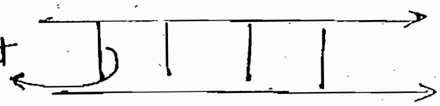
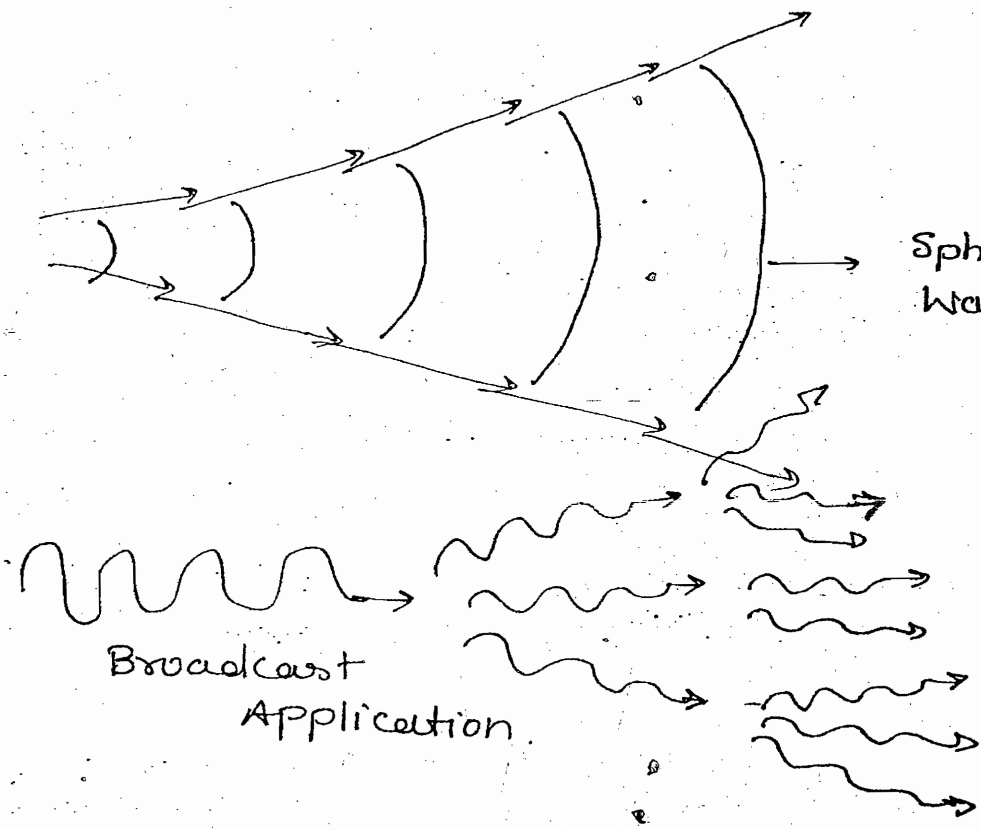
Wave Guides :-

Uniform Plane Wave → $E(z,t)_x$, $H(z,t)_y$

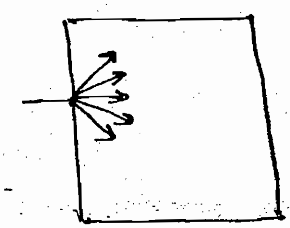
Wavefront



Dispersive Beams → $E(x,y,z,t)_{(x,y,z)}$
$H(x,y,z,t)_{(x,y,z)}$



Spherical Wavefront

Broadcast Application



Scattering
Diffraction
Diffusion

→ All practical EM waves are dispersive in nature and hence they obey Huygen's wave principle that every ray is a source of secondary emission. This is the cause of diffusion, diffraction and scattering properties of EM Waves

→ This is an advantage in broadcast application but a serious limitation in point to point communication. Hence waveguide are used to restrict the wave with a specific bound

$$\left.\begin{array}{l} E(x,z,t) \\ H(x,z,t) \end{array}\right\}$$ one dimension restriction in x
one dimension propogation in z

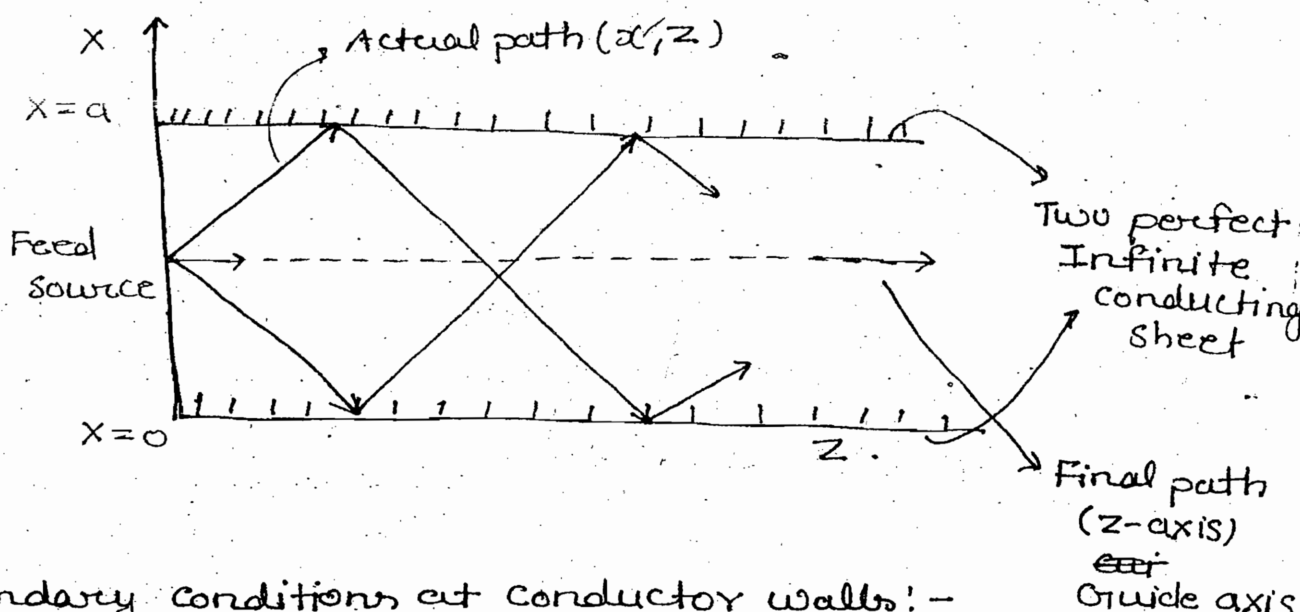
Parallel plane waveguide

eg:- earth and ionosphere guiding

$$\left.\begin{array}{l} E(x,y,z,t) \\ H(x,y,z,t) \end{array}\right\}$$ 2 dimension restriction in x
1 dimension propogation in z

