Chapter 10. Quadratic And Exponential Functions

Answer 1PT3.

Consider the data:

(1) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (a) Exponential decay equation

(2) $y = c(1+r)^t$ (b) Exponential growth equation

(3) $y = c(1-r)^t$ (c) Quadratic formula

The objective is to choose the letter of the term that matches given formula.

The formula for Exponential growth is

$y = c(1+r)^t$, where ' y ' represents the final amount, ' c ' represents the initial amount, ' r ' represents the rate of change expressed as a decimal, and ' t ' represents time.

The formula for exponential decay is

$y = c(1-r)^t$, where ' y ' represents the final amount, ' c ' represents the initial amount, ' r ' represents the rate of change expressed as a decimal, and ' t ' represents time.

The Quadratic formula for the quadratic equation:

$$ax^2 + bx + c = 0$$

$$a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = 0$$ Take a as common.

$$x^2 + 2\left(\frac{b}{2a}\right)x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = 0$$

$$\left(x + \frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{(2a)^2}$$

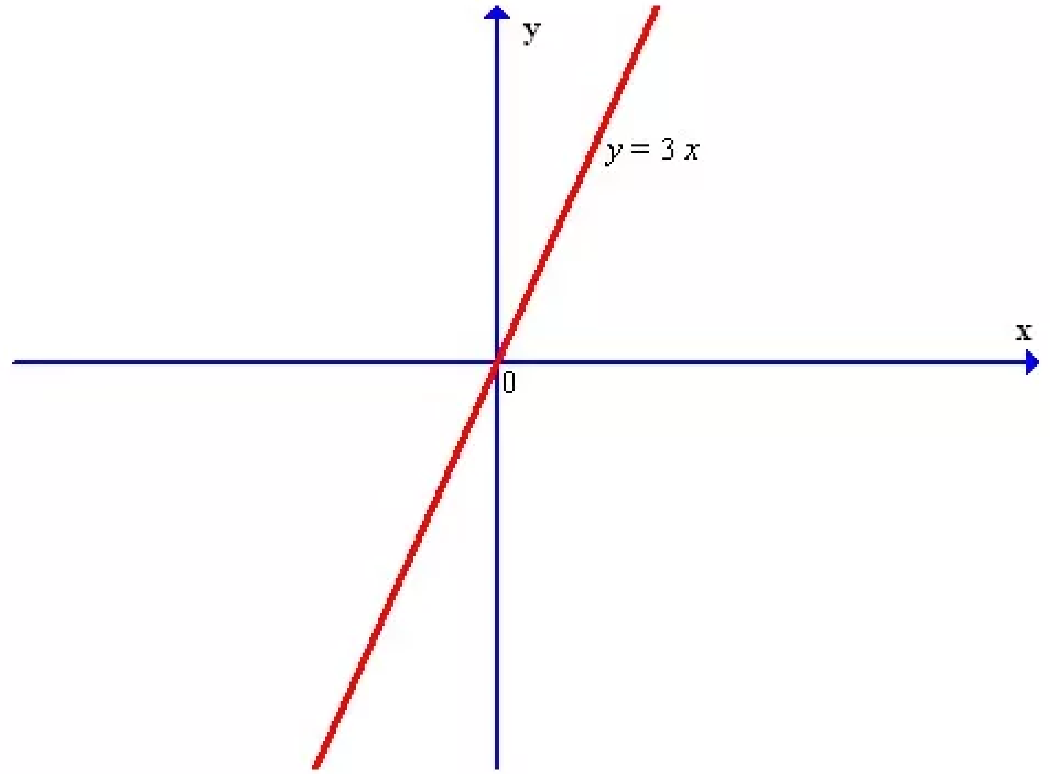
$$\left(x + \frac{b}{2a}\right) = \frac{\pm\sqrt{b^2 - 4ac}}{(2a)}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Therefore, the letter of the term that matches given formulae is $\boxed{1-(c), 2-(b), 3-(a)}$.

Answer 1STP.

Consider the following graph of the function $y = 3x$



The objective is to write the equation when the line is translated 2 units down.

When the line

$y = 3x$ is translated 2 units down, then the new line thus formed is

$y = 3x - 2$

Since from the graph of

$y = f(x)$, the graph of

$y = f(x) - c$ is shifted down ' c ' units.

Thus, the equation obtained when

$y = 3x$ is translated 2 units down is

$y = 3x - 2$

Therefore, the answer is <u>option(B)</u>.

Answer 2STP.

Consider $a = 21$ when

$b = 6$

' a ' varies directly as ' b '

The objective is to find ' a ' when

$b = 28$

The word ' y ' varies directly with ' x ' means that

$y = kx$, where ' k ' is constant of variation.

Then, the word ' a ' varies directly as ' b ' means

$a = kb$ where ' k ' is constant of variation.

Step (1): First find the value of ' k ' when

$a = 21,$
$b = 6$

$21 = k(6)$ (Substitute $a = 21, b = 6$ in $a = kb$)

$\frac{21}{6} = k$ (Divide both sides by 6)

Step (2): Substitute

$k = \frac{21}{6}$ in

$a = kb$ then

$a = \frac{21}{6}b$

Step (3): Substitute the value of ' b ' in

$a = \frac{21}{6}b$ to find the value of ' a '

$a = \frac{21}{6}(28)$ (Substitute $b = 28$ in $a = \frac{21}{6}b$)

$= \frac{3 \cdot 7 \cdot 2 \cdot 7 \cdot 2}{2 \cdot 3}$

(Write numerator and denominator as product of factors)

$= 98$ (Divide out common factors and multiply)

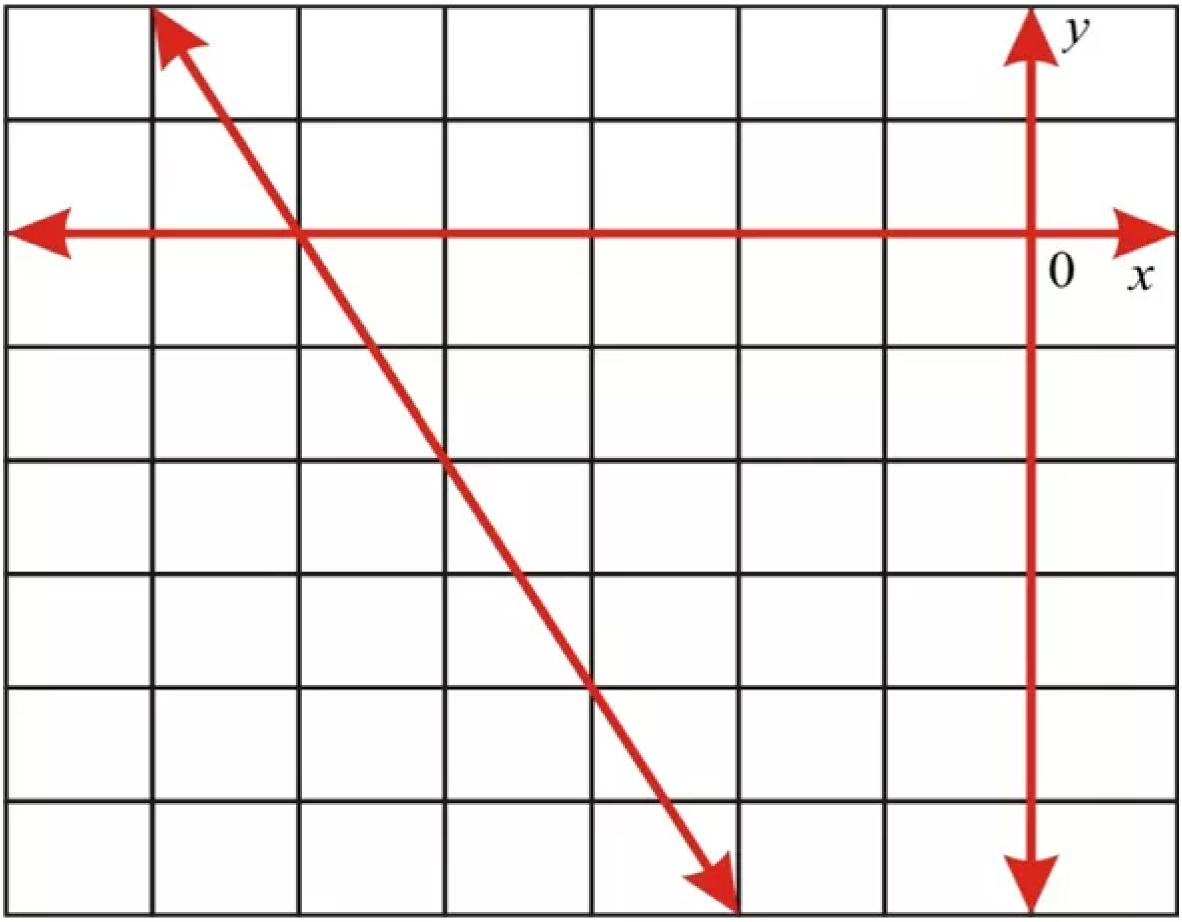
Therefore, the value of ' a ' is $\boxed{98}$ when

$b = 28$

Hence, the answer is option (C):

Answer 3STP.

Consider the following graph.



The objective is to find the equation that represents the given graph among the given options.

$(A)\quad y=-2x-10$

$(B)\quad y=-2x-5$

$(C)\quad y=2x+10$

$(D)\quad y=2x-5$

From the graph, line passes through the points $(-5,0)$ and $(-4,-2)$.

Substituting these points in the given options.

Consider

$y=-2x-10$ (Option (A))

$\Rightarrow \quad 0=-2(-5)-10$ (Replace $(x,y)=(-5,0)$)

$\Rightarrow \quad 0=0$ (True)

And $y=-2x-10$ (Option (A))

$\Rightarrow \quad -2=-2(-4)-10$ (Replace $(x,y)=(-4,-2)$)

$\Rightarrow \quad -2=-2$ (True)

Therefore, the equation that represents the given graph is

$\boxed{(A)\quad y=-2x-10}$.

Answer 4PT.

Consider the equation

$$y = x^2 - 4x + 13$$

The objective is to write the equation of the axis of symmetry, and to find the coordinates of the vertex of the graph of the given function.

And identify the vertex as a maximum or minimum and then graph the function.

The standard form of a quadratic function is

$$y = ax^2 + bx + c$$

The equation of the axis of symmetry for the graph of

$y = ax^2 + bx + c$, where

$a \neq 0$, is

$$x = \frac{-b}{2a}$$

In the equation

$y = x^2 - 4x + 3,$ and

$a = 1$

$b = -4$

Substitute these values into the of the axis of symmetry.

$x = \frac{-b}{2a}$ (Equation of the axis of symmetry)

$= \frac{-(-4)}{2(1)}$ (Replace $a = 1$ and $b = -4$)

$= 2$ (Simplify)

The equation of the axis of symmetry is

$$x = 2$$

Since the equation of the axis of symmetry is

$x = 2$ and the vertex lies on the axis, the $x-$coordinate for the vertex is 2.

$y = x^2 - 4x + 13$ (Original equation)

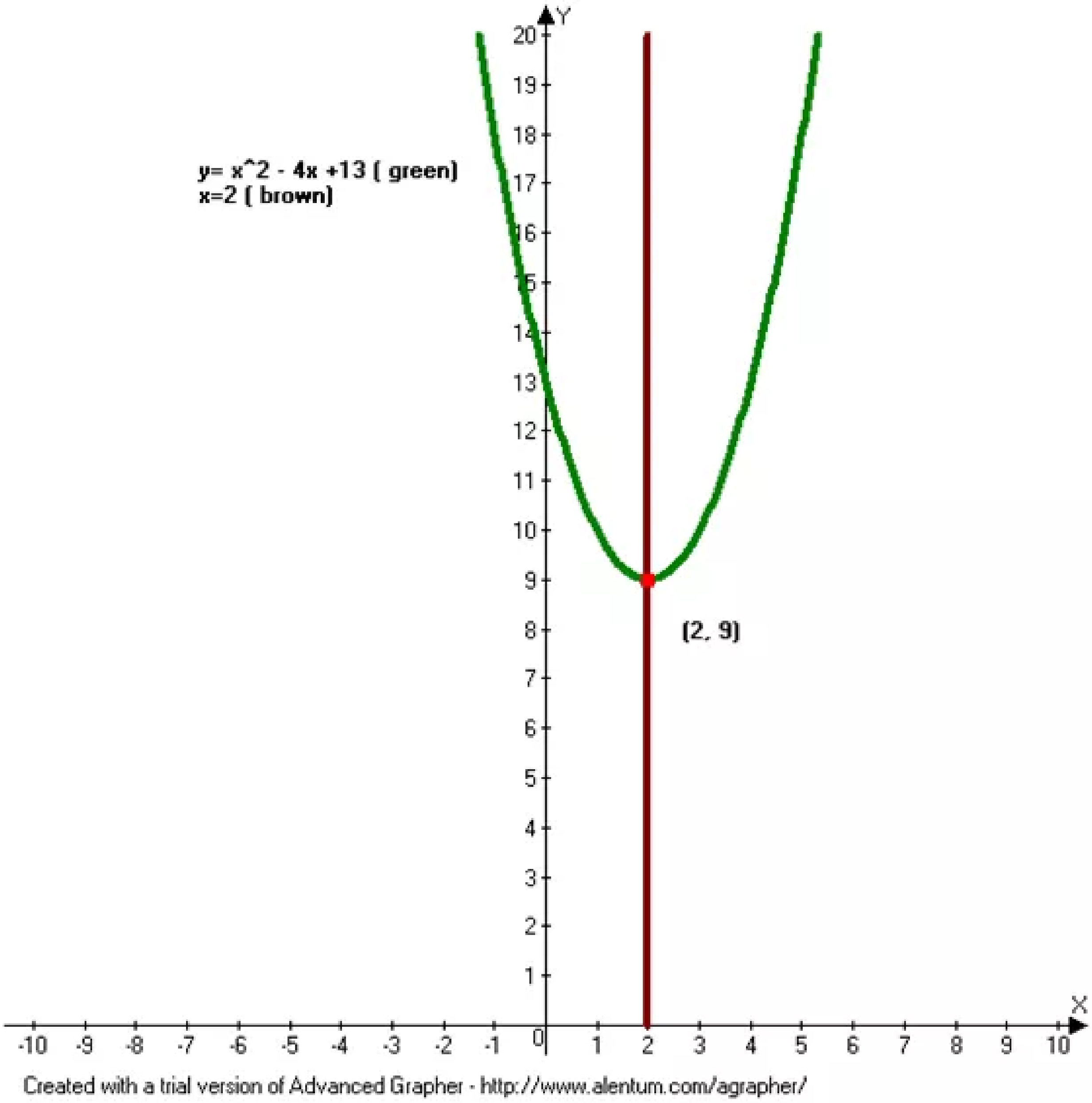
$\Rightarrow \quad y = (2)^2 - 4(2) + 13$ (Replace $x = 2$)

$\Rightarrow \quad y = 4 - 8 + 13$ (Simplify)

$\Rightarrow \quad y = 9$ (Add)

The vertex is at $(2, 9)$.

Since the coefficient of x^2 term is positive, the parabola opens upward and the vertex is a minimum point.



Therefore, the axis of symmetry and the coordinates of the vertex of the graph are

$\boxed{x = 2; (2, 9)}$ minimum point.

Answer 5PT.

Consider the equation

$$y = -3x^2 - 6x + 4$$

The objective is to write the equation of the axis of symmetry, and to find the coordinates of the vertex of the graph of the given function.

And identify the vertex as a maximum or minimum and then graph the function.

The standard form of a quadratic function is

$$y = ax^2 + bx + c$$

The equation of the axis of symmetry for the graph of

$y = ax^2 + bx + c$, where

$a \neq 0$, is

$$x = \frac{-b}{2a}$$

In the equation

$y = -3x^2 - 6x + 4$, and

$a = -3$

$b = -6$

Substitute these values into the of the axis of symmetry.

$x = \frac{-b}{2a}$ (Equation of the axis of symmetry)

$= \frac{-(-6)}{2(-3)}$ (Replace $a = -3$ and $b = -6$)

$= -1$ (Simplify)

The equation of the axis of symmetry is

$$x = -1$$

Since the equation of the axis of symmetry is

$x = -1$ and the vertex lies on the axis, the $x-$coordinate for the vertex is -1.

$y = -3x^2 - 6x + 4$ (Original equation)

$\Rightarrow \quad y = (-3)(-1)^2 - 6(-1) + 4$

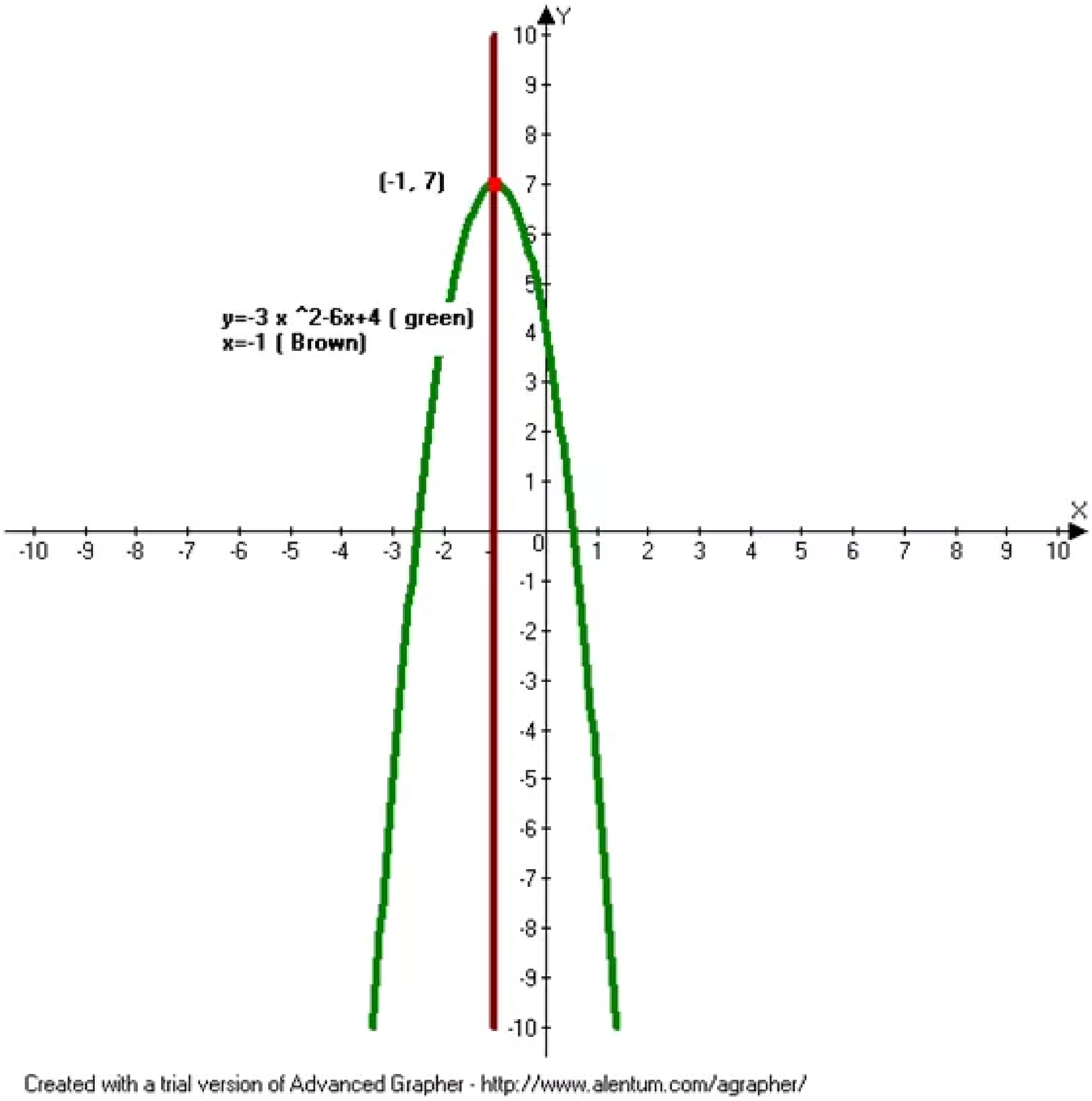
(Replace $x = -1$)

$\Rightarrow \quad y = -3 + 6 + 4$ (Simplify)

$\Rightarrow \quad y = 7$ (Add)

The vertex is at $(-1, 7)$.

Since the coefficient of x^2 term is positive, the parabola opens upward and the vertex is a minimum point.



Therefore, the axis of symmetry and the coordinates of the vertex of the graph are

$\boxed{x=-1;(-1,7)}$ maximum point.

Answer 6PT.

Consider the equation

$y = 2x^2 + 3$

The objective is to write the equation of the axis of symmetry, and to find the coordinates of the vertex of the graph of the given function.

And identify the vertex as a maximum or minimum and then graph the function.

The standard form of a quadratic function is

$y = ax^2 + bx + c$

The equation of the axis of symmetry for the graph of

$y = ax^2 + bx + c$, where

$a \neq 0$, is

$x = \frac{-b}{2a}$

In the equation

$y = 2x^2 + 3$, and

$a = 2$

$b = 0$

Substitute these values into the of the axis of symmetry.

$x = \frac{-b}{2a}$ (Equation of the axis of symmetry)

$= \frac{-(0)}{2(2)}$ (Replace $a = 2$ and $b = 0$)

$= 0$ (Simplify)

The equation of the axis of symmetry is

$x = 0$

Since the equation of the axis of symmetry is

$x = 0$ and the vertex lies on the axis, the $x-$coordinate for the vertex is 0.

$y = 2x^2 + 3$ (Original equation)

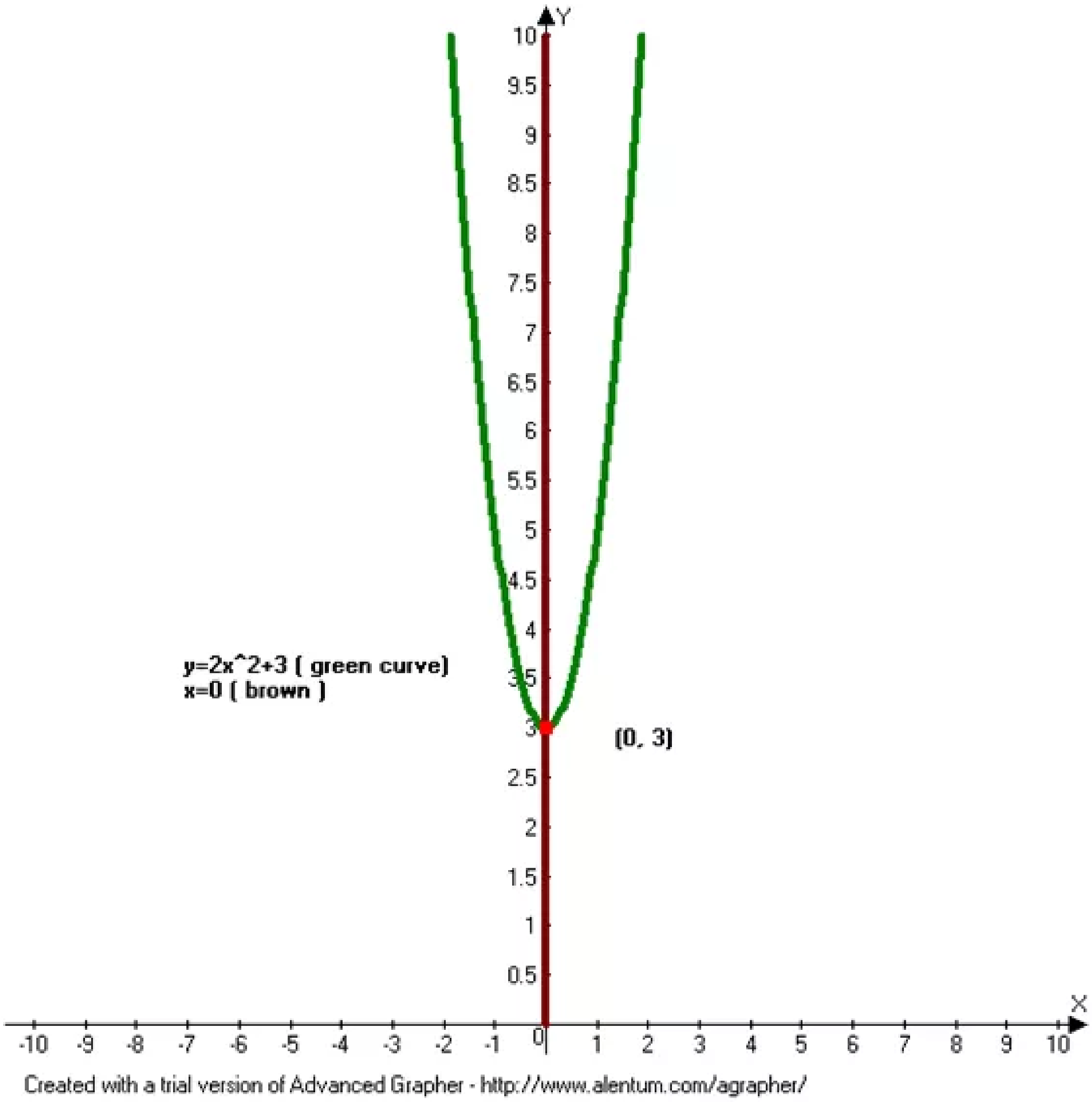
$\Rightarrow \quad y = 2(0)^2 + 3$ (Replace $x = 0$)

$\Rightarrow \quad y = 0 + 3$ (Simplify)

$\Rightarrow \quad y = 3$ (Add)

The vertex is at $(0,3)$.

