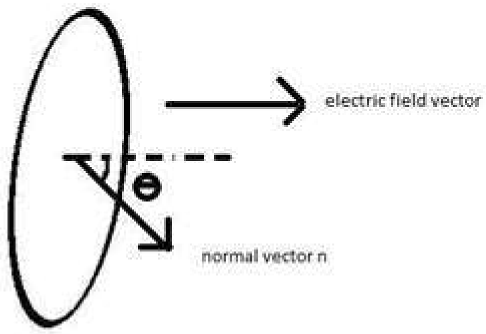
30. Gauss's Law

Short Answer

Answer.1

The flux of electric field through an area is defined as the number of field lines that pass through that area in the direction of the electric field.



In mathematical terms,

Flux, $\varphi = \int \vec{E}.ds$

Where,

E = electric field vector,

ds = small area element.

Now, **ds = n**da, where **n** is the unit normal in the direction of the area, and da is a small area element.

By the law of dot product in vector calculus,

E.ds = |E||ds|cos θ,

Where θ is the angle between the normal to the area and the electric field.

|E| = magnitude of electric field vector,

|ds| = magnitude of small area element.

Now, the flux is maximum when cos θ = 0 or 180°, since cos 0 or cos 180° = +1 or – 1.

In this case, the flux will be **|E||ds|**. In all other cases, the flux will be less than this.

The flux is minimum when cos 90°, since cos 90° = 0.

$\Phi = |E||ds| \cos(90^{0}) = 0$

Thus, when the unit normal vector perpendicular to the direction of the electric field the flux is **ZERO**.

Answer.2

No, the flux will remain the same.

We know, Flux = $\Phi = \int \mathbf{E}.\mathbf{ds}$

Where

E = electric field vector, **ds =** small area element.

Now, **ds = n.**da,

where **n** is the unit normal in the direction of the area, and da is a small area element.

By the law of dot product in vector calculus,

E.ds = |E||ds|cosθ

Where

θ is the angle between the normal to the area and the electric field.

|E| = magnitude of electric field vector,

|ds| = magnitude of small area element.

In the first case, since the axis of the ring is parallel to the electric field, the angle θ between the normal to the surface is 0°, since the normal is parallel to the axis.

When θ=180

$\Phi = |E||ds| \cos(180^{0}) = |E||ds| (-1) = -|E||ds|$

Therefore, the flux is just **|E||ds|**. When it is rotated about its diameter by 180°, the normal just rotates by 180°, and thus the angle changes to 180°. The flux becomes negative, but its value remains the same.

Answer.3

If we consider a thin spherical shell, then all its charge is concentrated in the centre. According to Gauss' law, the flux through a closed surface is $\frac{1}{\varepsilon_0}$time the charge enclosed.

flux through the closed surface, $\Phi = \int E \,.ds = \frac{q}{\epsilon_0}$,

where

ϵ_0 is the electric permittivity of vacuum,

E = electric field,

ds = surface element,

q = charge enclosed.

Now in this case since all the charge is bound to the surface, the charge enclosed by the sphere (at the centre) is 0.

Hence,

$\Phi = \int E \,.ds = \frac{0}{\epsilon_0} = 0$ and hence the electric field is also 0.

If a point charge is brought close to the shell, the charge enclosed the sphere will still be 0. Hence the field inside the sphere will still be 0.

The flux will remain 0 if the shell is conducting, since in a conductor, the charge always remains on the surface.

Thus, for a non-conducting shell, there might be some charge induced inside, due to which the electric field might change.

Answer.4

The charge Q is uniformly distributed over its surface, according to Gauss' law,
$\Phi = \int E \,.ds = \frac{q}{\epsilon_0}$,

Where

E = electric field,

Φ = electric flux flowing out through the surface

ds = surface area element, q = charge enclosed, ϵ_0 = electric permittivity in vacuum.

Here, the charge enclosed inside the shell is 0, since the electric charge on the shell is accumulated on the surface of the shell and so the electric field inside the shell will also be 0 since q in Gauss' equation is 0.

Even if the shell is hammered to reshape it, the charge is not altered, it still there is no charge inside in the center of the shell. Hence, the electric field will remain zero inside the shell even after the reshaping the shell.

Even if the charge is made of metal, the field inside will remain 0 since the charge remains on the surface.

Answer.5

Metals are conductors of electric charge, the charge placed in metal will always collect on the surface of the metal. However, it has an electrostatic shielding effect. That is, any charge in a cavity inside the metal block will be completely isolated from the outside world. This is because, if we take a Gaussian surface surrounding the cavity, the charge enclosed is zero. By Gauss' law,

$$\Phi = \int E.ds = \frac{q}{\epsilon_0},$$

Where

Φ= electric flux, E = electric field, ds = area element, q = charge enclosed, ϵ_0= electric permittivity of vacuum.

Here, q = 0 since the charge inside a conductor is 0. This justifies why the cavity in an external electric field will be completely shielded or isolated.

There will be no lines of force that can penetrate into the cavity. Therefore, even if a charge Q is brought outside the metal, the charge q inside the cavity will experience no force. Since charge distributed on the surface hence total charge will be on the surface is q+Q.

Answer.6

If the balloon does not have a spherical surface, the charge enclosed by the surface still remains 0 since the charge Q is distributed only over its surface. Hence, by Gauss' law,

$$\Phi = E.ds = \frac{q}{\epsilon_0},$$

where ϵ_0 is the electric permittivity of vacuum,

Φ= flux through closed surface, **E =** electric field, **ds** = surface element, q = charge enclosed.

Here, q(charge enclosed) = 0. Hence, the flux is 0, which give us that the electric field is also 0 everywhere.

Answer.7

A property of conductors is that all the free electrons come to its surface. We can explain this in the following way. If there was any charge inside the conductor, it would create an internal electric field. To compensate for that, an equal electric field would be created in the opposite direction to cancel out its effect. In this way, the inside of the conductor is absent of any free charges.

These are the free electrons that are responsible for all electrical phenomenon, so, if a conductor is a negative charge the free electron come to the surface of the conductor. If the conductor is given a positive charge, electron move away from the surface and leave a positive charge on the surface of the conductor.

Objective I

Answer.1

A large plastic plate and copper plate is an example of infinite long plates.

As we know,

The electric field due to infinite long sheet is $E = \frac{\sigma}{2\varepsilon_0}$

Where

σ is the surface charge density of the thin conducting sheet

ε_0 is the permittivity of the free space

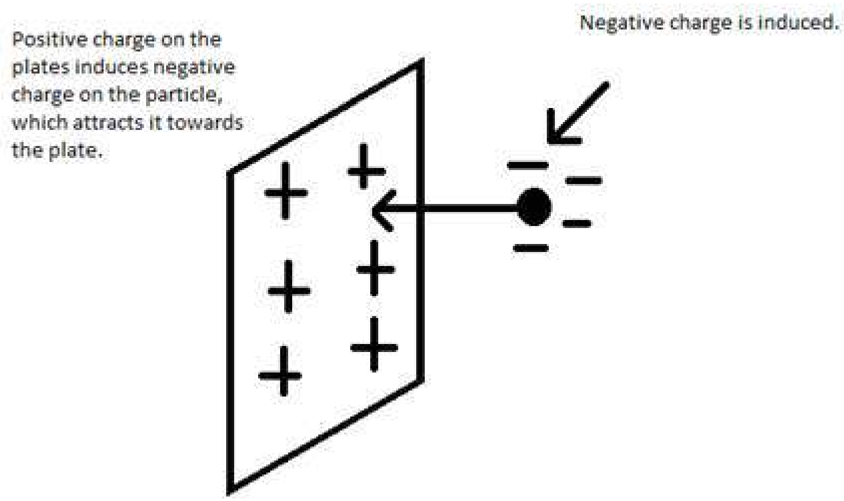
The electric field due to the plastic plate is 10 Vm^{-1}

Thus, the electric field for the copper would be same as that of the plastic plate.

The electric field of the copper plate id 10 Vm^{-1}

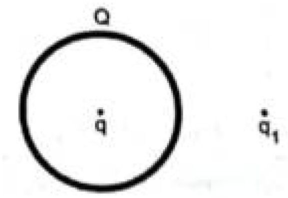
Answer.2

The metal plate carries a positive charge; it will induce an equal and opposite negative charge on the metallic particle following the law of electric induction. Hence, since electric field lines move from negative charge to positive charge, the force will pull the particle towards the plate.



Thus, option A is the correct option.

A thin, metallic spherical shell contains a charge Q on it. A point charge q is placed at the center of the shell, and another charge q_1 is placed outside it as shown in the figure. All three charges are positive. The force on the charge at the centre is:



A. towards left

B. towards right

C. upward

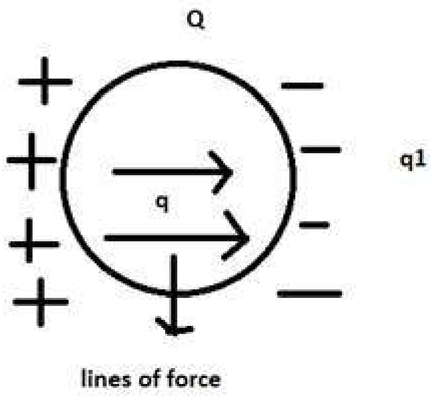
D. zero

Answer.3

Charge q_1 will only affect the outside charge Q. However, a charge can only induce charge on the surface of the conductor, but not on any charge inside it. Hence, there will be no force on the charge inside the conductor, q.