Rectangular Waveguide

Parallel Plane Waveguides :-



Boundary conditions at conductor walls :-

$E_{tang} = 0$ at $x=0$, $X=a$

$E(x)_{tang} = 0$ at $X=0$, $X=a$

$E(x)_y$ & $E(x)_z = 0$ at Guide walls

Propogation along the Guide Axis:-

Using $\nabla^2 E = \gamma^2 E$ Helmoltz's Equation

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial z^2} = -\omega^2 \mu \epsilon E$$

Let $E(z)$ be any natural harmonic $e^{-\bar{\gamma} z}$ as the z side is un-restricted

where $\bar{\gamma} = \gamma_z$ = Propogation consist in guide axis.

$$\frac{\partial^2 E}{\partial x^2} + \bar{\gamma}^2 E = -\omega^2 \mu \epsilon E$$

$$\frac{\partial^2 E}{\partial x^2} = -(\gamma^2 + \omega^2 \mu \epsilon) E$$

where $\bar{\gamma}^2 + \omega^2 \mu \epsilon = \gamma_x^2$

where γ_x = Propogation constant in x side or restricted side

$$\frac{\partial^2 E}{\partial x^2} = -\gamma_x^2 E$$

The $E(x)$ solution is also harmonic

$$E(x) = c_1 \sin(\gamma_x x) + c_2 \cos(\gamma_x x)$$

The restricted side propogation has to be trignometric harmonic only

Applying the Boundary conditions

at $x = 0$

$$E(0)_{tang} = 0 + c_2 = 0$$

$$\Rightarrow c_2 = 0$$

Note:- The tangential E field harmonic has to be a 'sin' only is the restricted side

at $x = a$

$$E(a)_{tang} = c_1 \sin(\gamma_x a) = 0$$

$$\boxed{\gamma_x = \frac{m\pi}{a}} \qquad m = 0, 1, 2, 3 \ldots$$

Note:-

The restricted wide propogation constant can take only discrete solutions but not any continous value

Finally $\bar{\gamma} = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu \epsilon}$

Concept 1:-

(f_c) cut off frequency of the guide

If $\left(\frac{m\pi}{a}\right)^2 > \omega^2 \mu \epsilon$

then $\bar{\gamma} = \bar{\alpha} + j0$ No propogation along the guide axis

If $\omega^2 \mu \epsilon > \left(\frac{m\pi}{a}\right)^2$

$$\omega > \frac{m\pi}{a\sqrt{\mu \epsilon}}$$

then $\bar{\gamma} = 0 + j\bar{\beta}$

The wave travels along the guide axis without attenuation.

$\omega \geq \frac{m\pi c}{a}$

Note:-

Every waveguide has a minimum cut-off frequency below which there cannot be propogation i.e. there is a max. wavelength above which there cannot be propogation and this wavelength is comparable to the guide dimensions

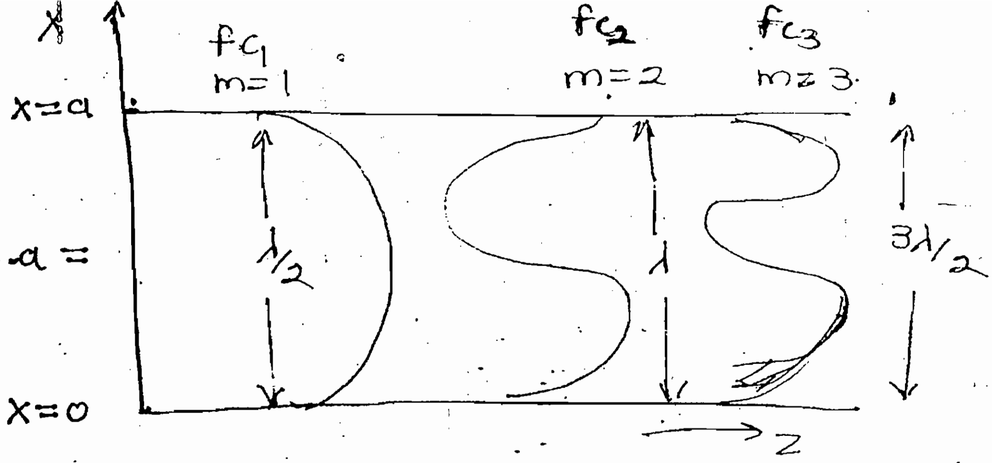
Hence $\omega_c > \frac{m\pi c}{a}$, $f_c > \frac{mc}{2a}$, $\lambda_c > \frac{2a}{m}$

$$\Rightarrow \omega_c = \frac{m\pi c}{a}$$

$$f_c = \frac{mc}{2a}$$

$$\lambda_c = \frac{2a}{m}$$

9p



At exact cut-off frequency where $\bar{\gamma} = 0$, the wave resonats b/w guide walls and oscillates b/w the walls for 'm' such frequencies.

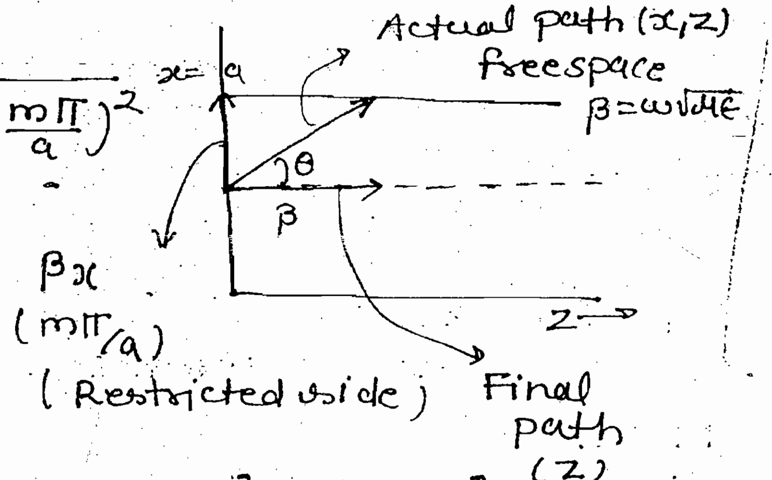
Concept 2 :-

Wave Angle or Tilt Angle :-

$$\bar{\beta} = \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}$$

$$\bar{\gamma} = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu \epsilon}$$

$$= j\sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}$$



$$\beta^2 = \beta_x^2 + \bar{\beta}^2$$

$$\bar{\beta} = \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}$$

Phase velocity along the guide axis, $\bar{V_p} = \frac{\omega}{\bar{\beta}} = \frac{\omega}{\sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2}}$

$$\bar{V_p} = \frac{1}{\sqrt{\mu \epsilon - \left(\frac{m\pi}{a\omega}\right)^2}}$$

$$= \frac{1/\sqrt{\mu \epsilon}}{\sqrt{1 - \left(\frac{m\pi}{a\omega\sqrt{\mu \epsilon}}\right)^2}}$$

$$\boxed{\bar{V}_p = \frac{c}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}}$$

