CBSE Class 09

Mathematics

Sample Paper 4 (2019-20)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. All the questions are compulsory.
ii. The question paper consists of 40 questions divided into 4 sections A, B, C, and D.
iii. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
iv. There is no overall choice. However, an internal choice has been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
v. Use of calculators is not permitted.

Section A

1. A terminating decimal is

    a. a natural number

    b. a rational number

    c. a whole number

    d. an integer.

2. The value of $(102)^3$ is

    a. 1820058

    b. 1001208

c. 1061280

d. 1061208

3. An angle is one-fifth of its supplement. The measure of the angle is:-

a. 15^0
b. 75^0
c. 150^0
d. 30^0

4. An external bisector of an angle measuring 70^0 will divide the angle into two angles measuring

a. 110^0

b. 70^0

c. 55^0

d. 35^0

5. The expanded form of $(3x-5)^3$ is

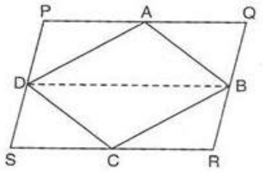
a. none of these

b. $27x^3+135x^2+225x-125$

c. $27x^3-135x^2+225x-125$

d. $27x^3+135x^2-225x-125$

6. A, B, C, D are mid-points of sides of parallelogram PQRS. If $ar\ (PQRS)=36\ cm^2$, then ar (ABCD) is



a. $24\ cm^2$.

b. $30\ cm^2$.

c. $36\ cm^2$.

d. $18\ cm^2$.

7. If $x^2 + \frac{1}{x^2} = 38$, then the value of $x - \frac{1}{x}$ is

a. 3

b. 4

c. 5

d. 6

8. The area of a triangle whose sides are 12 cm, 16 cm and 20 cm is

a. 72 cm^2

b. 320 cm^2

c. 240 cm^2

d. 96 cm^2

9. The curved surface area of a cylinder whose circumference of the base is 22 m and height 3 m is

a. 99m^2

b. 33m^2

c. 132m^2

d. 66m^2.

10. Two coins are tossed simultaneously. The probability of getting no head is :

a. 1

b. $\frac{1}{4}$

c. $\frac{3}{4}$

d. $\frac{1}{2}$

11. Fill in the blanks: Every ________ number is a whole number.
12. Fill in the blanks: Any point on the X-axis is of the form of ________.

OR

Fill in the blanks: Any point on the X-axis is of the form of ________.

13. Fill in the blanks: The positive abscissa lies in ________, ________ quadrants.
14. Fill in the blanks: The perpendicular from the centre of a circle to a chord ________ the chord.
15. Fill in the blanks: The lateral surface area of a cube is 512 cm^2, then the side of the cube is ________ cm.
16. Classify the following number as rational or irrational.1.101001000100001......
17. Whether the following are zero of the polynomial, indicated against them.p(x) = $3x^2 - 1$, x = $-\frac{1}{\sqrt{3}}, \frac{2}{3}$.
18. Find the area of the four walls of a room whose length is 6 m, breadth 5 m and height 4 m. Also find the cost of white-washing the walls, if the rate of white-washing is Rs.5 per square metre. (Doors, windows and other openings ignored).

OR

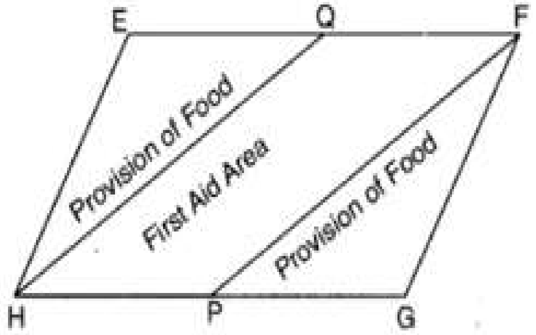
A granary is in the shape of a cuboid of size 8 m $\times$ 6 m $\times$ 3 m. If a bag of grain occupies a space of 0.65 m^3, how many bags can be stored in the granary?

19. In a parallelogram ABCD diagonals AC and BD intersect at O and AC = 6.8 cm and BD = 5.6 cm. Find the measures of OC and OD.
20. 2x + y = 3 passes from origin. Is this statement true or false?
21. Simplify: $(\sqrt{3} - \sqrt{2})^2$
22. Give the equations of two lines passing through (4, -2). How many more such lines are there, and why?
23. Find $p(0)$, $p(1)$ and $p(2)$ for each of the polynomials: $p(t) = 2 + t + 2t^2 - t^3$

OR

Find the cube of the following binomial expression: $4 - \frac{1}{3x}$

24. A flood relief camp was organized by state government for the people affected by the natural calamity near a city. Many school students volunteered to participate in the relief work. In the camp, the food items and first aid centre kits were arranged for the flood victims. The piece of land used for this purpose is shown in the figure.



If EFGH is a parallelogram with P and Q as mid-points of sides GH and EF respectively, then show that area used for first aid is half of the total area.

25. Prepare a continuous grouped frequency distribution from the following data:

Mid-point	Frequency
5	4
15	8
25	13
35	12
45	6

Also, find the size of class intervals.

OR

Given below is a cumulative frequency distribution table showing the age of people living in a locality.

Age in years	No. of persons
Above 108	0
Above 96	1
Above 84	3
Above 72	5

Above 60	20
Above 48	158
Above 36	427
Above 24	809
Above 12	1026
Above 0	1124

Prepare a frequency distribution table.

26. Following a lecture on waste management, a school decided to keep two dustbins in each class, one for biodegradable and other for non-biodegradable waste. One of the dustbins is cylindrical in shape with radius 35cm and height 40 cm, while the other is cuboidal in shape with dimensions 30 cm $\times$ 30 cm $\times$ 30 cm.
    i. Which dustbin would occupy less base area?
    ii. Which dustbin has more capacity?

