CBSE Class 10th Mathematics

Standard Sample Paper- 08

Maximum Marks:

Time Allowed: 3 hours

General Instructions:

i. All the questions are compulsory.
ii. The question paper consists of 40 questions divided into 4 sections A, B, C, and D.
iii. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
iv. There is no overall choice. However, an internal choice has been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
v. Use of calculators is not permitted.

Section A

1. The LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one number is 280, then the other number is

    a. 150

    b. 80

    c. 100

    d. 120

2. Every positive odd integer is of the form ________ where ‘q’ is some integer.

a. 2q + 2

b. 5q + 1

c. 3q + 1

d. 2q + 1

3. $Mode + \frac{3}{2}(Median - Mode)$=

a. None of these

b. Median

c. Mean

d. Mode

4. Rohan's mother is 26 years older than him. The product of their ages 3 years from now will be 360, then Rohan's present age is

a. 6 years

b. 7 years

c. 10 years

d. 8 years

5. Given that $\sin\theta = \frac{a}{b}$, then $cos\ \theta$ is equal to

a. $\frac{\sqrt{b^2-a^2}}{b}$

b. $\frac{b}{a}$

c. $\frac{\sqrt{b^2+a^2}}{b}$

d. $\frac{b}{\sqrt{b^2-a^2}}$

6. If 5 tan α = 4, then the value of $\frac{5\sin\alpha-3\cos\alpha}{5\sin\alpha+2\cos\alpha}$ is

a. $\frac{1}{5}$

b. $\frac{4}{5}$

c. $\frac{1}{4}$

d. $\frac{1}{6}$

7. A round balloon of radius ‘r’ subtends an angle ‘α’ at the eye of the observer while the angle of elevation of its centre is ‘β’. The height of the centre of the balloon is

   a. $r \cos \beta \cos ec \frac{\alpha}{2}$

   b. $r \cos \beta \sec \frac{\alpha}{2}$

   c. $r \sin \beta \sec \frac{\alpha}{2}$

   d. $r \sin \beta \cos ec \frac{\alpha}{2}$

8. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is

   a. 15 units

   b. 10 units

   c. 9 units

   d. 12 units

9. If the vertices of a triangle are (1, 1), (– 2, 7) and (3, – 3), then its area is

   a. 0 sq. units

   b. 2 sq. units

   c. 24 sq. units

   d. 12 sq. units

10. The probability of guessing the correct answer to certain text questions is. $\frac{x}{12}$. If the probability of not guessing the answer is $\frac{5}{8}$, then the value of x is

a. 1

b. 0

c. 4

d. 4.5

11. Fill in the blanks:

    A funnel is the combination of ________ and a cylinder.

12. Fill in the blanks:

    If 'x + a' is a factor (zero) of the polynomial $2x^2 + 2ax + 5x + 10$, the value of 'a' is ________.

    OR

    Fill in the blanks:

    The degree of polynomial p(x) $= x + \sqrt{2+1}$ is ________.

13. Fill in the blanks:

    $\triangle$ABC and $\triangle$DEF are similar. Area of $\triangle$ABC is $9cm^2$ and Area of $\triangle$DEF is $64cm^2$. If DE = 5.1cm, then the value of AB is ________.

14. Fill in the blanks:

    If S_n and S_{n-1} is the sum of first n and (n - 1) term of an AP, then its n^{th} term, a_n is given by ________.

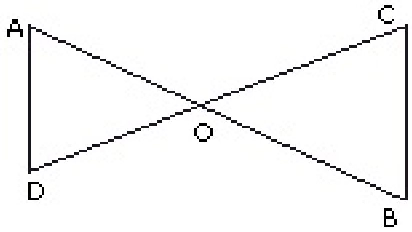
15. Fill in the blanks:

    The Abscissa is ________ to the right of y-axis and is ________ to the left of y-axis.

