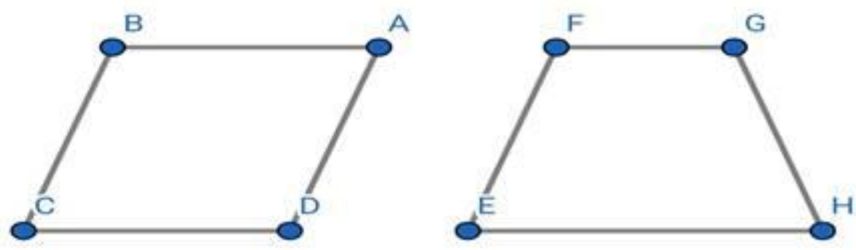
Quadrilaterals

Exercise 8.1

Q. 1. State whether the statements are True or False.

(i) Every parallelogram is a trapezium ( )
(ii) All parallelograms are quadrilaterals ( )
(iii) All trapeziums are parallelograms ( )
(iv) A square is a rhombus ( )
(v) Every rhombus is a square ( )
(vi) All parallelograms are rectangles ( )

Answer : (i) [True]



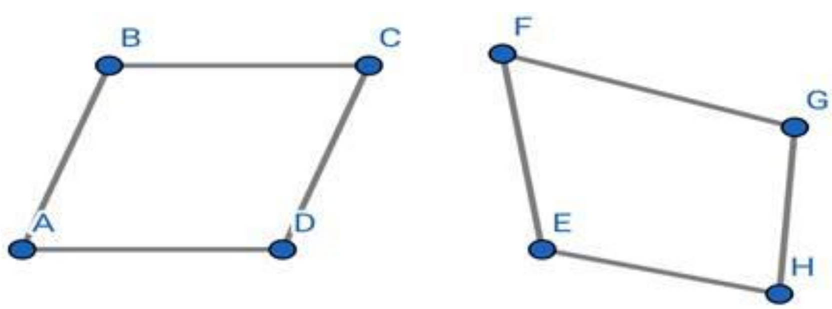
⇒ The trapezium has a property

One pair of opposite sides are parallel

This is conveyed by parallelogram as they have 2 pair of parallel sides

∴ Every parallelogram is a trapezium

(ii) [True]



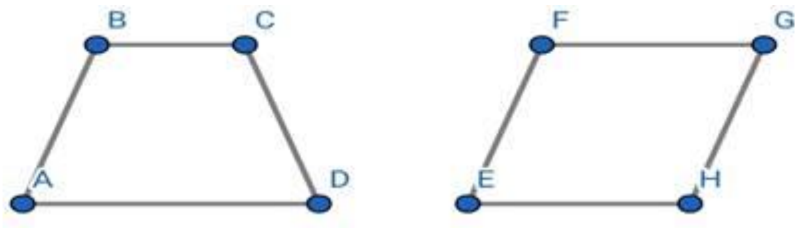
⇒ The quadrilateral is known as

Polygon having four sides

This is conveyed by parallelogram as they are also four sided polygon.

∴ Every parallelogram is a Quadrilateral

(iii) [False]



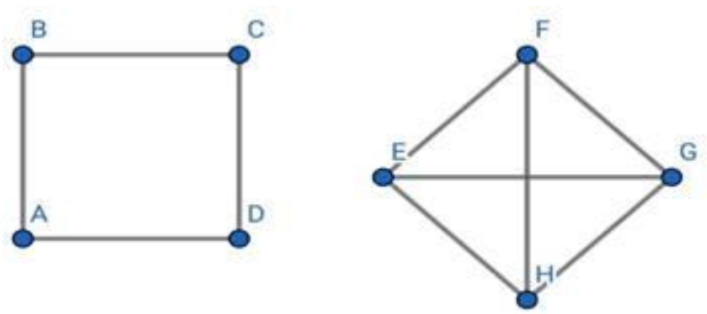
⇒ The parallelogram has a property

Both pair of opposite sides are parallel and equal

Which is not conveyed by trapezium as they have only 1 pair of parallel sides

∴ Every trapezium is not parallelogram

(iv) [True]



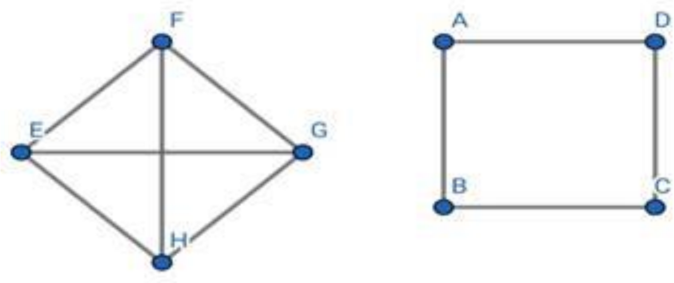
⇒ The Rhombus has a property

Diagonal of rhombus bisect each other at 90°

This is conveyed by square as their diagonal also bisect each other at 90°

∴ A square is a rhombus

(v) [False]



⇒ The square has a property

All angles of square are equal and 90°

And diagonals are equal and perpendicular bisector to each

other

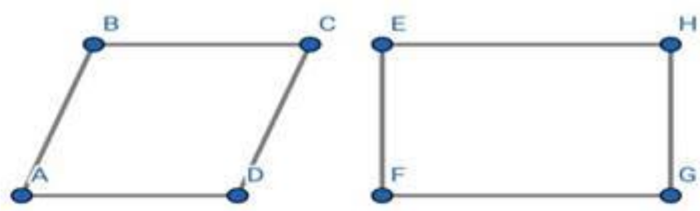
Which is not conveyed by Rhombus

as their diagonal are perpendicular bisector but not equal

And their angles are also not equal to 90°

∴ Every rhombus is not a square.