27. Simplify $\left\{\left[625^{\frac{-1}{2}}\right]^{-\frac{1}{4}}\right\}^2$

OR

Represent $\sqrt{3}$ on a number line.

28. In which quadrant or on which axis do each of the points (- 2, 4), (3, - 1), (- 1, 0), (1, 2) and (- 3, - 5) lie? Verify your answer by locating them on the Cartesian plane.

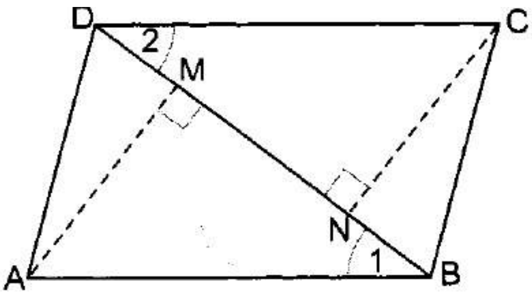
29. Write two solutions of the form x = 0, y = a and x = b, y = 0 for each of the following equation: 5x - 2y = 10

OR

If the work done by a body on application of a constant force is directly proportional to the distance traveled by the body, express this in the form of an equation in two variables and draw the graph of the same by taking the constant force as 5 units. Also read from the graph the work done when the distance travelled by the body is 0 units.

30. Construct an angle of 105^{o} at O.

31. In Figure AM and CN are perpendiculars to the diagonal BD of a parallelogram ABCD. Prove that AM = CN.

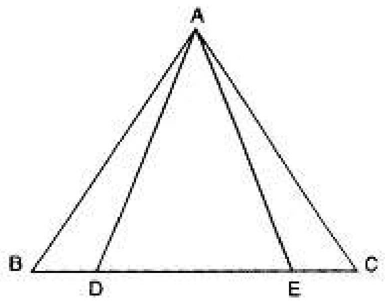


32. In $\triangle$ABC, side AB is produced to D so that BD = BC. If $\angle$B = 60° and $\angle$A = 70°, prove that:

i. AD > CD
ii. AD > AC

OR

In figure, AD = AE and D and E are points on BC such that BD = EC. Prove that AB = AC.



33. If the side of a rhombus is 10 cm and one diagonal is 16 cm, then the area of the rhombus is 96 cm^2. State whether the statement is True or False and justify your answer.

34. Over the past 200 working days, the number of defective parts produced by a machine is given in the following table:

Number of defective parts	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Days	50	32	22	18	12	12	10	10	10	8	6	6	2	2

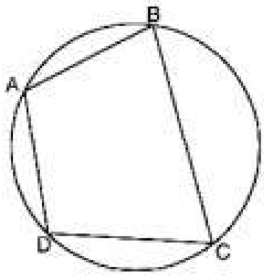
Determine the probability that tomorrow output will have

i. no defective part,
ii. at least one defective part,
iii. not more than 5 defective parts.

35. ABCD is a cyclic trapezium with AD | | BC. If $\angle B = 70^\circ$, determine other three angles of the trapezium.

OR

In figure, ABCD is a cyclic quadrilateral in which $\angle A = (x + y + 10)^0$, $\angle B = (y + 20)^0$,$\angle C = (x + y - 30)^0$ and $\angle D = (x + y)^0$ Find x and y.



36. The opposite sides of a quadrilateral are parallel. If one angle of the quadrilateral is 60°, find the other angles.

37. The polynomial $p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$ when divided by x + 1 leave 19 as remainder. Also, find the remainder when p(x) is divided by x + 2.

OR

Find the values of a and b so that the polynomial $x^3 - ax^2 - 13x + b$ has (x - 1) and (x + 3) as factors.

38. A cylinder is within the cube touching all the vertical faces. A cone is inside the cylinder. If their heights are the same with the same base, find the ratio of their volumes.

OR

The external length, breadth and height of a closed rectangular wooden box are 18 cm, 10 cm and 6 cm respectively and thickness of wood is 111 cm. When the box is empty, it weighs 15 kg and when filled with sand it weighs 100 kg. Find the weight of one cubic cm of wood and cubic cm of sand.

39. Prove that the perimeter of a triangle is greater than the sum of its altitudes.

40. A random survey of the number of children of various age groups playing in a park was found as follows

Age (in years)	**Number of children**
1-2	5
2-3	3
3-5	6
5-7	12
7-10	9
10-15	10
15-17	4

Draw a histogram to represent the data above.

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Solution
Section A

1. (b) a rational number

   Explanation: a rational number because it can be written in fraction

2. (d) 1061208

   Explanation:

   $(102)^3 = (100+2)^3$

   = $(100)^3 + (2)^3 + 3 \times 100 \times 2\,(100+2)$

   = 1000000+8+60000+1200

   = 1061208

3. (d) 30^0

   Explanation:

   Let one angle be x^0

   Its supplementary angle will be 180^0 - x^0

   According to question

   x = $\frac{1}{5}$ (180^0 - x)

   x + $\frac{1}{5}$ x = 36^0

   x = 30^0

4. (c)

   55^0

Explanation:

Let $\angle ABC = 70^0$, side BC is extended to point N and MB be the external bisector of $\angle ABC$.

$\implies \angle ABN = 180^0 - 70^0 = 110^0,$

Now, MB is the external bisector of $\angle ABC$, which divides $\angle ABN$ in two equal parts.

$\implies \angle ABM = \angle MBN = \frac{110^0}{2} = 55^0.$

5. (c) $27x^3 - 135x^2 + 225x - 125$ **Explanation:**

   $(3x-5)^3$

   = $(3x)^3 - (5)^3 - 3 \times 3x \times 5\,(3x-5)$

   = $27x^3 - 125 - 45x\,(3x-5)$

   = $27x^3 - 125 - 135x^2 + 225x$

   = $27x^3 - 135x^2 + 225x - 125$

6. (d) 18 cm^2.

