

Chapter 4

Controllers and Compensators

LEARNING OBJECTIVES

After reading this chapter, you will be able to understand:

- Compensators and controllers
- Compensators
- Lag compensator
- Lead-compensator
- Lead–Lag compensator
- Controllers
- Proportional controller
- Proportional plus integral controller
- Proportional plus derivative controller
- Proportional plus integral plus derivative controller

COMPENSATORS AND CONTROLLERS

Every control system is designed for a specific application and to meet certain performance parameters or specifications. System specifications in time domain and/or in frequency domain such as peak overshoot, settling time, gain margin, phase margin. A device inserted into the system for the purpose of satisfying the specification is called a compensator.

There are two types of compensation schemes.

1. Series compensation
2. Feedback or Parallel compensation

Series Compensation

In series compensation, a compensator is introduced in series with the plant to change the system behaviour and to meet the desired specifications.

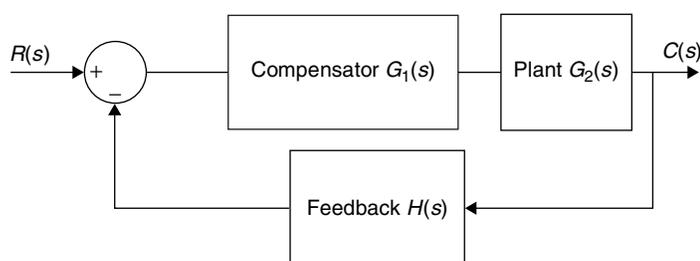


Figure 1 Block diagram for series compensation

Feedback Compensation

In feedback compensation, a compensator is introduced in the feedback path to meet the desired specifications.

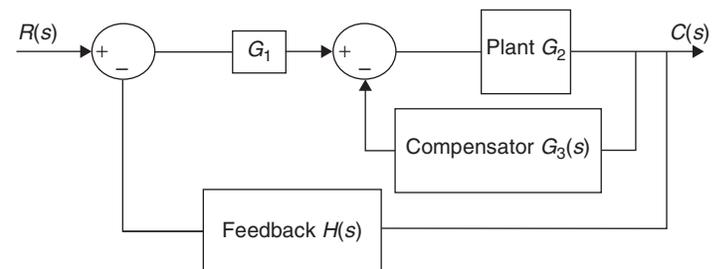


Figure 2 Block diagram of feedback compensation

COMPENSATORS

Different types of compensators are

1. Lag compensator
2. Lead compensator
3. Lead–lag compensator

Lag-Compensator

The low-pass filter is known as phase-lag compensator. The ideas to filter and phase shift are useful if designs are carried in the frequency domain.

The transfer function of a simple lag compensator is given by $\Rightarrow G_c(s) = \frac{1+Ts}{1+aTs}$; $a > 1$

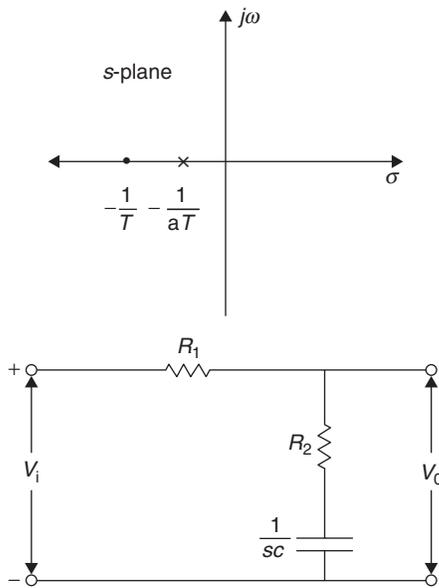


Figure 3 Pole zero configuration of electrical lag compensator

Lag compensator improves the steady-state behaviour of a system, while transient behaviour remains unchanged.