Answer.4

We know that electric field lines originate from a positive charge and terminate towards a negative charge. Now, the charge Q outside will try to nullify the field lines that emerge from charge q1, and hence, negative charge will be induced on the face nearer to q1. This will induce positive charge on the farther face of the sphere. Hence, a force acts on the central charge from left to right, that is, from negative to positive.



Answer.5

Gauss' law states that the electric flux through a closed surface is equal to $1/\epsilon 0$ times the total charge enclosed by the surface.

$$\Phi = \int E.ds = \frac{q}{\epsilon_0},$$

where

Φ = electric flux

E = electric field

ds = area element

q = charge enclosed

ϵ_0 = electric permittivity of vacuum

Hence, the flux depends on the charge enclosed.

In this case, even though the volume of the sphere enclosing the charge is increasing, the net charge enclosed inside it is remaining the same.

Hence, the electric field will remain the same = 25 V m. (Option a)

Answer.6

Graph d

According to Gauss' law,

We know, Electric flux = **E**.d**s** = q/ϵ_0,

Where

q = total charge enclosed by the surface,

ϵ_0 = electric permittivity of vacuum.

Now at t=0, the left end just touches the centre of the face of the cube opposite the left face.

Initially, charge inside the cube = 0.

Hence, the graph starts from the origin.

Now when the rod is introduced inside, the charge inside the cube slowly increases linearly, since the length of the rod going inside the cube increases, and $q_{inside} = \lambda l$, where λ = linear charge density of the rod, and l = length of rod inside the cube. This indicates the rise in the graph.

When a length of L/2 is inside the cube, that is its maximum length is filled, then the charge inside the cube is maximum, which will be L/2 times the total charge carried by the rod.

As long as the inside of the cube stays filled with a portion of length L/2, the flux remains constant, indicating the horizontal line in the graph.

Now when the length of the rod inside slowly keeps decreasing as the rod moves out of the, the charge inside it slowly decreases which also leads to a linear decrease in the flux.

Hence, the graph is completed with the straight line with negative slope until the flux becomes 0(rod is completely out of the cube).

Answer.7

Gauss' law states the electric flux through an enclosed surface is $1/\epsilon_0$ times the charge enclosed by the surface**.**

$$\Phi = \int E.ds = \frac{q}{\epsilon_0}$$

where

Φ = electric flux,

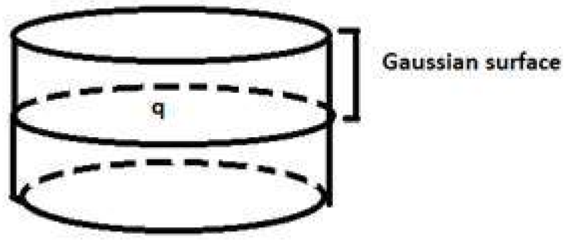
E = electric field,

ds = area element,

q is the charged enclosed,

ϵ_0 is the electric permittivity of vacuum.

In this case, the surface is not closed, since we are considering the open end of a cylindrical vessel. We construct another identical cylindrical vessel on top of it.



Now, the charge enclosed by two of the surfaces is q.

Hence, the charge enclosed by only of the surfaces will be q/2.

Therefore, the net flux will be half of the total = $q/2\epsilon_0$.

Objective II

Answer.1

Gauss' law states that the flux through a surface is equal to $1/\epsilon$ times the total charge enclosed by the surface(if the surface enclosing is in a medium other than vacuum)

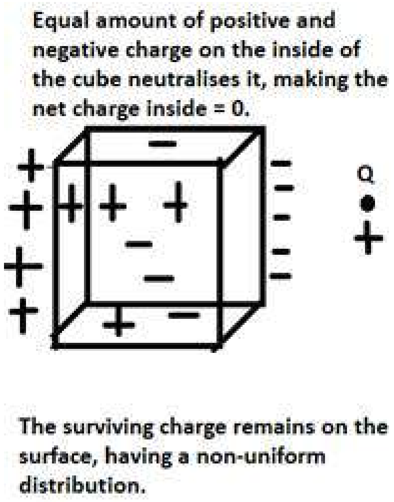
$$\Phi = \int E.ds = \frac{q}{\epsilon_o},$$

where ϵ is the electric permittivity of that medium.

Hence, the flux through the Gaussian surface will be due to charges enclosed by it (option d)

Answer.2

The face of the metal cube closest to the point charge, when brought near to it, had negative charge induced on it. This negative charge induces an equal and opposite positive charge on its immediate adjacent surface in the interior. In this way, the faces of the cube induce charges in the interior such that the total charge in the interior becomes zero. The net charge is on the surface of the cube, and it has a non-uniform charge distribution, consisting of both positive and negative charges.

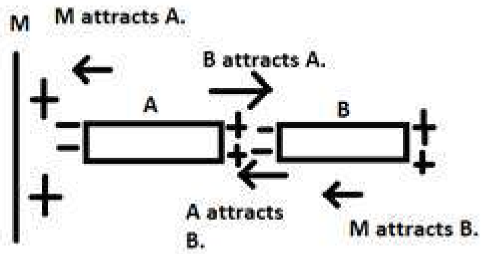


Answer.3

The face of rod A which faces M will develop an induced charge on its surface, equal and opposite to the charge on M. Hence it will attract A(option a).

Now on the far end of A, charge equal and opposite to the nearer end of A will be induced, and this will induce charge opposite to M as before, on the nearer end of B. Hence, B will have charge of same polarity as that on the nearer end of A, and will be attracted by M(Option b).

A and B have opposite polarities of charges on the sides facing each other, and hence they will attract each other as well(Options c and d).



The attraction between M, A and B if M had positive charge.

Answer.4

Gauss' law states that electric flux through an enclosed surface is equal to $1/\epsilon_0$ times the charge enclosed by the surface.

$$\Phi = \int E . ds = \frac{q}{\epsilon_0},$$

where,

Φ = electric flux,

ds = area element,

q = charge enclosed,

E = electric field,

ϵ_0 = electric permittivity of vacuum

Now, this law does not include the flux which is located on the surface. So it may or may not be zero, even though the flux through the surface is zero(option b).

Since the flux through the surface is zero, by Gauss' law, the charge inside the surface must be zero(option c).

Answer.5

A dipole consists of an equal positive and negative charge separated by a distance. Hence, the net charge due to a dipole is 0. Since the sphere encloses the dipole, by Gauss' law,

$$\Phi = \int E\,.ds = \frac{q}{\epsilon_o}$$

where

Φ = electric flux,

ds = area element,

q = charge enclosed,

E = electric field,

ϵ = electric permittivity of vacuum.

q(enclosed charge) = 0. Hence, flux = 0(option a).

At any point on the sphere, the electric field is given by

$E = \frac{p}{4\pi\epsilon_0 r^3}\sqrt{1+cos^2\theta}$, where

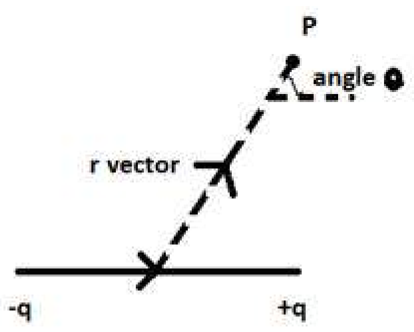
p = dipole moment

r = distance of the point from the centre of the dipole

E = electric field

Ɵ = angle between the dipole and the line joining the dipole with that point

ϵ = electric permittivity of vacuum



dipole with dipole moment p

Hence, the electric field anywhere on the sphere is not 0, since $\cos^2\theta$ cannot be equal to -1(option c).

Answer.6

As we know, the electric flux through a surface element ds is

$$\Phi = \vec{E}.\Delta\vec{S} = |E||\Delta S \cos\theta$$

Where,

$\vec{E}$ is the electric field passing through the area

ΔS is the area element

θ is the angle between the area element and electric field

The point A and C lie in the straight line with the charge q,

While the points B and D make an angle θ with the area element,

Thus, Electric flux at point A and C is

$\varphi = \overline{\vec{E}}.\Delta\vec{S} = |E||\Delta S|\cos\theta$ ($\theta=0^o$ for the point in the straight line)

$$\varphi = \overline{\vec{E}}.\Delta\vec{S} = |E||\Delta S|\cos\theta = |E||\Delta S|$$

Hence, the flux of the electric field through the hemisphere remains unchanged when second charge is placed at point A and C.

Answer.7

At any interval of time, the net charge enclosed by the surface S is zero, since an equal number of electrons enter and leave the surface. So the charge enclosed is 0.

According to Gauss' law, electric flux through a closed surface is equal to $1/\epsilon_0$ times the charge enclosed by the surface.

$$\Phi = \int E.ds = \frac{q}{\epsilon_0},$$

Where

Φ = electric flux

E = electric field

ds = area element

q = charge enclosed.

ϵ_0 = electric permittivity of vacuum

Here,

Q (total charge enclosed by the surface) = 0.

Hence, flux of electric field is zero and remains unchanged.

Answer.8

The conducting sphere will have a negative charge induced on the surface which is closer to the point P. However, it will have equal positive charge induced on the farther end by electric induction. Hence, by Gauss' law, the flux through a closed surface is equal to 1/ϵ0 times the total charge enclosed by it.

$$\Phi = \int E.ds = \frac{q}{\epsilon_0}$$

E = electric field,

q = charge enclosed.

The charge enclosed by the closed surface will not be zero, but will consist of the charge in the intersecting region of the sphere and the surface, which is positive. Hence, the flux of electric field will be positive.

