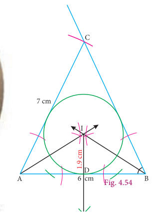




4 GEOMETRY

Inspiration is needed in geometry just as much as in poetry.

- Alexander Pushkin

Thales
(BC (BCE) 624 – 546)

Thales (Pronounced THAYLEES) was born in the Greek city of Miletus. He was known for theoretical and practical understanding of geometry, especially triangles. He used geometry to solve many problems such as calculating the height of pyramids and the distance of ships from the sea shore. He was one of the so-called Seven Sagas or Seven Wise Men of Greece and many regarded him as the first philosopher in the western tradition.

Learning Outcomes

- To understand theorems on linear pairs and vertically opposite angles.
- To understand the angle sum property of triangle.
- To understand the properties of quadrilaterals and use them in problem solving.
- To understand, interpret and apply theorems on the chords and the angles subtended by arcs of a circle
- To understand, interpret and apply theorems on the cyclic quadrilaterals.
- To construct and locate centroid, orthocentre, circumcentre and incentre of a triangle.

4.1 Introduction

In geometry, we study **shapes**. But what is there to *study* in shapes, you may ask. Think first, what are all the things we do with shapes? We draw shapes, we compare shapes, we *measure* shapes. *What* do we measure in shapes?

Take some shapes like this:

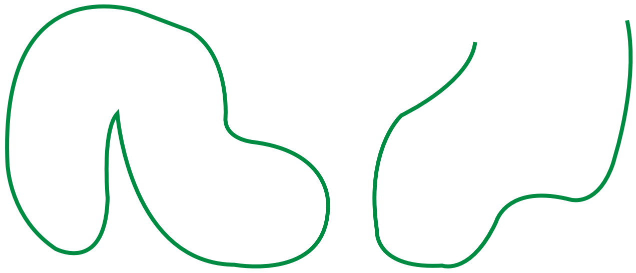


Fig. 4.1

In both of them, there is a *curve* forming the shape: one is a closed curve, enclosing a region, and the other is an open curve. We can use a rope (or a thick string) to measure the **length** of the open curve and the length of the boundary of the region in the case of the closed curve.

Curves are tricky, aren't they? It is so much easier to measure length of straight lines using the scale, isn't it? Consider the two shapes below.

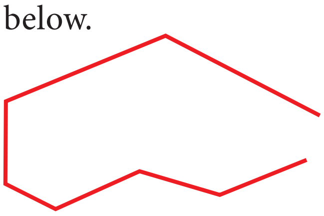


Fig. 4.2

Now we are going to focus our attention only on shapes made up of straight lines and on closed figures. As you will see, there is plenty of interesting things to do. Fig.4.2 shows an open figure.

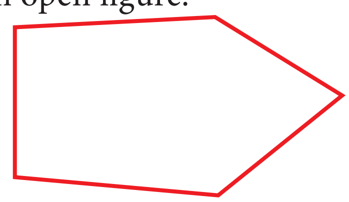
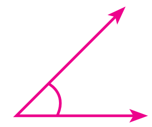


Fig. 4.3

We not only want to draw such shapes, we want to compare them, measure them and do much more. For doing so, we want to **describe** them. How would you describe these closed shapes? (See Fig 4.3) They are all made up of straight lines and are closed.

4.2 Types of Angles-Recall

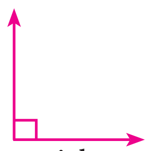
Plumbers measure the angle between connecting pipes to make a good fitting. Wood workers adjust their saw blades to cut wood at the correct angle. Air Traffic Controllers (ATC) use angles to direct planes. Carom and billiards players must know their angles to plan their shots. An angle is formed by two rays that share a common end point provided that the two rays are non-collinear.



acute

an angle that is less than 90°

Fig. 4.4



right

an angle that is exactly 90°

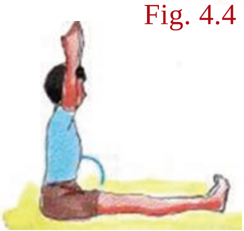
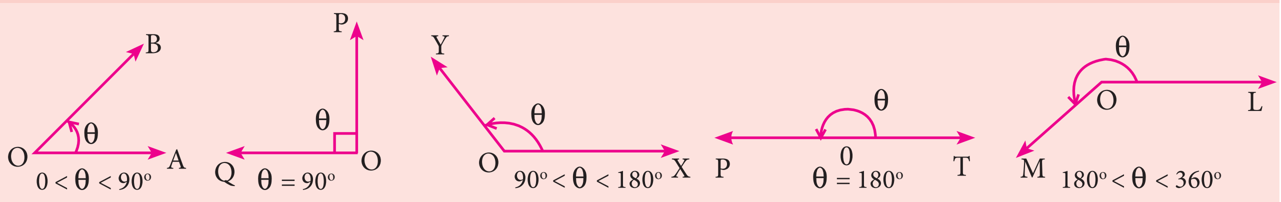


Fig. 4.5

Acute Angle	Right Angle	Obtuse Angle	Straight Angle	Reflex Angle



Complementary Angles

Two angles are Complementary if their sum is 90°. For example, if $\angle ABC$=64° and $\angle DEF$=26°, then angles $\angle ABC$ and $\angle DEF$ are complementary to each other because $\angle ABC + \angle DEF = 90°$

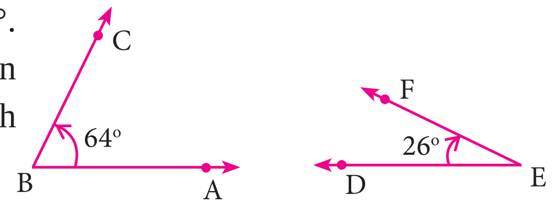


Fig. 4.6

Supplementary Angles

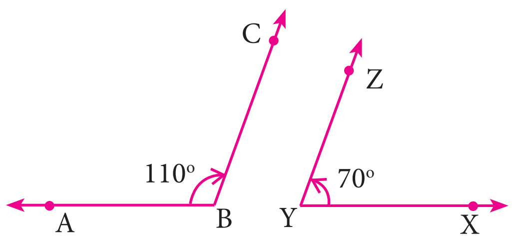


Fig. 4.7

Two angles are Supplementary if their sum is 180°. For example if $\angle ABC$=110° and $\angle XYZ$=70°

Here $\angle ABC + \angle XYZ = 180°$

$\therefore \angle ABC$ and $\angle XYZ$ are supplementary to each other

Adjacent Angles

Two angles are called adjacent angles if

(i) They have a common vertex.

(ii) They have a common arm.

(iii) The common arm lies between the two non-common arms.

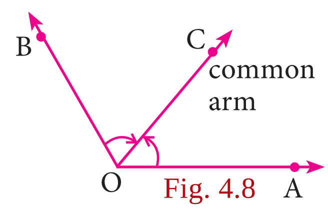


Fig. 4.8

Linear Pair of Angles

If a ray stands on a straight line then the sum of two adjacent angle is 180°. We then say that the angles so formed is a linear pair.

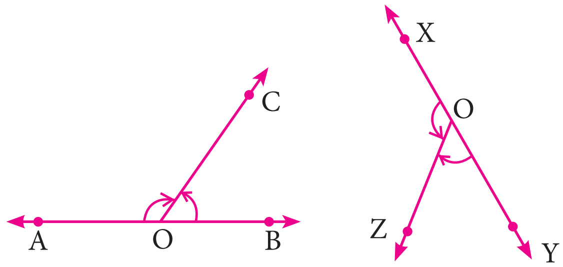


Fig. 4.9

$\angle AOC + \angle BOC$=180°

$\therefore \angle AOC$ and $\angle BOC$ form a linear pair

$\angle XOZ + \angle YOZ = 180°$

$\angle XOZ$ and $\angle YOZ$ form a linear pair

Vertically Opposite Angles

If two lines intersect each other, then vertically opposite angles are equal.

In this figure $\angle POQ = \angle SOR$

$\angle POS = \angle QOR$

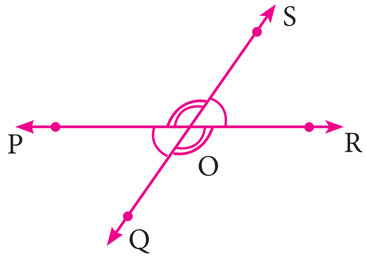


Fig. 4.10

4.2.1 Transversal

A line which intersects two or more lines at distinct points is called a transversal of those lines.

Case (i) When a transversal intersect two lines, we get eight angles.

In the figure the line *l* is the transversal for the lines *m* and *n*

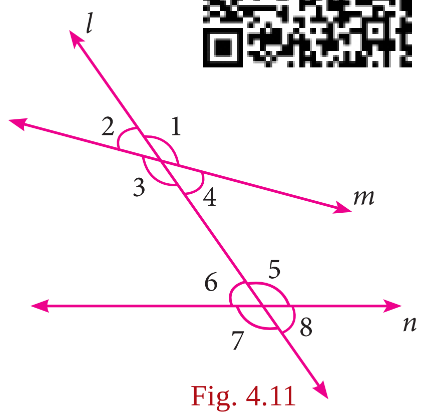


Fig. 4.11

(i) Corresponding Angles: ∠1 and ∠5, ∠2 and ∠6, ∠3 and ∠7, ∠4 and ∠8

(ii) Alternate Interior Angles: ∠4 and ∠6, ∠3 and ∠5

(iii) Alternate Exterior Angles: ∠1 and ∠7, ∠2 and ∠8

(iv) ∠4 and ∠5,∠3 and ∠6 are interior angles on the same side of the transversal.

(v) ∠1 and ∠8, ∠2 and ∠7 are exterior angles on the same side of the transversal.

Case (ii) If a transversal intersects two parallel lines. The transversal forms different pairs of angles.

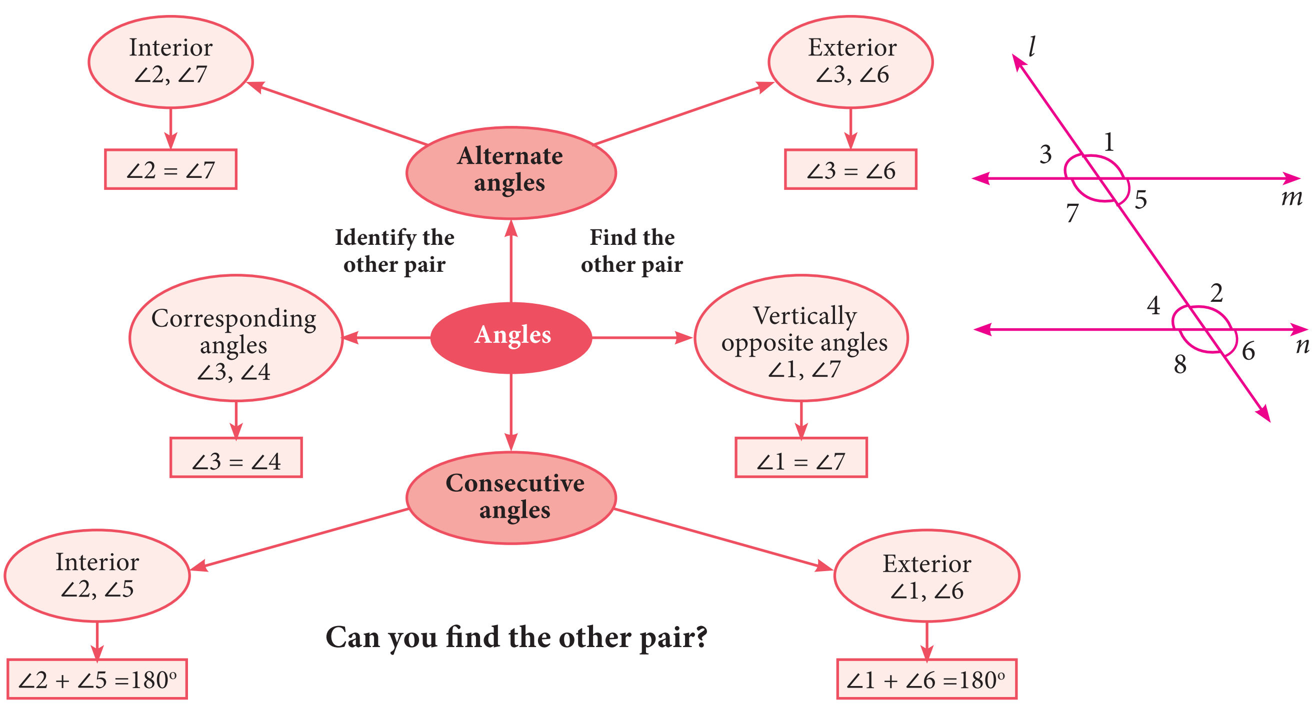


Fig. 4.12

4.2.2 Triangles

Activity 1

1. Take three different colour sheets; place one over the other and draw a triangle on the top sheet. Cut the sheets to get triangles of different colour which are identical. Mark the vertices and the angles as shown. Place the interior angles ∠1, ∠2 and ∠3 on a straight line, adjacent to each other, without leaving any gap. What can you say about the total measure of the three angles ∠1, ∠2 and ∠3?

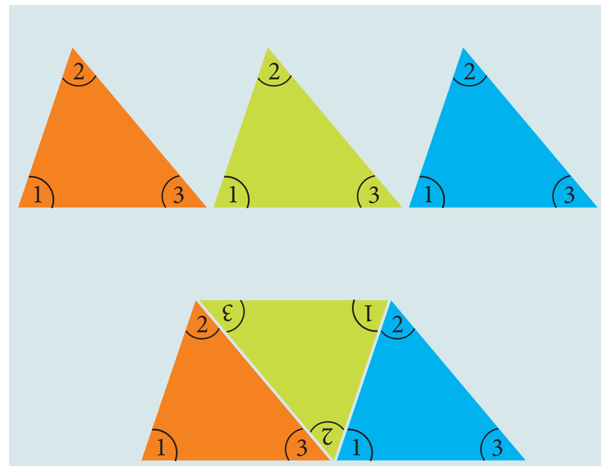


Fig. 4.13

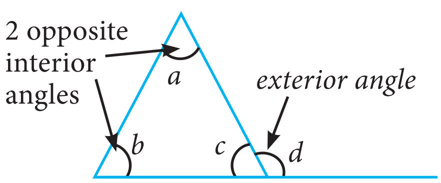


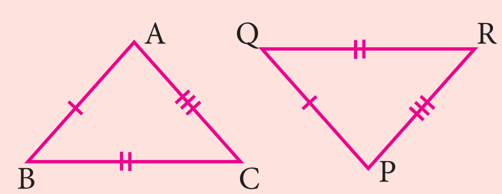
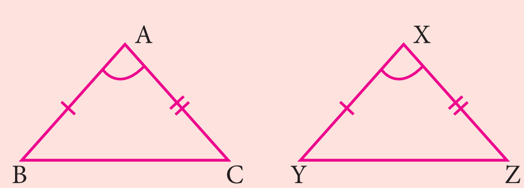
Fig. 4.14

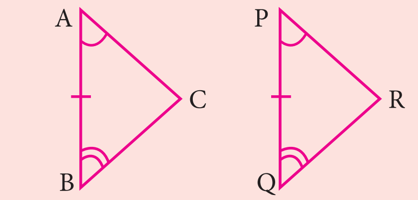
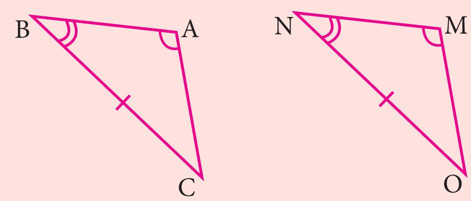
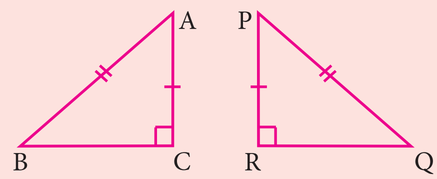
Can you use the same figure to explain the "**Exterior angle property**" of a triangle?

If a side of a triangle is stretched, the exterior angle so formed is equal to the sum of the two interior opposite angles. That is $d=a+b$ (see Fig 4.14)

4.2.3 Congruent Triangles

Two triangles are congruent if the sides and angles of one triangle are equal to the corresponding sides and angles of another triangle.