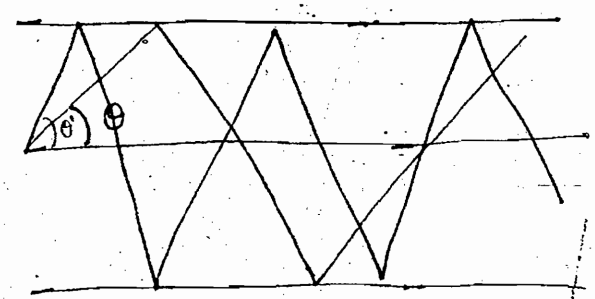
$$\bar{\beta} = \beta\cos\theta$$

$$\frac{2\pi}{\bar{\lambda}} = \frac{2\pi}{\lambda}\cos\theta$$

$$\Rightarrow \bar{\lambda} = \frac{\lambda}{\cos\theta}$$

$$\Rightarrow \bar{\lambda} f = \frac{\lambda f}{\cos\theta}$$

$$\boxed{\bar{V}_p = \frac{c}{\cos\theta}}$$

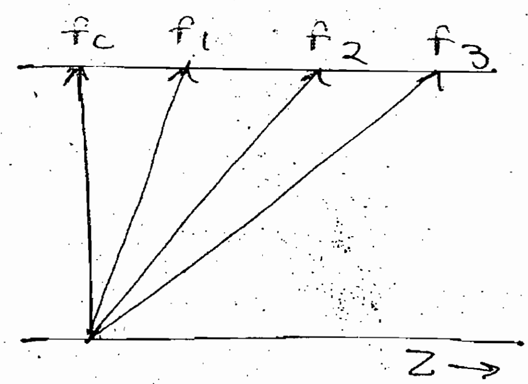


By comparison

$$\boxed{\sin\theta = \frac{f_c}{f}}$$

Note:-

Every frequency has a unique tilt angle and unique velocity inside the guide so that two different frequencies never overlap to each other



$$f_3 > f_2 > f_1 > f_c$$

Concept 3:-

Group Velocity ($\bar{V}_g$):-

$$\bar{V}_p = \frac{c}{\cos\theta} \Rightarrow \bar{V}_p > c \rightarrow \text{Always}$$

In linear conditions, $\beta \propto \omega$

velocity is phase ~~vel~~ velocity $V_p = \frac{\omega}{\beta}$

eg:- lossless EM Waves in free space

lossless VI Waves in transmission line

$$\beta = \omega\sqrt{\mu\epsilon}$$
$$\beta = \omega\sqrt{LC}$$

But in dispersive condition β is not linear with ω

eg:- $V_g = \frac{d\omega}{d\bar{\beta}}$ = Group velocity

$$\bar{\beta} = \sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2} \longrightarrow \text{dispersive conditions}$$

β is not linear with ω

$$\frac{d\bar{\beta}}{d\omega} = \frac{1}{\cancel{2}\sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2}} \cdot \cancel{2}\omega\mu\epsilon$$

$$= \frac{\mu\epsilon}{\sqrt{\mu\epsilon - \left(\frac{m\pi}{a\omega}\right)^2}}$$

$$= \frac{\sqrt{\mu\epsilon}}{\sqrt{1 - \left(\frac{m\pi}{a\omega\sqrt{\mu\epsilon}}\right)^2}}$$

$$= \frac{1}{c\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}}$$

$$\bar{V_g} = c\cos\theta$$

$\bar{V_g} < c \longrightarrow$ Always

$\boxed{\bar{V_g} \cdot \bar{V_p} = c^2} \longrightarrow$ Always

Concept 4:-

Modes of Operation

Concept 4 :-

Modes of Operation :-

Note :-

Mode stands for physical connection of field which can be axial or longitudinal feed

→ The no. of field connection decides the integer m of that mode and this m decides the cut-off freq of the mode

→ The mode can be identified from field such that m stands for no. of half cycles b/w the guide walls ~~and~~ or no. of maxima b/w the guide walls.

Transverse Electric (TE), TM and TEM Modes: —

$$E(x, z, t)_{(x, y, z)}$$
$$H(x, z, t)_{(x, y, z)}$$

Using Maxwells equation

$$\nabla \times E = -j\omega\mu H \qquad \nabla \times H = j\omega\epsilon E$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$$

$$\begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \nabla \times E$$

→ All y derivatives are zero

→ All z derivatives are back the same function with $\bar{\gamma}$ scaling

$$\nabla \times E = -j\omega\mu H \qquad \nabla \times H = j\omega\epsilon E$$

$$\bar{\gamma} E_y = -j\omega\mu H_x \qquad \bar{\gamma} H_y = j\omega\epsilon E_x$$

$$-\bar{\gamma} E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \qquad -\bar{\gamma} H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y$$

$$\frac{\partial E_y}{\partial x} = -j\omega\mu H_z \qquad \frac{\partial H_y}{\partial x} = j\omega\epsilon E_z$$

$$H_x = \frac{-\bar{\gamma} E_y}{j\omega\mu}$$

$$-\bar{\gamma}\left(\frac{-\bar{\gamma} E_y}{j\omega\mu}\right) - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y$$

$$\bar{\gamma}^2 E_y - j\omega\mu \frac{\partial H_z}{\partial x} = -\omega^2 \mu \epsilon E_y$$

$$(\bar{\gamma}^2 + \omega^2 \mu\epsilon) E_y = j\omega\mu \frac{\partial H_z}{\partial x}$$

(I) $$\frac{\partial H_z}{\partial x} = \frac{\gamma_x^2}{j\omega\mu} E_y$$

(II) $$\frac{\partial H_z}{\partial x} = \frac{-\gamma_x^2}{\bar{\gamma}} H_x$$

(III) $$\frac{\partial E_z}{\partial x} = \frac{-\gamma_x^2}{\bar{\gamma}} H_y$$

(IV) $$\frac{\partial E_z}{\partial x} = \frac{-\gamma_x^2}{\bar{\gamma}} E_x$$

Note :-

The axially directed field components determine the complete standing of the wave.