(vi) [False]



⇒ The rectangle has a property

Every angle of rectangle is 90°

Which is not conveyed by parallelogram

as it have only opposite side angles are equal not 90°

∴ Every parallelogram are not rectangle.

Q. 2. Complete the following table by writing (YES) if the property holds for the particular Quadrilateral and (NO) if property does not holds.

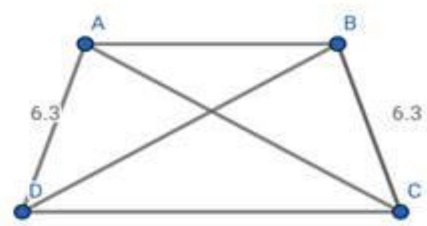
Properties	Trapezium	Parallelogram	Rhombus	Rectangle	square
a. One pair of opposite sides are parallel	YES				
b. Two pairs of opposite sides are parallel					
c. Opposite sides are equal					
d. Opposite angles are equal					
e. Consecutive angles are supplementary					
f. Diagonals bisect each other					
g. Diagonals are equal					
h. All sides are equal					
i. Each angle is a right angle					
j. Diagonals are perpendicular to each other.					

Answer :

Properties	Trapezium	Parallelogram	Rhombus	Rectangle	square
a. One pair of opposite sides are parallel	YES	No	No	No	No
b. Two pairs of opposite sides are parallel	No	Yes	Yes	Yes	Yes
c. Opposite sides are equal	No	Yes	Yes	Yes	Yes
d. Opposite angles are equal	No	Yes	Yes	Yes	Yes
e. Consecutive angles are supplementary	No	Yes	Yes	Yes	Yes
f. Diagonals bisect each other	No	Yes	Yes	Yes	Yes
g. Diagonals are equal	No	No	No	Yes	Yes
h. All sides are equal	No	No	Yes	No	Yes
i. Each angle is a right angle	No	No	No	Yes	Yes
j. Diagonals are perpendicular to each other.	No	No	Yes	No	Yes

Q. 3. ABCD is trapezium in which AB || CD. If AD = BC, show that ∠A = ∠B and ∠C = ∠D.

Answer :



Given:- ABCD is a isosceles trapezium

Trapezium with one pair of sides is parallel

And other pair of sides is equal[AD=BC].

Formula used:- SSS congruency property

If all 3 sides of triangle are equal to all 3 sides of other triangle

Then; Both triangles are congruent

Solution:- In trapezium ABCD

If AB||CD

And; AD = BC

∴ ABCD is a isosceles trapezium

⇒ If ABCD is a isosceles trapezium

Then;

Diagonals of ABCD must be equal

∴ AC=BD

In Δ ABD and Δ ABC

AC=BD [∵ ABCD is isosceles trapezium]

AD=BC [Given]

AB=AB [Common line in both triangle]

∴ Both triangles are congruent by SSS property

$\Delta ABD \cong \Delta ABC$

⇒ If both triangles are congruent

Then there were also be equal.

∴ $\angle A = \angle B$

As ABCD is trapezium and AB||CD

$\angle A + \angle D = 180°$ and $\angle C + \angle B = 180°$

$\angle A = 180° - \angle D$ and $\angle B = 180° - \angle C$

If $\angle A = \angle B$;

$180° - \angle D = 180° - \angle C$

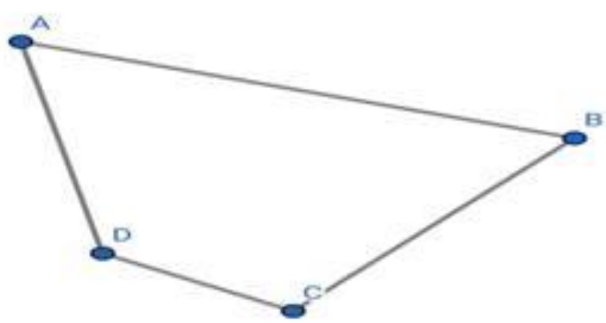
$180° + \angle C - 180° = \angle D$

∴ $\angle C = \angle D$;

Conclusion:- If ABCD is trapezium in which AD = BC, then $\angle A = \angle B$ and $\angle C = \angle D$.

