11

Elementary Trigonometric Identities

1. Trigonometric Identities
Three basic identities in trigonometry are

1.1 $\sin^2\theta + \cos^2\theta = 1$

1.2 $\sec^2\theta - \tan^2\theta = 1$

1.3 $\text{cosec}^2\theta - \cot^2\theta = 1$

All these identities can be proved with the Pythagoras theorem $p^2 + b^2 = h^2$

e.g.,

$$\sin^2\theta + \cos^2\theta = \left(\frac{p}{h}\right)^2 + \left(\frac{b}{h}\right)^2 = \frac{p^2+b^2}{h^2} = \frac{h^2}{h^2} = 1 \text{ etc.}$$

Exchange the sides of these identities to obtain more identities, which are as follows,

1.4 $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$ or, $\cos^2\theta = 1 - \sin^2\theta$

$\Rightarrow \sin\theta = \sqrt{1-\cos^2\theta}$ or, $\cos\theta = \sqrt{1-\sin^2\theta}$

1.5 $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \sec^2\theta = 1 + \tan^2\theta$ or, $\tan^2\theta = \sec^2\theta - 1$

$\Rightarrow \sec\theta = \sqrt{1+\tan^2\theta}$ or, $\tan\theta = \sqrt{\sec^2\theta - 1}$

1.6 $\text{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow \text{cosec}^2\theta = 1 + \cot^2\theta$ or, $\cot^2\theta = \text{cosec}^2\theta - 1$

$\Rightarrow \text{cosec}\,\theta = \sqrt{1+\cot^2\theta}$ or, $\cot\theta = \sqrt{\text{cosec}^2\theta - 1}$

2. Other Identities : Following identities given in the previous chapter are also helpful in this chapter. Learn it properly.

2.1 $\sin\theta \cdot \text{cosec}\theta = 1 \Rightarrow \sin\theta = \frac{1}{\text{cosec}\,\theta} \Rightarrow \text{cosec}\theta = \frac{1}{\sin\theta}$

2.2 $\cos\theta \cdot \sec\theta = 1 \Rightarrow \cos\theta = \frac{1}{\sec\theta} \Rightarrow \sec\theta = \frac{1}{\cos\theta}$

2.3 $\tan\theta \cdot \cot\theta = 1 \Rightarrow \tan\theta = \frac{1}{\cot\theta} \Rightarrow \cot\theta = \frac{1}{\tan\theta}$

2.4 $\tan\theta = \frac{\sin\theta}{\cos\theta}$ 2.5 $\cot\theta = \frac{\cos\theta}{\sin\theta}$

2.6 $\sin(90° - \theta) = \cos\theta$, $\cos(90° - \theta) = \sin\theta$

2.7 $\tan(90° - \theta) = \cot\theta$, $\cot(90° - \theta) = \tan\theta$

2.8 $\sec(90° - \theta) = \text{cosec}\,\theta$, $\text{cosec}(90° - \theta) = \sec\theta$

Points to remember

1. $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta \Rightarrow \cos^2\theta = 1 - \sin^2\theta$
2. $\sec^2\theta - \tan^2\theta = 1 \Rightarrow 1 + \tan^2\theta = \sec^2\theta \Rightarrow \sec^2\theta - 1 = \tan^2\theta$
3. $\text{cosec}^2\theta - \cot^2\theta = 1 \Rightarrow 1 + \cot^2\theta = \text{cosec}^2\theta \Rightarrow \text{cosec}^2\theta - 1 = \cot^2\theta$
4. $\sin\theta \cdot \text{cosec}\,\theta = 1 \Rightarrow \sin\theta = \frac{1}{\text{cosec}\,\theta} \Rightarrow \text{cosec}\,\theta = \frac{1}{\sin\theta}$
5. $\cos\theta \cdot \sec\theta = 1 \Rightarrow \cos\theta = \frac{1}{\sec\theta} \Rightarrow \sec\theta = \frac{1}{\cos\theta}$
6. $\tan\theta \cdot \cot\theta = 1 \Rightarrow \tan\theta = \frac{1}{\cot\theta} \Rightarrow \cot\theta = \frac{1}{\tan\theta}$
7. $\cot\theta = \frac{\cos\theta}{\sin\theta}$
8. $\tan\theta = \frac{\sin\theta}{\cos\theta}$
9. (i) $\sin(90° - \theta) = \cos\theta$ (ii) $\cos(90° - \theta) = \sin\theta$
10. (i) $\tan(90° - \theta) = \cot\theta$ (ii) $\cot(90° - \theta) = \tan\theta$
11. (i) $\sec(90° - \theta) = \text{cosec}\,\theta$ (ii) $\text{cosec}(90° - \theta) = \sec\theta$

Solved example

1. Prove that $\frac{1+\cos A}{\sin A} + \frac{\sin A}{1+\cos A} = 2\,\text{cosec}\,A$

Solution : LHS $= \frac{1+\cos A}{\sin A} + \frac{\sin A}{1+\cos A}$

$$= \frac{(1+\cos A)^2 + (\sin A)^2}{\sin A(1+\cos A)} = \frac{1+2\cos A+\cos^2 A+\sin^2 A}{\sin A(1+\cos A)}$$

$$= \frac{1+2\cos A+1}{\sin A(1+\cos A)} = \frac{2+2\cos A}{\sin A(1+\cos A)} \qquad (\because \sin^2 A + \cos^2 A = 1)$$

$$= \frac{2(1+\cos A)}{\sin A(1+\cos A)} = \frac{2}{\sin A} = 2\,\text{cosec}\,A = \text{RHS}.$$

2. Prove that $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}} + \sqrt{\frac{\sec\theta+1}{\sec\theta-1}} = 2\,\text{cosec}\,\theta$

Solution : LHS $= \sqrt{\frac{\sec\theta-1}{\sec\theta+1}} + \sqrt{\frac{\sec\theta+1}{\sec\theta-1}}$

$$= \frac{(\sqrt{\sec\theta-1})^2 + (\sqrt{\sec\theta+1})^2}{\sqrt{\sec\theta+1}\cdot\sqrt{\sec\theta-1}}$$

$$= \frac{\sec\theta-1+\sec\theta+1}{\sqrt{\sec^2\theta-1}}$$

$$= \frac{2\sec\theta}{\sqrt{\tan^2\theta}} \qquad (\because \sec^2\theta - 1 = \tan^2\theta)$$

$$= \frac{2\sec\theta}{\tan\theta} = 2\cdot\frac{1}{\cos\theta\cdot\frac{\sin\theta}{\cos\theta}} = \frac{2}{\sin\theta} = 2\,\text{cosec}\,\theta$$

If $\cot\theta + \cos\theta = m$ and $\cot\theta - \cos\theta = n$ then prove that $m^2 - n^2 = 4\sqrt{mn}$

Solution : $\text{LHS} = m^2 - n^2$

$$= (\cot\theta + \cos\theta)^2 - (\cot\theta - \cos\theta)^2$$

$$= (\cot^2\theta + \cos^2\theta + 2\cot\theta\cos\theta) - (\cot^2\theta + \cos^2\theta - 2\cot\theta\cos\theta)$$

$$= 4\cot\theta\cos\theta \quad \ldots \text{(i)}$$

$$\text{RHS} = 4\sqrt{mn} = 4\sqrt{(\cot\theta + \cos\theta)(\cot\theta - \cos\theta)}$$

$$= 4\sqrt{\cot^2\theta - \cos^2\theta}$$

$$= 4\sqrt{\frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta} = 4\sqrt{\frac{\cos^2\theta - \cos^2\theta\sin^2\theta}{\sin^2\theta}}$$

$$= 4\sqrt{\frac{\cos^2\theta(1-\sin^2\theta)}{\sin^2\theta}} = 4\sqrt{\frac{\cos^2\theta\cdot\cos^2\theta}{\sin^2\theta}}$$

$$= 4\sqrt{\cot^2\theta\cos^2\theta} \quad \left(\because \frac{\cos\theta}{\sin\theta} = \cot\theta\right)$$

$$= 4\cot\theta\cos\theta \quad \ldots \text{(ii)}$$

From (i) and (ii), LHS = RHS, **Proved.**

4. Prove that $(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta) = 2$ *[SSC Tier-I 2014]*

Solution : $\text{LHS} = \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)$

$$= \left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)\left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)$$

$$= \frac{\{(\sin\theta + \cos\theta) - 1\}\{(\cos\theta + \sin\theta) + 1\}}{\sin\theta\cos\theta}$$

$$= \frac{(\sin\theta + \cos\theta)^2 - 1^2}{\sin\theta\cos\theta}$$

[Let $\sin\theta + \cos\theta = a$, $1 = b$ and use $(a - b)(a + b) = a^2 - b^2$]

$$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$$

$$= \frac{1 + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta} \quad (\because \sin^2\theta + \cos^2\theta = 1)$$

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2 = \text{RHS};$$ **Proved.**

5. If $\sec\theta + \tan\theta = p$ then find the value of $\sin\theta$

Solution : since $\sec^2\theta - \tan^2\theta = 1$

$\therefore \quad (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

or, $(\sec\theta - \tan\theta)\,p = 1$ or, $\sec\theta - \tan\theta = \frac{1}{p}$ … (i)

But, $\sec\theta + \tan\theta = p$

Adding, $2\sec\theta = \frac{1}{p} + p = \frac{1+p^2}{p}$

or, $\sec\theta = \frac{1+p^2}{2p} = \frac{h}{b}$

$\therefore \quad p^2 = h^2 - b^2 = (1+p^2)^2 - (2p)^2 = (1-p^2)^2$

Hence $\sin\theta = \frac{p}{h} = \frac{1-p^2}{1+p^2}$

Trick Learn that : If $\sec\theta + \tan\theta = p$ then $\sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$

If $\sec\theta - \tan\theta = p$ then also $\sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$

If $\text{cosec}\,\theta + \cot\theta = p$ then $\text{cosec}\,\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$

If $\text{cosec}\,\theta - \cot\theta = p$ then also $\text{cosec}\,\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$

6. (i) Express $\cos\theta$ in terms of $\tan\theta$
(ii) Express $\cos\theta$ in terms of $\text{cosec}\,\theta$

Solution : (i) $\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\sqrt{1+\tan^2\theta}}$

(ii) $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1 - \frac{1}{\text{cosec}^2\theta}}$

7. Prove that $\sec^2\theta + \text{cosec}^2\theta = \sec^2\theta\,\text{cosec}^2\theta$

Solution : LHS $= \sec^2\theta + \text{cosec}^2\theta$

$$= \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} = \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\sin^2\theta}$$

$$= \frac{1}{\cos^2\theta\sin^2\theta} = \frac{1}{\cos^2\theta}\cdot\frac{1}{\sin^2\theta} = \sec^2\theta\cdot\text{cosec}^2\theta$$

Second method : RHS

$\sec^2\theta\cdot\text{cosec}^2\theta$

$= (1+\tan^2\theta)(1+\cot^2\theta)$

$= 1 + \cot^2\theta + \tan^2\theta + \tan^2\theta\cot^2\theta$

$= 1 + \cot^2\theta + \tan^2\theta + 1$

$= (1+\cot^2\theta) + (1+\tan^2\theta)$

$= \text{cosec}^2\theta + \sec^2\theta$

8. Prove that

$$\frac{\sec\theta + \tan\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{\cos\theta}{1-\sin\theta}$$

Solution : LHS $= \dfrac{\sec\theta + \tan\theta - 1}{\tan\theta - \sec\theta + 1}$

$$= \frac{(\sec\theta + \tan\theta) - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1} \quad (\because \sec^2\theta - \tan^2\theta = 1)$$

$$= \frac{(\sec\theta + \tan\theta) - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\sec\theta + \tan\theta)(1 - \sec\theta + \tan\theta)}{(\tan\theta - \sec\theta + 1)} = \sec\theta + \tan\theta$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \frac{1+\sin\theta}{\cos\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$$

$$= \frac{1-\sin^2\theta}{\cos\theta(1-\sin\theta)} = \frac{\cos^2\theta}{\cos\theta(1-\sin\theta)} = \frac{\cos\theta}{1-\sin\theta}; \text{ Proved}$$

[Note : To write $1 = \sec^2\theta - \tan^2\theta$ is very important. In some question we may write $1 = \operatorname{cosec}^2\theta - \cot^2\theta$]

9. Prove that $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

[SSC Tier-I 2014]

Solution : We know that $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

$$\therefore \sin^6\theta + \cos^6\theta = (\sin^2\theta)^3 + (\cos^2\theta)^3$$

$$= (\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)$$

$$= 1^3 - 3\sin^2\theta\cos^2\theta \cdot 1 = 1 - 3\sin^2\theta\cos^2\theta$$

10. Prove that $\sin\theta(1 + \tan\theta) + \cos\theta(1 + \cot\theta) = \sec\theta + \operatorname{cosec}\theta$

Solution : LHS $= \sin\theta(1 + \tan\theta) + \cos\theta(1 + \cot\theta)$

$$= \sin\theta\left(1 + \frac{\sin\theta}{\cos\theta}\right) + \cos\theta\left(1 + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \sin\theta\left(\frac{\cos\theta + \sin\theta}{\cos\theta}\right) + \cos\theta\left(\frac{\sin\theta + \cos\theta}{\sin\theta}\right)$$

$$= (\cos\theta + \sin\theta)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right) = \cos\theta + \sin\theta)\left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)$$

$$= (\cos\theta + \sin\theta)\frac{1}{\cos\theta\sin\theta} = \frac{\cos\theta}{\cos\theta\sin\theta} + \frac{\sin\theta}{\cos\theta\sin\theta}$$

$$= \frac{1}{\sin\theta} + \frac{1}{\cos\theta} = \operatorname{cosec}\theta + \sec\theta$$

$$= \sec\theta + \operatorname{cosec}\theta = \text{RHS; Proved}$$

11. Prove that $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$

Solution : LHS $= \dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

dividing numerator and denominator by $\sin A$

$$\text{LHS} = \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A} = \frac{(\cot A + \operatorname{cosec} A) - 1}{(\cot A - \operatorname{cosec} A) + 1} \times \frac{\cot A - \operatorname{cosec} A}{\cot A - \operatorname{cosec} A}$$

$$= \frac{(\cot^2 A - \text{cosec}^2 A) - (\cot A - \text{cosec } A)}{\{(\cot A - \text{cosec } A) + 1\}(\cot A - \text{cosec } A)}$$

$$= \frac{-1 - \cot A + \text{cosec } A}{(\cot A - \text{cosec } A + 1)(\cot A - \text{cosec } A)}$$

$$= \frac{-1}{\cot A - \text{cosec } A} = \frac{1}{\text{cosec } A - \cot A} = \text{RHS}$$;Proved

12. If $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$ then find the value of cosθ. Find θ if $0^\circ < \theta < 90^\circ$