Exercises

Answer.1

Given:

Electric field : $\frac{3}{5}E_0 i + \frac{4}{5}E_0 j$

E_0=2.0× 10^3 N/C

Surface area of plane=0.2m^2

The plane is parallel to y-z plane. The normal to this plane is parallel to x-axis. Therefore, area vector for this plane is given by $0.2m^2\hat{i}$

∴ $\Delta S = 0.2m^2\hat{i}$

We know that,

Flux of electric field through a surface of area ΔS is given by dot product of electric field $\vec{E}$ with surface area $\Delta\vec{S}$

$$\phi = \vec{E}.\Delta\vec{S}$$

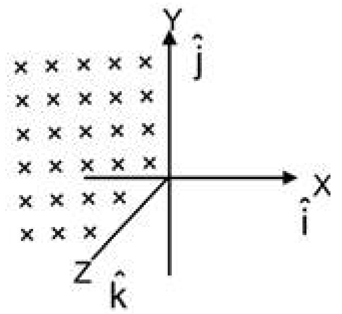
$$\Rightarrow \phi = \left(\frac{3}{5}E_0\hat{\imath} + \frac{4}{5}E_0\hat{\jmath}\right).(0.2\hat{\imath}) Nm^2/C$$

$$\Rightarrow \phi = \frac{3}{5} \times 0.2 \times E_0 \ Nm^2/C$$

$$\Rightarrow \phi = \frac{3}{5} \times 0.2 \times 2 \times 10^3 \ Nm^2/C \text{ (putting the value of } E_0\text{)}$$

$$\phi = 240 \ Nm^2/C$$

Therefore the flux of this electric field through the given plane surface is 240Nm²/C.



Answer.2

Given:

Length of rode=edge of cube=l

Portion of rod inside cube=l/2

Total charge =Q

Linear charge density of rod = Q/l of rod=λ

We know that,

By Gauss's law, flux of net electric field through a closed surface equals the net charge enclosed by the surface divided by ϵ_0

$$\oint \vec{E}.d\vec{S} = \frac{q_{in}}{\epsilon_0} \text{.....(i)}$$

Where q_{in} is the net charge enclosed by the surface through which the flux is calculated.

$\vec{E}$ =net electric field at the surface

$d\vec{S}$ =area of differential surface element

Using gauss law flux through the cube is given by Q_{in}/ϵ_0 where

Q_{in} is the charge enclosed by the cube

Charge enclosed by the cube is given by charge density × length of rod inside cube

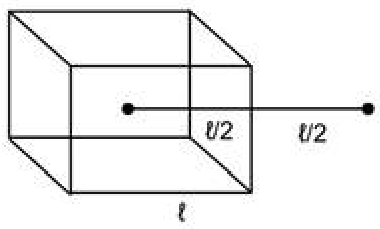
$$Q_{in} = \frac{Q}{l} \times \frac{l}{2}$$

$$\Rightarrow Q_{in} = \frac{Q}{2}$$

∴ by (i)

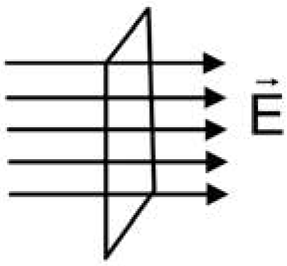
$$\text{Flux}\phi = \frac{Q}{2\epsilon_0}$$

Therefore flux of electric field through the entire surface of cube is given by $Q/2\epsilon_0$



Answer.3

Given the electric field is uniform, if we consider a plane perpendicular to electric field we can say that this plane is an **equipotential surface**(as equipotential surfaces are perpendicular to the electric field lines.)



Equipotential surface: A surface on which all points on the surface have the same electric potential.

That means by definition of potential difference (between two points) which is defined as work done in moving a unit test charge from one point to another point,

We can say that zero work is done in moving a unit test charge on this surface.

Let us assume that some charge is present in this region where electric field is uniform. Now due to this charge our test charge when introduced experiences a force (repulsive or attractive) due to which non-zero work needs to be done to move our test charge from one point to another.

And if non zero work is done that means the potential difference between these two points is finite (and not zero) and the surface can't be considered as equipotential.

This is a contradiction to the fact that the surface is equipotential.

Therefore net charge in the region of uniform electric field is zero.

Answer.4

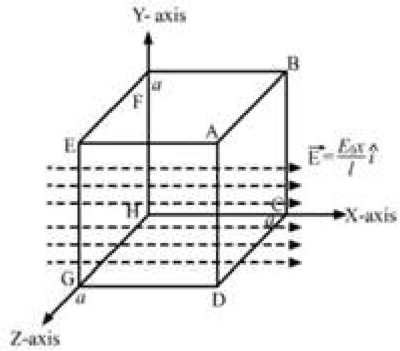
Given:

$\vec{E} = \frac{E_0 x}{l}\hat{i}$

$E_0 = 5\times 10^3 N/C$

$l = 2cm = 2\times 10^{-2}m$

$a = 1cm = 1\times 10^{-2}m$



We know that,

Flux of electric field through a surface of area ΔS is given by dot product of electric field $\vec{E}$ with surface area $\Delta\vec{S}$

$$\phi = \vec{E}.\Delta\vec{S}$$

From the fig. we can see that electric field lines are parallel to the normal of the surfaces ABCD and EFGH therefore flux is nonzero through these surfaces, whereas for other faces of the cube electric field is perpendicular to normal of faces therefore flux through these surfaces is zero

For face ABCD:

Area vector $\Delta\vec{S} = a^2\hat{i}$

Electric field $\vec{E} = \frac{E_0(x=a)}{l}\hat{\imath} = \frac{E_0 a}{l}\hat{\imath}$

∴ flux from (i) is given by

$$\phi_{ABCD} = \frac{E_0 a}{l}\hat{\imath}.(a^2\hat{\imath})$$

$$\Rightarrow \phi_{ABCD} = \frac{E_0 a^3}{l}$$

For face EFGH

Area vector $\Delta\vec{S} = -a^2\hat{i}$

Electric field $\vec{E} = \frac{E_0(x=0)}{l}\hat{\imath} = 0\hat{\imath}$

∴ flux from (i) is given by

$$\phi_{EFGH} = 0$$

∴ Total flux = $\frac{E_0 a^3}{l}$

$$\Rightarrow \phi = 5\times 10^3 \times \frac{(1\times10^{-2})^3}{2\times10^{-2}}\,\text{Nm}^2/\text{C}$$

$$\Rightarrow \phi = 2.5\times 10^{-1}\text{Nm}^2/\text{C}$$

We know that,

By Gauss's law, flux of net electric field through a closed surface equals the net charge enclosed by the surface divided by ϵ_0

$$\oint \vec{E}.d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

$\Rightarrow q_{in} = \phi \times \epsilon_0$

$\Rightarrow q_{in} = 2.5 \times 10^{-1} \times 8.85 \times 10^{-12}$ C

$\therefore q_{in} = 2.2125 \times 10^{-12}$ C

Therefore the net charge contained inside the cubical volume is given by 2.2125×10^{-12} C

Answer.5

Given:

Total charge in the cube =Q at the center

We know that,

By Gauss's law, flux of net electric field through a closed surface equals the net charge enclosed by the surface divided by ϵ_0

$$\oint \vec{E}.d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

From Gauss's law we can conclude that total flux enclosed by the cube is given by

$$\phi = \frac{Q_{in}}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

Since charge is placed at centre it is placed symmetrically with respect to all the faces of the cube . therefore we can say that equal flux passes through all the six surfaces.

$\therefore$ flux passing through each surface $\phi_0 = \frac{\phi}{6}$

$\Rightarrow \phi_0 = \frac{Q}{6\epsilon_0}$

Therefore flux of electric field passing through each of the six surfaces are equal and is given by $Q/6\epsilon_0$

Answer.6

Given:

Length of square edge=a

Distance of charge Q above the center=a/2 **Concept: Imagine the**

square to be one face of a cube of side a
center at a point where charge Q is placed.
with the

Since charge is placed at centre it is placed symmetrically with respect to all the faces of the cube . therefore we can say that equal flux passes through all the six surfaces.

∴ flux passing through each surface $\phi_0 = \frac{\phi}{6\epsilon_0}$

We know that,

By Gauss's law, flux of net electric field through a closed surface equals the net charge enclosed by the surface divided by ϵ_0

$$\oint \vec{E}.d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

$$\Rightarrow \phi_0 = \frac{Q}{6\epsilon_0}$$

(as q_{in}=Q)

Therefore flux of electric field through square surface is given by Q/6ϵ_0

Answer.7

Given:

Magnitude of charges=10^{-7}C

One at centre and another at a point 2R from centre.