If $E_z = 0$ → physical connection

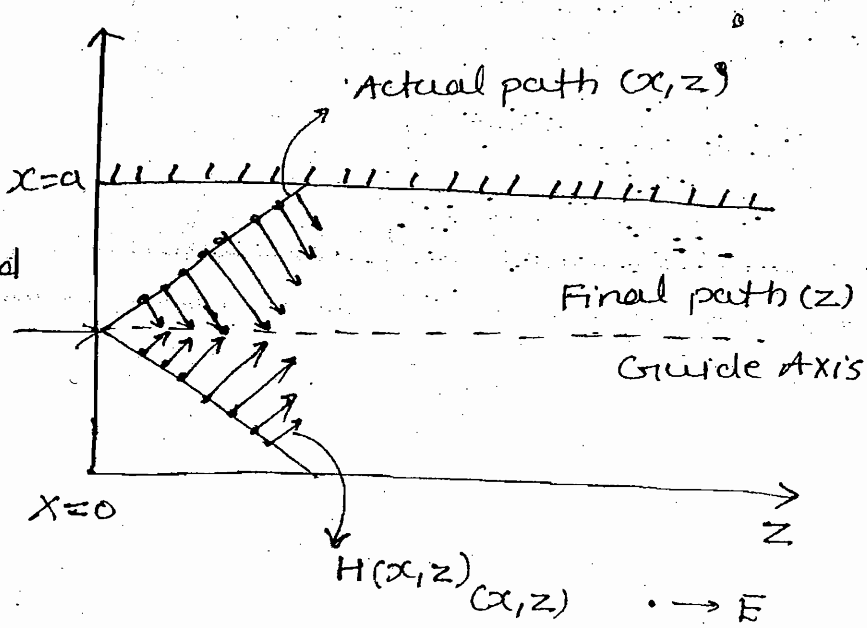
then $E_x = H_y = 0$ The waves becomes

$E(x, z, t)_y$

$H(\cancel{x}, \cancel{z}, t)_{(x,z)}$

$E \perp$ Guide Axis

The wave is called as TE wave



If $H_z = 0$. then $H_x = E_y = 0$

the wave becomes $H(x,z,t)_y$

$E(x,z,t)_{x,z}$

~~E ⊥~~ $\boxed{H \perp \text{Guide Axis}}$

The wave is called as TM Waves

$\boxed{E \perp H \perp \text{Propogation}}$

TE Wave solutions in Parallel - Plane Waveguides :-

The waves has $(E_x = E_z = H_y = 0)$

$$E(x,z,t)_y = E_{y0} \cdot \sin\left(\frac{m\pi}{a}x\right) e^{-\bar{\gamma} z} e^{j\omega t} a_y$$

$$E(x,z,t)_x = H_{x0} \cdot \sin\left(\frac{m\pi}{a}x\right) e^{-\bar{\gamma} z} e^{j\omega t} a_x$$

$$H(x,z,t)_z = H_{z_0} \cos\left(\frac{m\pi}{a}x\right) e^{-\bar{\gamma} z} e^{j\omega t} a_z$$

It is a product (AND) solution of time, x & z

Harmonics

$E(t)/H(t) \longrightarrow e^{j\omega t}$ Source harmonic

$E(z)/H(z) \longrightarrow e^{-\bar{\gamma} z}$ Natural Harmonic

$E(x)/H(x) \longrightarrow$ Trignometric Harmonic

$E(x)_{tang} \longrightarrow$ 'Sin' Harmonic

$E(x)_y$ or $E(x)_z$

$$\frac{\partial H_z}{\partial x} = (\ \ ) H_x$$
$$= (\ \ ) E_y$$

If $m = 0$ the TE wave does not exist and hence $m = 1$ is the least value required for propogation

TM Wave Solutions in Parallel Plane Waveguides :-

($H_z = H_x = E_y = 0$)

$$H(x,z,t)_y = H_{y_0} \cos\left(\frac{m\pi}{a}x\right) e^{-\bar{\gamma}z} e^{j\omega t} a_y$$

$$E(x,z,t)_x = E_{x0} \cos\left(\frac{m\pi}{a}x\right) e^{-\bar{\gamma}z} e^{j\omega t} a_x$$

$$E(x,z,t)_z = E_{z0} \sin\left(\frac{m\pi}{a}x\right) e^{-\bar{\gamma}z} e^{j\omega t} a_z$$

If $m = 0$, $E_z = 0$

The wave becomes $E(z,t)_x$
$H(z,t)_y$

This wave is called as ~~TEM waves~~ TEM Waves with $E_z = H_z = 0$

Properties of TEM Waves :-

1) It has $m = 0$ $\bar{\gamma} = \gamma = j\omega\sqrt{\mu\epsilon}$
- i.e. Propogates only along guide axis i.e. $\theta = 0$ → Always

II) It has $f_c = 0$ i.e. No cut off frequency

III) It has $m = 0$ i.e. No multi-mode connection mechanism

Summary :-

$\left.\begin{matrix} E(x,t) \\ H(x,t) \end{matrix}\right\} \longrightarrow$ Wave at f_c, $\theta = 90°$, $\bar{\gamma} = 0$

(f_{c_1}, f_{c_2} ----- f_{c_m} exists)

$\left.\begin{matrix} E(z,t)_x \\ H(z,t)_y \end{matrix}\right\} \longrightarrow$ TEM wave, $\theta = 0°$ $\bar{\gamma} = \gamma = j\omega\sqrt{\mu\epsilon}$

$m = 0$, $f_c = 0$

3. $E(x,z,t)$, $H(x,z,t)$ → TE → E_y, H_x, H_z → θ, f_c, mode, $\bar{V_p}$, $\bar{V_g}$

$E(x,z,t)$, $H(x,z,t)$ → TM → H_y, E_x, E_z → θ, f_c, mode, $\bar{V_p}$, $\bar{V_g}$

Waveguides (Workbook) :-

1) $f = 22\ GHz \qquad \sin\theta = \frac{f_c}{f} \qquad f_c = \frac{mc}{2a}$

$$f_{c_1} = \frac{1.38 \times 10^8}{2.25 \times 10^{-2}} = 6\ GHz$$

1st Mode - $6\ GHz \longrightarrow \infty$

1st Mode $\sin\theta = \frac{6}{22}$

IInd Mode $12\ GHz \longrightarrow \infty$

III Mode $\sin\theta = \frac{18}{22}$

IIIrd Mode $18\ GHz \longrightarrow \infty$

II) $f = 40\ GHz$

$$f_c = 36 \times 10^9 = \frac{3.3 \times 10^8}{2 \times a} \Rightarrow a = 1.25\ cm$$

III) $$\sin\theta = \frac{36}{40} = \frac{f_{c_3}}{f_3} = \frac{f_{c_7}}{f_7} = \frac{84}{f_7}$$