Solution : Given $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$

or, $\frac{\cos\theta(1+\sin\theta) + \cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} = 4$

or, $\frac{\cos\theta + \cos\theta\sin\theta + \cos\theta - \cos\theta\sin\theta}{1-\sin^2\theta} = 4$

or, $\frac{2\cos\theta}{\cos^2\theta} = 4$ or, $\frac{2}{\cos\theta} = 4$

or, $\cos\theta = \frac{2}{4} = \frac{1}{2}$ or, $\cos\theta = \cos 60^\circ$

or, $\theta = 60^\circ$

13. If $5\cos\theta + 12\sin\theta = 13$ then find the value of tan θ and cosec θ.

Solution : given, $12\sin\theta = 13 - 5\cos\theta$

or, $144\sin^2\theta = 169 + 25\cos^2\theta - 130\cos\theta$ (squaring)

or, $144(1 - \cos^2\theta) = 169 + 25\cos^2\theta - 130\cos\theta$

or, $144 - 144\cos^2\theta = 169 + 25\cos^2\theta - 130\cos\theta$

or, $169\cos^2\theta - 130\cos\theta + 25 = 0$

or, $(13\cos\theta)^2 - 2\cdot(13\cos\theta)\cdot 5 + 5^2 = 0$

or, $(13\cos\theta - 5)^2 = 0$

or, $13\cos\theta - 5 = 0$

or, $\cos\theta = \frac{5}{13} = \frac{b}{h}$ (say)

From Pythagoras theoram

$p = \sqrt{h^2 - b^2} = \sqrt{13^2 - 5^2} = 12$

$\therefore$ $\tan\theta = \frac{p}{b} = \frac{12}{5}$ and $\text{cosec }\theta = \frac{h}{p} = \frac{13}{12}$

14. If $\text{cosec}^2\theta + 2\cot^2\theta = 10$ then find the value of $\sin\theta + \cos\theta$ when $0^\circ < \theta < 90^\circ$

Solution : Given, $\text{cosec}^2\theta + 2\cot^2\theta = 10$

or, $1 + \cot^2\theta + 2\cot^2\theta = 10$

or, $3\cot^2\theta = 9$

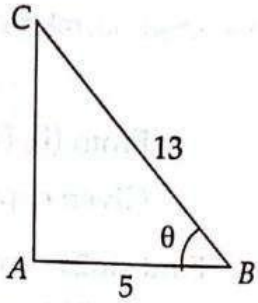
or, $\cot^2\theta = 3$

or, $\cot\theta = \sqrt{3} = \cot 30°$

or, $\theta = 30°$

$\therefore$ $\sin\theta + \cos\theta = \sin 30° + \cos 30°$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{\sqrt{3}+1}{2}$$



5. Prove that $\tan 1° \tan 2° \tan 3° \ldots\ldots\ldots \tan 88° \tan 89° = 1$

Solution: $\because \tan(90° - \theta) = \cot\theta$

$\therefore$ $\tan 89° = \tan(90° - 1°) = \cot 1°$

$\tan 88° = \tan(90° - 2°) = \cot 2°$

....

$\tan 46° = \tan(90° - 44°) = \cot 44°$

$\therefore$ LHS $= \tan 1° \tan 2° \tan 3° \ldots\ldots\ldots \tan 44° \tan 45° \tan 46° \ldots\ldots\ldots \tan 88° \cot 44°$

$= \tan 1° \tan 2° \tan 3° \ldots \tan 44° \tan 45° \cot 44° \ldots \cot 2° \cot 1°$

$= (\tan 1° \cot 1°)(\tan 2° \cot 2°) \ldots\ldots\ldots (\tan 44° \cot 44°) \tan 45°$

$= 1 \cdot 1 \cdot 1 \ldots\ldots\ldots 1$ $\quad (\because \tan\theta \cot\theta = 1)$

$= 1$

This question is based on complementary angle. Two angles are called complementary when their sum is 90° For such question we must not that $\sin\theta \sin(90 - \theta) = 1$, $\cos\theta \cos(90 - \theta) = 1$ etc.

e.g. $\sin 40 \cdot \sin 50 = 1$, $\cos 35 \cdot \cos 65 = 1$ etc]

16. Evaluate

$$\frac{\sec 29°}{\csc 61°} + 2\cot 8° \cot 17° \cot 45° \cot 73° \cot 82° - 3(\sin^2 38° + \sin^2 52°) + 4\{\cos\theta \sin(90° - \theta) + \sin\theta \cos(90° - \theta)\}$$

Solution: $\frac{\sec 29°}{\csc 61°} = \frac{\sec 29°}{\csc(90° - 29°)} = \frac{\sec 29°}{\sec 29°} = 1$ ... (i)

$2\cot 8° \cot 17° \cot 45° \cot 73° \cot 82°$

$= 2\cot 8° \cot 17° \cdot 1 \cot(90° - 17°) \cot(90° - 8°)$ $\quad (\because \cot 45° = 1)$

$= 2\cot 8° \cot 17° \tan 17° \tan 8°$ $\quad (\because \cot(90° - \theta) = \tan\theta)$

$= 2(\cot 8° \tan 8°)(\cot 17° \tan 17°)$ $\quad (\because \cot\theta \cdot \tan\theta = 1)$

$= 2 \cdot 1 \cdot 1$ ... (ii)

$= 2$

$3(\sin^2 38° + \sin^2 52°) = 3\{\sin^2 38° + \sin^2(90° - 38°)\}$

$= 3\{\sin^2 38° + \cos^2 38°\} = 3 \cdot 1 = 3$ ... (iii)

$4\{\cos\theta \cdot \sin(90° - \theta) + \sin\theta \cos(90° - \theta)\}$

$= 4(\cos\theta\cos\theta + \sin\theta\sin\theta)$

$= 4(\cos^2\theta + \sin^2\theta) = 4$ ... (iv)

From (i), (ii), (iii) and (iv)

Given expression $= 1 + 2 - 3 + 4 = 4$. Ans.

17. Find $\sin 2x + \cos 4x$ if $\tan 2x \cdot \tan 4x = 1$ [SSC Tier-I 2014]

Solution : (This question is also based on complementary angle. See the solution carefully)

$\tan 2x . \tan 4x = 1$

$\Rightarrow \tan 2x = \frac{1}{\tan 4x} = \cot 4x$

$\Rightarrow \tan 2x = \tan(90° - 4x))$ or, $2x = 90° - 4x$

or, $6x = 90°$ or, $x = \frac{90°}{6} = 15°$

$\therefore \sin 2x + \cos 4x = \sin 30° + \cos 60°$

$= \frac{1}{2} + \frac{1}{2} = 1$

18. Prove that $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} - \sec\theta = \sec\theta - \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

Solution : L.H.S. $= \sqrt{\frac{1+\sin\theta}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta}} - \sec\theta$

$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} - \sec\theta = \frac{1+\sin\theta}{\cos\theta} - \frac{1}{\cos\theta}$

$= \frac{1+\sin\theta-1}{\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$ ... (i)

R.H.S. $= \sec\theta - \sqrt{\frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}}$

$= \sec\theta - \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}} = \frac{1}{\cos\theta} - \frac{1-\sin\theta}{\cos\theta}$

$= \frac{1-(1-\sin\theta)}{\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$ ... (ii)

From equation (i) & (ii), L.H.S. = R.H.S.

19. If $a\cos\theta - b\sin\theta = c$,

then prove than $a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$

Solution : Given that $a\cos\theta - b\sin\theta = c$

squaring both sides,

$a^2\cos^2\theta + b^2\sin^2\theta - 2ab\cos\theta\sin\theta = c^2$

or, $a^2(1-\sin^2\theta) + b^2(1-\cos^2\theta) - 2ab\cos\theta\sin\theta = c^2$

or, $a^2 + b^2 - (a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cos\theta) = c^2$

or, $a^2 + b^2 - (a\sin\theta + b\cos\theta)^2 = c^2$

or, $(a\sin\theta + b\cos\theta)^2 = a^2 + b^2 - c^2$

or, $a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$

If $\operatorname{cosec}\theta - \sin\theta = m$ and $\sec\theta - \cos\theta = n$ then find a relation between m and n, independent of θ

Solution : $\operatorname{cosec}\theta - \sin\theta = m \Rightarrow \dfrac{1}{\sin\theta} - \sin\theta = m$

$$\Rightarrow \frac{1-\sin^2\theta}{\sin\theta} = m \Rightarrow \frac{\cos^2\theta}{\sin\theta} = m \quad \text{... (i)}$$

and $\sec\theta - \cos\theta = n \Rightarrow \dfrac{1}{\cos\theta} - \cos\theta = n$

$$\Rightarrow \frac{1-\cos^2\theta}{\cos\theta} = n \Rightarrow \frac{\sin^2\theta}{\cos\theta} = n \quad \text{... (ii)}$$

Eliminating $\cos\theta$ from (i) and (ii)

$$\frac{\cos^2\theta}{\sin\theta}\cdot\left(\frac{\sin^2\theta}{\cos\theta}\right)^2 = mn^2$$

$$\Rightarrow \sin^3\theta = mn^2 \Rightarrow \sin\theta = \left(mn^2\right)^{\frac{1}{3}} \quad \text{... (iii)}$$

Again, eliminating $\sin\theta$ from (i) and (ii)

$$\left(\frac{\cos^2\theta}{\sin\theta}\right)^2 \frac{\sin^2\theta}{\cos\theta} = m^2 n$$

$$\Rightarrow \cos^3\theta = m^2 n \Rightarrow \cos\theta = \left(m^2 n\right)^{\frac{1}{3}} \quad \text{... (iv)}$$

From (iii) and (iv)

$$\sin^2\theta + \cos^2\theta = \left\{\left(mn^2\right)^{\frac{1}{3}}\right\}^2 + \left\{\left(m^2 n\right)^{\frac{1}{3}}\right\}^2$$

$\therefore \left(mn^2\right)^{\frac{2}{3}} + \left(m^2 n\right)^{\frac{2}{3}} = 1$. which is required relation.

If $x > 0$ and $2\cos^2\left(x - \frac{1}{x}\right) = x + \frac{1}{x}$ then prove that $x^2 + \frac{1}{x^2} = 2$

Solution : $\because \cos^2\theta \le 1$

$$\because 2\cos^2\left(x - \frac{1}{x}\right) \le 2 \quad \text{... (i)}$$

again, from $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 \ge 0$ से

$$x + \frac{1}{x} - 2 \ge 0$$

or $x + \frac{1}{x} \ge 2$ ... (ii)

From (i)and (ii)

$2\cos^2\left(x - \frac{1}{x}\right) = x + \frac{1}{x}$ is possible only when

$2\cos^2\left(x - \frac{1}{x}\right) = 2$ and $x + \frac{1}{x} = 2$ simultaneously.

Clearly at $x = 1$ each of them is 2.

$$\therefore x^2 + \frac{1}{x^2} = 1^2 + 1^2 = 2$$

Exercise–11A

1. Expression $\frac{\tan x}{1+\sec x}-\frac{\tan x}{1-\sec x}$ is equal to

(a) cosec$x$ (b) 2 cosec$x$ (c) 2 sin$x$ (d) 2 cos$x$

2. Expression $(\sin^4 x-\cos^4 x+1)\operatorname{cosec}^2 x$ is eaqual to

(a) 1 (b) 2 (c) 0 (d) –1

3. If $l\cos^2\theta+m\sin^2\theta=\frac{\cos^2\theta(\operatorname{cosec}^2\theta+1)}{\operatorname{cosec}^2\theta-1}$ then what is the value of $\tan^2\theta$?

(a) $\frac{l-2}{m-1}$ (b) $\frac{l-1}{2-m}$ (c) $\frac{l-2}{l-m}$ (d) $\frac{2-l}{1-m}$

4. Assertion (A) : $\sec^2 23°-\tan^2 23°=1$

Reason (R) : For every real value of θ, $\sec^2\theta-\tan^2\theta=1$

(a) both A and R are true and R is a correct explanation of A.

(b) both A and R are true but R is not a correct explanation of A

(c) A is true, R is false

(d) A is false, R is true.

5. If $\sin x\cos x=\frac{1}{2}$ then the value of $\sin x-\cos x$ is

(a) 2 (b) 1 (c) 0 (d) –1

6. If $\tan^2 y\operatorname{cosec}^2 x-1=\tan^2 y$ then which one of the following is true.

(a) $x-y=0$ (b) $x=2y$ (c) $y=2x$ (d) $x-y=1°$

7. If $\frac{\cos x}{1+\operatorname{cosec} x}+\frac{\cos x}{\operatorname{cosec} x-1}=2$ then which one is a value of x ?

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

8. If $\sin x+\sin y=a$ and $\cos x+\cos y=b$ then value of $\sin x\cdot\sin y+\cos x\cdot\cos y$ is

(a) $a+b-ab$ (b) $a+b+ab$ (c) a^2+b^2-2 (d) $\left(\frac{a^2+b^2-2}{2}\right)$

9. If α is an angle in first quadrant such that $\operatorname{cosec}^4\alpha=17+\cot^4\alpha$, then what is the value of sinα ?

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{9}$ (d) $\frac{1}{16}$

10. If $x+\left(\frac{1}{x}\right)=2\cos\alpha$ then the value of $x^2+\left(\frac{1}{x^2}\right)$ is

(a) $4\cos^2\alpha$ (b) $4\cos^2\alpha-1$

(c) $2\cos^2\alpha-2\sin^2\alpha$ (d) $\cos^2\alpha-\sin^2\alpha$

11. If $\sin\theta+\cos\theta=a$ and $\sec\theta+\operatorname{cosec}\theta=b$ then which of the following relation is true ?

(a) $a=b(a^2-1)$ (b) $b=a(b^2-1)$

(c) $2a=b(a^2-1)$ (d) $2b=a(a^2-1)$

Among given values of θ which one satisfies the equation $\frac{\cos\theta}{1-\sin\theta} - \frac{\cos\theta}{1+\sin\theta} = 2$?

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

If $7\cos^2\theta + 3\sin^2\theta = 4$ and $0 < \theta < \frac{\pi}{2}$, then value of tanθ is.

(a) $\sqrt{7}$ (b) $\frac{7}{3}$ (c) 3 (d) $\sqrt{3}$

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14. When $0 < \theta < 90°$ then value of $[(1 - \sin^2\theta)\sec^2\theta + \tan^2\theta](\cos^2\theta + 1)$ is

(a) 2 (b) > 2 (c) ≥ 2 (d) < 2

15. What is the value of $\sin^2 15° + \sin^2 20° + \sin^2 25° + \ldots\ldots + \sin^2 75°$?

(a) $\tan^2 15° + \tan^2 20° + \tan^2 25° + \ldots\ldots + \tan^2 75°$

(b) $\cos^2 15° + \cos^2 20° + \cos^2 25° + \ldots\ldots + \cos^2 75°$

(c) $\cot^2 15° - \cot^2 20° + \cot^2 25° + \ldots\ldots + \cot^2 75°$

(d) $\sec^2 15° + \sec^2 20° + \sec^2 25° + \ldots\ldots + \sec^2 75°$

16. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$ is equal to

(a) $\sec\theta - \tan\theta$ (b) $\sec\theta + \tan\theta$ (c) $\operatorname{cosec}\theta + \cot\theta$ (d) $\operatorname{cosec}\theta - \cot\theta$

17. If $0° < \theta < 90°$ and $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 2$, then which of the following is equal to θ ?