   Explanation:

   Since area of the figure obtained by joining mid-points of adjacent sides of a parallelogram is half of the area of parallelogram.
   Therefore,
   $\text{area}\,(\text{ABCD}) = \frac{1}{2} \times \text{area}\,(\|gm\text{PQRS})$
   $\Rightarrow \text{area}\,(\text{ABCD}) = \frac{1}{2} \times 36 = 18\ \text{cm}^2$

7. (d) 6

   Explanation:

   $\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x}$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 38 - 2$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 36$

$\Rightarrow x - \frac{1}{x} = \pm 6$

8. (d) 96 cm^2

Explanation:

Here, s = $\frac{12+16+20}{2} = 24$ cm

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{24(24-12)(24-16)(24-20)}$

= 96 sq. cm

9. (d) 66m^2.

Explanation:

The curved surface area of cylinder=2πrh

given, 2πr=22 height=3

so CSA= 22*3=66 m^2

10. (b) $\frac{1}{4}$

Explanation:

When two different coins are tossed randomly, the sample space is given by

S = {HH, HT, TH, TT}

Therefore, n(S) = 4

Let E = event of getting no head.

Then, E = {TT} and, therefore, n(E) = 1.

Therefore, P(getting no head) = P(E) = $\frac{n(E)}{n(S)}$ = $\frac{1}{4}$

11. natural

12. (x, 0)

OR

(x, 0)

13. I, IV

14. bisects

15. $8\sqrt{2}$

16. $\because$ The decimal expansion is non-terminating non-recurring.
1.101001000100001...... is an irrational number.

17. $P\left(-\frac{1}{\sqrt{3}}\right) = 3\left(-\frac{1}{\sqrt{3}}\right)^2 - 1 = 3(\frac{1}{3}) - 1 = 1 - 1 = 0$
$p\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^2 - 1 = 3\left(\frac{4}{9}\right) - 1 = \frac{4}{3} - 1 = \frac{1}{3} \neq 0$
$\therefore -\frac{1}{\sqrt{3}}$ is a zero of p(x) but $\frac{2}{3}$ is not a zero of p(x).

18. Here, l = 6 m, b = 5m and h = 4m
$\therefore$ Area of the walls = 2h(l + b) = $\{2 \times 4 \times (6 + 5)\}m^2 = (8 \times 11)m^2 = 88m^2$
Cost of white washing of 1 square metre = Rs.5
$\therefore$ Cost of white washing the walls = Rs.(5×88) = Rs.440

OR

We have,
Volume of the granary = $(8 \times 6 \times 3)m^3 = 144\ m^3$
The volume of each bag of grain = $0.65\ m^3$
$\therefore$ Number of bags which can be stored in the granary
$= \frac{Volume\ of\ the\ granary}{Volume\ of\ a\ bag} = \frac{144}{0.65} = \frac{14400}{65} = 221.53 = 221$

19. Since the diagonals of a parallelogram bisect each other. Therefore, O is the mid-point of AC and BD.

OC = $\frac{1}{2}$ AC = $\frac{1}{2}$ $\times$ 6.8 = 3.4 cm
and, OD = $\frac{1}{2}$ BD = $\frac{1}{2}$ $\times$ 5.6 = 2.8 cm

20. When the line passes through origin, co-ordinates will be $(0,0)$
$\therefore 2(0)+0=3$
$0\neq 3$
Since $LHS\neq \quad RHS$,

$\therefore$ the given statement is false.

21. $(\sqrt{3}-\sqrt{2})^2 = (\sqrt{3})^2+(\sqrt{2})^2-2(\sqrt{3})(\sqrt{2})$
$=3+2-2\sqrt{3\times 2}=5-2\sqrt{6}$

22. Here x =4 and y = -2 So possible equations of the lines passing through (4, - 2) can be
$x+y=2$, $\{4+2(-2)=2\}$
$3x+2y=8$, $\{3\times 4 \ +2(-2)=8\}$
Since, infinitely many lines pass through a given point
$\therefore$ There are infinitely many lines passing through (4, -2).

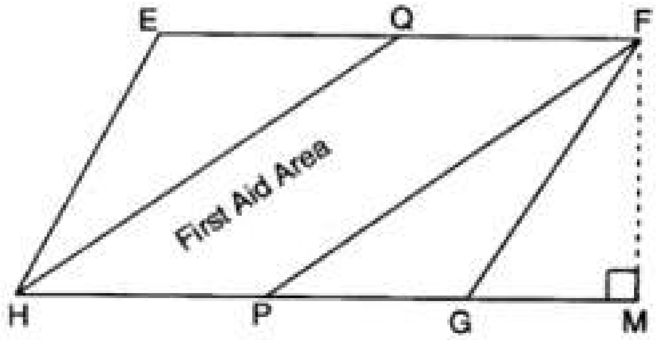
23. $p(t)=2+t+2t^2-t^3$
At $p(0)$:
$p(0)=2+(0)+2(0)^2-(0)^3 \ =2$
At $p(1)$:
$p(1)=2+(1)+2(1)^2-(1)^3 \ =2+1+2-1 \ =4$
At $p(2)$:
$p(2)=2+(2)+2(2)^2-(2)^3 \ =4+8-8 \ =4$

OR

We know that,

(a - b)^3 = a^3 - b^3 - 3ab(a - b)
Replacing a by 4 and b by $\frac{1}{3x}$,
We have,

$(4-\frac{1}{3x})^3=(4)^3-(\frac{1}{3x})^3-3\times 4\times \frac{1}{3x}(4-\frac{1}{3x})$
$=64-\frac{1}{27x^3}-\frac{4}{x}(4-\frac{1}{3x})$
$=64-\frac{1}{27x^3}-\frac{16}{x}+\frac{4}{3x^2}$

24. 