3^{rd} Mode $\longrightarrow 36$

7^{th} Mode $\longrightarrow 84$

$f_7 = 93.3\ GHz$

IV) $f = 2\ GHz$

$$f_{c_1} = \frac{1.3 \times 10^8}{\sqrt{9} \cdot 2 \times 3 \times 10^{-2}} = \frac{5}{3} = 1.67\ GHz$$

1st $\rightarrow 1.67 \rightarrow \infty$

2nd $\rightarrow 3.34 \rightarrow \infty$

$$\sin\theta = \frac{5/3}{2} = \frac{5}{6}$$

Note:-

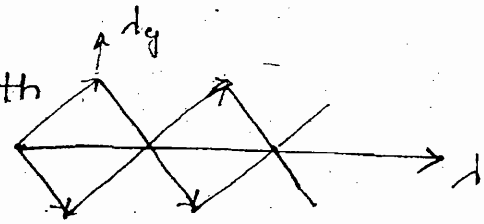
1. $\overline{\gamma}, \overline{\beta}, \overline{\lambda}, \overline{V_p}, \overline{V_g}$, TRANSVERSE $\longrightarrow$ w.r.t guide axis

$$\eta_{TE} = \frac{E_{trans}}{H_{trans}} = \frac{E_y}{H_x} = \frac{E_{total}}{H_{total}\cos\theta} = \frac{120\pi}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

9n:

$$\eta_{TM} = \frac{E_x}{H_y} = \frac{E_{total}\cos\theta}{H_{total}} = 120\pi\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

3. $\bar{\lambda} = \frac{\lambda}{\cos\theta} = \lambda_g$ = Guide Wavelength



$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{\lambda}{\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}}$$

$$1-\left(\frac{\lambda}{\lambda_c}\right)^2 = \left(\frac{\lambda}{\lambda_g}\right)^2$$

$$\boxed{\frac{1}{\lambda^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}}$$

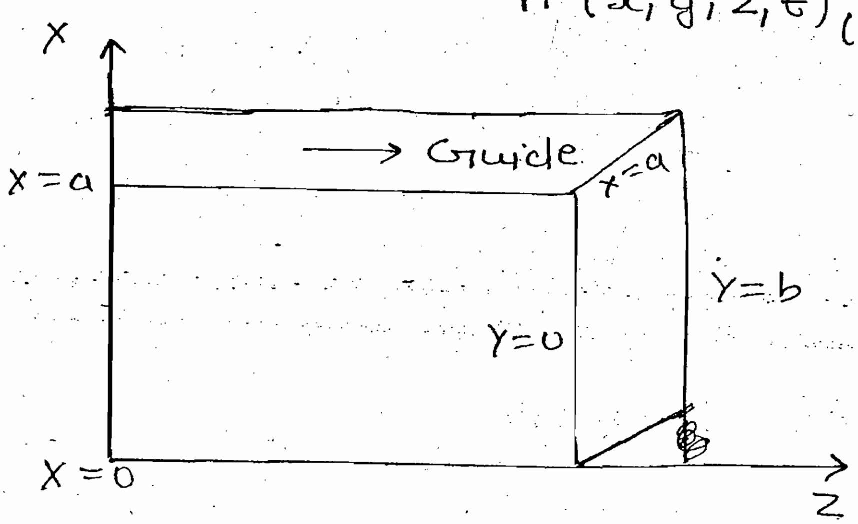
$\frac{1}{\lambda^2} \to \frac{c}{f}$ free space wavelength

$\frac{1}{\lambda_c^2} \to \frac{2a}{m}$ cut-off wavelength

Rectangular Waveguides:-

The waves are $E(x,y,z,t)\,(x,y,z)$

$H(x,y,z,t)\,(x,y,z)$



→ It is a single conductor hollow structure used to confine EM wave in 2 dimension using 4 walls

$x = 0 \qquad x = a$

$y = 0 \qquad y = b$

It has $\quad \gamma_x = \frac{m\pi}{a}$

$\gamma_y = \frac{n\pi}{b}$

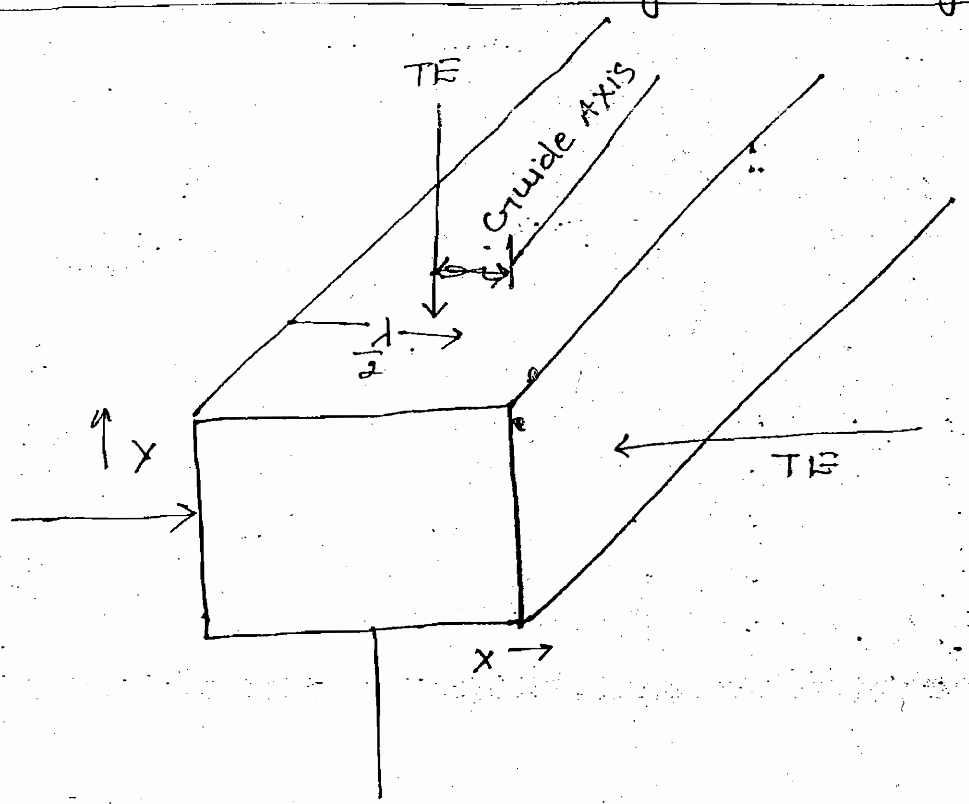
Applying $\nabla^2 E = -\omega^2 \mu \epsilon E$

$$\bar{\gamma} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \epsilon}$$

$$\omega_c = \text{cut-off frequency} = \left(\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}\right) c$$

$$f_c = \left(\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}\right) \frac{c}{2}$$

Modes and feeds in Rectangular Waveguides:-



E_y or E_x

but $E_z = 0$

A horizontal or vertical field always gives a suitable E field but not along the guide axis

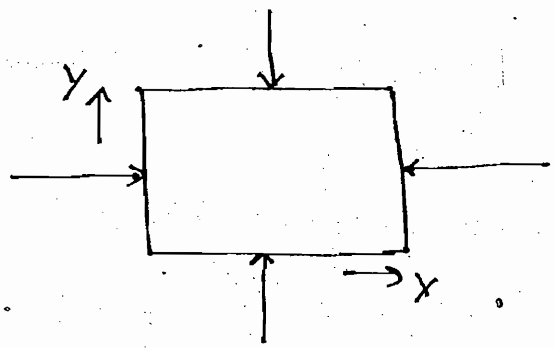
The no. of out of phase feed connections decides the integers m and n.