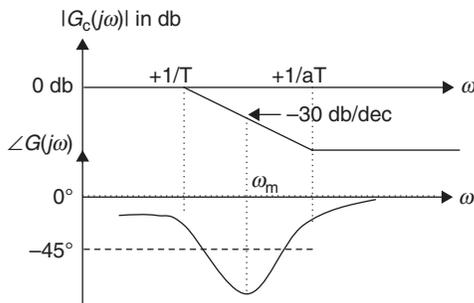


Figure 4 Bode plot of the lag compensator

The value of phase angle is maximum at a frequency of

$$\omega_m = \sqrt{\omega_{c1} \times \omega_{c2}} = \sqrt{\frac{1}{aT} \times \frac{1}{T}} = \frac{1}{T\sqrt{a}}$$

∴ Maximum phase angle

$$\angle G(j\omega) \Big|_{\omega=\omega_m} = \phi_m = \tan^{-1} \left(\frac{1-a}{2\sqrt{a}} \right)$$

$$\angle G(j\omega) \Big|_{\omega=\omega_m} = \phi_m = \sin^{-1} \left(\frac{1-a}{1+a} \right)$$

Stability of the system relatively reduces with addition of Lag-compensator. The gain crossover frequency of the system decreases and thus the bandwidth of the system is reduced. The raise time and settling time of the system are usually longer, because the bandwidth is usually decreased. System is more sensitive to parameter variations.

Lead Compensator

The high-pass filter is known as phase-lead compensator. These ideas to filter and phase shift are useful if designs are carried in the frequency domain. The transfer function of a simple lead compensator is given by

$$G_c(s) = \frac{1+\zeta s}{1+a\zeta s}; a < 1$$

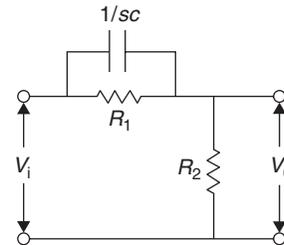


Figure 5 Electrical lead compensator

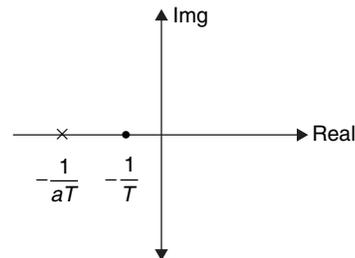


Figure 6 Pole zero configuration of lead compensator

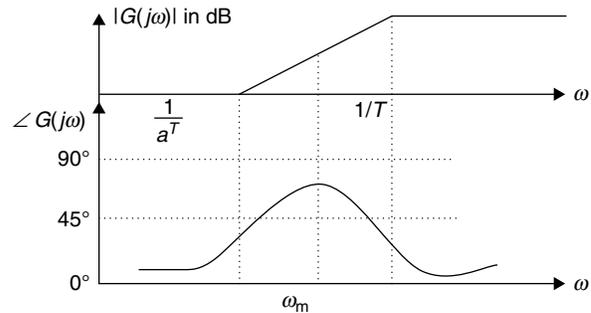


Figure 7 Bode plot of lead compensator

The value of phase angle is maximum at frequency of

$$\omega_m = \sqrt{\omega_{c1} \times \omega_{c2}} = \frac{1}{T\sqrt{a}}$$

$$\text{Maximum phase angle } \phi_m = \tan^{-1} \left(\frac{1-a}{2\sqrt{a}} \right).$$

The lead compensator affects the transient response of the system. It adds damping to the system and thus the rise time and settling time are reduced. The gain cross over frequency is increased and improves the phase margin of the closed loop system. The relative stability of the system is improved with improvement in gain and phase margins.

The bandwidth of the closed loop system is increased and results in fast response.

The steady-state error of the system is not affected.

Lead-Lag Compensator

The combination of lag and lead compensators is used to utilize the advantages of both the schemes. The transfer function of a simple lead-lag compensator is given by

$$G_c(s) = \left(\frac{1+a_1T_1s}{1+T_1s} \right) \left(\frac{1+a_2T_2s}{1+T_2s} \right) \quad (a_1 > 1, a_2 < 1)$$