We know that,

By Gauss's law, flux of net electric field ($\vec{E}$) through a closed surface S equals the net charge enclosed (q_{in}) by the surface divided by ϵ_0

$$\oint \vec{E}.d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

Here,

$Q_{in} = q = 10^{-7}$C

(Note that gauss's law include only those charges which are enclosed by the closed surfaces and not the charges inside the surfaces)

Therefore flux through the sphere is given by

$$\phi = \frac{q}{\epsilon_0} = \frac{10^{-7}}{8.85\times10^{-12}}\text{Nm}^2/\text{C}$$

$$\Rightarrow \phi = 1.1 \times 10^4\text{Nm}^2/\text{C}$$

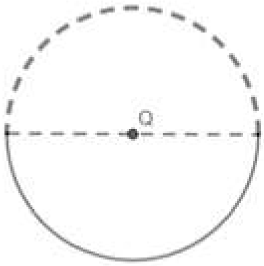
Therefore flux of electric field through a spherical surface of radius R is given by 1.1× 10^4Nm^2/C

Answer.8

Given:

Charge Q placed at centre of imaginary hemispherical surface

Concept: Consider the hemisphere to be one half of an imaginary sphere in which charge Q is at the centre



We know that,

By Gauss's law, flux of net electric field ($\vec{E}$) through a closed surface S equals the net charge enclosed (q_{in}) by the surface divided by ϵ_0

$$\oint \vec{E}.d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

Net charge enclosed by the sphere Q_{in}=Q

Therefore from Gauss's law flux of electric field through this sphere

$$\phi = \frac{Q}{\epsilon_0}$$

Since charge is symmetrically located with respect to sphere equal flux pass through each part of sphere of equal area. Therefore flux passing through both the hemispherical surfaces at the top and bottom are the same

So the flux passing through the hemispherical surface $=\frac{Q}{2\epsilon_0}$

Therefore flux passing of electric field due to this charge through the surface of the hemisphere is $Q/2\epsilon_0$

Answer.9

Given:

Volume charge density $\rho = 2.0\times 10^{-4} C/m^3$

We have to find the electric field at a point at a distance 4cm from the centre.

Assume a spherical gaussian surface inside the sphere of radius 4cm=r

Volume of this spherical surface $=\frac{4}{3}\pi r^3$

Charge enclose by this gaussian spherical surface=(volume charge density)× (volume of gaussian surface containing charge)

$Q_{in} = \rho \times \frac{4}{3}\pi r^3$ ……(i)

Surface area of this spherical gaussian surface=$4\pi r^2$

All the points on this surface are equivalent and by symmetry we can say that field at every point on this surface is equal in magnitude and radial in direction.

Therefore flux through this surface can be written as

$\phi = \oint \vec{E}.d\vec{S} = E\oint d\vec{S} = E.4\pi r^2$ …..(ii)

We know that,

By Gauss's law, flux of net electric field ($\vec{E}$) through a closed surface S equals the net charge enclosed (q_{in}) by the surface divided by ϵ_0

$$\oint \vec{E}.d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

Using gauss's law and eqn.(i) and eqn.(ii)

$$E.4\pi r^2 = \rho \times \frac{4\pi r^3}{\epsilon_0}$$

$$\Rightarrow E = \frac{\rho r}{\epsilon_0}$$

Putting the value of ρ, r and ϵ_0

$$E = (2 \times 10^{-4}) \times \frac{4\times10^{-2}}{8.85\times10^{-12}} \text{N/C}$$

$$E = 3 \times 10^5 \text{N/C}$$

Therefore electric field at a point inside volume at a distance 4cm from the centre is given by 3× 10^5N/C

Answer.10

Given:

Radius of gold nucleus=7.0× 10^{-15}m=r

Charge on the nucleus=79× 1.6× 10^{-19}C=q

(a)

Consider a gaussian spherical surface of radius of the nucleus=r

By symmetry all points on this surface are equivalent and electric field at all points have same magnitude and is in radial direction

Therefore flux passing through this surface is given by

$$\phi = \oint \vec{E}.d\vec{S} = E\oint d\vec{S} = E.4\pi r^2 \text{...(i)}$$

We know that,

By Gauss's law, flux of net electric field ($\vec{E}$) through a closed surface S equals the net charge enclosed (q_{in}) by the surface divided by ϵ_0

$$\oint \vec{E}.d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

Charge enclosed by this sphere =total charge on the nucleus=q

Using gauss's law and eqn.(i)

$$E.4\pi r^2 = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{4\pi r^2\epsilon_0}$$

Putting the values of r,q and ϵ_0

$$E = 79 \times 1.6 \times \frac{10^{-19}}{4 \times 3.14 \times (7 \times 10^{-10})^2 \times 8.85 \times 10^{-12}}$$

$$E = 2.31 \times 10^{21}\text{N/C}$$

Therefore the strength of electric field at the surface of nucleus is 2.31× 10^{21}N/C

(b)for calculating electric field at middle point of radius consider similar gaussian surface with radius half of that of nucleus=r/2

Charge enclosed by volume $\frac{4}{3}\pi r^3$ =Q

Charge enclosed by volume $\frac{4}{3\pi\left(\frac{r}{2}\right)^3}$ =Q'

We get

$$Q' = \frac{Q}{8}$$

Using gauss law and eqn.(i)

$$E.4\pi r^2 = \frac{Q}{8\epsilon_0}$$

$$E = \frac{Q}{32\pi r^2 \epsilon_0}$$

$$E = 2.31 \times \frac{10^{21}}{8} \text{N/C}$$

$$E = 1.16 \times 10^{21} \text{N/C}$$

Therefore electric field at middle point of radius of nucleus is given by 1.16× 10^{21}N/C

We know that when electric charge is given to a conductor it comes on its surface

But nucleons are bound by strong nuclear force inside nucleus which holds them and prevents them from coming out of conductor.

Therefore it is justified to assume that electric charge is uniformly distributed in its entire volume.

Answer.11

Given:

Amount of charge distributed within the hollow sphere=Q

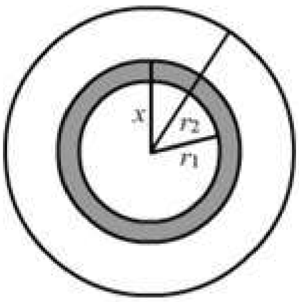
Inner and outer radius of hollow sphere=r_1 and r_2

Volume charge density of hollow sphere is given by

$$\rho = \frac{total\ charge}{volume\ of\ charge}$$

$$\rho = \frac{Q}{\frac{4}{3}\pi(r_2^3 - r_1^3)} \text{ ....(i)}$$

To find the electric field at a point P at a distance x away from the centre consider a spherical gaussian surface of radius x($r_1 < x < r_2$)



By symmetry all points on this surface are equivalent and electric field at all points have same magnitude and is in radial direction

Therefore flux through this surface is given by

$$\oint \vec{E}, d\vec{S} = E \oint d\vec{S} = E.4\pi x^2 \text{ ..(ii)}$$

Charge enclosed by this surface is given by

$$Q_{in} = volume\ charge\ density \times volume\ of\ gaussian\ surface\ containing\ charge$$

$$Q_{in} = \frac{Q}{\frac{4}{3}\pi(r_2^3 - r_1^3)} \times \frac{4}{3}\pi(x^3 - r_1^3)$$

$$Q_{in} = \frac{x^3 - r_1^3}{r_2^3 - r_1^3} Q \text{ ...(iii)}$$

We know that,

By Gauss's law, flux of net electric field ($\vec{E}$) through a closed surface S equals the net charge enclosed (q_{in}) by the surface divided by ϵ_0

$$\oint \vec{E}.d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

Using gauss's law and eqn.(ii) and (iii)

$$E.4\pi x^2 = \frac{x^3 - r_1^3}{r_2^3 - r_1^3} Q$$

$$\boldsymbol{E = \frac{(x^3 - r_1^3)Q}{(r_2^3 - r_1^3)\ 4\pi x\text{^}2}}$$

This electric field is directly proportional to x for $r_1<x<r_2$

For $r_2 < x < 2r_2$

Considering a similar gaussian spherical surface of radius x such that $r_2 < x < 2r_2$

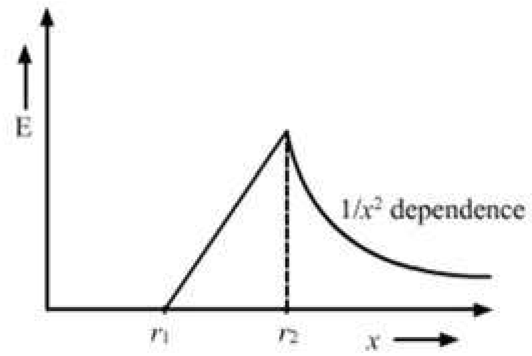
Charge enclosed by this surface =Q

Total electric flux through this surface = $E.4\pi x^2$

Using gauss law we can write

$$E = \frac{Q}{4\pi x^2 \epsilon_0}$$

The graph showing electric field as a function of x is shown as follows



Answer.12

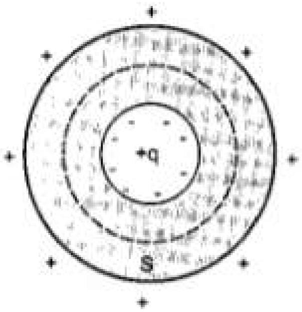
Given:

Charge present at the centre of hollow metallic sphere=Q

Radius of sphere=a

Surface area of sphere=$4\pi a^2$

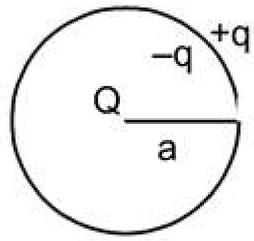
We know that,



Electric fields at all points inside the conductor is zero

So,

If a charge q is placed within the cavity of a metallic sphere then taking the Gaussian surface S as shown in fig. electric field $\vec{E}$ =0 at all points on this surface and hence $\oint \vec{E}.d\vec{S} = 0$ which ensures that charge contained in S is zero (by gauss's law)



And if a charge +q is place din the cavity , there must be a charge -q on the inner surface of the conductor. If the conductor is neutral i.e. no charge is placed on it,a charge +q will appear on the outer surface.