Q. 4. The four angles of a quadrilateral are in the ratio 1: 2:3:4. Find the measure of each angle of the quadrilateral.

Answer :



Given . Angles of a quadrilateral are in the ratio 1: 2:3:4

<u>Solution .</u>

Let any quadrilateral be ABCD

Then

Sum of all angles of quadrilateral is 360°

∴ $\angle$ A + $\angle$ B + $\angle$ C + $\angle$ D=360°

If angle A,B,C,D are in ratio 1:2:3:4

Then,

1x + 2x + 3x + 4x =360°

10x = 360°

X = $\frac{360^\circ}{10}$

X = 36°

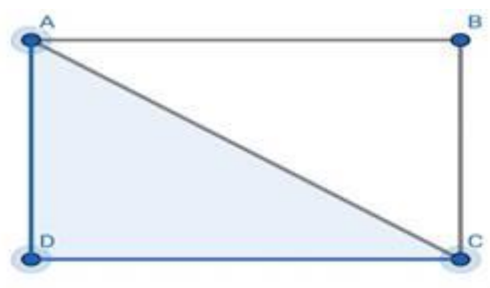
all angles are 1x,2x,3x,4x

<u>Conclusion.</u>

∴ Angles are 36° , 72° , 108° , 144°

Q. 5. ABCD is a rectangle AC is diagonal. Find the angles of ΔACD. Give reasons.

Answer :



<u>Given:-</u> ABCD is a rectangle

<u>Formula used:-</u> All angles of rectangle are 90°

In right angle triangle

Pythagoras theorem $a^2+b^2=c^2$

Sin (θ) = $\frac{\text{height}}{\text{hypotenuse}}$

Solution:-

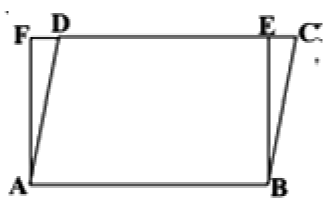
In ΔACD

∠ D = 90° [All angles of rectangle are 90°]

∠ ACD = θ_1

Exercise 8.2

Q. 1. In the adjacent figure ABCD is a parallelogram ABEF is a rectangle show that △AFD ≅ △BEC.



Answer : Given :- ABCD is a parallelogram and ABEF is a rectangle

Formula used :-

SAS congruency rule

If two sides of triangle and angle made by the 2 sides are equal then both the triangles are congruent

Solution :-

In ΔAFD and ΔBEC

*AF=BE [opposite sides of rectangle are equal]

*AD=BC [opposite sides of parallelogram are equal]

As angle of rectangle is 90°

∠ DAB +∠ FAD=90°

∠ DAB=90° - ∠ FAD -----1

Sum of corresponding angles of parallelogram is 180°

∠ DAB+∠ ABC =180°

∠ DAB+∠ ABE+∠ EBC=180°

90° - ∠ FAD + 90° +∠ EBC=180° ∵ putting value from 1

180° +∠ EBC-∠ FAD=180°

∠ EBC - ∠ FAD=0

∠ EBC = ∠ FAD

⇒ Hence;

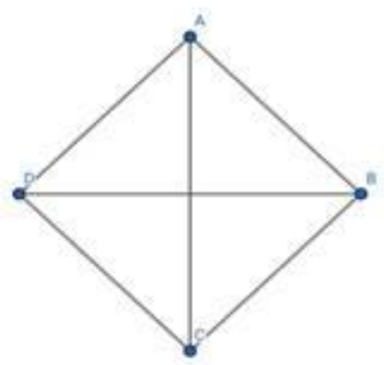
By SAS property both triangles are congruent

Conclusion :-

∆AFD ≅ ∆BEC

Q. 2. Show that the diagonals of a rhombus divide it into four congruent triangles.

Answer :



Given :- ABCD is a rhombus

Formula used :-

*SSS congruency rule

If all sides of both triangles are equal then both triangles are congruent

*properties of rhombus

<u>Solution :-</u>

In Δ AOD and Δ COB

AO=OC [diagonal of Rhombus bisect each other]

OD=OB [diagonal of Rhombus bisect each other]

AD=BC [all sides of rhombus are equal]

∴ Δ AOD ≅ Δ COB

In Δ AOB and Δ COD

AO=OC [diagonal of Rhombus bisect each other]

OD=OB [diagonal of Rhombus bisect each other]

CD=BA [all sides of rhombus are equal]

∴ Δ AOB ≅ Δ COD

In Δ AOB and Δ AOD

AO=AO [Common in both triangles]

OD=OB [diagonal of Rhombus bisect each other]

AD=AB [all sides of rhombus are equal]

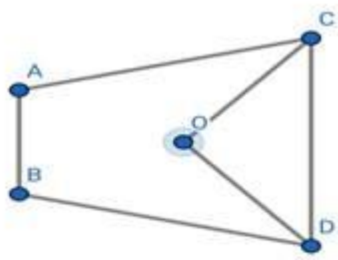
∴ Δ AOB ≅ Δ AOD

∴ All four triangles divide by diagonals of triangle are congruent

Q. 3. In a quadrilateral ABCD, the bisector of ∠C and ∠D intersect at O.

Prove that $\angle COD = \frac{1}{2}(\angle A + \angle B)$

Answer :



Given :- ABCD is a quadrilateral

∠ OCB = ∠ OCD=∠C/2 [OC is bisector of ∠ C]

∠ ODA = ∠ ODC=∠D/2 [OD is bisector of ∠ D]

Formula Used:- ∠ A+ ∠ B+∠ C+ ∠ D=360°

Solution :-

In Δ COD

∠ OCD+ ∠ COD +∠ ODC=180°

∠D/2 + ∠C/2 +∠ COD = 180°

(∠ D+∠ C)/2 + ∠ COD = 180°

If;

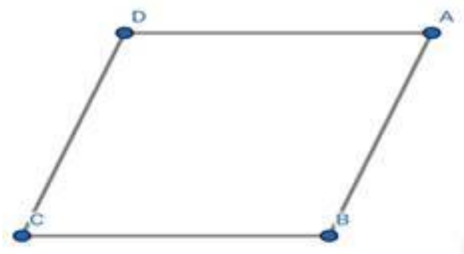
∠ A+ ∠ B+∠ C+ ∠ D=360°

∠ C+ ∠ D=360° -(∠ A+ ∠ B)

Exercise 8.3

Q. 1. The opposite angles of a parallelogram are $(3x-2)^{\circ}$ and $(x+48)^{\circ}$. Find the measure of each angle of the parallelogram.

