Unit -1

Chapter -1

Units & Measurement

* Need of Measurement
* Units of Measurement
* System of Units
* S.I Units
* Fundamental & Derived Units
* Significant figures
* Dimension of Physical Quantities
* Dimensional Analysis
* & its Applications

Need of Measurement

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- Measurement makes Communication Easier.
- By Measuring Physical Quantity we can define property of material
- We can measure mass, weight & different physical properties like distance, speed, force & Energy etc.
- Measurement is the system by which we can compare the physical quantity with a Unit.

Physical Quantities

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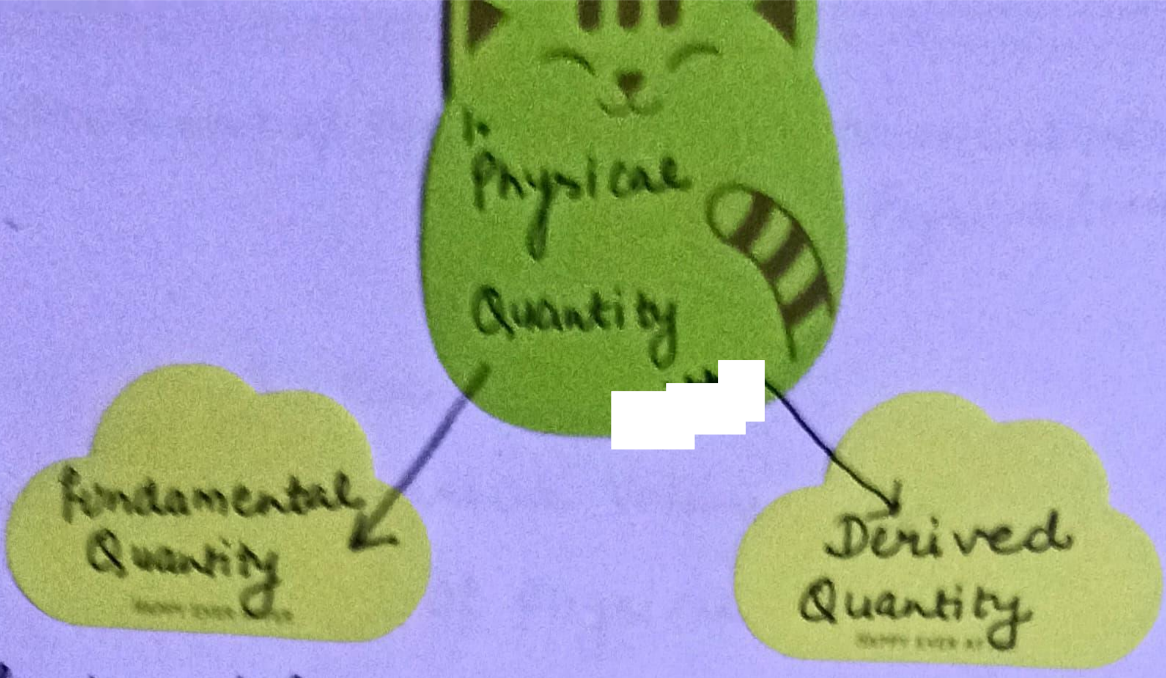
Physical Quantities are those by which laws of Physics can be expressed Easily.

for eg: Temperature, Speed, electric current etc

- **Types of Physical Quantities**

Physical Quantities are of two types

1. Fundamental Quantities
2. Derived Quantities



- fundamental Quantities are those which are not dipandent on other physical Quantities

- Seven fundamental & Base Quantities are
  1. Mass
  2. length
  3. Time
  4. Electric current
  5. Temperature
  6. Luminous Intensity
  7. Amount of Substance

- Derived Quantities are those which are depandent on fundamental Quantities.

- All Physical quantities other than 7 fundamental Quantities are known as Derived Quantities

for eg: Velocity
Density
Momentum
Power etc.

What do you mean by Measurement of a Physical Quantity?

- Measurement of a Physical Quantity is the process of comparing this Quantity with a standard Amount of Physical quantity.

Measurement of Physical Quantity = Numerical value of Physical Quantity × Size of Unit

$\Rightarrow Q = nu$

Where n = numerical value

U = Unit of Physical Quantity

$Q = n_1 u_1 = n_2 u_2$

Let length of a Table

$\Rightarrow$ 5m = 500cm

if $u_1 > u_2$

$\Rightarrow n_1 < n_2$

- This means smaller the size of Unit, larger will be numerical value.

$n \propto \frac{1}{u}$ or $\boxed{nu = \text{Constant}}$

2. Units of Measurement

Ques: What are fundamental & Derived Units? Give some Examples.

- **Fundamental Units** - fundamental Units are simpler Units which can't be further resolved into simpler Units.

for eg: Units of fundamental Quantities such as

Mass, length & time are known as fundamental Units

Derived Units - All other physical Unit other than fundamental Units which can be expressed in terms of fundamental Unit are known as derived Units.

for eg: Unit of speed = $\frac{\text{metre}}{\text{second}}$ is a derived Unit which is expressed in the terms of fundamental Units of length & time.

Q. What is a System of Units? Mention the Various types of System of Units.

A. A complete set of Units which is Used to measure all kinds of fundamental & derived quantities is known as System of Units.

1. **cgs system**: It is based on Centimetre, gram & second as fundamental Units of mass, length & time respectively.

2. **fps System**: It is based on foot, pound & second as fundamental Unit of length, mass & time respectively.

3. **mks System**: It is based on metre, Kilogram & second as fundamental Units of length mass & time respectively.