The modes are always designated as TE_{mn}

H_x & H_y but $H_z = 0$

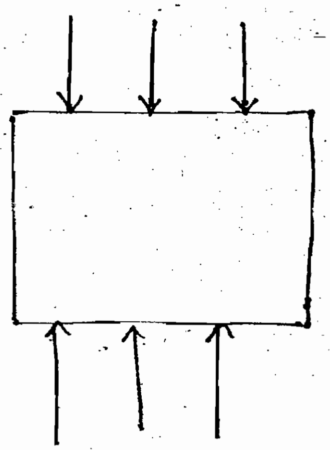
A lateral or axial feed always gives a suitable H field but not along the guide axis

(I)



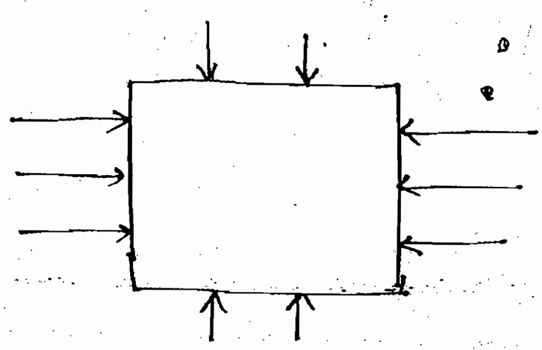
TE_{11}

(II)



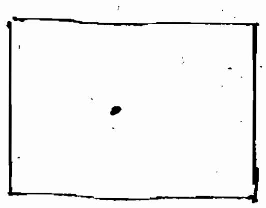
TE_{03}

(III)



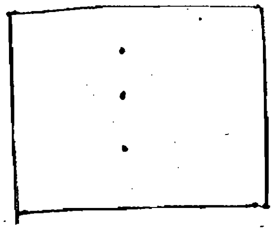
$\cancel{TE_{32}}$ TE_{23}

(IV)



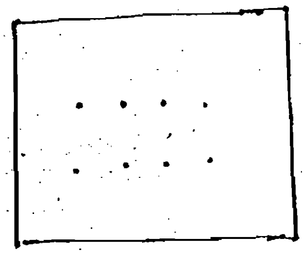
TM_{11}

v)



TM_{13}

vi)



TM_{42}

Note:-

TM_{M0} or TM_{0N} modes are even sent and donot exist in rectangular waveguides.

Such a connection mechanism doesn't exist.

i) $f_c = \left(\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}\right)\frac{c}{2}$

For both TE & TM are same so TE_{74}, TM_{74} are same cutoff called degenerate mode

Two different modes having the same f_c but different connection mechanism) are said to be degenerate modes.

iii) For $m=1$, $n=0$
or $m=0$, $n=1$
i.e. TE_{10} & TE_{01} mode have the least f_c

$$\left.\begin{aligned} f_c\,(TE_{10}) &= \frac{c}{2a} \\ f_c\,(TE_{01}) &= \frac{c}{2b} \end{aligned}\right\}$$

If $a>b$ TE_{10} is dominent mode
$a<b$ TE_{01} is dominent mode

→ Broadside dimensions decides the dominent mode

→ Narrowside dimensions decides the maximum operable voltage and power handling ability

TM Wave Solutions in Rectangular Waveguide ($H_z=0$) :-

The wave is $E(x,y,z,t)\,(x,y,z)$

$H\,(x,y,z,t)\,(x,y)$

$$E(x,y,z,t)_z = E_{z0}\cdot \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-\gamma z}\,e^{j\omega t}\,a_z$$

$$E(x,y,z,t)_x = E_{x0}\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-\gamma z}e^{j\omega t}\,a_x$$

$$E(x,y,z,t)_y = E_{y0}\left(\frac{n\pi}{b}\right)\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-\gamma z}e^{j\omega t}\,a_y$$

$$H(x,y,z,t)_x = H_{x0}\left(\frac{n\pi}{b}\right)\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-\gamma z}e^{j\omega t}\,a_x$$

$$H(x,y,z,t)_y = H_{y0}\left(\frac{n\pi}{a}\right)\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-\gamma z}e^{j\omega t}\,a_y$$

It is a product solution of x, y, z, t harmonics.

$E(t)/H(t) \longrightarrow e^{j\omega t} \longrightarrow$ Source harmonic

$E(z)/H(z) \longrightarrow e^{-\gamma z} \longrightarrow$ Natural Harmonic

$H/E(x \text{ or } y) \longrightarrow$ Trignometric harmonic

$E(x)_y$ or $E(x)_z$] Tangential harmonic } 'sin' Harmonics

$E(y)_x$ or $E(y)_z$]

If $m=0$ & $n\neq 0$ or $m\neq 0$ & $n=0$

TM waves donot exist i.e. they are even sent modes

TE Wave Solutions in Rectangular Waveguide ($E_z = 0$):-

The wave is $E(x,y,z,t)_{(x,y)}$

$H(x,y,z,t)_{(x,y,z)}$

$$H(x,y,z,t)_z = H_{z0} \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-\bar{\gamma} z} e^{j\omega t} a_z$$

$$E(x,y,z,t)_x = E_{x0} \left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\bar{\gamma} z} e^{j\omega t} a_x$$

$$E(x,y,z,t)_y = E_{y0} \left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-\bar{\gamma} z} e^{j\omega t} a_y$$

$$H(x,y,z,t)_x = H_{x0} \left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-\bar{\gamma} z} e^{j\omega t} a_x$$

$$H(x,y,z,t)_y = H_{y0} \left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\bar{\gamma} z} e^{j\omega t} a_y$$

If $m \neq 0$ & $n = 0$

The TE_{m0} wave exists $E(x,z,t)_y$

$H(x,z,t)_{(x,z)}$

If $m = 0$ & $n \neq 0$

The TE_{0n} wave exists $E(y,z,t)_x$

$H(y,z,t)_{(y,z)}$

If $E_z = H_z = 0$ the wave cannot exist

or $m = n = 0$

i.e. TEM waves donot exist in rectangular waveguide.