16. If - 1 is a zero of the polynomial $f(x) = x^2 - 7x - 8$, then calculate the other zero.

17. In the given figure, OA $\times$ OB = OC $\times$ OD or $\frac{OA}{OC} = \frac{OD}{OB}$ prove that $\angle$A = $\angle$C and $\angle$B

= ∠ D

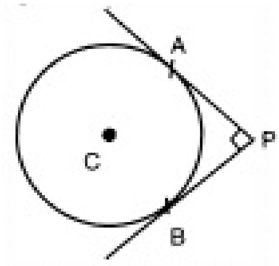


18. For the following APs, write the first term and the common difference : 3, 1, –1, –3, ….

OR

Find the 9th term from the end (towards the first term) of the A.P. 5, 9, 13, …, 185.

19. In fig., PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If PA $\perp$ PB, then find the length of each tangent.



20. Find the positive root of $\sqrt{3x^2+6}=9$.

Section B

21. A child has a die whose six faces show the letters as given below:

A		**B**		**C**		**D**		**E**		**A**

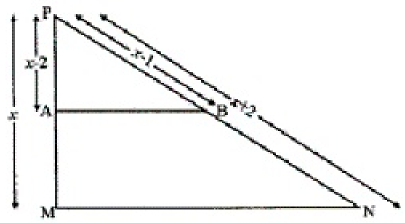
The die is thrown once. What is the probability of getting (i) A? (ii) D?

22. Find the roots of the equation, if they exist, by applying the quadratic formula: $x^2 + x + 2 = 0$.

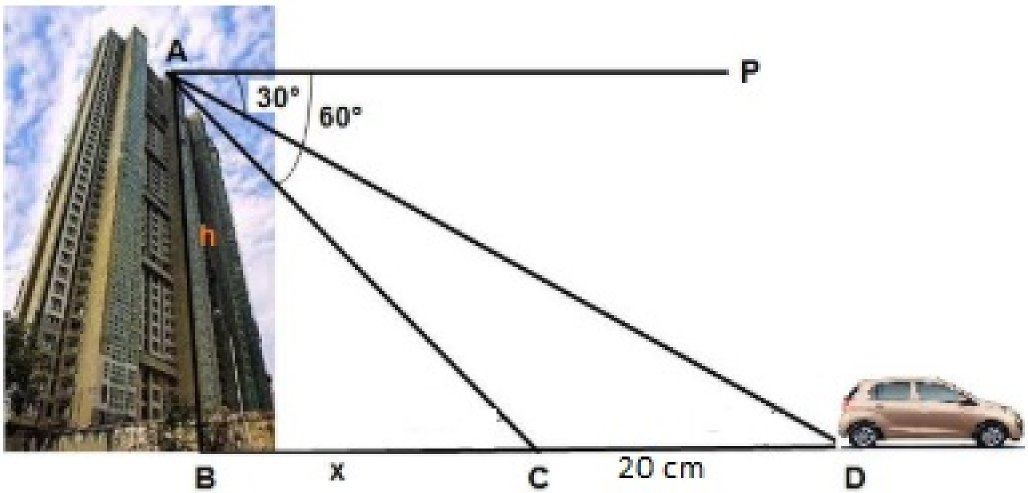
23. In $\triangle ABC$, AD is the bisector of $\angle A$. If AB = 5.6 cm, AC = 4 cm and DC = 3 cm, find BC.

OR

AB || MN. If PA = x – 2, PM = x, PB = x – 1 and PN = x + 2, Find the value of x.