Since the coefficient of x^2 term is positive, the parabola opens upward and the vertex is a minimum point.



Therefore, the axis of symmetry and the coordinates of the vertex of the graph are

$\boxed{x = 0; (0, 3)}$ minimum point.

Therefore, $x=\dfrac{-1\pm\sqrt{(1)^2-4(1)(-12)}}{2(1)}$

(Substitute $a=1, b=1$ and $c=-12$ in $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$)

$=\dfrac{-1\pm\sqrt{1+48}}{2}$

(Simplify)

$=\dfrac{-1\pm\sqrt{49}}{2}$

(Do addition: $1+48=49$)

$=\dfrac{-1\pm 7}{2}$ (Evaluate square - root)

$=\dfrac{-1+7}{2},\dfrac{-1-7}{2}$

$=\dfrac{6}{2},\dfrac{-8}{2}$ (Do addition: $-1+7=6,-1-7=-(1+7)=-8$)

$=3,-4$ (Divide)

Therefore, the solution set is $\{-4,3\}$.

Hence, the answer is option (B).

Answer 7PT.

Consider the equation

$$y=-1(x-2)^2+1$$

The objective is to write the equation of the axis of symmetry, and to find the coordinates of the vertex of the graph of the given function.

And identify the vertex as a maximum or minimum and then graph the function.

The standard form of a quadratic function is

$$y=ax^2+bx+c$$

The equation of the axis of symmetry for the graph of

$y=ax^2+bx+c$, where

$a\neq 0$, is

$$x=\frac{-b}{2a}$$

Consider

$y=-1(x-2)^2+1$ (Original equation)

$\Rightarrow \quad y=-x^2+4x-3$ (Simplify)

In the equation

$y=-x^2+4x-3,$ and

$a=-1$

$b=4$

Substitute these values into the equation of the axis of symmetry.

$x = \frac{-b}{2a}$ (Equation of the axis of symmetry)

$= \frac{-4}{2(-1)}$ (Replace $a = -1$ and $b = 4$)

$= 2$ (Simplify)

The equation of the axis of symmetry is

$x = 2$

Since the equation of the axis of symmetry is

$x = 2$ and the vertex lies on the axis, the $x-$coordinate for the vertex is 2.

$y = -1(x-2)^2 + 1$ (Original equation)

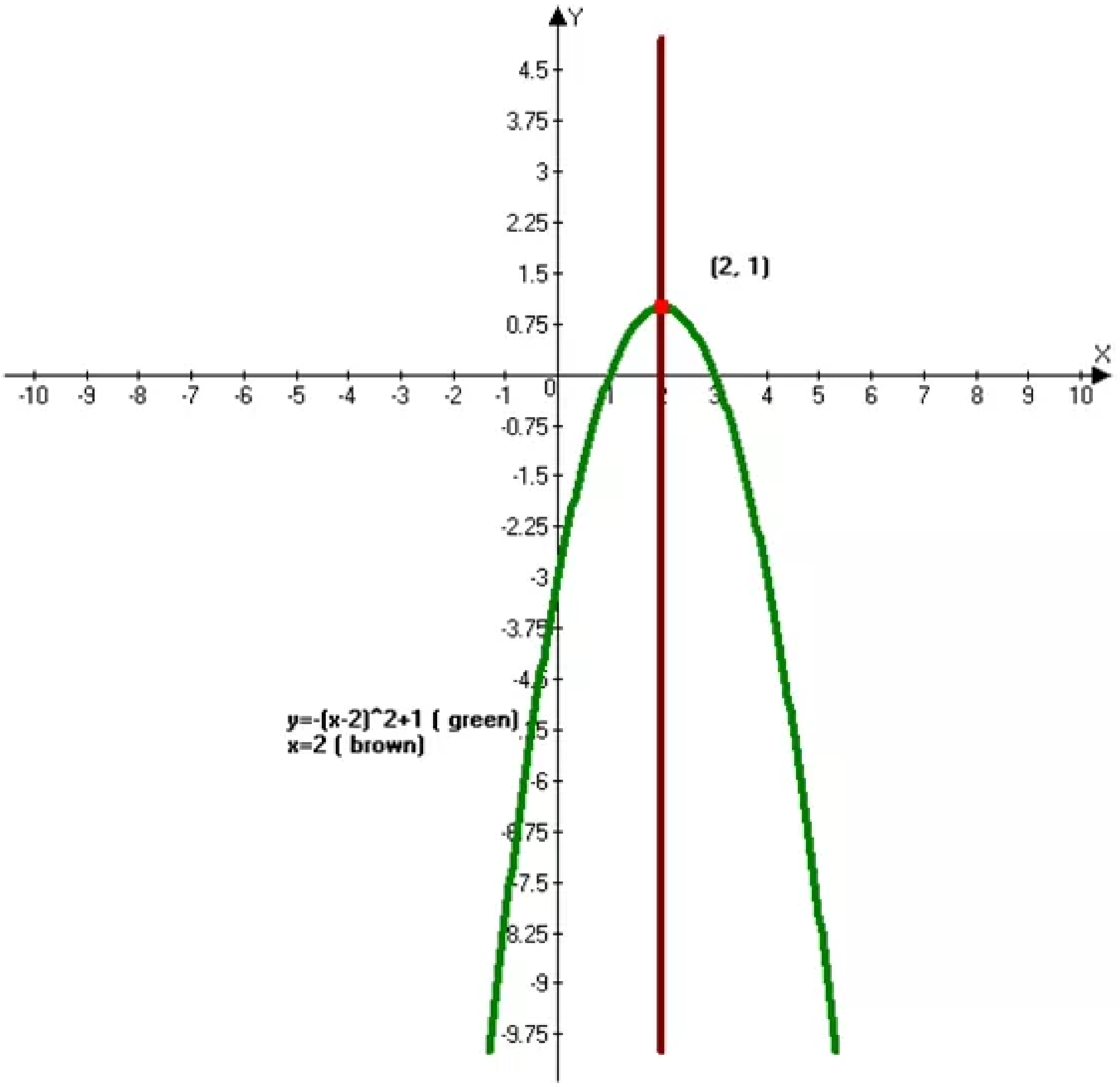
$\Rightarrow \quad y = -1(2-2)^2 + 1$ (Replace $x = 2$)

$\Rightarrow \quad y = 0 + 1$ (Simplify)

$\Rightarrow \quad y = 1$ (Add)

The vertex is at $(2,1)$.

Since the coefficient of x^2 term is positive, the parabola opens upward and the vertex is a minimum point.



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Therefore, the axis of symmetry and the coordinates of the vertex of the graph are

$\boxed{x = 2;(2,1)}$ maximum point.

Answer 7STP.

Consider the following data

x	y
-3	0
-1	8
0	9
2	5
3	0
4	-7

The objective is to find the equation that best represents the given data in the table among the options

$(A)\quad y=-x^2+3$

$(B)\quad y=-x^2+9$

$(C)\quad y=x^2-3$

$(D)\quad y=x^2+9$

Consider the equation

$y=-x^2+3$

Substitute the values of ' x ' and ' y ' from the table in

$y=-x^2+3$ to check whether

$y=-x^2+3$ fit for the given data.

When $x=-3,$

$y=0$

$0 \overset{?}{=} -(-3)^2 + 3$ (Substitute $x = -3, y = 0$ in $y = -x^2 + 3$)

$0 \overset{?}{=} -9 + 3$ (Evaluate exponent)

$0 \neq -6$ (Do subtraction: $-9 + 3 = -6$)

$(-3, 0)$ does not satisfies

$y = -x^2 + 3$

Therefore,

$y = -x^2 + 3$ does not fit for the given data.

Consider the eqution

$y = -x^2 + 3$:

When $x = -3,$

$y = 0$

$0 \overset{?}{=} (-3)^2 - 3$ (Substitute $x = -3, y = 0$ in $y = x^2 - 3$)

$0 \overset{?}{=} 9 - 3$ (Evaluate exponent)

$0 \neq 6$ (Do subtraction: $9 - 3 = 6$)

$(-3, 0)$ does not satisfied

$y = x^2 - 3$

Therefore,

$y = x^2 - 3$ does not fit for the given data.

Consider the equation

$y = x^2 + 9$:

When $x=-3,$

$y=0$

$0\stackrel{?}{=}(-3)^2+9$ (Substitute $x=-3, y=0$ in $y=x^2+9$)

$0\stackrel{?}{=}9+9$ (Evaluate exponent)

$0\neq 18$ (Do addition: $9+9=18$)

$(-3,0)$ does not satisfied

$y=x^2+9$

Therefore,

$y=x^2+9$ does not fit for the given data

Consider the equation

$y=-x^2+9$:

When $x=-3,$

$y=0$

$0\stackrel{?}{=}-(-3)^2+9$ (Substitute $x=-3, y=0$ in $y=-x^2+9$)

$0\stackrel{?}{=}-9+9$ (Evaluate exponent)

$0=0$

Thus, $(-3,0)$ satisfied

$y=-x^2+9$

When $x=-1,$

$y=8$

$8\stackrel{?}{=}-(-1)^2+9$ (Substitute $x=-1, y=8$ in $y=-x^2+9$)

$8\stackrel{?}{=}-1+9$ (Evaluate exponent: $(-1)^2=1$)

$8=+8$ (Do addition: $-1+9=8$)

Thus, $(-1,8)$ satisfied

$y=-x^2+9$

When $x=0$,

$y=9$

$9\stackrel{?}{=}-0^2+9$ (Substitute $x=0, y=9$ in $y=-x^2+9$)

$9\stackrel{?}{=}0+9$ (Evaluate exponent: $0^2=0$)

$9=9$ (Do addition: $0+9=9$)

Thus, $(0,9)$ satisfied

$y=-x^2+9$

When $x=2$,

$y=5$

$5\stackrel{?}{=}-2^2+9$ (Substitute $x=2, y=5$ in $y=-x^2+9$)

$5\stackrel{?}{=}-4+9$ (Evaluate exponent: $2^2=4$)

$5=5$

Thus, $(2,5)$ satisfied

$y=-x^2+9$

When $x=3$,

$y=0$

$0\stackrel{?}{=}-(3)^2+9$ (Substitute $x=3, y=0$ in $y=-x^2+9$)

$0\stackrel{?}{=}-9+9$ (Evaluate exponent: $3^2=9$)

$0=0$ (Do addition: $-9+9=0$)

Thus, $(3,0)$ satisfied

$y=-x^2+9$

When $x=4$,

$y=-7$

$-7\stackrel{?}{=}-4^2+9$ (Substitute $x=4, y=-7$ in $y=-x^2+9$)

$-7\stackrel{?}{=}-16+9$ (Evaluate exponent: $4^2=16$)

$-7=-7$ (Do addition: $-16+9=-7$)

Thus, $(4,-7)$ satisfied

$y=-x^2+9$.

Therefore,

$y=-x^2+9$ fit for the given data in the table.

Hence, the answer is option (B).

Answer 8PT.

Consider the equation

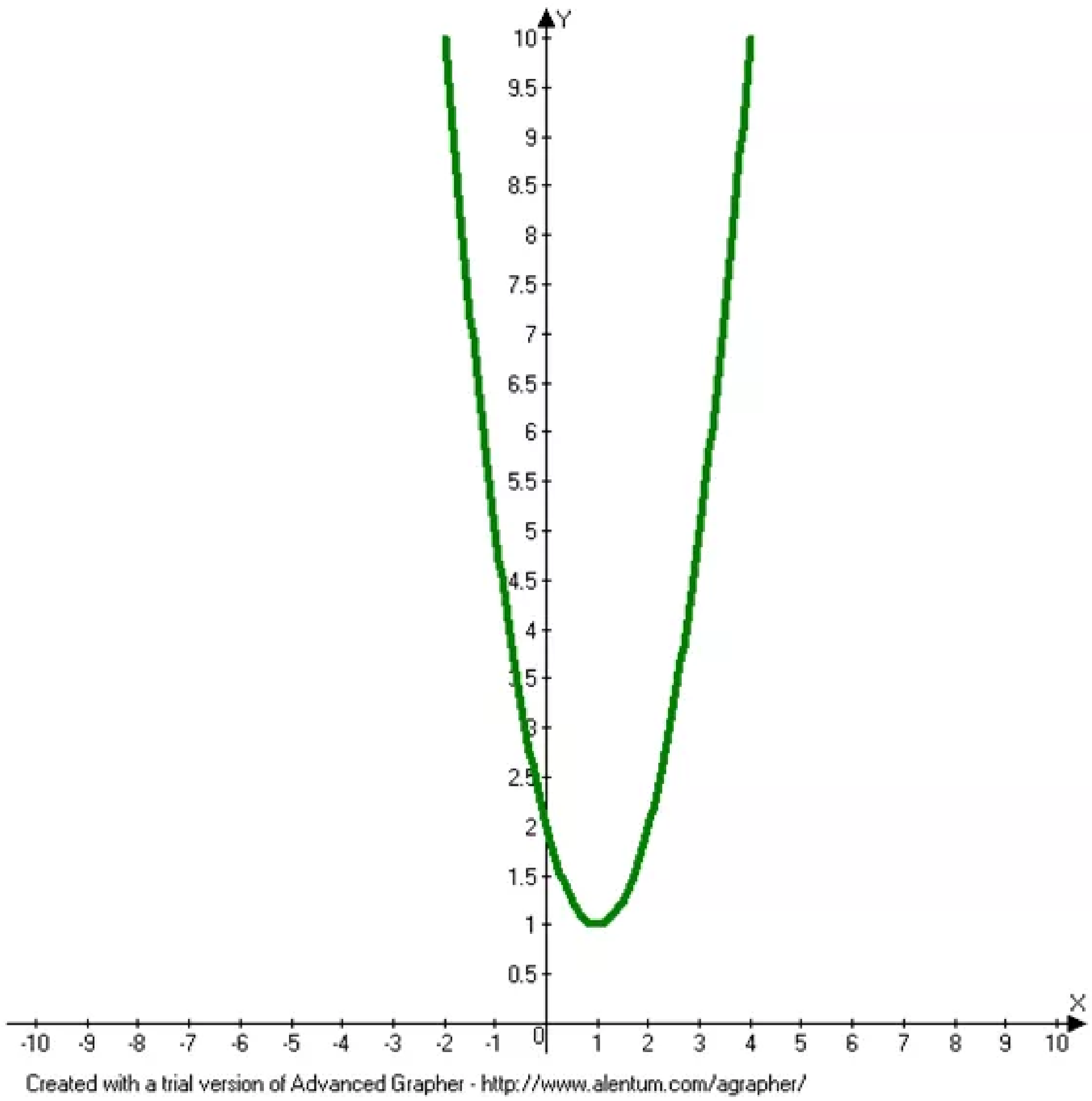
$$x^2 - 2x + 2 = 0$$

The objective is to solve the given equation by graphing. If the integral roots cannot be found, then to estimate the roots by stating the consecutive integers between which the roots lie.

Since the roots of a quadratic equation are the x – intercepts of the related quadratic function.

Graph the related function

$$f(x) = x^2 - 2x + 2$$



The graph has no $x-$intercept.

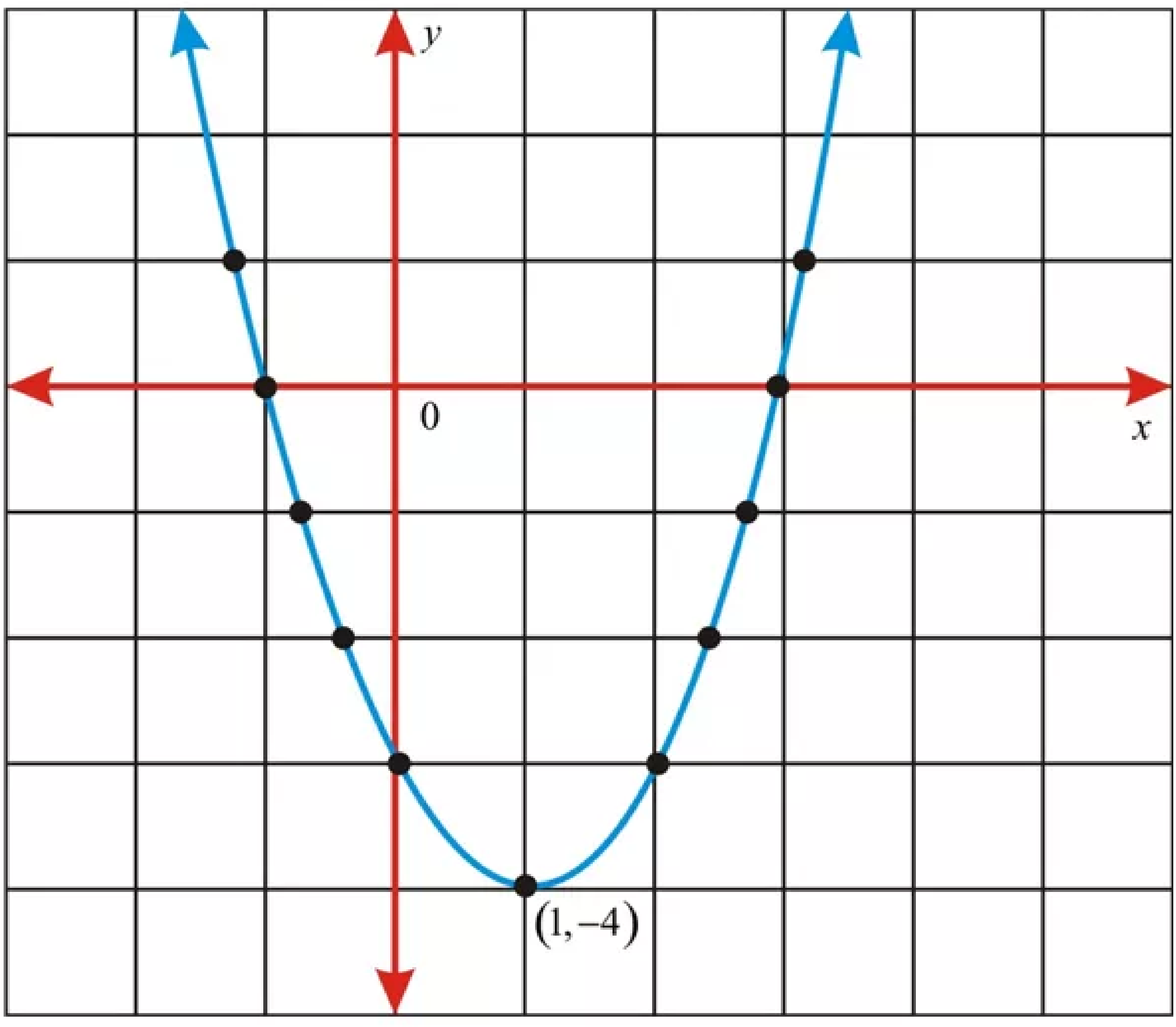
Thus there are no real number solutions for this equation.

x	$f(x)$
-1	5
0	2
1	1
2	2

Therefore, there are no real solutions for the given equation.

Answer 8STP.

Consider the graph of the parabola.



The objective is to find the equation that represents the parabola among the given options.

(A) $y=x^2-2x-4$

(B) $y=x^2-2x-3$

(C) $y=x^2+2x-3$

(D) $y=x^2+2x+3$

From the graph, the parabola passes through the points $(-1,0),(1,-4)$ and $(3,0)$.

Consider $y=x^2-2x-4$ (Option (A))

$\Rightarrow \quad 0=(-1)^2-2(-1)-4$ (Replace $(x,y)=(-1,0)$)

$\Rightarrow \quad 0=-1$ (False)

Consider $y=x^2-2x-3$ (Option (B))

$\Rightarrow \quad 0=(-1)^2-2(-1)-3$ (Replace $(x,y)=(-1,0)$)

$\Rightarrow \quad 0=0$ (True)

Consider $y=x^2-2x-3$ (Option (B))

$\Rightarrow \quad -4=(1)^2-2(1)-3$ (Replace $(x,y)=(1,-4)$)

$\Rightarrow \quad -4=-4$ (True)

Consider $y=x^2-2x-3$ (Option (B))

$\Rightarrow \quad 0=(3)^2-2(3)-3$ (Replace $(x,y)=(3,0)$)

$\Rightarrow \quad 0=0$ (True)

Therefore, the equation that represents the given graph is

$\boxed{(B) \quad y=x^2-2x-3}$.

Answer 9PT.

Consider the equation

$x^2+6x=-7$

$\Rightarrow \quad x^2+6x+7=0$

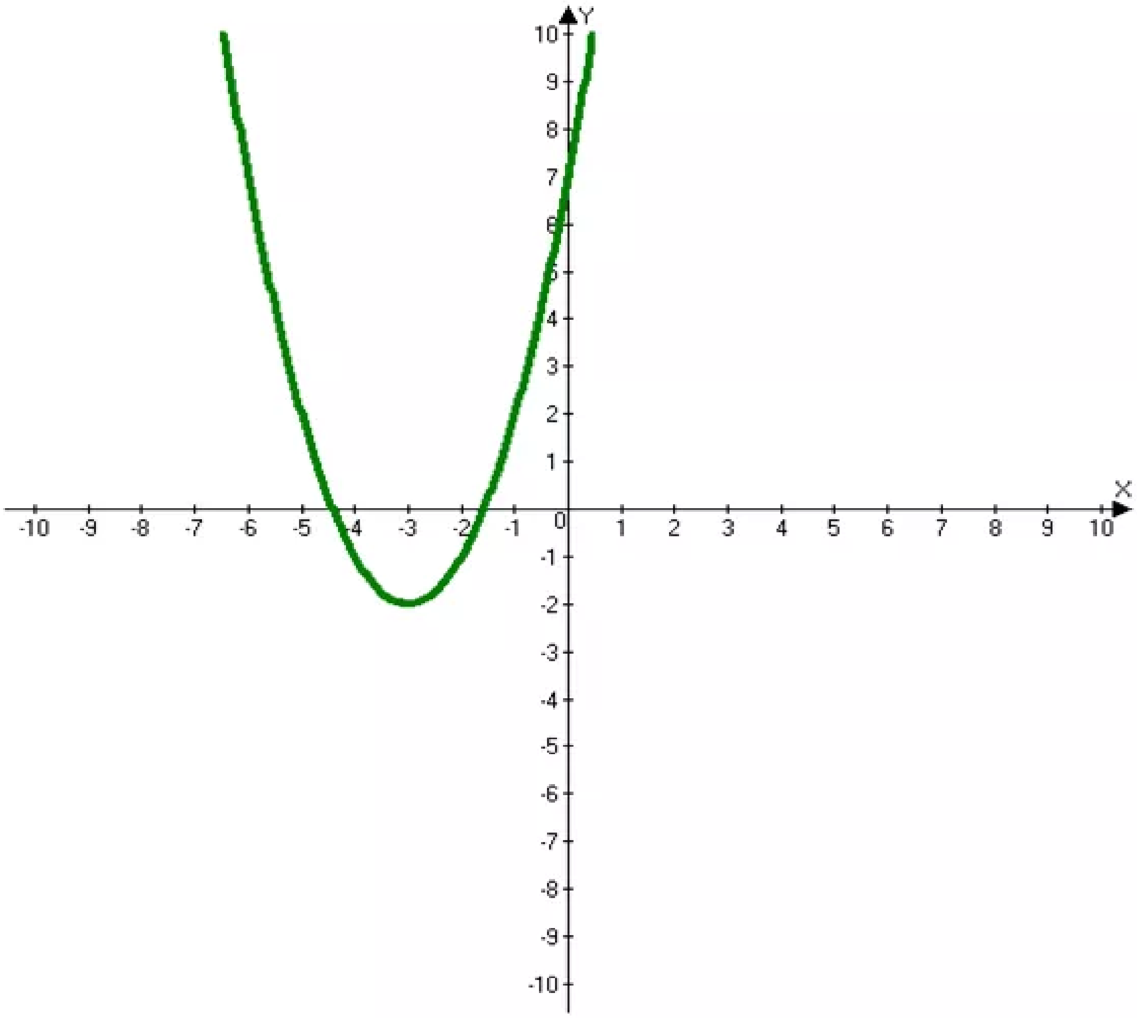
The objective is to solve the given equation by graphing.

If the integral roots cannot be found, then to estimate the roots by stating the consecutive integers between which the roots lie.

Since the roots of a quadratic equation are the $x-$intercepts of the related quadratic function.

Graph the related function

$f(x) = x^2 + 6x + 7$



From the graph, it is clear that the $x-$intercepts are not integers.

x	$f(x)$
-5	2
-4	-1
-3	-2
-2	-1
-1	2
0	7

The value of the function changes from negative to positive between the x values of -5 and -4 and between -2 and -1.

The $x-$intercepts of the graph are between -5 and -4, and between -2 and -1.

So, one root is between -5 and -4, and the other root is between -2 and -1.

Therefore, one root lies between $\boxed{-5}$ and $\boxed{-4}$ and the other root lies between $\boxed{-2}$ and $\boxed{-1}$.

Answer 9STP.

Consider the function $f(x)=2x^2+8x+6$

The objective is to find the points at which the graph of $f(x)$ intersect the $x-$axis.

Since, the graph of $f(x)$ intersect $x-$axis at the points where

$f(x)=0$

Equate $f(x)=0$ to get the values of 'x' where the graph of $f(x)$ intersect the $x-$axis.