Answer :



Given:- Opposite angles of parallelogram $(3x-2)^{\circ}$ and $(x+48)^{\circ}$.

Formula used:- opposite angles of a parallelogram are equal

Corresponding angles sum is 180°

Solution:-

Let ∠ A and ∠ C

∠ A = ∠ C [opposite angles of a parallelogram are equal]

3x-2=x+48

3x – x=48+2

2x=50

x=25

Then;

∠ A=∠ C=x+48=25+ 48 = 73°

∠ A+∠ B=180°

∠ B=180° - 73°

∠ B= 107°

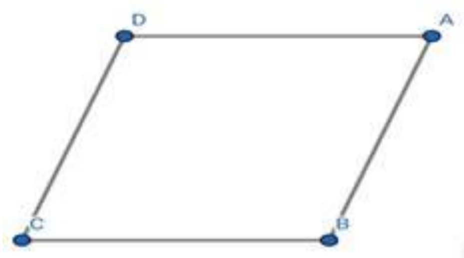
∠ B = ∠ D [opposite angles of a parallelogram are equal]

Conclusion:-

Angles of parallelogram = 73°, 107°, 73°, 107°

Q. 2. Find the measure of all the angles of a parallelogram, if one angle is 24° less than the twice of the smallest angle.

Answer :



Given:- one angle is 24° less than the twice of the smallest angle

Formula used:- opposite angles of a parallelogram are equal

Corresponding angles sum is 180°

Solution:-

Let ∠ A and ∠ C be smallest because

∠ A = ∠ C [opposite angles of a parallelogram are equal]

2∠ A+ 24° =∠ B

∠ A+∠ B=180° [Sum of corresponding angles is 180°]

∠ B=180°-∠ A

Then,

2∠ A+ 24° =180° -∠ A

3∠ A =180° - 24°

3∠ A=156°

$\angle A = \frac{156^\circ}{3} = 52^\circ$

∠ B=180° -∠ A

∠ B= 180° -52°

∠ B=128°

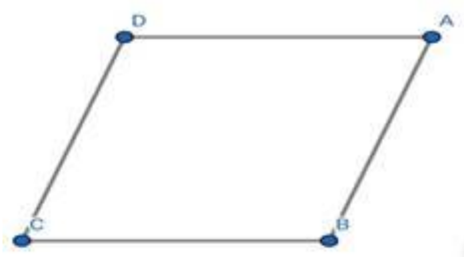
∠ B = ∠ D [opposite angles of a parallelogram are equal]

Conclusion:-

∴ Angles of parallelogram = 52°, 128°, 52°, 128°

Q. 2. Find the measure of all the angles of a parallelogram, if one angle is 24° less than the twice of the smallest angle.

Answer :



Given:- one angle is 24° less than the twice of the smallest angle

Formula used:- opposite angles of a parallelogram are equal

Corresponding angles sum is 180°

Solution:-

Let ∠ A and ∠ C be smallest because

∠ A = ∠ C [opposite angles of a parallelogram are equal]

2∠ A+ 24° =∠ B

∠ A+∠ B=180° [Sum of corresponding angles is 180°]

∠ B=180°-∠ A

Then,

2∠ A+ 24° =180° -∠ A

3∠ A =180° - 24°

3∠ A=156°

$\angle A = \frac{156°}{3} = 52°$

$\angle B = 180° - \angle A$

$\angle B = 180° - 52°$

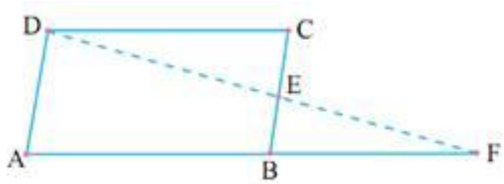
$\angle B = 128°$

$\angle B = \angle D$ [opposite angles of a parallelogram are equal]

Conclusion:-

∴ Angles of parallelogram = 52°, 128°, 52°, 128°

Q. 3. In the adjacent figure ABCD is a parallelogram and E is the midpoint of the side BC. If DE and AB are produced to meet at F, show that AF = 2AB.



Answer : Given :- ABCD is a parallelogram

And CE=EB

Formula used:- ASA congruency property

If 2 angles and one side between them in two triangles are equal then both triangle are congruent

Solutions :-

In Δ DCE and Δ FBE

$\angle DEC = \angle BEF$ [Vertically opposite angles]

As DC || BF

$\angle ECD = \angle EBF$ [Alternate angles]

And;

CE = EB [Given]

∴ Δ DCE ≅ Δ FBE

As Δ DCE ≅ Δ FBE

Then

DC = BF

And DC=AB [opposite sides of parallelogram are equal]

Hence ;

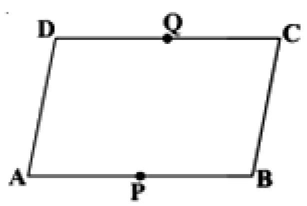
AB = BF

AF = AB + BF

AF = 2AB.