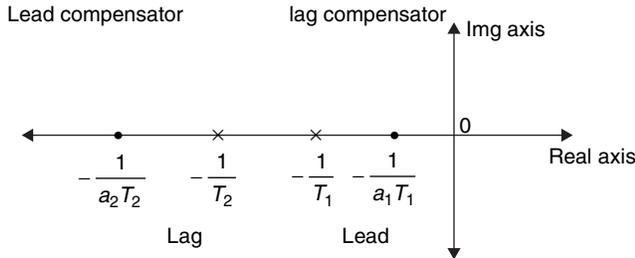


Figure 8 Pole-zero configuration of Lead-Lag compensator

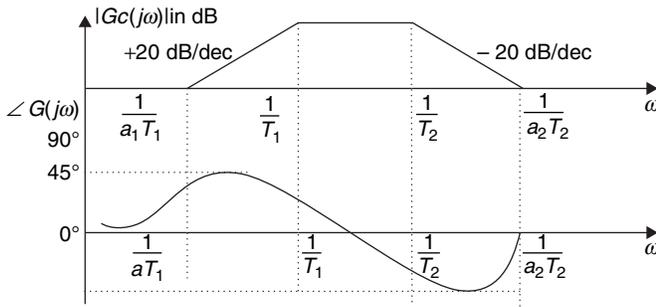
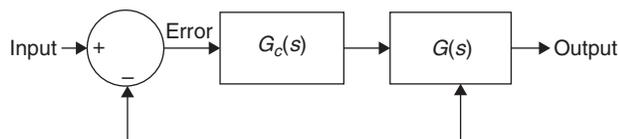


Figure 9 Bode plot of lead-lag compensator

CONTROLLERS

The cascaded controllers are used to modify the transient and the steady-state response of the system.



The different types of controllers available in control system are given as follows:

1. Proportional controller
2. Proportional plus integral controller (PI-Controller)
3. Proportional plus derivative controller (PD-Controller)
4. Proportional plus derivative plus integral controller (PID-Controller)

Proportional Controller

A controller that produces output proportional to the input signal (error $e(t)$) is called a proportional controller.

$$\therefore u(t) = K_p e(t)$$

where

$u(t)$ = Input to the plant

$e(t)$ = Input to the controller

K_p = Gain of the proportional controller

The proportional controller improves the steady-state accuracy, disturbance signal rejection and relative stability.

The proportional controller decreases the sensitivity of the system to parameter variation.

The proportional controller is not used alone because it produces a constant steady-state error.

Proportional Plus Integral Controller (PI)

A controller that produces output signal $u(t)$ proportional to the input signal (error $e(t)$) and to the integral of input signal.

$$\therefore u(t) = K_p e(t) + K_I \int e \cdot dt$$

K_p = Proportional controller gain

K_I = Integral controller gain

The PI controller increases the order and type of the system. It acts as a low-pass filter. The PI controller reduces the steady-state error.

Proportional Plus Derivative Controller (PD)

A controller that produces output signal $u(t)$ proportional to the input signal (error $e(t)$) and derivative of the input signal $\left(\frac{de(t)}{dt} \right)$.

$$u(t) = K_p e(t) + K_D \frac{de(t)}{dt}$$

The PD controller increases damping of the system which results in reducing the peak overshoot.

The PD controller acts as a high-pass filter. It improves gain margin and phase margin.

The PD controller increases bandwidth, reduces rise time and settling time system stability is improved.

Proportional Plus Integral Plus Derivative Controller

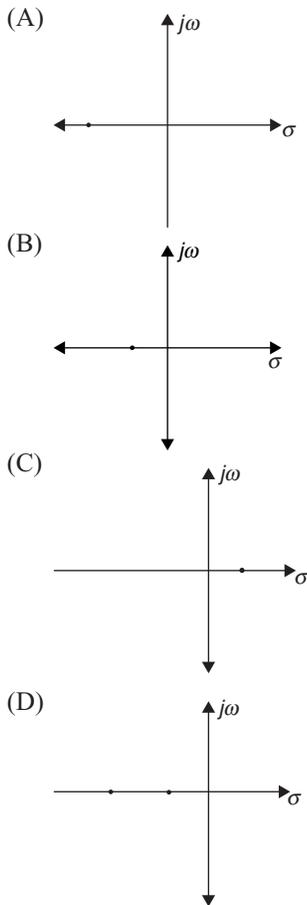
This is a combination of proportional, Integral and derivative controllers.

$$u(t) = K_p e(t) + K_I \int e(t) dt + K_D \frac{de(t)}{dt}$$

The PID controller decreases the steady-state error and increases stability.