So here,

Charge induced at inner surface =-Q

Charge induced at outer surface=+Q

Surface charge density is given by charge/total surface area

∴ surface charge density on inner surface= $-\frac{Q}{4\pi a^2}$

Surface charge density on outer surface= $+\frac{Q}{4\pi a^2}$

Therefore surface charge densities on inner and outer surfaces are given by -$Q/4\pi a^2$ and $Q/4\pi a^2$ respectively.

(b) If a charge q is put on the sphere all the charge will move to the outer surface of the sphere(since, total charge enclosed by the sphere is zero)

Therefore inner charge density will remain same and outer charge density will increase as the new outer charge is Q+q

∴ surface charge density on inner surface= $-\frac{Q}{4\pi a^2}$

∴ surface charge density on outer surface= $\frac{Q+q}{4\pi a^2}$

Therefore surface charge densities of inner and outer surfaces after putting charge q on the sphere is given by -$Q/4\pi a^2$ and $(Q+q)/4\pi a^2$ respectively

(c)

To find the electric field inside the sphere at a distance x from the sphere consider a spherical Gaussian surface of radius x

Applying Gauss law on this sphere

$$\oint \vec{E}.d\vec{S} = \frac{Q_{in}}{\epsilon_0}$$

$$E\oint d\vec{S} = E.4\pi x^2 = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi\epsilon_0 x^2}$$

Where q is charge enclosed by the sphere

For situation(b) the charge enclosed by the Gaussian surface remains the same as the charge provided to the metallic sphere it will move to the outer surface of the sphere

$$\therefore E = \frac{Q}{4\pi\epsilon_0 x^2}$$

Therefore for situations (a) and (b) the electric field inside the sphere at a distance x from the centre is same and given by $Q/4\pi\epsilon_0 x^2$

Answer.13

Given:

Radius of nucleus=10^{-15}m

Radius of 1-s charge cloud=1.3×10^{-11}m=r_1

Radius of 2-s charge cloud =5.2×10^{-11}m=r_2

(a)for calculating electric field at a point just inside 1s cloud

Consider a gaussian spherical surface of radius just equal to the radius of 1s cloud=r_1

By symmetry all points on this surface are equivalent and electric field at all points have same magnitude and is in radial direction

Therefore flux passing through this surface is given by

$\phi = \oint \vec{E}.d\vec{S} = E\oint d\vec{S} = E.4\pi r_1^2$...(i)

We know that,

By Gauss's law, flux of net electric field ($\vec{E}$) through a closed surface S equals the net charge enclosed (q_{in}) by the surface divided by ϵ_0

$$\oint \vec{E}.d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

Charge enclosed by this sphere =total charge on the nucleus=q

Now,

q= $(charge\ of\ 4protons\ inside\ nucleus) = (4)\times 1.6\times 10^{-19}$C

$\boldsymbol{q = 4\times 1.6\times 10^{-19}}$**C**

Using gauss's law and eqn.(i)

$$E.4\pi r^2 = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{4\pi r_1^2\epsilon_0}$$

Putting the values of r_1 ,q and ϵ_0

$$E = \frac{2\times 1.6\times 10^{-19}}{4\times 3.14\times(1.3\times 10^{-11})^2\times 8.85\times 10^{-12}}\text{N/C}$$

$\boldsymbol{E = 3.4\times 10^{13}}$**N/C**

Therefore electric field at a point just inside the 1s cloud is given by 3.4× $\boldsymbol{10^{13}}$N/C

(b)for calculating electric field at a point just inside the 2s cloud

Consider a gaussian spherical surface of radius just equal to the radius of 2s cloud=r_2

By symmetry all points on this surface are equivalent and electric field at all points have same magnitude and is in radial direction

Therefore flux passing through this surface is given by

$\phi = \oint \vec{E}.d\vec{S} = E\oint d\vec{S} = E.4\pi r_2^2$...(i)

We know that,

By Gauss's law, flux of net electric field ($\vec{E}$) through a closed surface S equals the net charge enclosed (q_{in}) by the surface divided by ϵ_0

$$\oint \vec{E}.d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

Charge enclosed by this sphere =total charge on the nucleus+ charge of 2 1s electrons=q

Now,

q= $(charge\ of\ 4protons\ inside\ nucleus) + (charge\ of\ two\ electrons\ inside\ 1s\ cloud) = (4-2) \times 1.6 \times 10^{-19}$ C

$\boldsymbol{q = 2 \times 1.6 \times 10^{-19}}$**C**

Using gauss's law and eqn.(i)

$$E.4\pi r^2 = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{4\pi r_2^2 \epsilon_0}$$

Putting the values of r_1 ,q and ϵ_0

$$E = \frac{2 \times 1.6 \times 10^{-19}}{4 \times 3.14 \times (5.2 \times 10^{-11})^2 \times 8.85 \times 10^{-12}} \text{ N/C}$$

$\boldsymbol{E = 1.1 \times 10^{12}}$**N/C**

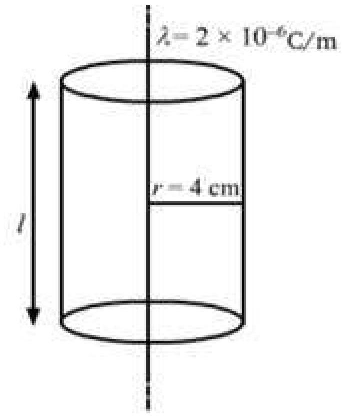
Therefore electric field at a point just inside the 2s cloud is given by 1.1× $\boldsymbol{10^{12}}$**N/C**

Answer.14

Given:

Charge density of line charge=2×10^{-6}C/m=λ

Distance of point from line charge=4cm=4×10^{-2}m=r



Consider a Gaussian cylindrical surface around the line charge of radius 4cm and height l.

All the points on the curved part of this gaussian surface are at the same perpendicular distance from the line charge. Therefore all these points are equivalent. The electric field at all these points will have the same magnitude E and direction of field at any point on the curved surface is normal to the line and hence normal to the cylindrical surface.

We know that,

By Gauss's law, flux of net electric field through a closed surface equals the net charge enclosed by the surface divided by ϵ_0

$\oint \vec{E}.d\vec{S} = \frac{q_{in}}{\epsilon_0}$ ...(i)

Through the circular faces at the top and bottom the angle between electric field and area vector is 90° therefore flux through these surfaces is zero.

By applying gauss's law for the curved cylindrical surface,

$$\oint \vec{E}.d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

Where $\vec{E}$=electric field vector

$d\vec{S}$ =area of differential surface element

q_{in}=charge enclosed by Gaussian surface

curved surface area of cylinder=2πrl

flux through the curved part is given by

$$\phi = \oint \vec{E}.d\vec{S} = E\oint d\vec{S} = E.2\pi rl$$

The total flux through the gaussian surface is given by 2πrl

The charge enclosed by the gaussian surface is (line charge density × height of cylindrical surface)

⇒ λ × l

Using gauss's law(i)

$$E.2\pi rl = \frac{\lambda l}{\epsilon_0}$$

$$\therefore E = \frac{\lambda}{2\pi r\epsilon_0}$$

$\Rightarrow E = 2 \times \frac{10^{-6}}{2\times3.14\times4\times10^{-2}\times8.85\times10^{-12}}$N/C (putting the values of λ ,r,π and ϵ_0)

$\Rightarrow E = 8.99 \times 10^5$N/C

Therefore the magnitude of electric field at a distance 4cm away from the line charge is given by 8.99× 10^5N/C

Answer.15

Given:

Linear charge density of wire=2.0× 10^{-6}C=λ

We know that,

Electric field E due to a linear charge distribution of linear charge density λ at a distance r from the line is given by

$E = \frac{\lambda}{2\pi\epsilon_0 r}$ ....(i)

Now,

Magnitude of Force experienced by a charge q in an electric field of intensity E is given by

$$F = q \times E$$

Here the charge particle is electron so the charge =q=e

Since this electron revolves around the wire in a circular path under the influence of this electrostatic force this force is equal to the centripetal force experienced by the electron

$\therefore q \times E = \frac{mv^2}{r}$ ...(ii)

Where,

m=mass of electron=9.1× 10^{-31}kg

r=radius of orbit in which electron revolves

q=e=charge of electron=1.6× 10^{-19}C

E=electric field due to line charge

V=velocity of electron

We know that ,

Kinetic energy of electron is given by

$K = \frac{1}{2}mv^2$ ...(iii)

Where,

m=mass of electron

v=velocity of electron

from eqn,(ii) ,

$$\frac{1}{2} \times mv^2 = \frac{1}{2} \times q \times E \times r$$

Therefore kinetic energy of electron is given by

$$K = \frac{qEr}{2}$$

Putting value of E from eqn.(i)

$$K = \frac{qr}{2} \times \frac{\lambda}{2\pi r \epsilon_0}$$

$$K = \frac{q\lambda}{4\pi \epsilon_0}$$

Thus kinetic energy is independent of radius(r)

$\Rightarrow K = \frac{1.6 \times 10^{-19}}{2} \times \frac{2 \times 10^{-6}}{2 \times 3.14 \times 8.85 \times 10^{-12}}$ J

$\Rightarrow K = 2.88 \times 10^{-17}$J

Therefore kinetic energy of electron while revolving around a cylindrical charge is given by 2.88×10^{-17}J and it is independent of radius.