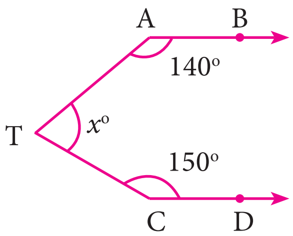
Rule	Diagrams	Reason
SSS		$AB = PQ$ $BC = QR$ $AC = PR$ $\Delta ABC \cong \Delta PQR$
SAS		$AB = XY$ $\angle BAC = \angle YXZ$ $AC = XZ$ $\Delta ABC \cong \Delta XYZ$

ASA		∠A = ∠P AB = PQ ∠B = ∠Q ΔABC ≅ ΔPQR
AAS		∠A = ∠M ∠B = ∠N BC = NO ΔABC ≅ ΔMNO
RHS		∠ACB = ∠PRQ = 90°(R) AB = PQ hypotenuse (H) AC = PR (S) ΔABC ≅ ΔPQR

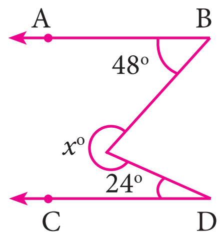
Exercise 4.1

1. In the figure, *AB* is parallel to *CD*, find x

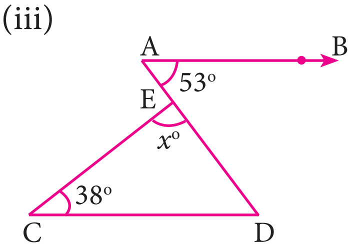
(i)



(ii)



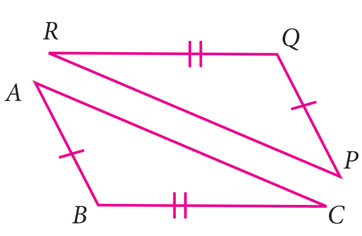
(iii)



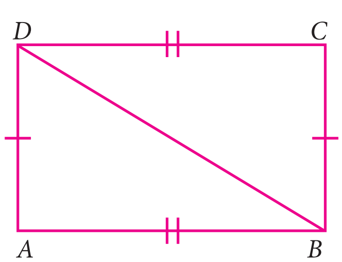
2. The angles of a triangle are in the ratio 1: 2 : 3, find the measure of each angle of the triangle.

3. Consider the given pairs of triangles and say whether each pair is that of congruent triangles. If the triangles are congruent, say 'how'; if they are not congruent say 'why' and also say if a small modification would make them congruent:

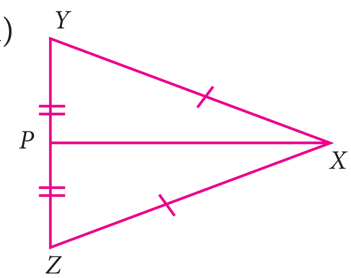
(i)



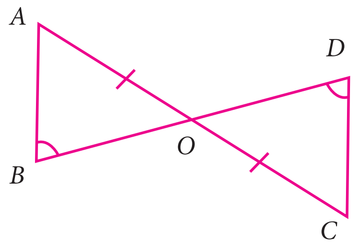
(ii)



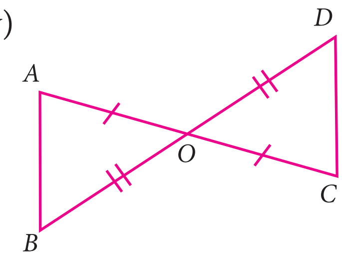
(iii)



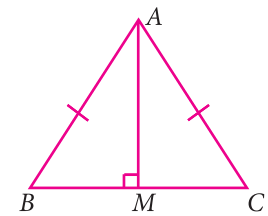
(iv)



(v)



(vi)

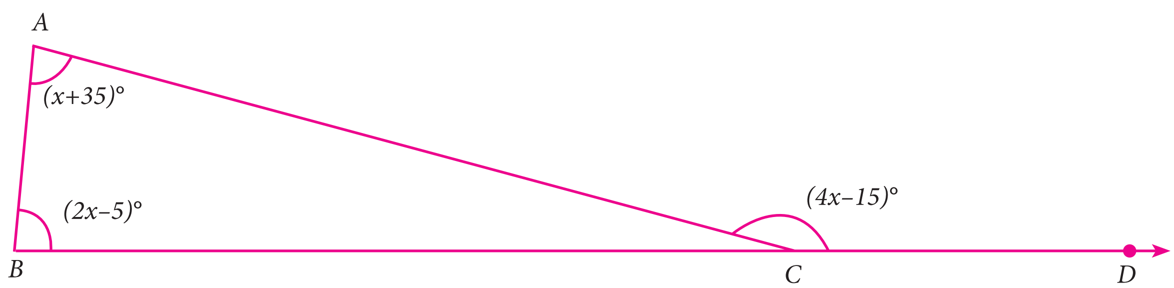


4. ΔABC and ΔDEF are two triangles in which

$AB=DF$, $\angle ACB=70°$, $\angle ABC=60°$; $\angle DEF=70°$ and $\angle EDF=60°$.

Prove that the triangles are congruent.

5. Find all the three angles of the ΔABC



4.3 Quadrilaterals

Activity 2

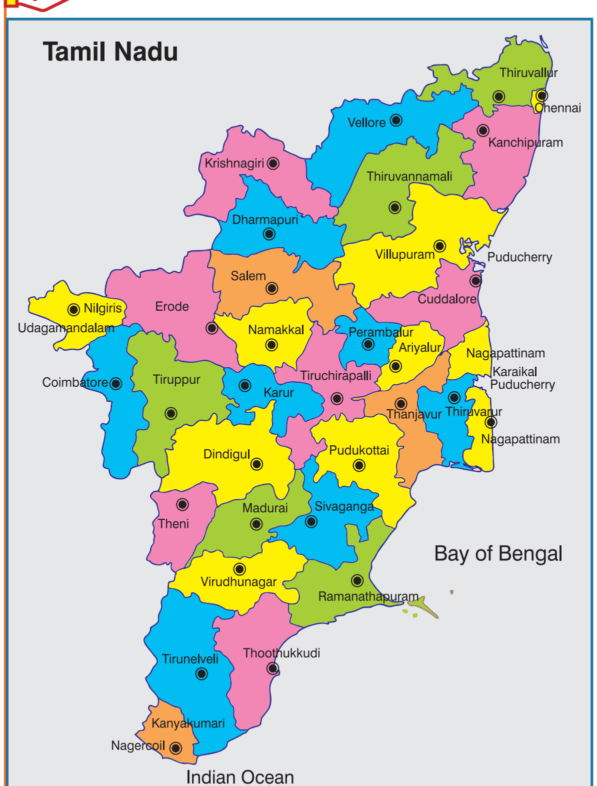


Fig. 4.15

Four Tamil Nadu State Transport buses take the following routes. The first is a one-way journey, and the rest are round trips. Find the places on the map, put points on them and connect them by lines to draw the routes. The places connecting four different routes are given as follows.

(i) Nagercoil, Tirunelveli, Virudhunagar, Madurai

(ii) Sivagangai, Puthukottai, Thanjavur, Dindigul, Sivagangai

(iii) Erode, Coimbatore, Dharmapuri, Karur, Erode

(iv) Chennai, Cuddalore, Krishnagiri, Vellore, Chennai

You will get the following shapes.

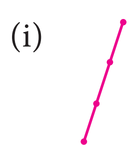


Fig. 4.16

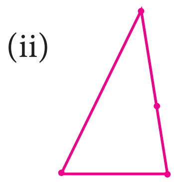


Fig. 4.17

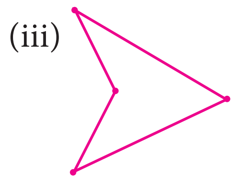


Fig. 4.18

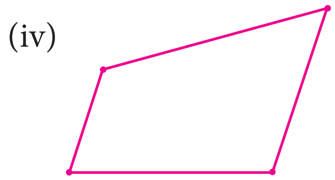


Fig. 4.19

Label the vertices with city names, draw the shapes exactly as they are shown on the map without rotations.

We observe that the first is a single line, the four points are collinear. The other three are closed shapes made of straight lines, of the kind we have seen before. We need names to call such closed shapes, we will call them **polygons** from now on.

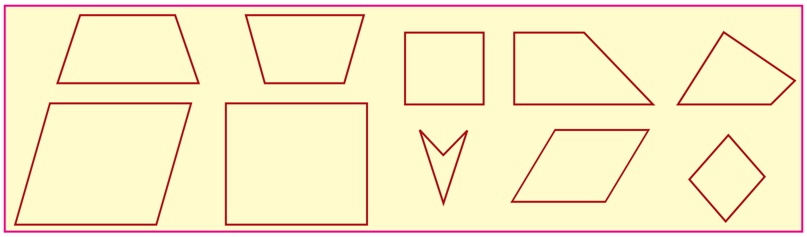


Fig. 4.20

How do polygons look? They have sides, with points at either end. We call these points as **vertices** of the polygon. The sides are line segments joining the vertices. The word *poly* stands for many, and a polygon is a many-sided figure.

Note

Concave polygon: Polygon having any one of the interior angle greater than 180°
Convex Polygon: Polygon having each interior angle less than 180°
(Diagonals should be inside the polygon)

How many sides can a polygon have? One? But that is just a line segment. Two ? But how can you get a closed shape with two sides? Three? Yes, and this is what we know as a triangle. Four sides?

Squares and rectangles are examples of polygons with 4 sides but they are not the only ones. Here (Fig. 4.20) are some examples of 4-sided polygons. We call them ***quadrilaterals.***

4.3.1 Special Names for Some Quadrilaterals

1. A **parallelogram** is a quadrilateral in which opposite sides are parallel and equal.
2. A **rhombus** is a quadrilateral in which opposite sides are parallel and all sides are equal.
3. A **trapezium** is a quadrilateral in which *one pair of* opposite sides are parallel.

Draw a few parallellograms, a few rhombuses (correctly called rhombii, like cactus and cactii) and a few trapeziums (correctly written trapezia).

Fig. 4.21

The great advantage of knowing properties of quadrilateral is that we can see the relationships among them immediately.

- Every parallelogram is a trapezium, but not necessarily the other way.
- Every rhombus is a parallelogram, but not necessarily the other way.
- Every rectangle is a parallelogram, but not necessarily the other way.
- Every square is a rhombus and hence every square is a parallelogram as well.

For "not necessarily the other way" mathematicians usually say "the converse is not true". A smart question then is: just *when* is the other way also true? For instance, when is a parallelogram also a rectangle? Any parallelogram in which all angles are also equal is a rectangle. (Do you see why?) Now we can observe many more interesting properties. For instance, we see that a rhombus is a parallelogram in which all **sides** are also equal.

Note

You know *bi*-cycles and *tri*-cycles? When we attach *bi or tri* to the front of any word, they stand for 2 (bi) or 3 (tri) of them. Similarly *quadri* stands for 4 of them. We should really speak of quadri-cycles also, but we don't. *Lateral* stands for sideways, thus quadrilateral means a 4-sided figure. You know *trilaterals*; they are also called triangles !
After 4 ? We have: 5 – *penta*, 6 – *hexa*, 7 – *hepta*, 8 – *octa*, 9 – *nano*, 10 – *deca*. Conventions are made by history. Trigons are called triangles, quadrigons are called quadrilaterals. Continuing in th same way we get pentagons, hexagons, heptagons, octagons, nanogons and decagons. Beyond these, we have 11-gons, 12-gons etc. Perhaps you can draw a 23-gon !

4.3.2 More Special Names

When all sides of a quadrilateral are equal, we call it **equilateral**. When all angles of a quadrilateral are equal, we call it **equiangular**. In triangles, we talked of equilateral triangles as those with all sides equal. Now we can call them equiangular triangles as well!

We thus have:

A rhombus is an equilateral parallelogram.
A rectangle is an equiangular parallelogram.
A square is an equilateral and equiangular parallelogram.

Here are two more special quadrilaterals, called **kite** and **isosceles trapezium**.

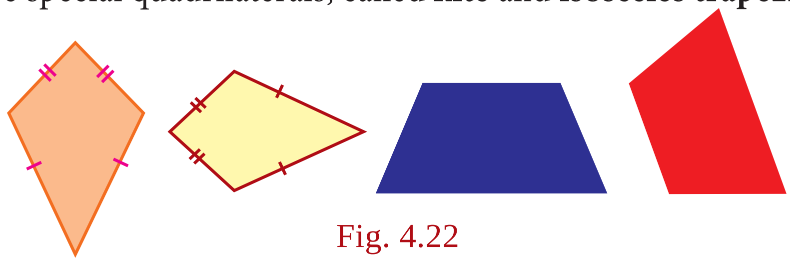


Fig. 4.22

4.3.3 Types of Quadrilaterals

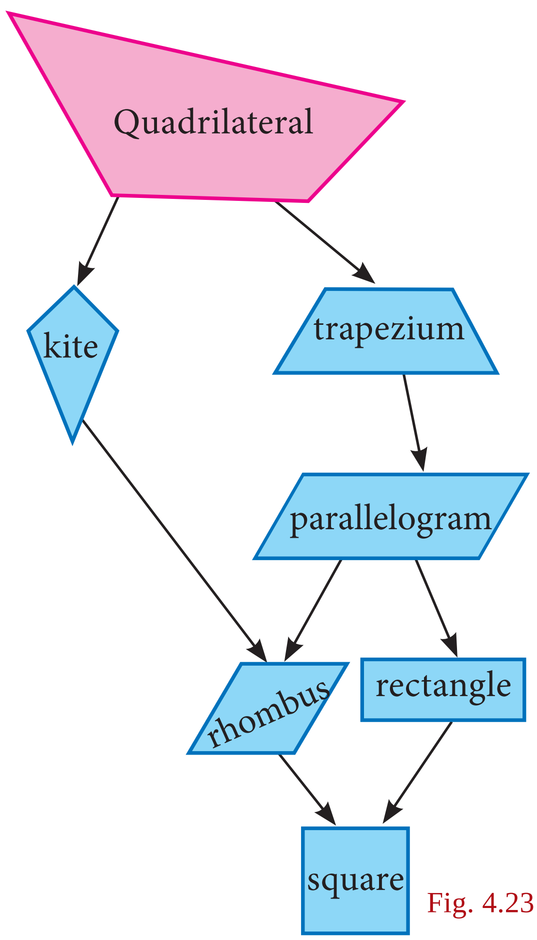


Fig. 4.23

Progress Check

Answer the following question.

(i) Are the opposite angles of a rhombus equal?

(ii) A quadrilateral is a ______________ if a pair of opposite sides are equal and parallel.

(iii) Are the opposite sides of a kite equal?

(iv) Which is an equiangular but not an equilateral parallelogram?

(v) Which is an equilateral but not an equiangular parallelogram?

(vi) Which is an equilateral and equiangular parallelogram?

(vii) _________ is a rectangle, a rhombus and a parallelogram.

Activity 3

Step – 1: Cut out four different quadrilaterals from coloured glazed papers.

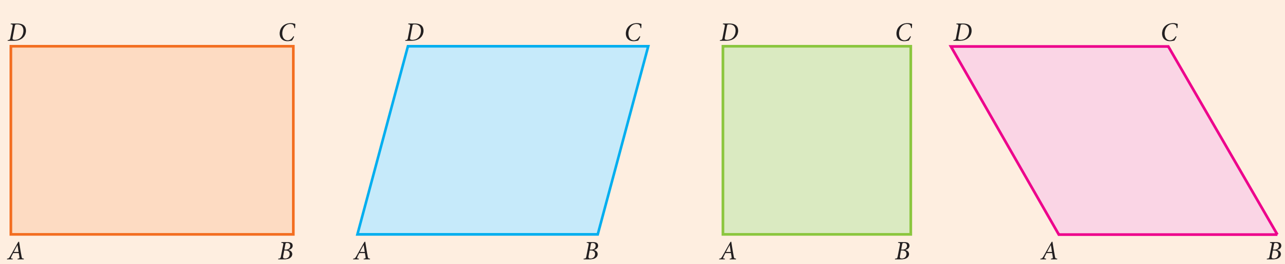


Fig. 4.24

Step – 2: Fold the quadrilaterals along their respective diagonals. Press to make creases. Here, dotted line represent the creases.

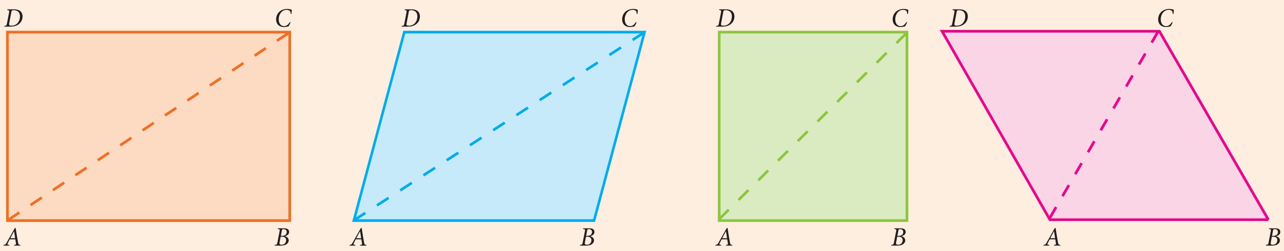


Fig. 4.25

Step – 3: Fold the quadrilaterals along both of their diagonals. Press to make creases.