Here, EFGH is a ||gm

$\therefore$ EF = GH and EF || GH

i.e. $\frac{1}{2}\mathrm{EF} = \frac{1}{2}\mathrm{GH}$ and $\frac{1}{2}\mathrm{EF} \| \frac{1}{2}\mathrm{GH}$

$\Rightarrow$ QF = PH and QF || PH

Thus, HPFQ is a ||gm

Now, $\mathrm{FM} \perp \mathrm{HM}$

ar(||gm HPFQ) = HP $\times$ FM

$= \frac{1}{2}\mathrm{HG} \times \mathrm{FM}$ [$\because$ P is the mid-point of HG]

$= \frac{1}{2}$ ar(||gm EFGH)

Hence, area used for first aid is half of the total area.

25. The mid-point are 5, 15, 25, 35 and 45. The common difference of class marks is 10. Therefore, we need to subtract and add $\frac{10}{2} = 5$ from and either side of each class marks. Hence, the class intervals corresponding to each class marks are 0 - 10, 10 - 20, 20 - 30, 30 - 40, 40 - 50.

A continuous grouped frequency distribution is given below:

Class	Frequency
0 – 10	4
10 – 20	8
20 – 30	13
30 – 40	12
40 – 50	6

OR

Age(in years)	No. of persons
0-12	98

12-24	217
24-36	382
36-48	269
48-60	138
60-72	15
72-84	2
84-96	2
96-108	0

26. i. The cylindrical dustbin will cover more base because its radius is 35 that means diameter will be 70 cm whereas cuboidal dustbin have base dimension of 30 cm. Thus, cuboidal dustbin would occupy less base area.

ii. More capacity means more volume.

Volume of Cuboidal dustbin = $30 \times 30 \times 30$ = 27000 cm^3

And Volume of Cylindrical dustbin = $3.14 \times 35^2 \times 40$ = 153860 cm^3

Thus, Cylindrical dustbin has more capacity.

27. $\left\{\left(625^{-\frac{1}{2}}\right)^{-\frac{1}{4}}\right\}^2$

$$= \left\{\left(\frac{1}{625^{\frac{1}{2}}}\right)^{-\frac{1}{4}}\right\}^2 = \left\{\left(\frac{1}{(25^2)^{\frac{1}{2}}}\right)^{-\frac{1}{4}}\right\}^2 \left(a^{-m} = \frac{1}{a^m}\right)$$

$$= \left\{\left(\frac{1}{25}\right)^{-\frac{1}{4}\times 2}\right\}$$

$$= \left(\frac{1}{25^{-\frac{1}{2}}}\right) = \frac{1}{(5^2)^{-\frac{1}{2}}} = \frac{1}{5^{-1}} = 5$$

OR

Take AB= BC = 1cm

$\angle B = 90^0$

In $\triangle$ABC

$AC^2 = AB^2 + BC^2$

$AC^2 = 1 + 1$

$AC^2 = 2$

$AC = \sqrt{2}$

In $\triangle OCD$

$\angle C = 90^0$

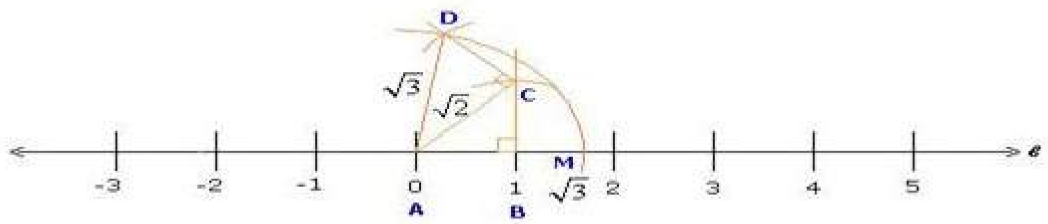
$OD^2 = OC^2 + DC^2$

$OD^2 = (\sqrt{2})^2 + 1$

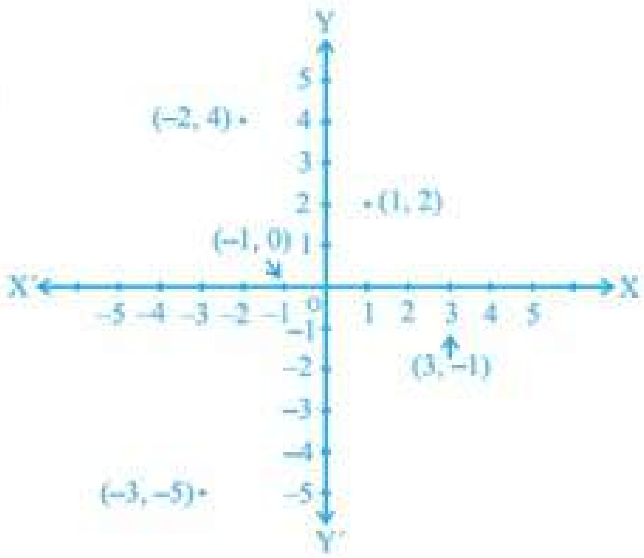
$OD^2 = 2 + 1$

$OD^2 = 3$

$OD = \sqrt{3}$



28. We need to determine the quadrant or axis of the points (– 2, 4), (3, – 1), (– 1, 0), (1, 2) and (– 3, – 5). First, we need to plot the points (– 2, 4), (3, – 1), (– 1, 0), (1, 2) and (– 3, – 5) on the graph, to get



We need to determine the quadrant, in which the points (– 2, 4), (3, – 1), (– 1, 0), (1, 2) and (– 3, – 5) lie.