4. SI : The International system of Units : It is based on seven basic Units & two supplementary Units.

Basic S.I quantities & Units

Basic Physical quantity	Basic Unit	Symbol
1. length	metre	m
2. Mass	Kilogram	kg
3. Time	second	s
4. Electric Current	ampere	A
5. Temperature	kelvin	K
6. Luminous Intensity	Candela	cd
7. Quantity of Matter	mole	mol

Supplementary S.I Units

Supplementary Quantity	Basic Unit	Symbol
1. Plane Angle	radian	rad
2. Solid Angle	steradian	sr

(a) Radian (rad) - It is plane angle subtended at centre of a circle by an arc equal to its length to the radius of the circle.

$$(\theta) \text{ in radian} = \frac{\text{Arc}}{\text{Radius}} = \frac{l}{r}$$

(b) Steradian (sr) : It is solid Angle subtended at Centre of a Sphere by surface of a sphere.

$$\Omega \text{ (In steradian)} = \frac{\text{Surface Area}}{(\text{radius})^2}$$

Advantages of S.I System

(i) **S.I is a Coherent system of Units** All derived Units can be obtained by simple multiplication or division of fundamental Units without introducing any numerical factor.

(ii) **S.I is a rational system of Units** It assigns only one Unit for a given physical quantity. for example all forms of Energy in S.I are measured in Joule & in mks System mechanical Energy is measured in Joule, heat Energy in Calorie & Electrical Energy in Kilowatt hour.

(iii) **S.I is a metric System** Multiples & Submultiples of S.I Units Can be expressed as power of 10.

(iv) **S.I is an Absolute System of Units**

It does not Use gravitational Units.

(v) **S.I is An Internationally accepted System of Units.**

Q. **What do you mean by dimensions of Physical Quantity? Explain with the help of an example.**

A. Dimensions of a Physical quantity are the powers to which the fundamental quantities must be raised to represent that quantity completely.

for example Density $= \frac{Mass}{Volume} = \frac{Mass}{L \times B \times H}$

Dimensions of density $= \frac{[M]}{[L][L][L]} = [M^1 L^{-3} T^0]$

* Dimension of density are 1 in mass
  -3 in length
  0 in Time.

Q. **What do you mean by dimensional formula & dimensional equation? Give examples.**

Expression which shows how & which of the fundamental quantities represent the dimensions of a physical quantity is known as dimensional formula of the given physical quantity.

Examples: Dimensional formula of velocity is $[M^0 L^1 T^{-1}]$ & that of work is $[M^1 L^2 T^{-2}]$.

Dimensional equation Equation obtained by Equating a physical quantity with its dimensional formula is known as Dimensional equation

Example:

Dimensional equation of force is Force $= [M^1 L^1 T^{-2}]$

Dimensional formulae & S.I Units

Mechanical Quantities

S. No.	Physical Quantity	Relation with other Quantities	Dimensional Formula	S.I Unit
1.	Area	L X B	$L \times L = [M^0 L^2 T^0]$	m^2
2.	Volume	L X B X H	$L \times L \times L = [M^0 L^3 T^0]$	m^3
3.	Density	$\frac{\text{Mass}}{\text{Volume}}$	$\frac{M}{L^3} = [M^1 L^{-3} T^0]$	$kg\,m^{-3}$
* 4.	Relative Density OR Specific gravity	$\frac{\text{Density of object}}{\text{Density of water}}$	$\frac{[M^1 L^{-3} T^0]}{[M^1 L^{-3} T^0]} = [M^0 L^0 T^0]$ * Dimensionless	Unitless
5.	Speed or velocity	$\frac{\text{Distance}}{\text{time}}$	$\frac{L^1}{T^1} = [M^0 L^1 T^{-1}]$	ms^{-1}
6.	Acceleration OR Acc. due to gravity	$\frac{\text{Change in velocity}}{\text{time}}$	$\frac{L^1 T^{-1}}{T} = [M^0 L^1 T^{-2}]$	ms^{-2}
7.	Momentum	Mass X Velocity	$M^1 \times [L^1 T^{-1}] = [M^1 L^1 T^{-1}]$	$kg\,ms^{-1}$
8.	Force imp	Mass X Acceleration	$M^1 L^1 T^{-2} = [M^1 L^1 T^{-2}]$	N
9.	Work imp	Force X distance	$M^1 L^1 T^{-2} \times L = [M^1 L^2 T^{-2}]$	J
10.	Energy	Amount of work	$[M^1 L^2 T^{-2}]$	J
11.	Power	$\frac{\text{Work}}{\text{Time}}$	$\frac{M^1 L^2 T^{-2}}{T} = [M^1 L^2 T^{-3}]$	W
12.	Pressure	$\frac{\text{Force}}{\text{Area}}$	$\frac{M^1 L^1 T^{-2}}{L^2} = [M^1 L^{-1} T^{-2}]$	Pa or Nm^{-2}
13.	Moment of force OR Torque(τ)	Force X ⊥ distance	$M^1 L^1 T^{-2} \times L = [M^1 L^2 T^{-2}]$	N.m
14.	Gravitational constant 'G'	$\frac{\text{Force} \times (\text{distance})^2}{\text{Mass} \times \text{Mass}}$	$\frac{M^1 L^1 T^{-2} \times L^2}{M^2} = [M^{-1} L^3 T^{-2}]$	$Nm^2 kg^{-2}$
15.	Impulse	Force X time	$M^1 L^1 T^{-2} \times T = [M^1 L^1 T^{-1}]$	N-s