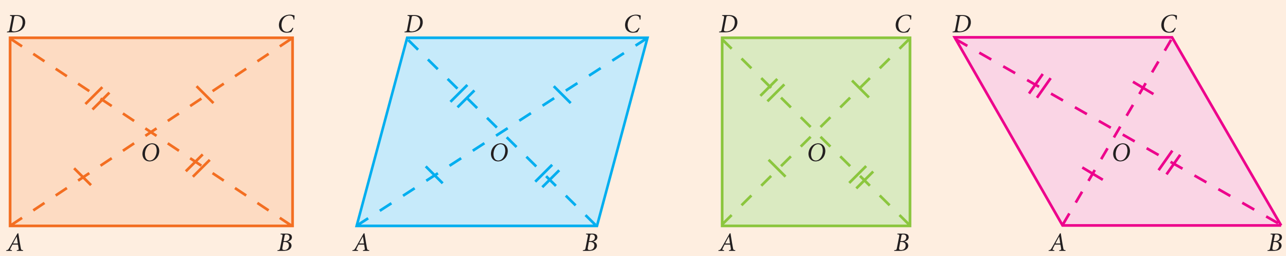


Fig. 4.26

We observe that two imposed triangles are congruent to each other. Measure the lengths of portions of diagonals and angles between the diagonals.

Also do the same for the quadrilaterals such as Trapezium, Isosceles Trapezium and Kite.

From the above activity, measure the lengths of diagonals and angles between the diagonals and record them in the table below:

S. No.	Name of the quadrilateral	Length along diagonals						Measure of angles			
		AC	*BD*	*OA*	*OB*	*OC*	*OD*	$\angle AOB$	$\angle BOC$	$\angle COD$	$\angle DOA$
1	Trapezium										
2	Isosceles Trapezium										
3	Parallelogram										
4	Rectangle										
5	Rhombus										
6	Square										
7	Kite										

Activity 4

Angle sum for a polygon

Draw any quadrilateral *ABCD*.

Mark a point *P* in its interior.

Join the segments *PA*, *PB*, *PC* and *PD*.

You have 4 triangles now.

How much is the sum of all the angles of the 4 triangles?

How much is the sum of the angles at P?

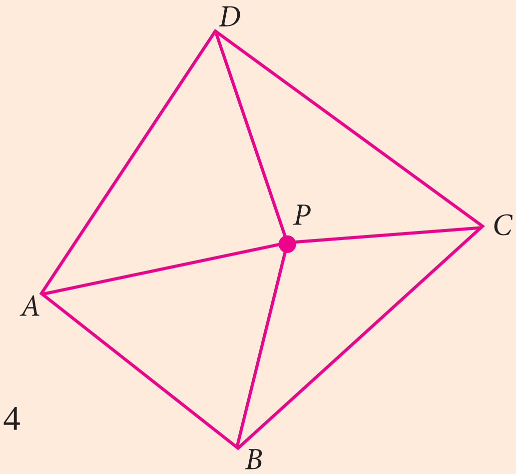


Fig. 4.27

Can you now find the 'angle sum' of the quadrilateral *ABCD*?

Can you extend this idea to any polygon?

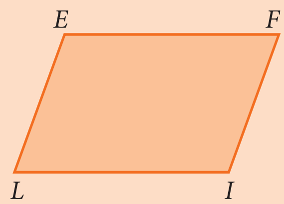
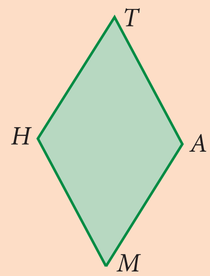
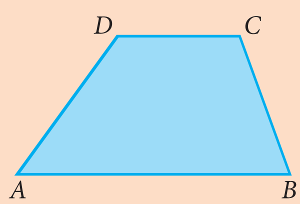
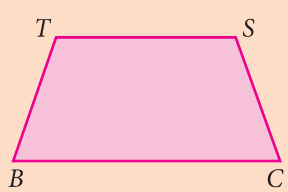
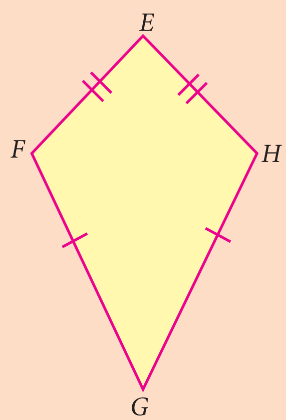
Thinking Corner

1. If there is a polygon of n sides ($n \geq 3$), then the sum of all interior angles is $(n-2) \times 180°$

2. For the regular polygon (All the sides of a polygon are equal in size)

   - Each interior angle is $\frac{(n-2)}{n} \times 180°$
   - Each exterior angle is $\frac{360°}{n}$
   - The sum of all the exterior angles formed by producing the sides of a convex polygon in the order is 360° .
   - If a polygon has 'n' sides , then the number of diagonals of the polygon is $\frac{n(n-3)}{2}$

4.3.4 Properties of Quadrilaterals

Name	Diagram	Sides	Angles	Diagonals
Parallelogram		Opposite sides are parallel and equal	Opposite angles are equal and sum of any two adjacent angles is 180°	Diagonals bisect each other.
Rhombus		All sides are equal and opposite sides are parallel	Opposite angles are equal and sum of any two adjacent angles is 180°	Diagonals bisect each other at right angle.
Trapezium		One pair of opposite sides are parallel	The angles at the ends of each non-parallel sides are supplementary	Diagonals need not be equal
Isosceles Trapezium		One pair of opposite sides are parallel and non-parallel sides are equal in length.	The angles at the ends of each parallel sides are equal.	Diagonals are of equal length.
Kite		Two pairs of adjacent sides are equal	One pair of opposite angles are equal	1. Diagonals intersect at right angle. 2. Shorter diagonal bisected by longer diagonal 3. Longer diagonal divides the kite into two congruent triangles

Note

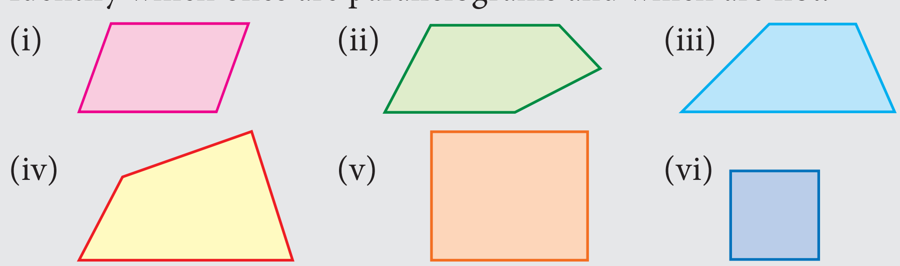
(i) A rectangle is an equiangular parallelogram.
(ii) A rhombus is an equilateral parallelogram.
(iii) A square is an equilateral and equiangular parallelogram.
(iv) A square is a rectangle , a rhombus and a parallelogram.

Progress Check

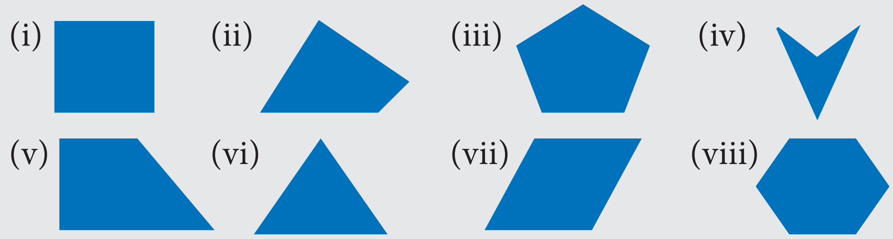
1. State the reasons for the following.
   (i) A square is a special kind of a rectangle.
   (ii) A rhombus is a special kind of a parallelogram.
   (iii) A rhombus and a kite have one common property.
   (iv) A square and a rhombus have one common property.

2. What type of quadrilateral is formed when the following pairs of congruent triangles are joined together?
   (i) Equilateral triangle.
   (ii) Right angled triangle.
   (iii) Isosceles triangle.

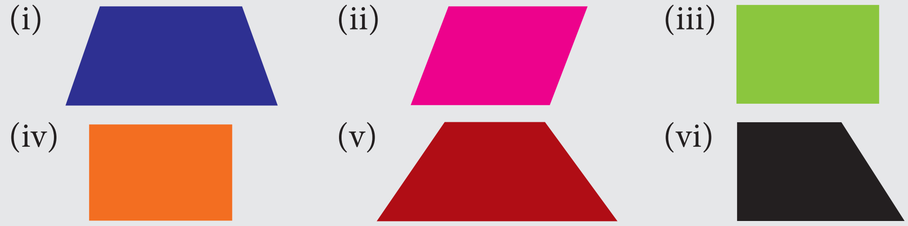
3. Identify which ones are parallelograms and which are not.



4. Which ones are not quadrilaterals?



5. Identify which ones are trapeziums and which are not.



4.3.5 Properties of Parallelogram

We can now embark on an interesting journey. We can tour among lots of quadrilaterals, noting down interesting properties. What properties do we look for, and how do we know they are true?

For instance, opposite sides of a parallelogram are parallel, but are they also **equal**? We could draw any number of parallelograms and verify whether this is true or not. In fact, we see that opposite sides are equal in **all of** them. Can we then conclude that opposite sides are equal in *all* parallelograms? No, because we might later find a parallelogram, one which we had not thought of until then, in which opposite sides are unequal. So, we need an argument, a **proof**.

Consider the parallelogram $ABCD$ in the given Fig. 4.28. We believe that $AB = CD$ and $AD = BC$, but how can we be sure? We know triangles and their properties. So we can try and see if we can use that knowledge. But we don't have any triangles in the parallelogram $ABCD$.

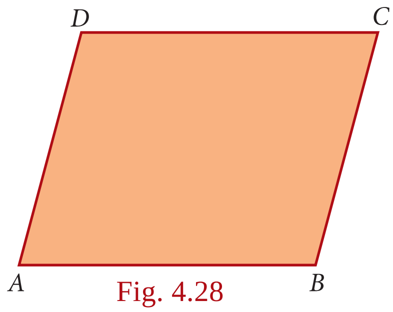


Fig. 4.28

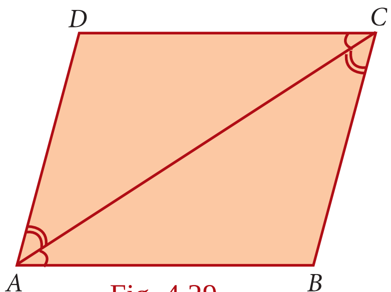


Fig. 4.29

This is easily taken care of by joining AC. (We could equally well have joined BD, but let it be AC for now.) We now have 2 triangles ADC and ABC with a common side AC. If we could somehow prove that these two triangles are congruent, we would get $AB = CD$ and $AD = BC$, which is what we want!

Is there any hope of proving that ΔADC and ΔABC are congruent? There are many criteria for congruence, it is not clear which one is relevant here.

So far we have not used the fact that $ABCD$ is a parallelogram at all. So we need to use the facts that $AB \parallel DC$ and $AD \parallel BC$ to show that ΔADC and ΔABC are congruent. From sides being parallel we have to get into some angles being equal. Do we know any such properties? we do, and that is all about **transversals!**

Now we can see it clearly. $AD \parallel BC$ and AC is a transversal, hence $\angle DAC = \angle BCA$. Similarly, $AB \parallel DC$, AC is a transversal, hence $\angle BAC = \angle DCA$. With AC as common side, the ASA criterion tells us that ΔADC and ΔABC are congruent, just what we needed. From this we can conclude that $AB = CD$ and $AD = BC$.

Thus opposite sides are indeed equal in a parallelogram.

The argument we now constructed is written down as a ***formal proof*** in the following manner.

Theorem 1

In a parallelogram, opposite sides are equal

Given $ABCD$ is a parallelogram

To Prove $AB=CD$ and $DA=BC$

Construction Join AC

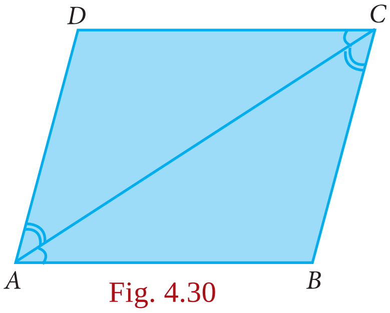


Fig. 4.30

Proof

Since $ABCD$ is a parallelogram

$AD \parallel BC$ and AC is the transversal

$\angle DAC = \angle BCA$ → (1) (alternate angles are equal)

$AB \parallel DC$ and AC is the transversal

$\angle BAC = \angle DCA$ → (2) (alternate angles are equal)

In ΔADC and ΔCBA

$\angle DAC = \angle BCA$ from (1)

AC is common

$\angle DCA = \angle BAC$ from (2)

$\Delta ADC \cong \Delta CBA$ (By ASA)

Hence $AD = CB$ and $DC = BA$ (Corresponding sides are equal)

Along the way in the proof above, we have proved another property that is worth recording as a theorem.

Theorem 2

A diagonal of a parallelogram divides it into two congruent triangles.

Notice that the proof above established that $\angle DAC = \angle BCA$ and $\angle BAC = \angle DCA$. Hence we also have, in the figure above,

$\angle BCA + \angle BAC = \angle DCA + \angle DAC$

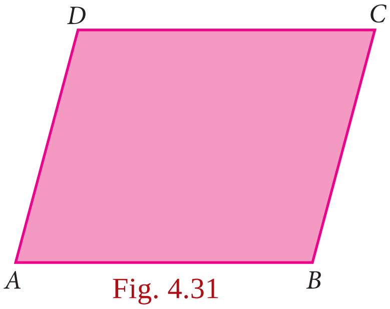


Fig. 4.31

But we know that:

$\angle B + \angle BCA + \angle BAC = 180$

and $\angle D + \angle DCA + \angle DAC = 180$

Therefore we must have that $\angle B = \angle D$.

With a little bit of work, proceeding similarly, we could have shown that $\angle A = \angle C$ as well. Thus we have managed to prove the following theorem:

Theorem 3

The opposite angles of a parallelogram are equal.

Now that we see congruence of triangles as a good "strategy", we can look for more triangles. Consider both diagonals AC and DB. We already know that ΔADC and ΔCBA are congruent. By a similar argument we can show that ΔDAB and ΔBCD are congruent as well. Are there more congruent triangles to be found in this figure ?

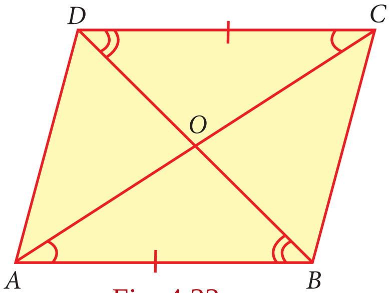


Fig. 4.32

Yes. The two diagonals intersect at point O. We now see 4 new ΔAOB, ΔBOC, ΔCOD and ΔDOA. Can you see any congruent pairs among them?

Since AB and CD are parallel and equal, one good guess is that ΔAOB and ΔCOD are congruent. We could again try the ASA crierion, in which case we want $\angle OAB = \angle OCD$ and $\angle ABO = \angle CDO$. But the first of these follows from the fact that $\angle CAB = \angle ACD$ (which we already established) and observing that $\angle CAB$ and $\angle OAB$ are the same (and so also $\angle OCD$ and $\angle ACD$). We now use the fact that BD is a transversal to get that $\angle ABD = \angle CDB$, but then $\angle ABD$ is the same as $\angle ABO$, $\angle CDB$ is the same as $\angle CDO$, and we are done.

Again, we need to write down the formal proof, and we have another theorem.

Theorem 4

The diagonals of a parallelogram bisect each other.

It is time now to reinforce our concepts on parallelograms. Consider each of the given statements, in the adjacent box, one by one. Identifiy the type of parallelogram which satisfies each of the statements. Support your answer with reason.

- Each pair of its opposite sides are parallel.
- Each pair of opposite sides is equal.
- All of its angles are right angles.
- Its diagonals bisect each other.
- The diagonals are equal.
- The diagonals are perpendicular and equal.
- The diagonals are perpendicular bisectors of each other.
- Each pair of its consecutive angles is supplementary.

Now we begin with lots of interesting properties of parallelograms. Can we try and prove some property relating to two or more parallelograms ? A simple case to try is when two parallelograms share the same base, as in Fig.4.33

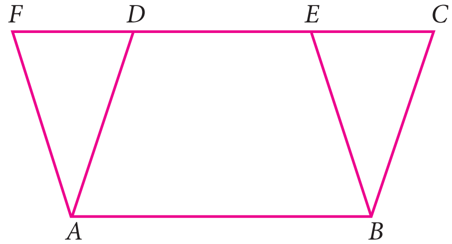


Fig. 4.33

We see parallelograms $ABCD$ and $ABEF$ are on the common base AB. At once we can see a pair of triangles for being congruent ΔADF and ΔBCE. We already have that $AD = BC$ and $AF = BE$. But then since $AD \parallel BC$ and $AF \parallel BE$, the angle formed by AD and AF must be the same as the angle formed by BC tand BE. Therefore $\angle DAF = \angle CBE$. Thus ΔADF and ΔBCE are congruent.