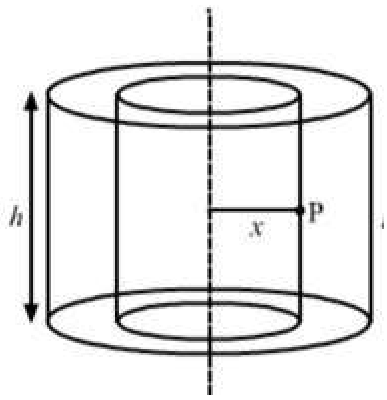
Answer.16

Given:

Volume charge density inside cylinder=ρ

We need to find the electric field at point P which is at a distance x from the axis of the cylinder

Consider a cylindrical gaussian surface of radius x and height h



We know that,

By Gauss's law, flux of net electric field ($\vec{E}$) through a closed surface S equals the net charge enclosed by the surface (q_{in}) divided by ϵ_0

$\oint \vec{E}.d\vec{S} = \frac{q_{in}}{\epsilon_0}$ ..(i)

Volume of this gaussian surface $=\pi x^2 h$

Curved surface area of this surface$=2\pi xh$

∴ charge enclosed by this surface=

$q_{in} = (volume\ charge\ density) \times volume\ of\ surface$

$\rightarrow q_{in} = \rho \times \pi x^2 h$ ....(ii)

Now,

All the points on the curved part of this gaussian surface are at the same perpendicular distance from the line charge. Therefore all these points are equivalent. The electric field at all these points will have the same magnitude E and direction of field at any point on the curved surface is normal to the line and hence normal to the cylindrical surface.

Therefore flux through this surface can be written as,

$\oint \vec{E}.d\vec{S} = E^{\rightarrow} \oint d\vec{S} = E.2\pi x h$ ......(iii)

From gauss law(i) and using eqns (ii) and (iii)

$$E.2\pi x h = \rho \times \frac{\pi x^2 h}{\epsilon_0}$$

$$\Rightarrow E = \frac{\rho x}{2\epsilon_0}$$

Therefore electric field at a point P inside cylindrical volume at a distance x from its axis is given by $\rho x/2\epsilon_0$

Answer.17

Given:

Thickness of sheet=d

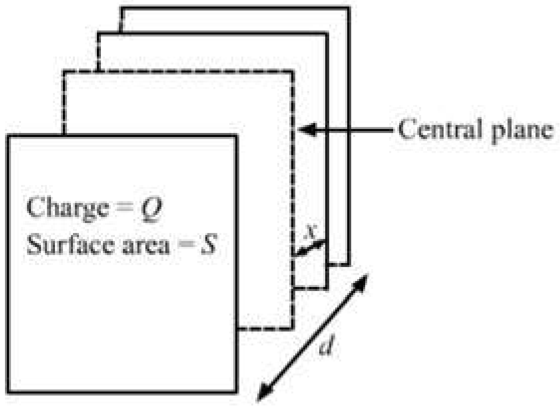
Volume charge density=ρ

Let the surface area of sheet be A

Consider a cuboidal Gaussian surface of width x from the central plane such that the central plane becomes one face of the cuboidal surface.

Volume of this cuboidal surface is given by

$V = A \times x$



Charge enclosed by this gaussian surface = (volume charge density)× (volume of gaussian surface containing charge)

$q_{in} = \rho \times A \times x$ ..(i)

We know that,

By Gauss's law, flux of net electric field ($\vec{E}$) through a closed surface S equals the net charge enclosed by the surface (q_{in}) divided by ϵ_0

$\oint \vec{E}.d\vec{S} = \frac{q_{in}}{\epsilon_0}$ ....(i)

By symmetry arguments we can say that electric field is normal to the plane and has same magnitude at all points which are at same distance from the central plane.

Here flux only pass through the face which is parallel to the central plane which is at distance x from the central plane . for remaining faces of the gaussian surfaces the angle between the electric field vector and area vector =90.

Therefore no flux pass through these faces .

Therefore total flux passing through the gaussian surface is given by

$\oint \vec{E}.d\vec{S} = E \oint d\vec{S} = E.A$ ....(ii)

Using gauss's law(i) and eqn.(ii) we can write

$$E.A = \frac{\rho \times A \times x}{\epsilon_0}$$

$\Rightarrow E = \frac{\rho x}{\epsilon_0}$ .....(iii)

Therefore , the electric field at a point P inside the plate at a distance x from the central plane is given by $\rho x/\epsilon_0$

Eqn(iii) is valid only inside the sheet i.e. when x<d/2

(as x is measured from the central plane)

Now for outside the sheet (i.e. x>d/2)

Consider again a similar Gaussian surface of width x where x is measured from the central plane

Charge enclosed by this gaussian surface =(volume charge density × volume of gaussian surface containing charge)

$q_{in} = \rho \times A \times \frac{d}{2}$ ...(iv)

Using gauss's law(i) and eqn(ii) and (iv)

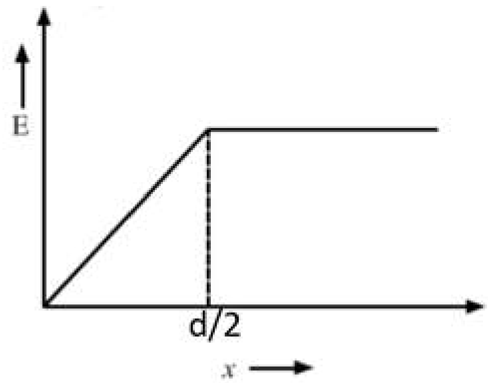
$$E.A = \frac{\rho \times A \times \frac{d}{2}}{\epsilon_0}$$

$$E = \frac{\rho \times d}{2\epsilon_0}$$

Which is independent of x

Therefore electric field at a point P outside the sheet at a distance x is constant and given by $\rho d/2\epsilon_0$

Graph of electric field for 0<x<d is given by



Answer.18

Given:

Charge of the particle= -2.0× 10^{-6}C=q

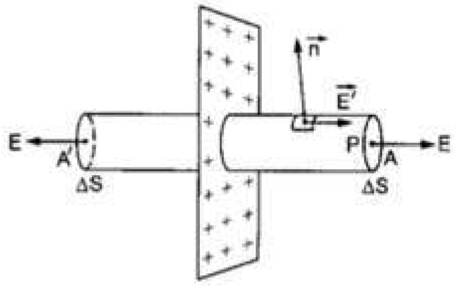
Surface charge density=4.0× 10^{-6}C/m^2 =σ

The electric field due to a plane thin sheet of charge density σ is given by

$$E = \frac{\sigma}{2\epsilon_0}$$

Proof:

To calculate the electric field at P we choose a cylindrical Gaussian surface as shown in the fig. in which the cross section A and A' are at equal distance from the plane.



The electric field at all points of A have equal magnitude E. and direction along positive normal. The flux of electric field through A is given by

$$\phi = E.\Delta S$$

Since A and A' are at equal distance from sheet the electric field at any point of A' is also equal to E and flux of electric field through A' is also given by

E.ΔS

At the points on curved surface the field and area make an angle of 90° with each other and hence $\vec{E}.\Delta\vec{S} = 0$

The total flux through the closed surface is given by

$\phi = E.\Delta S + E.\Delta S = 2E.\Delta S$ ..,(i)

The area of sheet enclosed in the cylinder is given by ΔS

So the charge contained in the cylinder is given by

$q_{in} = \sigma.\Delta S$ ...(ii)

We know that,

By Gauss's law, flux of net electric field ($\vec{E}$) through a closed surface S equals the net charge enclosed by the surface (q_{in}) divided by ϵ_0

$$\oint \vec{E}.d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

Using gauss law and eqns(i) and (ii)

$$2E.\Delta S = \sigma.\frac{\Delta S}{\epsilon_0}$$

$\Rightarrow E = \frac{\sigma}{2\epsilon_0}$ ..(iii)

Now,

Force $\vec{F}$ on a charge particle of charge q in presence of electric field $\vec{E}$ is given by

$$\vec{F} = q\vec{E}$$

Using eqn(iii),we get

$$F = \frac{q\sigma}{2\epsilon_0}$$

Putting the values of σ and q we get

$$F = (-2.0 \times 10^{-6}) \times \frac{4.0\times10^{-6}}{2\times8.85\times10^{-12}} N$$

$F = -0.45$N(-ve sign indicates that the force is attractive in nature)

Therefore force of attraction between the particle and the plate is given by 0.45N

Answer.19

Given:

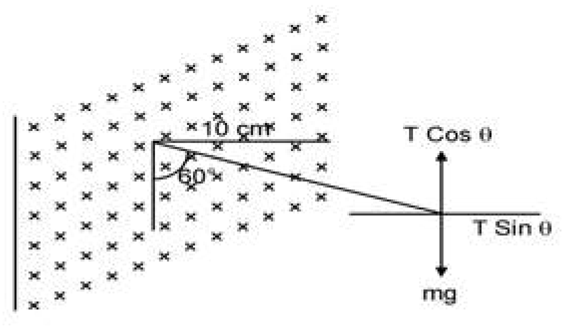
Length of silk thread=10cm

Mass of ball=10g

Charge of ball=4.0×10^{-6}C

Equilibrium angle of thread with vertical=60°

Let the surface charge density on the plate be σ



The forces acting on the ball are

• Weight of the ball downwards (W=mg)

Where m=mass of ball

g=acceleration due to gravity

• Electric force due to non-conducting plate producing electric field E (F=qE)

Where q=charge of the ball

E=electric field intensity

• Tension force (T) due to string at an angle of 60° from vertical

Electric field due to a thin non-conducting plate of surface charge density σ is given by

$E = \frac{\sigma}{2\epsilon_0}$......(i)

The tension force T due to string is divided into horizontal and vertical components given by Tsin60° and Tcos60°