S. No	Physical Quantity	Relation with other Quantities	Dimensional Formula	S.I Unit
16.	Stress	$\frac{\text{Force}}{\text{Area}}$	$\frac{M^1L^1T^{-2}}{L^2} = [M^1L^{-1}T^{-2}]$	$N\text{-}m^{-2}$
17.	Strain *	$\frac{\text{Change in dimension}}{\text{Original dimension}}$	$[M^0L^0T^0]$ Dimensionless	Unitless
18.	Coeff. of Elasticity	$\frac{\text{Stress}}{\text{Strain}}$	$\frac{[M^1L^{-1}T^{-2}]}{[M^0L^0T^0]} = [M^1L^{-1}T^{-2}]$	$N\text{-}m^{-2}$
19.	Surface Tension	$\frac{\text{Force}}{\text{length}}$	$\frac{[M^1L^1T^{-2}]}{[M^0L^1T^0]} = [M^1L^0T^{-2}]$	Nm^{-1}
20	Surface Energy	$\frac{\text{Work}}{\text{Area}}$	$\frac{[M^1L^2T^{-2}]}{L^2} = [M^1L^0T^{-2}]$	Jm^{-2}
21.	Angle *	$\frac{\text{Arc}}{\text{Radius}}$	$\frac{L}{L} = [M^0L^0T^0]$ Unitless	rad
22.	Angular (ω) Velocity	$\omega = \frac{\theta}{t} = \frac{\text{Angle}}{\text{time}}$	$\frac{1}{T} = [M^0L^0T^{-1}]$	$rad\ s^{-1}$
23.	Ang. Acceleration	$\alpha = \frac{\text{Ang. velocity}}{\text{Time}}$	$\frac{T^{-1}}{T} = T^{-2} = [M^0L^0T^{-2}]$	$rad\ s^{-2}$
24.	Mom. of Inertia	Mass × (distance)^2	$[M^1L^2T^0]$	kgm^2
25.	Radius of gyration	Distance	$L = [M^0L^1T^0]$	m
26.	Ang. Momentum L	L = mvr L = mass × velocity × radius	$M \times L^1T^{-1} \times L = [M^1L^2T^{-1}]$	kgm^2s^{-1}
27.	T-ratios (Sinθ, Cosθ, tanθ)	$\frac{\text{length}}{\text{length}}$	$[M^0L^0T^0]$ Dimensionless	—
28.	Time period	Time	$[M^0L^0T^1]$	s

S. No	Physical Quantity	Relation with other Quantities	Dimensional Formula	S.I Unit
29.	Coefficient of Viscosity	$\frac{Fr}{Av} = \frac{\text{Force} \times \text{distance}}{\text{Area} \times \text{velocity}}$	$\frac{M^1 \cancel{L} T^{-2} \times \cancel{L}}{\cancel{L^2} \times L^1 T^{-1}} = [M^1 L^{-1} T^{-1}]$	$N m^{-2} s$ or $Pa s$
30.	frequency	$\frac{1}{\text{Time period}}$	$\frac{1}{T} = [M^0 L^0 T^{-1}]$	s^{-1} or Hz
31.	Planck's constant (h)	$h = \frac{\text{Energy}}{\text{frequency}}$	$\frac{M^1 L^2 T^{-2}}{T^{-1}} = [M^1 L^2 T^{-1}]$	J s
32.	Velocity gradient	$\frac{\text{Velocity}}{\text{distance}}$	$\frac{[M^0 L^1 T^{-1}]}{[L]} = [M^0 L^0 T^{-1}]$	s^{-1}
33.	Pressure gradient	$\frac{\text{Pressure}}{\text{Distance}}$	$\frac{M^1 L^{-1} T^{-2}}{L} = [M^1 L^{-2} T^{-2}]$	$Pa\ m^{-1}$
34.	Force Constant	$\frac{\text{Force}}{\text{displacement}}$	$\frac{M^1 L^1 T^{-2}}{L} = [M^1 L^0 T^{-2}]$	$N m^{-1}$
		Thermal Quantities		
35.	Heat or Enthalpy	Energy	$[M^1 L^2 T^{-2}]$	J
36.	Specific heat	$\frac{\text{Heat}}{\text{Mass} \times \text{Temp}}$	$\frac{M^1 L^2 T^{-2}}{M \cdot K} = [M^0 L^2 T^{-2} K^{-1}]$	$J kg^{-1} K^{-1}$
37.	Latent Heat	$\frac{\text{Heat}}{\text{Mass}}$	$\frac{M^1 L^2 T^{-2}}{M} = [M^0 L^2 T^{-2}]$	$J kg^{-1}$
38.	Boltzman's Constant	$\frac{\text{Energy}}{\text{Temperature}}$	$\frac{M^1 L^2 T^{-2}}{K} = [M^1 L^2 T^{-2} K^{-1}]$	$J K^{-1}$

Electrical Quantities

S. No	Physical Quantity	Relation with other Quantities	Dimensional Formula	S.I Unit
39.	Electric charge	$q = I \times t$ = Time×current	$[M^0 L^0 A T]$	C
40	Electrical Potential V	$V = \frac{W}{Q} = \frac{\text{Work}}{\text{Charge}}$	$\frac{M^1 L^2 T^{-2}}{AT} = [M^1 L^2 T^{-3} A^{-1}]$	V (volt)
41.	Resistance	$\frac{\text{Potential difference}}{\text{Current}}$	$\frac{M^1 L^2 T^{-3} A^{-1}}{A}$ $[M^1 L^2 T^{-3} A^{-2}]$	Ω (ohm)
42.	Capacitance	$\frac{\text{Charge}}{\text{Pot. difference}} = \frac{q}{V}$	$\frac{[AT]}{[M^1 L^2 T^{-3} A^{-1}]}$ $[M^{-1} L^{-2} T^{4} A^{2}]$	F (farad)
43.	Inductance	$\frac{\text{EMF}}{\text{current/time}} = \frac{\text{Voltage}}{I/t}$	$\frac{M^1 L^2 T^{-3} A^{-1}}{A/T} = [M^1 L^2 T^{-2} A^{-2}]$	H (henry)
44.	Permittivity of free space	$\epsilon_0 = \frac{q_1 q_2}{F r^2}$	$\frac{AT \cdot AT}{M^1 L^1 T^{-2} \times L^2} = \frac{A^2 T^2}{M^1 L^3 T^{-2}} = [M^{-1} L^{-3} T^{4} A^{2}]$	$C^2 N^{-1} m^{-2}$
45.	Electric dipole moment	$p = q \times 2l$	$AT \times L$ $[M^0 L^1 T^1 A^1]$	Cm

Magnetic Quantities

S.No	Physical quantity	Relation with Other Quantities	Dimensional Formula	S.I Units
46.	Magnetic field	$B = \frac{F}{qv\sin\theta}$	$\frac{M^1 \cancel{L^1} T^{-2}}{A T \cancel{L^1} T^{-1}} = M^1 L^0 T^{-2} A^{-1}$	T (tesla)
47.	Magnetic flux	$\phi = B.A$ Magnetic field Area	$M^1 L^0 T^{-2} A^{-1} \times L^2$ $[M^1 L^2 T^{-2} A^{-1}]$	Wb (weber)
48.	Magnetic Moment	Current X Area	$A \times L^2$ $[M^0 L^2 T^0 A^1]$	Am^2
49.	Pole Strength	$\frac{\text{Magnetic Moment}}{\text{Magnetic length}}$	$\frac{AL^2}{L} = [M^0 L T^0 A^1]$	Am