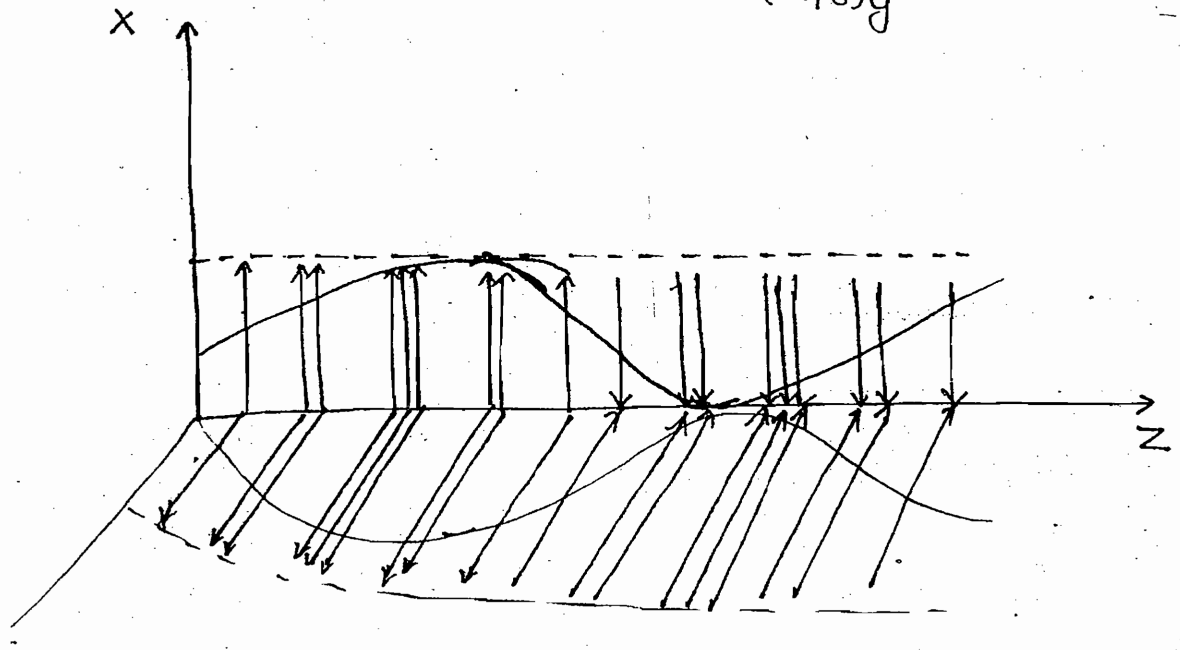
In general TEM waves donot exist in all single conductor guides

TEM needs two distinct conductor for its existence.

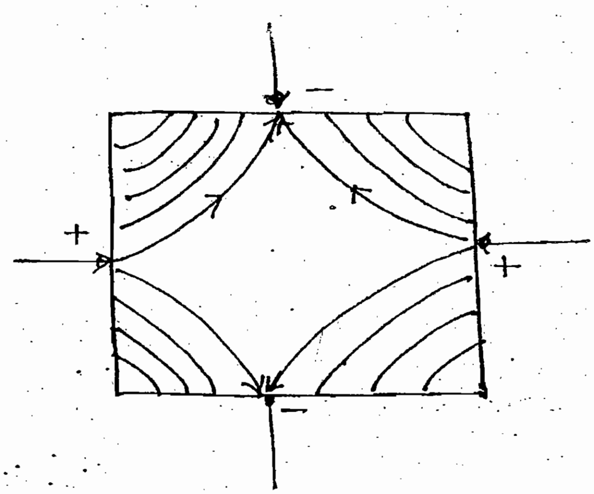
Field line Representations of guided waves:-

(i) Uniform plane Wave — $E(z,t)_x$

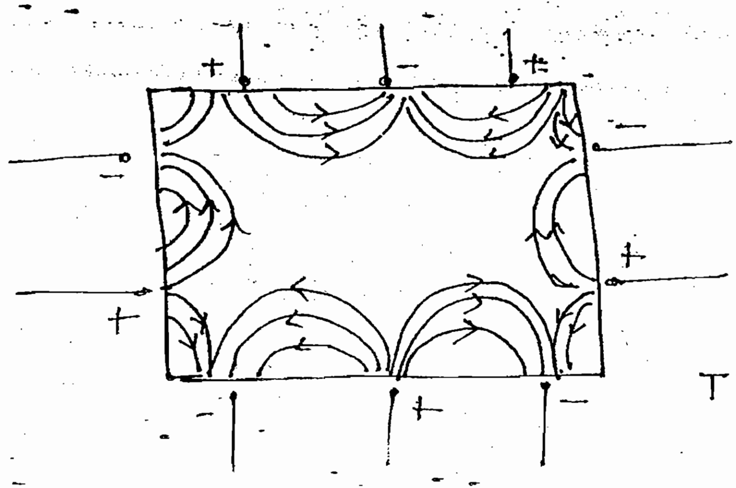
$H(z,t)_y$



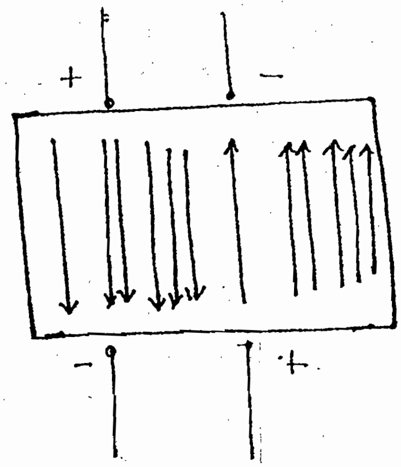
TE Waves:- (E_x, E_y)