$2x^2+8x+6=0$

$\Rightarrow \quad 2(x^2+4x+3)=0$ (Take out '2' as common factor)

$\Rightarrow \quad x^2+4x+3=0$

The solution of the equation

$ax^2+bx+c=0$ can be found by using the quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Compare the

$x^2+4x+3=0$ with

$ax^2+bx+c=0$

Then $a=1,$

$b=4$ and

$c=3$

Therefore, $x=\frac{-4\pm\sqrt{(4)^2-4(1)(3)}}{2(1)}$

(Substitute $a=1, b=4$ and $c=3$ in $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$)

$=\frac{-4\pm\sqrt{16-12}}{2}$

(Simplify)

$=\frac{-4\pm\sqrt{4}}{2}$ (Do subtraction: $16-12=4$)

$=\frac{-4\pm 2}{2}$ (Evaluate square - root)

$=\frac{-4+2}{2}, \frac{-4-2}{2}$

$=\frac{-2}{2}, \frac{-6}{2}$ (Do addition: $-4-2=-(4+2)=-6$, Do subtraction: $-4+2=-2$)

$=-1,-3$ (Divide)

Therefore, the graph of $f(x)$ intersect $x-$axis at $(-1,0)$ and $(-3,0)$.

Hence, the answer is option (B).

Answer 10PT.

Consider the equation

$x^2+24x+144=0$

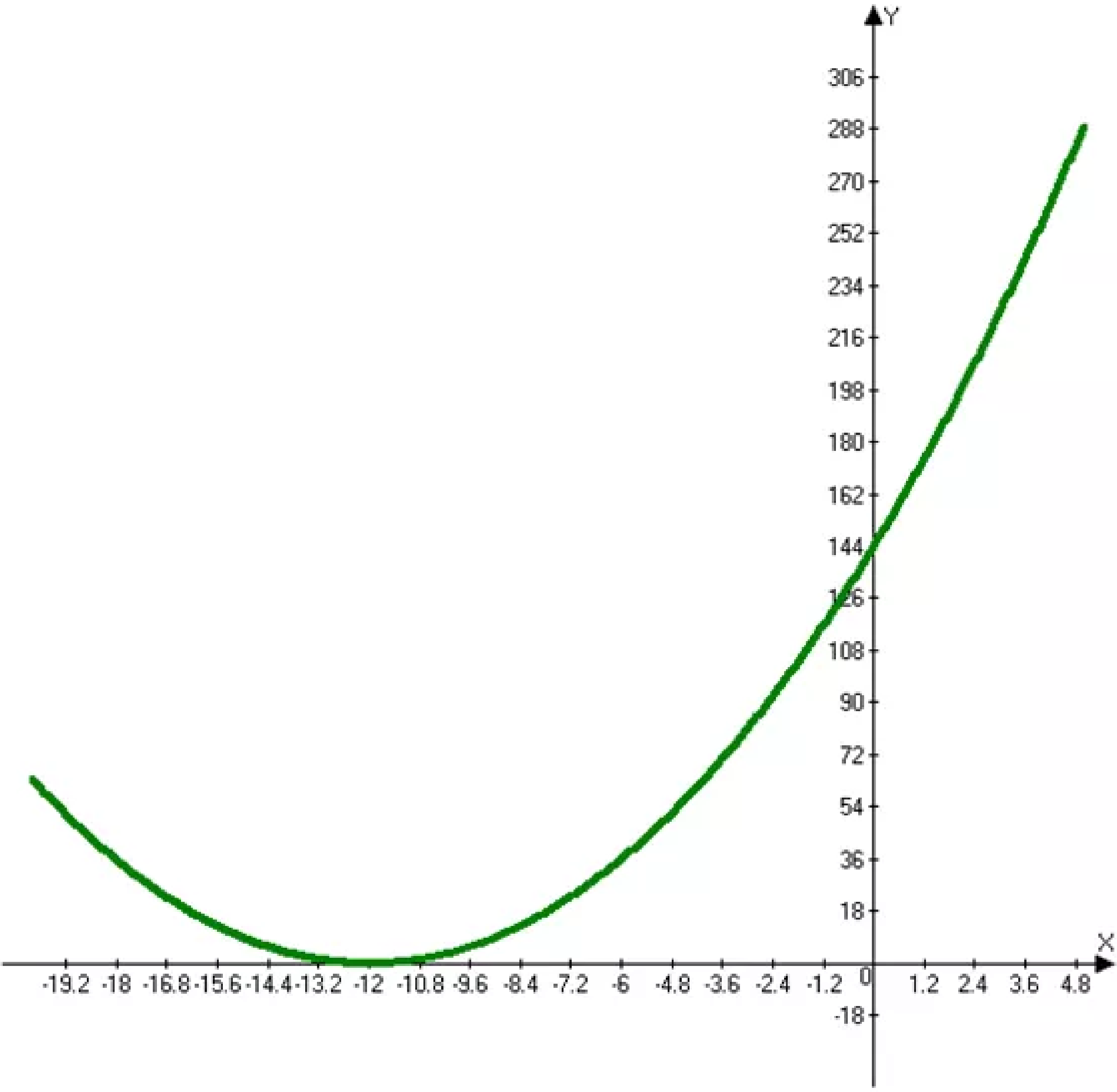
The objective is to solve the given equation by graphing.

If the integral roots cannot be found then to estimate the roots by stating the consecutive integers between which the roots lie.

Since the roots of a quadratic equation are the $x-$intercepts of the related quadratic function.

Graph the related function

$f(x)=x^2+24x+144$


Y
306
288
270
252
234
216
198
180
162
144
126
108
90
72
54
36
18
-18
X
-19.2 -18 -16.8 -15.6 -14.4 -13.2 -12 -10.8 -9.6 -8.4 -7.2 -6 -4.8 -3.6 -2.4 -1.2 0 1.2 2.4 3.6 4.8

x	$f(x)$
-14	4
-13	1
-12	0
-11	1
-10	4

Note that the vertex of the parabola is the $x-$intercept.

Thus, one solution is -12.

To find the other solution, solve the equation by factoring.

$x^2+24x+144=0$ (Original equation)

$\Rightarrow \quad (x+12)(x+12)=0$ (Factor)

$\Rightarrow \quad x+12=0$

Or, $x+12=0$ (Zero product property)

$\Rightarrow \quad x=-12$

Or, $x=-12$ (Solve for x)

There are two identical factors for the quadratic function.

So there is only one root, called a double root.

Therefore, the solution is $\boxed{-12}$.

Answer 10STP.

Consider the following data

Monica earned $\$18.50, \$23.00$ and $\$15.00$

Mowing lawns for 3 consecutive weeks.

She wanted to earn an average of at least $\$18$ per week.

The objective is to find the minimum that she should earn during the 4th week to meet her goal.

Since Monica wanted to earn at least $\$18$ per week, she should earn at least

$4(\$18) = \$72$ for 4 weeks.

Total amount she earned for 3 weeks.

$=$ Amount earned for 1st week $+$ Amount earned for 2nd week $+$ Amount earned for 3rd week.

$= \$18.50 + \$23.00 + \$15.00$

$= \$56.5$ (Do addition)

Amount she should earn 4th week.

$=$ Total amount she should earn for 4 weeks $-$ Total amount she earned for 3 weeks.

$= \$72 - \$56.5$

$= \$15.5$ (Do subtraction)

Therefore, she should earn minimum of $\boxed{\$15.5}$ during the 4th week to meet her goal.

Answer 11E.

Consider the equation

$y = x^2 + 2x$

The objective is to write the equation of the axis of symmetry, and to find the coordinates of the vertex of the graph of the give function. And to indentify the vertex as a maximum or minimum and then graph the function.

The standard form of a quadratic function is

$y = x^2 + 2x$

The equation of the axis of symmetry for the graph of

$y = ax^2 + bx + c$, where

$a \neq 0$, is

$$x = \frac{-b}{2a}$$

In the equation

$y = x^2 + 2x$, and

$a = 1$

$b = 2$

Substitute these values into the equation of the axis of symmetry.

$x = \frac{-b}{2a}$ (Equation of the axis of symmetry)

$= \frac{-(2)}{2(1)}$ (Replace: $a = 1$ and $b = 2$)

$= -1$ (Simplify)

The equation of the axis of symmetry is

$x = -1$

Since the equation of the axis of symmetry is

$x = -1$ and the vertex lies on the axis, the $x-$coordinate for the vertex is -1.

$y = x^2 + 2x$ (Original equation)

$\Rightarrow \quad y = (-1)^2 + 2(-1)$

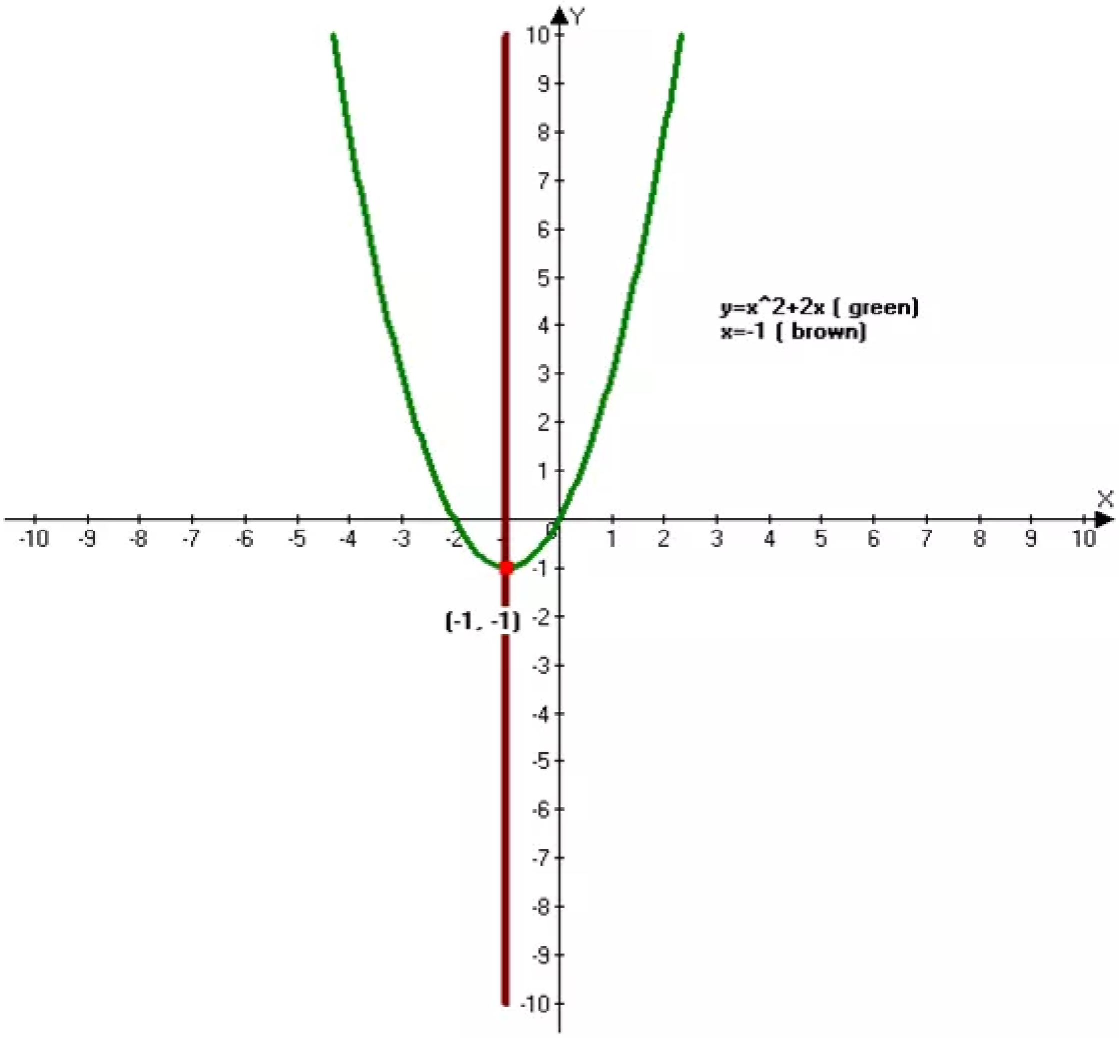
(Replace $x = -1$)

$\Rightarrow \quad y = +1 - 2$ (Simplify)

$\Rightarrow \quad y = -1$ (Add)

The vertex is at $(-1, -1)$.

Since the coefficient of x^2 term is positive, the parabola opens upward and the vertex is a minimum point.



Therefore, the axis of symmetry and the coordinates of the vertex of the graph are

$\boxed{x=-1;(-1,-1)}$, minimum point.

Answer 11PT.

Consider the equation

$$2x^2-8x=42$$

The objective is to solve the given equation by graphing.

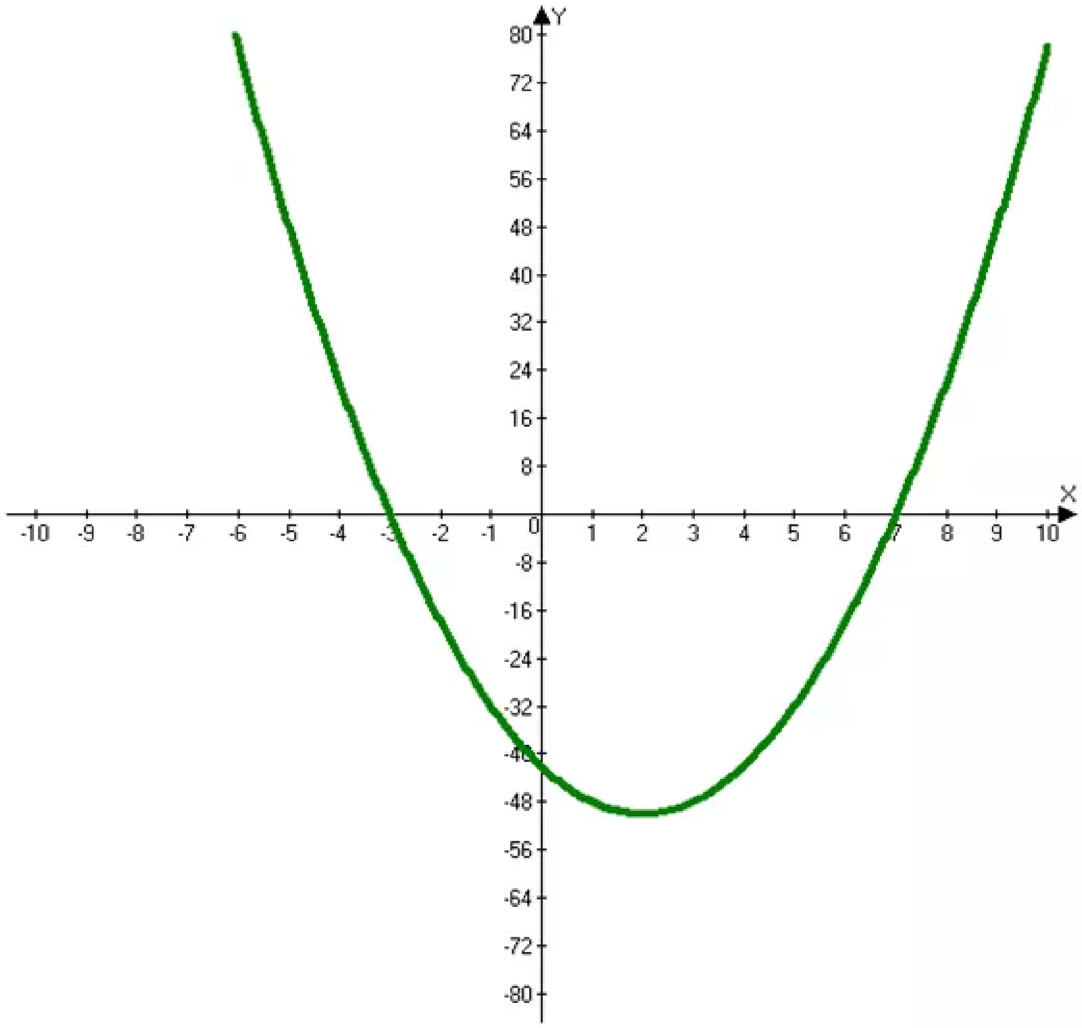
If the integral roots cannot be found then to estimate the roots by stating the consecutive integers between which the roots lie.

Since the roots of a quadratic equation are the $x-$intercepts of the related quadratic function.

Graph the related function

Graph the related function

$f(x) = 2x^2 - 8x - 42$



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From the graph, the $x-$intercepts are 7 and -3.

Check: Solve by factoring.

$2x^2 - 8x = 42$ (Original equation)

$\Rightarrow \quad 2x^2 - 8x - 42 = 0$ (Subtract 42 from both sides)

$\Rightarrow \quad (2x+6)(x-7) = 0$ (Factor)

$\Rightarrow \quad 2x + 6 = 0$

Or, $x - 7 = 0$ (Zero product property)

$\Rightarrow \quad 2x = -6$

Or, $x = 7$ (Solve for x)

$\Rightarrow \quad x = -3$

Or, $x = 7$

The solutions of the equation are -3 and 7.

Therefore, the solutions are $\boxed{-3}$ and $\boxed{7}$.

Answer 11STP.

Consider the line

$8x-4y+9=0$

The objective is to find the slop – intercept form of the line that is perpendicular to the given line and passes through the point $(2,3)$.

Equation of the line that passes through the point (x_1, y_1) is

$y-y_1=m(x-x_1)$

Then, equation of the line that passes through $(2,3)$ is

$7-3=m(x-2)$

For the equation

$ax+by+c=0$, slope is

$m=\frac{-a}{b}$.

Compare

$8x-4y+9=0$ with

$ax+by+c=0$

Then $a=8,$

$b=-4,$

$c=9$

Slope $(m_1)=\frac{-a}{b}$

$=\frac{-8}{-4}$ (Substitute $a=8, b=-4$)

$=2$ (Simplify)

Two lines are perpendicular if and only if product of their slopes is -1.

Then, slope of the perpendicular line

$$= \frac{-1}{\text{slope of the given line}}$$

$= \frac{-1}{2}$ (Since slope of given line is 2)

Equation of the line that is perpendicular to the given line and passing through the point $(2,3)$ is $y-3 = \frac{-1}{2}(x-2)$

$2(y-3) = -1(x-2)$

(Multiply both sides with 2)

$2y-6 = -x+2$ (Multiply)

$x+2y-6 = 2$ (Add ' x ' to both sides)

$x+2y-6-2 = 0$ (Subtract 2 from both sides)

$x+2y-8 = 0$ (Do addition: $-6-2 = -(6+2) = -8$)

$2y = -x+8$ (Subtract ' x ' from both sides. Add 8 to both sides)

$y = \frac{-x}{2}+4$ (Divide both sides by 2)

$y = \left(-\frac{1}{2}\right)x+4$

Therefore, the slope – Intercept form of the line is

$\boxed{y = \left(-\frac{1}{2}\right)x+4}$.

Answer 12E.

Consider the following equation

$y=-3x^2+4$ (Green curve)

The objective is to write the equation of the axis of symmetry and to find the coordinates of the vertex of the graph of the given function and to identify the vertex as a maximum or minimum and then graph the function.

The standard form of a quadratic function is

$y=ax^2+bx+c$

The equation of axis of symmetry for the graph of

$y=ax^2+bx+c$, where

$a \neq 0$ is

$x=\frac{-b}{2a}$

In the equation $y=-3x^2+4,$

$a=-3,$
$b=0$

Substitute these values into the equation of the axis of symmetry

$x=\frac{-b}{2a}$ (Equation of axis of symmetry)

$=\frac{-0}{2(-3)}$ (Replace $a=-3, b=0$)

$=0$ (Simplify)

The equation of the axis of symmetry is

$x=0$

Since the equation of the axis of symmetry is

$x=0$ and the vertex lies on the axis. The $x-$coordinate for the vertex is 0.

$y=-3x^2+4$ (Original equation)

$\Rightarrow \quad y=-3(0)^2+4$

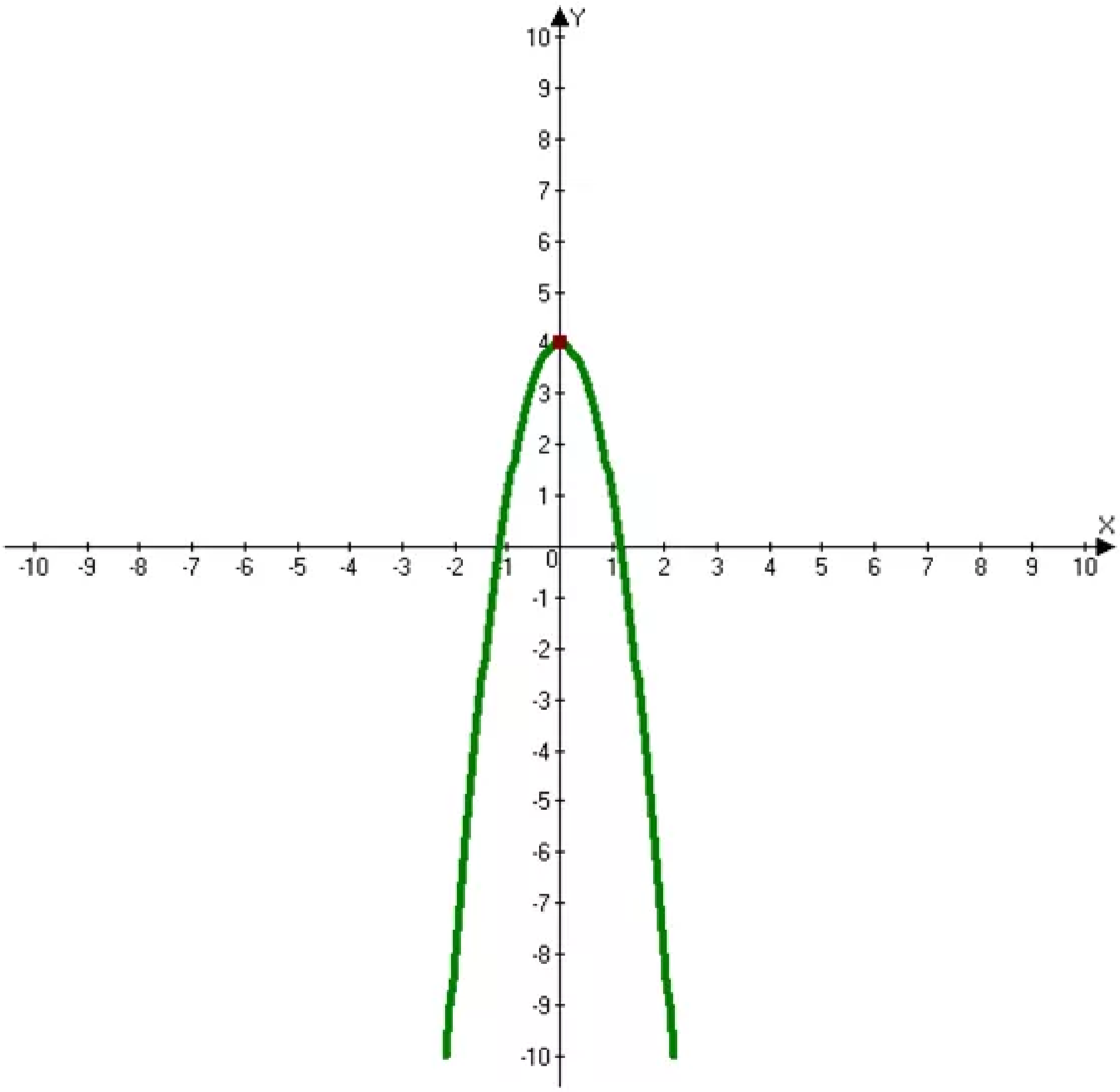
(Substitute 0 for ' x ')

$\Rightarrow \quad y=0+4$ (Simplify)

$\Rightarrow \quad y=4$ (Add)

The vertex is $(0,4)$ (brown dot).

Since the coefficient of x^2 term is negative, the parabola opens downward and the vertex is a maximum point (brown dot),



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Therefore, the axis of symmetry and the coordinates of the vertex are

$\boxed{x = 0; (0, 4)\ \text{maximum}}$.

Answer 12PT.

Consider the following equation

$$x^2 + 7x + 6 = 0$$

The objective is to solve the equation.

Since the solution of equation

$ax^2 + bx + c = 0$ can be found by using the Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$x^2 + 7x + 6 = 0$ (This is the given equation)

Now compare the above equation with

$$ax^2 + bx + c = 0$$

Then $a = 1$,

$b = 7$ and

$c = 6$

Therefore, $x = \dfrac{-7 \pm \sqrt{(7)^2 - 4(1)(6)}}{2(1)}$

(Substitute $a = 1, b = 7$ and $c = 6$ in $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$)

$= \dfrac{-7 \pm \sqrt{49 - 24}}{2}$

(Simplify)

$= \dfrac{-7 \pm \sqrt{25}}{2}$

(Do subtraction: $49 - 24 = 25$)

$= \dfrac{-7 \pm 5}{2}$ (Evaluate square - root)

$= \dfrac{-7 + 5}{2}, \dfrac{-7 - 5}{2}$

$= \dfrac{-2}{2}, \dfrac{-12}{2}$

(Do addition: $-7 + 5 = -2, -7 - 5 = -(7 + 5) = -12$)

$= -1, -6$ (Simplify)

Therefore, the solutions are $\boxed{-1}$ and $\boxed{-6}$.

Answer 13E.

Consider the following equation

$y = x^2 - 3x - 4$ (green curve)

The objective is to write the equation of the axis of symmetry and to find the coordinates of the vertex of the graph of the given function and to identify the vertex as a maximum or minimum and then graph the function.