That is an interesting observation; can we infer anything more from this ? Yes, we know that congruent triangles have the *same area*. This makes us think about the areas of the parallelograms $ABCD$ and $ABEF$.

$$\begin{aligned}\text{Area of } ABCD &= \text{area of quadrilateral } ABED + \text{area of } \Delta BCE \\ &= \text{area of quadrilateral } ABED + \text{area of } \Delta ADF \\ &= \text{area of } ABEF\end{aligned}$$

Thus we have proved another interesting theorem:

Theorem 5:

Parallelograms on the same base and between the same parallels are equal in area.

In this process, we have also proved other interesting statements. These are called *Corollaries*, which do not need separate detailed proofs.

Corollary 1: Triangles on the same base and between the same parallels are equal in area.

Corollary 2: A rectangle and a parallelogram on the same base and between the same parallels are equal in area.

These statements that we called Theorems and Corollaries, hold for all parallelograms, however large or small, with whatever be the lengths of sides and angles at vertices.

Example 4.1

In a parallelogram $ABCD$, the bisectors of the consecutive angles $\angle A$ and $\angle B$ insersect at P. Show that $\angle APB = 90°$

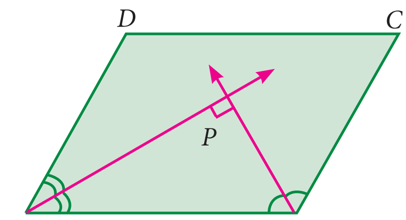


Fig. 4.34

Solution

$ABCD$ is a parallelogram AP and BP are bisectors of consecutive angles $\angle A$ and $\angle B$.

Since the consecutive angles of a parallelogram are supplementary

$$\angle A + \angle B = 180°$$

$$\frac{1}{2}\angle A + \frac{1}{2}\angle B = \frac{180^{\circ}}{2}$$

$$\Longrightarrow \angle PAB + \angle PBA = 90°$$

In ΔAPB,

$$\angle PAB + \angle APB + \angle PBA = 180° \text{ (angle sum property of triangle)}$$

$$\angle APB = 180° - [\angle PAB + \angle PBA]$$

$$= 180° - 90° = 90°$$

Hence Proved.

Example 4.2

In the Fig.4.35 $ABCD$ is a parallelogram, P and Q are the mid-points of sides AB and DC respectively. Show that $APCQ$ is a parallelogram.

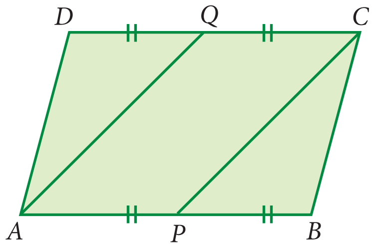


Fig. 4.35

Solution

Since P and Q are the mid points of

AB and DC respectively

Therefore $AP = \frac{1}{2} AB$ and

$QC = \frac{1}{2} DC$ (1)

But $AB = DC$ (Opposite sides of a parallelogram are equal)

$\Longrightarrow \frac{1}{2} AB = \frac{1}{2} DC$

$\Longrightarrow AP = QC$ (2)

Also , $AB \parallel DC$

$\Longrightarrow AP \parallel QC$ (3) [$\because$ $ABCD$ is a parallelogram]

Thus , in quadrilateral $APCQ$ we have $AP = QC$ and $AP \parallel QC$ [from (2) and (3)]

Hence , quadrilateral $APCQ$ is a parallelogram.

Example 4.3

$ABCD$ is a parallelogram Fig.4.36 such that $\angle BAD = 120^\circ$ and AC bisects $\angle BAD$ show that $ABCD$ is a rhombus.

Solution

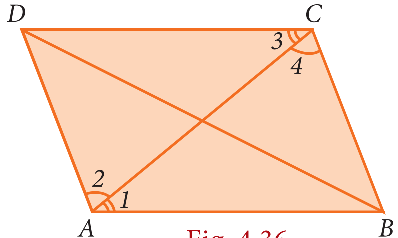


Fig. 4.36

Given $\angle BAD = 120^\circ$ and AC bisects $\angle BAD$

$\angle BAC = \frac{1}{2} \times 120^\circ = 60^\circ$

$\angle 1 = \angle 2 = 60^\circ$

$AD \parallel BC$ and AC is the traversal

$\angle 2 = \angle 4 = 60^\circ$

ΔABC is isosceles triangle [$\because \angle 1 = \angle 4 = 60^\circ$]

$\Longrightarrow AB = BC$

Parallelogram $ABCD$ is a rhombus.

Example 4.4 In a parallelogram *ABCD* , *P* and *Q* are the points on line *DB* such that $PD = BQ$ show that *APCQ* is a parallelogram

Solution

ABCD is a parallelogram.

$OA = OC$ and

$OB = OD$ (∵ Diagonals bisect each other)

now $OB + BQ = OD + DP$

$OQ = OP$ and $OA = OC$

APCQ is a parallelogram.

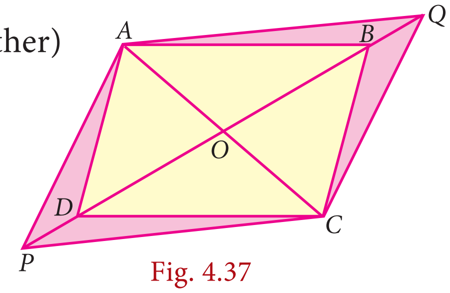


Fig. 4.37

Exercise 4.2

1. The angles of a quadrilateral are in the ratio 2 : 4 : 5 : 7. Find all the angles.
2. In a quadrilateral *ABCD*, $\angle A = 72°$ and $\angle C$ is the supplementary of $\angle A$. The other two angles are $2x–10$ and $x + 4$. Find the value of x and the measure of all the angles.
3. *ABCD* is a rectangle whose diagonals *AC* and *BD* intersect at *O*. If $\angle OAB = 46°$, find $\angle OBC$
4. The lengths of the diagonals of a Rhombus are 12 cm and 16 cm . Find the side of the rhombus.
5. Show that the bisectors of angles of a parallelogram form a rectangle .
6. If a triangle and a parallelogram lie on the same base and between the same parallels, then prove that the area of the triangle is equal to half of the area of parallelogram.
7. Iron rods *a*, *b*, *c*, *d*, *e*, and *f* are making a design in a bridge as shown in the figure. If $a \parallel b$, $c \parallel d$, $e \parallel f$, find the marked angles between
   (i) *b* and *c*
   (ii) *d* and *e*
   (iii *d* and *f*
   (iv) *c* and *f*

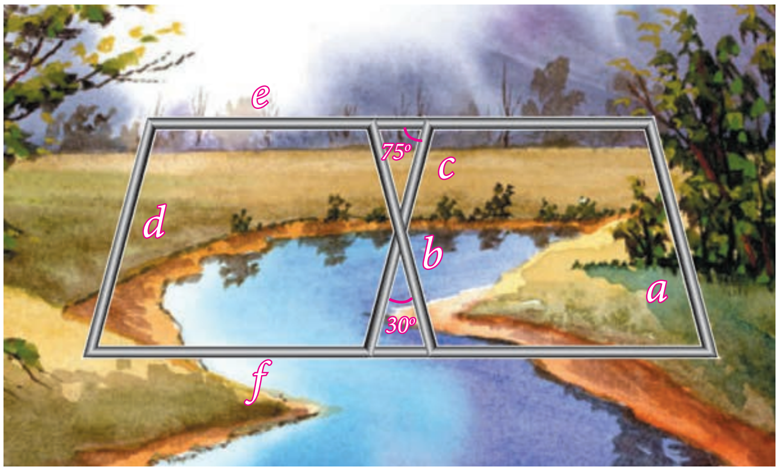


Fig. 4.38

8. In the given Fig. 4.39, $\angle A = 64°$, $\angle ABC = 58°$. If BO and CO are the bisectors of $\angle ABC$ and $\angle ACB$ respectively of ΔABC, find $x°$ and $y°$

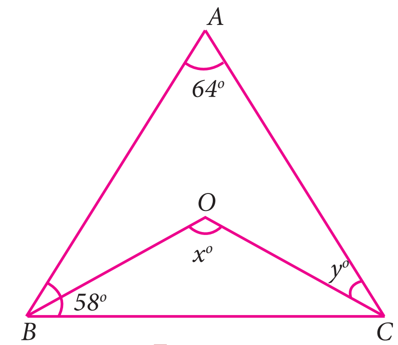


Fig. 4.39

9. In the given Fig. 4.40, if $AB = 2$, $BC = 6$, $AE = 6$, $BF = 8$, $CE = 7$, and $CF = 7$, compute the ratio of the area of quadrilateral $ABDE$ to the area of ΔCDF. (Use congruent property of triangles).

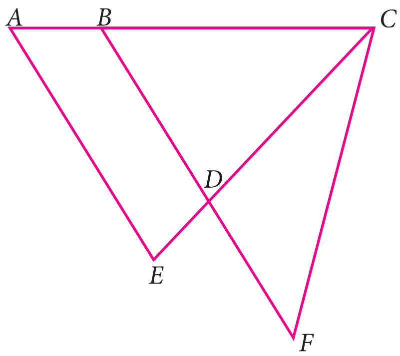


Fig. 4.40

10. In the Fig. 4.41 , $ABCD$ is a rectangle and $EFGH$ is a parallelogram. Using the measurements given in the figure, what is the length d of the segment that is perpendicular to $\overline{HE}$ and $\overline{FG}$?

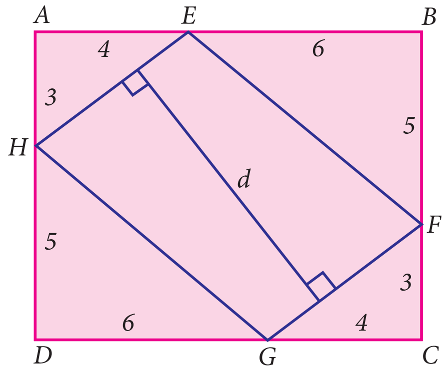


Fig. 4.41

11. In parallelogram $ABCD$ of the accompanying diagram, line DP is drawn bisecting BC at N and meeting AB (extended) at P. From vertex C, line CQ is drawn bisecting side AD at M and meeting AB (extended) at Q. Lines DP and CQ meet at O. Show that the area of triangle QPO is $\frac{9}{8}$ of the area of the parallelogram $ABCD$.

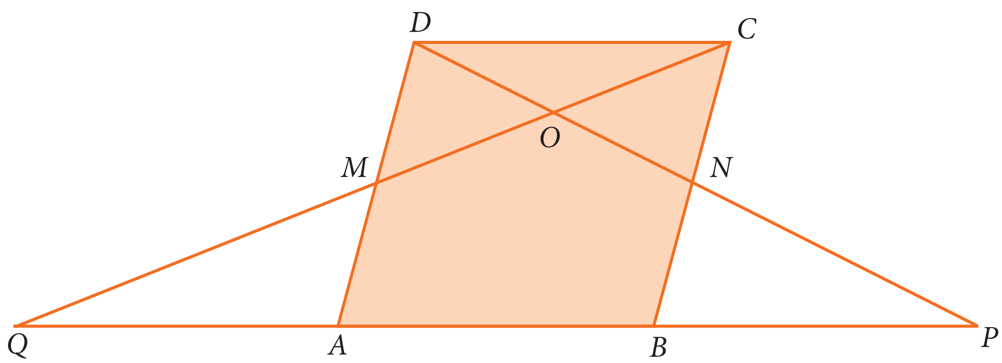


Fig. 4.42

4.4 Parts of a Circle

Circles are geometric shapes you can see all around you. The significance of the concept of a circle can be well understood from the fact that the wheel is one of the ground-breaking inventions in the history of mankind.

Fig. 4.43

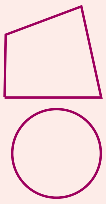
A circle, you can describe, is the set of all points in a plane at a constant distance from a fixed point. The fixed point is the centre of the circle; the constant distance corresponds to a radius of the circle.

A line that cuts the circle in two points is called a secant of the circle.

A line segment whose end points lie on the circle is called a chord of the circle.

Note

A circle notably differs from a polygon. A polygon (for example, a quadrilateral) has edges and corners while, a circle is a 'smooth' curve.



A chord of a circle that has the centre is called a diameter of the circle. The circumference of a circle is its boundary. (We use the term perimeter in the case of polygons).

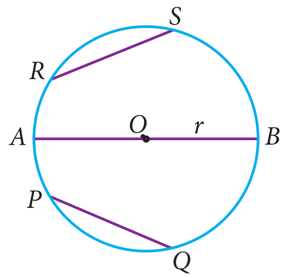


Fig. 4.44

In Fig.4.44, we see that all the line segments meet at two points on the circle. These line segments are called the chords of the circle. So, a line segment joining any two points on the circle is called a chord of the circle. In this figure AB, PQ and RS are the chords of the circle.

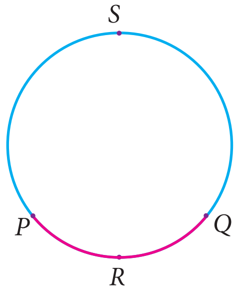


Fig. 4.45

Now place four points P, R, Q and S on the same circle (Fig.4.45), then PRQ and QSP are the continuous parts (sections) of the circle. These parts (sections) are to be denoted by $\overset{\frown}{PRQ}$ and $\overset{\frown}{QSP}$ or simply by $\overset{\frown}{PQ}$ and $\overset{\frown}{QP}$. This continuous part of a circle is called an arc of the circle. Usually the arcs are denoted in anti-clockwise direction.

Now consider the points P and Q in the circle (Fig.4.45). It divides the whole circle into two parts. One is longer and another is shorter. The longer one is called major arc $\overset{\frown}{QP}$ and shorter one is called minor arc $\overset{\frown}{PQ}$.

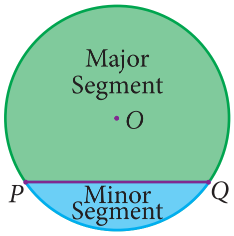


Fig. 4.46

Now in (Fig.4.46), consider the region which is surrounded by the chord PQ and major arc $\overparen{QP}$. This is called the major segment of the circle. In the same way, the segment containing the minor arc and the same chord is called the minor segment.

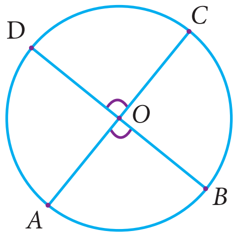


Fig. 4.47

In (Fig.4.47), if two arcs $\overparen{AB}$ and $\overparen{CD}$ of a circle subtend the same angle at the centre, they are said to be congruent arcs and we write,

$$\overparen{AB} \equiv \overparen{CD} \text{ implies } m\overparen{AB} = m\overparen{CD}$$

$$\text{implies } \angle AOB = \angle COD$$

Note

A diameter of a circle is:

- the line segment which bisects the circle.
- the largest chord of a circle.
- a line of symmetry for the circle.
- twice in length of a radius in a circle.

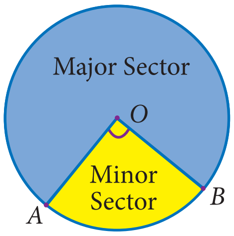


Fig. 4.48

Now, let us observe (Fig.4.48). Is there any special name for the region surrounded by two radii and arc? Yes, its name is sector. Like segment, we find that the minor arc corresponds to the minor sector and the major arc corresponds to the major sector.

Concentric Circles

Circles with the same centre but different radii are said to be concentric.

Here are some real-life examples:

An Archery target

A carom board coin

Water ripples

Fig. 4.49

Congruent Circles

Two circles are congruent if they are copies of one another or identical. That is, they have the same size. Here are some real life examples:

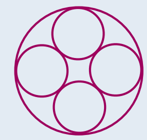
The two wheels of a bullock cart

The Olympic rings

Fig. 4.50

Thinking Corner

Draw four congruent circles as shown. What do you infer?



Position of a Point with respect to a Circle

Consider a circle in a plane (Fig.4.51). Consider any point P on the circle. If the distance from the centre O to the point P is OP, then

(i) OP = radius (If the point P lies on the circle)

(ii) $OP <$ radius (If the point P Point lies inside the circle)

(iii) $OP >$ radius(If the point P lies outside the circle)

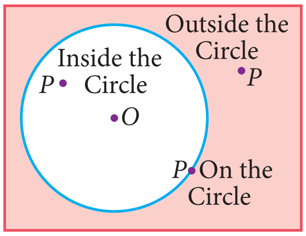


Fig. 4.51

So, a circle divides the plane on which it lies into three parts.