Abbreviations in Powers of ten

Multiple	Prefix	Symbol	Sub Multiple	Prefix	Symbol
10^1	deca	da	10^{-1}	deci	d
10^2	hecto	h	10^{-2}	centi	c
10^3	kilo	k	10^{-3}	milli	m
10^6	Mega	M	10^{-6}	micro	μ
10^9	giga	G	10^{-9}	nano	n
10^{12}	tera	T	10^{-12}	pico	p
10^{15}	Peta	P	10^{-15}	femto	f
10^{18}	exa	E	10^{-18}	atto	a

Four types of Dimensional Quantities

1. Dimensional Constants

Quantities whose values are constant & they possess dimensions.

for eg: Universal Gravitational Constant G
Planck's Constant
Universal Gas Constant.

2. Dimensionless Constants

Quantities which are constants & Dimensionless.

for eg: Pure number 1, 2, 3.......
Mathematical constants π, e (exponential) etc

3. Dimensional Variables:

Quantities which are variable & they have dimensions.

for eg: Area, Volume, density etc.

4. Dimensionless Variables

Quantities whose values are Variable & they do not have dimensions.

for eg: Angle, Specific gravity & strain

Uses of Dimensional Equations : (Applications)

1. Conversion from One System to Another
2. Checking the correctness of formulae
3. Derivation of formulae.

Checking the correctness of formulae

To check correctness of formulae we use

Principle of Homoginity of dimensions

- According to this principle only that formula is correct in which dimensions of Units on One side of relation are equal to their respective dimensions on other side

Concept Used

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Q.1 Check the Correctness of given formulae

1. de-Broglie wavelength $\lambda = \frac{h}{mv}$
2. Escape Velocity $v = \sqrt{\frac{2GM}{R}}$

Sol. 1

$\lambda = \frac{h}{mv}$

λ = Wavelength
m = mass
v = Velocity

$\lambda = [L]$

$h = [M^1L^2T^{-1}]$

$v = [M^0L^1T^{-1}]$

L.H.S	R.H.S
λ	$\frac{h}{mv}$
$[L]$	$\frac{[M^1 L^2 T^{-1}]}{[M][L^1 T^{-1}]}$
$[L]$	$[L]$

L.H.S = R.H.S [Correct]

2. $V = \sqrt{\frac{2GM}{R}}$

V = Velocity $= [M^0 L^1 T^{-1}]$

G = Gravitational Constant $= [M^{-1} L^3 T^{-2}]$

M = Mass $= [M]$

R = Radius $= [L]$

L.H.S	R.H.S
V	$\sqrt{\frac{2GM}{R}}$
$[M^0 L^1 T^{-1}]$	$\sqrt{\frac{[M^{-1} L^3 T^{-2}][M]}{L}}$
	$\sqrt{L^2 T^{-2}} = L^1 T^{-1}$

L.H.S = R.H.S [Correct]

Q.2 Check the Correctness of $E = mc^2$

Sol. E = Energy $= [M^1 L^2 T^{-2}]$

m = Mass $= [M^1]$

C = velocity of light $= [M^0 L^1 T^{-1}]$

L.H.S	R.H.S
$E = [M^1 L^2 T^{-2}]$	$mc^2 = [M^1][L^1 T^{-1}]^2$
	$= [M^1 L^2 T^{-2}]$
L.H.S	= R.H.S [Correct]

Q.3 Critical velocity (V) of flow of a liquid through a pipe of radius (r) is given by

$$V = \frac{\eta}{\rho r}$$

Where ρ is density & η is Coefficient of viscosity

Check if the relation is correct dimensionally?

Sol. V = velocity $= [M^0 L^1 T^{-1}]$

r = radius $= [M^0 L^1 T^0]$

η = Co-efficient of viscosity $= [M^1 L^{-1} T^{-1}]$

ρ = density $= [M^1 L^{-3} T^0]$

L.H.S	R.H.S
Velocity	$\frac{\eta}{\rho r} = \frac{[M^1 L^{-1} T^{-1}]}{[M^1 L^{-3}][L^1]}$
$[M^0 L^1 T^{-1}]$	$= [M^0 L^1 T^{-1}]$

L.H.S = R.H.S [Correct]

Q.4 Check the Correctness of relation

$$h = \frac{2\sigma \cos\theta}{r^2 d g}$$

Sol. h = height = $[M^0 L^1 T^0]$

σ = Surface Tension = $[M^1 L^0 T^{-2}]$

$\cos\theta$ = $[M^0 L^0 T^0]$

r = $[M^0 L^1 T^0]$

d = density = $[M^1 L^{-3} T^0]$

g = Acc. due to gravity = $[M^0 L^1 T^{-2}]$

L.H.S

$[M^0 L^1 T^0]$

R.H.S

$$\frac{2\sigma\cos\theta}{r^2 d g} = \frac{[\cancel{M^1} L^0 \cancel{T^{-2}}]}{[\cancel{L^2}][\cancel{M^1} \cancel{L^{-3}} T^0][M^0 \cancel{L^1} \cancel{T^{-2}}]}$$

$[M^0 L^1 T^0]$

$= [M^0 L^0 T^0]$

L.H.S $\neq$ R.H.S [Incorrect]

Q.5 Rate of flow (V) of a liquid through a pipe of radius (r) Under a pressure gradient $\left(\frac{P}{l}\right)$ is given by $V = \frac{\pi}{8}\frac{P r^4}{\eta l}$

where η is coefficient of viscosity. Check whether formula is correct or not?