The standard form of a quadratic function is

$y = ax^2 + bx + c$

The equation of axis of symmetry for the graph of

$y = ax^2 + bx + c$, where

$a \neq 0$ is

$$x = \frac{-b}{2a}$$

In the equation $y = x^2 - 3x - 4,$

$a = 1,$
$b = -3$

Substitute these values into the equation of the axis of symmetry

$x = \frac{-b}{2a}$ (Equation of axis of symmetry)

$= \frac{-(-3)}{2(1)}$ (Replace $a = 1, b = -3$)

$= \frac{3}{2}$ (Simplify)

$= 1\frac{1}{2}$ (Convert proper fraction to mixed fraction)

The equation of the axis of symmetry is

$$x = 1\frac{1}{2}$$

Since the equation of the axis of symmetry is

$x = 1\frac{1}{2}$ and the vertex lies on the axis. The $x-$coordinate for the vertex is $1\frac{1}{2}$. i.e, $\frac{3}{2}$.

$$y = x^2 - 3x - 4$$

(Original equation)

$$\Rightarrow \quad y = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) - 4$$

(Substitute $\frac{3}{2}$ for ' x')

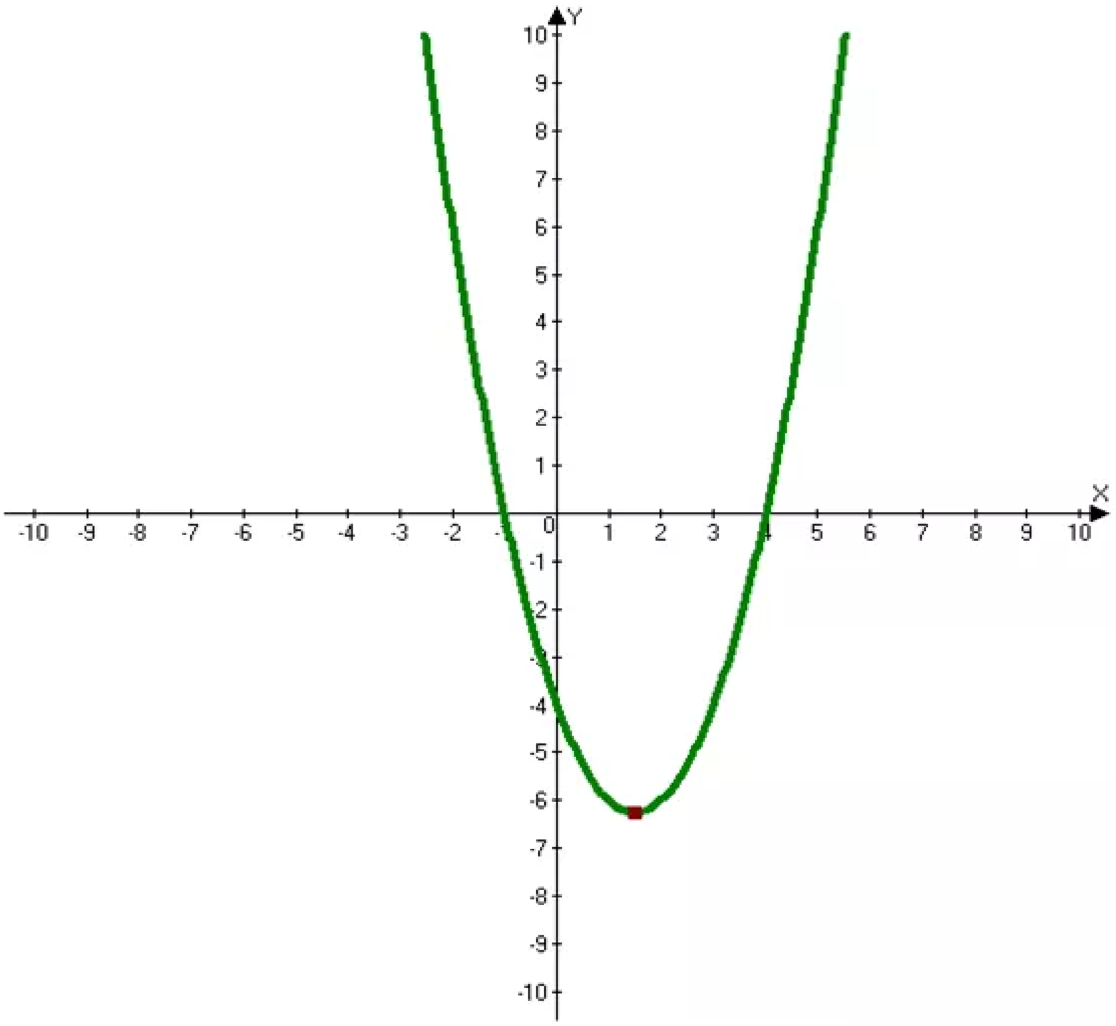
$= \frac{9}{4} - \frac{9}{2} - 4$ (Evaluate exponent)

$= \frac{-25}{4}$ (Simplify)

$= -6\frac{1}{4}$ (Convert proper fraction to mixed fraction)

The vertex is $\left(1\frac{1}{2}, -6\frac{1}{4}\right)$(brown point).

Since the coefficient of x^2 term is positive, the parabola opens upward and the vertex is a minimum point (brown dot).



Therefore, the axis of symmetry, the coordinates of the vertex are

$$\boxed{x = 1\frac{1}{2};\left(1\frac{1}{2}, -6\frac{1}{4}\right)\text{minimum}}.$$

Answer 13PT.

Consider the following equation

$$2x^2 - 5x - 12 = 0$$

The objective is to solve the equation.

The solution of equation

$ax^2 + bx + c = 0$ can be found by using the Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$2x^2 - 5x - 12 = 0$ (This is the given equation)

Now compare the above equation with

$$ax^2 + bx + c = 0$$

Then $a = 2$,

$b = -5$ and

$c = -12$

Therefore $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-12)}}{2(2)}$

(Substitute $a = 2, b = -5$ and $c = -12$ in $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$)

$= \frac{5 \pm \sqrt{25 + 96}}{4}$

(Simplify)

$= \frac{5 \pm \sqrt{121}}{4}$

(Do addition)

$= \frac{5 \pm 11}{4}$ (Evaluate square - root)

$= \frac{5+11}{4}, \frac{5-11}{4}$

$= \frac{16}{4}, \frac{-6}{4}$

(Do addition: $5 + 11 = 16$, Do subtraction: $5 - 11 = -6$)

$= 4, \frac{-3}{2}$ (Simplify)

$= 4, -1.5$

Therefore, the solutions are $\boxed{4}$ and $\boxed{-1.5}$.

Answer 13STP.

Consider the equation

$5a + 4b = 25$ and

$3a - 8b = 41$

The objective is to solve these equations for a and b.

Consider the second equation,

$3a - 8b = 41$

$\Rightarrow \quad -8b = 41 - 3a$ (Subtract $3a$ from both sides)

$\Rightarrow \quad b = \frac{-1}{8}(41 - 3a)$ (Divide both sides by -8)

Consider the first equation

$5a + 4b = 25$

$\Rightarrow$ $5a+4\left(\frac{-1}{8}(41-3a)\right)$ (Replace $b=\frac{-1}{8}(41-3a)$)
$=25$

$\Rightarrow$ $5a-\frac{1}{2}(41-3a)$ (Simplify)
$=25$

$\Rightarrow$ $10a-41+3a$
$=50$ (Multiply both sides by 2, to clear the equation fractions)

$\Rightarrow$ $13a-41=50$ (Combine like terms)

$\Rightarrow$ $13a=91$ (Add 41 to both sides)

$\Rightarrow$ $a=7$ (Divide both sides by 13)

Replace $a=7$ in the first equation.

$5a+4b=25$ (First equation)

$\Rightarrow$ $5(7)+4b=25$ (Replace a by 7)

$\Rightarrow$ $35+4b=25$ (Multiply: $5(7)=35$)

$\Rightarrow$ $4b=-10$ (Subtract 35 from both sides)

$\Rightarrow$ $b=\frac{-10}{4}$ (Divide both sides by 4)

$\Rightarrow$ $b=\frac{-5}{2}$ (Simplify)

Or, $b=-2.5$

Therefore, the values of a and b are $\boxed{7\ \&\ -2.5}$.

Answer 14E.

Consider the equation

$y = 3x^2 + 6x - 12$

The objective is to write the equation of the axis of symmetry, and to find the coordinates of the vertex of the graph of the give function.

And indentify the vertex as a maximum or minimum and then graph the function.

The standard form of a quadratic function is

$y = ax^2 + bx + c$

The equation of the axis of symmetry for the graph of

$y = ax^2 + bx + c$, where

$a \neq 0$, is

$x = \frac{-b}{2a}$

In the equation

$y = 3x^2 + 6x - 12,$ and

$a = 3$

$b = 6$

Substitute these values into the equation of the axis of symmetry.

$x = \frac{-b}{2a}$ (Equation of the axis of symmetry)

$= \frac{-(6)}{2(3)}$ (Replace: $a = 3$ and $b = 6$)

$= -1$ (Simplify)

The equation of the axis of symmetry is

$x = -1$

Since the equation of the axis of symmetry is

$x = -1$ and the vertex lies on the axis, the $x-$coordinate for the vertex is -1.

$y = 3x^2 + 6x - 12$

(Original equation)

$\Rightarrow \quad y = 3(-1)^2 + 6(-1) - 12$

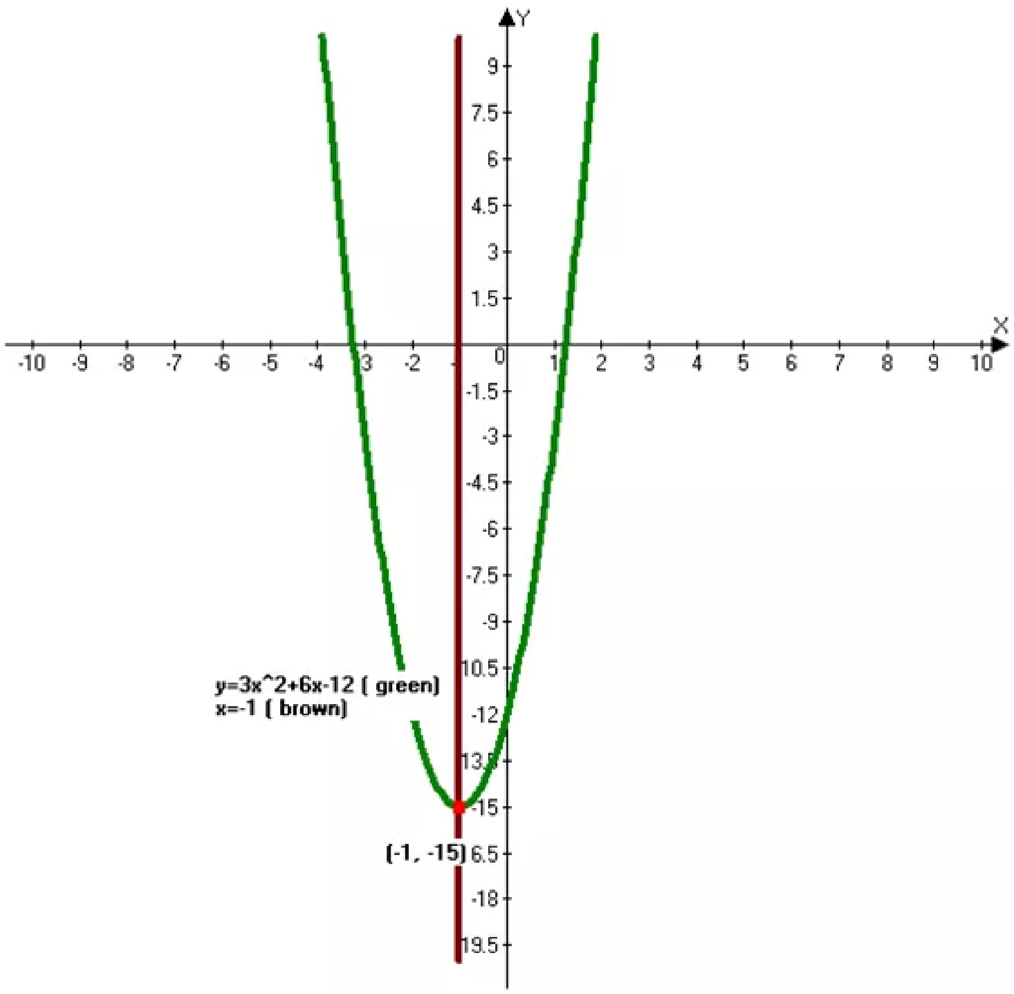
(Replace $x = -1$)

$\Rightarrow \quad y = 3 - 6 - 12$ (Simplify)

$\Rightarrow \quad y = -15$ (Add)

The vertex is at $(-1, -15)$.

Since the coefficient of x^2 term is positive, the parabola opens upward and the vertex is a minimum point.



Therefore, the axis of symmetry and the coordinates of the vertex of the graph are

$\boxed{x=-1;(-1,-15)}$ minimum point.

Answer 14PT.

Consider the following equation

$6n^2+7n=20$

The objective is to solve the equation.

The solution of equation

$ax^2+bx+c=0$ can be found by using the quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$6n^2+7n=20$ (This is the given equation)

$6n^2+7n-20=0$ (Subtract 20 from both sides)

Now compare the above equation with

$ax^2+bx+c=0$

Then $a=6$,

$b=7,$ and
$c=-20$

$x=n$

Therefore $n = \frac{-7 \pm \sqrt{(7)^2 - 4(6)(-20)}}{2(6)}$

(Substitute $a = 6, b = 7$ and $c = -20, x = n$ in $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$)

$= \frac{-7 \pm \sqrt{49 + 480}}{12}$

(Simplify)

$= \frac{-7 \pm \sqrt{529}}{12}$

(Do addition: $49 + 480 = 529$)

$= \frac{-7 \pm 23}{12}$ (Evaluate square - root)

$= \frac{-7 + 23}{12}, \frac{-7 - 23}{12}$

$= \frac{16}{12}, \frac{-30}{12}$ (Do addition: $-7 + 23 = 16, -7 - 23 = -(7 + 23) = -30$)

$= \frac{4}{3}, \frac{-5}{2}$ (Divide out common factors)

$= 1.3, -2.5$ (Do division)

Therefore, the solutions are $\boxed{1.3}$ and $\boxed{-2.5}$.

Answer 14STP.

Consider the rational expression $\left(\frac{4d^3}{3a^7}\right)^{-3}$

The objective is to simplify the given expression.

$$\left(\frac{4d^3}{3a^7}\right)^{-3} = \left(\frac{3a^7}{4d^3}\right)^{3} \quad \text{(Because } \left(\frac{A}{B}\right)^{-n} = \left(\frac{B}{A}\right)^{n}\text{)}$$

$$= \frac{\left(3a^7\right)^3}{\left(4d^3\right)^3} \quad \left(\left(\frac{A}{B}\right)^{n} = \frac{A^n}{B^n}\right)$$

$$= \frac{3^3\left(a^7\right)^3}{4^3\left(d^3\right)^3} \quad \left((AB)^n = A^n \cdot B^n\right)$$

$$= \frac{3^3 \cdot a^{21}}{4^3 \cdot d^9} \quad \left(\left(A^n\right)^m = A^{mn}\right)$$

$$= \frac{27a^{21}}{64d^9} \quad \text{(Evaluate the exponents)}$$

Therefore,

$$\left(\frac{4d^3}{3a^7}\right)^{-3} = \boxed{\frac{27a^{21}}{64d^9}}$$

Answer 15E.

Consider the equation

$$y=-2x^2+1$$

The objective is to write the equation of the axis of symmetry, and to find the coordinates of the vertex of the graph of the give function. And to indentify the vertex as a maximum or minimum and then graph the function.

The standard form of a quadratic function is

$$y=ax^2+bx+c$$

The equation of the axis of symmetry for the graph of

$y=ax^2+bx+c$, where

$a\neq 0$, is

$$x=\frac{-b}{2a}$$

In the equation

$y=-2x^2+1$, and

$a=-2$

$b=0$

Substitute these values into the equation of the axis of symmetry.

$x=\frac{-b}{2a}$ (Equation of the axis of symmetry)

$=\frac{-(0)}{2(-2)}$ (Replace: $a=-2$ and $b=0$)

$=0$ (Simplify)

The equation of the axis of symmetry is

$$x=0$$

Since the equation of the axis of symmetry is

$x=0$ and the vertex lies on the axis, the $x-$coordinate for the vertex is 0.

$y=-2x^2+1$ (Original equation)

$\Rightarrow \quad y=-2(0)^2+1$

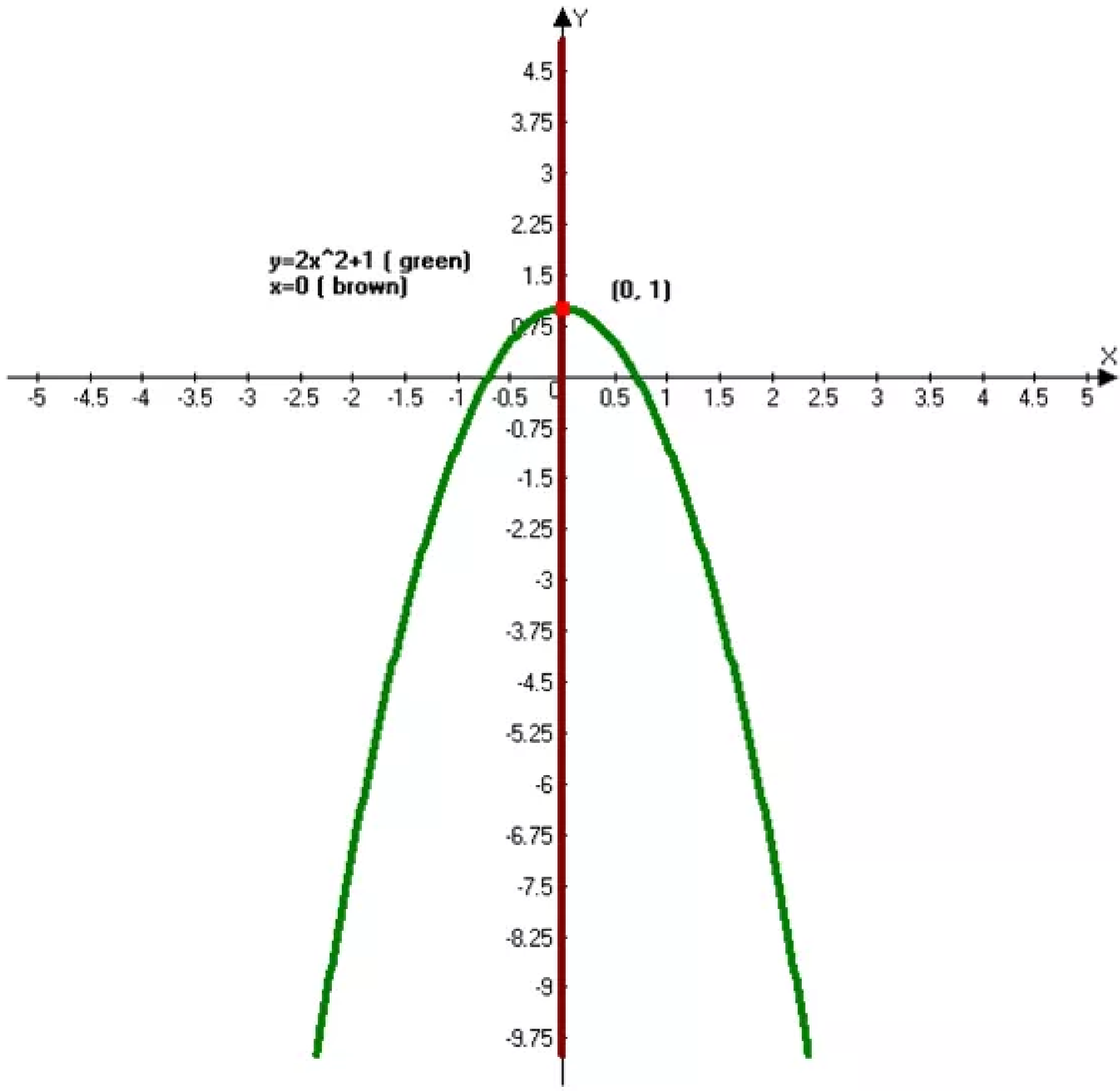
(Replace $x=0$)

$\Rightarrow \quad y=0+1$ (Simplify)

$\Rightarrow \quad y=1$ (Add)

The vertex is at $(0,1)$.

Since the coefficient of x^2 term is positive, the parabola opens upward and the vertex is a minimum point.



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Therefore, the axis of symmetry and the coordinates of the vertex of the graph are

$\boxed{x = 0; (0,1)}$ maximum point.

Answer 15PT.

Consider the following equation

$$3k^2 + 2k = 5$$

The objective is to solve the equation

The solution of the equation

$ax^2 + bx + c = 0$ can be found by using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$3k^2 + 2k = 5$ (This is the given equation)

$3k^2 + 2k - 5 = 0$ (Subtract ' 5 ' from both sides)

Now compare the above equation with

$$ax^2 + bx + c = 0$$

Then $a = 3$,

$b = 2$, and
$c = -5$

$$x = k$$

Therefore $k = \frac{-2 \pm \sqrt{(2)^2 - 4(3)(-5)}}{2(3)}$

(Substitute $a = 3, b = 2, c = -5$ and $x = k$ in $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$)

$= \frac{-2 \pm \sqrt{4+60}}{6}$

(Simplify)

$= \frac{-2 \pm \sqrt{64}}{6}$ (Do addition: $4 + 60 = 64$)

$= \frac{-2 \pm 8}{6}$ (Evaluate square - root)

$= \frac{-2+8}{6}, \frac{-2-8}{6}$

$= \frac{6}{6}, \frac{-10}{6}$ (Do addition: $-2+8=6, -2-8=-(2+8)=-10$)

$= 1, \frac{-5}{3}$ (Divide out common factors)

$= 1, -1.7$ (Simplify)

Therefore, the solutions are $\boxed{1}$ and $\boxed{-1.7}$.

Consider $x^2 + 4x - 5 = (x+h)^2 + k$

The objective is to complete the square of $x^2 + 4x - 5$ by finding numbers h and k such that

$x^2 + 4x - 5 = (x+h)^2 + k$

$x^2 + 4x - 5 = (x+h)^2 + k$ (Original equation)

$\Rightarrow \quad x^2 + 4x + 4 - 4 - 5 = (x+h)^2 + k$ (Add and subtract the number 4)

$\Rightarrow \quad (x+2)^2 - 9 = (x+h)^2 + k$ (Simplify)

By the comparison, $h = 2$ and

$k = -9$

Therefore, the values of h and k are $\boxed{2 \,\&\, -9}$.

Answer 16E.

Consider the following equation

$y=-x^2-3x$ (Green curve)

The objective is to write the equation of the axis of symmetry and to find the co-ordinates of the vertex of the graph of the given function and to identify the vertex as a maximum or minimum and then graph the function.

The standard form of a quadratic function is

$y=ax^2+bx+c$

The equation of axis of symmetry for the graph of

$y=ax^2+bx+c$, where

$a\neq 0$ is

$x=\frac{-b}{2a}$

In the equation $y=-x^2-3x$,

$a=-1,$

$b=-3$

Substitute these values into the equation of the axis of symmetry

$x=\frac{-b}{2a}$ (Equation of axis of symmetry)

$=\frac{-(-3)}{2(-1)}$ (Replace $a=-1, b=-3$)

$=\frac{3}{-2}$ (Simplify)

$=-1\frac{1}{2}$ (Convert proper fraction of mixed fraction)

The equation of the axis of symmetry is

$x = -1\frac{1}{2}$

Since the equation of axis of symmetry is

$x = -1\frac{1}{2}$ (or) $\frac{-3}{2}$, and the vertex lies on the axis. The $x-$coordinate for the vertex is $\frac{-3}{2}$.