Progress Check

Say True or False

1. Every chord of a circle contains exactly two points of the circle.
2. All radii of a circle are of same length.
3. Every radius of a circle is a chord.
4. Every chord of a circle is a diameter.
5. Every diameter of a circle is a chord.
6. There can be any number of diameters for a circle.
7. Two diameters cannot have the same end-point.
8. A circle divides the plane into three disjoint parts.
9. A circle can be partitioned into a major arc and a minor arc.
10. The distance from the centre of a circle to the circumference is that of a diameter

Thinking Corner

1. How many sides does a circle have ?
2. Is circle, a polygon?

4.4.1 Circle Through Three Points

We have already learnt that there is one and only one line passing through two points. In the same way, we are going to see how many circles can be drawn through a given point, and through two given points. We see that in both cases there can be infinite number of circles passing through a given point P (Fig.4.52) , and through two given points A and B (Fig.4.53).

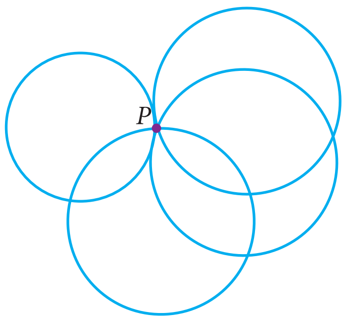


Fig. 4.52

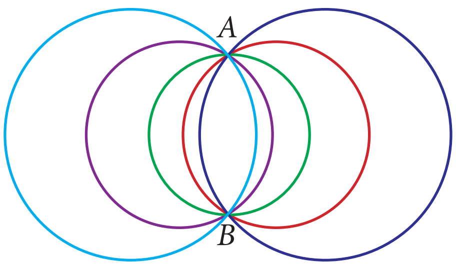


Fig. 4.53

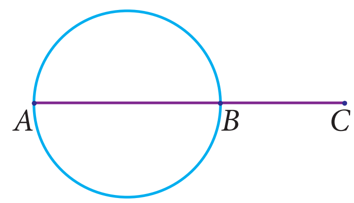


Fig. 4.54

Now consider three collinear points A, B and C (Fig.4.14). Can we draw a circle passing through these three points? Think over it. If the points are collinear, we can't?

If the three points are non collinear, they form a triangle (Fig.4.55). Recall the construction of the circumcentre. The intersecting point of the perpendicular bisector of the sides is the circumcentre and the circle is circumcircle.

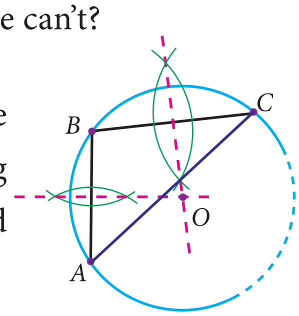


Fig. 4.55

Therefore from this we know that, there is a unique circle which passes through A, B and C. Now, the above statement leads to a result as follows.

Theorem 6 There is one and only one circle passing through three non-collinear points.

4.5 Properties of Chords of a Circle

In this chapter, already we come across lines, angles, triangles and quadrilaterals. Recently we have seen a new member circle. Using all the properties of these, we get some standard results one by one. Now, we are going to discuss some properties based on chords of the circle.

Considering a chord and a perpendicular line from the centre to a chord, we are going to see an interesting property.

4.5.1 Perpendicular from the Centre to a Chord

Consider a chord AB of the circle with centre O. Draw $OC \perp AB$ and join the points OA, OB . Here, easily we get two triangles ΔAOC and ΔBOC (Fig.4.56).

Can we prove these triangles are congruent? Now we try to prove this using the congruence of triangle rule which we have already learnt. $\angle OCA = \angle OCB = 90°(OC \perp AB)$ and $OA = OB$ is the radius of the circle. The side OC is common. RHS criterion tells us that ΔAOC and ΔBOC are congruent. From this we can conclude that $AC = BC$. This argument leads to the result as follows.

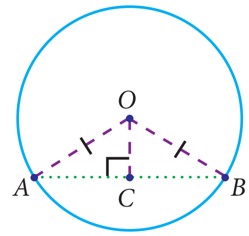


Fig. 4.56

Theorem 7 The perpendicular from the centre of a circle to a chord bisects the chord.

Converse of Theorem 7 The line joining the centre of the circle and the midpoint of a chord is perpendicular to the chord.

Example 4.5 Find the length of a chord which is at a distance of $2\sqrt{11}$ cm from the centre of a circle of radius 12cm.

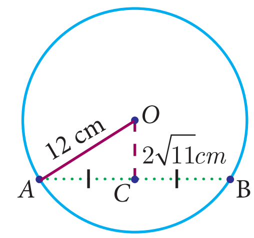


Fig. 4.57

Solution

Let AB be the chord and C be the mid point of AB

Therefore, $OC \perp AB$

Join OA and OC.

OA is the radius

Given $OC = 2\sqrt{11}$cm and OA = 12cm

In a right ΔOAC,

using Pythagoras Theorem, we get,

$$AC^2 = OA^2 - OC^2$$
$$= 12^2 - (2\sqrt{11})^2$$
$$= 144 - 44$$
$$= 100\text{cm}$$
$$AC^2 = 100\text{cm}$$
$$AC = 10\text{cm}$$

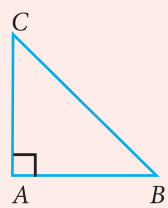
Therefore, length of the chord $AB = 2AC$

$$= 2 \times 10\text{cm} = 20\text{cm}$$

Note

Pythagoras theorem

One of the most important and well known results in geometry is Pythagoras Theorem. "In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides".



In right Δ ABC, $BC^2 = AB^2 + AC^2$.
Application of this theorem is most useful in this unit.

Example 4.6 In the concentric circles, chord AB of the outer circle cuts the inner circle at C and D as shown in the diagram. Prove that, $AB - CD = 2AC$

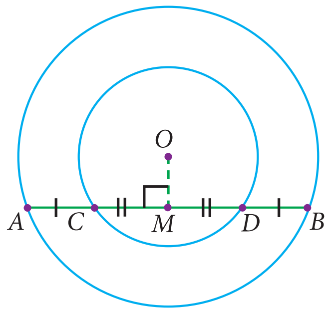


Fig. 4.58

Solution

Given : Chord AB of the outer circle cuts the inner circle at C and D.

To prove : $AB - CD = 2AC$

Construction : Draw $OM \perp AB$

Proof : Since, $OM \perp AB$ (By construction)

Also, $OM \perp CD$

Therefore, $AM = MB$ ... (1) (Perpendicular drawn from centre to chord bisect it)

$CM = MD$ ... (2)

Now, $AB - CD = 2AM - 2CM$

$= 2(AM - CM)$ from (1) and (2)

$AB - CD = 2AC$

Progress Check

1. The radius of the circle is 25 cm and the length of one of its chord is 40cm. Find the distance of the chord from the centre.
2. Draw three circles passing through the points P and Q, where PQ = 4cm.

4.5.2 Angle Subtended by Chord at the Centre

Instead of a single chord we consider two equal chords. Now we are going to discuss another property.

Let us consider two equal chords in the circle with centre O. Join the end points of the chords with the centre to get the triangles ΔAOB and ΔOCD, chord AB = chord CD (because the given chords are equal). The other sides are radii, therefore $OA=OC$ and $OB=OD$. By SSS rule, the triangles are congruent, that is $\Delta OAB \equiv \Delta OCD$. This gives $m\angle AOB = m\angle COD$. Now this leads to the following result.

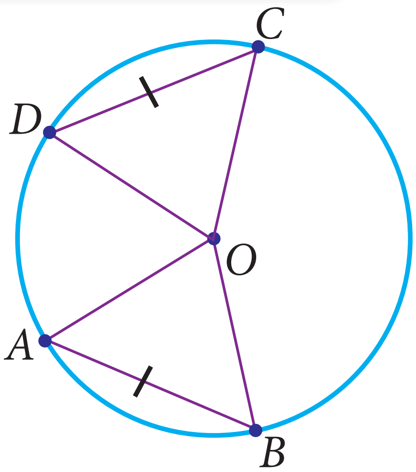


Fig. 4.59

Theorem 8 Equal chords of a circle subtend equal angles at the centre.

Activity - 5

Procedure

1. Draw a circle with centre O and with suitable radius.
2. Make it a semi-circle through folding. Consider the point A, B on it.
3. Make crease along AB in the semi circles and open it.
4. We get one more crease line on the another part of semi circle, name it as CD ($observe$ $AB = CD$)
5. Join the radius to get the ΔOAB and ΔOCD.
6. Using trace paper, take the replicas of triangle ΔOAB and ΔOCD.
7. Place these triangles ΔOAB and ΔOCD one on the other.

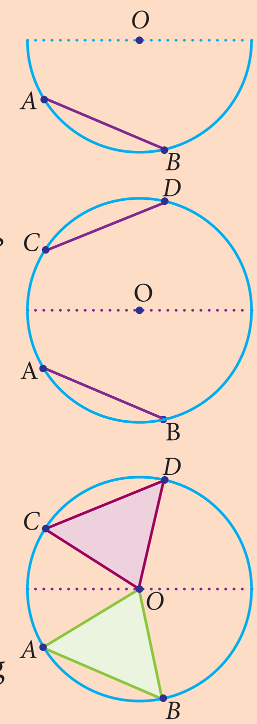


Fig. 4.60

Observation

1. What do you observe? Is $\Delta OAB \equiv \Delta OCD$?
2. Construct perpendicular line to the chords AB and CD passing through the centre O. Measure the distance from O to the chords.

Now we are going to find out the length of the chords AB and CD, given the angles subtended by two chords at the centre of the circle are equal. That is, $\angle AOB = \angle COD$ and the two sides which include these angles of the ΔAOB and ΔCOD are radii and are equal.

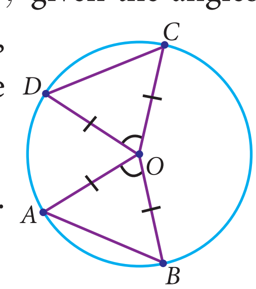


Fig. 4.61

By SAS rule, $\Delta AOB \equiv \Delta COD$. This gives chord AB = chord CD. Now let us write the converse result as follows:

Converse of theorem 8

If the angles subtended by two chords at the centre of a circle are equal, then the chords are equal.

In the same way we are going to discuss about the distance from the centre, when the equal chords are given. Draw the perpendicular $OL \perp AB$ and $OM \perp CD$. From theorem 7, these perpendicular divides the chords equally. So $AL = CM$. By comparing the ΔOAL and ΔOCM, the angles $\angle OLA = \angle OMC = 90°$ and $OA = OC$ are radii. By RHS rule, the $\Delta OAL \equiv \Delta OCM$. It gives the distance from the centre $OL = OM$ and write the conclusion as follows.

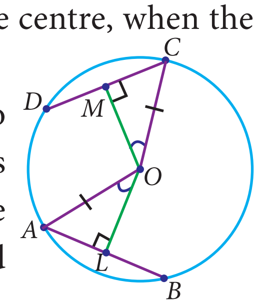


Fig. 4.62

Theorem 9 Equal chords of a circle are equidistant from the centre.

Let us know the converse of theorem 9, which is very useful in solving problems.

Converse of theorem 9

The chords of a circle which are equidistant from the centre are equal.

4.5.3 Angle Subtended by an Arc of a Circle

Activity - 6

Procedure :

1. Draw three circles of any radius with centre O on a chart paper.
2. From these circles, cut a semi-circle, a minor segment and a major segment.
3. Consider three points on these segment and name them as A, B and C.

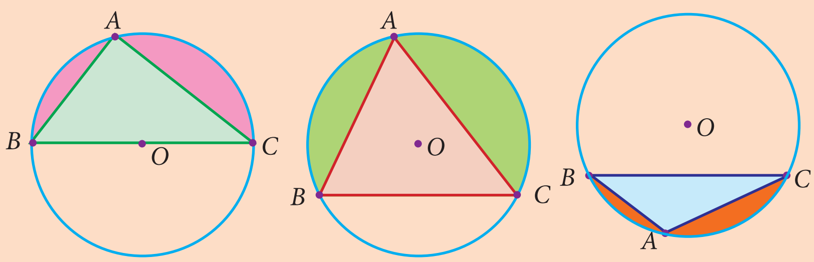


Fig. 4.63

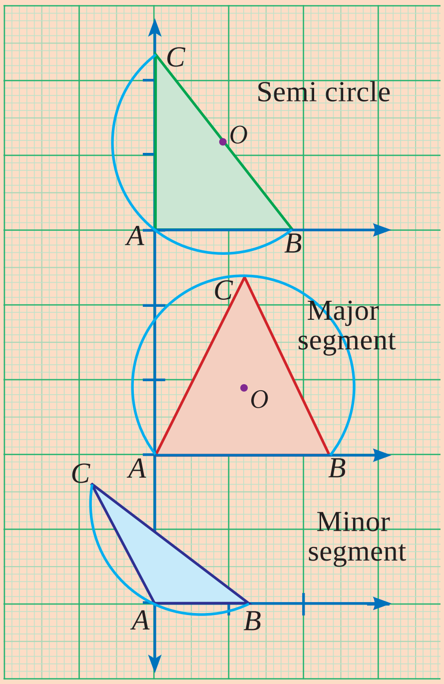


Fig. 4.64

4. (iv) Cut the triangles and paste it on the graph sheet so that the point A coincides with the origin as shown in the figure.

Observation :

(i) Angle in a Semi-Circle is _______ angle.

(ii) Angle in a major segment is _____ angle.

(iii) Angle in a minor segment is _____ angle.

Now we are going to verify the relationship between the angle subtended by an arc at the centre and the angle subtended on the circumference.

4.5.4 Angle at the Centre and the Circumference

Let us consider any circle with centre O. Now place the points A, B and C on the circumference.

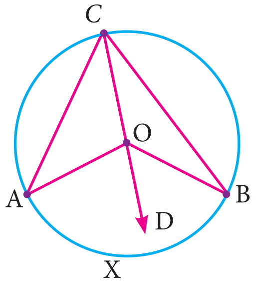


Fig. 4.65

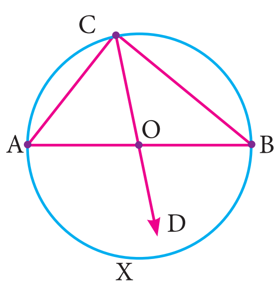


Fig. 4.66

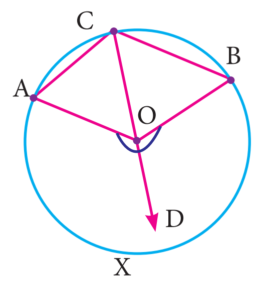


Fig. 4.67

Here $\overset{\frown}{AB}$ is a minor arc in Fig.4.65, a semi circle in Fig.4.66 and a major arc in Fig.4.67. The point C makes different types of angles in different positions (Fig. 4.65 to 4.67). In all these circles, $\overset{\frown}{AXB}$ subtends $\angle AOB$ at the centre and $\angle ACB$ at a point on the circumference of the circle.

We want to prove $\angle AOB = 2\angle ACB$. For this purpose extend CO to D and join CD.

$\angle OCA = \angle OAC$ since $OA = OC$ (radii)

Exterior angle = sum of two interior opposite angles.

$$\angle AOD = \angle OAC + \angle OCA$$
$$= 2\angle OCA \quad \text{.... (1)}$$

Similarly,

$$\angle BOD = \angle OBC + \angle OCB$$
$$= 2\angle OCB \quad \text{.... (2)}$$

From (1) and (2),

$$\angle AOD + \angle BOD = 2(\angle OCA + \angle OCB)$$

Finally we reach our result $\angle AOB = 2\angle ACB$.

From this we get the result as follows :

Progress Check

i. Draw the outline of different size of bangles and try to find out the centre of each using set square.

ii. Trace the given cresent and complete as full moon using ruler and compass.

Fig. 4.68

Theorem 10

The angle subtended by an arc of the circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Note

- Angle inscribed in a semicircle is a right angle.
- Equal arcs of a circle subtend equal angles at the centre.

Example 4.7 Find the value of $x°$ in the following figures:

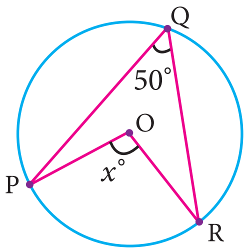
(i)


Fig. 4.69

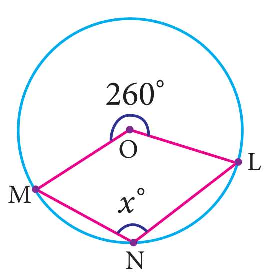
(ii)


Fig. 4.70

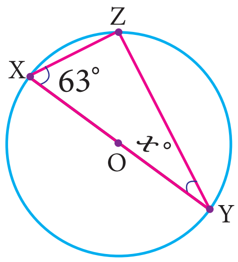
(iii)


Fig. 4.71

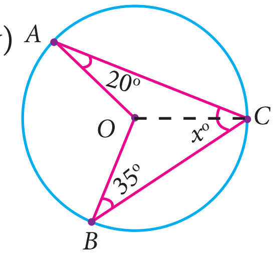
(iv)


Fig. 4.72

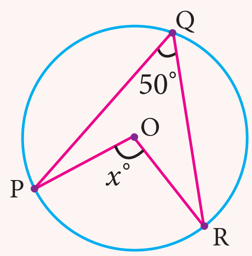
Solution

Using the theorem the angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of a circle.