Solved Examples

Example 1: The pole zero configuration of a phase Lag Compensator is given by



Solution: (A)
Lag compensator has a dominant pole with a single pole zero configuration.

Example 2: A lead compensator used for a closed loop controller has the following transfer function

$$\frac{k(1+as)}{(1+bs)}$$

- For such a lead compensator
- (A) $a < b$
 - (B) $a > b$
 - (C) $a > kb$
 - (D) $a < kb$

Solution: (B)
Pole zero values of given compensator are

$$-\frac{1}{a} \text{ and } -\frac{1}{b}$$

For a lead compensator, pole is dominating; so

$$-\frac{1}{a} > -\frac{1}{b} \Rightarrow \frac{1}{a} < \frac{1}{b} \Rightarrow a > b.$$

Hence, the correct option is (B).

Example 3: The transfer function of PID controller is given by

- (A) $\frac{s}{K_I s^2 + K_D s + K_P}$
- (B) $\frac{K_D s^2 + K_P s + K_I}{s}$
- (C) $\frac{s}{K_D s^2 + K_P s + K_I}$
- (D) $\frac{K_I s^2 + K_D s + K_P}{s}$

Solution: (B)
The relation between input and output of a PID controller is given by

$$u(t) = K_P e(t) + K_I \int e(t) + K_D \frac{de(t)}{dt}$$

Apply Laplace transform on both sides

$$u(s) = \left(K_P + \frac{K_I}{S} + K_D s \right) E(s)$$

$$\frac{u(s)}{E(s)} = \frac{K_D s^2 + K_P s + K_I}{S}$$

Example 4: Maximum phase lead of the compensator $D(s)$ is

$$D(s) = \frac{0.4s+1}{0.04s+1}$$

- (A) 50°
- (B) 55°
- (C) 60°
- (D) None of the above

Solution: (B)

$$\text{Compensator } D(s) = \frac{0.4s+1}{0.04s+1} = \frac{1+aTs}{1+Ts}$$

$$aT = 0.4; 0.04 = T$$

\therefore

$$a = 10$$

$$\text{Maximum phase angle } \phi_m = \sin^{-1} \left(\frac{a-1}{a+1} \right) = 55^\circ$$

Example 5: Phase angle of the PID controller at high frequencies is (as frequency tends to infinity)

- (A) ∞
- (B) 90°
- (C) -90°
- (D) 180°

Solution: (B)

Transfer function of the PID controller

$$= \frac{K_D s^2 + K_P s + K_I}{s}$$

Phase angle of

$$\text{controller} = \tan^{-1} \left(\frac{\omega K_p}{K_i - \omega^2 K_D} \right) - \tan^{-1} \left(\frac{\omega}{\infty} \right)$$

$$\phi = 180^\circ - 90^\circ = 90^\circ \text{ (as } \omega \rightarrow \infty \text{)}$$

Example 6: The transfer function of two compensators are given by

$$C_1 = \frac{20(1+s)}{(20+s)}, C_2 = \frac{s+10}{10(s+1)}$$

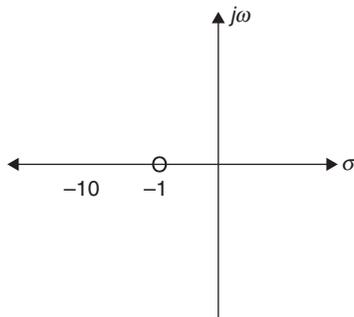
Which of the following statement is correct?

- (A) C_1 is a lag compensator and C_2 is a lead compensator.
- (B) C_1 is a lead compensator and C_2 is a lag compensator.
- (C) Both C_1 and C_2 are lead compensators.
- (D) Both C_1 and C_2 are lag compensators.

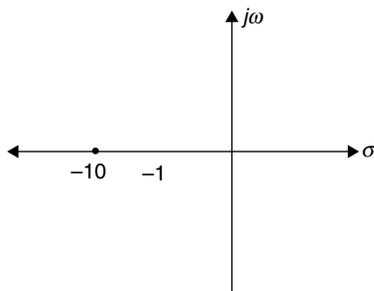
Solution: (B)

$$\text{Compensator } C_1 = \frac{10(s+1)}{(s+10)}$$

From pole zero configuration, C_1 is lead compensator.



$$\text{Compensator } C_2 = \frac{s+10}{10(s+1)}$$



From the above pole zero,

Configuration C_2 is lag compensator.