Hence Proved;

Q. 4. In the adjacent figure ABCD is a parallelogram P, Q are the midpoints of sides AB and DC respectively. Show that PBCQ is also a parallelogram.



Answer : Given:-ABCD is a parallelogram

P,Q are the midpoints of sides AB and DC respectively.

Solution:-

In parallelogram ABCD

If P,Q are the midpoints of sides AB and DC respectively

∴ DC=2QC=2QD

∴ AB=2AP=2PB

If DC||AB

Then;

QD||AP and QC||PB

Line PQ divides the parallelogram in 2 equal parts

Because P and Q are midpoints

∴ AD||PQ||CB

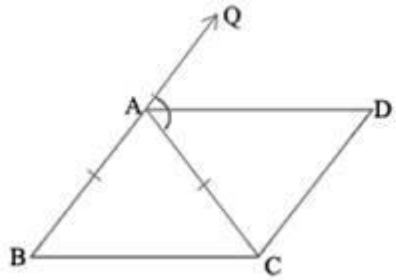
If;

PQ||CB and QC||PB

Then, PBCQ is also a parallelogram

Q. 5. ABC is an isosceles triangle in which AB = AC. AD bisects exterior angle QAC and CD || BA as shown in the figure. Show that

(i) ∠DAC = ∠BCA
(ii) ABCD is a parallelogram



Answer : Given:- AB=AC and ABC is isosceles triangle

∠ QAD=∠ DAC

CD||BA

Formula used:- sum of angles of triangle is 180°

Solution:-

As Δ ABC is a isosceles triangle

∠ ABC=∠ ACB

∠ ABC+∠ ACB+∠ CAB=180°

2∠ ACB+∠ CAB=180°

*As BAQ is a straight line

$\angle CAB + \angle DAC + \angle QAD = 180°$

*$\angle QAD = \angle DAC$ [Given]

$\angle CAB = 180° - 2\angle DAC$

Putting value of $\angle CAB$ in above equation 1

$2\angle ACB + 180° - 2\angle DAC = 180°$

$2\angle ACB = 2\angle DAC$

$\angle ACB = \angle DAC$

If;

There is $\angle ACB = \angle DAC$ and AC is the transverse

∴ These are equal by alternate angles

And AD||BC

In ABCD

If;

AD||BC & CD||BA

If both pair of sides of quadrilateral are parallel

Then the quadrilateral is parallelogram

<u>Conclusion:-</u>

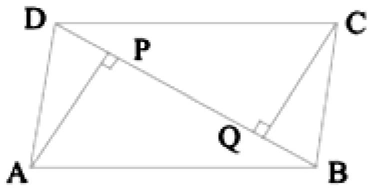
ABCD is a parallelogram

And $\angle DAC = \angle BCA$

Q. 6. ABCD is a parallelogram AP and CQ are perpendiculars drawn from vertices A and C on diagonal BD (see figure) show that

(i) $\Delta APB \cong \Delta CQD$

(ii) AP = CQ



Answer : <u>Given:-</u> ABCD is parallelogram

<u>Formula used :</u>- AAS property

If 2 angles and any one side is are equal in both triangle

Then both triangles are congruent

<u>Solution:-</u>

In Δ APB and Δ CQD

∠ CQD=∠ APB=90°

If ABCD is a parallelogram

∠ CDB=∠ DBA [alternate angles as DC||AB]

AB=DC [opposite sides of parallelogram are equal]

By AAS property

∴ Δ APB ≅ Δ CQD

If Δ APB ≅ Δ CQD

Then

AP=CQ

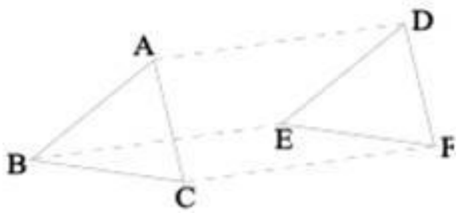
<u>Conclusion:-</u>

In parallelogram ABCD; Δ APB ≅ Δ CQD

Q. 7. In $\Delta^{s}ABC$ and DEF, AB || DE, AB=DE; BC = EF and BC || EF. Vertices A, B and C are joined to vertices D, E and F respectively (see figure). Show that

(i) ABED is a parallelogram

(ii) BCFE is a parallelogram
(iii) AC = DF
(iv) △ABC ≅ △DEF



Answer : Given:- AB||DE , AB=DE; BC||EF, BC=EF

Formula used:- SSS congruency rule

If all 3 sides of both triangle are equal then

Both triangles are congruent.

Solution:-

(1) In ABED

AB||DE

AB=DE

∴ ABED is a parallelogram

∵ If one pair of side in quadrilateral is equal and parallel

Then the quadrilateral is parallelogram.

(2) In BCFE

BC||EF

BC=EF

∴ BCEF is a parallelogram

∵ If one pair of side in quadrilateral is equal and parallel

Then the quadrilateral is parallelogram.

