CHAPTER -4

DETERMINANTS

One marks questions;

1. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find$|2A|$.
2. Find the values of x for which $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$.
3. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, Find $|2A|$.
4. If A is a square matrix with $|A| = 6$. Find the value of $|AA'|$.
5. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find$|A^{-1}|$.
6. If A is a square matrix of order 3 and $|A| = 4$. Find the value of $|2A|$.
7. If A is a square matrix and $|A| = 2$, then find the value of $|AA'|$.
8. If A is a invertible matrix of order 2, then find $|A^{-1}|$.
9. If A is a square matrix of order 3 and $|A| = 4$, then find$|adjA|$.
10. Find x if $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$.
11. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, find $A \cdot adjA$.
12. Without expansion find the value of $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$.
13. Evaluate $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$.
14. Without expanding, prove that $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$. (2 Marks Question)
15. Evaluate $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$.
16. If $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$, find x.
17. If A is a square matrix of order 3×3, find the value of $|KA|$.
18. If $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$, find the values of x.
19. Evaluate $\begin{vmatrix} \sin 30° & \cos 30° \\ -\sin 60° & \cos 60° \end{vmatrix}$.
20. If $A = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then find $|3A|$.
21. Examine the consistency of the system of linear equations $x + 2y = 2$ and $2x + 3y = 3$.
22. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that $|2A| = 4|A|$.
23. Evaluate $\begin{vmatrix} 0 & \sin\alpha & -\cos\alpha \\ -\sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{vmatrix}$.

24. If $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$, find $|A|$.

25. Evaluate $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$.

Two marks questions;

1. Find the equation of the line joining the points $(3, 1)$ and $(9, 3)$ using determinants.
2. Find the equation of the line joining the points $(1, 2)$ and $(3, 6)$ using determinants.
3. If each element of a row is expressed as the sum of two elements then verify for a third order determinant that the determinant can be expressed as sum of two determinants.
4. Find the area of the triangle whose vertices are $(3, 8), (-4, 2)$and $(5, 1)$ using determinants.
5. Prove that the value of the determinant remains unaltered if its rows and columns are interchanged by considering a third order determinant.
6. Without expansion, prove that $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = 0.$
7. If the area of the triangle with vertices $(-2, 0)(0, 4)$ and $(0, k)$ is 4 square units. Find the value of k using determinants.
8. Examine the consistency of the system of equations $x + 3y = 5$ and $2x + 6y = 8.$
9. Without expansion find the value of $\begin{vmatrix} 4 & a & b + c \\ 4 & b & c + a \\ 4 & c & a + b \end{vmatrix}$.
10. Find the area of the triangle whose vertices are $(1, 0)(6, 0)$and $(4, 3)$ using determinants.
11. Find$k$, if the area of the triangle is 3 square units and whose vertices are $(k, 0)(1, 3)$ and $(0, 0)$ using determinants.
12. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$ and hence find A^{-1}. (4 Marks Question)
13. Prove that $|adjA| = |A|^2$, where A is the matrix of order 3×3.
14. Find the area of the triangle whose vertices are $(3, 8), (-4, 2)$ and $(5, 1)$ using determinants.
15. Find the area of the triangle whose vertices are $(2, 7), (1, 1)$ and (10,8) using determinants.
16. Without expansion, prove that $\begin{vmatrix} x & a & x + a \\ y & b & y + b \\ z & c & z + c \end{vmatrix} = 0.$
17. If in a determinant, any two rows or columns are interchanged, then prove that the sign of the determinant changes.
18. Prove that $\begin{vmatrix} x + 4 & 2x & 2x \\ 2x & x + 4 & 2x \\ 2x & 2x & x + 4 \end{vmatrix} = (5x + 4)(4 - x)^2$. (4 Marks Question)
19. If each element of a row or a column of a determinant is multiplied by a constant k, then prove that the whole determinant is multiplied by the same constant k.
20. Solve the system of linear equations using matrix method:
    (i) $2x + 5y = 1,\ 3x + 2y = 7$ (ii) $5x + 2y = 4,\ 7x + 3y = 5$
    (iii) $5x + 2y = 3,\ 3x + 2y = 5$ (iv) $4x - 3y = 3,\ 3x - 5y = 7$.
21. Prove that $\begin{vmatrix} y + k & y & y \\ y & y + k & y \\ y & y & y + k \end{vmatrix} = k^2(3y + 4)$. (4 Marks Question)

Four marks questions;

1. Prove that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc.$
2. Prove that $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3.$
3. Prove that $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx).$
4. Prove that $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2.$
5. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c).$
6. Prove that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3.$
7. Show that $\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca).$
8. Prove that $\begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix} = (1-x^2)\begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}.$
9. Prove that $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3.$
10. Prove that $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+2b & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2.$
11. Prove that $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}.$
12. Prove that $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}.$
13. If x, y, z are all different from zero and $\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then show that $1+xyz=0$.
14. If $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = 0$ and x, y, z are all different from zero

    Then prove that $1+\sum\left(\frac{1}{x}\right)=0.$
15. Show that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right).$
16. Prove that $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = (a-b)(b-c)(c-a).$

17. Prove that $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$.

18. Prove that $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4\,a^2b^2c^2$.

19. Prove that $\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$.

20. Prove that $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$.

21. Prove that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$.

22. Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$.

23. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$.

24. Prove that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ.

25. Evaluate $\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$.

26. Prove that $\begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) \\ \sin\beta & \cos\beta & \cos(\beta+\delta) \\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta) \end{vmatrix} = 0$.

27. Prove that $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1$.

28. Solve the equation $\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0, a \neq 0$.

Five marks questions;

1. Solve the system of equations $x+y+z=6$, $y+3z=11$ and $x-2y+z=0$ by matrix method.
2. Solve the system of equations $3x-2y+3z=8$, $2x+y-z=1$ and $4x-3y+2z=4$ by matrix method.
3. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the system of equations $2x-3y+5z=11, 3x+2y-4z=-5$ and $x+y-2z=-3$.
4. The cost of 4 kg onion, 3 kg wheat and 2 kg rice is Rs 60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is Rs 90. The cost of 6 kg onion, 2 kg wheat and 3 kg rice is Rs 70. Find the cost of each item per kg by matrix method.

5. The sum of three numbers is 6. If we multiply the third number by 3 and add the second number to it we get 11. By adding the first and third numbers, we get double the second number. Represent it algebraically and find the numbers using matrix method.
6. Solve the equations $2x + y + z = 1, x - 2y - z = \frac{3}{2}$ and $3y - 5z = 9$ by matrix method.
7. Solve the equations $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$ and $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$ by matrix method.
8. Use the product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations $x - y + 2z = 1, 2y - 3z = 1, 3x - 2y + 4z = 2$.
9. Solve the system of equations $x - y + 2z = 7, 3x + 4y - 5z = -5$ and $2x - y + 3z = 12$ by matrix method.
10. Solve the system of equations $x - y + z = 4,\ 2x + y - 3z = 0$ and $x + y + z = 2$ by matrix method.
11. Solve the system of equations $2x + 2y + 3z = 4,\ x - 2y + z = -4$ and $3x - 4y - 2z = 3$ by matrix method.