Since the ball is in equilibrium, the net horizontal and vertical force on the ball is zero

Applying equilibrium along horizontal direction, we get

$Tsin60° = qE$ ...(ii)

Similarly, applying equilibrium along vertical direction, we get

$Tcos60° = mg$ ....(iii)

Dividing eqn. (ii) by (iii)

$$tan60° = \frac{qE}{mg}$$

$$E = \frac{mgtan60°}{q}$$

Putting the value of E from eqn.(i), we get,

$$\frac{\sigma}{2\epsilon_0} = \frac{mgtan60°}{q}$$

$$\Rightarrow \sigma = \frac{2\epsilon_0 \times mgtan60°}{q}$$

Putting the values of ϵ_0,m,g and q in the above equation

$$\sigma = \frac{2 \times 8.85 \times 10^{-12} \times 10 \times 10^{-3} \times 9.8 \times \sqrt{3}}{4 \times 10^{-6}}$$

$$\sigma = 7.5 \times 10^{-7} \mathrm{C/m^2}$$

Therefore the surface charge density on the plate is given by $7.5 \times 10^{-7} \mathrm{C/m^2}$

Answer.20

Given:

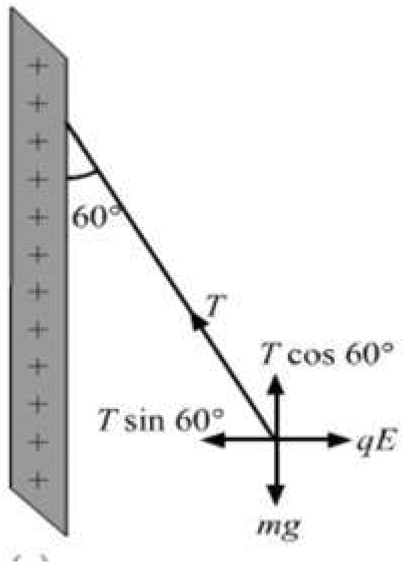
Length of silk thread=10cm

Mass of ball=10g

Charge of ball=4.0× 10^{-6}C

Equilibrium angle of thread with vertical=60°

Let the surface charge density on the plate be σ



The forces acting on the ball are

- Weight of the ball downwards (W=mg)

Where m=mass of ball

g=acceleration due to gravity

- Electric force due to non-conducting plate producing electric field E (F=qE)

Where q=charge of the ball

E=electric field intensity

- Tension force (T) due to string at an angle of 60° from vertical

Electric field due to a thin non-conducting plate of surface charge density σ is given by

$E = \frac{\sigma}{2\epsilon_0}$......(i)

The tension force T due to string is divided into horizontal and vertical components given by Tsin60° and Tcos60°

Since the ball is in equilibrium, the net horizontal and vertical force on the ball is zero

Applying equilibrium along horizontal direction, we get

$Tsin60° = qE$ ...(ii)

Similarly, applying equilibrium along vertical direction, we get

$Tcos60° = mg$ ....(iii)

Dividing eqn. (ii) by (iii)

$$tan60° = \frac{qE}{mg}$$

$$E = \frac{mgtan60°}{q}$$

Putting the value of E from eqn.(i), we get,

$$\frac{\sigma}{2\epsilon_0} = \frac{mgtan60°}{q}$$

$$\Rightarrow \sigma = \frac{2\epsilon_0 \times mgtan60°}{q}$$

Putting the values of ϵ_0,m,g and q in the above equation

$$\sigma = \frac{2 \times 8.85 \times 10^{-12} \times 10 \times 10^{-3} \times 9.8 \times \sqrt{3}}{4 \times 10^{-6}}$$

$$\sigma = 7.5 \times 10^{-7} \text{C/m}^2$$

(a)Now to find the tension in the string in equilibrium condition we can use eqn.(iii)

$$Tcos60° = mg$$

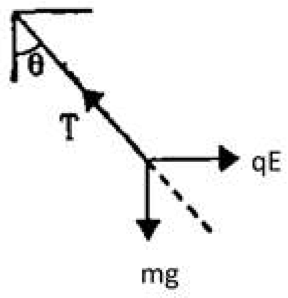
$$T = 2mg$$

$$T = 2 \times (10 \times 10^{-3}) \times 9.8$$

$$T = 0.196\text{N}$$

Therefore tension in the string in equilibrium is given by 0.196N

(b)when the ball is slightly pushed aside and released, the ball will undergo small oscillations due to the restoring forces. When the ball will come in mean position, tension, weight and electric force will balance.



The forces acting on the ball are $m\vec{g}$ in vertical direction and $q\vec{E}$ in horizontal direction

From the fig, tension is given by

$$T = \sqrt{(mg)^2 + (qE)^2}$$

So the net acceleration g_{eff} is given by

$$g_{eff} = \frac{T}{m}$$

$$g_{eff} = \sqrt{g^2 + \left(\frac{qE}{m}\right)^2}$$

Therefore time period of small oscillations T is given by

$$T = 2\pi\sqrt{\frac{l}{g_{eff}}}$$

$$T = 2\pi\sqrt{\frac{l}{\left(g^2 + \left(\frac{qE}{m}\right)^2\right)^{0.5}}}$$

Putting the value of E from eqn.(i) we get

$$T = 2\pi\sqrt{\frac{l}{\left(g^2 + \left(\frac{q\sigma}{2\epsilon_0 m}\right)^2\right)^{0.5}}}$$

Putting the values of g, l, q, σ ϵ_0 and m and solving we get,

$$T = 0.45 sec$$

Therefore the time period of small oscillations of ball is given by 0.45sec

Answer.21

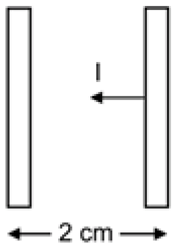
Given:

Distance travelled by the electron=2cm=2× 10^{-2}m=s

Time taken by the electron =2μs=2× 10^{-6}s=t

Let the surface charge density of the plate be σ

Let the acceleration of electron be a



Using 2nd law of motion

$$s = ut + \frac{1}{2}at^2$$

Where,

u=initial velocity

t=time taken to travel

a=acceleration of particle

s=displacement of particle

it is given that electron starts from rest ∴ u=0

$$s = \frac{1}{2}at^2$$

$$a = \frac{2s}{t^2}$$

This acceleration is provided by the force due to electric field between plates

Force applied to the particle is given by

$F = m \times a = m \times \frac{2s}{t^2}$ ...(i)

Where,

m=mass of electron=9.1× 10^{-31}kg

this force is provided by the electric field (E) between the plates which is given by

$F = qE$ .....(ii)

Equating eqns.(i) and (ii)

$qE = m \times \frac{2s}{t^2}$ ..(iii)

We know that

Electric field due to a conducting plate of surface charge density σ is given by

$$E = \frac{\sigma}{\epsilon_0}$$

Putting this value of E in eqn.(iii)

$$\frac{q\sigma}{\epsilon_0} = \frac{2ms}{t^2}$$

$$\sigma = \frac{2ms\epsilon_0}{qt^2}$$

Putting values of s, m, t and q

$$\sigma = 2 \times 9.1 \times 10^{-31} \times 8.85 \times \frac{10^{-12}}{(1.6\times10^{-19})\times(2\times10^{-6})^2} \text{C/m}^2$$

$$\boldsymbol{\sigma = 0.503 \times 10^{-12}\text{C/m}^2}$$

Therefore the surface charge density of the plate is given by

0.503× 10^{-12}C/m^2

Answer.22

Given:

Surface charge density of both plate=σ

Both plates carry equal and opposite charges

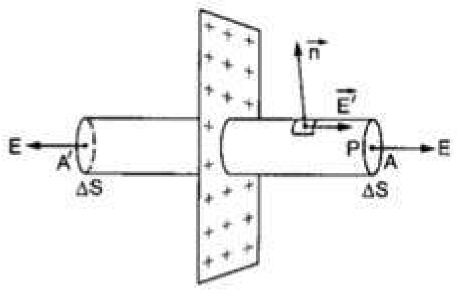
We know that,

The electric field due to a plane thin sheet of charge density σ is given by

$$E = \frac{\sigma}{2\epsilon_0}$$

<u>Proof:</u>

To calculate the electric field at P we choose a cylindrical Gaussian surface as shown in the fig. in which the cross section A and A' are at equal distance from the plane.