(a) 30° (b) 45° (c) 60° (d) 75°

18. If $\sin 3\theta = \cos(\theta - 2°)$ where 3θ and $(\theta - 2°)$ are acute angle then θ is ?

(a) 22° (b) 23° (c) 24° (d) 25°

19. Expression $\frac{\sin^6\theta - \cos^6\theta}{\sin^2\theta - \cos^2\theta}$ is equal to

(a) $\sin^4\theta - \cos^4\theta$ (b) $1 - \sin^2\theta\cos^2\theta$

(c) $1 + \sin^2\theta\cos^2\theta$ (d) $1 - 3\sin^2\theta\cos^2\theta$

20. If $\sin^4 x + \sin^2 x = 1$, then the value of $\cot^4 x + \cot^2 x$ is

(a) $\cos^2 x$ (b) $\sin^2 x$ (c) $\tan^2 x$ (d) 1

21. If $x\cos\theta + y\sin\theta = 2$ and $x\cos\theta - y\sin\theta = 0$, then which one of the following is true ?

(a) $x^2 + y^2 = 1$ (b) $\frac{1}{x^2} + \frac{1}{y^2} = 1$ (c) $xy = 1$ (d) $x^2 - y^2 = 1$

22. Expression $\sin A(1 + \tan A) + \cos A(1 + \cot A)$ is equal to

(a) $\sec A + \operatorname{cosec} A$ (b) $2\operatorname{cosec} A(\sin A + \cos A)$

(c) $\tan A + \cot A$ (d) $\sec A \operatorname{cosec} A$

23. If $0° < \theta < 90°$ and $\cos^2\theta - \sin^2\theta = \frac{1}{2}$, then value of θ is ?

(a) 30° (b) 45° (c) 60° (d) 90°

24. If $3\sin\theta + 4\cos\theta = 5$, then $3\cos\theta - 4\sin\theta$ is equal to ?

(a) 0 (b) 3 (c) 4 (d) 5

25. On simplification $\frac{(1-\sin A\cos A)(\sin^2 A-\cos^2 A)}{\cos A(\sec A-\operatorname{cosec} A)(\sin^3 A+\cos^3 A)}$ equals
(a) $\sin A$ (b) $\cos A$ (c) $\sec A$ (d) $\operatorname{cosec} A$

26. For $0° < \theta < 90°$ which of the following expression is/are independent of θ ?
(i) $\cos\theta\,(1-\sin\theta)^{-1}+\cos\theta\,(1+\sin\theta)^{-1}$
(ii) $\cos\theta\,(1+\operatorname{cosec}\theta)^{-1}+\cos\theta\,(\operatorname{cosec}\theta-1)^{-1}$
Choose the correct code among following .
(a) Only (i) (b) Only(ii)
(c) Both (i) and (ii) (d) Neither (i) Nor (ii)

27. If $a\cos\theta - b\sin\theta = c$, then the value of $a\sin\theta + b\cos\theta$ is
(a) $\pm\sqrt{a^2+b^2+c^2}$ (b) $\pm\sqrt{a^2-b^2+c^2}$
(c) $\pm\sqrt{a^2+b^2-c^2}$ (d) $\pm\sqrt{b^2-c^2-a^2}$

28. Expression $\tan^2\alpha + \cot^2\alpha$ is
(a) ≥ 2 (b) ≤ 2
(c) ≥ -2 (d) None of these

29. Maximum value of $\sin^8\theta + \cos^{14}\theta$ is
(a) $\sqrt{2}$ (b) 2 (c) 1 (d) $\frac{1}{\sqrt{2}}$

30. If $P = \frac{1}{2}\sin^2\theta + \frac{1}{3}\cos^2\theta$, then
(a) $\frac{1}{3} \leq P \leq \frac{1}{2}$ (b) $P \geq \frac{1}{2}$
(c) $2 \leq P \leq 3$ (d) $-\frac{\sqrt{13}}{6} \leq P \leq \frac{\sqrt{13}}{6}$

31. Minimum value of $5\cos\theta + 12$ is
(a) 5 (b) 12 (c) 7 (d) 17

32. If $a\sin^3\theta + b\cos^3\theta = \sin\theta\cos\theta$, $0 < \theta < 90°$ and $a\sin\theta = b\cos\theta$ then the value of $a^2 + b^2$ is
(a) ab (b) $2ab$ (c) 1 (d) 2

33. $\sin^2 17.5° + \sin^2 72.5°$ is equal to
(a) $\cos^2 90°$ (b) $\tan^2 45°$ (c) $\cos^2 30°$ (d) $\sin^2 45°$

34. A cow is tied in a pole with a rope. The cow moves in a circular path keeping rope straight. When it covers a distance of 44 meter an angle of 72° is subtended at the centre. The length of the rope is
(a) 45 m (b) 35 m (c) 22 m (d) 56m

35. $(\sin\theta + \cos\theta)(\tan\theta + \cot\theta) =$
(a) 1 (b) $\sin\theta\cdot\cos\theta$ (c) $\sec\theta\cdot\operatorname{cosec}\theta$ (d) $\sec\theta + \operatorname{cosec}\theta$

37. If $\sec\alpha$, $\operatorname{cosec}\alpha$ are roots of equation $x^2 + px + q = 0$ then
(a) $p^2 = p + 2q$ (b) $q^2 = p + 2q$ (c) $p^2 = q(q+2)$
(d) $q^2 = p(p+2)$ (e) $p^2 = q(q-2)$

If sec θ and tan θ are roots of equation $ax^2 + bx + c = 0$ $(a, b \neq 0)$ then the value of sec θ − tan θ is

(a) $-\frac{a}{b}$ (b) $\frac{\sqrt{b^2 - 4ac}}{a}$ (c) $1 - \frac{a}{b}$

(d) $1 + \frac{a^2}{b^2}$ (e) $\frac{a}{b}$

If $x = h + a \sec\theta$ and $y = k + b \operatorname{cosec}\theta$ then

(a) $\frac{a^2}{(x+h)^2} - \frac{b^2}{(y+k)^2} = 01$ (b) $\frac{a^2}{(x-h)^2} + \frac{b^2}{(y-k)^2} = 1$

(c) $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ (d) $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

If $\sin A - \sqrt{6} \cos A = \sqrt{7} \cos A$, then the value of $\cos A + \sqrt{6} \sin A$ is

(a) $\sqrt{6} \sin A$ (b) $-\sqrt{7} \sin A$

(c) $\sqrt{6} \cos A$ (d) $\sqrt{7} \cos A$

If sin θ and cos θare roots of equation $ax^2 + bx + c = 0$ then

(a) $(a-c)^2 = b^2 - c^2$ (b) $(a-c)^2 = b^2 + c^2$

(c) $(a+c)^2 = b^2 - c^2$ (d) $(a+c)^2 = b^2 + c^2$

42. Maximum value of sin (cos$x$) is—

(a) sin 1 (b) 1 (c) $\sin\left(\frac{1}{\sqrt{2}}\right)$ (d) $\sin\left(\frac{\sqrt{3}}{2}\right)$

43. If $\cos x + \cos^2 x = 1$ then the value of $\sin^{12} x + 3 \sin^{10} x + 3 \sin^8 x + \sin^6 x - 1$ is

(a) 2 (b) 1 (c) −1 (d) 0

44. If $3 \sin\theta + 5 \cos\theta = 5$ then the value of $5 \sin\theta - 3 \cos\theta$ is

(a) 5 (b) 3 (c) 4 (d) None of these

45. If $\tan\theta + \sec\theta = p$ then the value of sec θ is

(a) $\frac{p^2+1}{p^2}$ (b) $\frac{p^2+1}{\sqrt{p}}$ (c) $\frac{p^2+1}{2p}$ (d) $\frac{p+1}{2p}$

46. If $\sin\theta - \cos\theta = \sqrt{2} \cos\theta$ then the value of $\sin\theta + \cos\theta$ is

(a) 2cosθ (b) 2sinθ (c) $\sqrt{2}\sin\theta$ (d) $\sqrt{2}\cos\theta$

47. If $\tan(\theta + 3\theta) \tan(2\theta + 3\theta) = 1$ then the value of $\sin(5\theta - 2\theta)$ is

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) $\frac{1}{\sqrt{2}}$

48. If sec x = cosec y then the value of cosec $(x + y)$ is

(a) 1 (b) 2 (c) $\sqrt{2}$ (d) undefined

49. If $\tan 2\theta = \cot(\theta - 18^\circ)$ then then value of $\sin\left(\frac{5\theta}{4}\right) + \cos\left(\frac{5\theta}{4}\right)$ is

(a) $\frac{1}{\sqrt{2}}$ (b) $2\sqrt{2}$ (c) 1 (d) $\sqrt{2}$

50. If $\sin\theta + \cos\theta = 1$ then the value of $\sin\theta - \cos\theta$ is
(a) 0 (b) $\pm\sqrt{2}$ (c) ± 1 (d) $\pm\frac{1}{\sqrt{2}}$

51. If $k = (1 - \sin\alpha)(1 - \sin\beta)(1 - \sin\gamma) = (1 + \sin\alpha)(1 + \sin\beta)(1 + \sin\gamma)$ then the value of k is
(a) $\pm \sin\alpha \sin\beta \sin\gamma$ (b) $\pm \cos\alpha \cos\beta \cos\gamma$
(c) $\pm \sec\alpha \sec\beta \sec\gamma$ (d) $\pm \text{cosec}\alpha\, \text{cosec}\beta\, \text{cosec}\gamma$

52. If $p = (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$ $= (\sec A + \tan A)(\sec B \times \tan B)(\sec C + \tan C)$ then value of p is
(a) $\pm \tan A \tan B \tan C$ (b) $\pm \sec A \sec B \sec C$
(c) ± 1 (d) None of these

53. **The value of** $(1 + \cot\theta + \text{cosec}\,\theta)(1 + \cot\theta - \text{cosec}\,\theta)$ is
(a) $2\tan\theta$ (b) $2\cot\theta$ (c) $2\sec\theta$ (d) $2\,\text{cosec}\,\theta$

54. **Which of the following is not equal to** $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$
(a) $\sec\theta + \tan\theta$ (b) $\frac{1}{\sec\theta - \tan\theta}$
(c) $\frac{1 + \sin\theta}{\cos\theta}$ (d) $\frac{1 - \sin\theta}{\cos\theta}$

55. If $m = \tan\theta + \sin\theta$ and $n = \tan\theta - \sin\theta$ then the value of $m^2 - n^2$ is
(a) $2\sqrt{mn}$ (b) $4\sqrt{mn}$ (c) $\sqrt{mn}$ (d) $\sqrt{2mn}$

56. The value of $\frac{\cos\theta}{\tan\theta + \sec\theta} - \frac{\cos\theta}{\tan\theta - \sec\theta}$ is
(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) $2\cos\theta$

57. $\sec^2\theta + \text{cosec}^2\theta$ is equal to which of the following ?
(a) $\sec\theta \tan\theta$ (b) $\sec\theta\, \text{cosec}\,\theta$
(c) $\sec^2\theta\, \text{cosec}^2\theta$ (d) $\sin^4\theta + \cos^4\theta$

58. The identity $(1 + \tan\theta - \sec\theta)(1 + \cot\theta - \text{cosec}\,\theta)$ equals
(a) 2 (b) 1 (c) $2\tan\theta$ (d) $2\cot\theta$

59. Which is equal to $\sec\theta.\text{cosec}\,\theta$?
(a) $\sin\theta + \cos\theta$ (b) $\tan\theta + \cot\theta$
(c) $2(\tan\theta + \cot\theta)$ (d) $2(\sin\theta + \cos\theta)$

60. The value of $\tan^4 A + \tan^2 A$ in terms of $\sec A$ is
(a) $\sec^4 A + \sec^2 A$ (b) $\sec^2 A + \sec^4 A$
(c) $\sec^4 A + \sec^2 A - 1$ (d) $\sec^4 A - \sec^2 A$

61. If $0 < \theta < 90°$ then what is the minimum value of $\sin^2\theta + \cos^2\theta + \tan^2\theta + \cot^2\theta + \sec^2\theta + \text{cosec}^2\theta$?
(a) 10 (b) 5 (c) 6 (d) 7

62. If $\cos^2\alpha + \cos^2\beta = 2$ then what is the value of $\tan^3\alpha + \sin^5\beta$?
(a) -1 (b) 0 (c) 1 (d) $\frac{1}{\sqrt{3}}$

63. If $A = \tan 11^\circ \tan 29^\circ$, $B = 2\cot 61^\circ \cot 79^\circ$ then which among the following is true ?

(a) $A = 2B$ (b) $A = -2B$ (c) $2A = B$ (d) $2A = -B$

64. On simplification $(\sec A - \cos A)^2 + (\operatorname{cosec} A - \sin A)^2 - (\cot A - \tan A)^2$ yields

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

65. The value of $\sin^2 1^\circ + \sin^2 5^\circ + \sin^2 9^\circ + \ldots\ldots + \sin^2 89^\circ$ is

(a) $11\frac{1}{2}$ (b) $11\sqrt{2}$ (c) 11 (d) $\frac{11}{\sqrt{2}}$

66. The numeric value of $\cot 18^\circ\left(\cot 72^\circ \cos^2 22^\circ + \frac{1}{\tan 72^\circ \sec^2 68^\circ}\right)$ is

(a) 1 (b) $\sqrt{2}$ (c) 3 (d) $\frac{1}{\sqrt{3}}$

67. If $\sin\alpha \sec(30^\circ + \alpha) = 1$ $(0 < \alpha < 60^\circ)$ then the value of $\sin\alpha + \cos 2\alpha$ is

(a) 1 (b) $\frac{2+\sqrt{3}}{2\sqrt{3}}$ (c) 0 (d) $\sqrt{2}$

68. If $\cos^4\theta - \sin^4\theta = \frac{2}{3}$, then the value of $2\cos^2\theta$ is

(a) $\frac{5}{3}$ (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

69. If θ is a positive acute angle and $\cos^2\theta + \cos^4\theta = 1$ then the value of $\tan^2\theta + \tan^4\theta$ is

(a) $\frac{3}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) 0

70. If θ is an acute angle and $\tan\theta + \cot\theta = 2$ then the value of $\tan^5\theta + \cot^{10}\theta$ is

(a) 1 (b) 2 (c) 3 (d) 4

71. $(\sin^2 1^\circ + \tan^2 3^\circ + \sin^2 5^\circ + \tan^2 7^\circ + \ldots\ldots + \tan^2 87^\circ + \sin^2 89^\circ)$ equals

(a) 23 (b) 22 (c) $22\frac{1}{2}$ (d) $23\frac{1}{2}$

72. If $2\cos\theta - \sin\theta = \frac{1}{\sqrt{2}}$, $(0^\circ < \theta < 90^\circ)$ then the value of $2\sin\theta + \cos\theta$ is

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) $\frac{3}{\sqrt{2}}$ (d) $\frac{\sqrt{2}}{3}$

73. If $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = 3$ then the value of $\sin^4\theta - \cos^4\theta$ is

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

74. If $\sec^2\theta + \tan^2\theta = 7$, then the value of θ is

(a) 60° (b) 30° (c) 0° (d) 90°

75. $(\sec x \cdot \sec y + \tan x \cdot \tan y)^2 - (\sec x \cdot \tan y\ \tan x \cdot \sec y)^2$ in its simplest form, is

(a) -1 (b) 0 (c) $\sec^2 x$ (d) 1

76. If $\frac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3$ and $0^\circ < \theta < 90^\circ$ then the value of θ is

