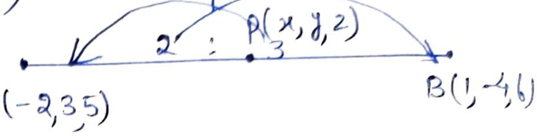
EXERCISE 12.3

QNo1 Find the coordinates of the point which divides the line segment joining the points $(-2,3,5)$ and $(1,-4,6)$ in the ratio (i) $2:3$ internally (ii) $2:3$ externally.

Sol: (i) Let $P(x,y,z)$ be the point which divides segment joining $A(-2,3,5)$ and $B(1,-4,6)$ internally in ratio $2:3$



$$\therefore \quad x = \frac{2(1)+3(-2)}{2+3} = \frac{2-6}{5} = \frac{-4}{5}$$

$$y = \frac{2(-4)+3(3)}{2+3} = \frac{-8+9}{5} = \frac{1}{5}$$

$$z = \frac{2(6)+3(5)}{2+3} = \frac{12+15}{5} = \frac{27}{5}. \quad \therefore \text{Reqd pt is } \left(-\frac{4}{5}, \frac{1}{5}, \frac{27}{5}\right)$$

(ii) Let $P(x,y,z)$ divides given pts $A(-2,3,5)$ and $B(1,-4,6)$ externally in ratio $2:3$

$\Rightarrow$ P divides AB internally in Ratio $\longrightarrow$ $2:(-3)$

$$\therefore \quad x = \frac{2(1)+(-3)(-2)}{2+(-3)} = \frac{2+6}{-1} = -8$$

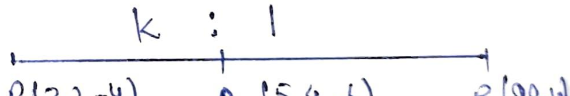
$$y = \frac{2(-4)+(-3)(+3)}{2+(-3)} = \frac{-8-9}{-1} = 17$$

$$z = \frac{2(6)+(-3)(5)}{2-3} = \frac{12-15}{-1} = \frac{-3}{-1} = 3.$$

$\therefore$ Required point is $(-8, 17, 3)$

QNo.2. Given that $P(3,2,-4)$, $Q(5,4,-6)$ and $R(9,8,-10)$ are collinear. Find the ratio in which Q divides PR.

Sol. Let Q divides PR in Ratio $k:1$



Using Section formula.

$$5 = \frac{9(k)+3(1)}{k+1}.$$

ie $5(k+1) = 9k+3 \Rightarrow 5-3 = 9k-5k \Rightarrow 2 = 4k$

$$\Rightarrow k = \frac{2}{4} = \frac{1}{2}$$

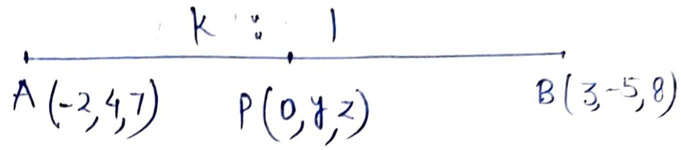
$\therefore$ Required Ratio is $k:1$

ie $\frac{1}{2}:1$

ie $1:2.$

Q No 3/12.3 Find the Ratio in which YZ-plane divides the line segment formed by joining the points $(-2,4,7)$ and $(3,-5,8)$

Sol: The equation of YZ-plane is $x = 0$



Let YZ-plane divides join of $A(-2,4,7)$ and $B(3,-5,8)$ at point $P(0,y,z)$ in Ratio $k:1$.

$\therefore$ By Section formula

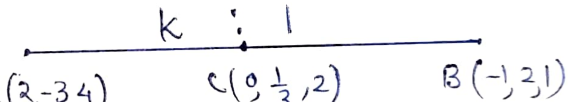
$$0 = \frac{k(3)+1(-2)}{k+1}$$

$$\Rightarrow 0 = 3k-2 \Rightarrow k = \frac{2}{3}$$

$\therefore$ YZ-plane divides AB in Ratio $k:1$ ie $\frac{2}{3}:1$ ie $2:3$.

Q No 4. Using Section formula, show that the points $A(2,-3,4)$, $B(-1,2,1)$ and $C(0,\frac{1}{3},2)$ are collinear.

Sol. Let C divides AB in Ratio $k:1$



$\therefore$ By Section formula.

$$0 = \frac{k(-1)+1(2)}{k+1} \Rightarrow -k+2=0 \Rightarrow k=2$$

$$\frac{1}{3} = \frac{k(2)+1(-3)}{k+1} \Rightarrow k+1 = 3(2k-3) \Rightarrow k+1 = 6k-9$$

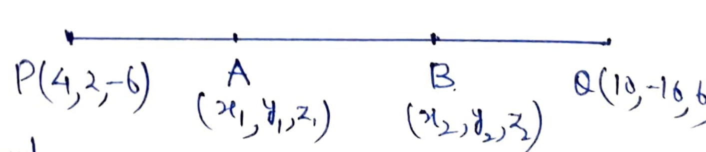
$$\Rightarrow 10 = 5k \Rightarrow k=2$$

$$2 = \frac{k(1)+1(4)}{k+1} \Rightarrow 2(k+1) = k+4 \Rightarrow 2k+2 = k+4 \Rightarrow k=2$$

$\Rightarrow$ C divides AB in ratio $2:1$

Q No 5. Find the coordinates of the points which trisect the line segments joining the points $P(4,2,-6)$ and $Q(10,-16,6)$

Sol: Let $A(x_1,y_1,z_1)$, $B(x_2,y_2,z_2)$ be the points which divides PQ in Ratio $1:2$ and $2:1$ respectively.



So A and B trisect segment PQ

Now $A(x_1, y_1, z_1)$ divides PQ internally in the ratio 1:2

$\therefore \quad x_1 = \frac{1(10)+2(4)}{1+2} = \frac{10+8}{3} = \frac{18}{3} = 6$

$y_1 = \frac{1(-16)+2(2)}{1+2} = \frac{-16+4}{3} = \frac{-12}{3} = -4$

$z_1 = \frac{1(6)+2(-6)}{1+2} = \frac{6-12}{3} = \frac{-6}{3} = -2.$

$\therefore$ A is $(6, -4, -2)$

Now $B(x_2, y_2, z_2)$ divides PQ internally in Ratio 2:1

$\therefore \quad x_2 = \frac{2(10)+1(4)}{2+1} = \frac{20+4}{3} = \frac{24}{3} = 8$

$y_2 = \frac{2(-16)+1(2)}{2+1} = \frac{-32+2}{3} = \frac{-30}{3} = -10$

$z_2 = \frac{2(6)+1(-6)}{2+1} = \frac{12-6}{3} = \frac{6}{3} = 2$

$\therefore B(8, -10, 2)$.

$\therefore$ Required points are $(6, -4, -2)$ and $(8, -10, 2)$

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