From the figure, – 2, 4) lie in IInd quadrant.

From the figure, we can conclude that the point (3, – 1) lie in IVth quadrant.

From the figure, we can conclude that the point (–1, 0) lie on x-axis.

From the figure, we can conclude that the point (1, 2) lie in Ist quadrant.

From the figure, we can conclude that the point (– 3, – 5) lie in IIIrd quadrant.

29. We have,

5x - 2y = 10 ...(i)

Substituting x = 0 in the equation 5x - 2y = 10, we get

$5 \times 0 - 2y = 10$

$\Rightarrow y = \frac{10}{-2} = -5$

Thus, x = 0 and y = -5 is a solution of 5x - 2y = 10.

Substituting y = 0 in (i), we get

$5x - 2 \times 0 = 10$

$\Rightarrow 5x = 10$

$\Rightarrow x = 2$

Thus, x = 2 and y = 0 is a solution of 5x - 2y = 10.

Thus, x = 0, y = -5 and x = 2, y = 0 are two solutions of 5x - 2y = 10

OR

Let the work done by the constant force be y units and the distance traveled by the body be x units.

Constant force = 5 units

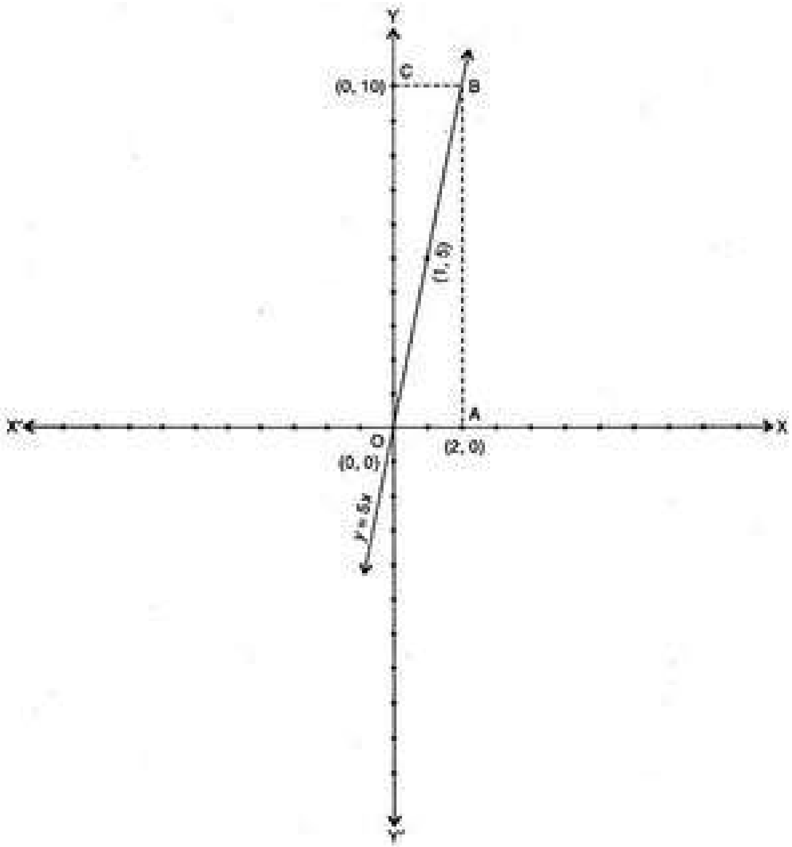
We know that

Work done = Force $\times$ Displacement

$\Rightarrow$ y = 5x

x	0	1
y	0	5

We plot the points (0, 0) and (1, 5) on the graph paper and join the same by a ruler to get the line which is the graph of the equation y = 5x.

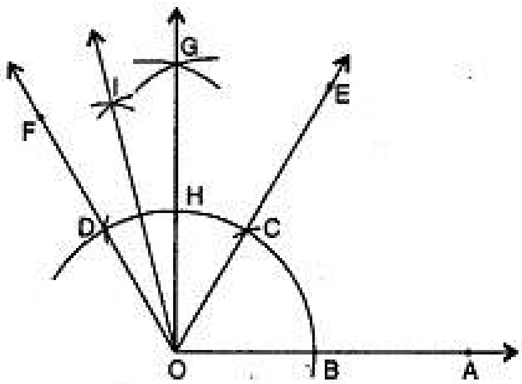


Clearly y = 0 when x = 0. So, the work done when the distance traveled by the body is 0 unit.

30. Given: A ray OA.

    Required: To construct an angle of 105° at O.
    Steps of construction :

    i. Taking O as centre and some radius, draw an arc of a circle, which intersects OA, say at a point B.
    ii. Taking B as centre and with the same radius as before, draw an arc intersecting the previously drawn arc, say at a point C.
    iii. Taking C as centre and with the same radius as before, draw an arc intersecting the arc drawn in step 1, say at D.
    iv. Draw the ray OE passing through C. Then $\angle EOA = 60^{\circ}$.
    v. Draw the ray OF passing through D. Then $\angle FOE = 60^{\circ}$.



vi. Taking C and D as centres and with the radius more than $\frac{1}{2}$ CD, draw arcs to intersect each other, say at G.

vii. Draw the ray OG intersecting the arc drawn in step 1 at H. This ray OG is the bisector of the angle FOE,

i.e. $\angle FOG = \angle EOG = \frac{1}{2} \angle FOE = \frac{1}{2}(60^o) = 30^o$.

Thus, $\angle GOA = \angle GOE + \angle EOA = 30^o + 60^o = 90^o$

viii. Taking H and D as centres and with the radius more than $\frac{1}{2}$ HD, draw arcs to intersect each other, say at I.

ix. Draw the ray OI. This ray OI is the bisector of the angle FOG, i.e. $\angle FOI = \angle GOI = \frac{1}{2} \angle FOG = (30^o) = 15^o$.

