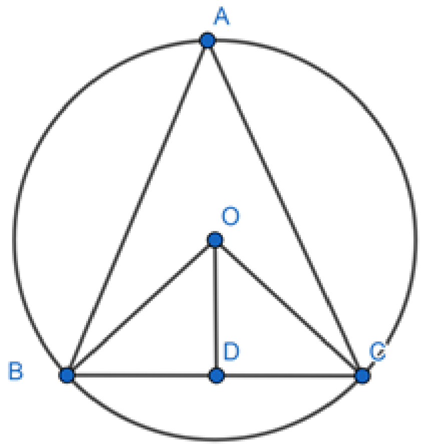
$$\Rightarrow \angle CBD + \angle CDB = \frac{1}{2}\angle BAD$$

Hence, proved.

13. Question

O is the circumcentre of Δ ABC and OD is perpendicular on the side BC; let us prove that ∠BOD = ∠BAC

Answer



By the theorem:-

The angle formed at the centre of a circle by an arc, is double of the angle formed by the same arc at any point on circle.

∠BOC = 2 ∠BAC ------------ (1)

In ΔBOD and ∠COD,

∠BDO = ∠CDO = 90° (given)

OD = OD (common)

OB = OC (radius)

Therefore, ΔBOD ≅ ∠COD by RHS congruency

⇒ ∠BOD = ∠COD ------------------- (2)

From (1) and (2), we get,

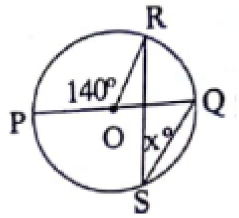
2 ∠BOD = 2 ∠BAC

⇒ ∠BOD = ∠BAC

Hence, proved.

14 A1. Question

In the adjoining figure, if ‘O’ is the centre of circle and PQ is a diameter, then the value of x is



A. 140

B. 40

C. 80

D. 20

Answer

By the theorem:-

The angle formed at the centre of a circle by an arc, is double of the angle formed by the same arc at any point on circle.

∠POR = 2 ∠PSR

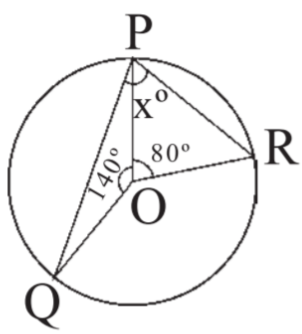
⇒ ∠PSR = 70°

As PQ is diameter, angle subtended by the diagonal at any point on the circle is equal to 90°

⇒ x = 90 – 70 = 20°

14 A2. Question

In the adjoining figure, if O is the centre of circle, the then the value of x is



A. 70

B. 60

C. 40

D. 200

Answer

∠QOR = 360 - 140 – 80

$\Rightarrow \angle QOR = 140°$

By the theorem:-

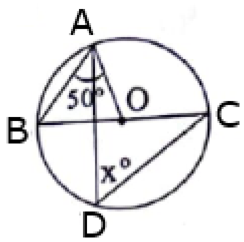
The angle formed at the centre of a circle by an arc, is double of the angle formed by the same arc at any point on circle.

$\angle QOR = 2x$

$\Rightarrow x = 70°$

14 A3. Question

In the adjoining figure, if O is centre of circle and BC is the diameter then the value of x is



A. 60

B. 50

C. 100

D. 80

Answer

In ΔABO, $2\angle BAO + \angle BOA = 180$

$\Rightarrow \angle BOA = 80°$

$\Rightarrow \angle AOC = 180 - 80 = 100$

By the theorem:-

The angle formed at the centre of a circle by an arc, is double of the angle formed by the same arc at any point on circle.

$\Rightarrow \angle AOC = 2x$

$\Rightarrow x = 50°$

14 A4. Question

O is the circumcentre of Δ ABC and $\angle OAB = 50°$, then the value of $\angle ACB$ is

A. 50

B. 100

C. 40

D. 80

Answer

In ΔABO, the value of ∠AOB = 80.