(i) $\angle POR = 2\angle PQR$

$x^\circ = 2\times 50^\circ$

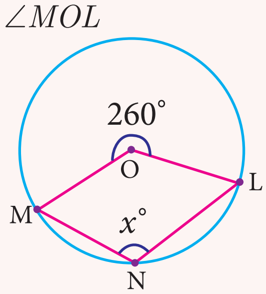
$x^\circ = 100^\circ$



(ii) $\angle MNL = \frac{1}{2}$ Reflex $\angle MOL$

$= \frac{1}{2}\times 260^\circ$

$x^\circ = 130^\circ$



(iii) XY is the diameter of the circle.

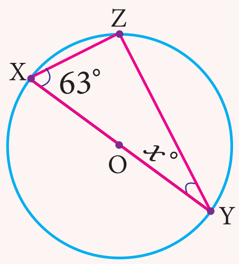
Therefore $\angle XZY = 90^\circ$

(Angle on a semi – circle)

In ΔXYZ

$x^\circ + 63^\circ + 90^\circ = 180^\circ$

$x^\circ = 27^\circ$

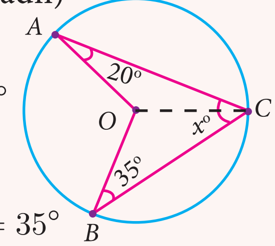


(iv) $OA = OB = OC$ (Radii)

In ΔOAC,

$\angle OAC = \angle OCA = 20^\circ$

In ΔOBC,



$\angle OBC = \angle OCB = 35^\circ$

(angles opposite to equal sides are equal)

$\angle ACB = \angle OCA + \angle OCB$

$x^\circ = 20^\circ + 35^\circ$

$x^\circ = 55^\circ$

Example 4.8 If O is the centre of the circle and $\angle ABC = 30^\circ$ then find $\angle AOC$. (see Fig. 4.73)

Solution

Given $\angle ABC = 30^\circ$

$\angle AOC = 2\angle ABC$ (The angle subtended by an arc at the centre is double the angle at any point on the circle)

$= 2\times 30^\circ$

$= 60^\circ$

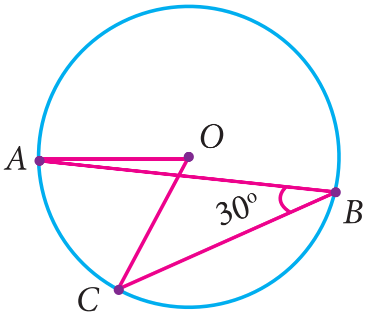


Fig. 4.73

Now we shall see, another interesting theorem. We have learnt that minor arc subtends obtuse angle, major arc subtends acute angle and semi circle subtends right angle on the circumference. If a chord AB is given and C and D are two different points on the circumference of the circle, then find $\angle ACB$ and $\angle ADB$. Is there any difference in these angles?

4.5.5 Angles in the same segment of a circle

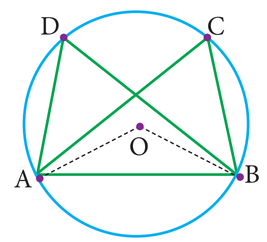


Fig. 4.74

Consider the circle with centre O and chord AB. C and D are the points on the circumference of the circle in the same segment. Join the radius OA and OB.

$$\frac{1}{2}\angle AOB = \angle ACB \quad \text{(by theorem 10)}$$

and $$\frac{1}{2}\angle AOB = \angle ADB \quad \text{(by theorem 10)}$$

$$\angle ACB = \angle ADB$$

This conclusion leads to the new result.

Theorem 11 Angles in the same segment of a circle are equal.

Example 4.9 In the given figure, O is the center of the circle. If the measure of $\angle OQR = 48°$, what is the measure of $\angle P$?

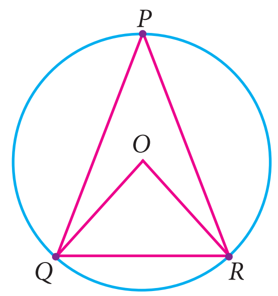


Fig. 4.75

Solution

Given $\angle OQR = 48°$.

Therefore, $\angle ORQ$ also is $48°$. (Why?__________)

$\angle QOR = 180° - (2 \times 48°) = 84°$.

The central angle made by chord QR is twice the inscribed angle at P.

Thus, measure of $\angle QPR = \frac{1}{2} \times 84° = 42°$.

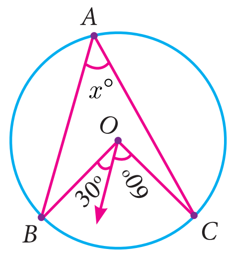
Exercise 4.3

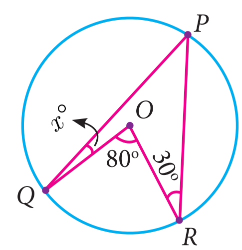
1. The diameter of the circle is 52cm and the length of one of its chord is 20cm. Find the distance of the chord from the centre.
2. The chord of length 30 cm is drawn at the distance of 8cm from the centre of the circle. Find the radius of the circle
3. Find the length of the chord AC where AB and CD are the two diameters perpendicular to each other of a circle with radius $4\sqrt{2}$ cm and also find $\angle OAC$ and $\angle OCA$.
4. A chord is 12cm away from the centre of the circle of radius 15cm. Find the length of the chord.

5. In a circle, AB and CD are two parallel chords with centre O and radius 10 cm such that $AB = 16$ cm and $CD = 12$ cm determine the distance between the two chords?

6. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

7. Find the value of $x°$ in the following figures:

(i)


(ii)


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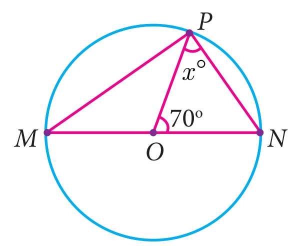

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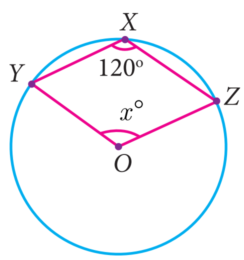
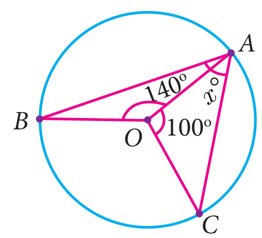
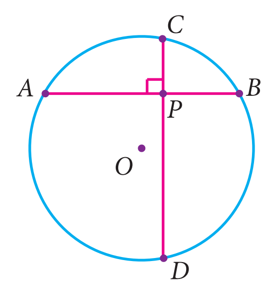


(v)


8. In the given figure, $\angle CAB = 25°$,

find $\angle BDC$, $\angle DBA$ and $\angle COB$



4.6 Cyclic Quadrilaterals

Now, let us see a special quadrilateral with its properties called "Cyclic Quadrilateral". A quadrilateral is called cyclic quadrilateral if all its four vertices lie on the circumference of the circle. Now we are going to learn the special property of cyclic quadrilateral.

Consider the quadrilateral $ABCD$ whose vertices lie on a circle. We want to show that its opposite angles are supplementary. Connect the centre O of the circle with each vertex. You now see four radii OA, OB, OC and OD giving rise to four isosceles triangles OAB, OBC, OCD and ODA. The sum of the angles around the centre of the circle is $360°$. The angle sum of each isosceles triangle is $180°$

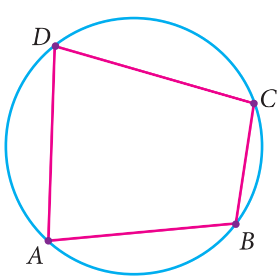


Fig. 4.76

Thus, we get from the figure,

$$2\times(\angle 1+\angle 2+\angle 3+\angle 4) + \text{Angle at centre } O = 4\times 180°$$

$$2\times(\angle 1+\angle 2+\angle 3+\angle 4) + 360° = 720°$$

Simplifying this, $(\angle 1+\angle 2+\angle 3+\angle 4) = 180°$.

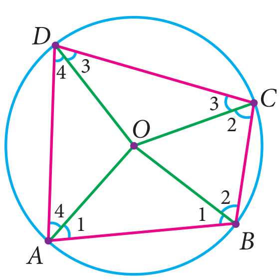


Fig. 4.77

You now interpret this as

(i) $(\angle 1+\angle 2) + (\angle 3+\angle 4) = 180°$ (Sum of opposite angles B and D)

(ii) $(\angle 1+\angle 4) + (\angle 2+\angle 3) = 180°$ (Sum of opposite angles A and C)

Now the result is given as follows.

Theorem 12 Opposite angles of a cyclic quadrilateral are supplementary.

Let us see the converse of theorem 12, which is very useful in solving problems

Converse of Theorem 12 If a pair of opposite angles of a quadrilateral is supplementary, then the quadrilateral is cyclic.

Activity - 7

Procedure

1. Draw a circle of any radius with centre O.
2. Mark any four points A, B, C and D on the boundary. Make a cyclic quadrilateral $ABCD$ and name the angles as in Fig. 4.78
3. Make a replica of the cyclic quadrilateral $ABCD$ with the help of tracing paper.
4. Make the cutout of the angles A, B, C and D as in Fig. 4.79
5. Paste the angle cutout $\angle 1, \angle 2$, $\angle 3$ and $\angle 4$ adjacent to the angles opposite to A, B, C and D as in Fig. 4.80
6. Measure the angles $\angle 1 + \angle 3$, and $\angle 2 + \angle 4$.

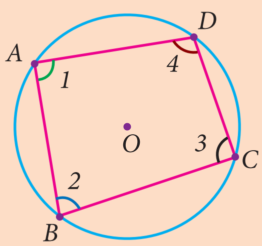


Fig. 4.78

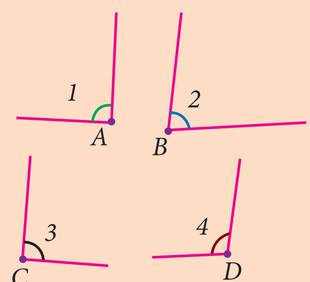


Fig. 4.79

Observe and complete the following:

1. (i) $\angle A + \angle C =$ _____ (ii) $\angle B + \angle D =$ _____

   (iii) $\angle C + \angle A =$ _____ (iv) $\angle D + \angle B =$ _____
2. Sum of opposite angles of a cyclic quadrilateral is __________.
3. The opposite angles of a cyclic quadrilateral is _______.

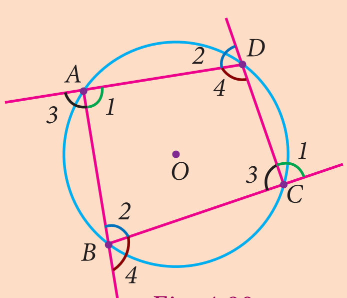


Fig. 4.80

Example 4.10

If $PQRS$ is a cyclic quadrilateral in which $\angle PSR = 70°$ and $\angle QPR = 40°$, then find $\angle PRQ$ (see Fig. 4.81).

Solution

$PQRS$ is a cyclic quadrilateral

Given $\angle PSR = 70°$

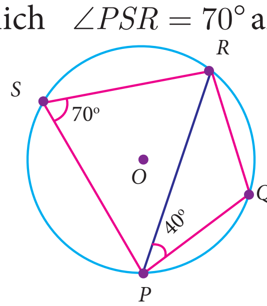


Fig. 4.81

$$\angle PSR + \angle PQR = 180^\circ \quad \text{(state reason\_\_\_\_\_\_\_\_)}$$
$$70^\circ + \angle PQR = 180^\circ$$
$$\angle PQR = 180^\circ - 70^\circ$$
$$\angle PQR = 110^\circ$$

In ΔPQR we have,

$$\angle PQR + \angle PRQ + \angle QPR = 180^\circ \quad \text{(state reason\_\_\_\_\_\_\_\_\_)}$$
$$110^\circ + \angle PRQ + 40^\circ = 180^\circ$$
$$\angle PRQ = 180^\circ - 150^\circ$$
$$\angle PRQ = 30^\circ$$

Exterior Angle of a Cyclic Quadrilateral

An exterior angle of a quadrilateral is an angle in its exterior formed by one of its sides and the extension of an adjacent side.

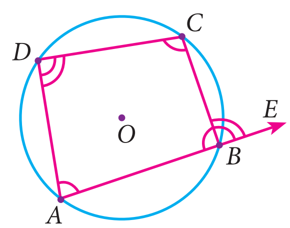


Fig. 4.82

Let the side AB of the cyclic quadrilateral $ABCD$ be extended to E. Here $\angle ABC$ and $\angle CBE$ are linear pair, their sum is 180° and the angles $\angle ABC$ and $\angle ADC$ are the opposite angles of a cyclic quadrilateral, and their sum is also 180°. From this, $\angle ABC + \angle CBE = \angle ABC + \angle ADC$ and finally we get $\angle CBE = \angle ADC$. Similarly it can be proved for other angles.

Theorem 13 If one side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle.

Progress Check

1. If a pair of opposite angles of a quadrilateral is supplementary, then the quadrilateral is ______________.
2. As the length of the chord decreases, the distance from the centre ______________.
3. If one side of a cyclic quadrilateral is produced then the exterior angle is ______________ to the interior opposite angle.
4. Opposite angles of a cyclic quadrilateral are ______________.

Example 4.11 In the figure given, find the value of x° and y°.

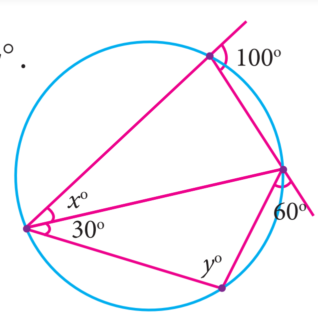


Fig. 4.83

Solution

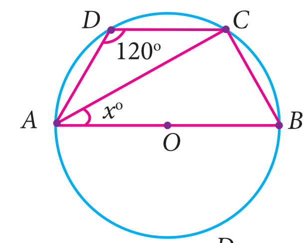
By the exterior angle property of a cyclic quadrilateral,

we get, $y^\circ = 100^\circ$ and

$$x^\circ + 30^\circ = 60^\circ \text{ and so } x^\circ = 30^\circ$$

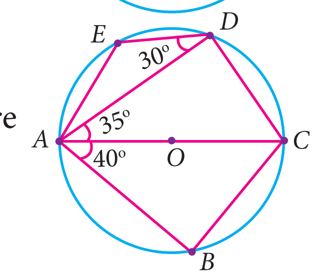
Exercise 4.4

1. Find the value of x in the given figure.

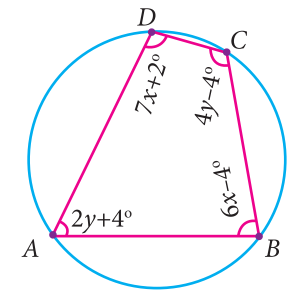


2. In the given figure, AC is the diameter of the circle with centre O. If $\angle ADE = 30°$; $\angle DAC = 35°$ and $\angle CAB = 40°$.

   Find (i) $\angle ACD$ (ii) $\angle ACB$ (iii) $\angle DAE$

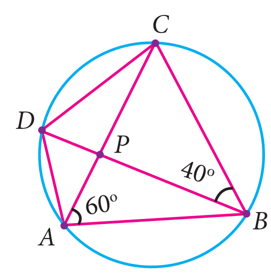


3. Find all the angles of the given cyclic quadrilateral $ABCD$ in the figure.

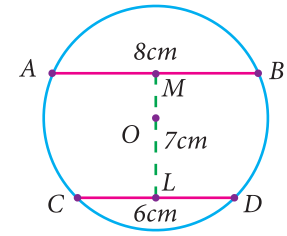


4. In the given figure, $ABCD$ is a cyclic quadrilateral where diagonals intersect at P such that $\angle DBC = 40°$ and $\angle BAC = 60°$ find

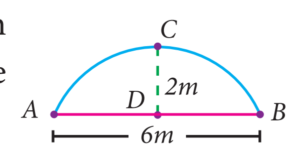
   (i) $\angle CAD$ (ii) $\angle BCD$



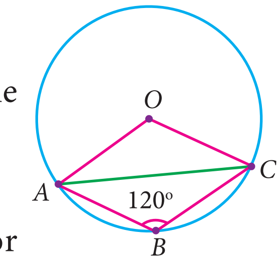
5. In the given figure, AB and CD are the parallel chords of a circle with centre O. Such that AB = 8cm and CD = 6cm. If $OM \perp AB$ and $OL \perp CD$ distance between LM is 7cm. Find the radius of the circle?