V = Rate of flow $= [M^0 L^3 T^{-1}]$

$\frac{P}{l} = \frac{M^1 L^{-1} T^{-2}}{L} = [M^1 L^{-2} T^{-2}]$

$r = [M^0 L^1 T^0]$

$\eta = [M^1 L^{-1} T^{-1}]$

L.H.S	R.H.S
V	$\frac{Pr^4}{\eta l}$
$[M^0 L^3 T^{-1}]$	$\frac{[\cancel{M^1} L^{-1} T^{-2}][L^4]}{[\cancel{M^1} \cancel{L^{-1}} T^{-1}][\cancel{L}]}$
$[M^0 L^3 T^{-1}]$	$[L^3 T^{-1}]$

L.H.S = R.H.S [Correct]

Q.6 Find the dimensions of the quantity q from the expression: $T = 2\pi\sqrt{\frac{ml^3}{3Yq}}$, where T is time period of a bar of length l, mass m & Young's modulus Y.

Sol. Given expression $T = 2\pi\sqrt{\frac{ml^3}{3Yq}}$

S.B.S

$$T^2 = 4\pi^2\left(\frac{ml^3}{3Yq}\right)$$

$$q = \frac{4\pi^2 ml^3}{3Y T^2}$$

$$q = \frac{[\cancel{M^1}][L^3]}{[\cancel{M^1} L^{-1} \cancel{T^{-2}}][\cancel{T^2}]}$$

$$q = [L^4]$$ Answer

Q.7 Check the correctness of relation

$$S_{nth} = u + \frac{a}{2}(2n-1)$$

Where

S_{nth} = distance travelled in n^{th} second

n = time

a = Acceleration

u = Initial velocity

Sol.

$S_{nth} = [L^1 T^{-1}]$

$n = [M^0 L^0 T^1]$

$a = [M^0 L^1 T^{-2}]$

$u = [M^0 L^1 T^{-1}]$

L.H.S	R.H.S
$[M^0 L^1 T^{-1}]$	$u + \frac{a}{2}(2n-1)$
	$= L^1 T^{-1} + L^1 T^{-2}(T)$
	$= L^1 T^{-1} + L^1 T^{-1}$
$L^1 T^{-1}$	$= L^1 T^{-1}$

L.H.S = R.H.S [Correct]

Checking the Correctness of formulae:

Try these Questions Yourself!

Q.1. An Artificial Satellite of mass m is revolving in a circular Orbit around a planet of M & radius R. if the radius of the orbit of the satellite be r, Justify by method of dimensions that time period of satellite is given by:

$$T = \frac{2\pi}{R}\sqrt{\frac{r^3}{g}}$$

Q.2 Test dimensional Consistency of the following equations:

(i) $V = u + at$ (ii) $S = ut + \frac{1}{2}at^2$

(iii) $v^2 - u^2 = 2as$

Q.3. The viscous force 'F' acting on a small sphere of radius 'r' moving with velocity v through a liquid is given by $F = 6\pi\eta r v$.
Calculate dimensions of η (coefficient of viscosity)

Q.4 Check the Correctness of relation of Time period of a Simple Pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Where l = length g = acc. due to gravity

Q.8. Justify L+L=L & L-L=L.

Sol. L+L=L & L-L=L means

only same type of physical quantities can be added & subtracted & again we will get same physical quantity.

for eg: distance can be added in distance only & we will get distance again.

Q.9 Pressure is defined as Momentum per unit volume. Is it true?

Sol. Pressure $= \dfrac{\text{Force}}{\text{Area}} = \dfrac{M^1L^1T^{-2}}{L^2}$. $\dfrac{\text{Momentum}}{\text{Volume}}$

$P = [M^1L^{-1}T^{-2}]$. $\dfrac{M^1L^1T^{-1}}{L^3}$

$M^1L^{-2}T^{-1}$

So Answer is NO. It's Not true.

Derivation of formulae:

Using Principle of Homogeneity we can derive formula of physical quantity. Provided we know the factors on which the physical quantity depends.

for eg: Q.1 Derive an expression for time period(t) of a simple pendulum which may depend upon: Mass, length of Pendulum & Acc. due to gravity.

Sol:

$$T \propto m^a l^b g^c$$

$$T = k\, m^a l^b g^c \quad \text{—} \ (1)$$

$$[M^0L^0T^1] = k\,[M^1L^0T^0]^a\,[M^0L^1T^0]^b\,[M^0L^1T^{-2}]^c$$

$$[M^0L^0T^1] = k\,[M^{a+0+0}]\,[L^{0+b+c}]\,[T^{0+b-2c}]$$

$$M^0L^0T^1 = k\,[M^a]\,[L^{b+c}]\,[T^{0-2c}]$$

According to Principle of Homogenity

$0 = a$	$0 = b + c$	$1 = -2c$
	$b = -c$	$1 = -2c$
	$b = -\left[\frac{-1}{2}\right]$	
$\boxed{a = 0}$	$\boxed{b = \frac{1}{2}}$	$\boxed{c = \frac{-1}{2}}$

$$T = k\, m^0 l^{1/2} g^{-1/2}$$

$$\boxed{T = k\sqrt{\frac{l}{g}}}$$

Q.2 The velocity of transverse waves on a String may depend upon length (l) of string
(ii) Tension T in the String
(iii) Mass per Unit length of String
Derive the formula dimensionally.

$$V \propto l^a T^b m^c$$

$$V = k\, l^a T^b m^c \quad -①$$

$V = [M^0 L^1 T^{-1}]$ $\quad l = [M^0 L^1 T^0]$ $\quad T = [M^1 L^1 T^{-2}]$

m = mass per Unit length = $[M^1 L^{-1} T^0]$

$$[M^0 L^1 T^{-1}] = k[M^0 L^1 T^0]^a [M^1 L^1 T^{-2}]^b [M^1 L^{-1} T^0]^c$$

$$[M^0 L^1 T^{-1}] = k[M^{0+b+c}][L^{a+b-c}][T^{0-2b+0}]$$

Compare L.H.S & R.H.S

$0 = b+c$ $\boxed{b=-c}$ $c=-b$ $\boxed{c=-\frac{1}{2}}$	$1 = a+b-c$ $1 = a-c-c$ $1 = a-2c$ $1 = a-2\left(-\frac{1}{2}\right)$ $1 = a+1$ $\boxed{a=0}$	$-1 = -2b$ $\boxed{b=\frac{1}{2}}$

Put all these values in eqⁿ ①

$$V = k\, l^0\, T^{1/2}\, m^{-1/2}$$

$$\boxed{V = k\sqrt{\frac{T}{m}}}$$

Q.3 Reynold number N_R (a dimensionless quantity) determines the condition of laminar flow of a viscous liquid through a pipe. N_R is a function of the density of the liquid 'ρ', its average speed 'v' and coefficient of viscosity 'η'.
Given that N_R is also directely proportional to 'D' (the diameter of pipe), show from dimensional considerations that $N_R \propto \frac{\rho v D}{\eta}$.