$y = -x^2 - 3x$ (Original equation)

$\Rightarrow \quad y = -\left(\frac{-3}{2}\right)^2 - 3\left(\frac{-3}{2}\right)$ (Substitute $\frac{-3}{2}$ for ' x')

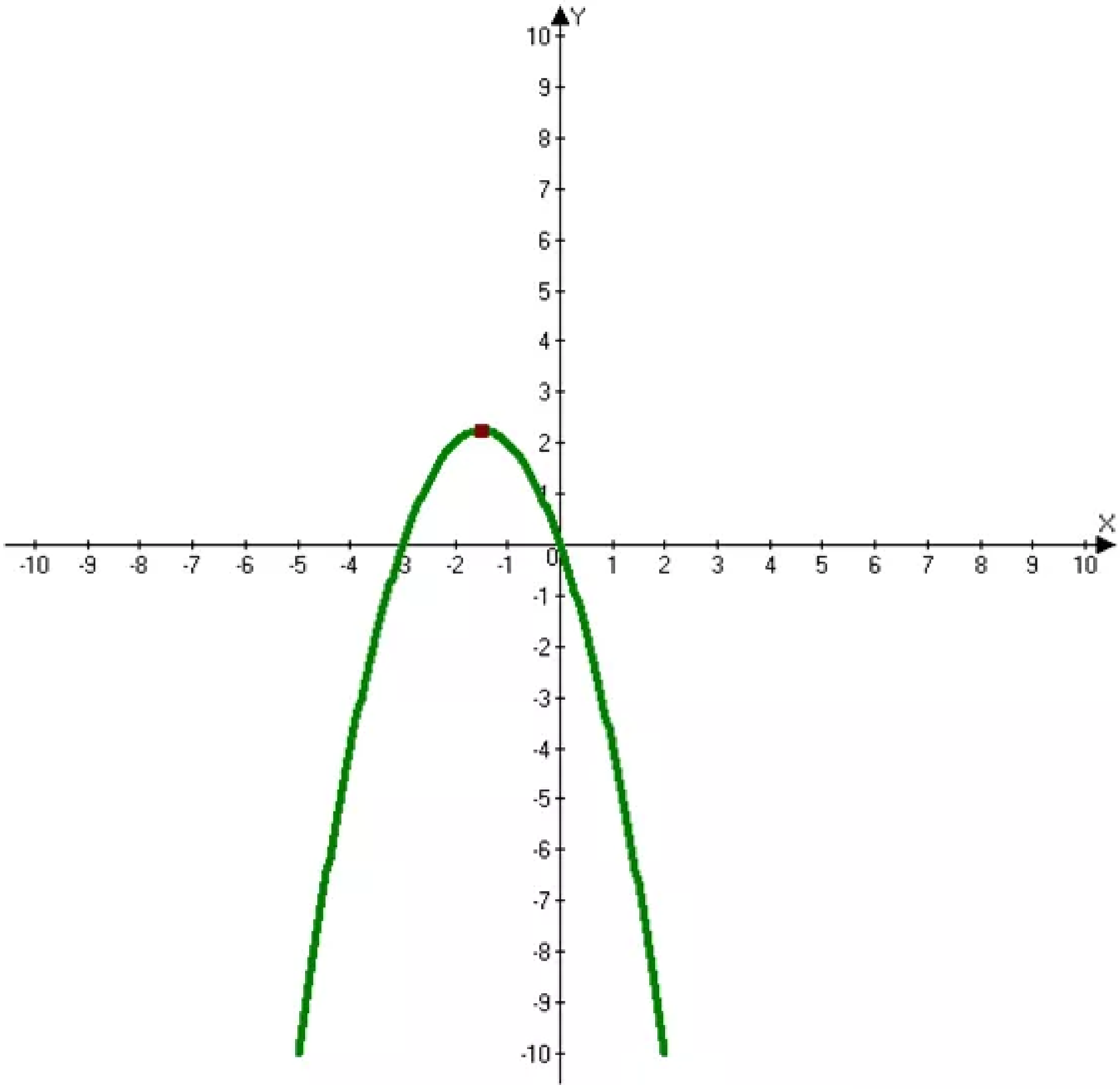
$\Rightarrow \quad y = \frac{-9}{4} + \frac{9}{2}$ (Simplify)

$\Rightarrow \quad y = \frac{9}{4}$ (Add)

$= 2\frac{1}{4}$ (Convert to mixed fraction)

The vertex is $\left(-1\frac{1}{2}, 2\frac{1}{4}\right)$ (brown dot).

Since the coefficient of x^2 term is negative, the parabola opens downward and the vertex is a maximum point (brown dot).



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Therefore, the axis of symmetry and coordinates of the vertex are

$$\boxed{x=-1\frac{1}{2};\left(-1\frac{1}{2},2\frac{1}{4}\right)\text{maximum}}.$$

Answer 16PT.

Consider the following equation

$$y^2-\frac{3}{5}y+\frac{2}{25}=0$$

The objective is to solve the equation.

The solution of the equation

$ax^2+bx+c=0$ can be found by using the quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$y^2-\frac{3}{5}y+\frac{2}{25}=0$ (This is the given equation)

$25y^2-15y+2=0$ (Multiply with 25 on both sides and simplify)

Now compare the above equation with

$$ax^2+bx+c=0$$

Then $a=25$,

$b=-15$, and

$c=2$

$x=y$

Therefore, $y=\frac{-(-15)\pm\sqrt{(-15)^2-4(25)(2)}}{2(25)}$

(Substitute $a=25, b=-15, c=2$ and $x=y$ in $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$)

$$=\frac{15\pm\sqrt{225-200}}{50}$$

(Simplify)

$$=\frac{15\pm\sqrt{25}}{50}$$

(Do subtraction: $225-200=25$)

$=\frac{15\pm5}{50}$ (Evaluate square - root)

$$=\frac{15+5}{50},\frac{15-5}{50}$$

$$=\frac{20}{50},\frac{10}{50}$$

(Do addition: $15+5=20$. Do subtraction: $15-5=10$)

$$=0.4, 0.2$$

(Do division)

Therefore, the solutions are $\boxed{0.4}$ and $\boxed{0.2}$.

Answer 16STP.

Consider the equation

$y = 6x^2 + 11x + 4$

The objective is to find the number of points that the graph of

$y = 6x^2 + 11x + 4$ intersect $x-$axis.

The points at which the graph intersect $x-$axis are the solutions of the Quadratic equation.

Use the discriminate to determine the number of solutions for a quadratic equation.

For the equation

$ax^2 + bx + c = 0$.

Quadratic formula is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The expression under the radical sign, $b^2 - 4ac$, is called the discriminate.

The value of the discriminate can be used to determine the number of real roots for a quadratic equation.

If $b^2 - 4ac < 0$, then number of real roots are 0.

$b^2 - 4ac = 0$, then number of real roots are 1.

$b^2 - 4ac > 0$, then number of real roots are 2.

Compare $6x^2 + 11x + 4$ with $ax^2 + bx + c$.

Then $a = 6,$

$b = 1$ and

$c = 4$

Consider

$b^2 - 4ac = (11)^2 - 4(6)(4)$ (Substitute a, b, c values)

$= 121 - 96$ (Multiply)

$= 25 > 0$ (Do subtraction)

Therefore, the number of real roots are 2.

Then, the graph of

$y = 6x^2 + 11x + 4$ intersect $x-$axis at $\boxed{2}$ points.

Answer 17E.

Consider the equation

$$x^2 - x - 12 = 0$$

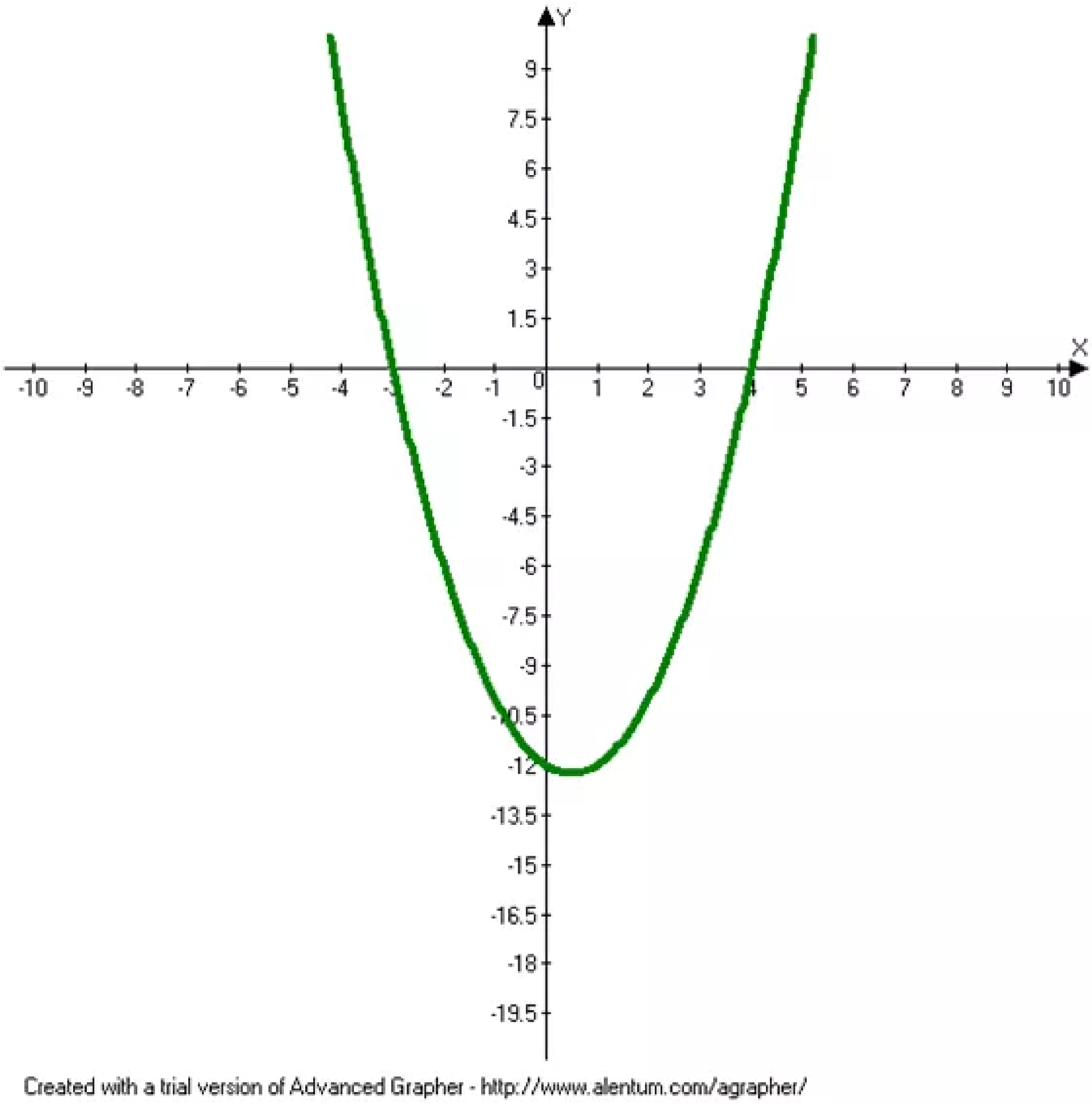
The objective is to solve the given equation by graphing.

If the integral roots cannot be found, then to estimate the roots by stating the consecutive integers between which the roots lie.

Since the roots of a quadratic equation are the x – intercepts of the related quadratic function.

Graph the related function

$$f(x) = x^2 - x - 12$$



From the graph, the $x-$intercepts are 4 and -3.

Check: solve by factoring.

$x^2-x-12=0$ (Original equation)

$\Rightarrow \quad (x-4)(x+3)=0$ (Factor)

$\Rightarrow \quad x-4=0$

Or, $x+3=0$ (Zero product property)

$\Rightarrow \quad x=4$

Or, $x=-3$ (Solve for x)

The solutions of the equation are 4 and -3.

Therefore, the solution are $\boxed{-3 \,\&\, 4}$.

Answer 17PT.

Consider the following equation

$-3x^2+5=14x$

The objective is to solve the equation.

The solution of the equation

$ax^2+bx+c=0$ can be found by using the quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$-3x^2+5=14x$ (This is the given equation)

$3x^2+14x-5=0$ (Add $3x^2$ to both sides and subtract 5 from both sides)

Now compare the above equation with

$ax^2+bx+c=0$

Then $a=3,$

$b=14,$

$c=-5$

Therefore, $x=\frac{-14\pm\sqrt{(14)^2-4(3)(-5)}}{2(3)}$

(Substitute $a=3, b=14$ and $c=-5$ in $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$)

$=\frac{-14\pm\sqrt{196+60}}{6}$

(Simplify)

$=\frac{-14\pm\sqrt{256}}{6}$

(Do addition: $196+60=256$)

$=\frac{-14\pm16}{6}$

(Evaluate square - root)

$=\frac{-14+16}{6},\frac{-14-16}{6}$

$=\frac{2}{6},\frac{-30}{6}$

(Do addition: $-14+16=2, -14-16=-(14+16)=-30$)

$=0.3,-5$ (Do division)

Therefore, the solutions are $\boxed{0.3}$ and $\boxed{-5}$.

Answer 17STP.

Consider the data

The length and width of a rectangle that measures 8 inches by 6 inches are both increased by the same amount.

The area of the larger rectangle is twice the area of the original rectangle.

The objective is to find the number that was added to each dimension of the original rectangle and to round to the nearest hundredth of an inch.

The area of the rectangle is length $\times$ breadth.

Let $x =$ The number that was added to each dimension.

Then $8+x =$ Length of the new rectangle.

$6+x =$ Width of the new rectangle.

Area of the new rectangle

$$=(8+x)(6+x)$$

Area of the new rectangle is twice the area of the original rectangle.

Area of the original rectangle

$$= \text{length} \times \text{breadth}$$
$$= 8 \times 6$$
$$= 48 \cdot \text{sq} \cdot \text{inches}$$

Area of new rectangle

$= 2 \times$ Area of original rectangle.

$$(8+x)(6+x) = 2 \times 48$$

$48 + x^2 + 14x = 96$ (Multiply)

$$x^2 + 14x + 48 = 96$$

$x^2 + 14x - 48 = 0$ (Subtract 96 from both sides)

Now solve the above equation by using quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$a=1,$
$b=14,$
$c=-48$

Therefore, $x=\dfrac{-14\pm\sqrt{(14)^2-4(1)(-48)}}{2(1)}$

(Substitute $a=1, b=14, c=-48$ in $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$)

$$=\frac{-14\pm\sqrt{196+192}}{2}$$

(Simplify)

$$=\frac{-14\pm\sqrt{388}}{2}$$

(Do addition)

$=2.85$ or -16.85

(Simplify)

Since dimensions cannot be negative,

$x=2.85$

Therefore, $\boxed{2.85}$ is added to each dimension.

Answer 18E.

Consider the following equation

$x^2+6x+9=0$ (Green curve)

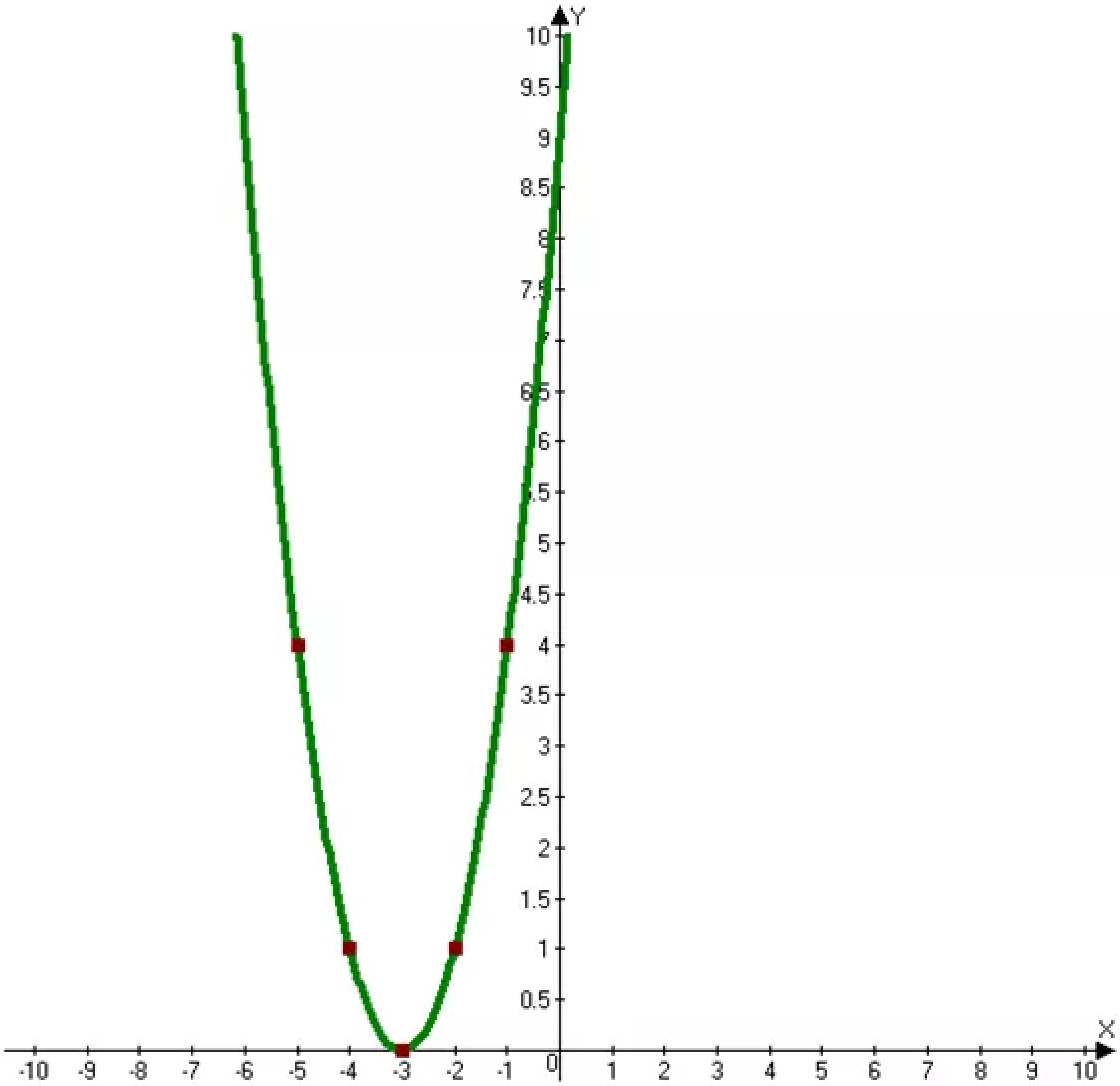
The objective is to solve the given equation by graphing.

If the integral roots cannot be found then to estimate the roots by stating the consecutive integers between which the roots lie.

Since the roots of a quadratic equation are the $x-$intercepts of the related quadratic function.

Graph the related function

$f(x) = x^2 + 6x + 9$ (Green curve)



x	$f(x)$
-5	4
-4	1
-3	0
-2	1
-1	4

The above table is showing with brown dots.

Note that the vertex of the parabola is the $x-$intercept.

Thus, one solution is -3.

To find the other solution, solve the equation by factoring.

$x^2+6x+9=0$ (Original equation)

$\Rightarrow \quad (x+3)(x+3)=0$ (Factor)

$\Rightarrow \quad x+3=0$

Or, $x+3=0$ (Zero product property)

$\Rightarrow \quad x=-3$

Or, $x=-3$ (Solve for x)

There are two identical factors for the quadratic function.

So there is only one root, called a double root.

Therefore, the solution is $\boxed{-3}$.

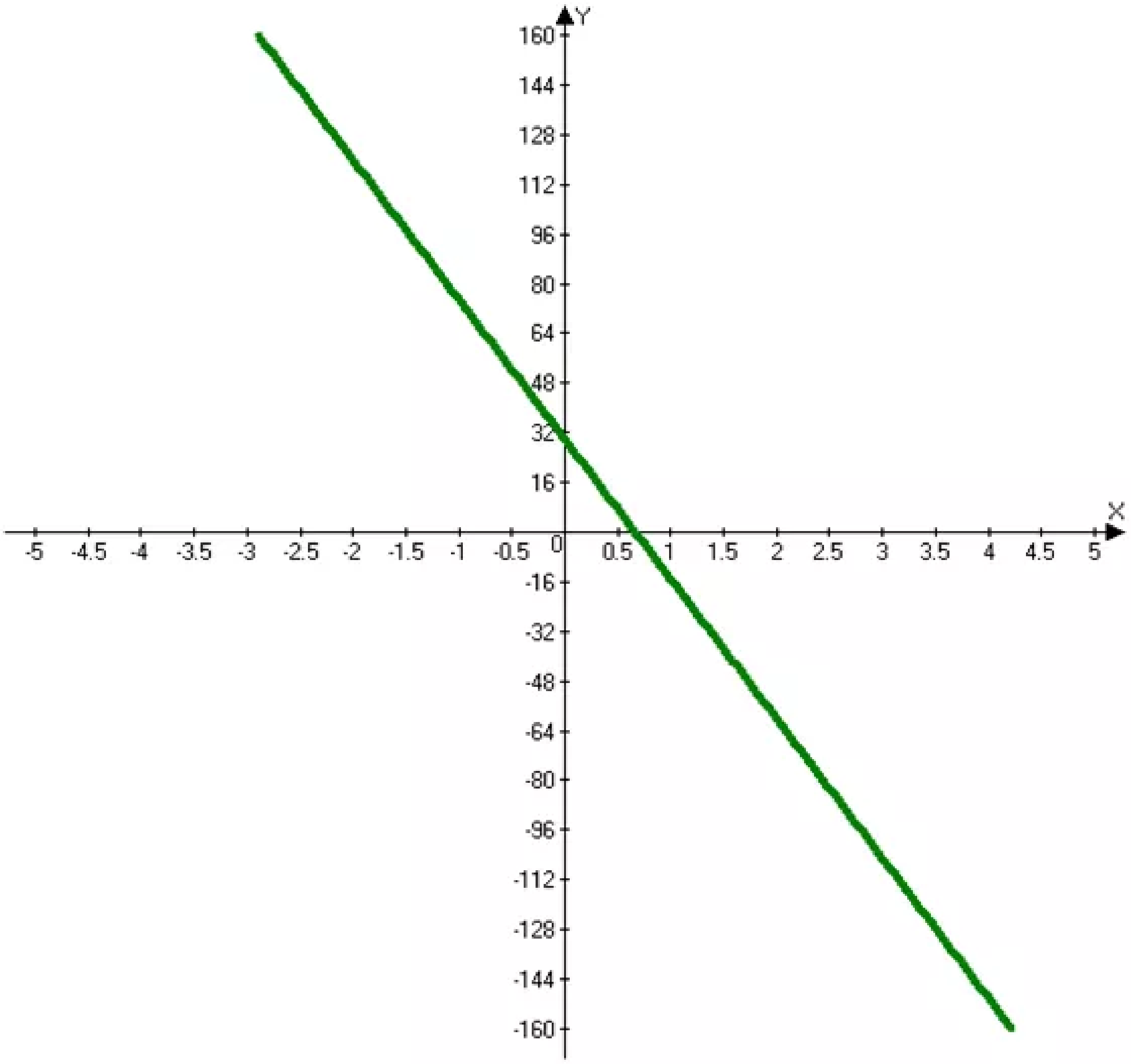
Answer 18PT.

Consider the function $y=5(6-9x)$

The objective is to graph the given function and to state the $y-$intercept.

x	$f(x)$
-2	$\frac{2425}{81}$
-1	$\frac{265}{9}$
0	25
1	-15
2	-375

Graph the ordered pairs and connect the points with a smooth curve.



From the graph, the $y-$intercept is $\boxed{25}$.

Answer 19E.

Consider the following equation

$$x^2 + 4x - 3 = 0$$

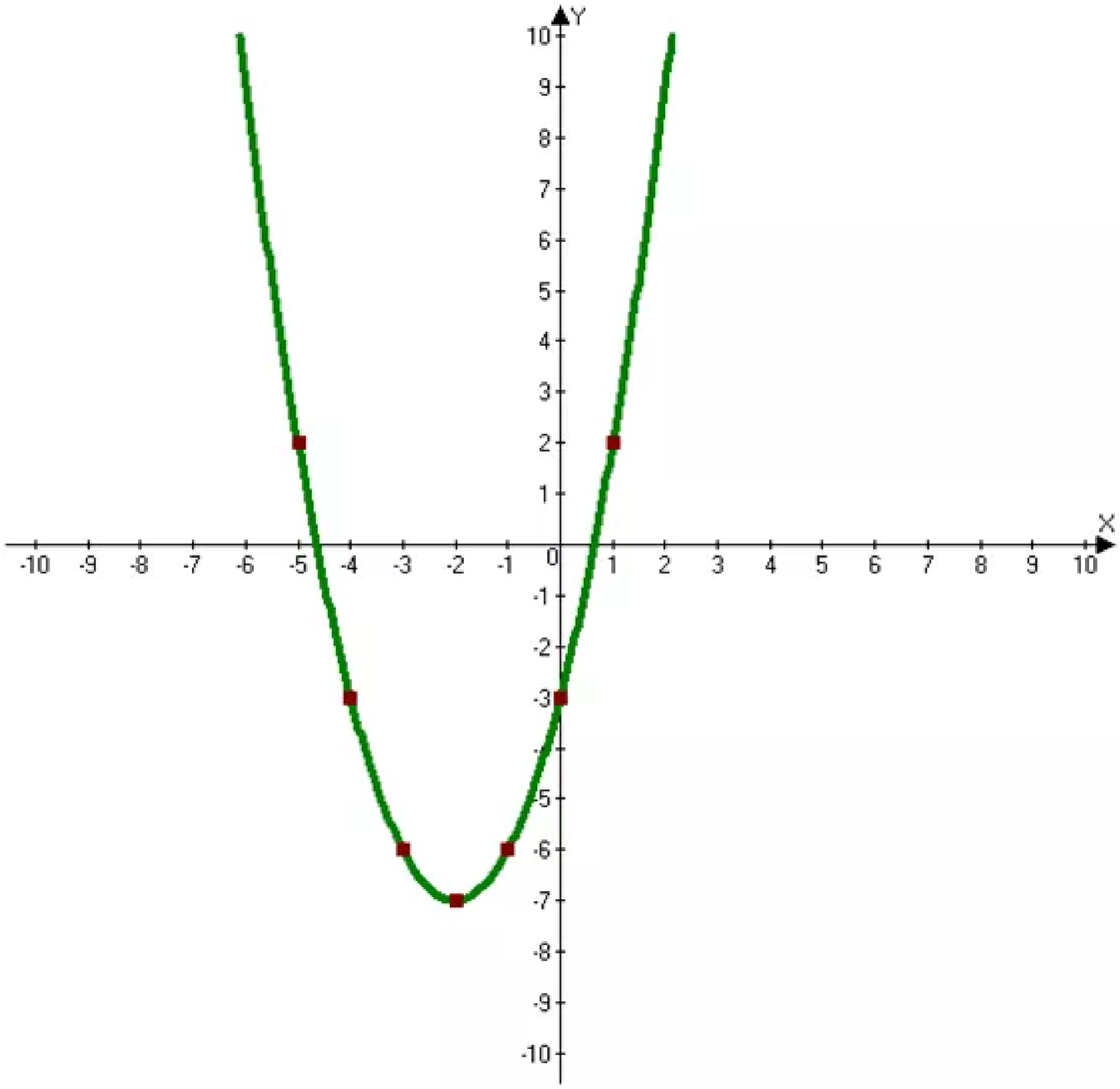
The objective is to solve the given equation by graphing.