The electric field at all points of A have equal magnitude E. and direction along positive normal. The flux of electric field through A is given by

$$\phi = E.\Delta S$$

Since A and A' are at equal distance from sheet the electric field at any point of A' is also equal to E and flux of electric field through A' is also given by

E.ΔS

At the points on curved surface the field and area make an angle of 90° with each other and hence $\vec{E}.\Delta\vec{S} = 0$

The total flux through the closed surface is given by

$\phi = E.\Delta S + E.\Delta S = 2E.\Delta S$ ..,(i)

The area of sheet enclosed in the cylinder is given by ΔS

So the charge contained in the cylinder is given by

$q_{in} = \sigma.\Delta S$ ...(ii)

We know that,

By Gauss's law, flux of net electric field ($\vec{E}$) through a closed surface S equals the net charge enclosed by the surface (q_{in}) divided by ϵ_0

$$\oint \vec{E}.d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

Using gauss law and eqns(i) and (ii)

$$2E.\Delta S = \sigma.\frac{\Delta S}{\epsilon_0}$$

$$\Rightarrow E = \frac{\sigma}{2\epsilon_0}$$

Now

Magnitude of Electric field due to plate 1 is given by $E_1 = \frac{\sigma}{2\epsilon_0}$

Magnitude of Electric field due to plate 2 is given by $E_2 = \frac{\sigma}{2\epsilon_0}$

(a)at the left of the plates

Electric field due to plate 1 is in left direction(-ve) whereas electric field due to plate 2 is in right direction(+ve)

∴ net electric field

$$E = E_2 - E_1 = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0$$

Therefore net electric field due to both plates at the left of plates is zero

(b)in between the plates,

The electric field due to plate 1 is in right direction(+ve) and electric field due to plate 2 is also in right direction(+ve)

∴ net electric field

$$E = E_1 + E_2 = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

Therefore net electric field due to both plates in between the plates is given by σ/ϵ_0

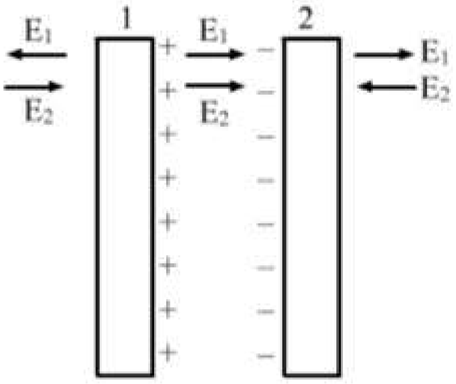
(c)at the right of the plates

Electric field due to plate 1 is in right direction (+ve) whereas electric field due to plate 2 is in left direction (-ve)

∴ net electric field

$$E = E_1 - E_2 = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0$$

Therefore net electric field due to both plates in the right of both plates is zero



Answer.23

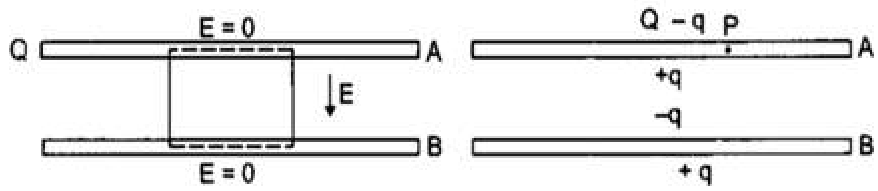
Given:

Charge on plate X=Q

Charge on plate Y=zero

Consider the gaussian surface as shown in fig.

Two faces of this closed surface lie completely inside the conductor where the electric field is zero. The other parts of closed surface which are outside are parallel to electric field and hence flux on these parts is zero. The total flux of electric field through this closed surface is zero. So, from gauss's law total charge inside the closed surface be zero. The charge on inner surface of X should be equal and opposite to charge on inner surface of Y.



To find the value of q consider electric field at point P

We know that electric field due to a thin plate of charge Q is given by

$$E = \frac{Q}{2A\epsilon_0}$$

Where A=area of plate

Electric field at P

Due to charge Q-q=$\frac{Q-q}{2A\epsilon_0}$(towards right)

Due to charge q= $\frac{q}{2A\epsilon_0}$(towards left)

Due to charge -q =$\frac{q}{2A\epsilon_0}$(towards right)

Due to charge q= $\frac{q}{2A\epsilon_0}$(towards left)

The net electric field at P due to all four charged surfaces is

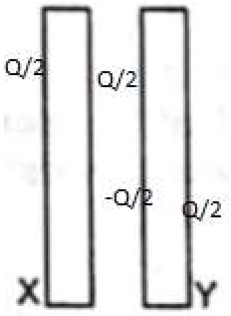
$$E = \frac{Q-2q}{2A\epsilon_0}$$

Which is zero as P is inside conductor

$\therefore \frac{Q-2q}{2A\epsilon_0} = 0$

$$\therefore q = +\frac{Q}{2}$$

The final charge distribution is as shown in fig



(a)surface charge density at inner surface of plate X is given by

$$\frac{charge\ at\ inner\ surface}{area\ of\ plate}$$

$\Rightarrow \frac{Q}{2A}$

Therefore surface charge density at inner surface of plate X is given by Q/2A

Consider Right direction as negative and left as positive

(b) at left of the plates

electric field due to,

outer surface of plate X=$\frac{Q}{4A\epsilon_0}$(towards left)

inner surface of plate X=$Q/4A\epsilon_0$(towards left)

inner surface of plate Y =$+Q/4A\epsilon_0$(towards right)

outer surface of plate Y =$Q/4A\epsilon_0$(towards left)

net electric field at left of plates is given by

$$E = \frac{Q}{4A\epsilon_0} + \frac{Q}{4A\epsilon_0} - \frac{Q}{4A\epsilon_0} + \frac{Q}{4A\epsilon_0} = \frac{Q}{2A\epsilon_0}$$

Therefore net electric field at a point to left to the plates is given by $Q/2A\epsilon_0$ towards left

(c)between the plates

electric field due to,

outer surface of plate X=$\frac{Q}{4A\epsilon_0}$(towards right)

inner surface of plate X=$Q/4A\epsilon_0$(towards right)

inner surface of plate Y =$Q/4A\epsilon_0$(towards right)

outer surface of plate Y =$Q/4A\epsilon_0$(towards left) net electric field at a point between the plates is given by

$$E = -\frac{Q}{4A\epsilon_0} - \frac{Q}{4A\epsilon_0} - \frac{Q}{4A\epsilon_0} + \frac{Q}{4A\epsilon_0} = -\frac{Q}{2A\epsilon_0}$$

Therefore net electric field at a point between the plates is given by $Q/2A\epsilon_0$ towards right

(d) to the right of the plates

electric field due to,

outer surface of plate X=$\frac{Q}{4A\epsilon_0}$(towards right)

inner surface of plate X=$Q/4A\epsilon_0$(towards right)

inner surface of plate Y =$Q/4A\epsilon_0$(towards left)

outer surface of plate Y =$Q/4A\epsilon_0$(towards right)

net electric field at a point between the plates is given by

$$E = -\frac{Q}{4A\epsilon_0} - \frac{Q}{4A\epsilon_0} + \frac{Q}{4A\epsilon_0} - \frac{Q}{4A\epsilon_0} = -\frac{Q}{2A\epsilon_0}$$

Therefore net electric field at a point to the right of the plates is given by $Q/2A\epsilon_0$ towards right

Answer.24

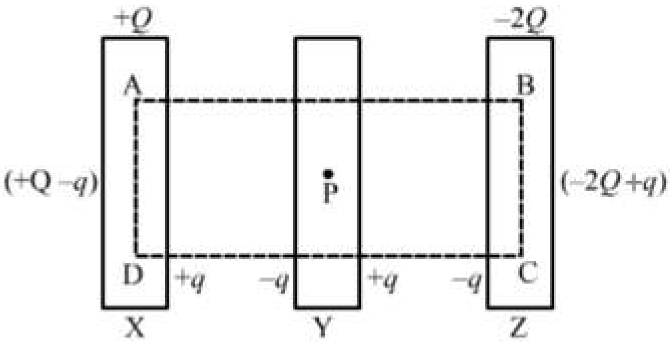
Given:

Charge on the left plate =Q

Charge on the rightmost plate =-2Q

Consider the gaussian surface as shown in fig.

Two faces of this closed surface lie completely inside the conductor where the electric field is zero. The other parts of closed surface which are outside are parallel to electric field and hence flux on these parts is zero. The total flux of electric field through this closed surface is zero. So, from gauss's law total charge inside the closed surface be zero



To find the value of q consider electric field at point P

We know that electric field due to a thin plate of charge Q is given by

$$E = \frac{Q}{2A\epsilon_0}$$

Where A=area of plate

Now electric field at point P due to,

Outer surface of plate X=$\frac{Q-q}{2A\epsilon_0}$(towards right)

Inner surface of plate X=$\frac{q}{2A\epsilon_0}$(towards right)

Left surface of plate Y=$\frac{q}{2A\epsilon_0}$

(towards left)

Right surface of plate Y=$\frac{q}{2A\epsilon_0}$

(towards left)

Inner surface of plate Z=$q/2A\epsilon_0$(towards right)

Outer surface of plate Z=$\frac{2Q-q}{2A\epsilon_0}$(towards right)

Consider field towards right as positive and towards left as negative

Net field at point P is given by

$\frac{Q-q}{2A\epsilon_0} + \frac{q}{2A\epsilon_0} - \frac{q}{2A\epsilon_0} - \frac{q}{2A\epsilon_0} + \frac{q}{2A\epsilon_0} + \frac{2Q-q}{2A\epsilon_0} = \frac{3Q-2q}{2A\epsilon_0}$

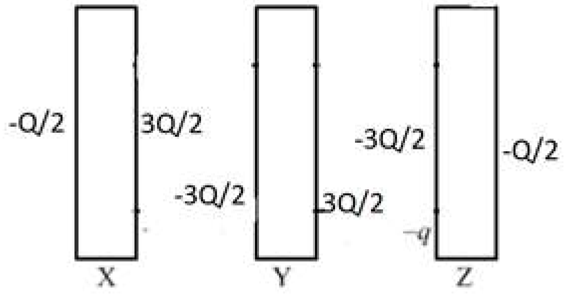
We know that field inside a point inside conductor should be zero

∴ net field at p=0

$$\frac{3Q - 2q}{2A\epsilon_0} = 0$$

$$q = \frac{3Q}{2}$$

The final charge distribution is shown in fig.



Charge on the outer surface of rightmost plate is given by
$-2Q + q = -2Q + \frac{3Q}{2} = -\frac{Q}{2}$

Therefore, the charge on the outer surface of the rightmost plate is given by -Q/2