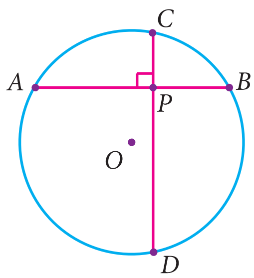
6. The arch of a bridge has dimensions as shown, where the arch measure 2m at its highest point and its width is 6m. What is the radius of the circle that contains the arch?



7. In figure, $\angle ABC = 120^0$, where A, B and C are points on the circle with centre O. Find $\angle OAC$?

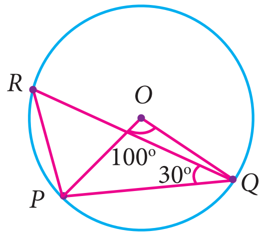


8. A school wants to conduct tree plantation programme. For this a teacher allotted a circle of radius 6m ground to nineth



standard students for planting sapplings. Four students plant trees at the points A, B, C and D as shown in figure. Here $AB = 8$m, $CD = 10$m and $AB \perp CD$. If another student places a flower pot at the point P, the intersection of AB and CD, then find the distance from the centre to P.

9. In the given figure, $\angle POQ = 100^0$ and $\angle PQR = 30^0$, then find $\angle RPO$.



4.7 Practical Geometry

Practical geometry is the method of applying the rules of geometry dealt with the properties of points, lines and other figures to construct geometrical figures. "Construction" in Geometry means to draw shapes, angles or lines accurately. The geometric constructions have been discussed in detail in Euclid's book 'Elements'. Hence these constructions are also known as Euclidean constructions. These constructions use only compass and straightedge (i.e. ruler). The compass establishes equidistance and the straightedge establishes collinearity. All geometric constructions are based on those two concepts.

It is possible to construct rational and irrational numbers using straightedge and a compass as seen in chapter II. In 1913 the Indian mathematical Genius, Ramanujam gave a geometrical construction for 355/113 =π. Today with all our accumulated skill in exact measurements. it is a noteworthy feature that lines driven through a mountain meet and make a tunnel.In the earlier classes, we have learnt the construction of angles and triangles with the given measurements.

In this chapter we are going to learn to construct Centroid, Orthocentre, Circumcentre and Incentre of a triangle by using concurrent lines.

4.7.1 Construction of the Centroid of a Triangle

Centroid

The point of concurrency of the medians of a triangle is called the centroid of the triangle and is usually denoted by G.

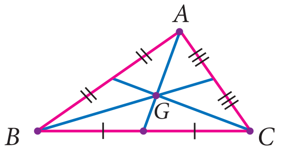


Fig. 4.84

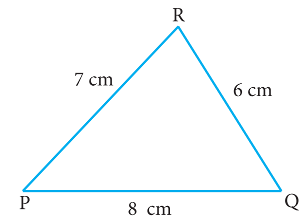
Activity 8

Objective To find the mid-point of a line segment using paper folding

Procedure Make a line segment on a paper by folding it and name it PQ. Fold the line segment PQ in such a way that P falls on Q and mark the point of intersection of the line segment and the crease formed by folding the paper as M. M is the midpoint of PQ.

Example 4.12 Construct the centroid of ΔPQR whose sides are $PQ = 8$cm; $QR = 6$cm; $RP = 7$cm.

Rough Diagram



Solution

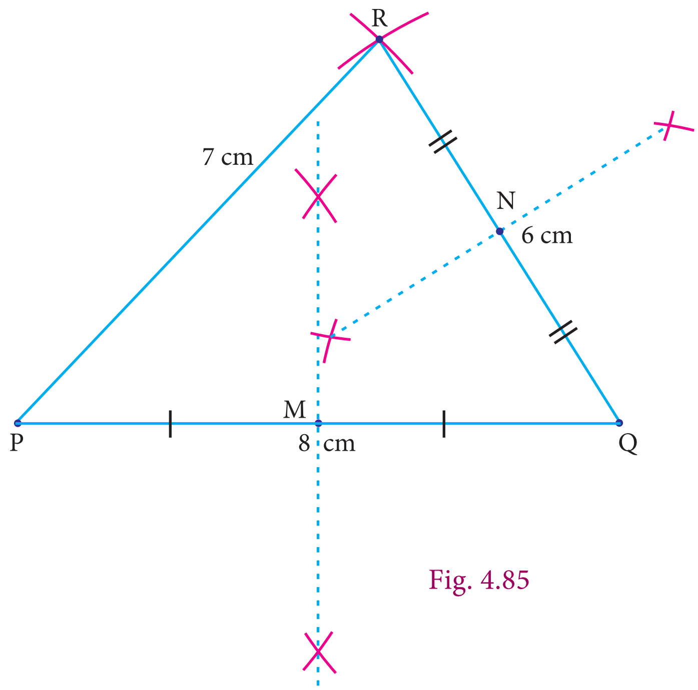


Fig. 4.85

Step 1 : Draw ΔPQR using the given measurements $PQ = 8$cm $QR = 6$cm and $RP = 7$cm and construct the perpendicular bisector of any two sides (PQ and QR) to find the mid-points M and N of PQ and QR respectively.

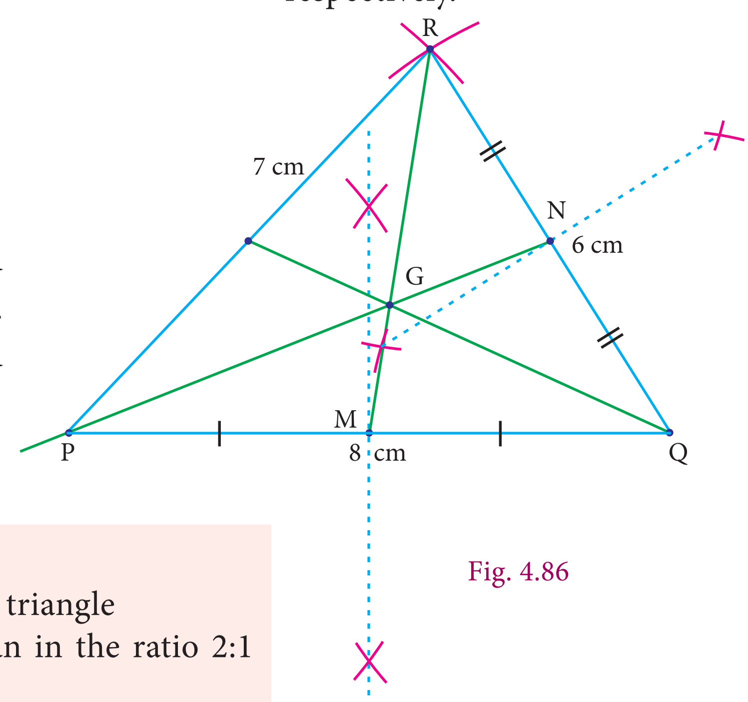


Fig. 4.86

Step 2 : Draw the medians PN and RM and let them meet at G. The point G is the centroid of the given ΔPQR.

Note

- Three medians can be drawn in a triangle
- The centroid divides each median in the ratio 2:1 from the vertex.
- The centroid of any triangle always lie inside the triangle.
- Centroid is often described as the triangle's centre of gravity (where the triangle balances evenly) and also as the barycentre.

4.7.2 Construction of Orthocentre of a Triangle

Orthocentre

The orthocentre is the point of concurrency of the altitudes of a triangle. Usually it is denoted by H.

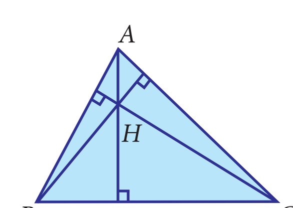


Fig. 4.87

Activity 9

Objective To construct a perpendicular to a line segment from an external point using paper folding.

Procedure Draw a line segment *AB* and mark an external point *P*. Move *B* along *BA* till the fold passes through *P* and crease it along that line. The crease thus formed is the perpendicular to *AB* through the external point *P*.

Activity 10

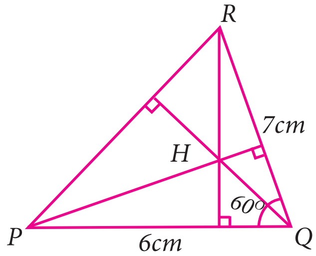
Objective To locate the Orthocentre of a triangle using paper folding.

Procedure Using the above Activity with any two vertices of the triangle as external points, construct the perpendiculars to opposite sides. The point of intersection of the perpendiculars is the Orthocentre of the given triangle.

Example 4.13

Construct ΔPQR whose sides are $PQ = 6$ cm $\angle Q = 60^0$ and $QR = 7$ cm and locate its Orthocentre.

Solution



Rough Diagram

Step 1 Draw the ΔPQR with the given measurements.

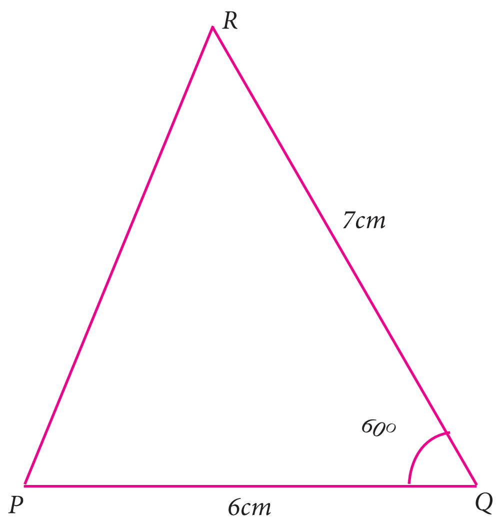


Fig. 4.88

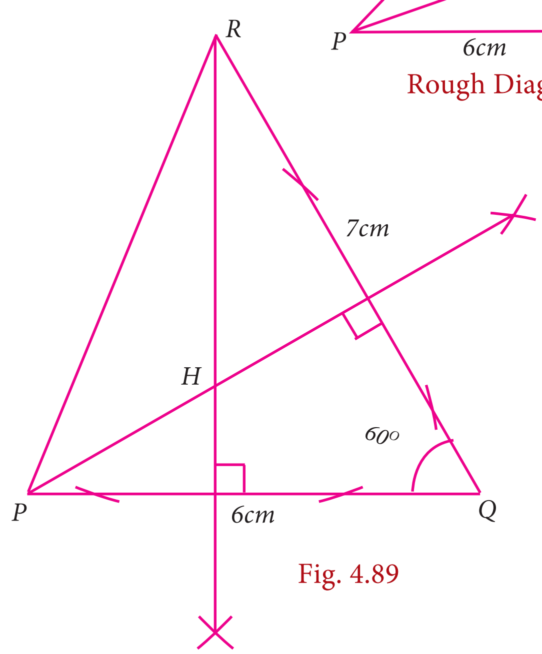


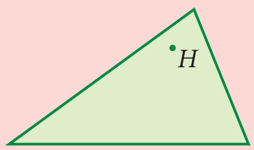
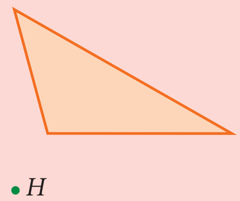
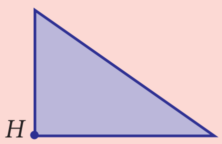
Fig. 4.89

Step 2:

Construct altitudes from any two vertices (say) R and P, to their opposite sides PQ and QR respectively.

The point of intersection of the altitude H is the Orthocentre of the given ΔPQR.

Note

Where do the Orthocentre lie in the given triangles.			
	Acute Triangle	**Obtuse Triangle**	**Right Triangle**
Orthocentre	Inside of Triangle 	Outside of Triangle 	Vertex at Right Angle 

Exercise 4.5

1. Construct the ΔLMN such that LM=7.5cm, MN=5cm and LN=8cm. Locate its centroid.
2. Draw and locate the centroid of the triangle ABC where right angle at A, AB = 4cm and AC = 3cm.
3. Draw the ΔABC, where AB = 6cm, $\angle B = 110°$ and AC = 9cm and construct the centroid.
4. Construct the ΔPQR such that PQ = 5cm, PR= 6cm and $\angle QPR = 60°$ and locate its centroid.
5. Draw ΔPQR with sides PQ = 7 cm, QR = 8 cm and PR = 5 cm and construct its Orthocentre.
6. Draw an equilateral triangle of sides 6.5 cm and locate its Orthocentre.
7. Draw ΔABC, where AB = 6 cm, $\angle B = 110^0$ and BC = 5 cm and construct its Orthocentre.
8. Draw and locate the Orthocentre of a right triangle PQR where PQ = 4.5 cm, QR = 6 cm and PR = 7.5 cm.

4.7.3 Construction of the Circumcentre of a Triangle

Circumcentre

The Circumcentre is the point of concurrency of the Perpendicular bisectors of the sides of a triangle.

It is usually denoted by S.

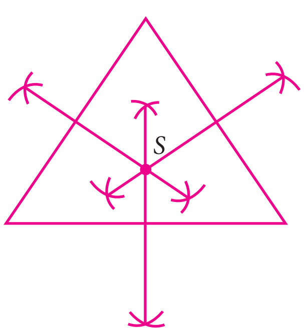


Fig. 4.90

Circumcircle

The circle passing through all the three vertices of the triangle with circumcentre (S) as centre is called circumcircle.

Circumradius

The line segment from any vertex of a triangle to the Circumcentre of a given triangle is called circumradius of the circumcircle.

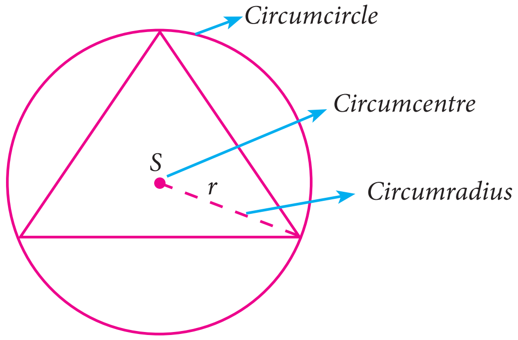


Fig. 4.91

Activity 11

Objective To construct a perpendicular bisector of a line segment using paper folding.

Procedure Make a line segment on a paper by folding it and name it as *PQ*. Fold *PQ* in such a way that *P* falls on *Q* and thereby creating a crease RS. This line *RS* is the perpendicular bisector of *PQ*.

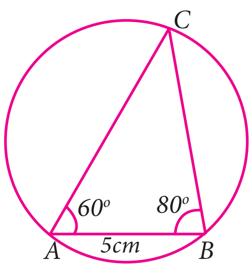
Activity 12

Objective To locate the circumcentre of a triangle using paper folding.

Procedure Using Activity 12, find the perpendicular bisectors for any two sides of the given triangle. The meeting point of these is the circumcentre of the given triangle.

Example 4.14

Construct the circumcentre of the ΔABC with $AB = 5$ cm, $\angle A = 60^0$ and $\angle B = 80^0$. Also draw the circumcircle and find the circumradius of the ΔABC.



Rough Diagram

Solution

Step 1 Draw the ΔABC with the given measurements

Step 2

Construct the perpendicular bisector of any two sides (*AC* and *BC*) and let them meet at *S* which is the circumcentre.

Step 3

S as centre and $SA = SB = SC$ as radius,

draw the Circumcircle to passes through *A*,*B* and *C*.

Circumradius = 3.9 cm.