If integral roots cannot be found, then to estimate the roots by stating the consecutive integers between which he roots lie.

Since the roots of a quadratic equation are the $x-$intercepts of the related quadratic function.

Graph the related function

$f(x) = x^2 + 4x - 3$ (Green curve)



From the graph, it is clear that the $x-$intercepts are not integers.

x	$f(x)$
-5	2
-4	-3
-3	-6
-2	-7
-1	-6
0	-3
1	2

The above table values are showing with brown dots in the above graph.

The value of the function changes from negative to positive between the ‘ x’ values of -5 and -4 and between 0 and 1.

The $x-$intercepts of the graph are between -5 and -4 and between 0 and 1.

So, one root is between -5 and -4, and the other root is between 0 and 1.

Therefore, one root lies between $\boxed{-5}$ and $\boxed{-4}$ and the other root lies between $\boxed{0}$ and $\boxed{1}$.

Answer 19PT.

Consider the following equation

$$3x^2+4x=8$$

The objective is to solve the equation.

The solution of the equation

$ax^2+bx+c=0$ can be found by sing the quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$3x^2+4x=8$ (This is the given equation)

$3x^2+4x-8=0$ (Subtract 8 from both sides)

Now compare the above equation with

$$ax^2+bx+c=0$$

Then $a=3,$

$b=4,$

$c=-8$

Therefore, $x=\dfrac{-4\pm\sqrt{(4)^2-4(3)(-8)}}{2(3)}$

(Substitute $a=3,b=4,c=-8$ in $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$)

$$=\frac{-4\pm\sqrt{16+96}}{6}$$

(Simplify)

$$=\frac{-4\pm\sqrt{112}}{6}$$

(Do addition: $16+96=112$)

$$=\frac{-4\pm10.58}{6}$$

(Evaluate square - root)

$$=-\frac{4+10.58}{6},\frac{-4-10.58}{6}$$

$=1.1,-2.4$ (Simplify)

Therefore, the solutions are $\boxed{1.1}$ and $\boxed{-2.4}$.

Answer 19STP.

Consider the following data

Mr. Ramirez bought a car for $\$27,000$.

The car depreciates 13% per year.

The objective is to find the value of the car after 8 years.

The general equation for exponential decay is

$$y = c(1-r)^t$$

Where ' y ' represents the final amount,

' c ' represents the initial amount

' r ' represents the rate of decay expressed as a decimal and ' t ' represents time.

Then, initial value of the car is

$$(c) = \$27,000$$

Rate of decay

$(r) = 13\%$ Or 0.13 and the time

$(t) = 8$ Years.

$y = c(1-r)^t$ (General equation for exponential decay)

$y = 27,000(1-0.13)^8$ (Substitute $c = 27,000, r = 0.13, t = 8$)

$= 27,000(0.87)^8$ (Do subtraction: $1-0.13 = 0.87$)

$= 8861.7$ (Simplify)

Therefore, the value of the car after 8 years is $\boxed{\$8861.7}$.

Answer 20E.

Consider the equation

$$2x^2 - 5x + 4 = 0$$

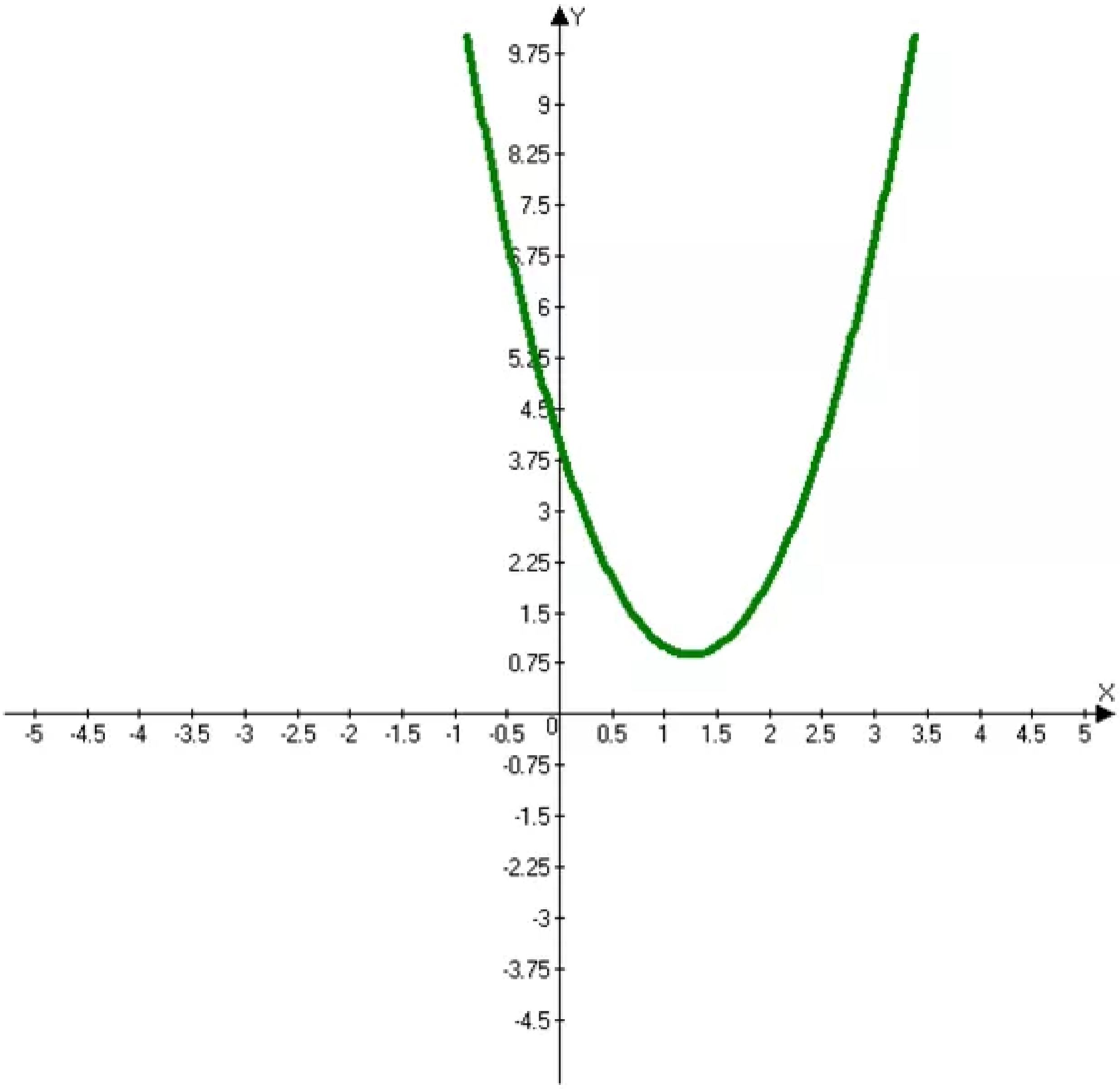
The objective is to solve the given equation by graphing.

If the integral roots cannot be found, then to estimate the roots by stating the consecutive integers between which the roots lie.

Since the roots of a quadratic equation are the $x-$intercepts of the related quadratic function.

Graph the related function

$$f(x) = 2x^2 - 5x + 4$$



The graph has no $x-$intercept.

Thus, there are no real number solutions for this equation.

x	$f(x)$
-1	11
0	4
1	1
2	2

Therefore, there are no real solutions for the given equation.

Answer 20PT.

Consider the following equation

$$7m^2 = m+5$$

The objective is to solve the equation.

The solution of the equation

$ax^2+bx+c=0$ can be found by using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$7m^2 = m+5$ (This is the given equation)

$7m^2 - m - 5 = 0$ (Subtract ' m ' and 5 from both sides)

Now compare the above equation with

$$ax^2+bx+c=0$$

Then $a=7$,

$b=-1$,

$c=-5$

And $x=m$

Therefore, $m=\frac{-(-1)\pm\sqrt{(-1)^2-4(7)(-5)}}{2(7)}$

(Substitute $a=7, b=-7, c=-5$ and $x=m$ in $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$)

$=\frac{1\pm\sqrt{1+140}}{14}$

(Simplify)

$=\frac{1\pm\sqrt{141}}{14}$

(Do addition: $1+140=141$)

$=\frac{1\pm 11.87}{14}$

(Evaluate square - root)

$=\frac{1+11.87}{14}, \frac{1-11.87}{14}$

$=0.919, -0.8$

(Simplify)

$\approx 0.9, -0.8$

Therefore, the nearest solutions are $\boxed{0.9}$ and $\boxed{-0.8}$.

Answer 20STP.

Consider the geometric sequence $3, -, 48$.

The objective is to find the geometric mean in the given sequence.

Missing term between two nonconsecutive terms in a geometric sequence is called geometric mean.

The nth term a_n of a geometric sequence with the first term a_1 and common ratio r is given by

$a_n = a_1 r^{n-1}$.

Use the formula for the nth term of a geometric sequence to find a geometric mean.

In the given sequence

$a_1 = 3$, and

$a_3 = 48$.

To find a_2, first find r.

$a_n = a_1 r^{n-1}$ (Formula for the nth term of a geometric sequence)

$\Rightarrow \quad a_3 = a_1 r^{3-1}$ (Replace n by 3)

$\Rightarrow \quad 48 = 3(r^2)$ (Replace $a_1 = 3$ and $a_3 = 48$)

$\Rightarrow \quad 16 = r^2$ (Divide both sides by 3)

$\Rightarrow \quad \pm 4 = r$ (Take square root of each side)

If $r = 4$, the geometric mean is

$4(3) = 12$

If $r = -4$, the geometric mean is

$(-4)(3) = -12$

Therefore the geometric mean is $\boxed{\pm 12}$.

Answer 21E.

Consider the equation

$$x^2 - 10x = -21$$

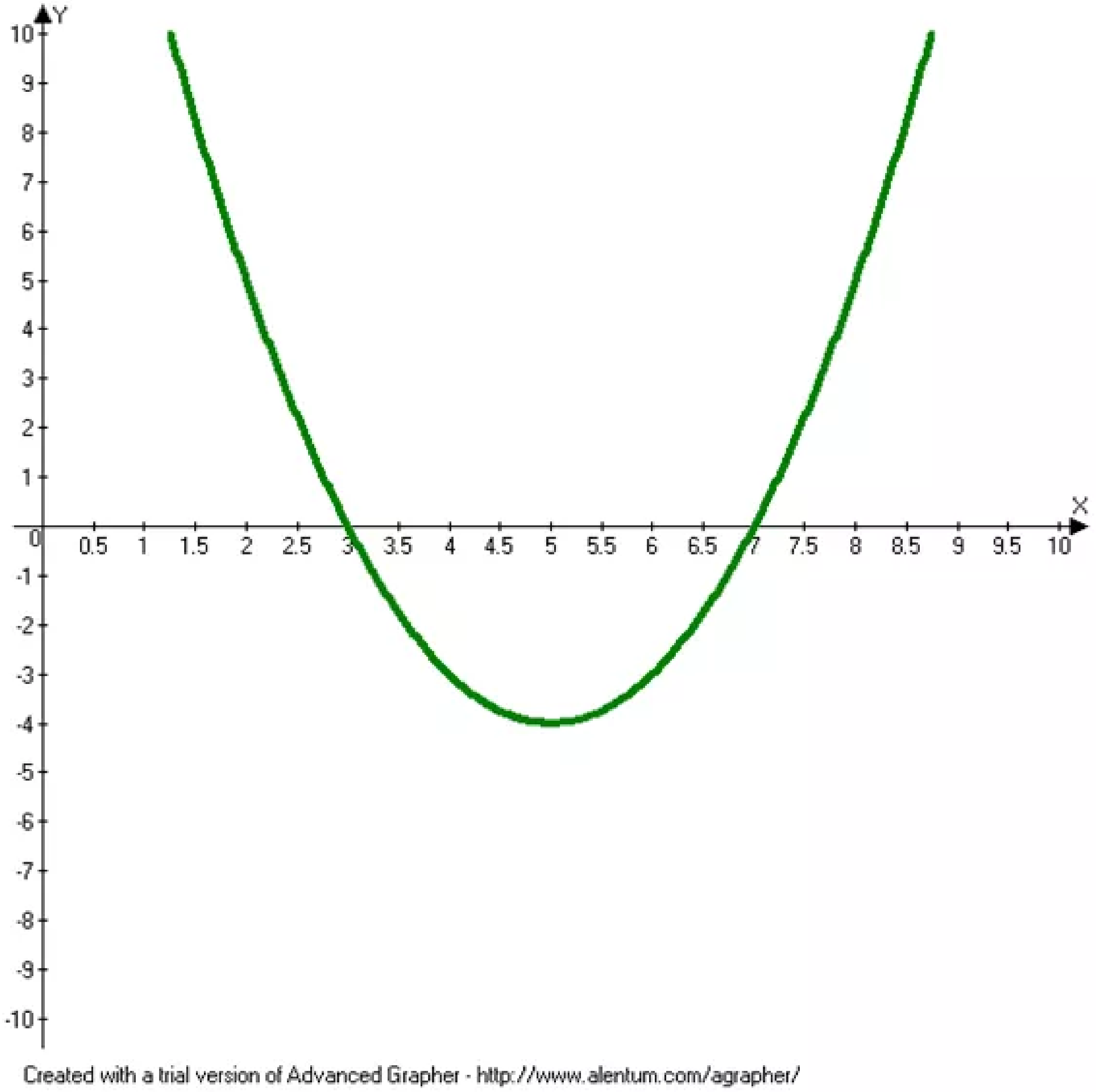
The objective is to solve the given equation by graphing.

If the integral roots cannot be found, then to estimate the roots by stating the consecutive integers between which the roots lie.

Since the roots of a quadratic equation are the x – intercepts of the related quadratic function.

Graph the related function

$$f(x) = x^2 - 10x + 21$$



From the graph, the $x-$intercepts are 3 and 7.

Check: Solve by factoring

$x^2-10x=-21$ (Original equation)

$\Rightarrow \quad x^2-10x+21=0$ (Add 21 to both sides)

$\Rightarrow \quad (x-3)(x-7)=0$ (Factor)

$\Rightarrow \quad x-3=0$

Or, $x-7=0$ (Zero product property)

$\Rightarrow \quad x=3$

Or, $x=7$ (Solve for x)

The solutions of the equation are 3 and 7.

Therefore, the solutions are $\boxed{3\,\&\,7}$.

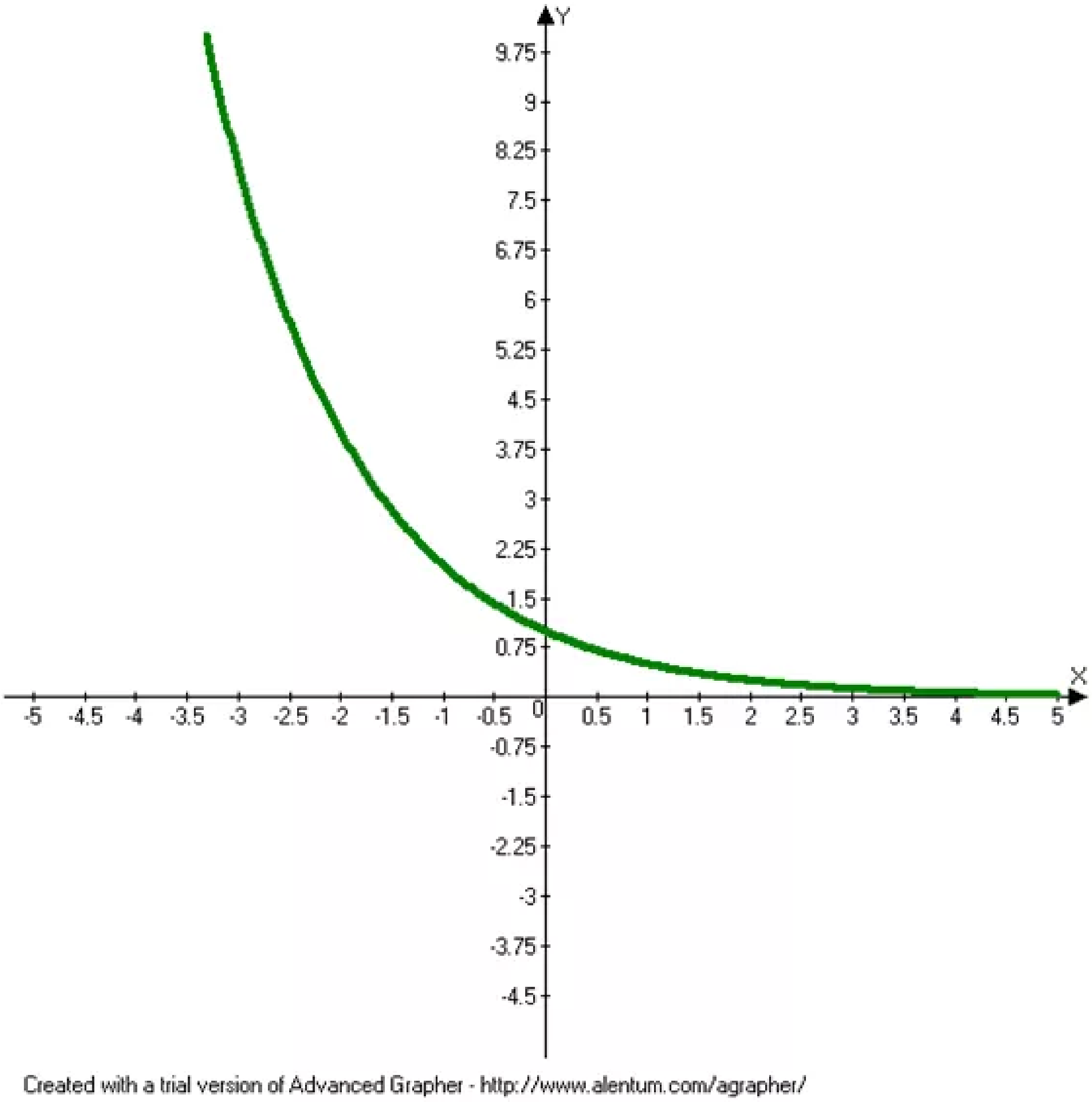
Answer 21PT.

Consider the function $y=\left(\frac{1}{2}\right)^x$

The objective is to graph the given function and to state the $y-$intercept.

x	$f(x)$
-3	8
-2	4
-1	2
0	1
1	$1/2$
2	$1/4$
3	$1/8$

Graph the ordered pairs and connect the points with a smooth curve.



From the graph, the $y-$intercept is $\boxed{1}$.

Answer 22E.

Consider the following equation

$$6x^2 - 13x = 15$$

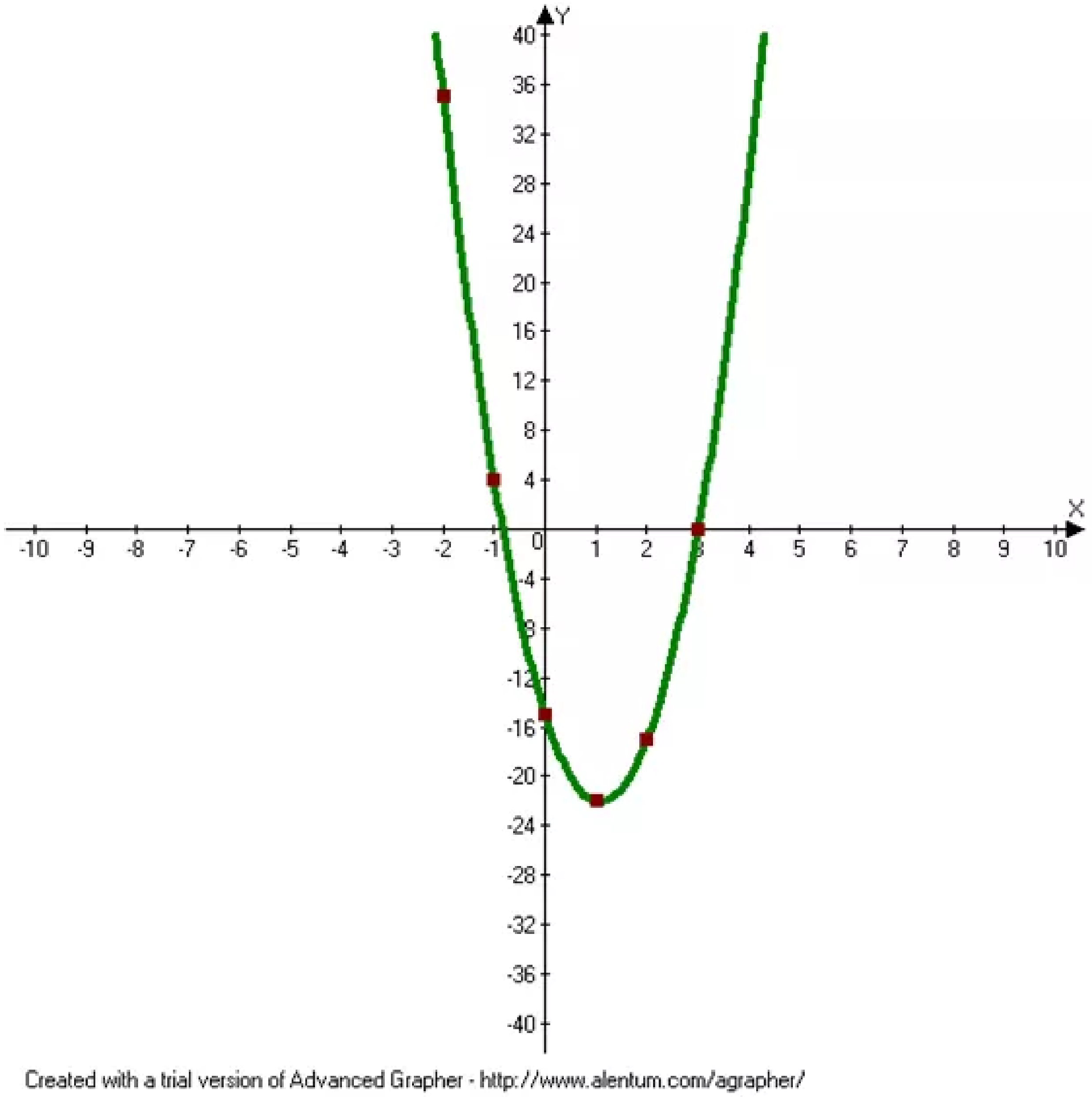
The objective is to solve the given equation by graphing.

If integral roots cannot be found, then to estimate the roots by stating the consecutive integers between which the roots lie.

Since the roots of a quadratic equation are the x – intercepts of the related quadratic function.

Graph the related function

$f(x) = 6x^2 - 13x - 15$ (Green curve)



From the graph, it is clear that one of the $x-$intercept is 3 and the other is not an integer.

x	$f(x)$
-2	35
-1	4
0	-15
1	-22
2	-17
3	0

The above table values are showing with brown dots in the above graph.

The value of the function changes from positive to negative between the ' x ' values of -1 and 0.

One of the $x-$intercept of the graph is 3 and the other is between -1 and 0.

The solutions of the given equation are $\boxed{3}$ and a value between $\boxed{-1}$ and $\boxed{0}$.

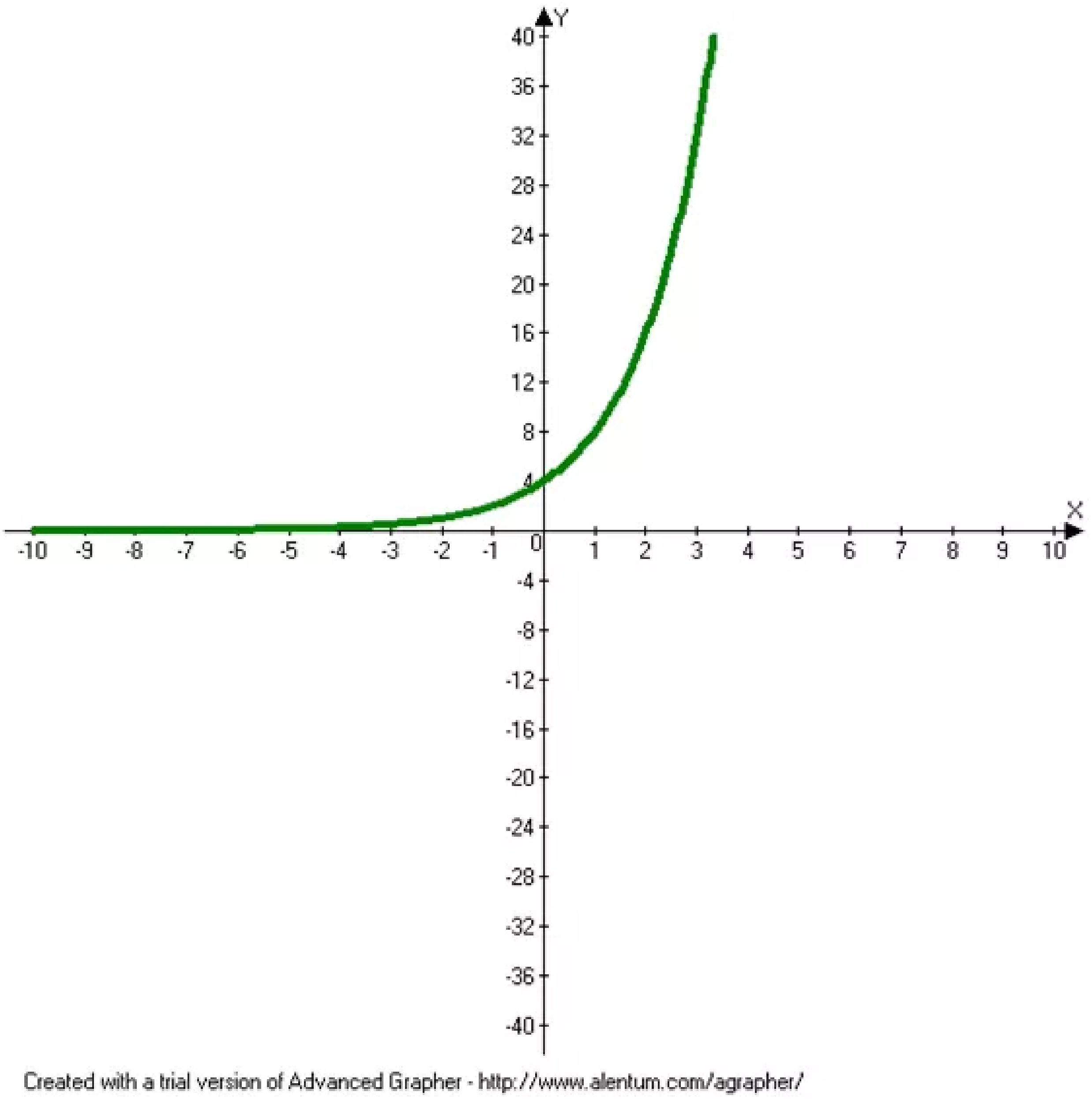
Answer 22PT.