Example 7: The system $\frac{900}{s(s+1)(s+9)}$ is to be such that its gain crossover frequency becomes same as its

uncompensated phase crossover frequency and provides 45° phase margin. To accomplish this, one may use

- (A) A lag-lead compensator that provides an amplification of 20 dB and a phase lead of 45° at the frequency of $\sqrt{3}$ rad/sec.
- (B) A lag-Lead compensator that provides an attenuation of 20 dB and phase lead of 45° at a frequency of 3 rad/s.
- (C) A lag compensator that provides an attenuation of 20 dB and a phase angle of 45° at the frequency of $3\sqrt{3}$ rad/s.
- (D) A lead compensator that provides an amplification of 20 dB and a phase lead of 45° at the frequency of 3 rad/s.

Solution: (B)

Phase crossover frequency of the system can be calculated as

$$\angle G(j\omega) \Big|_{\omega=\omega_{pc}} = -180$$

$$\omega_{pc} = \frac{1}{\sqrt{T_1 T_2}} = \frac{1}{\sqrt{1 \times \frac{1}{9}}} = 3 \text{ rad/sec}$$

Gain margin of the system (GM)

$$= 20 \log \frac{1}{|G(j\omega_{pc})H(j\omega_{pc})|}$$

$$|G(j\omega_{pc})H(j\omega_{pc})| = \left| \frac{900}{s(s+1)(s+9)} \right|$$

$$\left| \frac{900}{\omega \sqrt{(\omega^2+1)} \sqrt{\omega^2+9^2}} \right| = 10$$

$$\text{Gainmargin(GM)} = 20 \log \frac{1}{|G(j\omega)H(j\omega)|} = 20 \text{ dB}$$

From the given data, phase crossover frequency is equal to gain crossover frequency. For this we need to make the magnitude of the system at $\omega = \omega_{pc}$ equal to zero. A lag compensator is used to reduce the gain of the system by 20 dB at $\omega = \omega_{pc}$.

To provide phase margin of 45° , we need to increase the phase angle of the system by 45° which is 0° at $\omega = \omega_{pc}$ 3 rad/s. A Lead compensator is used to obtain 45° of phase margin at $\omega = \omega_{pc} = 3$ rad/s.

EXERCISES

Practice Problems I

Directions for questions 1 to 9: Select the correct alternative from the given choices.

1. Match List-I with List-II and select the correct answer using the codes given below the lists:

List-I	List-II
(A) Phase lag controller	(1) Improvement in transient response
(B) Addition of zero at origin	(2) Reduction in steady-state error
(C) Derivative output compensation	(3) Reduction in settling time
(D) Derivative error compensation	(4) Increase in damping constant

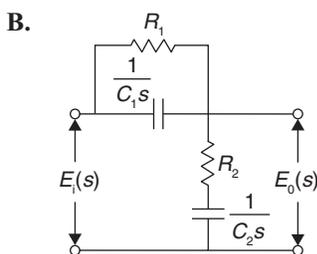
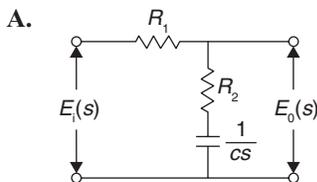
Codes:

	A	B	C	D
(a)	4	3	1	2
(b)	2	1	3	4
(c)	4	1	3	2
(d)	2	3	1	4

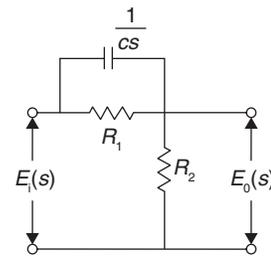
2. The effect of phase lead compensator on gain cross over frequency (ω_{gc}) and on bandwidth (ω_b) is that
- both increases.
 - ω_{gc} increases but ω_b decreases.
 - ω_{gc} decreases but ω_b increases.
 - both decreases.

Match List-I (circuits) with List-II and select the correct answer using the codes given below the lists for questions (3) and (4).

3.



C.