(3) As ABED is a parallelogram

BE||AD , BE=AD ;

As BCFE is a parallelogram

BE||CF , BE=CF ;

∴ By concluding both above statements

AD||CF and AD=CF

∴ ACFD is a parallelogram

∵ If one pair of side in quadrilateral is equal and parallel

Then the quadrilateral is parallelogram

If ACDF is a parallelogram ;

Then

AC=DF [opposite sides of parallelogram are equal]

(4) In Δ ABC and Δ DEF

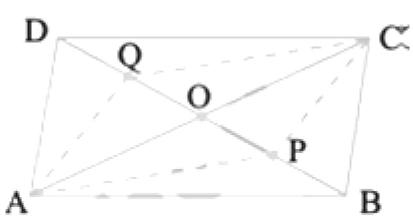
AB=DE [Given]

BC=EF [Given]

AC=DF [Proved above]

Δ ABC ≅ Δ DEF [SSS congruency rule]

Q. 8. ABCD is a parallelogram. AC and BD are the diagonals intersect at O. P and Q are the points of tri section of the diagonal BD. Prove that CQ || AP and also AC bisects PQ (see figure).



Answer : <u>Given:-</u> ABCD is parallelogram

BP=BD/3

DQ=BD/3

<u>Formula used:-</u> SAS congruency property

If 2 sides and angle between the two sides of both the triangle are equal, then both triangle are congruent

<u>Solution:-</u>

⇒ In parallelogram ABCD

AO=OC and DO=OB [diagonal of parallelogram bisect each other]

As DO=OB

Where DO=DQ+OQ

OB=OP+PB

∴ DQ+OQ=OP+PB

$\Rightarrow \frac{BD}{3}+OQ=OP+\frac{BD}{3}$

$\Rightarrow OQ=OP+\frac{BD}{3} - \frac{BD}{3}$

⇒ OQ=OP

PQ = OP+OQ = 2(OQ) = 2(OP)

∴ AC diagonal bisect PQ

⇒ In Δ QOC and Δ POA

OQ=PO [Proven above]

AO=OC [Diagonal of parallelogram bisect each other]

∠ QOC=∠ AOP [vertically opposite angles]

Hence Δ QOC ≅ Δ POA

∴ ∠ CQO=∠ OPA

If QC and PA are 2 lines

And QP is the transverse

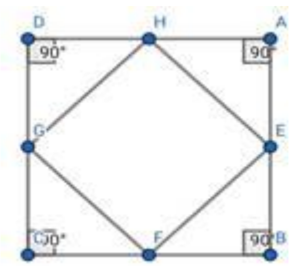
And ∠ CQO=∠ OPA by Alternate angles

∴ QC||PA

<u>Conclusion:-</u>

CQ||PA and CA is bisector of PQ.

Q. 9. ABCD is a square. E, F, G and H are the mid points of AB, BC, CD and DA respectively. Such that AE = BF = CG = DH. Prove that EFGH is a square.



Answer : <u>Given:-</u> *ABCD is a square

*E, F, G and H are the mid-points of AB, BC, CD and DA

Respectively

*AE = BF = CG = DH

<u>Formula used:-</u> *Isosceles Δ property

⇒ If 2 sides of triangle are equal then

Corresponding angle will also be equal

*Properties of quadrilateral to be square

⇒ All sides are equal

⇒ All angles are 90°

<u>Solution:-</u>

In Δ AHE, Δ EBF, Δ FCG, Δ DHG

⇒ AE = BF = CG = DH [Given]

If;

⇒ AE=EB [E is midpoint of AB]

⇒ BF=FC [F is midpoint of BC]

⇒ CG=GD [G is midpoint of CD]

⇒ DH=HA [H is midpoint of DA]

⇒ AE = BF = CG = DH

On replacing every part we get;

⇒ EB=FC=GD=HA;

⇒ ∠ A+∠ B+∠ C+∠ D=90° [All angles of square are 90°]

Hence;

All triangles Δ AHE, Δ EBF, Δ FCG, Δ DHG

are congruent by SAS property

∴ Δ AHE≅ Δ EBF≅ Δ FCG ≅ Δ DHG

⇒ HE=EF=FG=GH [All triangles are congruent]

In Δ AHE, Δ EBF, Δ FCG, Δ DHG

∵ all sides of square are equal and after the midpoint of each sides

Every half side of square are equal to half of other sides.

HA=AE , EB=FB ,FC=GC ,HD=DG

∴ All Δ AHE, Δ EBF, Δ FCG, Δ DHG are isosceles

⇒ as central angle of all triangle is 90°

It makes all Δ AHE, Δ EBF, Δ FCG, Δ DHG are right angle isosceles Δ

∴ all corresponding angles of equal side will be 45°

∠ AHE=∠ BEF=∠ CFG=∠ DHG=∠ AEH=∠ BFE=∠ CGF=∠ DGH=45°

⇒ as AB is straight line

Then; ∠ AEH+∠ HEF+∠ BEF=180°

45°+∠ HEF+45° =180°

∠ HEF=180° -90° =90°

Similarly ;

∠ EFG=90°

∠ FGH=90°

∠ GHE=90°

Conclusion:-

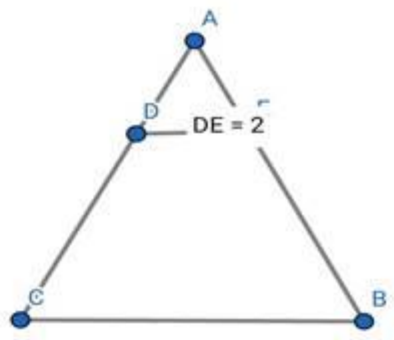
All angles are 90° and all sides are equal of quadrilateral

Hence quadrilateral is square

Exercise 8.4

Q. 1. ABC is a triangle. D is a point on AB such that $AD = \frac{1}{4}AB$ **and E is a point on AC such that** $AE = \frac{1}{4}AC,$ **If DE = 2 cm find BC.**

Answer :



Given:- DE=2cm

$$AE = \frac{1}{4}AC,$$

$$AD = \frac{1}{4}AB$$

Formula used:- Line joining midpoints of 2 sides of triangle will be half of its 3rd side.