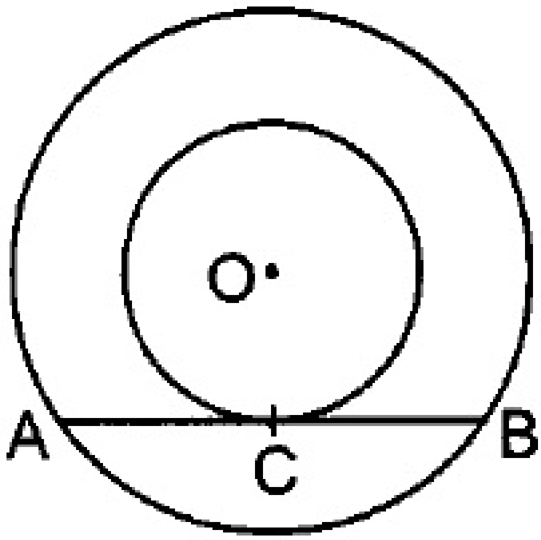
24. Vijay lives in a flat in a multi-story building. His driving was rough so his father keeps eye on his driving. Once he drives from his house to Faridabad. His father was standing on the top of the building at point A as shown in the figure. At point C, the angle of depression of a car from the building was 60^{o}. After accelerating 20 m from point C, Vijay stops at point D to buy ice-cream and the angle of depression changed to 30^{o}.



By analysing the above given situation answer the following questions:

i. Find the value of x.
ii. Find the height of the building AB.

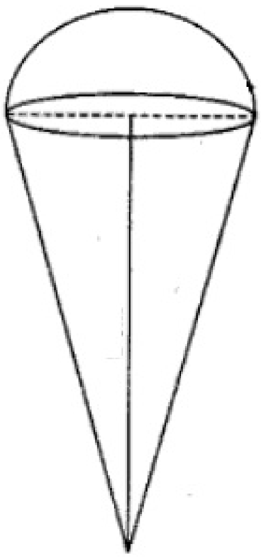
25. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact. Using the above, do the following: In figure, O is the centre of the two concentric circles. AB is a chord of the larger circle touching the smaller circle at C. Prove that AC = BC.



OR

PA and PB are tangents from P to the circle with centre O. At the point M, a tangent is drawn cutting PA at K and PB at N. Prove that KN = AK + BN.

26. An 'ice-cream seller used to sell different kinds and different shapes of ice-cream like rectangular shaped with one end hemispherical, cone-shaped and rectangular brick, etc. One day a child came to his shop and purchased an ice-cream which has the following shape: ice-cream cone as the union of a right circular cone and a hemisphere that has the same (circular) base as the cone. The height of the cone is 9 cm and the radius of its base is 2.5 cm.



By reading the above-given information, find the following:

i. The volume of the ice-cream without hemispherical end.

ii. The volume of the ice-cream with a hemispherical end.

Section C

27. Express the number $0.3\overline{178}$ in the form of rational number $\frac{a}{b}$.

OR

Find the LCM of the following polynomials:$a^8 - b^8$ and $\left(a^4 - b^4\right)(a + b)$