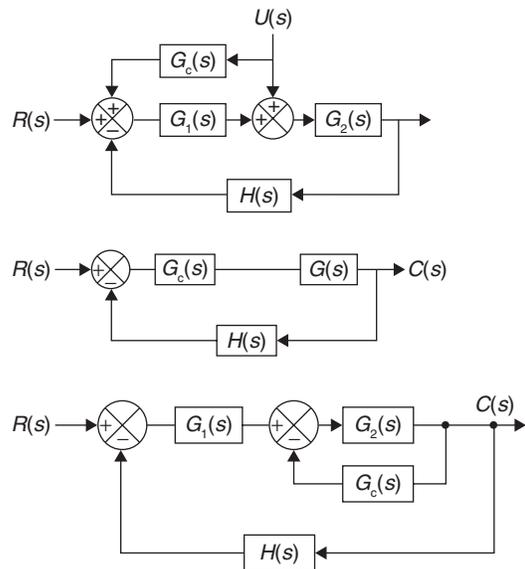


List-II

- Lag network
- Lead network
- Lag-lead network

	A	B	C
(A)	1	2	3
(B)	1	3	2
(C)	2	3	1
(D)	2	1	3

4.



List-II

- Cascade compensation
- Feedback compensation
- Feedforward compensation

	A	B	C
(A)	1	2	3
(B)	2	1	3
(C)	2	3	1
(D)	3	1	2

3.1052 | Control Systems

- (A) $Z_1 > P_1$ and $Z_2 > P_2$ (B) $Z_1 > P_1$ and $Z_2 < P_2$
 (C) $Z_1 < P_1$ and $Z_2 > P_2$ (D) $Z_1 < P_1$ and $Z_2 < P_2$

16. A system gain crossover frequency is less than its phase crossover frequency then
 (A) Lag-lead compensator which will decrease the gain and increase the phase angle can be used to stabilize the system.
 (B) Lead-lag compensator which will increase the gain and decrease the phase angle can be used to stabilize the system.
 (C) Lead compensator which will increase the gain can stabilize the system.
 (D) None of the above.

17. Match the following

Transfer Function	Type of Damping
1. PI Controller	P. Improves damping of the system
2. PD controller	Q. Increases bandwidth
3. Lead compensator	R. Decreases bandwidth
4. Lag compensator	S. Decreases steady-state error

Code:

- | | P | Q | R | S |
|-----|---|---|---|---|
| (A) | 2 | 4 | 1 | 3 |
| (B) | 3 | 2 | 4 | 1 |
| (C) | 2 | 3 | 4 | 1 |
| (D) | 3 | 4 | 1 | 2 |

18. Transfer function of a compensator is given by $\frac{10(s + .001)}{(s + 0.1)}$. The compensator offers maximum frequency at
 (A) 0.01 rad/s
 (B) 1 rad/s
 (C) 10 rad/s
 (D) 100 rad/s

Practice Problems 2

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. Which of the following are effects of PD control?

- (i) Increases rise time and setting time.
 - (ii) Improves GM, PM, and M_r .
 - (iii) Decreases bandwidth.
 - (iv) Improves damping and reduces maximum overshoot.
- (A) i, ii and iii (B) ii and iv
 (C) i, iii and iv (D) i, and iv

2. Which of the following are effects of PI control?

- (i) Filters out high-frequency noise.
 - (ii) Increases bandwidth.
 - (iii) Improves damping and reduces maximum overshoot.
 - (iv) By proper design PI control can improve transient and the steady-state performances.
- (A) i, ii and iii (B) ii and iii
 (C) iii and iv (D) i, iii and iv

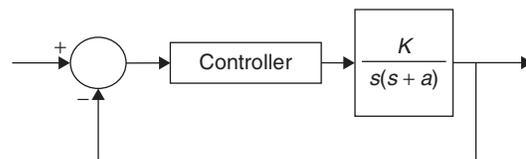
3. Phase-lag compensation results in

- (i) Increase in gain-crossover frequency.
 - (ii) Reduction of bandwidth.
 - (iii) More sensitivity.
 - (iv) Improvement of the relative stability.
- (A) i, ii and iv (B) ii and iv
 (C) ii, iii and iv (D) iii and iv

4. A double integrator plant $G(s) = \frac{k}{s^2}$, $H(s) = 1$ is to be compensated to achieve the damping ratio $\zeta = 0.5$ and an undamped natural frequency $\omega_n = 5$ rad/s. Which one of the following compensator $G_c(s)$ will be suitable?