Solution:-

Assume mid points of line AB and AC is M and N

The formation of new triangle will be there Δ AMN

As;

$$AE = \frac{1}{4}AC \Rightarrow AC=4AE$$

and $AM = \frac{1}{2}AC$

∴ AM=2AE

Hence;

E is the midpoint of AM

Similarly

D is the midpoint of AN

→ In Δ AMN

If E,D are midpoints

ED=$\frac{1}{2}$MN

MN=2ED

MN=2×2cm=4cm

→ In Δ ABC

If M,N are midpoints

AB = $\frac{1}{2}$MN

AB = 2MN

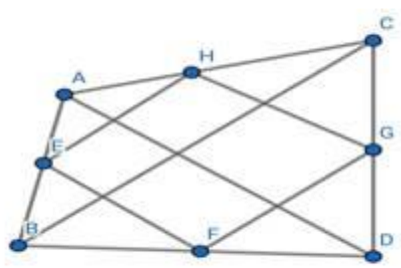
AB = 2 × 4cm = 8cm

<u>Conclusion:-</u>

BC = 8cm

Q. 2. ABCD is quadrilateral E, F, G and H are the midpoints of AB, BC, CD and DA respectively. Prove that EFGH is a parallelogram.

Answer :



<u>Given:-</u> ABCD is quadrilateral E, F, G and H are the midpoints of

AB, BC, CD and DA respectively

<u>Formula used:-</u> Line joining midpoints of 2 sides of triangle

Is parallel to 3rd side

<u>Solution:-</u>

BD is diagonal of quadrilateral

EH is the line joined by midpoints of triangle ABD,

∴EH is parallel to BD

GF is line joined by midpoints of side BC&BD of triangle BCD

∴GF is parallel to BD

⇒ If HE is parallel to BD and BD is parallel to GF

∴ It gives HE is parallel to GF

⇒ AC is another diagonal of quadrilateral

GH is the line joined by midpoints of triangle ADC,

∴GH is parallel to AC

FE is line joined by midpoints of side BC&AB of triangle ABC

∴FE is parallel to AC

⇒ If GH is parallel to AC and AC is parallel to FE

∴ It gives GH is parallel to FE

As HE||GF and GH||FE

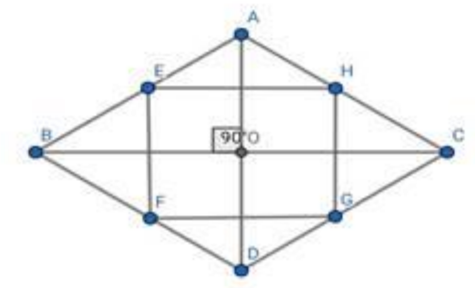
∴ EFGH is a parallelogram

<u>Conclusion:-</u>

EFGH is a parallelogram

Q. 3. Show that the figure formed by joining the midpoints of sides of a rhombus successively is a rectangle.

Answer :



Given:- ABCD is a rhombus

E,F,G,H are the mid points of AB,BC,CD,DA

Formula used:- Line joining midpoints of 2 sides of triangle

Is parallel and half of 3rd side

Solution:-

BD is diagonal of rhombus

EH is the line joined by midpoints of triangle ABD,

∴ EH is parallel and half of BD

GF is line joined by midpoints of side BC&BD of triangle BCD

∴GF is parallel and half BD

⇒ If HE is parallel to BD and BD is parallel to GF

∴ It gives HE is parallel to GF

⇒ If HE is half of BD and GF is also half of BD

∴ It gives HE is equal to GF

⇒ AC is another diagonal of rhombus

GH is the line joined by midpoints of triangle ADC,

∴ GH is parallel to AC

∴ GH is half of AC

FE is line joined by midpoints of side BC&AB of triangle ABC

∴ FE is parallel to AC

∴ FE is half of AC

⇒ If GH is parallel to AC and AC is parallel to FE

∴ It gives GH is parallel to FE

⇒ If GH is half of AC and FE is also half of AC

∴ It gives GH is equal to FE

As diagonals of rhombus intersect at 90°

And AC is parallel to FE and GH

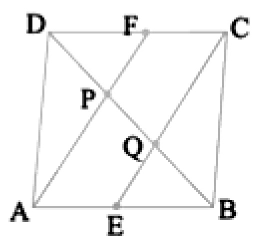
All angles of EFGH is 90°

All angles are 90° and opposite sides are equal and parallel

Conclusion:-

EFGH is a rectangle

Q. 4. In a parallelogram ABCD, E and F are the midpoints of the sides AB and DC respectively. Show that the line segments AF and EC trisect the diagonal BD.