28. In an AP: d = 5, S_9 = 75, find a and a_9 .

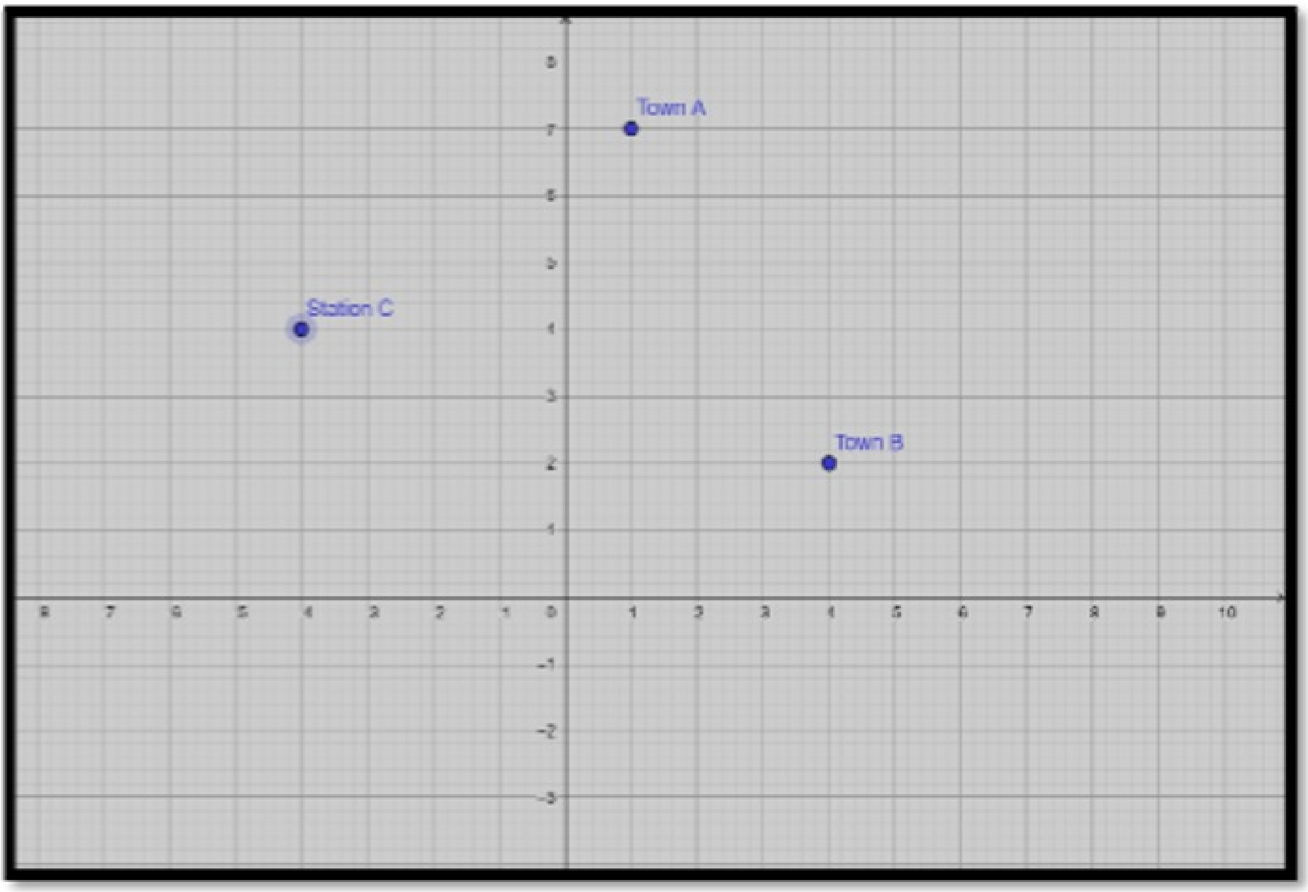
29. Solve the following pairs of equations by reducing them to a pair of linear equations:
$\frac{1}{(3x+y)} + \frac{1}{(3x-y)} = \frac{3}{4}$ and $\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$

OR

The sum of a two digit number and the number formed by interchanging its digits is 110. If 10 is subtracted from the first number. The new number is 4 more than 5 times the sum of the digits in the first number. Find the first number.

30. Obtain all zeros of the polynomial ($2x^3 - 4x - x^2 + 2$), if two of its zeros are $\sqrt{2}$ and -$\sqrt{2}$

31. Two friends Seema and Aditya work in the same office in Delhi. In the Christmas vacations, both decided to go their hometowns represented by Town A and Town B respectively in the figure given below. Town A and Town B are connected by trains from the same station C (in the given figure) in Delhi. Based on the given situation answer the following questions:

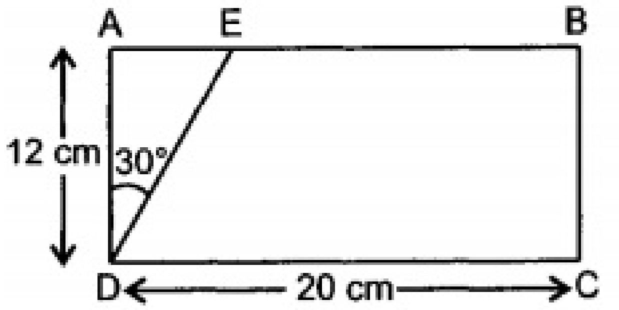


i. Who will travel more distance, Seema or Aditya, to reach to their hometown?
ii. Seema and Aditya planned to meet at a location D situated at a point D represented by the mid-point of the line joining the point represented by Town A and Town B. Find the coordinates of the point represented by the point D.
iii. Find the area of the triangle formed by joining the points represented by A, B and C.