(a) 30° (b) 45° (c) 60° (d) None of these

77. If $\sin\theta - \cos\theta = \frac{7}{13}$ and $0 < \theta < 90°$ then the value of $\sin\theta + \cos\theta$ is

(a) $\frac{17}{13}$ (b) $\frac{13}{17}$ (c) $\frac{1}{13}$ (d) $\frac{1}{17}$

Answer–11A

1. (b)	2. (b)	3. (b)	4. (a)	5. (c)	6. (a)	7. (c)	8. (d)
9. (a)	10. (c)	11. (c)	12. (c)	13. (d)	14. (b)	15. (b)	16. (b)
17. (b)	18. (b)	19. (b)	20. (d)	21. (b)	22. (a)	23. (a)	24. (a)
25. (b)	26. (d)	27. (c)	28. (a)	29. (c)	30. (a)	31. (c)	32. (a)
33. (b)	34. (b)	35. (d)	37. (c)	38. (b)	39. (b)	40. (b)	41. (d)
42. (a)	43. (d)	44. (b)	45. (c)	46. (c)	47. (a)	48. (a)	49. (d)
50. (c)	51. (b)	52. (c)	53. (b)	54. (d)	55. (b)	56. (c)	57. (c)
58. (a)	59. (b)	60. (d)	61. (d)	62. (b)	63. (c)	64. (c)	65. (a)
66. (a)	67. (a)	68. (a)	69. (b)	70. (b)	71. (c)	72. (c)	73. (c)
74. (a)	75. (d)	76. (c)	77. (a)				

Explanation

1. **(b)** $\frac{\tan x}{1+\sec x} - \frac{\tan x}{1-\sec x}$

$$= \frac{\tan x(1-\sec x - 1 - \sec x)}{1-\sec^2 x}$$

$$= \frac{\tan x(-2\sec x)}{-\tan^2 x} \qquad (\because \; -\sec^2 x + 1 = -\tan^2 x)$$

$$= \frac{-2\tan x \sec x}{-\tan^2 x} = \frac{\frac{2}{\cos x}}{\frac{\sin x}{\cos x}} = \frac{2}{\sin x} = 2\,\text{cosec}\,x$$

2. **(b)** $(\sin^4 x - \cos^4 x + 1)\,\text{cosec}^2 x$

$$= \{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) + 1\}\,\text{cosec}^2 x$$

$$[\because \; a^2 - b^2 = (a+b)(a-b)]$$

$$= \{\sin^2 x - \cos^2 x + 1\}\,\text{cosec}^2 x$$

$$= (\sin^2 x + \sin^2 x)\,\text{cosec}^2 x \qquad (\because \; 1 - \cos^2 x = \sin^2 x)$$

$$= 2\sin^2 x \frac{1}{\sin^2 x} = 2$$

3. **(b)** Given, $l\cos^2\theta + m\sin^2\theta = \frac{\cos^2\theta\left(\text{cosec}^2\theta + 1\right)}{\cot^2\theta}$ $(\because \; \text{cosec}^2\theta - 1 = \cot^2\theta)$

or, $l\cos^2\theta + m\sin^2\theta = \frac{\cos^2\theta\left(\text{cosec}^2\theta + 1\right)\cdot\sin^2\theta}{\cos^2\theta}$

or, $l\cos^2\theta + m\sin^2\theta = \sin^2\theta(\text{cosec}^2\theta + 1) = 1 + \sin^2\theta$

or $l\cos^2\theta + m\sin^2\theta = \sin^2\theta + \cos^2\theta + \sin^2\theta = 2\sin^2\theta + \cos^2\theta$ $(\because 1 = \sin^2\theta + \cos^2\theta)$

or $(l-1)\cos^2\theta = (2-m)\sin^2\theta$

or $\frac{l-1}{2-m} = \tan^2\theta$

(a) $\sec^2\theta - \tan^2\theta = 1$ is not true when $\theta = 90°$, because $\tan\theta$ and $\sec\theta$ are not defined at $= 90°$

(c) Now, $(\sin x - \cos x)^2 = (\sin^2 x + \cos^2 x) - 2\sin x\cos x = 1 - 2\left(\frac{1}{2}\right) = 0$

$\Rightarrow \sin x - \cos x = 0$

(a) Given, $\tan^2 y\,\text{cosec}^2 x - 1 = \tan^2 y$

$\Rightarrow \tan^2 y(\text{cosec}^2 x - 1) = 1$

$\Rightarrow \tan^2 y\cot^2 x = 1 \Rightarrow \tan^2 y = \frac{1}{\cot^2 x} = \tan^2 x$ $\quad\therefore\ x = y$

(c) Given, $\frac{\cos x}{1+\text{cosec}\,x} + \frac{\cos x}{\text{cosec}\,x - 1} = 2$

$\Rightarrow \frac{2\cos x\,\text{cosec}\,x}{\text{cosec}^2 x - 1} = 2 \quad \Rightarrow \frac{\cos x\,\text{cosec}\,x}{\cot^2 x} = 1$

$\Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}$

(d) Given, $\sin x + \sin y = a$ and $\cos x + \cos y = b$, squaring

$\Rightarrow \sin^2 x + \sin^2 y + 2\sin x\sin y = a^2$ ...(i)

and $\cos^2 x + \cos^2 y + 2\cos x\cos y = b^2$ ... (ii)

adding (i) and (ii)

$(\sin^2 x + \cos^2 x) + (\sin^2 y + \cos^2 y) + 2(\sin x\sin y + \cos x\cos y) = a^2 + b^2$

$\Rightarrow (\sin x\sin y + \cos x\cos y) = \frac{a^2 + b^2 - 2}{2}$

(a) Given, $\text{cosec}^4\alpha - \cot^4\alpha = 17$

$\Rightarrow (\text{cosec}^2\alpha - \cot^2\alpha)(\text{cosec}^2\alpha + \cot^2\alpha) = 17$

$\Rightarrow 1.\left(\frac{1}{\sin^2\alpha} + \frac{\cos^2\alpha}{\sin^2\alpha}\right) = 17$

$\Rightarrow \left(\frac{1+\cos^2\alpha}{\sin^2\alpha}\right) = 17$

$\Rightarrow 2 - \sin^2\alpha = 17\sin^2\alpha \quad \Rightarrow \sin^2\alpha = \frac{1}{9} \Rightarrow \sin\alpha = \frac{1}{3}$

10. (c) Given, $x + \frac{1}{x} = 2\cos\alpha$

$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 4\cos^2\alpha$

$\Rightarrow x^2 + \frac{1}{x^2} = 2(2\cos^2\alpha - 1) = 2(\cos^2\alpha - (1 - \cos^2\alpha)) = 2\cos^2\alpha - 2\sin^2\alpha$

11. **(c)** $b = \frac{1}{\cos\theta} + \frac{1}{\sin\theta} = \frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} = \frac{a}{\sin\theta\cos\theta}$

$\therefore \quad \frac{a}{b} = \sin\theta\cos\theta$

$\sin\theta + \cos\theta = a$ squaring

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = a^2$

or, $1 + 2\sin\theta\cos\theta = a^2$

or, $\sin\theta\cos\theta = \frac{a^2-1}{2}$

From (i) and (ii)

$$\frac{a}{b} = \frac{a^2-1}{2} \quad \Rightarrow 2a = b(a^2-1)$$

12. **(c)** Given, $\frac{\cos\theta}{1-\sin\theta} - \frac{\cos\theta}{1+\sin\theta} = 2$

$\Rightarrow \frac{\cos\theta + \sin\theta\cos\theta - \cos\theta + \cos\theta\sin\theta}{1-\sin^2\theta} = 2$

$\Rightarrow 2\sin\theta\cdot\cos\theta = 2\cos^2\theta \Rightarrow 2\sin\theta = 2\cos\theta$

$\Rightarrow \tan\theta = 1 \qquad \Rightarrow \tan\theta = \frac{\pi}{4}$

13. **(d)** Given, $7\cos^2\theta + 3\sin^2\theta = 4$

$\Rightarrow 7(1-\sin^2\theta) + 3(\sin^2\theta) = 4$

$\Rightarrow 7 - 4\sin^2\theta = 4$

$\Rightarrow 4\sin^2\theta = 3$

$\Rightarrow \sin\theta = \frac{\sqrt{3}}{2} \qquad \Rightarrow \theta = 60° \qquad \therefore \quad \tan\theta = \tan 60° = \sqrt{3}$

14. **(b)** $[(1-\sin^2\theta)\sec^2\theta + \tan^2\theta](\cos^2\theta + 1)$

$= (\sec^2\theta - \tan^2\theta + \tan^2\theta)(\cos^2\theta + 1)$

$= 1 + \sec^2\theta > 1 + 1 > 2 \qquad [\because \sec^2\theta > 1, 0 < \theta < 90°]$

15. **(b)** $\sin^2 15° + \sin^2 20° + \sin^2 25° + \ldots + \sin^2 75°$

$= \sin^2(90° - 75°) + \sin^2(90° - 70°) + \ldots + \sin^2(90° - 15°)$

$= \cos^2 75° + \cos^2 70° + \ldots + \cos^2 15°$

16. **(b)** $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sqrt{\frac{(1+\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}}$

$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} = \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$

$= \frac{1+\sin\theta}{\cos\theta}$

$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \sec\theta + \tan\theta$

(a) If $0^\circ < \theta < 90^\circ$

and $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 2$

$\Rightarrow \sin^2\theta + \cos^2\theta = 2\sin\theta\cdot\cos\theta$

$\Rightarrow (\sin^2\theta + \cos^2\theta - 2\sin\theta\cdot\cos\theta) = 0$

$\Rightarrow (\sin\theta - \cos\theta)^2 = 0$

$\Rightarrow \sin\theta - \cos\theta = 0 \quad \Rightarrow \tan\theta = 1 = \tan\frac{\pi}{4}$

$\Rightarrow \theta = \frac{\pi}{4}$ or, 45°

(b) $\sin 3\theta = \cos(\theta - 2^\circ)$

$\Rightarrow \sin 3\theta = \sin(90^\circ - (\theta - 2^\circ))]$

$\Rightarrow 3\theta = 90^\circ - \theta + 2^\circ$

$\Rightarrow 4\theta = 92^\circ \quad \Rightarrow \theta = \frac{92}{4} = 23^\circ$

(b) $\frac{\sin^6\theta - \cos^6\theta}{\sin^2\theta - \cos^2\theta} = \frac{(\sin^2\theta)^3 - (\cos^2\theta)^3}{\sin^2\theta - \cos^2\theta}$

$= \frac{(\sin^2\theta - \cos^2\theta)(\sin^4\theta + \cos^4\theta + \sin^2\theta\cos^2\theta)}{\sin^2\theta - \cos^2\theta}$

$= \sin^4\theta + \cos^4\theta + \sin^2\theta\cos^2\theta$

$= \sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta$

$= (\sin^2\theta + \cos^2\theta)^2 - \sin^2\theta\cos^2\theta$

$= 1 - \sin^2\theta\cos^2\theta$

1. (d) $\sin^4 x + \sin^2 x = 1$

$\Rightarrow \sin^4 x = 1 - \sin^2 x = \cos^2 x \quad \ldots \text{(i)}$

$\therefore \cot^4 x + \cot^2 x = \cot^2 x(1 + \cot^2 x) = \cot^2 x\cdot\text{cosec}^2 x$

$\frac{\cos^2 x}{\sin^2 x}\cdot\frac{1}{\sin^2 x} = \frac{\cos^2 x}{\sin^4 x} = 1 \quad (\because \sin^4 x = \cos^2 x)$

1. (b) Given,

$x\cos\theta + y\sin\theta = 2 \quad \ldots \text{(i)}$

and $x\cos\theta - y\sin\theta = 0 \quad \ldots \text{(ii)}$

Solving equation (i) and (ii),

$\Rightarrow x\cos\theta = 1$ and $y\sin\theta = 1$

$\Rightarrow \cos\theta = \frac{1}{x}$ and $\sin\theta = \frac{1}{y}$

$\because \cos^2\theta + \sin^2\theta = 1$

$\therefore \frac{1}{x^2} + \frac{1}{y^2} = 1$

22. (a) $\sin A(1+\tan A)+\cos A(1+\cot A)$

$$=\sin A\left(\frac{\sin A+\cos A}{\cos A}\right)+\cos A\left(\frac{\cos A+\sin A}{\sin A}\right)$$

$$=(\sin A+\cos A)\left(\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}\right)$$

$$=(\sin A+\cos A)\left(\frac{\sin^2 A+\cos^2 A}{\sin A\cos A}\right)$$

$$=\frac{\sin A}{\sin A\cos A}+\frac{\cos A}{\sin A\cos A}=\sec A+\operatorname{cosec} A$$

23. (a) $\because\ \cos^2\theta-\sin^2\theta=\frac{1}{2}$

or, $(1-\sin^2\theta)-\sin^2\theta=\frac{1}{2}$

or, $1-\frac{1}{2}=2\sin^2\theta$

or, $\sin^2\theta=\frac{1}{4}$ or $\theta=30^\circ$

24. (a) $\because\ (3\sin\theta+4\cos\theta)=5$

Squaring both sides $(3\sin\theta+4\cos\theta)^2=25$

$\Rightarrow\ 9\sin^2\theta+16\cos^2\theta+24\sin\theta\cos\theta=25$

$\Rightarrow\ 9(1-\cos^2\theta)+16(1-\sin^2\theta)+24\sin\theta\cos\theta=25$

$\Rightarrow\ 9-9\cos^2\theta+16-16\cos^2\theta+24\sin\theta\cos\theta=25$

$\Rightarrow\ 9\cos^2\theta+16\sin^2\theta-24\sin\theta\cos\theta=0$

$\Rightarrow\ (3\cos\theta-4\sin\theta)^2=0 \qquad \Rightarrow\ 3\cos\theta-4\sin\theta=0$

25. (b) $\dfrac{(1-\sin A\cos A)(\sin^2 A-\cos^2 A)}{\cos A(\sec A-\operatorname{cosec} A)(\sin^3 A+\cos^3 A)}$

$$=\frac{(1-\sin A\cos A)(\sin^2 A-\cos^2 A)}{\cos A\left(\frac{1}{\cos A}-\frac{1}{\sin A}\right)(\sin A+\cos A)\left(\sin^2 A+\cos^2 A-\sin A\cos A\right)}$$

$$=\frac{(1-\sin A\cos A)(\sin A-\cos A)}{\cos A(\sin A-\cos A)(\sin A+\cos A)(1-\sin A\cos A)}=\sin A$$

26. (d) (i) $\dfrac{\cos\theta}{1-\sin\theta}+\dfrac{\cos\theta}{1+\sin\theta}$

$$=\frac{\cos\theta(1+\sin\theta+1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}=\frac{2\cos\theta}{1-\sin^2\theta}=\frac{2\cos\theta}{\cos^2\theta}=\frac{2}{\cos\theta}$$

(ii) $\dfrac{\cos\theta}{1+\operatorname{cosec}\theta}+\dfrac{\cos\theta}{\operatorname{cosec}\theta-1}$

$$=\frac{\cos\theta[\operatorname{cosec}\theta-1+\operatorname{cosec}\theta+1]}{(\operatorname{cosec}\theta+1)(\operatorname{cosec}\theta-1)}$$

$$=\frac{2\cos\theta\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta-1}=\frac{2\cot\theta}{\cot^2\theta}=\frac{2}{\cot\theta}$$

Neither 1 nor 2 is independent θ

[illegible] $a\cos\theta - b\sin\theta = c$,

Squaring

$a^2\cos^2\theta + b^2\sin^2\theta - 2ab\cos\theta.\sin\theta = c^2$

$a^2(1-\sin^2\theta) + b^2(1-\cos^2\theta) - 2ab\cos\theta.\sin\theta = c^2$

$-a^2\sin^2\theta - b^2\cos^2\theta - 2ab\cos\theta.\sin\theta = c^2 - a^2 - b^2$

$a^2\sin^2\theta + b^2\cos^2\theta + 2ab\cos\theta.\sin\theta = a^2 + b^2 - c^2$

$(a\sin\theta + b\cos\theta)^2 = a^2 + b^2 - c^2$

$a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$

[illegible] (a) $(\tan\alpha - \cot\alpha)^2 \geq 0$

$\tan^2\alpha + \cot^2\alpha - 2\tan\alpha\cot\alpha \geq 0$

$\tan^2\alpha + \cot^2\alpha - 2 \geq 0$ $\quad (\because \tan\alpha\cot\alpha = 1)$

[illegible] (c) Since values of $\sin^2\theta$ and $\cos^2\theta$ lie between and 0 and 1, therefore its value decreases as power of $\sin\theta$ and $\cos\alpha$ increases.