Consider the function $y = 4 \cdot 2^x$

The objective is to graph the given function and to state the $y-$intercept.

x	$f(x)$
-3	$\frac{1}{2}$
-2	1
-1	2
0	4
1	8
2	16
3	32

Graph the ordered pairs and connect the points with a smooth curve.



From the graph, the $y-$intercept is $\boxed{4}$.

Answer 23E.

Consider the quadratic equation

$-3x^2 + 4 = 0$

The objective is to solve the given equation by completing the square.

$-3x^2 + 4 = 0$ (Original equation)

$\Rightarrow \quad x^2 - \frac{4}{3} = 0$ (Divide the equation by -3 and simplify)

$\Rightarrow \quad x^2 - \frac{4}{3} + \frac{4}{3} = 0 + \frac{4}{3}$ (Add $\frac{4}{3}$ to each side)

$\Rightarrow \quad x^2 = \frac{4}{3}$ (Simplify)

$\Rightarrow \quad x = \pm\sqrt{\frac{4}{3}}$ (Take square root of each side)

Use a calculator to evaluate each value of x

$x = \sqrt{\frac{4}{3}}$

Or, $x = -\sqrt{\frac{4}{3}}$

$\Rightarrow \quad x \approx 1.2$

Or, $x = -1.2$

Therefore, the solution set is $\boxed{\{-1.2, 1.2\}}$.

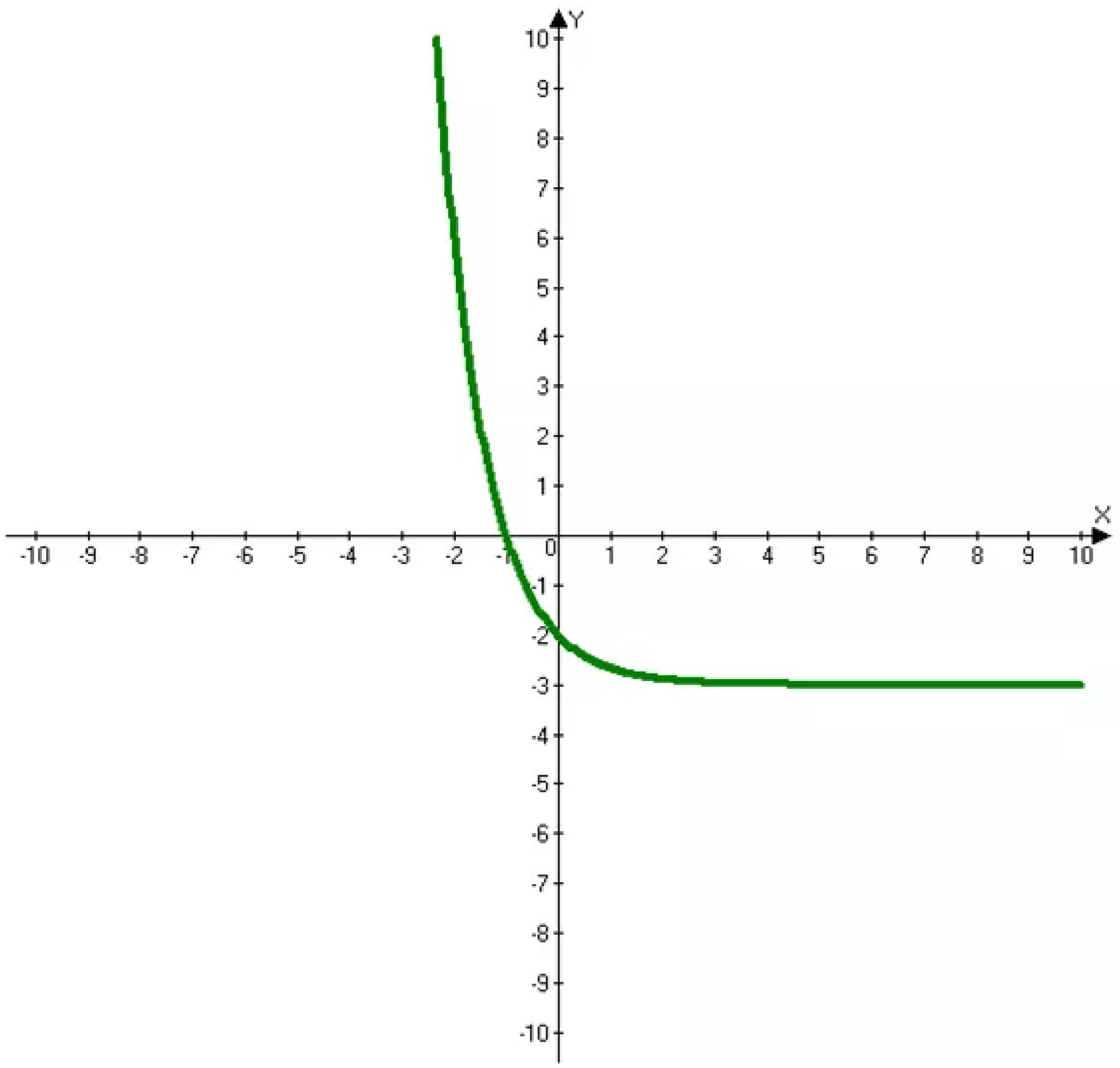
Answer 23PT.

Consider the function $y=\left(\frac{1}{3}\right)^{x}-3$

The objective is to graph the given function and to state the $y-$intercept.

x	$f(x)$
-3	24
-2	6
-1	0
0	-2
1	$-8/3$
2	$-26/9$

Graph the ordered pairs and connect the points with a smooth curve.



From the graph, the $y-$intercept is $\boxed{-2}$.

Answer 24E.

Consider the quadratic equation

$x^2 - 16x + 32 = 0$

The objective is to solve the given equation by completing the square.

$x^2 - 16x + 32 = 0$ (Original equation)

$\Rightarrow \quad x^2 - 16x + 32 - 32 = 0 - 32$ (Subtract 32 from each side)

$\Rightarrow \quad x^2 - 16x = -32$ (Simplify)

$\Rightarrow \quad x^2 - 16x + 64 = -32 + 64$ (Since $\left(\frac{16}{2}\right)^2 = 64$, add 64 to each side)

$\Rightarrow \quad (x-8)^2 = 32$ (Factor $x^2 - 16x + 64$)

$\Rightarrow \quad x - 8 = \pm\sqrt{32}$ (Take the square root of each side)

$\Rightarrow \quad x - 8 + 8 = \pm\sqrt{32} + 8$

(Add 8 to each side)

$\Rightarrow \quad x = 8 \pm \sqrt{32}$ (Simplify)

Use a calculator to evaluate each value of y.

$x = 8 + \sqrt{32}$

Or, $x = 8 - \sqrt{32}$

$\Rightarrow \quad x \approx 13.7$

Or, $x \approx 2.3$

Therefore, the solution set is $\boxed{\{2.3, 13.7\}}$.

Answer 24PT.

Consider $a_1 = 12,$

$n = 6,$
$r = 2$

The objective is to find nth term of given geometric sequence.

The nth term a_n of a geometric sequence with the first term a_1 and common ratio r is given by $a_n = a_1 r^{n-1}$.

Here $n = 6$, so the objective is to find 6th term of geometric sequence.

$a_n = a_1 r^{n-1}$ (Formula for the nth term of a geometric sequence)

$\Rightarrow \quad a_6 = (12)(2)^{6-1}$

(Replace $a_1 = 12, n = 6, r = 2$)

$\Rightarrow \quad a_6 = (12)(2)^5$ (Do subtraction: $6 - 1 = 5$)

$\Rightarrow \quad a_6 = (12)(32)$ (Evaluate exponent: $2^5 = 32$)

$\Rightarrow \quad a_6 = 384$ (Multiply: $(12)(32) = 384$)

Therefore, the 6th term of the geometric sequence is $\boxed{384}$.

Answer 25E.

Consider the quadratic equation

$m^2 - 7m = 5$

The objective is to solve the given equation by completing the square.

$m^2 - 7m = 5$ (Original equation)

$\Rightarrow \quad m^2 - 7m + \frac{49}{4} = 5 + \frac{49}{4}$ (Since $\left(\frac{7}{2}\right)^2 = \frac{49}{4}$, add $\frac{49}{4}$ to each side)

$\Rightarrow \quad m^2 - \frac{7m}{2} + \frac{49}{4} = \frac{69}{4}$ (Simplify)

$\Rightarrow \quad \left(m - \frac{7}{2}\right)^2 = \frac{69}{4}$ (Factor $m^2 - 7m + \frac{49}{4}$)

$\Rightarrow \quad m - \frac{7}{2} = \pm\sqrt{\frac{69}{4}}$ (Take square root of each side)

$\Rightarrow \quad m - \frac{7}{2} + \frac{7}{2} = \pm\sqrt{\frac{69}{4}} + \frac{7}{2}$ (Add $\frac{7}{2}$ to each side)

$\Rightarrow \quad m = \frac{7}{2} \pm \sqrt{\frac{69}{4}}$ (Simplify)

Use a calculator to evaluate each value of m.

$$m = \frac{7}{2} + \sqrt{\frac{69}{4}}$$

Or, $m = \frac{7}{2} - \sqrt{\frac{69}{4}}$

$\Rightarrow \quad m = 3.5 + 4.2$

Or, $m = 3.5 - 4.2$

$\Rightarrow \quad m = 7.7$

Or, $m = -0.7$

Therefore, the solution set is $\boxed{\{-0.7, 7.7\}}$.

Answer 25PT.

Consider $a_1 = 20,$

$n = 4,$

$r = 3$

The objective is to find the nth term of given geometric sequence.

The nth term ' a_n ' of a geometric sequence with the first term ' a_1 ' and common ratio ' r ' is given by

$a_n = a_1 r^{n-1}$ (Formula for the nth term of a geometric sequence)

$a_4 = (20)(3)^{4-1}$

(Replace $n = 4, a = 20, r = 3$)

$a_4 = 20(3)^3$ (Do subtraction: $4 - 1 = 3$)

$a_4 = 20(27)$ (Evaluate exponent: $3^3 = 27$)

$a_4 = 540$ (Multiply: $(20)(27) = 540$)

Therefore, the 4th term of the geometric sequence is $\boxed{540}$.

Answer 26E.

Consider the quadratic equation

$$4a^2+16a+15=0$$

The objective is to solve the given equation by completing the square.

$4a^2+16a+15=0$ (Original equation)

$\Rightarrow \quad a^2+4a+\frac{15}{4}=0$ (Divide the equation by 4)

$\Rightarrow \quad a^2+4a+\frac{15}{4}-\frac{15}{4}=0-\frac{15}{4}$ (Subtract $\frac{15}{4}$ from each side)

$\Rightarrow \quad a^2+4a=\frac{-15}{4}$ (Simplify)

$\Rightarrow \quad a^2+4a+4=\frac{-15}{4}+4$ (Since $\left(\frac{4}{2}\right)^2=4$, add 4 to each side)

$\Rightarrow \quad (a+2)^2=\frac{1}{4}$ (Factor a^2+4a+4)

$\Rightarrow \quad a+2=\pm\sqrt{\frac{1}{4}}$ (Take square root of each side)

$\Rightarrow \quad a+2=\pm\frac{1}{2}$ (Simplify)

$\Rightarrow \quad a+2-2=\pm\frac{1}{2}-2$ (Subtract 2 from both sides)

$\Rightarrow \quad a=-2\pm\frac{1}{2}$ (Simplify)

$\Rightarrow \quad a=-2+\frac{1}{2}$

Or, $a=-2-\frac{1}{2}$

$\Rightarrow \quad a=\frac{-3}{2}$

Or, $a=\frac{-5}{2}$

$\Rightarrow \quad a=-1.5$

Or, $a=-2.5$

Therefore, the solution set is $\boxed{\{-2.5,-1.5\}}$.

Answer 26PT.

Consider the finite geometric sequence $7, -, 63$.

The objective is to find the geometric mean in the given sequence.

Missing term between two nonconsecutive terms in a geometric sequence is called geometric mean.

The nth term 'a_n' of a geometric sequence with the first term 'a_1' and common ratio 'r' is given by $a_n = a_1 \cdot r^{n-1}$

Use the formula for the nth term of a geometric sequence to find a geometric mean.

In the given sequence

$a_1 = 7$ and

$a_3 = 63$

To find a_3, first find 'r'

$a_n = a_1 r^{n-1}$ (Formula for the nth term of a geometric sequence)

$\Rightarrow \quad a_3 = a_1 r^{3-1}$ (Replace 'n' by 3)

$\Rightarrow \quad 63 = 7r^2$ (Replace $a_1 = 7$ and $a_3 = 63$)

$\Rightarrow \quad \frac{63}{7} = \frac{7r^2}{7}$ (Divide each side by 7)

$\Rightarrow \quad 9 = r^2$ (Simplify)

$\Rightarrow \quad \pm 3 = r^2$ (Take square root of each side)

If $r = 3$, the geometric mean is

$7(3) = 21$

If $r = -3$, the geometric mean is

$7(-3) = -21$

Therefore, the geometric mean is $\boxed{\pm 21}$.

Answer 27E.

Consider the quadratic equation

$$\frac{1}{2}y^2+2y-1=0$$

The objective is to solve the given equation by completing the square.

$\frac{1}{2}y^2+2y-1=0$ (Original equation)

$\Rightarrow \quad y^2+4y-2=0$ (Multiply both sides by 2)

$\Rightarrow \quad y^2+4y-2+2=0+2$ (Add 2 to each side)

$\Rightarrow \quad y^2+4y=2$ (Simplify)

$\Rightarrow \quad y^2+4y+4=2+4$ (Since $\left(\frac{4}{2}\right)^2=4$, add 4 to each side)

$\Rightarrow \quad (y+2)^2=6$ (Factor y^2+4y+4)

$\Rightarrow \quad y+2=\pm\sqrt{6}$ (Take square root of each side)

$\Rightarrow \quad y+2-2=\pm\sqrt{6}-2$ (Subtract 2 to both sides)

$\Rightarrow \quad y=-2\pm\sqrt{6}$ (Simplify)

Use a calculator to evaluate each value of y.

$y=-2+\sqrt{6}$

Or, $y=-2-\sqrt{6}$

$\Rightarrow \quad y\approx 0.4$

Or, $y\approx -4.4$

Therefore, the solution set is $\boxed{\{-4.4, 0.4\}}$.

Answer 27PT.

Consider the geometric sequence $-\frac{1}{3},-,-12$.

The objective is to find the geometric mean in the given sequence.

Missing term between two nonconsecutive terms in a geometric sequence is called geometric mean.

The nth term a_n of a geometric sequence with the first term a_1 and common ratio r is given by $a_n = a_1 r^{n-1}$.

Use the formula for the nth term of a geometric sequence to find a geometric mean.

In the given sequence

$a_1 = -\frac{1}{3}$, and

$a_3 = -12$.

To find a_2, first find r.

$a_n = a_1 r^{n-1}$ (Formula for the nth term of a geometric sequence)

$\Rightarrow \quad a_3 = a_1 r^{3-1}$ (Replace n by 3)

$\Rightarrow \quad (-12) = \left(\frac{-1}{3}\right) r^2$ (Replace $a_1 = -\frac{1}{3}$ and $a_3 = -12$)

$\Rightarrow \quad 36 = r^2$ (Multiply each side by -3)

$\Rightarrow \quad \pm 6 = r$ (Take square root of each side)

If $r = 3$, the geometric mean is

$$(3)\left(\frac{-1}{3}\right) = -1$$

If $r = -3$, the geometric mean is

$$(-3)\left(\frac{-1}{3}\right) = 1.$$

Therefore, the geometric mean is $\boxed{\pm 1}$.

Answer 28E.

Consider the quadratic equation

$$n^2 - 3n + \frac{5}{4} = 0$$

The objective is to solve the given equation by completing the square.

$n^2 - 3n + \frac{5}{4} = 0$ (Original equation)

$\Rightarrow \quad n^2 - 3n + \frac{5}{4} - \frac{5}{4} = 0 - \frac{5}{4}$ (Subtract $\frac{5}{4}$ from each side)

$\Rightarrow \quad n^2 - 3n = -\frac{5}{4}$ (Simplify)

$\Rightarrow \quad n^2 - 3n + \frac{9}{4} = \frac{-5}{4} + \frac{9}{4}$ (Since $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$, add $\frac{9}{4}$ to each side)

$\Rightarrow \quad \left(n - \frac{3}{2}\right)^2 = 1$ (Factor $n^2 - 3n + \frac{9}{4}$)

$\Rightarrow \quad n - \frac{3}{2} = \pm 1$ (Take square root of each side)

$\Rightarrow \quad n - \frac{3}{2} + \frac{3}{2} = \pm 1 + \frac{3}{2}$ (Add $\frac{3}{2}$ to each side)

$\Rightarrow \quad n = \frac{3}{2} \pm 1$ (Simplify)

$\Rightarrow \quad n = \frac{3}{2} + 1$

Or, $n = \frac{3}{2} - 1$

$\Rightarrow \quad n = \frac{5}{2}$

Or, $n = \frac{1}{2}$

$\Rightarrow \quad n = 2.5$

Or, $n = 1.5$

Therefore, the solution set is $\boxed{\{1.5, 2.5\}}$.

Answer 29E.

Consider the quadratic equation

$x^2 - 8x = 20$

The objective is to solve the given equation by using quadratic formula.

The solutions of a quadratic equation in the form

$ax^2 + bx + c = 0$, where

$a \neq 0$, are given by the quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Consider $x^2 - 8x = 20$

$\Rightarrow \quad x^2 - 8x - 2x = 0$

For this equation $a = 1$,

$b = -8$, and

$c = -20$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ (Quadratic formula)}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-20)}}{2(1)}$$

(Replace $a = 1, b = -8,$ and $c = -20$)

$$x = \frac{8 \pm \sqrt{64 + 80}}{2} \text{ (Evaluate exponent and multiply)}$$

$$x = \frac{8 \pm \sqrt{144}}{2} \text{ (Simplify)}$$

$$x = \frac{8 + 12}{2}$$

Or, $x = \dfrac{8 - 12}{2}$

$x = 10$

Or, $x = -2$

Therefore the solution set is $\boxed{\{-2, 10\}}$.

Answer 29PT.

Consider the following data

The amount invested is $\$1500$.

Rate of interest is 6%

The Compounded interest calculated quarterly for 10 years.

The objective is to determine the total amount for the given investment.

Use the formula

$A = P\left(1+\frac{r}{n}\right)^{nt}$ to determine the final amount where

' A ' represents the amount of the investment

' P ' represents the principal

' r ' represents the annual rate of interest expressed as a decimal.

' n ' represents the number of times that the interest is compounded each year and

' t ' represents the number of years that the money is invested.

Then, $P = \$1500$,

$r = 6\%$ or 0.06,

$n = 4,$

$t = 10$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$ (Compound interest equation)

$$A = 1500\left(1+\frac{0.06}{4}\right)^{4\cdot 10}$$

(Substitute $p = 1500, r = 6\%$ or 0.06, $n = 4$ and $t = 10$)

$A = 1500(1+0.015)^{40}$ (Divide: $\frac{0.06}{4} = 0.015$, Multiply: $4\cdot 10 = 40$)

$= 1500(1.015)^{40}$ (Do addition: $1+0.015 = 1.015$)

$= 2721.08$ (Simplify)

Therefore, the final amount in the account is about $\boxed{\$2721.08}$.

Answer 30E.

Consider the quadratic equation

$$r^2 + 10r + 9 = 0$$

The objective is to solve the given equation by using quadratic formula.

The solutions of a quadratic equation in the form

$ar^2 + br + c = 0$, where

$a \neq 0$, are given by the quadratic formula,

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For the given quadratic equation

$a = 1,$ and
$b = 10$

$c = 9$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{(Quadratic formula)}$$

$$\Rightarrow \quad r = \frac{-10 \pm \sqrt{(10)^2 - 4(1)(9)}}{2(1)}$$

(Replace $a = 1, b = 10$ and $c = 9$)

$$\Rightarrow \quad r = \frac{-10 \pm \sqrt{64}}{2} \quad \text{(Simplify)}$$

$$\Rightarrow \quad r = \frac{-10 + 8}{2}$$

Or, $r = \dfrac{-10 - 8}{2}$

$$\Rightarrow \quad r = -1$$

Or, $r = -9$

The solution set is $\boxed{\{-9, -1\}}$.

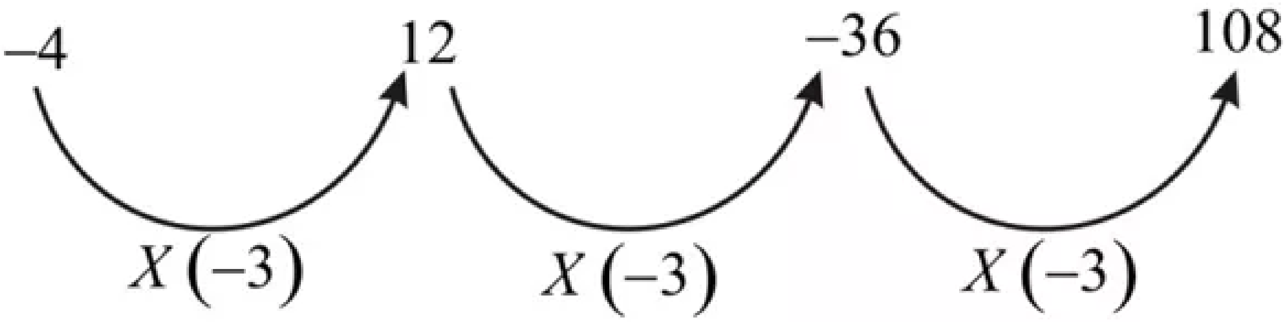
Answer 30PT.

Consider the sequence $-4, 12, -36, 108, \ldots$

The objective is to find the next value of the given pattern.

First check whether the given sequence is geometric or not.

A geometric sequence is a sequence in which each term after the nonzero first term is found by multiplying the previous term by a constant called the common ratio r, where $r \neq 0, 1$.



In this sequence, each term is found by multiplying the previous term times -3.

This sequence is geometric.

To find the next value of the given sequence multiply -3 to 108, the next value is -324.

Therefore, the next value of the pattern is $\boxed{(A) - 324}$.

Answer 31E.

Consider the quadratic equation

$$4p^2 + 4p = 15$$

$$\Rightarrow \quad 4p^2 + 4p - 15 = 0$$

The objective is to solve the given equation by using quadratic formula.

The solutions of a quadratic equation in the form

$ap^2 + bp + c = 0$, where

$a \neq 0$, are given by the quadratic formula,

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For the given quadratic equation

$a = 4,$ and
$b = 4$

$c = -15$

$$p=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$ (Quadratic formula)

$$\Rightarrow \quad p=\frac{-4\pm\sqrt{(4)^2-4(4)(-15)}}{2(4)}$$

(Replace $a=4, b=4$ and $c=-15$)

$$\Rightarrow \quad p=\frac{-4\pm\sqrt{256}}{8}$$ (Simplify)

$$\Rightarrow \quad p=\frac{-4+16}{8}$$

Or, $p=\frac{-4-16}{8}$

$$\Rightarrow \quad p=1.5$$

Or, $p=-2.5$

Therefore, the solution set is $\boxed{\{-2.5,1.5\}}$.

Answer 32E.

Consider the quadratic equation

$$2y^2+3=-8y$$

$$\Rightarrow \quad 2y^2+8y+3=0$$

The objective is to solve the given equation by using quadratic formula.

The solutions of a quadratic equation in the form

$ay^2+by+c=0$, where

$a\neq 0$, are given by the quadratic formula,

$$y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

For the given quadratic equation

$a=2,$ and
$b=8$

$c=3$

$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (Quadratic formula)

$\Rightarrow \quad y = \frac{-8 \pm \sqrt{(8)^2 - 4(2)(3)}}{2(2)}$

(Replace $a = 2, b = 8$ and $c = 3$)

$\Rightarrow \quad y = \frac{-8 \pm \sqrt{40}}{4}$ (Simplify)

$\Rightarrow \quad y = \frac{-8 \pm \sqrt{40}}{4}$

(or) $y = \frac{-8 - \sqrt{40}}{4}$

$\Rightarrow \quad y = -0.418861170$

(or) $y = -3.581138830$

$\Rightarrow \quad y \approx -0.4$

(or) $y = -3.6$

Therefore, the solution set is $\boxed{\{-3.6, -0.4\}}$.