TE_{11}

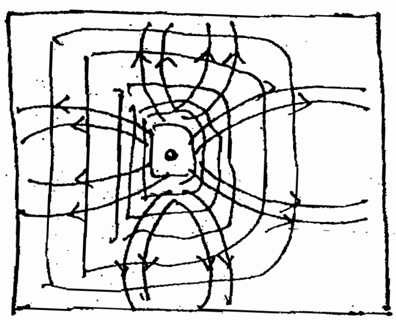


TE_{32}



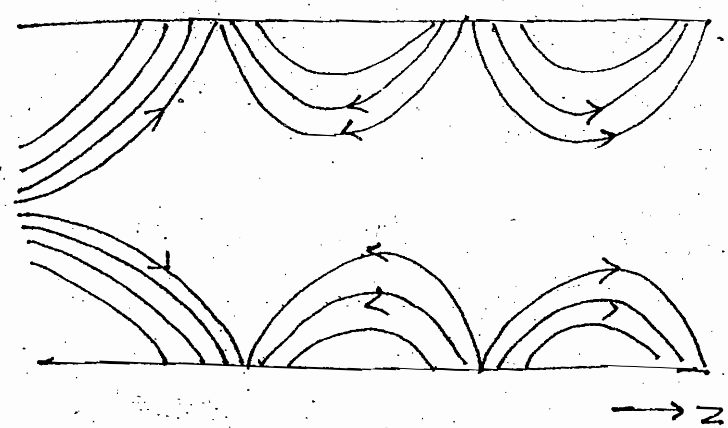
TE_{20}

TM Waves (E_x, E_y, E_z) :-

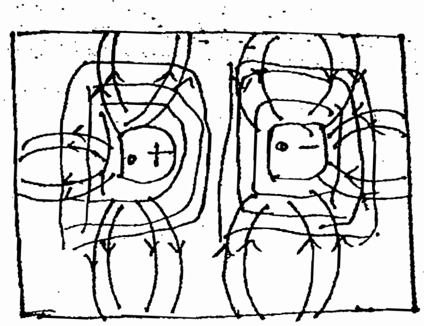


~~TE~~

TM_{11}



TM_{21} :-



TM_{21}

10K

With a guide axis out of board and both E & H fields traced on the board the wave is called as TEM waves else TE or TM

eg:- Co-axial cable and all other low frequency Transmission line. So TEM waves is low frequency energy transfer format

TM/TE $\rightarrow$ High frequency transmission waveguide

<u>Workbook:-</u>

5. $$\frac{1}{\lambda^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}$$

$$\frac{3\times10^8}{2.5\times10^9} = 12\,cm$$

$TE_{10} \rightarrow 2a = 20\,cm$

$\lambda_g =$

6. $$f_c = \frac{1\times3\times10^8}{\sqrt{4}\times2\times3\times10^{-2}}$$

$$= 2.5\,GH$$

7. $v_p > c$

3. $$f_c = 10\times10^9 = \frac{1\times3\times10^8}{2\times a}$$

$a = 1.5\,cm$

$a\cdot b > 2$

$a > b$

9. $$\eta_{TE} = \frac{120\pi}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$= \frac{377}{\sqrt{1-\left(\frac{90}{30}\right)^2}}$$

$$= 400\,\Omega$$

$$f_c = \frac{c}{2a} = 10\,GHz$$

10. $E(x,z,t)_y$

$\rightarrow$ Not depends on y

$\Rightarrow n=0$

TE_{m0}

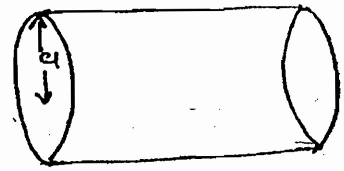
$\sin\left(\frac{2\pi}{a}x\right)$

$\sin(\omega t - \beta z)$

TE_{20}

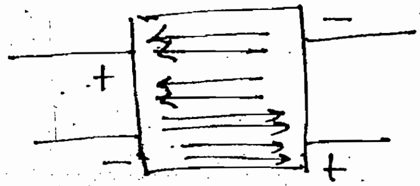
11. TEM exists $\rightarrow$ single conductor

(B)



12. No cut off freq. $\longrightarrow$ TEM exists   Ans - A

13. TE_{02}



14. $f_c = \dfrac{3\times10^8}{2\times 2\times10^{-2}} = \dfrac{3\times10^8}{2\times\sqrt{\epsilon_R}\times1\times10^{-2}} \Rightarrow \epsilon_R = 4$

15 **Note!—** Parallel wave guide

TEM — $(0-\infty)$

TE_1/TM_1 — $(f_{c_1} - \infty)$

TE_2/TM_2 — $(f_{c_2} - \infty)$

15. ① $m=1 \quad n=0$
② $m=0 \quad n=1$
③ $m=1 \quad n=1$

$$\left.\begin{array}{l} \\ \\ \end{array}\right\} f_c = \left(\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}\right)\frac{c}{2}$$

4x3 cm

① $\longrightarrow$ TE_{10} $\quad f_{c_1} = \dfrac{c}{2a} = 3.75\ GHz \quad (3.75 \rightarrow \infty)$ GHz

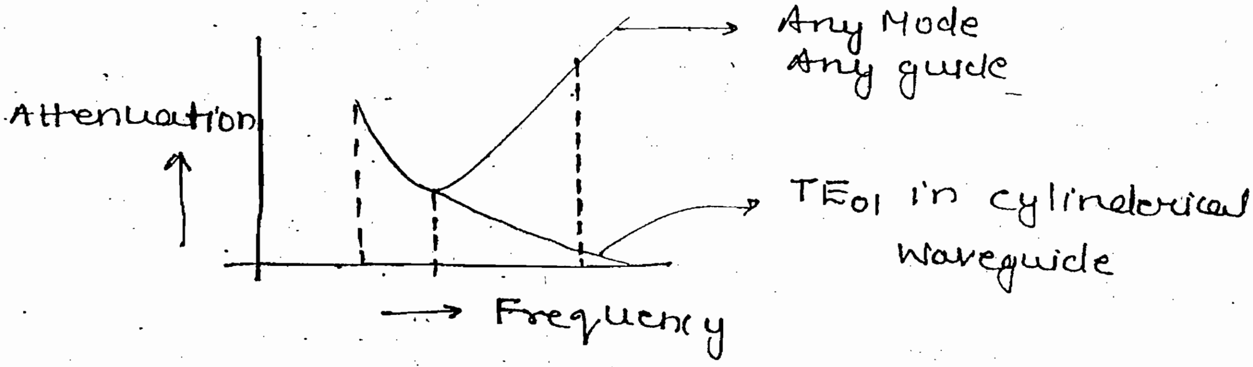
TE_{01} $\quad f_{c_2} = \dfrac{c}{2b} = 5\ GHz \quad (5\ GHz \rightarrow \infty$

$\dfrac{TE_{11}}{TM_{11}}$ $\quad f_{c_3} = \quad 6.25\ GHz \quad (6.25\ GHz \rightarrow \infty)$

From $(3.75-5)$ GHz there is strictly single mode operation.

Note:-

Practically Graph :-



With practical non-ideal conducting walls, higher freq. are not preferred in lower modes

16. → D

17. → TE_{10} only

$$\frac{c}{2a} < \frac{c}{\lambda} < \frac{c}{2b}$$

$$\underset{TE_{10}}{f_c} < f < \underset{TE_{01}}{f_c}$$

$$2a > \lambda > 2b$$