Thus, $\angle IOA = \angle IOG + \angle GOA = 15^o + 90^o = 105^o$. On measuring the $\angle IOA$ by protractor, we find that $\angle FOA = 105^o$.

31. Since ABCD is a parallelogram, we can write

$AB \parallel DC$

Now, AB | | DC and transversal BD intersects them at B and D.

$\therefore \angle 1 = \angle 2$ ....(Alternate interior angles)

Now, in triangles ABM and CDN, we have

$\angle 1 = \angle 2$ ....(Proved above)

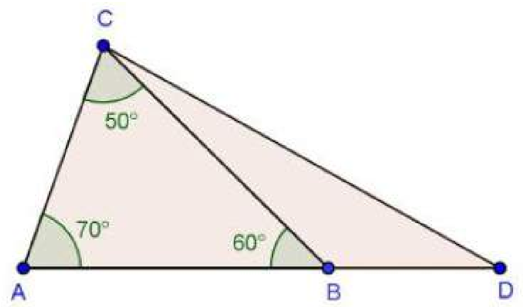
$\angle AMB = \angle CND$ ....(Each 90^o)

AB = DC ....(Opposite sides of parallelogram)

$\therefore \triangle ABM \cong \triangle CDN$ ....(By AAS criterion of congruence)

$\therefore$ AM = CN (by CPCT).

Hence proved

32. 

Given: In $\triangle$ABC, side AB is produced to D so that BD = BC and $\angle$B = 60° and $\angle$A = 70°

i. Since $\angle A = 70^o$, $\angle B = 60^o$ (Given)

   $\angle A + \angle B + \angle C = 180^o$ [Angle sum property of $\triangle$]

   $\therefore \angle C = 180^o - \angle A - \angle B$

   $= 180^o - 70^o - 60^o$

   $= 50^o$

   Again, $\angle ABC + \angle CBD = 180^o$ [linear pair]

   $\angle CBD = 180^o - \angle ABC$

   $\angle CBD = 180^o - 60^o = 120^o$

   Now, BD = BC, so

   $\therefore \angle BCD = \angle BDC = \frac{180-120}{2} = 30^o$

   $\therefore \angle ACD = 50^o + 30^o = 80^o$

   $\angle CAD = 70^o$

   $\therefore \angle ACD > \angle CAD$

   $\Rightarrow$ AD > CD

   Hence Proved.

ii. Again $\angle ACD = 80^o$

   $\angle ABC = 60^o$

   $\therefore \angle ACD > \angle ABC$

   $\Rightarrow$ AD > AC

   Hence Proved

OR

In $\triangle$ADE,

AD = AE . . . [Given]

$\angle$AED = $\angle$ADE . . . .[$\angle$s opposite to equal side of a $\triangle$ADE]

180^o – $\angle$AED = 180^o – $\angle$ADE

$\angle$AEC = $\angle$ADB

In $\triangle$ADB and $\triangle$AEC,

AD = AE . . . [Given]

BD = EC . . . [Given]

$\angle$ADB = $\angle$AEC . . . .[From (1)]

$\therefore \triangle ADB \cong \triangle AEC$ .... [By SAS property]

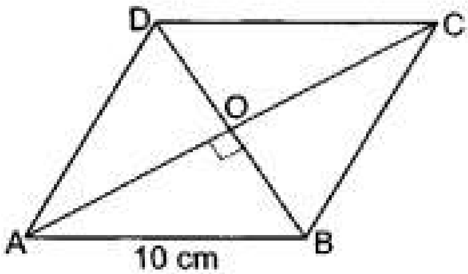
$\therefore$ AB = AC .... [c.p.c.t]

33. If the side of a rhombus is 10 cm and one diagonal is 16 cm

So, AC = 16 cm

BD =? and AB = 10 cm

As the diagonals of a rhombus bisect each other at 90°



$\therefore \quad OA = \frac{1}{2}AC = \frac{1}{2} \times 16 = 8\text{cm}$

$OB = \frac{1}{2}BD$

$\therefore$ $OA^2 + OB^2 = AB^2$

$8^2 + OB^2 = 10^2$

$\Rightarrow OB^2 = 100 - 64$

$\Rightarrow OB^2 = 36$

$\Rightarrow$ OB = 6cm

$\therefore$ BD = 2 $\times$ OB = 2 $\times$ 6 = 12 cm

Area of rhombus = $\frac{1}{2}$ (product of diagonals)

So, area of rhombus $= \frac{1}{2}AC \times BD = \frac{1}{2} \times 16 \times 12 = 96\text{cm}^2$

So, given statement is true.

34. i. $\text{P (no defective part)} = \frac{50}{200} = 0.25$

ii. P (at least one defective part) $= 1$ - P (no defective part)

$= 1 - 0.25 = 0.75$

iii. P (not more than 5 defective parts) $=$ P (no defective part) + P (1 defective part) + P (2 defective parts) + P (3 defective parts)

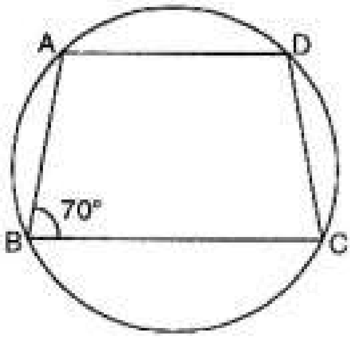
+ P (4 defective parts) + P (5 defective parts)

= $\frac{50}{200} + \frac{32}{200} + \frac{22}{200} + \frac{18}{200} + \frac{12}{200} + \frac{12}{200} = \frac{146}{200} = 0.73$