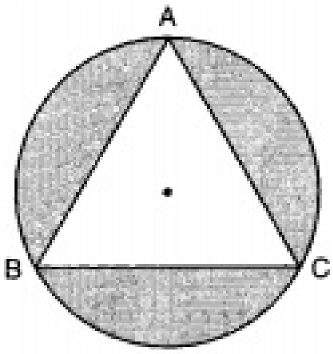
32. If tan A = n tanB and sin A = m sinB, then prove that $\cos^2 A = \frac{m^2-1}{n^2-1}$

OR

In the given figure, ABCD is a rectangle with AD = 12 cm and DC = 20 cm. Line segment DE is drawn making an angle of 30° with AD, intersecting AB at E. Find the length of DE and AE.



33. In fig., an equilateral triangle ABC of side 6 cm has been inscribed in a circle. Find the area of the shaded region. (Take π = 3.14).



34. Find the mean of the following data :

Class	Less than 20	Less than 40	Less than 60	Less than 80	Less than 100
Cumulative Frequency	15	37	74	99	120

Section D

35. Construct a triangle similar to a given ΔABC such that each of its sides is $(5/7)^{th}$ of the corresponding sides of $\Delta ABC.$. It is given that AB = 5 cm, BC = 7 cm and $\angle ABC = 50^{\circ}$.

OR

Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths.

36. In a $\triangle ABC$, P and Q are points on AB and AC respectively such that $PQ \| BC$. Prove that the median AD. drawn form A to BC, bisects PQ.

37. Solve the following pair of equations graphically:(i.e. has no solution):
3x - 4y -1 = 0
$2x - \frac{8}{3}y + 5 = 0$

OR

Solve the following system of equations graphically:
x + 2y - 7 = 0
2x - y - 4 = 0

38. A tent consists of a frustum of a cone, surmounted by a cone. If the diameters of the upper and lower circular ends of the frustum be 14 m and 26 m respectively, the height of the frustum be 8 m and the slant height of the surmounted conical portion be 12 m, find the area of canvas required to make the tent. (Assume that the radii of the upper circular end of the frustum and the base of the surmounted conical portion are equal.)

OR

From a solid cylinder of height 14 cm and base diameter 7 cm, two equal conical holes each of radius 2.1 cm and height 4 cm are cut off. Find the volume of the remaining solid.

39. An aeroplane flying horizontally 1 km above the ground is observed at an elevation of 60^{o}. After 10 seconds, its elevation is observed to 30^{o}. Find the speed of the aeroplane in km/hr.

40. A survey regarding the heights (in cm) of 50 girls of a class was conducted and the following data was obtained:

Height(in cm)	120 - 130	130 - 140	140 - 150	150 - 160	160 - 170	170 -180
Number of girls	2	8	12	20	8	50

Find the mean, median and mode of the above data.

Solution

Section A

1. (b) 80

   Explanation:

   Given: LCM =$14 \times HCF$

   and LCM + HCF = 600 ....(i)

   Putting LCM = $14 \times HCF$ in eq. (i),

   $14 \times HCF$ + HCF = 600

   $\Rightarrow 15 \times HCF$ = 600

   $\Rightarrow$ HCF = 40

   And LCM = 14×40 = 560

   $\therefore$ Using the result,

   $HCF \times LCM$ = Product of two natural numbers

   Other number = $\frac{40\times560}{280}$ = 80

2. (d) 2q + 1

   Explanation:

   Let a be any positive integer and b = 2
   Then by applying Euclid's Division Lemma,
   we have, a = 2q +r,
   where $0 \leqslant r < 2 \Rightarrow r$ = 0 or 1 $\therefore$ a = 2q or 2q + 1 .
   Therefore, it is clear that a = 2q i.e., a is an even integer.
   Also, 2q and are two 2q + 1 consecutive integers, therefore, 2q + 1 is an odd integer.