Hence $\sin^8\theta \leq \sin^2\theta$ and $\cos^{14}\theta \leq \cos^2\theta$; adding

$\sin^8\theta + \cos^{14}\theta \leq \sin^2\theta + \cos^2\theta$ or, $\sin^8\theta + \cos^{14}\theta \leq 1$

[illegible] (a) $P = \frac{1}{2}\sin^2\theta + \frac{1}{3}\cos^2\theta = \frac{3\sin^2\theta + 2\cos^2\theta}{6} = \frac{\sin^2\theta + 2}{6}$

$\because 0 \leq \sin^2\theta \leq 1$

$\therefore$ When $\sin^2\theta = 0, P = \frac{2}{6} = \frac{1}{3}$, $\quad \sin^2\theta = 1, P = \frac{3}{6} = \frac{1}{2}$

[illegible] (c) $\because$ Minimum value of $\cos\theta$ is -1

$\therefore$ Minimum value of $5\cos\theta + 12 = -5 + 12 = 7$

[illegible] (a) $a\sin\theta = b\cos\theta \Rightarrow \tan\theta = \frac{b}{a}$

$\therefore \sin\theta = \frac{b}{\sqrt{a^2+b^2}}$ and $\cos\theta = \frac{a}{\sqrt{a^2+b^2}}$

Given relation $a\sin^3\theta + b\cos^3\theta = \sin\theta\cdot\cos\theta$

$\Rightarrow \frac{ab^3}{\left(\sqrt{a^2+b^2}\right)^3} + \frac{ba^3}{\left(\sqrt{a^2+b^2}\right)^3} = \frac{ba}{\left(\sqrt{a^2+b^2}\right)^2}$

$\Rightarrow \frac{ab\left(b^2+a^2\right)}{\left(\sqrt{a^2+b^2}\right)^3} = \frac{ab}{\left(\sqrt{a^2+b^2}\right)^2}$

or, $\frac{b^2+a^2}{\sqrt{a^2+b^2}} = 1$

$\Rightarrow \sqrt{a^2+b^2} = 1 \Rightarrow a^2 + b^2 = 1$

33. (b) $\sin^2 17.5^\circ + \sin^2 72.5^\circ$

$= \sin^2 17.5 + \cos^2 17{,}5^\circ = 1 = \tan^2 45^\circ$

34. (b) ...

$\Rightarrow 360° = \frac{44}{72} \times 360 = 220$ meters

Perimeter $2\pi r = 220$ meters

$\therefore \quad r = \frac{220}{2\pi} = 110 \times \frac{7}{22} = 35$ meters

35. (d) $(\sin\theta + \cos\theta)(\tan\theta + \cot\theta) = (\sin\theta + \cos\theta)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$

$$= (\sin\theta + \cos\theta)\left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)$$

$$= \frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta} = \sec\theta + \text{cosec}\theta$$

37. (c) Sum of roots $= \sec\alpha + \text{cosec}\alpha = p$ ...(i)

Product of roots $= \sec\alpha\ \text{cosc}\alpha = q$ ...(ii)

from (i) $\frac{1}{\cos\alpha} + \frac{1}{\sin\alpha} = p \Rightarrow \frac{\sin\alpha + \cos\alpha}{\cos\alpha\sin\alpha} = p$ ...(iii)

from (ii) $\frac{1}{\cos\alpha\sin\alpha} = q \Rightarrow \sin\alpha\cos\alpha = \frac{1}{q}$ ...(iv)

from (iii) and (iv) $\sin\alpha + \cos\alpha = \frac{p}{q}$ ...(v)

but $(\sin\alpha + \cos\alpha)^2 = \sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha$

$\Rightarrow \left(\frac{p}{q}\right)^2 = 1 + \frac{2}{q} \quad \Rightarrow \frac{p^2}{q^2} = \frac{q+2}{q} \quad \Rightarrow p^2 = q^2 + 2q = q(q+2$

38. (b) $\sec\theta - \tan\theta = \sqrt{(\sec\theta + \tan\theta)^2 - 4\sec\theta\tan\theta}$

$$= \sqrt{\left(\frac{-b}{a}\right)^2 - \frac{4c}{a}} = \sqrt{\frac{b^2 - 4ac}{a^2}} = \frac{\sqrt{b^2 - 4ac}}{a}$$

39. (b) $x - h = a\sec\theta \Rightarrow \cos\theta = \frac{a}{x-h}$

$y - k = b\,\text{cosec}\theta \Rightarrow \sin\theta = \frac{b}{y-k}$

$\because \quad \cos^2\theta + \sin^2\theta = 1$

$\therefore \quad \left(\frac{a}{x-h}\right)^2 + \left(\frac{b}{y-k}\right)^2 = 1$

40. (b) $\sin A - \sqrt{6}\cos A = \sqrt{7}\cos A$ squaring both sides

$\sin^2 A + 6\cos^2 A - 2\sqrt{6}\sin A\cos A = 7\cos^2 A$

$\Rightarrow \sin^2 A = \cos^2 A + 2\sqrt{6}\sin A\cos A$

Adding $6\sin^2 A$ both sids

$7\sin^2 A = \cos^2 A + 2\sqrt{6}\sin A\cos A + 6\sin^2 A$

or, $7\sin^2 A = (\cos A + \sqrt{6}\sin A)^2$

$\therefore \quad \cos A + \sqrt{6}\sin A = \pm\sqrt{7}\sin A$

(a) Sum of roots $\sin\theta + \cos\theta = \frac{-b}{a}$

product of roots $\sin\theta \cdot \cos\theta = \frac{c}{a}$

$(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cdot\cos\theta$

$\therefore \left(\frac{-b}{a}\right)^2 = 1 + 2\frac{c}{a}$

or, $a^2 - b^2 + 2ac = 0$

or, $a^2 + 2ac = b^2$

Adding c^2, $(a + c)^2 = b^2 + c^2$

42. (a) $\because$ Maximum value of $\cos x$ is 1

$\therefore$ Maximum value of $\sin(\cos x)$ is $\sin 1$

43. (d) Given $\cos x + \cos^2 x = 1$

$\Rightarrow \cos x = 1 - \cos^2 x = \sin^2 x$ ... (i)

Now, $\sin^{12}x + 3\sin^{10}x + 3\sin^8 x + \sin^6 x - 1$

$= \cos^6 x + 3\cos^5 x + 3\cos^4 x + \cos^3 x - 1$ $(\because \sin^2 x = \cos x)$

$= (\cos^2 x + \cos x)^3 - 1 = 1^3 - 1 = 0$ $(\because \cos^2 x + \cos x = 1)$

44. (b) Given $3\sin\theta + 5\cos\theta = 5$, Squaring

$9\sin^2\theta + 25\cos^2\theta + 30\sin\theta\cos\theta = 25$

or, $9(1 - \cos^2\theta) + 25(1 - \sin^2\theta) + 30\sin\theta\cos\theta = 25$

or, $9 + 25 - (9\cos^2\theta + 25\sin^2\theta - 30\sin\theta\cos\theta) = 25$

or, $9 = (5\sin\theta - 3\cos\theta)^2$

$\therefore 5\sin\theta - 3\cos\theta = 3$

45. (c) $\because \sec^2\theta - \tan^2\theta = 1$

$\therefore (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$\therefore (\sec\theta - \tan\theta)\, p = 1$

$\therefore \sec\theta - \tan\theta = \frac{1}{p}$ ... (i)

and $\sec\theta + \tan\theta = p$ ... (ii)

Adding $2\sec\theta = \frac{1}{p} + p = \frac{1 + p^2}{p}$ or, $\sec\theta = \frac{1 + p^2}{2p}$

46. (c) $(\sin\theta - \cos\theta) = \sqrt{2}\cos\theta$

Squaring $\sin^2\theta + \cos^2\theta - 2\sin\theta.\cos\theta = 2\cos^2\theta$

or, $\sin^2\theta = \cos^2\theta + 2\sin\theta\cos\theta$

Adding $\sin^2\theta$ both sids

$2\sin^2\theta = \cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta$

or, $2\sin^2\theta = (\sin\theta + \cos\theta)^2$

$\therefore \sin\theta + \cos\theta = \sqrt{2}\sin\theta$

47. (a) Given $\tan 4\theta \tan 5\theta = 1$

or, $\tan 4\theta = \frac{1}{\tan 5\theta} = \cot 5\theta$

or, $\tan 4\theta = \tan(90^\circ - 5\theta)$

or, $4\theta = 90^\circ - 5\theta$

or, $9\theta = 90^\circ$

or, $\theta = 10^\circ$

$\therefore \sin(5\theta - 2\theta) = \sin 3\theta = \sin 30^\circ = \frac{1}{2}$

48. (a) $\sec x = \text{cosec } y = \sec(90^\circ - y)$

$\therefore x = 90^\circ - y \Rightarrow x + y = 90^\circ$

$\therefore \text{cosec}(x + y) = \text{cosec } 90^\circ = 1$

49. (d) $\tan 2\theta = \cot(\theta - 18^\circ) = \tan(90^\circ - (\theta - 18^\circ))$

$\therefore 2\theta = 90^\circ - \theta + 18^\circ$

or, $3\theta = 108^\circ$

or, $\theta = 36^\circ$ or, $\frac{5\theta}{4} = 45^\circ$

Hence $\sin\left(\frac{5\theta}{4}\right) + \cos\left(\frac{5\theta}{4}\right) = \sin 45^\circ + \cos 45^\circ = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$

50. (c) Solve as Question number 46

51. (b) $k^2 = (1 - \sin\alpha)(1 - \sin\beta)(1 - \sin\gamma)(1 + \sin\alpha)(1 + \sin\beta)(1 + \sin\gamma)$

$= (1 - \sin^2\alpha)(1 - \sin^2\beta)(1 - \sin^2\gamma)$

$= \cos^2\alpha \cos^2\beta \cos^2\gamma$

52. (c) Solve as Question number 51 and use $\sec^2\theta - \tan^2\theta = 1$

53. (b) Required value $= (1 + \cot\theta)^2 - (\text{cosec}^2\theta)$

$= 1 + \cot^2\theta + 2\cot\theta - \text{cosec}^2\theta$

$= \text{cosec}^2\theta + 2\cot\theta - \text{cosec}^2\theta = 2\cot\theta$

54. (d) See solved example 8.

55. (b) Do as in solved example 3

56. (c) Take L C M and apply $\sec^2\theta - \tan^2\theta = 1$

57. (c) $\sec^2\theta + \text{cosec}^2\theta = \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} = \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta \sin^2\theta}$

$= \frac{1}{\cos^2\theta \sin^2\theta} = \sec^2\theta \, \text{cosec}^2\theta$

58. (a) See solved example -4

59. (b) $\sec\theta \, \text{cosec}\theta = \frac{1}{\cos\theta \sin\theta} = \frac{\cos^2\theta + \sin^2\theta}{\cos\theta \sin\theta} = \cot\theta + \tan\theta$

60. (d) $\tan^4 A + \tan^2 A = (\sec^2 A - 1)^2 + (\sec^2 A - 1)$

$= \sec^4 A - 2\sec^2 A + 1 + \sec^2 A - 1$

$= \sec^4 A - \sec^2 A$

61. (d) Givan expression $= \sin^2\theta + \cos^2\theta + \tan^2\theta + \cot^2\theta + \sec^2\theta + \text{cosec}^2\theta$

$$= (\sin^2\theta + \cos^2\theta) + \tan^2\theta + \cot^2\theta + (1+\tan^2\theta)+(1+\cot^2\theta)$$

$$= 1 + 1 + 1 + 2(\tan^2\theta + \cot^2\theta)$$

$$= 3 + 2\{(\tan\theta - \cot\theta)^2 + 2\}$$

But $(\tan\theta - \cot\theta)^2 \geq 0$

$\therefore$ Given expression $\geq 3 + 2(0+2) \Rightarrow$ Given expression ≥ 7

62. (b) $\cos^2\alpha + \cos^2\beta = 2$

It is possible only when each of $\cos^2\alpha$ and $\cos^2\beta$ is equal to 1 as their individual value cannot exeed 1.

$\therefore\ \cos^2\alpha = 1$ and $\cos^2\beta = 1 \quad \Rightarrow \quad \alpha = \beta = 0^\circ$

Hence $\quad \tan^3\alpha + \sin^5\beta$

$$= (\tan 0^\circ)^3 + (\sin 0^\circ)^5 = 0$$

63. (c) $A = \tan 11^\circ \cdot \tan 29^\circ \quad \ldots$ (i)

$$B = 2\cot 61^\circ \cdot \cot 79^\circ$$

$$= 2\cot(90^\circ - 29^\circ)\cdot\cot(90^\circ - 11^\circ)$$

$$= 2\tan 29^\circ \cdot \tan 11^\circ = 2\tan 11^\circ \cdot \tan 29 = 2A$$

64. (c) $(\sec A - \cos A)^2 + (\text{cosec} A - \sin A)^2 - (\cot A - \tan A)^2$

$$= \sec^2 A + \cos^2 A - 2\sec A\cdot\cos A + \text{cosec}^2 A + \sin^2 A - 2\sin A\cdot\text{cosec} A - \cot^2 A - \tan^2 A + 2\cot A\cdot\tan A$$

$$= \sin^2 A + \cos^2 A + \sec^2 A - \tan^2 A + \text{cosec}^2 A - \cot^2 A - 2 - 2 + 2$$

$$= 1 + 1 + 1 - 2 = 1$$

65. (a) $\sin^2 1^\circ + \sin^2 5^\circ + \sin^2 9^\circ + \ldots + \sin^2 89^\circ$

$$= \sin^2 1^\circ + \sin^2 89^\circ + \sin^2 5^\circ + \sin^2 85^\circ + \ldots + \sin^2 41^\circ + \sin^2 49^\circ + \sin^2 45^\circ$$

$$= \sin^2 1^\circ + \sin^2(90-1)^\circ + \sin^2 5^\circ + \sin^2(90-5)^\circ + \ldots\ldots + \sin^2 41^\circ + \sin^2(90^\circ - 41^\circ) + \sin^2 45^\circ$$

$$= (\sin^2 1^\circ + \cos^2 1^\circ) + (\sin^2 5^\circ + \cos^2 5^\circ) + \ldots\ldots + \sin^2 45^\circ$$

$$= 1 + 1 + \ldots\ldots 11 \text{ to term} + \left(\frac{1}{\sqrt{2}}\right)^2 = 11 + \frac{1}{2} = 11\frac{1}{2}$$