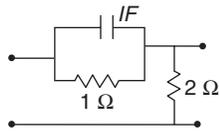
- (A) $\frac{(s+3)}{(s+9.9)}$ (B) $\frac{(s+9.9)}{(s+3)}$
 (C) $\frac{(s-6)}{(s+8.33)}$ (D) $\frac{(s+6)}{(s+9.99)}$

5. In the control system shown in given figure, the controller which can give zero steady-state error to a ramp input, with $k = 9$ is



- (A) Proportional type
 (B) Integral type
 (C) Derivative type
 (D) PD type
6. The transfer function of a simple RC network functioning as a controller $G_c(s) = \frac{s + z_1}{s + P_1}$; the condition of RC network to act as a phase lead controller is
 (A) $P_1 < z_1$ (B) $P_1 = 0$
 (C) $P_1 = z_1$ (D) $P_1 > z_1$

7. For a given phase-lead network, the maximum possible phase-lead is



- (A) 90° (B) 45°
(C) 30° (D) 15°

8. Consider the following statements: In a feedback control system, lead compensator

- (1) Speeds up the transient response
(2) High-frequency gain increases
(3) Bandwidth increases. of these statements, which one is correct?
(A) (1) and (2)
(B) (1) and (3)
(C) (2) and (3)
(D) (1), (2) and (3)

9. The effect of cascade lag compensation on the transient response of a control system can be neutralized by choosing

- (A) A slightly higher value of the static position error constant
(B) A slightly higher value of the static velocity error constant
(C) A slightly higher value of damping ratio.
(D) A slightly higher value of undamped natural frequency

10. Select the statements regarding the properties of phase-lead compensation.

- (1) It improves phase margin of the closed loop system.
(2) It increases bandwidth of the closed loop system.
(3) It gives slow response.

Among these, which one is correct?

- (A) (1) and (2)

- (B) (1) and (3)
(C) (2) and (3)
(D) (1), (2) and (3)

11. Maximum phase lead of $\frac{4(1+0.15s)}{(1+0.05s)}$ is

- (A) 45° (B) 60°
(C) 30° (D) 90°

12. The maximum value of phase lead for which a single stage cascade lead compensator should be designed is

- (A) 90° (B) 65°
(C) 135° (D) 180°

13. For $G_c(s) = K \frac{1+6s}{1+2s}$.

The minimum phase lead and corresponding frequencies are

- (A) $45^\circ, \frac{1}{2\sqrt{3}}$ (B) $30^\circ, \frac{1}{2}$
(C) $30^\circ, \frac{1}{2\sqrt{3}}$ (D) $45^\circ, \frac{1}{2}$

14. The open loop transfer function of a plant is given as

$G(s) = \frac{1}{s^2 - 1}$. If the plant is operated in a unity feedback configuration, then the lead compensator that can stabilize this control system is

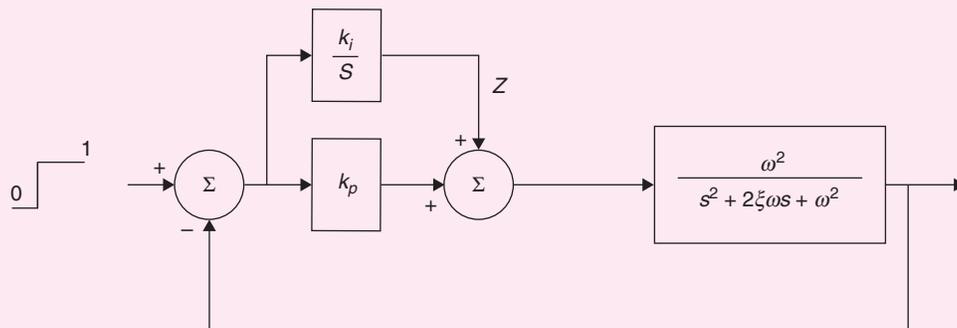
- (A) $\frac{10(s-1)}{s+2}$ (B) $\frac{10(s+4)}{s+2}$
(C) $\frac{10(s+2)}{s+10}$ (D) $\frac{10(s+10)}{s+2}$

15. The transfer function of a phase lead compensator is $\frac{1+3Ts}{1+Ts}$. The maximum value of phase provided by this compensator is

- (A) 90° (B) 60°
(C) 45° (D) 30°

PREVIOUS YEARS' QUESTIONS

1. Consider the feedback control system shown below which is subjected to a unit step input. The system is stable and has the following parameters $k_p = 4, k_i = 10, \omega = 500$ and $\xi = 0.7$.