Sol. K = a dimensionless constant

$N_R = 1 = [M^0 L^0 T^0]$

$\rho = [M^1 L^{-3} T^0]$

$v = [M^0 L^1 T^{-1}]$

$\eta = [M^1 L^{-1} T^{-1}]$

$D = [L]$

$N_R \propto \rho^a v^b \eta^c D^1$

$N_R = K \rho^a v^b \eta^c D^1$ —①

$[M^0 L^0 T^0] = K [M^1 L^{-3} T^0]^a [M^0 L^1 T^{-1}]^b [M^1 L^{-1} T^{-1}]^c [L]$

$[M^0 L^0 T^0] = K [M^{a+0+c}][L^{-3a+b-c+1}][T^{-b-c}]$

Equating the Powers of M, L & T we get

$a + c = 0$	$-3a + b - c + 1 = 0$	$-b - c = 0$
$a = -c$	$-3(-c) + -c - c + 1 = 0$	$-b = c$
$a = -(-1)$	$3c - 2c + 1 = 0$	$b = -c$
$\boxed{a = 1}$	$c + 1 = 0$	$b = -(-1)$
	$\boxed{c = -1}$	$\boxed{b = 1}$

Put all values in eqⁿ ①

$N_R = K \rho^1 v^1 \eta^{-1} D$

$\Rightarrow \boxed{N_R = \frac{K \rho v D}{\eta}}$

Q.4 Derive by method of dimensions an expression for Volume of a liquid flowing out per second through a narrow pipe. Assume that rate of flow of liquid depends on

(i) coefficient of viscosity (η) of the liquid

(ii) radius ('r') of pipe

(iii) pressure gradient $\left(\frac{P}{l}\right)$ along the pipe.

Take $K = \pi/8$

Sol. Let Volume flowing out per second through pipe be given by!

$$V = k\,\eta^a r^b \left(\frac{P}{l}\right)^c \quad -①$$

$V = [L^3 T^{-1}]$

$\eta = [M^1 L^{-1} T^{-1}]$

$r = [M^0 L^1 T^0]$

$P = M^1 L^{-1} T^{-2}$

$l = [M^0 L^1 T^0]$

Put all these values in eqⁿ ①

$$[M^0 L^3 T^{-1}] = k\,[M^1 L^{-1} T^{-1}]^a [M^0 L^1 T^0]^b [M^1 L^{-2} T^{-2}]^c$$

Equating the Powers of M, L & T

$0 = a+0+c$	$3 = -a+b-2c$	$-1 = -a+0-2c$
$a+c=0$	$-a+b-2c=3$	$-1 = -a-2c$
$\boxed{a=-c}$	$-(-1)+b-2(1)=3$	$-1 = -(-c)-2c$
$\boxed{a=-1}$	$1+b-2=3$	$-1 = c-2c$
	$b-1=3$, $\boxed{b=4}$	$-1 = -c \Rightarrow \boxed{c=1}$

Put all these values in eqⁿ ①

$$V = K\,\eta^{-1} r^{4} \left(\frac{P}{l}\right)^{1}$$

$$\boxed{V = K\,\frac{P}{l}\frac{r^4}{\eta}}$$

Try These Questions Your Self

Q.1 The escape velocity v of a body depends upon
(i) the acceleration due to gravity of planet
(ii) Radius of Planet R
Establish dimensionally the relationship between v, g & R.

$\boxed{V = K\sqrt{gR}}$

Q.2 The frequency ν of an oscillating drop may depend upon radius r of the drop, density ρ of the liquid & surface tension S of liquid. Establish an expression for ν dimensionally.

$\boxed{\nu = K\sqrt{\frac{S}{\rho r^3}}}$

Q.3 Obtain an expression for the centripetal force F acting on a particle of mass m moving with velocity v in a circle of radius r. Take dimensionless Constant K=1

$\boxed{F = \frac{mv^2}{r}}$

Q.4 A small spherical ball of radius r falls with velocity through a liquid having coefficient of viscosity η. Find the viscous drag F on the ball assuming it depends on η, r & v. Take $K = 6\pi$

Conversion from one system to another

This is based on the fact that

Magnitude of a physical quantity remains the same, whatever be the system of its measurement i.e.

$$Q = n_1 u_1 = n_2 u_2$$

Formula Used

HAPPY EVER AFTER

$$n_2 = n_1 \left[\frac{u_1}{u_2}\right]$$

$$n_2 = n_1 \frac{[M_1]^a [L_1]^b [T_1]^c}{[M_2]^a [L_2]^b [T_2]^c}$$

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

Ques 1 Convert 1 newton into dyne.

Sol^n Newton is S.I Unit of force & dyne is c.g.s Unit of force.

SI	$\longrightarrow$	c.g.s
kg	$\longrightarrow$	g
m	$\longrightarrow$	cm
s	$\longrightarrow$	s

$$n_2 = 1\left[\frac{1kg}{1g}\right]^a \left[\frac{1m}{1cm}\right]^b \left[\frac{1s}{1s}\right]^c$$

Force = $M^1 L^1 T^{-2}$

$a=1,\ b=1,\ c=-2$

$$n_2 = 1\left[\frac{1000g}{1g}\right]^1 \left[\frac{100cm}{1cm}\right]^1 \left[\frac{1s}{1s}\right]^{-2}$$

$$n_2 = 1\left[10\right]^3 \left[10^2\right]^1 \left[1\right]^{-2}$$

$$\boxed{n_2 = 10^5 \text{ dyne}}$$

Q.2 Convert an Energy of 5 Joule into ergs.