66. (a) $\cot 18^\circ\left(\cot 72^\circ \cos^2 22^\circ + \dfrac{1}{\tan 72^\circ \sec^2 68^\circ}\right)$

$$= \cot 18^\circ(\cot 72^\circ \cos^2 22^\circ + \cot 72^\circ \cos^2 68^\circ)$$

$$= \cot 18^\circ \cot 72^\circ[\cos^2 22^\circ + \cos^2(90-22)^\circ]$$

$$= \cot 18^\circ \cot 72^\circ[\cos^2 22^\circ + \sin^2 22^\circ]$$

$$= \cot 18^\circ \cot(90-18)^\circ \times 1 = \cot 18^\circ \tan 18^\circ = 1$$

67. (a) ∵ $\frac{\ldots}{\cos(30^\circ+\alpha)} = 1$

$\Rightarrow \frac{\sin\alpha}{\sin(90^\circ - 30 - \alpha)} = 1$

$\Rightarrow \frac{\sin\alpha}{\sin(60^\circ - \alpha)} = 1$

$\Rightarrow \sin\alpha = \sin(60^\circ - \alpha)$

$\Rightarrow 2\alpha = 60^\circ \quad \therefore \alpha \Rightarrow 30^\circ$

$\therefore \sin\alpha + \cos 2\alpha = \sin 30^\circ + \cos 60^\circ = \frac{1}{2} + \frac{1}{2} = 1$

68. (a) ∵ $\cos^4\theta - \sin^4\theta = \frac{2}{3}$

$\Rightarrow (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta) = \frac{2}{3}$

$\Rightarrow \cos^2\theta - \sin^2\theta = \frac{2}{3}$

$\Rightarrow \cos^2\theta - (1 - \cos^2\theta) = \frac{2}{3}$

$\Rightarrow 2\cos^2\theta - 1 = \frac{2}{3} \qquad \Rightarrow 2\cos^2\theta = \frac{2}{3} + 1 = \frac{5}{3}$

69. (b) ∵ $\cos^2\theta + \cos^4\theta = 1$

$\Rightarrow \cos^2\theta + \cos^4\theta = \sin^2\theta + \cos^2\theta$

$\therefore \cos^4\theta = \sin^2\theta \Rightarrow \cos^2\theta = \frac{\sin^2\theta}{\cos^2\theta} \Rightarrow \cos^2\theta = \tan^2\theta$

Hence $\tan^2\theta + \tan^4\theta = \cos^2\theta + \cos^4\theta = 1$

70. (b) ∵ $\tan\theta + \cot\theta = 2$

$\Rightarrow \tan\theta + \frac{1}{\tan\theta} = 2$

$\Rightarrow \tan^2\theta + 1 = 2\tan\theta \qquad \Rightarrow (\tan\theta - 1)^2 = 0$

$\Rightarrow \tan\theta = 1 \qquad \Rightarrow \cot\theta = 1$

$\therefore \tan^5\theta = \cot^{10}\theta = 1 + 1 = 2$

71. (c) $(\sin^2 1^\circ + \sin^2 89^\circ) + (\tan^2 3^\circ + \tan^2 87^\circ) + (\sin^2 5^\circ + \sin^2 85^\circ) + \ldots\ldots (\tan^2 43^\circ + \tan^2 47^\circ) + \sin^2 45^\circ$

$= 1 + 1 \ldots\ldots$ to 22 terms $+ \frac{1}{2}$

($\because \sin^2 1^\circ + \sin^2 89^\circ = \sin^2 1^\circ + \cos^2 1^\circ = 1$ etc)

$= 22 + \frac{1}{2} = 22\frac{1}{2}$

72. (c) $2\cos\theta - \sin\theta = \frac{1}{\sqrt{2}}$, Squaring $4\cos^2\theta + \sin^2\theta - 4\cos\theta\sin\theta = \frac{1}{2}$

$\Rightarrow 4(1 - \sin^2\theta) + (1 - \cos^2\theta) - 4\cos\theta\cdot\sin\theta = \frac{1}{2}$

$\Rightarrow 5 - \frac{1}{2} = 4\sin^2\theta + \cos^2\theta + 4\cos\theta\cdot\sin\theta$

$\Rightarrow \frac{9}{2} = (2\sin\theta + \cos\theta)^2 \qquad \therefore 2\sin\theta + \cos\theta = \sqrt{\frac{9}{2}} = \frac{3}{\sqrt{2}}$

73. (c) $\because \dfrac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = 3$

$\Rightarrow \sin\theta + \cos\theta = 3\sin\theta - 3\cos\theta$

$\Rightarrow 4\cos\theta = 2\sin\theta$

$\Rightarrow \tan\theta = 2 = \dfrac{p}{b}$

$\therefore \sin^4\theta - \cos^4\theta$

$$= \left(\frac{p}{h}\right)^4 - \left(\frac{b}{h}\right)^4 = \frac{p^4 - b^4}{h^4} = \frac{p^4 - b^4}{\left(p^2 + b^2\right)^2} = \frac{16-1}{5^2} = \frac{15}{25} = \frac{3}{5}$$

74. (a) $\because \sec^2\theta + \tan^2\theta = 7$

$\Rightarrow 1 + \tan^2\theta + \tan^2\theta = 7$

$\Rightarrow 2\tan^2\theta = 7 - 1 = 6$

$\Rightarrow \tan^2\theta = 3$

$\Rightarrow \tan\theta = \sqrt{3} \qquad \therefore \theta = 60°$

75. (d) $(\sec x\cdot\sec y + \tan x\cdot\tan y)^2 - (\sec x\cdot\tan y + \tan x\cdot\sec y)^2$

$$= \left(\frac{1}{\cos x\cdot\cos y} + \frac{\sin x\cdot\sin y}{\cos x\cdot\cos y}\right)^2 - \left(\frac{1\cdot\sin y}{\cos x\cdot\cos y} + \frac{\sin x}{\cos x\cdot\cos y}\right)^2$$

$$= \left(\frac{1 + \sin x\cdot\sin y}{\cos x\cdot\cos y}\right)^2 - \left(\frac{\sin x + \sin y}{\cos x\cdot\cos y}\right)^2$$

$$= \frac{1 + \sin^2 x\sin^2 y + 2\sin x\cdot\sin y - \sin^2 x - \sin^2 y - 2\sin x\cdot\sin y}{\cos^2 x\cdot\cos^2 y}$$

$$= \frac{1 + \sin^2 x\cdot\sin^2 y - \sin^2 x - \sin^2 y}{\cos^2 x\cdot\cos^2 y} = \frac{\left(1 - \sin^2 x\right)\left(1 - \sin^2 y\right)}{\cos^2 x\cos^2 y}$$

$$= \frac{\cos^2 x\cdot\cos^2 y}{\cos^2 x\cdot\cos^2 y} = 1$$

76. (c) $\because \dfrac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3 \quad \Rightarrow \cos^2\theta = 3\cot^2\theta - 3\cos^2\theta$

$\Rightarrow 4\cos^2\theta = \dfrac{3\cos^2\theta}{\sin^2\theta} \quad \Rightarrow \sin\theta = \dfrac{\sqrt{3}}{2} \quad \therefore \theta = 60°$

77. (a) $(\sin\theta - \cos\theta)^2 = \dfrac{49}{169}$

$\Rightarrow \sin^2\theta + \cos^2\theta - 2\sin\theta\cdot\cos\theta = \dfrac{49}{169} \Rightarrow 2\sin\theta\cos\theta = 1 - \dfrac{49}{169} = \dfrac{120}{169}$

Now $(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cdot\cos\theta = 1 + \dfrac{120}{169} = 289$

$\Rightarrow \sin\theta + \cos\theta = \sqrt{\dfrac{287}{169}} = \dfrac{17}{13}$

1. If $4x = \sec\theta$ and $\frac{4}{x} = \tan\theta$ then the value of $8\left(x^2 - \frac{1}{x^2}\right)$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{16}$ (d) $\frac{1}{8}$

[SSC Tier-I 2012]

2. $2 - \cos^2\theta = 3\sin\theta\cos\theta$, $\sin\theta \neq \cos\theta$ then the value of $\tan\theta$ is

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 0

[SSC Tier-I 2012]

3. If $\sin\theta + \cos\theta = \sqrt{2}\cos(90° - \theta)$ then the value of $\cot\theta$ is

(a) $\sqrt{2}$ (b) $\sqrt{2} - 1$ (c) $\sqrt{2} + 1$ (d) 0

[SSC Tier-I 2012]

4. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$; $\sin\theta \neq 0$, $\cos\theta \neq 0$ then the value of $x^2 + y^2$ is

(a) 1 (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2}$

[SSC Tier-I 2012]

5. If A and B are complementary angle then the value of $\sin A\cos B + \cos A\sin B - \tan A\tan B + \sec^2 A - \cot^2 B$ is

(a) 1 (b) -1 (c) 2 (d) 0

[SSC Tier-I 2012]

6. Minimum value of $2\sin^2\theta + 3\cos^2\theta$ is

(a) 1 (b) 2 (c) 3 (d) 5

[SSC Tier-I 2012]

7. $\sec^4\theta - \sec^2\theta$ equals

(a) $\cos^4\theta - \cos^2\theta$ (b) $\cos^2\theta - \cos^4\theta$ (c) $\tan^2\theta - \tan^4\theta$ (d) $\tan^2\theta + \tan^4\theta$

[SSC Tier-I 2012]

8. If $\cos A + \cos^2 A = 1$ then the value $\sin^2 A + \sin^4 A$ is

(a) 0 (b) -1 (c) 1 (d) $\frac{1}{2}$

[SSC Tier-I 2012]

9. If $\sec\theta - \operatorname{cosec}\theta = 0$ then the value of $(\sec\theta + \operatorname{cosec}\theta)$ is

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{2}{\sqrt{3}}$ (c) 0 (d) $2\sqrt{2}$

[SSC Tier-I 2012]

10. If $P\sin\theta = \sqrt{3}$ and $P\cos\theta = 1$ then the value of P is

(a) $\frac{1}{2}$ (b) $\frac{2}{\sqrt{3}}$ (c) $\frac{-1}{\sqrt{3}}$ (d) 2

[SSC Tier-I 2012]

11. If $u_n = \cos^n\alpha + \sin^n\alpha$ then the value of $2u_6 - 3u_4 + 1$ is

(a) 1 (b) 4 (c) 6 (d) 0

12. If $\sin(x + y) = \cos[3(x + y)]$ then the value of $\tan[2(x + y)]$ is

(a) $\sqrt{3}$ (b) 1 (c) 0 (d) $\frac{1}{\sqrt{3}}$

[SSC Tier-I 2012]

13. The value of $(1 + \sec 20^\circ + \cot 70^\circ)(1 - \operatorname{cosec} 20^\circ + \tan 70^\circ)$ is
(a) 0 (b) −1 (c) 2 (d) 1
[SSC Tier-I 2012]

14. If $0 \le \alpha \le \frac{\pi}{2}$ and $2\sin\alpha + 15\cos^2\alpha = 7$ then then value of $\cot\alpha$ is
(a) $\frac{1}{2}$ (b) $\frac{5}{4}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$
[SSC Tier-I 2012]

15. The value of θ $[0^\circ < \theta < 90^\circ]$, for which $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$, is
(a) 30° (b) 45° (c) 60° (d) None of these
[SSC Tier-I 2012]

16. If $\sec\theta + \tan\theta = 2$, then the value of $\sec\theta$ is
(a) $\frac{7}{4}$ (b) $\frac{7}{2}$ (c) $\frac{5}{2}$ (d) $\frac{5}{4}$
[SSC Tier-I 2012]

17. If $\tan 2\theta \cdot \tan 4\theta = 1$ then the value of $\tan 3\theta$ is
(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) 2
[SSC Tier-I 2012]

18. If $\cos\theta + \sec\theta = \sqrt{3}$, then the value of $\cos^3\theta + \sec^3\theta$ is
(a) 0 (b) 1 (c) −1 (d) $\sqrt{3}$
[SSC Tier-I 2012]

19. If $2y\cos\theta = x\sin\theta$ and $2x\sec\theta - y\operatorname{cosec}\theta = 3$, then the relation between x and y is
(a) $2x^2 + y^2 = 2$ (b) $x^2 + 4y^2 = 4$ (c) $x^2 + 4y^2 = 1$ (d) $4x^2 + y^2 = 4$
[SSC Tier-I 2012]

20. If $\sec\theta + \tan\theta = \sqrt{3}$, then the positive value of $\sin\theta$ is
(a) 0 (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1
[SSC Tier-I 2012]

21. If $\frac{\cos^4\alpha}{\cos^2\beta} + \frac{\sin^4\alpha}{\sin^2\beta} = 1$, then the value of $\frac{\cos^4\beta}{\cos^2\alpha} + \frac{\sin^4\beta}{\sin^2\alpha}$ is
(a) 4 (b) 0 (c) $\frac{1}{8}$ (d) 1
[SSC Tier-I 2012]

22. Value of $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$ $\left(\text{where } \theta \ne \frac{\pi}{2}\right)$ is
(a) $\frac{1+\sin\theta}{\cos\theta}$ (b) $\frac{1-\sin\theta}{\cos\theta}$ (c) $\frac{1-\cos\theta}{\sin\theta}$ (d) $\frac{1+\cos\theta}{\sin\theta}$
[SSC Tier-I 2012]

23. If x, y are acute positive angle $x + y < 90^\circ$ and $\sin(2x - 20^\circ) = \cos(2y + 20^\circ)$ then the value of $\sec(x + y)$ is
(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) 0
[SSC Tier-I 2012]

24. Minimum value of $(4\sec^2\theta + 9\csc^2\theta)$ is
(a) 1 (b) 19 (c) 25 (d) 7
[SSC Tier-II 2012]

25. If $\tan(x+y)\tan(x-y) = 1$ then the value of $\tan\left(\frac{2x}{3}\right)$ is
(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{2}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) 1
[SSC Tier-II 2012]

26. If $x = \csc\theta - \sin\theta$ and $y = \sec\theta - \cos\theta$ then the value of $x^2y^2(x^2+y^2+3)$ is
(a) 0 (b) 1 (c) 2 (d) 3
[SSC Tier-I 2012]

27. If $\sin\theta + \sin^2\theta = 1$ then the value of $\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta - 1$ is
(a) 0 (b) 1 (c) –1 (d) 2
[SSC Tier-I 2012]

28. If $\tan(x+y)\tan(x-y) = 1$ then the value of $\tan x$ is
(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$
[SSC Tier-I 2012]

29 If $\cot A + \csc A = 3$ and A is an acute angle then the value of $\cos A$ is
(a) 1 (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{4}{5}$
[SSC Tier-I 2012]

30. The simplified value of $1 - \frac{\sin^2 A}{1+\cos A} + \frac{1+\cos A}{\sin A} - \frac{\sin A}{1-\cos A}$ is
(a) 0 (b) 1 (c) $\sin A$ (d) $\cos A$
[SSC Tier-I 2012]

31. If α is an acute angle and $2\sin\alpha + 15\cos^2\alpha = 7$ then value of $\cot\alpha$ is
(a) $\frac{4}{3}$ (b) $\frac{\sqrt{5}}{2}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{3}{4}$
[SSC Tier-I 2012]

32. If $\tan\theta - \cot\theta = a$ and $\cos\theta - \sin\theta = b$ then value of $(a^2+4)(b^2-1)^2$ is
(a) 1 (b) 2 (c) 3 (d) 4
[SSC Tier-I 2012]