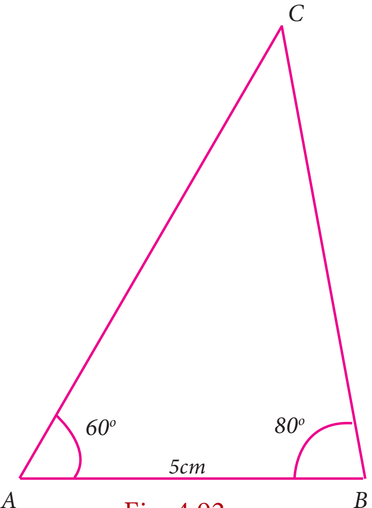


Fig. 4.92

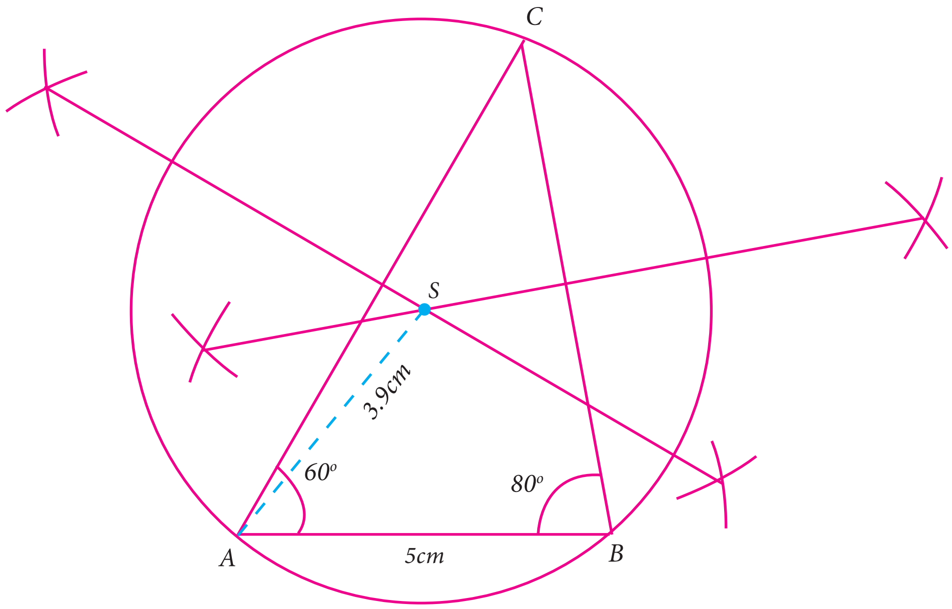
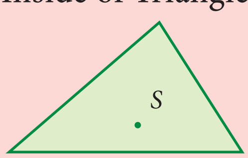
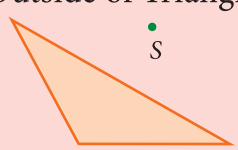
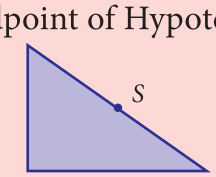


Fig. 4.93

Note

Where do the Circumcentre lie in the given triangles.			
	Acute Triangle	**Obtuse Triangle**	**Right Triangle**
Circumcentre	Inside of Triangle 	Outside of Triangle 	Midpoint of Hypotenuse 

4.7.4 Construction of the Incircle of a Triangle

Incentre

The incentre is (one of the triangle's points of concurrency formed by) the intersection of the triangle's three angle bisectors.

The incentre is the centre of the incircle ; It is usually denoted by I; it is the one point in the triangle whose distances to the sides are equal.

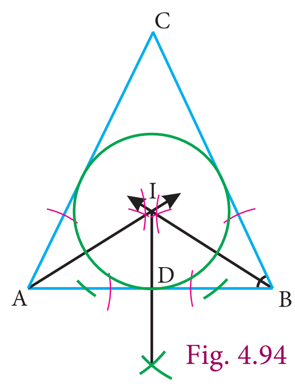
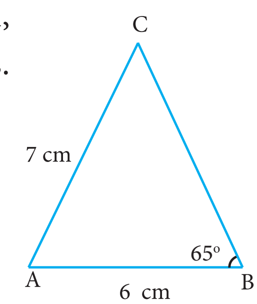


Fig. 4.94

Example 4.15 Construct the incentre of ΔABC with $AB = 6$ cm, $\angle B = 65°$ and $AC = 7$ cm Also draw the incircle and measure its radius.

Solution

Rough Diagram



Step 1 : Draw the ΔABC with AB = 6cm, $\angle B = 65°$ and $AC = 7$cm

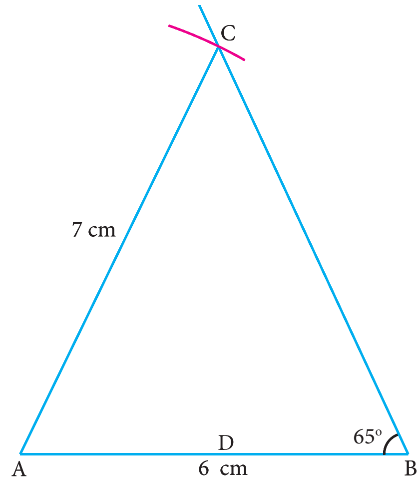


Fig. 4.95

Step 2 : Construct the angle bisectors of any two angles (A and B) and let them meet at I. Then I is the incentre of ΔABC. Draw perpendicular from I to any one of the side (AB) to meet AB at D.

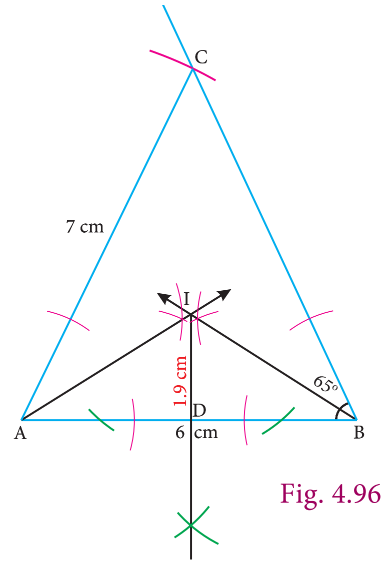


Fig. 4.96

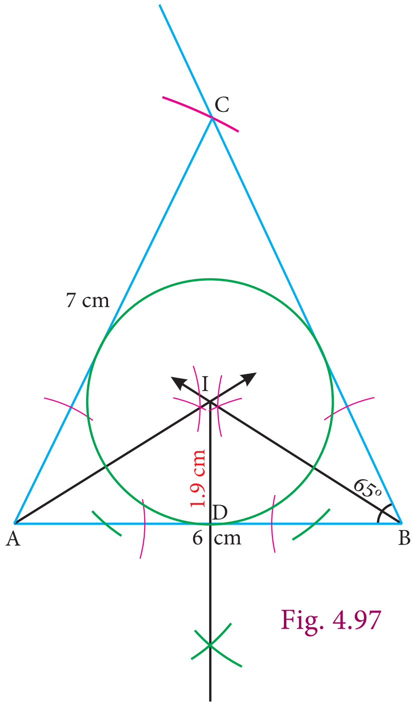


Fig. 4.97

Step 3: With I as centre and ID as radius draw the circle. This circle touches all the sides of the triangle internally.

Step 4: Measure inradius
In radius = 1.9 cm.

Exercise 4.6

1 Draw a triangle ABC, where AB = 8 cm, BC = 6 cm and $\angle B = 70^0$ and locate its circumcentre and draw the circumcircle.

2 Construct the right triangle PQR whose perpendicular sides are 4.5 cm and 6 cm. Also locate its circumcentre and draw the circumcircle.

3. Construct ΔABC with $AB = 5$ cm $\angle B = 100^0$ and $BC = 6$ cm. Also locate its circumcentre draw circumcircle.

4. Construct an isosceles triangle PQR where $PQ = PR$ and $\angle Q = 50^0$, QR = 7cm. Also draw its circumcircle.

5. Draw an equilateral triangle of side 6.5 cm and locate its incentre. Also draw the incircle.

6. Draw a right triangle whose hypotenuse is 10 cm and one of the legs is 8 cm. Locate its incentre and also draw the incircle.
7. Draw ΔABC given $AB = 9$ cm, $\angle CAB = 115°$ and $\Delta ABC = 40°$. Locatge its incentre and also draw the incircle. (Note: You can check from the above examples that the incentre of any triangle is always in its interior).
8. Construct ΔABC in which $AB = BC = 6$cm and $\angle B = 80°$. Locate its incentre and draw the incircle.

Exercise 4.7

Multiple Choice Questions

1. The exterior angle of a triangle is equal to the sum of two

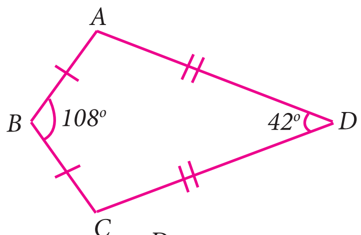
(1) Exterior angles (2) Interior opposite angles

(3) Alternate angles (4) Interior angles

2. In the quadrilateral $ABCD$, $AB = BC$ and $AD = DC$ Measure of $\angle BCD$ is

(1) 150° (2) 30°

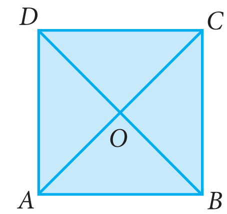
(3) 105° (4) 72°



3. $ABCD$ is a square, diagonals AC and BD meet at O. The number of pairs of congruent triangles with vertex O are

(1) 6 (2) 8

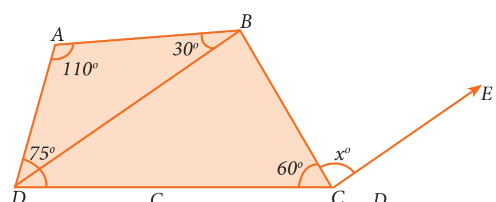
(3) 4 (4) 12



4. In the given figure $CE \parallel DB$ then the value of $x°$ is

(1) 45° (2) 30°

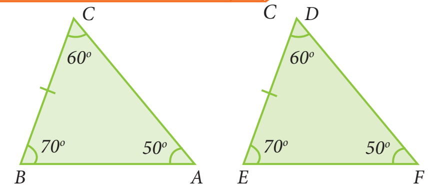
(3) 75° (4) 85°



5. The correct statement out of the following is

(1) $\Delta ABC \cong \Delta DEF$ (2) $\Delta ABC \cong \Delta DEF$

(3) $\Delta ABC \cong \Delta FDE$ (4) $\Delta ABC \cong \Delta FED$



6. If the diagonal of a rhombus are equal, then the rhombus is a

(1) Parallelogram but not a rectangle

(2) Rectangle but not a square

(3) Square

(4) Parallelogram but not a square

7. If bisectors of $\angle A$ and $\angle B$ of a quadrilateral $ABCD$ meet at O, then $\angle AOB$ is

(1) $\angle C + \angle D$ (2) $\frac{1}{2}(\angle C + \angle D)$

(3) $\frac{1}{2}\angle C + \frac{1}{3}\angle D$ (4) $\frac{1}{3}\angle C + \frac{1}{2}\angle D$

8. The interior angle made by the side in a parallelogram is 90° then the parallelogram is a

(1) rhombus (2) rectangle (3) trapezium (4) kite

9. Which of the following statement is correct?

(1) Opposite angles of a parallelogram are not equal.
(2) Adjacent angles of a parallelogram are complementary.
(3) Diagonals of a parallelogram are always equal.
(4) Both pairs of opposite sides of a parallelogram are always equal.

10. The angles of the triangle are $3x-40$, $x+20$ and $2x-10$ then the value of x is

(1) 40° (2) 35° (3) 50° (4) 45°

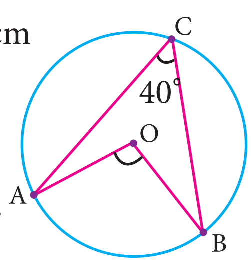
11. PQ and RS are two equal chords of a circle with centre O such that $\angle POQ = 70°$, then $\angle ORS =$

(1) 60° (2) 70° (3) 55° (4) 80°

12. A chord is at a distance of 15cm from the centre of the circle of radius 25cm. The length of the chord is

(1) 25cm (2) 20cm (3) 40cm (4) 18cm

13. In the figure, O is the centre of the circle and $\angle ACB = 40°$ then $\angle AOB =$

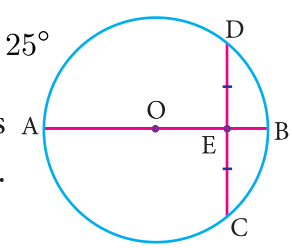


(1) 80° (2) 85° (3) 70° (4) 65°

14. In a cyclic quadrilaterals $ABCD$, $\angle A = 4x$, $\angle C = 2x$ the value of x is

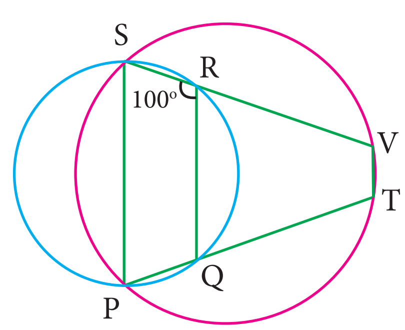
(1) 30° (2) 20° (3) 15° (4) 25°

15. In the figure, O is the centre of a circle and diameter AB bisects the chord CD at a point E such that CE=ED=8 cm and EB=4cm. The radius of the circle is



(1) 8cm (2) 4cm (3) 6cm (4)10cm

16. In the figure, $PQRS$ and $PTVS$ are two cyclic quadrilaterals, If $\angle QRS$=100°, then $\angle TVS =$



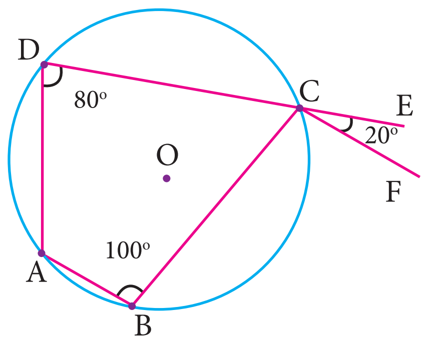
(1) 80° (2) 100° (3) 70° (4) 90°

17. If one angle of a cyclic quadrilateral is 75°, then the opposite angle is

(1) 100° (2) 105° (3) 85° (4) 90°

18. In the figure, $ABCD$ is a cyclic quadrilateral in which DC produced to E and CF is drawn parallel to AB such that $\angle ADC = 80°$ and $\angle ECF = 20°$, then $\angle BAD = ?$

(1) 100° (2) 20° (3) 120° (4) 110°

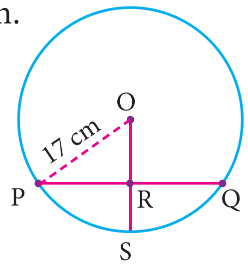


19. AD is a diameter of a circle and AB is a chord If $AD = 30$ cm and $AB = 24$ cm then the distance of AB from the centre of the circle is

(1) 10cm (2) 9cm (3) 8cm (4) 6cm.

20. In the given figure, If $OP = 17$cm, $PQ = 30$cm and OS is perpendicular to PQ, then RS is

(1) 10cm (2) 6cm
(3) 7cm (4) 9cm.

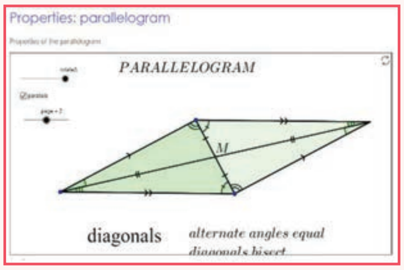


Points to Remember

- In a parallelogram the opposite sides and opposite angles are equal
- The diagonals of a parallelogram bisect each other.
- The diagonals of a parallelogram divides it into two congruent triangles
- A quadrilateral is a parallelogram if its opposite sides are equal .
- Parallelogram on the same base and between same parallel are equal in area.
- Triangles on the same base and between same parallel are equal in area.
- Parallelogram is a rhombus if its diagonals are perpendicular.
- There is one and only one circle passing through three non-collinear points.
- Equal chords of a circle subtend equal angles at the centre.
- Perpendicular from the centre of a circle to a chord bisects the chord.
- Equal chords of a circle are equidistant from the centre.
- The angle substended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.
- The angle in a semi circle is a right angle.
- Angles in the same segment of a circle are equal.
- The sum of either pair of opposite angle of a cyclic quadrilateral is 180°.
- If one side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle.

ICT Corner-1

Expected Result is shown in this picture



Step – 1

Open the Browser and copy and paste the Link given below (or) by typing the URL given (or) Scan the QR Code.

Step - 2

GeoGebra worksheet "Properties: Parallelogram" will appear. There are two sliders named "Rotate" and "Page"

Step-3

Drag the slider named "Rotate "and see that the triangle is doubled as parallelogram.

Step-4

Drag the slider named "Page" and you will get three pages in which the Properties are explained.

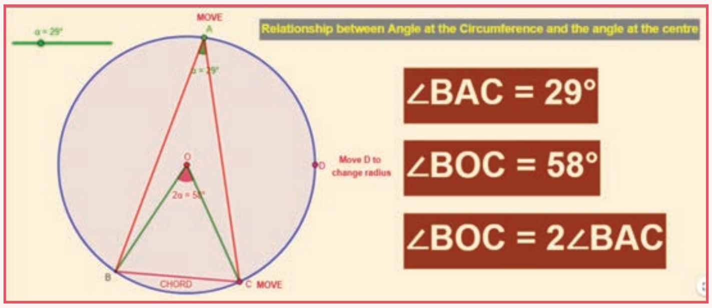
Z1MKN

Browse in the link

Properties: Parallelogram: https://www.geogebra.org/m/m9Q2QpWD

ICT Corner-2

Expected Result is shown in this picture



Step – 1

Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "Angles in a circle" will open. In the work sheet there are two activities on Circles.

The first activity is the relation between Angle at the circumference and the angle at the centre. You can change the angle by moving the slider. Also, you can drag on the point *A*, *C* and *D* to change the position and the radius. Compare the angles at *A* and *O*.

Step - 2

The second activity is "Angles in the segment of a circle". Drag the points *B* and *D* and check the angles. Also drag "Move" to change the radius and chord length of the circle.

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Browse in the link

Angle in a circle: https://ggbm.at/yaNUhv9S or Scan the QR Code.

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