Joule —— S.I Units of Energy

ergs —— c.g.s Unit of Energy

S.I $\longrightarrow$ c.g.s.

kg $\longrightarrow$ g

m $\longrightarrow$ cm

s $\longrightarrow$ s.

Energy = $M^1 L^2 T^{-2}$

$$n_2 = 5\left[\frac{1000g}{1g}\right]^1 \left[\frac{100\,cm}{1cm}\right]^2 \left[\frac{1s}{1s}\right]^{-2}$$

$$n_2 = 5\left[1000\right]^1 \left[100\right]^2 \left[1\right]^{-2}$$

$$\boxed{n_2 = 5\times10^7 \text{ erg}}$$

Q.3 Convert a power of one mega watt on a system whose fundamental Units are 10kg, 1dm, & 1minute.

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

Given → Power = 1 MW = 10^6 Watt

$$\text{Power} = \frac{\text{Work}}{\text{time}} = \frac{M^1L^2T^{-2}}{T} = M^1L^2T^{-3}$$

kg ——— 10 kg

1m ——— 1dm

1s ——— 1minute

$$n_2 = 10^6\left[\frac{1kg}{10kg}\right]^1\left[\frac{1m}{1dm}\right]^2\left[\frac{1s}{1minute}\right]^{-3}$$

$$n_2 = 10^6\left[\frac{1}{10}\right]^1\left[\frac{10dm}{1dm}\right]^2\left[(60)^{-1}\right]^{-3}$$

$$n_2 = 10^6 \times 10^2 \times (60)^3 \times 10^{-1}$$

$$n_2 = 216 \times 10^3 \times 10^6 \times 10^2 \times 10^{-1}$$

$$= 216 \times 10^{10}$$

$$\boxed{n_2 = 2.16 \times 10^{12} \text{ new Units of Power}}$$

Conversion from one System to another.

Try these questions yourself: sciphy.in

Q.1 The value of Stefan's constant is $\sigma = 5.67 \times 10^{-8}$ $J s^{-1} m^{-2} K^{-4}$

Find its value in c.g.s system.

Ans 5.67×10^{-5} $erg\, s^{-1} cm^{-2} K^{-4}$.

Q.2 When one metre, one kg & one minute are taken as fundamental Units the magnitude of a force is 36 Units. What is the value of this force on c.g.s system?

Ans 10^3 dyne.

Very Short answer type of questions:

1. Can a quantity have constant value & be dimensionless?

Ans. Yes, Joule's mechanical equivalent of heat has a constant value, but is dimensionless.

2. Can a quantity have Units but still be dimensionless?

Ans-2 Yes. For example angle is dimensionless but it has Units.

3. Does a quantity have different dimensions in different system of Units?

A-3. No a quantity has same dimensions in different system of Units.

Q.4 All constants are dimensionless. Comment.

A.4. No all constants are not dimensionless.

for eg: Universal gravitational constant.

Q.5 Can there be a physical quantity which has no units & no dimensions.

A.5 Yes, for example strain has units & no dimensions.

Q.6 Calculate x in equation:

$$(\text{Velocity})^x = (\text{Pressure diff})^{3/2} \times (\text{density})^{-3/2}$$

$$[M^0L^1T^{-1}]^x = (M^1L^{-1}T^{-2})^{3/2} \times (M^1L^{-3}T^0)^{-3/2}$$

$$[M^0L^1T^{-1}]^x = M^{3/2-3/2}\, L^{-3/2} \cdot L^{9/2}\, T^{-3}$$

$$[M^0L^1T^{-1}]^x = [M^0L^3T^{-3}]$$

$$[M^0L^1T^{-1}]^x = [M^0L^1T^{-1}]^3$$

$$\boxed{x = 3}$$

Q.7 Find value of x in the relation $Y = \dfrac{T^x \cos\theta \cdot \tau}{L^3}$

Where Y is Young's Modulus, T is Time period, τ = Torque

L = length

$$T^x = \frac{YL^3}{\cos\theta \cdot \tau} = \frac{[M^1L^{-1}T^{-2}][L^3]}{[M^1L^2T^{-2}]} \Rightarrow \boxed{x = 0}$$

The fact that quantities having different dimensions cannot be added or subtracted.

Q-1 The velocity v of a particle depends upon time t, according to the eq^n $v = a + bt + \frac{c}{d+t}$

Write the dimensions of a, b, c & d

Sol= $L^1T^{-1} = a + b \times T + \frac{c}{d+t}$

$d+t \Rightarrow \quad d = [T]$

$\frac{c}{d+t} = L^1T^{-1}$

$\frac{c}{T} = L^1T^{-1} \Rightarrow L^1T^{-1} \times T = [L^1]$

$bt = L^1T^{-1}$

$b = \frac{L^1T^{-1}}{T^1} = [L^1T^{-2}]$

$a = [L^1T^{-1}]$

Q-2 Write the dimensions of a/b in the relation $F = a\sqrt{x} + bt^2$ Where F is Force, x is distance & t is time.

$a\sqrt{x} = F$

$a = \frac{F}{\sqrt{x}} = \frac{M^1L^1T^{-2}}{L^{1/2}} = [M^1L^{1/2}T^{-2}]$

$bt^2 = F$

$b = \frac{F}{t^2} = \frac{M^1 L^1 T^{-2}}{T^2} \Rightarrow [M^1 L^1 T^{-4}]$

$\frac{a}{b} = \frac{M^1 L^{1/2} T^{-2}}{M^1 L^1 T^{-4}} = [M^0 L^{-1/2} T^2]$

Q3 Write dimensions of a/b in the relation

$P = \frac{a - t^2}{bx}$ $\qquad P = \text{Power}$

$a = [T^2]$

$M^1 L^2 T^{-3} = \frac{T^2}{b \times L} \Rightarrow b = \frac{T^2}{M^1 L^3 T^{-3}}$

$b = M^{-1} L^{-3} T^5$

$\frac{a}{b} = \frac{T^2}{M^{-1} L^{-3} T^5} = [M^1 L^3 T^{-3}]$ Ans

Q4 In the equation $y = A \sin(\omega t - kx)$, obtain dimensional formula of ω & k. Given x is distance & t is time.