33. If $(a^2 - b^2)\sin\theta + 2ab\cos\theta = a^2 + b^2$ then the value of $\tan\theta$ is
(a) $\frac{1}{2}(a^2 - b^2)$ (b) $\frac{1}{2ab}(a^2 - b^2)$ (c) $\frac{1}{2}(a^2 + b^2)$ (d) $\frac{1}{2ab}(a^2 + b^2)$
[SSC Tier-I 2012]

34. $\sin^2 21^\circ + \sin^2 69^\circ$ is equal to
(a) $2\sin^2 21^\circ$ (b) $2\sin^2 69^\circ$ (c) 1 (d) 0

35. $\sin^2 5^\circ + \sin^2 25^\circ + \sin^2 45^\circ + \sin^2 65^\circ + \sin^2 85^\circ$ is equal to
(a) 2.5 (b) 3 (c) 1.5 (d) 2
[SSC Tier-I 2012]

$3\sin^2\alpha + 7\cos^2\alpha =$ [redacted] then the value of $\tan\alpha$ is

$\sqrt{3}$ (b) $\sqrt{6}$ (c) $\sqrt{2}$ (d) $\sqrt{5}$

[SSC Tier-I 2012]

for all real values of α, $x = \cos^4\alpha + \sin^2\alpha$, then range of x is

) $\frac{3}{4} \le x \le 1$ (b) $\frac{3}{4} \le x \le \frac{13}{15}$ (c) $\frac{13}{16} \le x \le 1$ (d) $\frac{1}{2} \le x \le 2$

$\sin^2\alpha = \cos^3\alpha$ then the value of $(\cot^6\alpha - \cot^2\alpha)$ is

) 1 (b) 0 (c) −1 (d) 2

[SSC Tier-I 2012]

Answers–11B

(a)	2. (c)	3. (b)	4. (a)	5. (a)	6. (b)	7. (d)	8. (c)
(d)	10. (d)	11. (d)	12. (b)	13. (c)	14. (c)	15. (c)	16. (d)
(b)	18. (a)	19. (b)	20. (b)	21. (d)	22. (a)	23. (a)	24. (c)
(a)	26. (b)	27. (a)	28. (a)	29. (d)	30. (d)	31. (d)	32. (d)
(b)	34. (c)	35. (a)	36. (a)	37. (a)	38. (a)		

Explanation

a) $\because \sec^2\theta - \tan^2\theta = 1 \quad \therefore (4x)^2 - \left(\frac{4}{x}\right)^2 = 1$

$\Rightarrow 16\left(x^2 - \frac{1}{x^2}\right) = 1 \quad \Rightarrow 8\left(x^2 - \frac{1}{x^2}\right) = \frac{1}{2}$

(c) $2 - \cos^2\theta = 3\sin\theta\cos\theta$

Dividing both sides by $\cos^2\theta$

$2\sec^2\theta - 1 = 3\tan\theta$

or, $2(1 + \tan^2\theta) - 1 = 3\tan\theta$

or, $2\tan^2\theta - 3\tan\theta + 1 = 0$

$\Rightarrow (2\tan\theta - 1)(\tan\theta - 1) = 0$

$\Rightarrow \tan\theta = \frac{1}{2}, 1$

but $\sin\theta \ne \cos\theta \quad \tan\theta \ne 1$

$\therefore \tan\theta = \frac{1}{2}$

(b) $\sin\theta + \cos\theta = \sqrt{2}\cos(90° - \theta)$

or, $\sin\theta + \cos\theta = \sqrt{2}\sin\theta$

or, $\cos\theta = (\sqrt{2} - 1)\sin\theta$

Dividing both sides by $\sin\theta$

$\cot\theta = \sqrt{2} - 1$

(a) $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

or, $x\sin\theta\sin^2\theta + y\cos\theta\cos^2\theta = \sin\theta\cos\theta$

or, $y\cos\theta\sin^2\theta + x\sin\theta\cos^2\theta = \sin\theta\cos\theta$ $(\because x\sin\theta = y\cos\theta)$

or, sin

or, $y\sin\theta + x\cos\theta = 1$

From second relation $x\sin\theta - y\cos\theta = 0$

Squaring and adding (1) and (2)

$$(y\sin\theta + x\cos\theta)^2 + (x\sin\theta - y\cos\theta)^2 = 1^2 + 0^2$$

$\Rightarrow$ $y^2(\sin^2\theta + \cos^2\theta) + x^2(\cos^2\theta + \sin^2\theta) = 1$

[$2xy\sin\theta\cos\theta$ will be cancelled out]

Second method (Trial Method) :

We can guess from $x\sin\theta = y\cos\theta$ and that $x = \cos\theta$ and $y = \sin\theta$

$x = \cos\theta$ and $y = \sin\theta$

If also satisfies $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$\therefore$ $x^2 + y^2 = 1$

(Third Method) :

Let $x\sin\theta = y\cos\theta = k$ then $\sin\theta = \frac{k}{x}$ and $\cos\theta = \frac{k}{y}$

Now from $\sin^2\theta + \cos^2\theta = 1$

$$\left(\frac{k}{x}\right)^2 + \left(\frac{k}{y}\right)^2 = 1$$

$\Rightarrow$ $k^2\left(\frac{1}{x^2} + \frac{1}{y^2}\right) = 1$ $\Rightarrow$ $k^2\left(\frac{y^2 + x^2}{x^2y^2}\right) = 1$

$\Rightarrow$ $k^2(x^2 + y^2) = x^2y^2$ ...(i)

Again from $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$$x\left(\frac{k}{x}\right)^3 + y\left(\frac{k}{y}\right)^3 = \frac{k}{x}\cdot\frac{k}{y}$$

$k^3\left(\frac{1}{x^2} + \frac{1}{y^2}\right) = \frac{k^2}{xy}$ $\Rightarrow$ $k^3\left(\frac{y^2 + x^2}{x^2y^2}\right) = \frac{k^2}{xy}$

$k(x^2 + y^2) = xy$ ...(ii)

Putting $xy = k(x^2 + y^2)$ in equation (i)

$$k^2(x^2 + y^2) = k^2(x^2 + y^2)^2$$

$\Rightarrow$ $1 = x^2 + y^2$

5. **(a)** Given $A + B = 90°$ or, $B = 90° - A$

$\therefore$ $\cos B = \sin A$, $\sin B = \cos A$, $\tan B = \cot A$

and $\cot B = \tan A$

Hence given expression

$$= \sin A \sin A + \cos A \cos A - \tan A \cot A + \sec^2 A - \tan^2 A$$

$$= \sin^2 A + \cos^2 A - 1 + 1 = 1 - 1 + 1 = 1$$

6. (b) $2\sin^2\theta + 3\cos^2\theta = 2(\sin^2\theta + \cos^2\theta) + \cos^2\theta = 2 + \cos^2\theta$

Since minimum value of $\cos^2\theta$ is zero

$\therefore$ Minimum value of given expression $= 2 + 0 = 2$

7. (d) $\sec^4\theta - \sec^2\theta = \sec^2\theta\,(\sec^2\theta - 1)$

$= \sec^2\theta \tan^2\theta = (1 + \tan^2\theta)\tan^2\theta = \tan^2\theta + \tan^4\theta$

8. (c) $\cos A + \cos^2 A = 1 \Rightarrow \cos A = 1 - \cos^2 A = \sin^2 A$

Now, $\sin^2 A + \sin^4 A = \cos A + \cos^2 A = 1$ $\quad (\because \sin^2 A = \cos A)$

9. (d) $\sec\theta = \text{cosec}\theta \Rightarrow \theta = 45^\circ$ $\quad (\because \sec 45^\circ = \sqrt{2}, \text{cosec } 45^\circ = \sqrt{2})$

$\therefore$ $\sec\theta + \text{cosec}\theta = \sec 45^\circ + \text{cosec}45^\circ = \sqrt{2} + \sqrt{2} = 2\sqrt{2}$

10. (d) Squaring and adding $(P\sin\theta)^2 + (P\cos\theta)^2 = \left(\sqrt{3}\right)^2 + 1^2$

$\Rightarrow P^2(\sin^2\theta + \cos^2\theta) = 3 + 1 \quad \Rightarrow P^2 = 4 \quad \therefore P = 2$

11. (d) $2u_6 - 3u_4 + 1 = 2(\cos^6\alpha + \sin^6\alpha) - 3(\cos^4\alpha + \sin^4\alpha) + 1$

$= 2\{(\cos^2\alpha + \sin^2\alpha)^3 - 3\sin^2\alpha\cos^2\alpha\,(\sin^2\alpha + \cos^2\alpha)\}$

$- 3\{(\cos^2\alpha + \sin^2\alpha)^2 - 2\cos^2\alpha\sin^2\alpha\} + 1$

$(\because a^3 + b^3 = (a+b)^3 - 3ab(a+b),\ a^2 + b^2 = (a+b)^2 - 2ab,$ here $a = \cos^2\alpha$ and $b = \sin^2\alpha)$

$= 2\{1 - 3\sin^2\alpha\cos^2\alpha\cdot 1\} - 3\{1 - 2\sin^2\alpha\cos^2\alpha\} + 1$

$= 2 - 6\sin^2\alpha\cos^2\alpha - 3 + 6\sin^2\alpha\cos^2\alpha + 1$

$2u_6 - 3u_4 + 1 = 2(\cos^6\alpha + \sin^6\alpha) - 3(\cos^4\alpha + \sin^4\alpha) + 1$

Trick, since all the options are idependent of α, putting $\alpha = 0$

$2u_6 - 3u_4 + 1 = 2(\cos^6 0 + \sin^6 0) - 3(\cos^4 0 + \sin^4 0) + 1$

$= 2(1 + 0) - 3(1 + 0) + 1 = 0$

We can put any value of α

12. (b) $\sin(x + y) = \cos(3(x + y) = \sin\left(\frac{\pi}{2} - 3(x+y)\right)$

$\therefore$ $(x + y) = 90^\circ - 3(x + y)$

or, $4(x + y) = 90^\circ$

or, $2(x + y) = 45^\circ$

$\therefore$ $\tan(2(x + y) = \tan 45^\circ = 1$

13. (c) $(1 + \sec 20^\circ + \cot 70^\circ)(1 - \text{cosec}20^\circ + \tan 70^\circ)$

$= (1 + \sec 20^\circ + \tan 20^\circ)(1 - \text{cosec}20^\circ + \cot 20^\circ)$

$= \left(1 + \frac{1 + \sin 20^\circ}{\cos 20^\circ}\right)\left(1 - \frac{1 - \cos 20^\circ}{\sin 20^\circ}\right)$

$= \left(\frac{\cos 20^\circ + 1 + \sin 20^\circ}{\cos 20^\circ}\right)\left(\frac{\sin 20^\circ - 1 + \cos 20^\circ}{\sin 20^\circ}\right)$

$= \frac{\cos 20^\circ + \sin 20^{\circ 2} - 1}{\cos 20^\circ \sin 20^\circ}$

$= \frac{1 + 2\sin 20^\circ \cos 20^\circ - 1}{\cos 20^\circ \sin 20^\circ}$

$= \frac{2\sin 20^\circ \cos 20^\circ}{\cos 20^\circ \sin 20^\circ} = 2$ $\quad (\because \sin^2 20^\circ + \cos^2 20^\circ = 1)$

14. (c) $2\sin\alpha + 15\cos^2\alpha = 7$

$\Rightarrow 2\sin\alpha + 15(1 - \sin^2\alpha) = 7$

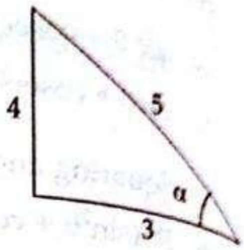
$\Rightarrow 15\sin^2\alpha - 2\sin\alpha - 8 = 0$

$\Rightarrow 15\sin^2\alpha - 12\sin\alpha + 10\sin\alpha - 8 = 0$

$\Rightarrow 3\sin\alpha(5\sin\alpha - 4) + 2(5\sin\alpha - 4) = 0$

$\Rightarrow (3\sin\alpha + 2)(5\sin\alpha - 4) = 0$

$\therefore \sin\alpha = \frac{-2}{3}, \frac{4}{5}$



But $0 \le \alpha \le \frac{\pi}{2}$

$\therefore \sin\alpha = \frac{4}{5} \Rightarrow \cot\alpha = \frac{3}{4}$

15. (c) $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$

or, $\frac{\cos\theta(1+\sin\theta) + \cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} = 4$

or, $\frac{2\cos\theta}{1-\sin^2\theta} = 4$

or, $\frac{\cos\theta}{\cos^2\theta} = 2$

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$

16. (d) Given $\sec\theta + \tan\theta = 2$ ...(i)

$\because \sec^2\theta - \tan^2\theta = 1$

$\therefore (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$2(\sec\theta - \tan\theta) = 1$

or, $\sec\theta - \tan\theta = \frac{1}{2}$ ...(ii)

Adding (1) and (2),

$2\sec\theta = 2 + \frac{1}{2} = \frac{5}{2}$

$\therefore \sec\theta = \frac{5}{4}$

17. (b) $\tan 2\theta \tan 4\theta = 1$

or, $\tan 2\theta = \frac{1}{\tan 4\theta} = \cot 4\theta$

or, $\tan 2\theta = \tan (90° - 4\theta)$

or, $2\theta = 90° - 4\theta$

or, $6\theta = 90°$

or, $\theta = 15°$

$\therefore \tan 3\theta = \tan 45° = 1$

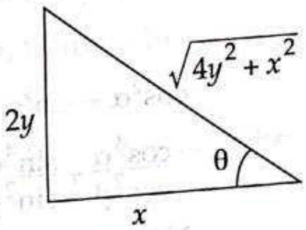
18. (a) $\cos\theta + \sec\theta = \sqrt{3}$

$\cos^3\theta + \sec^3\theta + 3(\cos\theta + \sec\theta)(\cos\theta \cdot \sec\theta) = 3\sqrt{3}$

or, $\cos^3\theta + \sec^3\theta + 3\sqrt{3} \cdot 1 = 3\sqrt{3}$

or, $\cos^3\theta + \sec^3\theta = 0$

19. (b) From first relation, $\tan\theta = \frac{2y}{x}$



Here $p = 2y$, $b = x$

$\therefore h = \sqrt{4y^2 + x^2}$

From second relation, $2x \sec\theta - y \operatorname{cosec}\theta = 3$

or, $2x \frac{\sqrt{4y^2 + x^2}}{x} - \frac{y\sqrt{4y^2 + x^2}}{2y} = 3$

or, $\left(2 - \frac{1}{2}\right)\sqrt{4y^2 + x^2} = 3$ or, $\frac{3}{2}\sqrt{4y^2 + x^2} = 3$

or, $\sqrt{4y^2 + x^2} = 2$ or, $x^2 + 4y^2 = 4$

20. (b) Given, $\sec\theta + \tan\theta = \sqrt{3}$ ... (i)

$\because \sec^2\theta - \tan^2\theta = 1$

$\therefore (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

or, $(\sec\theta - \tan\theta)\sqrt{3} = 1$

or, $\sec\theta - \tan\theta = \frac{1}{\sqrt{3}}$ ... (ii)

(i) and (ii) adding $2\sec\theta = \sqrt{3} + \frac{1}{\sqrt{3}}$

$\Rightarrow \sec\theta = \frac{1}{2}\left(\frac{3+1}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}}$