Answer 33E.

Consider the quadratic equation

$$2d^2+8d+3=3$$

$$\Rightarrow \quad 2d^2+8d=0$$

The objective is to solve the given equation by using quadratic formula.

The solutions of a quadratic equation in the form

$ad^2+bd+c=0$, where

$a \neq 0$, are given by the quadratic formula,

$$d=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

For the given quadratic equation

$a=2,$ and
$b=8$

$c=0$

$$d=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \text{ (Quadratic formula)}$$

$$\Rightarrow \quad d=\frac{-8\pm\sqrt{(8)^2-4(2)(0)}}{2(2)} \text{ (Replace } a=2, b=8 \text{ and } c=0\text{)}$$

$$\Rightarrow \quad d=\frac{-8\pm\sqrt{64}}{4} \text{ (Simplify)}$$

$$\Rightarrow \quad d=\frac{-8+8}{4}$$

(or) $d=\frac{-8-8}{4}$

$$\Rightarrow \quad d=0$$

(or) $d=-4$

Therefore, the solution set is $\boxed{(-4,0)}$.

Answer 34E.

Consider the quadratic equation

$21a^2 + 5a - 7 = 0$

The objective is to solve the given equation by using quadratic formula.

The solutions of a quadratic equation in the form

$ax^2 + bx + c = 0$, where

$a \neq 0$, are given by quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For the given quadratic equation, the variable is a.

And $a = 21,$

$b = 5,$ and

$c = -7$

Therefore, $a = \dfrac{-5 \pm \sqrt{(5)^2 - 4(21)(-7)}}{2(21)}$ (Replace corresponding values)

$\Rightarrow \quad a = \dfrac{-5 \pm \sqrt{613}}{42}$ (Simpllify)

$\Rightarrow \quad a = \dfrac{-5 + 24 \cdot 75883680}{42}$

(or) $a = \dfrac{-5 - 24 \cdot 75883680}{42}$

$\Rightarrow \quad a = 0.470448495$

(or) $a = -0.708543733$

$\Rightarrow \quad a \approx 0.5$

(or) $a \approx -0.7$

Therefore, the solution set is $\boxed{\{-0.7, 0.5\}}$.

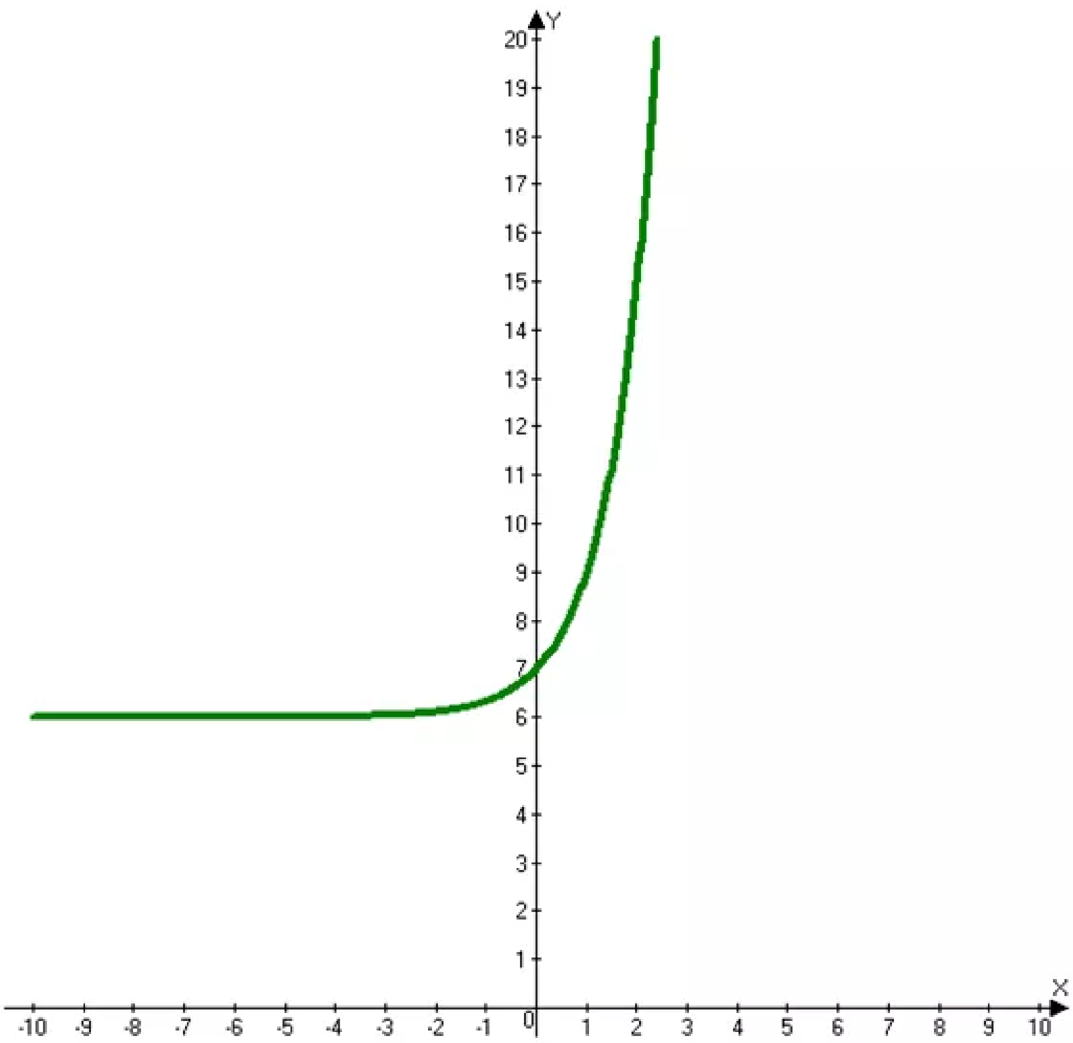
Answer 35E.

Consider the function $y = 3^x + 6$

The objective is to graph the given function and to state the $y-$intercept.

x	$f(x)$
-3	$\frac{163}{27}$
-2	$\frac{55}{9}$
-1	$\frac{19}{3}$
0	7
1	9
2	15
3	33

Graph the ordered pairs and connect the points with a smooth curve.



From the graph, the $y-$intercept is $\boxed{7}$.

Answer 36E.

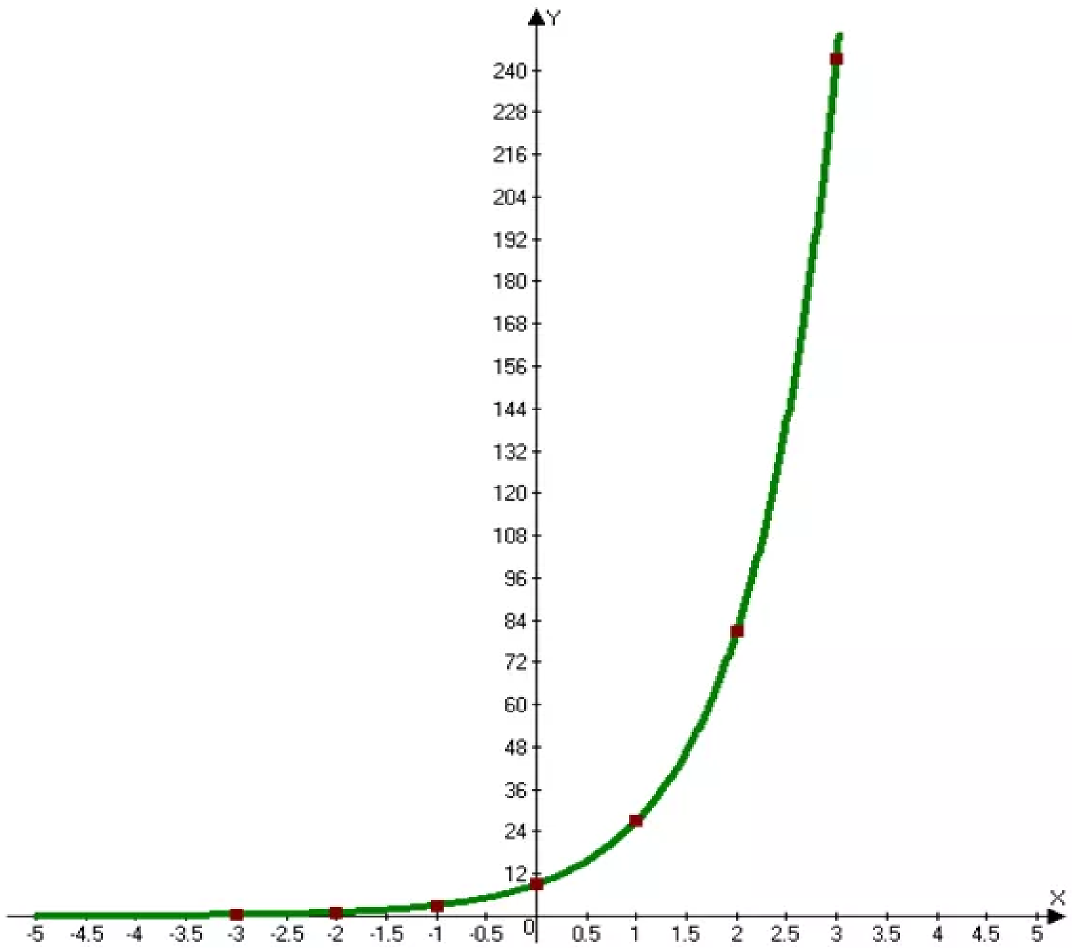
Consider the function $y = 3^{x+2}$.

The graph of the function is shown with green curve in below graph.

The objective is to graph the given function and to state the $y-$intercept.

x	$f(x)$
-3	$\frac{1}{3}$
-2	1
-1	3
0	9
1	27
2	81
3	243

Graph the ordered pairs (brown dots) and connect the points with a smooth curve.



From the graph, the $y-$Intercept is $\boxed{9}$.

Answer 37E.

Consider the function $y = 2\left(\frac{1}{2}\right)^x$.

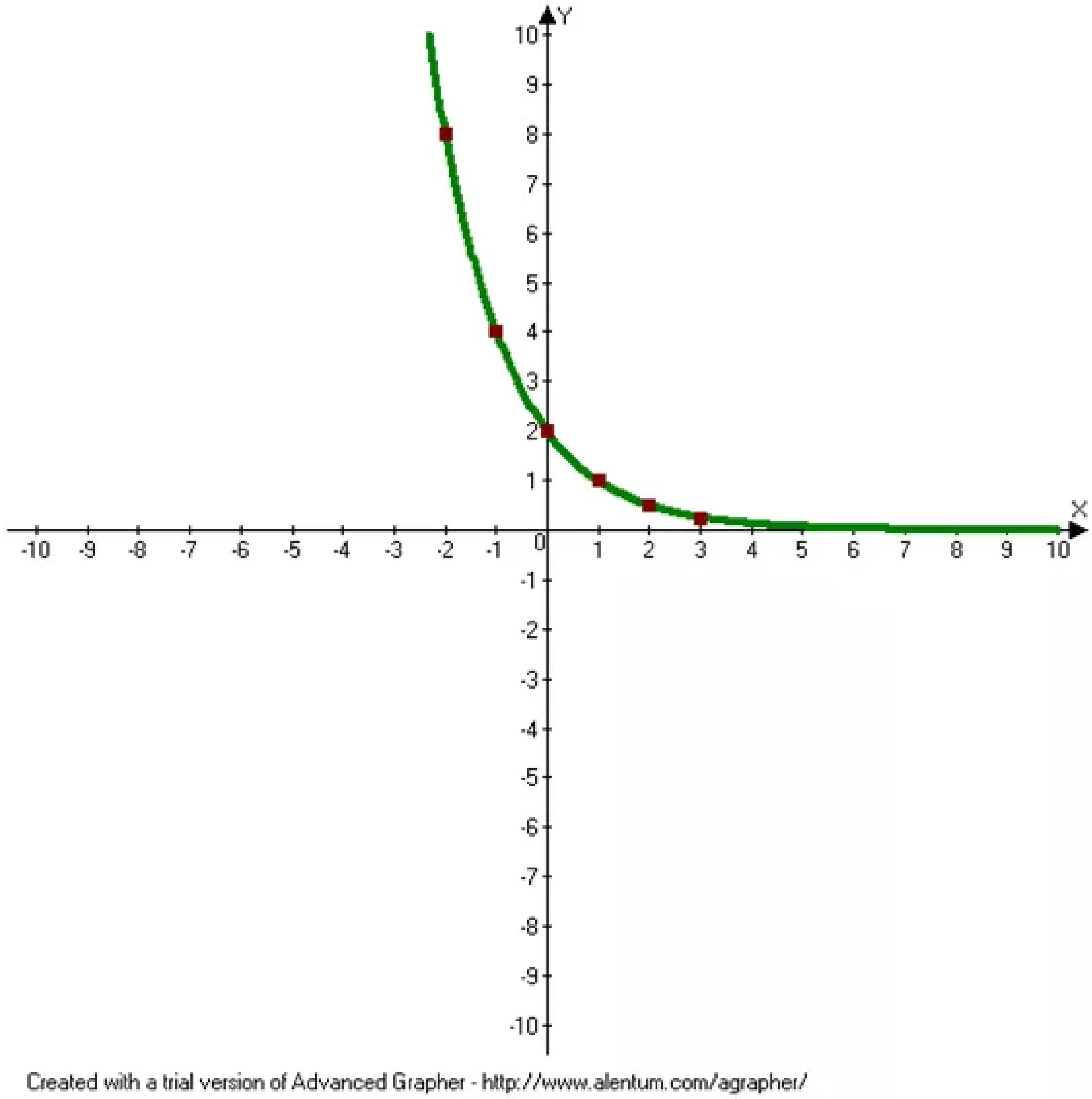
The graph of the function is shown with green curve.

The objective is to graph the given function and to state the $y-$intercept.

x	$f(x)$
-3	16
-2	8
-1	4
0	2
1	1
2	$\frac{1}{2}$
3	$\frac{1}{4}$

The above table values are shown with brown dots in the below graph.

Graph the ordered pairs and connect the points with a smooth curve.



From the graph, the $y-$intercept is $\boxed{2}$.

Answer 38E.

Consider the following data

The amount invested is $\$2000$.

Rate of interest is 8%

Compounded quarterly for 8 years

The objective is to determine the final amount for the given investment.

Use the formula

$A = P\left(1+\frac{r}{n}\right)^{nt}$ to determine the final amount where

'A' represents the amount of the investment

'P' represents the principal

'r' represents the annual rate of interest expressed as a decimal.

'n' represents the number of times that the interest is compounded each year and

't' represents the number of years that the money is invested.

Then, $P = \$2000,$

$r = 8\%$ or 0.08

$n = 4,$

$t = 8$

$A = P\left(1+\frac{r}{n}\right)^{nt}$ (Compound interest equation)

$A = 2000\left(1+\frac{0.08}{4}\right)^{4\cdot 8}$

(Substitute $P = 2000, r = 8\%$ or $0.08, n = 4$ and $t = 8$)

$A = 2000(1+0.02)^{32}$

(Simplify: $\frac{0.08}{4} = 0.02$, $4\cdot 8 = 32$)

$A = 2000(1.02)^{32}$ (Do addition: $1+0.02 = 1.02$)

$A \approx 3769.08$ (Simplify)

Therefore, the final amount in the account is about $\boxed{\$3769.08}$.

Answer 39E.

Consider the following data

The amount invested is $\$5500$.

Rate of interest is 5.25%

The compounded quarterly for 15 years

The objective is to determine the final amount for the given investment.

Use the formula

$A = P\left(1+\frac{r}{n}\right)^{nt}$ to determine the final amount where

' A ' represents the amount of the investment

' P ' represents the principal

' r ' represents the annual rate of interest expressed as a decimal.

' n ' represents the number of times that the interest is compounded each year and

' t ' represents the number of years that the money is invested.

Then, $P = \$5500$,

$r = 5.25\%$ or 0.0525,

$n = 12,$

$t = 15$

$A = P\left(1+\frac{r}{n}\right)^{nt}$ (Compound interest equation)

$$A = 5500\left(1+\frac{0.0525}{12}\right)^{12\cdot 15}$$

(Substitute the values of ' P ',' r ',' n 'and' t ',compound interest equation)

$$A = 5500(1+0.004375)^{180}$$

(Multiply: $12\cdot 15 = 180$)

$$A = 5500(1.004375)^{180}$$

(Do addition)

$A \approx 12067.67$ (Simplify)

Therefore, the final amount in the account is about $\boxed{\$12067.68}$.

Answer 40E.

Consider the following data

The amount invested is $\$15,000$

Rate of interest is 7.5%

The Compounded interest calculated monthly for 25 years.

The objective is to determine the final amount for the given investment.

Use the formula

$A = P\left(1+\frac{r}{n}\right)^{nt}$ to determine the final amount where.

' A ' represents the amount of the investment

' P ' represents the principal

' r ' represents the annual rate of interest expressed as a decimal.

' n ' represents the number of times that the interest is compounded each year and

' t ' represents the number of years that the money is invested.

Then,

$$P = \$15,000,$$
$$r = 7.5\% \text{ or } 0.075,$$
$$n = 12,$$
$$t = 25$$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$A = 15,000\left(1+\frac{0.075}{12}\right)^{12.25} \quad \text{(Compound interest equation)}$$

$$A = 15,000\left(1+\frac{0.075}{12}\right)^{300} \quad \text{(Multiply)}$$

$$A = 15,000(1.00625)^{300} \quad \text{(Do addition)}$$

$$A \approx 97243.21 \quad \text{(Simplify)}$$

Therefore, the final amount in the account is about $\boxed{\$97243.21}$.

Answer 41E.

Consider the following data

The amount invested is $\$500$.

Rate of interest is 9.75%

Compounded daily for 40 years

The objective is to determine the final amount for the given investment.

Use the formula

$A = P\left(1+\frac{r}{n}\right)^{nt}$ to determine the final amount where

‘ A ’ represents the amount of the investement

‘ P ’ represents the principal

‘ r ’ represents the annual rate of interest expressed as a decimal.

‘ n ’ represents the number of times that the interest is compounded each year and

‘ t ’ represents the number of years that the money is invested.

Then, $P = \$500$,

$r = 9.75\%$ or 0.0975,

$n = 365$

$t = 40$

$A = P\left(1+\frac{r}{n}\right)^{nt}$ (Compound interest equation)

$$A = 500\left(1+\frac{0.0975}{365}\right)^{(365)\cdot(40)}$$

(Substitute the values of ‘ P ’,‘ r ’,‘ n ’and‘ t ’,compound interest equation)

$A \approx 24688.36$ (Simplify)

Therefore, the final amount for the given investment is $\boxed{\$24688.36}$.

Answer 42E.

Consider $a_1 = 2,$

$n = 5,$

$r = 2$

The objective is to find nth term of given geometric sequence.

The nth term a_n of a geometric by

$$a_n = a_1 r^{n-1}$$

Here $n = 5$, so the objective is to find 5th term of geometric sequence.

$a_n = a_1 r^{n-1}$ (Formula for the nth term of a geometric sequence)

$\Rightarrow \quad a_5 = (2)(2)^{5-1}$ (Replace $a_1 = 2, n = 5, r = 2$)

$\Rightarrow \quad a_5 = (2)(2)^4$ (Do subtraction: $5 - 1 = 4$)

$\Rightarrow \quad a_5 = (2)(16)$ (Evaluate exponent: $2^4 = 16$)

$\Rightarrow \quad a_5 = 32$ (Multiply: $(2)(16) = 32$)

Therefore, the 5th term of the geometric sequence is $\boxed{32}$.

Answer 43E.

Consider $a_1 = 7,$

$n = 4,$

$r = \frac{2}{3}$

The objective is to find nth term of given geometric sequence.

The nth term a_n of a geometric sequence with the first term a_1 and common ratio r is given by

$$a_n = a_1 r^{n-1}$$

Here $n = 4$, so the objective is to find 4th term of geometric sequence.

$a_n = a_1 r^{n-1}$ (Formula for the nth term of a geometric sequence)

$\Rightarrow \quad a_4 = (7)\left(\frac{2}{3}\right)^{4-1}$ (Replace $a_1 = 7, n = 4, r = \frac{2}{3}$)

$\Rightarrow \quad a_4 = (7)\left(\frac{2}{3}\right)^{3}$ (Do subtraction: $4 - 1 = 3$)

$\Rightarrow \quad a_4 = (7)\left(\frac{8}{27}\right)$ (Evaluate exponent: $\left(\frac{2}{3}\right)^3 = \frac{8}{27}$)

$\Rightarrow \quad a_4 = \frac{56}{27}$ (Multiply: $(7)\left(\frac{8}{27}\right) = \frac{56}{27}$)

Therefore, the 4th term of the geometric sequence is $\boxed{\frac{56}{27}}$.

Answer 44E.

Consider $a_1 = 243$,

$n = 5,$

$r = \frac{-1}{3}$

The objective is to find nth term of given geometric sequence.

The nth term a_n of a geometric sequence with the first term a_1 and common ratio r is given by

$a_n = a_1 r^{n-1}$

Here $n = 5$, so the objective is to find 5th term of geometric sequence.

$a_n = a_1 r^{n-1}$ (Formula for the nth term of a geometric sequence)

$\Rightarrow \quad a_5 = (243)\left(\frac{-1}{3}\right)^{5-1}$ (Replace $a_1 = 243, n = 5, r = -\frac{1}{3}$)

$\Rightarrow \quad a_5 = (243)\left(-\frac{1}{3}\right)^{4}$ (Do subtraction: $5-1=4$)

$\Rightarrow \quad a_5 = (243)\left(\frac{1}{81}\right)$ (Evaluate exponent: $\left(\frac{-1}{3}\right)^4 = \frac{1}{81}$)

$\Rightarrow \quad a_5 = 3$ (Simplify)

Therefore, the 5th term of the geometric sequence is $\boxed{3}$.

Answer 45E.

Consider the geometric sequence $5, -, 20$.

The objective is to find the geometric mean in the given sequence.

Missing term between two nonconsecutive terms in a geometric sequence is called geometric mean.

The nth term a_n or a geometric sequence with the first term a_1 and common ratio r is given by

$$a_n = a_1 r^{n-1}$$

Use the formula for the nth term of a geometric sequence to find a geometric mean.

In the given sequence

$a_1 = 5$ and

$a_3 = 20$.

To find a_2, first find r.

$a_n = a_1 r^{n-2}$ (Formula for the nth term of a geometric sequence)

$\Rightarrow \quad a_3 = a_1 r^{3-1}$ (Replace n by 3)

$\Rightarrow \quad 20 = (5)(r^2)$ (Replace $a_1 = 5$ and $a_3 = 20$)

$\Rightarrow \quad 4 = r^2$ (Divide both sides by 5)

$\Rightarrow \quad \pm 2 = r$ (Take square root of each side)

If $r = 2$, the geometric mean is

$(2)(5) = 10$

If $r = -2$, the geometric mean is

$(-2)(5) = -10$.

Therefore, the geometric mean is $\boxed{\pm 10}$.

Answer 46E.

Consider the geometric sequence $-12, -, -48$.

The objective is to find the geometric mean in the given sequence.

Missing term between two nonconsecutive terms in a geometric sequence is called geometric mean.

The nth term a_n or a geometric sequence with the first term a_1 and common ratio r is given by

$$a_n = a_1 r^{n-1}$$

Use the formula for the nth term of a geometric sequence to find a geometric mean.

In the given sequence

$a_1 = -12$ and

$a_3 = -48$.

To find a_2, first find r.

$a_n = a_1 r^{n-2}$ (Formula for the nth term of a geometric sequence)

$\Rightarrow \quad a_3 = a_1 r^{3-1}$ (Replace n by 3)

$\Rightarrow \quad -48 = (-12) r^2$ (Replace $a_1 = -12$ and $a_3 = -48$)

$\Rightarrow \quad 4 = r^2$ (Divide both sides by -12)

$\Rightarrow \quad \pm 2 = r$ (Take square root of each side)

If $r = 2$, the geometric mean is

$$(-12)(2) = -24$$

If $r = -2$, the geometric mean is

$$(-12)(-2) = 24.$$

Therefore, the geometric mean is $\boxed{\pm 24}$.

Answer 47E.

Consider the geometric sequence $1, -, \frac{1}{4}$.

The objective is to find the geometric mean in the given sequence.

Missing term between two nonconsecutive terms in a geometric sequence is called geometric mean.

The nth term a_n or a geometric sequence with the first term a_1 and common ratio r is given by

$$a_n = a_1 r^{n-1}$$

Use the formula for the nth term of a geometric sequence to find a geometric mean.

In the given sequence

$a_1 = 1$ and

$a_3 = \frac{1}{4}$.

To find a_2, first find r.

$a_n = a_1 r^{n-2}$ (Formula for the nth term of a geometric sequence)

$\Rightarrow \quad a_3 = a_1 r^{3-1}$ (Replace n by 3)

$\Rightarrow \quad \frac{1}{4} = (1) r^2$ (Replace $a_1 = 1$ and $a_3 = \frac{1}{4}$)

$\Rightarrow \quad \pm\frac{1}{2} = r$ (Take square root of each side)

If $r = \frac{1}{2}$, the geometric mean is

$$(1)\left(\frac{1}{2}\right) = \frac{1}{2}$$

If $r = -\frac{1}{2}$, the geometric mean is

$$(1)\left(\frac{-1}{2}\right) = \frac{-1}{2}.$$

Therefore, the geometric mean is $\boxed{\pm\frac{1}{2}}$.