The steady-state value of z is [2007]
 (A) 1 (B) 0.25
 (C) 0.1 (D) 0

2. The system $900/s(s+1)(s+9)$ is to be compensated such that its gain-crossover frequency becomes same as its uncompensated phase-crossover frequency and provides a 45° phase margin. To achieve this, one may use [2007]
- (A) A lag compensator that provides an attenuation of 20 dB and a phase lag of 45° at the frequency of $3\sqrt{3}$ rad/s.
 - (B) A lead compensator that provides an amplification of 20 dB and a phase lead of 45° at the frequency of 3 rad/s.
 - (C) A lag-lead compensator that provides an amplification of 20 dB and a phase lag of 45° at the frequency of $\sqrt{3}$ rad/s.
 - (D) A lag-lead compensator that provides an attenuation of 20 dB and phase lead of 45° at the frequency of 3 rad/s.

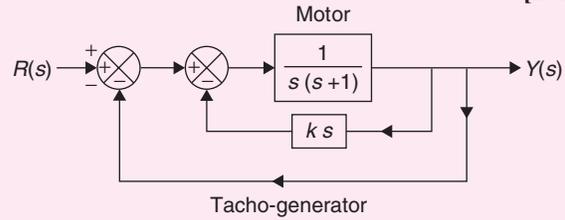
3. The transfer function of two compensators are given below.

$$C_1 = \frac{10(s+1)}{(s+10)}, C_2 = \frac{s+10}{10(s+1)}$$

Which one of the following statements is correct? [2008]

- (A) C_1 is a lead compensator and C_2 is a lag compensator.
- (B) C_1 is a lag compensator and C_2 is a lead compensator.
- (C) Both C_1 and C_2 are lead compensators.
- (D) Both C_1 and C_2 are lag compensators.

4. A two-loop position control system is shown below. [2011]



The gain k of the **Tacho**-generator influences mainly the

- (A) Peak overshoot
 - (B) Natural frequency of oscillation
 - (C) Phase shift of the closed loop transfer function at very low frequencies ($\omega \rightarrow 0$)
 - (D) Phase shift of the closed loop transfer function at very high frequencies ($\omega \rightarrow \infty$)
5. An open loop system represented by the transfer function $G(s) = \frac{(s-1)}{(s+2)(s+3)}$ is [2011]
- (A) Stable and of the minimum phase type
 - (B) Stable and of the non-minimum phase type
 - (C) Unstable and of the minimum phase type
 - (D) Unstable and of the non-minimum phase type

Common Data for Questions 6 and 7:

The transfer function of a compensator is given as

$$G_c(s) = \frac{s+a}{s+b}$$

6. $G_c(s)$ is a lead compensator if [2012]
- (A) $a = 1, b = 2$ (B) $a = 3, b = 2$
 - (C) $a = -3, b = -1$ (D) $a = 3, b = 1$
7. The phase of the above lead compensator is maximum at [2012]
- (A) $\sqrt{2}$ rad/s (B) $\sqrt{3}$ rad/s
 - (C) $\sqrt{6}$ rad/s (D) $1/\sqrt{3}$ rad/s

ANSWER KEYS

EXERCISES

Practice Problems 1

1. B 2. A 3. B 4. D 5. B 6. D 7. B 8. B 9. B 10. A
 11. A 12. C 13. B 14. D 15. B 16. A 17. C 18. A

Practice Problems 2

1. B 2. D 3. C 4. A 5. B 6. D 7. C 8. D 9. D 10. A
 11. C 12. B 13. C 14. A 15. D

Previous Years' Questions

1. A 2. D 3. A 4. A 5. B 6. A 7. A