35. Given: ABCD is a cyclic trapezium with AD | | BC. $\angle B = 70^\circ$

To determine: Other three angles of the trapezium

Determination :



$\angle B + \angle D = 180^\circ$

[∵ Opposite angles of a cyclic quadrilateral are supplementary]

$\Rightarrow 70^\circ + \angle D = 180^\circ \Rightarrow \angle D = 180^\circ - 70^\circ$

$\Rightarrow \angle D = 110$

Again, AD | | BC and transversal AB intersects them

$\therefore \angle A + \angle B = 180^\circ$

| ∵ The sum of the consecutive interior angles on the same side of a transversal is 180^o

$\Rightarrow \angle A + 70^\circ = 180^\circ \Rightarrow \angle A = 180^\circ - 70^\circ$

$\Rightarrow \angle A = 110^\circ$

Also, $\angle A + \angle C = 180^\circ$ | ∵ Opposite angles of a cyclic quadrilateral are supplementary

$\Rightarrow 110^\circ + \angle C = 180^\circ \Rightarrow \angle C = 180^\circ - 110^\circ$

$\Rightarrow \angle C = 70^\circ$

OR

$\angle A + \angle C = 180^\circ$

|∵ Opposite angles of a cyclic quadrilateral are supplementary

$\Rightarrow (x+y+10)^0 + (x+y-30)^0 = 180^\circ$

$\Rightarrow$ x + y + 10 + x + y - 30 = 180

$\Rightarrow$ 2x + 2y - 20 = 180

$\Rightarrow$ 2x + 2y = 20 + 180

$\Rightarrow$ 2x + 2y = 200

$\Rightarrow$ x + y = 100 ...... (1) |Dividing both sides by 2

Again, $\angle B + \angle D = 180^\circ$

|∵ Opposite angles of a cyclic quadrilateral are supplementary

$\Rightarrow (y + 20)^0 + (x + y)^0 = 180^0$

$\Rightarrow$ y + 20 + x + y = 180

$\Rightarrow$ x + 2y + 20 = 180

$\Rightarrow$ x + 2y = 180 - 20

$\Rightarrow$ x + 2y = 160 ...... (2)

|Subtracting (1) from (2), we get

y = 60

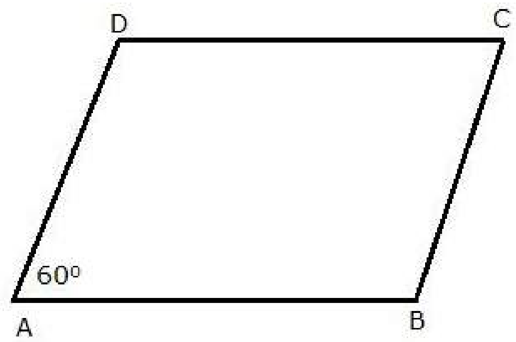
Putting y = 60 in (1), we get

x + 60 = 100

$\Rightarrow$ x = 100 - 60

$\Rightarrow$ x = 40

Hence, the required values of x and y are 40 and 60 respectively.

36. 

Given: AB || CD and AD || BC. Let one angle i.e. $\angle A$ of the quadrilateral is 60°

To find: $\angle B$, $\angle C$, $\angle D$

Solution: Since we know that if opposite sides of a quadrilateral are parallel i.e. AB || CD then ABCD is a parallelogram.

$\therefore \angle A + \angle D = 180^0$ [Sum of adjacent angles of ||gm are supplementary]

$60^o + \angle D = 180^o$ [given]

$\angle D = 180^o - 60^o$

$\angle D = 120^o$

$\angle D = \angle B$ [Opposite angles of ||gm are equal]

$\Rightarrow \angle B = 120^o$

Similary, $\angle A = \angle C$ [Opposite angles of ||gm are equal]

$\therefore \angle C = 60^o$

Hence, $\angle A = \angle C = 60^o$

and $\angle B = \angle D = 120^o$

37. We know that when $p(x)$ is divided by x + a , then the $remainder = p(-a)$.

Now, $p(x) = x^4 - 2x^3 + 3x^3 - ax + 3a - 7$ is divided by $x + 1$, then the $remainder = p(-1)$

Now, $p(-1) = (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + 3a - 7$

$= 1 - 2(-1) + 3(1) + a + 3a - 7$

$= 1 + 2 + 3 + 4a - 7$

$= -1 + 4a$

Also, remainder = 19

$\therefore -1 + 4a = 19$

$\Rightarrow 4a = 20, a = 20 \div 4 = 5$

When $p(x)$ is divided by $x + 2$, then

Remainder $= p(-2) = (-2)^4 - 2(-2)^3 + 3(-2)^2 - a(-2) + 3a - 7$

$= 16 + 16 + 12 + 2a + 3a - 7$

$= 37 + 5a$

$= 37 + 5(5) = 37 + 25 = 62$

OR

Let p(x) = $x^3 - ax^2 - 13x + b$ be the given polynomial. If (x - 1) and (x + 3) are factors of p(x), then

$\Rightarrow$ p(1) = 0 and p(-3) = 0

Thus, $1^3 - a \times 1^2 - 13 \times 1 + b = 0$ and $(-3)^3 - a(-3)^2 - 13 \times (-3) + b = 0$

$\Rightarrow$ 1 - a - 13 + b = 0 and -27 - 9a + 39 + b = 0

$\Rightarrow$ -12 - a + b = 0 and - 9a + b + 12 = 0

$\Rightarrow$ a - b = -12 and 9a - b = 12

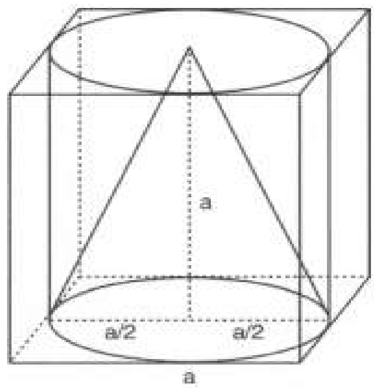
Subtracting second equation from first, we get

(a - b) - (9a - b) = - 12 - 12

$\Rightarrow$ a - b - 9a + b = -24 $\Rightarrow$ -8 a = -24 $\Rightarrow$ a = 3

Putting a = 3 in a - b = -12, we get, 3 - b = -12 $\Rightarrow$ b = 15

Hence, a = 3 and b = 15

38. 