By the theorem:-

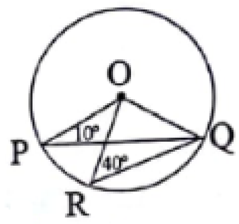
The angle formed at the centre of a circle by an arc, is double of the angle formed by the same arc at any point on circle.

⇒ ∠AOB = 2 ∠ACB

⇒ ∠ACB = 40°

14 A5. Question

In the adjoining figure, if O is centre of circle, the value of ∠POR is



A. 20

B. 40

C. 60

D. 80

Answer

In ΔORQ,

∠ROQ = 180 – 40 – 40

⇒ ∠ROQ = 100°

In ΔPOQ,

∠POQ = 180 – 10 – 10

⇒ ∠ROQ = 160°

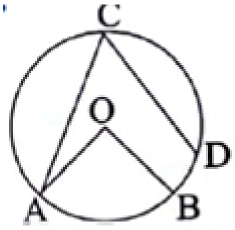
∠POR = 160 – 100

⇒ ∠POR = 60°

14 B. Question

Let us write whether the following statements are true of false”

(i) In the adjoining figure, if O is centre of circle, then ∠AOB = 2 ∠ACD



(ii) The point O lies within the triangular region ABC in such a way that OA = OB and ∠AOB = 2∠ACB. If we draw a circle with centre O and length of radius OA, then the point c lies on the circle.

Answer

(i) False

For the given condition to happen, the angles should be subtended by the same arc only.

(ii) True

The given statement is true, due to the following theorem:-

The angle formed at the centre of a circle by an arc, is double of the angle formed by the same arc at any point on circle. Therefore, point C lies on circle.

14 C. Question

Let us fill in the blanks

(i) The angle at the centre is ______ the angle on the circle, subtended by the same arc.

(ii) The length of two chord AB and CD are equal of a circle with centre O. If ∠APB and ∠DQC are angles on the circle, then the values of the two angles are_______.

(iii) If O is the circumcentre of equilateral triangle, then the value of the front angle formed by any side of the triangle is_______.

Answer

(i) Double

By the theorem:-

The angle formed at the centre of a circle by an arc, is double of the angle formed by the same arc at any point on circle.

(ii) Equal

By the theorem:-

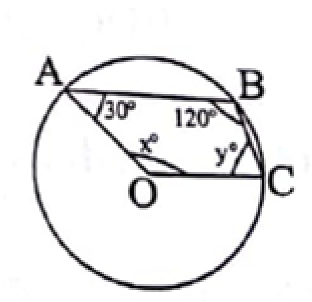
The angle formed at the centre of a circle by an arc, is double of the angle formed by the same arc at any point on circle.

(iii) 60°

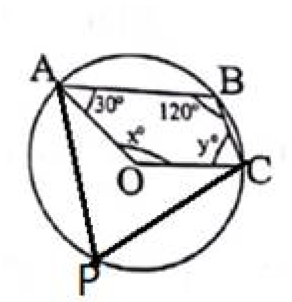
The front angle is the angle subtended by the arc at any point on the circle in its opposite arc.

15 A. Question

In the adjoining figure, O is the centre of the circle, if ∠ACB = 30°, ∠ABC = 60°, ∠DAB = 35° and ∠DBC = x°, the value of x is



Answer



Let P be any Point in major arc of circle.

By the theorem:-

The angle formed at the centre of a circle by an arc, is double of the angle formed by the same arc at any point on circle.

⇒ x = 2 ∠APC

AS APCB is a cyclic quadrilateral, so sum of opposite sides is equal to 180°

⇒ ∠APC + ∠ABC = 180

$$\Rightarrow \frac{x}{2} + 120 = 180$$

⇒ x = 120°

Also, ABCO is a quadrilateral whose sum of interior angles is equal to 360°.

$\Rightarrow$ y = 360 – x – 120 – 30

$\Rightarrow$ y = 90°

15 B. Question

O is circumcentre of the triangle ABC and D is the mid point of the side BC. If ∠BAC = 40°, let us find the value of ∠BOD.