3. (c) Mean

Explanation:

3 Median = 2 Mean + Mode

$\Rightarrow$ 3 Median – Mode = 2 Mean

$\Rightarrow$ Mean = $\frac{3}{2}\text{Median} - \frac{\text{Mode}}{2}$

$\Rightarrow \text{Mode} + \frac{3}{2}\text{Median} - \frac{\text{Mode}}{2} - \text{Mode}$ = Mean

$\Rightarrow \text{Mode} + \frac{3}{2}(\text{Median} - \text{Mode})$ = Mean

4. (b) 7 years

Explanation:

Let Rohan’s present age be x years.

Then Rohan’s mother age will be $(x + 26)$ years.

And after 3 years their ages will be $(x + 3)$ and $(x + 29)$ years. According to question,

$(x + 3)(x + 29) = 360$

$\Rightarrow x^2 + 29x + 3x + 87 = 360$

$\Rightarrow x^2 + 32x - 273 = 0$

$\Rightarrow x^2 + 39x - 7x - 273 = 0$

$\Rightarrow x(x + 39) - 7(x + 39) = 0$

$\Rightarrow (x - 7)(x + 39) = 0$

$\Rightarrow x - 7 = 0$ and $x + 39 = 0$

$\Rightarrow x = 7$ and $x = -39$ [$x = -39$ is not possible]

Therefore, Rohan’s present is 7 years.

5. (a) $\frac{\sqrt{b^2 - a^2}}{b}$

Explanation:

Given: $\sin\theta = \frac{a}{b}$

we know that cosθ = $\sqrt{1 - sin^2\theta}$

[∵, sin²θ+cos²θ=1]

or, cosθ = $\sqrt{1-a^2/b^2}$

or, cosθ = $\frac{\sqrt{b^2-a^2}}{b}$

6. (d) $\frac{1}{6}$

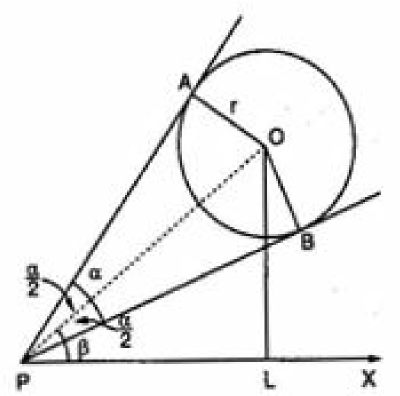
Explanation:

Given: $5\tan\alpha = 4$

Dividing all terms of $\frac{5\sin\alpha-3\cos\alpha}{5\sin\alpha+2\cos\alpha}$ by $\cos\alpha$
= $\frac{5\tan\alpha-3}{5\tan\alpha+2}$ = $\frac{4-3}{4+2} = \frac{1}{6}$

7. (d) $r\sin\beta\cos ec\frac{\alpha}{2}$

Explanation:



Let O be the centre of the balloon of radius r and P the eye of the observer. Let PA, PB be tangents from P to the balloon. Then $\angle\text{APB} = \alpha$.
$\therefore\angle\text{APO} = \angle\text{BPO} = \frac{\alpha}{2}$
Let OL be perpendicular from O on the horizontal PX. We are given that the angle of the elevation of the centre of the balloon is β i.e., $\angle\text{OPL} = \beta$ In triangle OAP, we have
$\sin\frac{\alpha}{2} = \frac{\text{OA}}{\text{OP}}$
$\Rightarrow\sin\frac{\alpha}{2} = \frac{r}{\text{OP}}$
$\Rightarrow\text{OP} = r\cos ec\frac{\alpha}{2}$ ....(i)
In triangle OPL, we have
$\sin\beta = \frac{\text{OL}}{\text{OP}}$
$\Rightarrow\text{OL} = \text{OP}\sin\beta$

$\Rightarrow \mathrm{OL} = r \cos ec \frac{\alpha}{2} \sin \beta$ [From eq. (i)]

Therefore, the height of the centre of the balloon is $r \cos ec \frac{\alpha}{2} \sin \beta$.