Answer : Given:- ABCD is a parallelogram

E and F are the midpoints of the sides AB and DC respectively

Formula used:- Line drawn through midpoint of one side of triangle

Parallel to other side, bisect the 3^{rd} side.

Solution:-

As ABCD is parallelogram

AB = CD and AB||CD;

⇒ AE = CF and AE||CF [E and F are the midpoints of AB and CD]

In quadrilateral AECF

⇒ AE = CF and AE||CF

AECF is a parallelogram.

∵ Quadrilateral having one pair of side equal and parallel are parallelogram

If AECF is a parallelogram

∴ AF||CE

Hence ;

PF||CQ and AP||QE [As AF=AP+PF and CE=CQ+QE]

In Δ DQC

DF = FC [F is midpoint]

PF||CQ

Then;

P is midpoint of DQ

DP = PQ

∵ Line drawn through midpoint of one side of triangle Parallel to other side , bisect the 3rd side

In Δ APB

AE = EB [E is midpoint]

AP||QE

Then;

Q is midpoint of PB

PQ = QB

∵ Line drawn through midpoint of one side of triangle Parallel to other side , bisect the 3rd side

If DP=PQ and PQ=QB

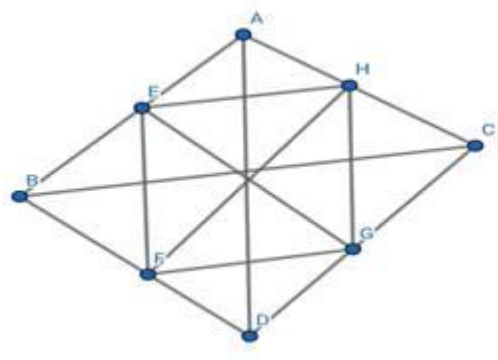
Then DP=PQ=QB

Conclusion:-

Line segments AF and EC trisect the diagonal BD.

Q. 5. Show that the line segments joining the midpoints of the opposite sides of a quadrilateral and bisect each other.

Answer :



Given:- ABCD is a quadrilateral

Formula used:- Line joining midpoints of 2 sides of triangle

Is parallel and half of 3rd side

Solution:-

BD is diagonal of quadrilateral

EH is the line joined by midpoints of triangle ABD,

∴ EH is parallel and half of BD

GF is line joined by midpoints of side BC&BD of triangle BCD

∴ GF is parallel and half BD

⇒ If HE is parallel to BD and BD is parallel to GF

∴ It gives HE is parallel to GF

⇒ If HE is half of BD and GF is also half of BD

∴ It gives HE is equal to GF

⇒ AC is another diagonal of quadrilateral

GH is the line joined by midpoints of triangle ADC,

∴ GH is parallel to AC

∴ GH is half of AC

FE is line joined by midpoints of side BC&AB of triangle ABC

∴ FE is parallel to AC

∴ FE is half of AC

⇒ If GH is parallel to AC and AC is parallel to FE

∴ It gives GH is parallel to FE

⇒ If GH is half of AC and FE is also half of AC

∴ It gives GH is equal to FE

If both opposite sides are parallel and equal

Then, the quadrilateral is parallelogram

If EFGH is parallelogram

Then their diagonal bisect each other

If diagonal of parallelogram is the line joining midpoint of opposite sides of quadrilateral

Then;

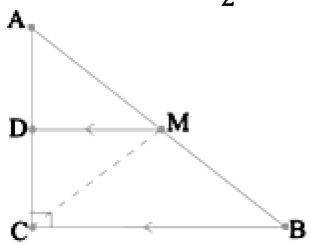
Line joining midpoints of opposite sides of quadrilateral bisect each other.

<u>Conclusion:-</u>

Lines joining midpoints of opposite sides of quadrilateral bisect each other

Q. 6. ABC is a triangle right angled at C. A line through the midpoint M of hypotenuse AB and Parallel to BC intersects AC at D. Show that

(i) D is the midpoint of AC
(ii) MD ⊥ AC
(iii) CM = MA = $\frac{1}{2}$ AB



Answer : Given:- ACB is right angle triangle

M is midpoint of AB

DM||CB

Formula used:- Line drawn through midpoint of one side of triangle

Parallel to other side, bisect the 3rd side

* SAS congruency property

If 2 sides and angle between the two sides of both the triangle are equal, then both triangle are congruent

Solution:-

(1) In Δ ABC

M is midpoint of AB

And DM||CB

∴ D is midpoint of AC

AD = DC

∵ Line drawn through midpoint of one side of triangle

Parallel to other side, bisect the 3rd side

(2) In Δ ABC

DM||CB

∠ ADM=∠ ACB [Corresponding angles]

∠ ACB = 90°

∠ ADM = 90°

(3) In Δ ADM and Δ DMC

⇒ DM=DM [Common in both triangles]

⇒ AD=DC [D is the midpoint]

As ADC is straight line

∠ ADM+∠ MDC=180°

∠ MDC=180° - ∠ ADM

∠ MDC=90°

⇒ ∠ ADM=∠ MDC

Hence;

In Δ ADM ≅ Δ DMC

∴ CM=MA

⇒ MA= $\frac{1}{2}$AB [M is midpoint of AB]

∴ CM=MA = $\frac{1}{2}$AB