Answer

Since D is the midpoint of BC,

In ΔBOD and ΔCOD, we have,

BO = CO (radius)

OD = OD (common)

BD = DC (given D as midpoint)

∴ ΔBOD ≅ ΔCOD by SSS congruency

$\Rightarrow$ ∠BOD = ∠COD ------------- (1)

By the theorem:-

The angle formed at the centre of a circle by an arc, is double of the angle formed by the same arc at any point on circle.

∠BOC = 2 ∠BAC

$\Rightarrow$ ∠BOD + ∠COD = 2 ∠BAC

From equation (1), we get,

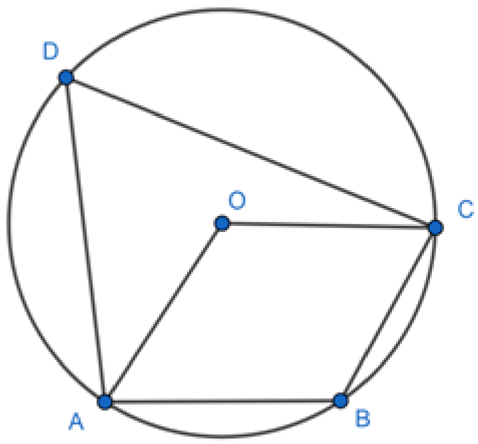
$\Rightarrow$ 2 ∠BOD = 2 ∠BAC

$\Rightarrow$ ∠BOD = ∠BAC = 40°

15 C. Question

Three points A, B and C lie on the circle with centre O in such a way that AOCB is a parallelogram, let us calculate the value of ∠AOC.

Answer



As OABC is a parallelogram, therefore, opposite angles will be equal.

⇒ ∠AOC = ∠ABC --------- (1)

Also, ABCD is a cyclic quadrilateral, therefore sum of opposite angles must be 180°

⇒ ∠ADC + ∠ABC = 180 ---------- (2)

By the theorem:-

The angle formed at the centre of a circle by an arc, is double of the angle formed by the same arc at any point on circle.

∠AOC = 2 ∠ADC -------- (3)

From (1), (2) and (3), we get,

$$\Rightarrow \frac{\angle AOC}{2} + \angle AOC = 180$$

$$\Rightarrow \frac{3}{2} \angle AOC = 180$$

⇒ ∠AOC = 120°

15 D. Question

O is the circumcentre of isosceles triangle ABC and ∠ABC = 120°; if the length of the radius of the circle is 5 cm, let us find the value of the side AB.

Answer

Since ∠ABC = 120° and therefore the angle subtended by arc ABC in major arc will be:

⇒ ∠APB = 180 – 120(By using Property of cyclic quadrilateral, where P is any point on circle)

⇒ ∠APB = 60°

By the theorem:-

The angle formed at the centre of a circle by an arc, is double of the angle formed by the same arc at any point on circle.

$\angle AOC = 2\angle APB = 120°$

In ΔAOB and ΔCOB,

AO = CO (radius)

OB = OB (given)

AB = BC (given)

$\therefore \Delta AOB \cong \Delta COB$ by SSS congruency

$\Rightarrow \angle ABO = \angle CBO = 60°$

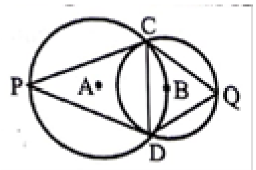
Also, $\angle AOB = \angle COB = 60°$.

This shows ΔOAB is an equilateral triangle.

$\Rightarrow AB = OA = 5$ cm

15 E. Question

Two circles with centres A and B interest each other at the points C and D. The centre lies on the circle with centre A. If $\angle CQD = 70°$, let us find the value of $\angle CPD$.



Answer

In circle with centre B, by the theorem:-

The angle formed at the centre of a circle by an arc, is double of the angle formed by the same arc at any point on circle.

$\angle CBD = 2\angle CQD = 140°$

Now, since B lies on the circle with centre A, therefore, quadrilateral CPDB is cyclic quadrilateral.

We also know that, the sum of opposite sides in a cyclic quadrilateral is equal to 180°

$\Rightarrow \angle CPD + \angle CBD = 180°$

On substituting the value in above equation, we get,

$\Rightarrow \angle CPD = 180 - 140$

$\Rightarrow \angle CPD = 40°$