Let the length of each edge of the cube be 'a' units. Then,

V_1 = Volume of the cube = a^3 cubic units.

Since a cylinder is within the cube and it touches all the vertical faces of the cube,

$\therefore$ r = Radius of the base of the cylinder = $\frac{a}{2}$, h = height of the cylinder = a

$\therefore V_2$ = Volume of the cylinder = $\pi r^2 h$

$\Rightarrow V_2 = \frac{22}{7} \times \frac{a^2}{4} \times a$

$\Rightarrow V_2 = \frac{11}{14} a^3$ cubic units

A cone is drawn inside the cylinder such that it has the same base and the same height,

$\therefore V_3$ = Volume of the cone = $\frac{1}{3} \pi r^2 h$

$\Rightarrow V_3 = \frac{1}{3} \times \frac{22}{7} \times (\frac{a}{2})^2 \times a$

$\Rightarrow V_3 = \frac{11}{42} a^3$ cubic units

$\therefore V_1 : V_2 : V_3 = a^3 : \frac{11}{14} a^3 : \frac{11}{42} a^3 = 42 : 33 : 11$

OR

We have,

Thickness of wood = $\frac{1}{2}$ cm

Internal length of wooden box = 18 - ($\frac{1}{2} + \frac{1}{2}$) = 17 cm

Internal breadth of wooden box = 10 - ($\frac{1}{2} + \frac{1}{2}$) = 9 cm

Internal depth of wooden box = 6 - ($\frac{1}{2} + \frac{1}{2}$) = 5 cm

$\therefore$ Internal volume of wooden box = $(17 \times 9 \times 5)\text{cm}^3 = 765\ \text{cm}^3$

External volume of wooden box = $(18 \times 10 \times 6)\text{cm}^3 = 1080\ \text{cm}^3$

Volume of wood = External volume - Internal volume

= $(1080 - 765)\ \text{cm}^3 = 315\ \text{cm}^3$

Weight of empty box = 15 kg

$\Rightarrow$ Weight of 315 cm^3 wood is 15 kg

$\therefore$ Weight of 1 cm^3 of wood = $(\frac{15}{315})$ kg = $\frac{1}{21}$ kg

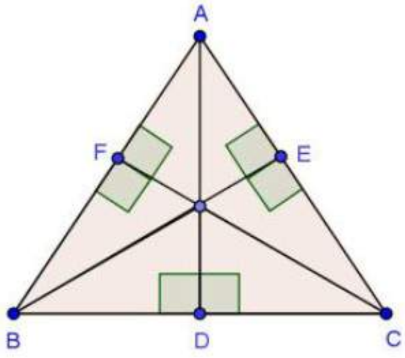
Now, Volume of sand = Internal volume of box = 765 cm^3

Weight of sand = Weight of box filled with sand - Weight of empty box

= (100 - 15)kg = 85 kg

volume of sand = 765 cm^3

$\therefore$ Weight of 1 cm^3 of sand = $(\frac{85}{765})$ kg = $\frac{1}{9}$ kg

39. 

Given: ΔABC in which $AD \perp BC$, $BE \perp AC$ and $CF \perp AB$

To prove: AB + BC + CA > AD + BE + CF

Proof: Since perpendicular is the shortest of all the line segment from a point not lying on it.

Now, we have AD $\perp$ BC

AB > AD and AC > AD

$\Rightarrow$ AB + AC > 2AD ...(i)

also, BE $\perp$ AC

$\Rightarrow$ BA > BE and BC > BE

$\Rightarrow$ BA + BC > 2BE...(ii)

Also CF $\perp$ AB

$\Rightarrow$ CA > CF and CB > CF

$\Rightarrow$ CA + CB > 2CF...(iii)

Now adding (i), (ii) and (iii)

We get,

AB + AC + BA + BC + CA + CB > 2AD + 2BE + 2CF

⇒ 2AB + 2BC + 2CA > 2(AD + BE + CF)

⇒ 2(AB + BC + CA) > 2(AD + BE + CF)

⇒ AB + BC + CA > AD + BE + CF

Hence, the perimeter of the triangle is greater than the sum of its altitudes. Hence proved.

40. Here, the widths of the rectangles are varying. So, we need to make certain modifications in the lengths of the rectangles, so that the areas are became proportional to the frequencies.

The minimum class size is 1.

Length of rectangle (Adjusted frequency) = $\frac{\text{Minimum class size}}{\text{Class size of this class}} \times$ Frequency

Then modified table of given data is shown below

Age (in years)	**Number of children (Frequency)**	**Width of the class**	**Length of the rectangle**
1-2	5	1	$\frac{1}{1} \times 5 = 5$
2-3	3	1	$\frac{1}{1} \times 3 = 3$
3-5	6	2	$\frac{1}{2} \times 6 = 3$

5-7	12	2	$\frac{1}{2} \times 12 = 6$
7-10	9	3	$\frac{1}{3} \times 9 = 3$
10-15	10	5	$\frac{1}{5} \times 10 = 2$
15-17	4	2	$\frac{1}{2} \times 4 = 2$

So, the histogram with varying width is given below