$\Rightarrow \theta = 30°$

$\therefore \sin\theta = \frac{1}{2}$

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21. (d) $\dfrac{\cos^4\alpha}{\cos^2\beta}+\dfrac{\sin^4\alpha}{\sin^2\beta}=1=\sin^2\alpha+\cos^2\alpha$

$\therefore\quad \dfrac{\cos^4\alpha}{\cos^2\beta}-\cos^2\alpha=\sin^2\alpha-\dfrac{\sin^4\alpha}{\sin^2\beta}$

or, $\dfrac{\cos^4\alpha-\cos^2\alpha\cos^2\beta}{\cos^2\beta}=\dfrac{\sin^2\alpha\sin^2\beta-\sin^4\alpha}{\sin^2\beta}$

or, $\dfrac{\cos^2\alpha\left(\cos^2\alpha-\cos^2\beta\right)}{\cos^2\beta}=\dfrac{\sin^2\alpha\left(\sin^2\beta-\sin^2\alpha\right)}{\sin^2\beta}$

or, $\dfrac{\cos^2\alpha}{\cos^2\beta}=\dfrac{\sin^2\alpha}{\sin^2\beta}$ $\quad(\because \cos^2\alpha-\cos^2\beta=\sin^2\beta-\sin^2\alpha)$

or, $\dfrac{\sin^2\beta}{\cos^2\beta}=\dfrac{\sin^2\alpha}{\cos^2\alpha}$ or, $\tan^2\beta=\tan^2\alpha$ or, $\alpha=\beta$

$\therefore\quad \dfrac{\cos^4\beta}{\cos^2\alpha}+\dfrac{\sin^4\beta}{\sin^2\alpha}=\dfrac{\cos^4\alpha}{\cos^2\alpha}+\dfrac{\sin^4\alpha}{\sin^2\alpha}=\cos^2\alpha+\sin^2\alpha=1$

Second method (tricky approach) :

$\cos^2\alpha+\sin^2\alpha=1$

$\dfrac{\cos^4\alpha}{\cos^2\beta}+\dfrac{\sin^4\alpha}{\sin^2\beta}=1$ is possible only when $\alpha=\beta$

Now put $\alpha=\beta$ and get requred value

22. (a) $\dfrac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=\dfrac{\sin\theta-(\cos\theta-1)}{\sin\theta+(\cos\theta-1)}\times\dfrac{\sin\theta-(\cos\theta-1)}{\sin\theta-(\cos\theta-1)}$

$=\dfrac{\left(\sin\theta-(\cos\theta-1)\right)^2}{\sin^2\theta-(\cos\theta-1)^2}$

$=\dfrac{\sin^2\theta+(\cos\theta-1)^2-2\sin\theta(\cos\theta-1)}{\sin^2\theta-\left(\cos^2\theta+1-2\cos\theta\right)}$

$=\dfrac{\sin^2\theta+\cos^2\theta+1-2\cos\theta-2\sin\theta\cos\theta+2\sin\theta}{\sin^2\theta-\cos^2\theta-1+2\cos\theta}$

$=\dfrac{1+1-2\cos\theta-2\sin\theta\cos\theta+2\sin\theta}{-\cos^2\theta-\cos^2\theta+2\cos\theta}$ $\quad(\because \sin^2\theta-1=-\cos^2\theta)$

$=\dfrac{2(1-\cos\theta-\sin\theta\cos\theta+\sin\theta)}{-2\left(\cos^2\theta-\cos\theta\right)}$

$=\dfrac{2(1-\cos\theta)(1+\sin\theta)}{2\cos\theta(1-\cos\theta)}=\dfrac{1+\sin\theta}{\cos\theta}$

(Quick approach) putting $\theta = 60^\circ$

$$\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{\frac{\sqrt{3}}{2} - \frac{1}{2} + 1}{\frac{\sqrt{3}}{2} + \frac{1}{2} - 1} = \frac{\frac{\sqrt{3}}{2} + \frac{1}{2}}{\frac{\sqrt{3}}{2} - \frac{1}{2}} = \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2\sqrt{3}$$

Now put $\theta = 60^\circ$ in each option, only option (a) gives $2\sqrt{3}$.

23. (a) $\sin(2x - 20^\circ) = \cos(2y + 20^\circ)$

$\Rightarrow \sin(2x - 20^\circ) = \sin(90^\circ - 2y - 20^\circ)$

$\therefore 2x - 20^\circ = 70^\circ - 2y$

or, $2(x + y) = 90^\circ$

or, $x + y = 45^\circ$

$\therefore \sec(x + y) = \sec 45^\circ = \sqrt{2}$

24. (c) $4\sec^2\theta + 9\,\text{cosec}^2\theta$

$= 4(1 + \tan^2\theta) + 9(1 + \cot^2\theta)$

$= 4\tan^2\theta + 9\cot^2\theta + 13$

$= (2\tan\theta - 3\cot\theta)^2 + 2\cdot 2\tan\theta \cdot 3\cot\theta + 13$

$= (2\tan\theta - 3\cot\theta)^2 + 12 \times 1 + 13$ $(\because \tan\theta\cdot\cot\theta = 1)$

But $(2\tan\theta - 3\cot\theta)^2 \geq 0$

$\therefore$ Required expression $\geq 12 + 13 = 25$ which is the minimum value.

Note : Do not work

$(2\tan\theta + 3\cot\theta)^2 - 2\cdot 2\tan\theta \cdot 3\cot\theta$ as $2\tan\theta + 3\cot\theta \neq 0$)

25. (a) $\tan(x + y)\tan(x - y) = 1$

$\Rightarrow \tan(x + y) = \frac{1}{\tan(x - y)} = \cot(x - y)$

$\Rightarrow \tan(x + y) = \tan(90^\circ - (x - y))$

$\Rightarrow x + y = 90^\circ - (x - y)$

$\Rightarrow 2x = 90^\circ \Rightarrow \frac{2x}{3} = 30^\circ$

$\therefore \tan\frac{2x}{3} = \tan 30^\circ = \frac{1}{\sqrt{3}}$

26. (b) $x = \text{cosec}\theta - \sin\theta$

$$= \frac{1}{\sin\theta} - \sin\theta = \frac{1 - \sin^2\theta}{\sin\theta} = \frac{\cos^2\theta}{\sin\theta}$$

similary $y = \frac{\sin^2\theta}{\cos\theta}$

$$\therefore x^2y^2(x^2 + y^2 + 3) = \frac{\cos^4\theta}{\sin^2\theta}\frac{\sin^4\theta}{\cos^2\theta}\left(\frac{\cos^4\theta}{\sin^2\theta} + \frac{\sin^4\theta}{\cos^2\theta} + 3\right)$$

$$= \cos^2\theta\sin^2\theta\left(\frac{\cos^6\theta + \sin^6\theta}{\sin^2\theta\cos^2\theta} + 3\right)$$

$$= \cos^2\theta\sin^2\theta\left\{\frac{(\cos^2\theta+\sin^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\cos^2\theta+\sin^2\theta)}{\sin^2\theta\cos^2\theta}+3\right\}$$

$$= \cos^2\theta\sin^2\theta\frac{(1-3\sin^2\theta\cos^2\theta+3\sin^2\theta\cos^2\theta)}{\sin^2\theta\cos^2\theta} = 1$$

27. (a) $\sin\theta + \sin^2\theta = 1 \Rightarrow \sin\theta = 1 - \sin^2\theta = \cos^2\theta$ ... (i)

Now, $\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta - 1$

$= (\cos^4\theta)^3 + 3(\cos^4\theta)^2\cos^2\theta + 3\cos^4\theta(\cos^2\theta)^2 + (\cos^2\theta)^3 - 1$

$= (\cos^4\theta + \cos^2\theta)^3 - 1$

$= (\sin^2\theta + \sin\theta)^3 - 1$ $(\because \cos^2\theta = \sin\theta)$

$= 1^3 - 1$ $(\because \sin\theta + \sin^2\theta = 1)$

$= 1 - 1 = 0$

28. (a) $\tan(x+y) = \tan(x-y)$

$\Rightarrow \tan(x+y) = \frac{1}{\tan(x-y)} = \cot(x-y)$

$\Rightarrow x + y = \frac{\pi}{2} - (x-y)$

or, $2x = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{4}$ $\therefore \tan x = 1$

29. (d) Given $\cot A + \text{cosec} A = 3$ ... (i)

We know that $\text{cosec}^2 A - \cot^2 A = 1$

or, $(\text{cosec} A + \cot A)(\text{cosec} A - \cot A) = 1$

or, $3(\text{cosec} A - \cot A) = 1$

or, $\text{cosec} A - \cot A = \frac{1}{3}$ ... (ii)

(i) and (ii) adding

$2\text{cosec} A = 3 + \frac{1}{3} = \frac{10}{3}$

or, $\text{cosec} A = \frac{5}{3} = \frac{h}{p}$

$\therefore b = \sqrt{h^2 - p^2} = \sqrt{25-9} = 4$

$\therefore \cos A = \frac{b}{h} = \frac{4}{5}$

30. (d) Given expression $= 1 - \frac{\sin^2 A}{1+\cos A} + \frac{1+\cos A}{\sin A} - \frac{\sin A}{1-\cos A}$

$$= 1 - \frac{1-\cos^2 A}{1+\cos A} + \frac{(1+\cos A)(1-\cos A) - \sin^2 A}{\sin A(1-\cos A)}$$

$$= 1 - \frac{(1+\cos A)(1-\cos A)}{(1+\cos A)} + \frac{1-\cos^2 A - \sin^2 A}{\sin A(1-\cos A)}$$

$$= 1 - (1 - \cos A) + \frac{1 - (\cos^2 A + \sin^2 A)}{\sin A(1 - \cos A)}$$

$$= 1 - 1 + \cos A + \frac{1-1}{\sin A(1 - \cos A)} = \cos A$$

$2\sin\alpha + 15\cos^2\alpha = 7$

$2\sin\alpha + 15(1 - \sin^2\alpha) = 7$

$15\sin^2\alpha - 2\sin\alpha - 8 = 0$

Solving, $\sin\alpha = \frac{4}{5}$

$\therefore \cot\alpha = \frac{3}{4}$

(d) $a^2 + 4 = (\tan\theta - \cot\theta)^2 + 4$

$= \tan^2\theta + \cot^2\theta - 2\tan\theta\cot\theta + 4$

$= \tan^2\theta + \cot^2\theta - 2 + 4$

$= \tan^2\theta + \cot^2\theta + 2 = (\tan\theta + \cot\theta)^2$

$$= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)^2 = \frac{1}{\cos^2\theta \cdot \sin^2\theta}$$

$(b^2 - 1)^2 = ((\cos\theta - \sin\theta)^2 - 1)^2$

$= (\cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta - 1)^2$

$= (1 - 2\sin\theta\cos\theta - 1)^2$

$= 4\sin^2\theta\cos^2\theta$

$$\therefore (a^2 + 4)(b^2 - 1)^2 = \frac{1}{\cos^2\theta\sin^2\theta} 4\sin^2\theta\cos^2\theta = 4$$

33. (b) $(a^2 - b^2)\sin\theta + 2ab\cos\theta = a^2 + b^2$

dividing by $a^2 + b^2$

$$\frac{a^2 - b^2}{a^2 + b^2}\sin\theta + \frac{2ab}{a^2 + b^2}\cos\theta = 1 \quad \ldots \text{(i)}$$

we will solve it by trial

$$\because \left(\frac{a^2 - b^2}{a^2 + b^2}\right)^2 + \left(\frac{2ab}{a^2 + b^2}\right)^2 = \frac{\left(a^2 - b^2\right) + 4a^2b^2}{\left(a^2 + b^2\right)^2} = \frac{\left(a^2 + b^2\right)^2}{\left(a^2 + b^2\right)^2} = 1$$

$\therefore$ From (i) $\frac{a^2 - b^2}{a^2 + b^2} = \sin\theta$ and $\frac{2ab}{a^2 + b^2} = \cos\theta$ can be considered.

so that $\sin^2\theta + \cos^2\theta = 1$

Hence $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{a^2 - b^2}{2ab}$

34. (c) $\sin^2 21° + \sin^2 69° = \sin^2 21° + \cos^2 21° = 1$

35. (a) $\sin^2 5° + \sin^2 25° + \sin^2 45° + \sin^2 65° + \sin^2 85°$

$= \sin^2 5° + \sin^2 25° + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2 25° + \cos^2 5°$ $(\because \cos(90° - \theta) = \sin\theta)$

$= (\sin^2 5° + \cos^2 5°) + (\sin^2 25° + \cos^2 25°) + \frac{1}{2}$

$= 1 + 1 + \frac{1}{2} = \frac{5}{2} = 2.5$

36. (a) $3\sin^2\alpha + 7\cos^2\alpha = 4$

or, $3(1 - \cos^2\alpha) + 7\cos^2\alpha = 4$

or, $4\cos^2\alpha = 1$

or, $\cos^2\alpha = \left(\frac{1}{2}\right)^2 \Rightarrow \alpha = 60°$

$\therefore \tan\alpha = \tan 60° = \sqrt{3}$

37. (a) $x = \cos^4\alpha + \sin^2\alpha$

$= \cos^4\alpha + 1 - \cos^2\alpha = 1 + \cos^4\alpha - \cos^2\alpha = 1 + \cos^2\alpha(\cos^2\alpha - 1)$

$= 1 - \cos^2\alpha \sin^2\alpha = 1 - \left(\frac{2\sin\alpha\cos\alpha}{2}\right)^2 = 1 - \frac{1}{4}\sin^2 2\alpha$

When $\sin^2 2\alpha = 0, x = 1$

When $\sin^2 2\alpha = 1, x = 1 - \frac{1}{4} = \frac{3}{4}$

Second method

$x = \cos^4\alpha + \sin^2\alpha = \cos^4\alpha + 1 - \cos^2\alpha$

$= \cos^4\alpha - 2\cdot\cos^2\alpha\cdot\frac{1}{2} + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1$

$= \left(\cos^2\alpha - \frac{1}{2}\right)^2 + \frac{3}{4}$

When $\cos^2\alpha = \frac{1}{2}$ then $x = \left(\frac{1}{2} - \frac{1}{2}\right)^2 + \frac{3}{4} = \frac{3}{4}$

When $\cos^2\alpha = 1$ then $x = \left(1 - \frac{1}{2}\right)^2 + \frac{3}{4} = \frac{1}{4} + \frac{3}{4} = 1$

$\therefore$ Minimum value $= \frac{3}{4}$, maximum value. $= 1$

38. (a) $\sin^2\alpha = \cos^3\alpha \Rightarrow \frac{1}{\cos\alpha} = \frac{\cos^2\alpha}{\sin^2\alpha} \Rightarrow \sec\alpha = \cot^2\alpha$ ... (i)

Now, $\cot^6\alpha - \cot^2\alpha = \sec^3\alpha - \sec\alpha$ (from (i))

$= \sec\alpha(\sec^2\alpha - 1) = \sec\alpha \cdot \tan^2\alpha$

$= \frac{1}{\cos\alpha}\cdot\frac{\sin^2\alpha}{\cos^2\alpha} = \frac{\sin^2\alpha}{\cos^3\alpha}$

$= \frac{\sin^2\alpha}{\sin^2\alpha} = 1$ $(\because \cos^3\alpha = \sin^2\alpha)$

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