$\omega t - kx = \theta$

$\omega t = \theta$

$\omega = \frac{\theta}{t} = [T^{-1}]$

$kx = \theta$

$k = \frac{\theta}{x} = [L^{-1}]$

Q-5 In the expression $P = E l^2 m^{-5} G^{-2}$

E = Energy
m = mass
l = Angular Momentum
G = gravitational constant resp.

Show that P is a dimensionless quantity.

$$P = [M^1L^2T^{-2}][M^1L^2T^{-1}]^2[M]^{-5}[M^{-1}L^3T^{-2}]^{-2}$$

$$P = [M^0L^0T^0]$$

Hence P is a dimensionless quantity.

Limitation of Dimensional Analysis

1. This method gives us no information about dimensionless constants in the formula.

2. If a quantity depends on more than 3 factors having dimensions, formula cannot be derived.

3. We can not derive the forimulae containing trignometrical functions, exponential functions etc which have no dimensions.

4. Method of dimensions can not be used to derive an exact form of relation when it consists of more than one part on any side
for eg. $S = ut + \frac{1}{2}at^2$ can not be obtained.

5. It gives no information whether a physical quantity is a scalar or a vector.

Significant Figures

Significant figure are digits known reliably plus the first Uncertain digit are known as Significant digits.

- for eg 374.5m has four Significant figures.
- 3, 7 & 4 are certain & reliable.
- digit 5 is Uncertain.

⇒ larger number of significant figures in a measurement, higher the accuracy of measurement & vice-versa.

Common Rules for Counting Significant figures

(i) All non-zero digits are significant. So 13.75 has four significant figures.

(ii) All zeros between two non-zero digits are significant.

for eg: 100.05 km has five significant figures.

(iii) All zeros to the right of a non-zero digit are not significant

for eg: 86400 has 3 significant figures

But such zeros will be significant if they come from a measurement.

for eg: 86400s has five significant figures.

(iv) All zeros to the right of a non-zero digit but to the left of a decimal point are significant.

for example 684 700. has six significant fig.

(v) All zeros to the right of a decimal point are significant

for eg: 161 cm, 161.0 cm & 161.00 cm have three four & five significant figures respectively.

(vi) All zeros to the right of a decimal point but to the left of a non zero digit are not significant. So 0.161cm & 0.0161 cm both have 3 significant figures.

(vii) Number of significant figures does not depend on the system of units.

for eg 16.4 cm
0.164 m
& 0.000164 km all have 3 significant figures.

(viii) Multiplying or dividing factors which are neither rounded number nor numbers representing measured values are exact. They have infinite number of significant digits.

for example -

circumference $S = 2\pi r$, factor 2 is an exact number

It can be written as 2.0

2.00

2.000

Arithmetic operations with significant figures.

1. Significant figures in the sum or difference of two numbers.

In addition or subtraction the final result should retain same number of decimal places as that of the original number with minimum number of decimal places.

for example: Sum of three measurements of length (2.1m, 1.78 m & 2.046m) = 5.926m

⇒ 5.926 is rounded off to 5.9 m (upto smallest number of decimal places in 3 measurements).

\# Similarly $x = 12.587m$, $y = 12.5m$

then $x - y$ ⇒ $12.587 - 12.5 = 0.087$ m

- So 0.087m is rounded off to 0.1m upto smallest number of decimal places in y.

Significant figures in multiplication & Division of numbers

In multiplication or division the final result should retain same number of Significant figures as that of Original number with minimum number of significant digits.

for example $x = 3.8$ $y = 0.125$

So $x \times y = 3.8 \times 0.125 = 0.475$

0.475 is rounded off to 0.48 (upto 2 significant figures)

Because least number of significant figures is 2 in ($x = 3.8$).

Ques: State the rules for rounding off a measurement.

(i) If the digit to be dropped is smaller than 5 then preceding digit is left Onunchanged

(ii) If the digit to be dropped is greater than 5, then preceding digit is increased by 1.

(iii) If the digit to be dropped is 5 followed by non-zero digits, then preceding digit is increased by 1.

(iv) If the digit to be dropped is 5 then preceding digit is left Unchanged it is even.

(V) If the digit to be dropped is 5, then preceding digit is increased by 1 if it is odd.

Ques-1 Write the result of the following with regard to significant figures.

(i) $876 + 0.4382 = 876.4382$

It is rounded off to 876.

(ii) $8.0 - 0.42 = 7.58$ (Rounded off to 7.6)

Ques-2 Subtract 2.5×10^4 from 3.9×10^5 with due regard to significant figures.

$$y - x = 3.9 \times 10^5 - 0.25 \times 10^5$$
$$= (3.9 - 0.25) \times 10^5$$
$$= 3.65 \times 10^5$$
$$= 3.6 \times 10^5 \text{ Ans.}$$

Ques-3. Write number of significant figures in the following:

(i) 5238 N — 4 significant figures

(ii) 4200 kg — 4 significant figures

(iii) 34.000 m — 5 significant figures

(iv) $0.02340 Nm^{-1}$ — 4 significant figures.

Ques-4 5.74 g of a substance occupies $12 cm^3$. Express its density by keeping the significant figures in view

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{5.74g}{1.2cm^3} = 4.783 g\,cm^{-3}.$$

$4.783\ g\,cm^{-3}$ is rounded off to $4.8\ g\,cm^{-3}$.

Q.5 Add 6.75×10^3 cm to 4.52×10^2 cm

$\Rightarrow$ $6.75 \times 10^3\ cm + 0.452 \times 10^3\ cm$

$= (6.75 + 0.452) \times 10^3$

$= 7.202 \times 10^3$ cm is rounded off to

$= 7.20 \times 10^3$ cm

Q.6 In the expression surface Area $= 4\pi r^2$, the factor 4 is an exact number. How many number of significant figures are there in the factor 4 ?

Sol. Factor 4 is an exact number so number of significant figures are infinite