8. (d) 12 units

Explanation:

Given: the vertices of a triangle ABC, A(0, 4), B (0, 0) and C (3, 0).

$\therefore$ Perimeter of triangle ABC = AB + BC + AC

= $\sqrt{(0-0)^2 + (0-4)^2}$ + $\sqrt{0-3)^2 + (0-0)^2}$ + $\sqrt{0-3)^2 + (4-0)^2}$

= $\sqrt{0+16} + \sqrt{9+0} + \sqrt{9+16}$

= $\sqrt{16} + \sqrt{9} + \sqrt{25}$

= 4 + 3 + 5 = 12 units

9. (a) 0 sq. units

Explanation:

Given: $(x_1, y_1) = (1, 1), (x_2, y_2) = (-2, 7)$ and $(x_3, y_3) = (3, -3)$, then the Area of triangle

$\therefore \frac{1}{2} |x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)|$

= $\frac{1}{2} |1 (7+3) + (-2) (-3-1) + 3 (1-7)|$

= $\frac{1}{2} |10 + 8 - 18|$

= $\frac{1}{2} |0|$ = 0 sq. units

Also therefore the three given points(vertices) are collinear.

10. (d) 4.5

Explanation:

Since, Total Probability = 1

$\therefore \frac{x}{12} + \frac{5}{8} = 1$

$\Rightarrow \frac{2x+15}{24} = 1$

$\Rightarrow 2x + 15 = 24$

$\Rightarrow 2x = 9$

$\Rightarrow x = 4.5$

11. frustum of a cone

12. a = 2 OR 1

13. AB = 1.91cm

14. $a_n = S_n - S_{n-1}$

15. +ve, -ve

16. $f(x) = x^2 - 7x - 8$

    As, one zero is -1 .

    Let, other zero be k,

    then, Sum of zeroes - 1 + k = $-\left(\frac{-7}{1}\right)$ = 7

    $\Rightarrow$ k = 8

17. In $\triangle$AOD and $\triangle$BOC,

    OA $\times$ OB = OC $\times$ OD

    i.e $\frac{OA}{OC} = \frac{OD}{OB}$

    And $\angle$ AOD = $\angle$ BOC [Vertically opposite Angles]

    $\therefore$ $\triangle$AOD $\sim\triangle$ BOC [By SAS]

    $\therefore$ $\angle$A = $\angle$C and $\angle$B = $\angle$D [Corresponding angles of similar $\triangle$]

18. 3, 1, -1, -3, ...

    First term (a) = 3

    Common difference (d) = 1 - 3 = -2

    OR

    Common difference, d, of the AP = 9 − 5 = 4

    Last term, l, of the AP = 185

    We know that in general the nth term from the end of an AP is given by l - (n - 1) d.

    Thus, the 9 th term from the end is

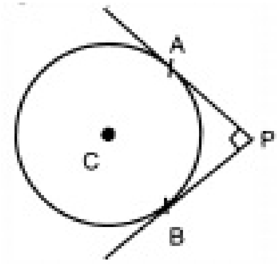
    185 - (9 - 1)4

= 185 - 4 $\times$ 8

= 185 - 32

= 153

Therefore required 9 th term from the end = 153

19. 

Construction: Join AC and BC

Now, $AC \perp AP \ and \ CB \perp BP$

$\angle APB = 90^0$

Therefore, CAPB will be a square

CA = AP = PB = BC = 4 cm

$\therefore$ Length of tangent = 4 cm.

20. $\sqrt{3x^2+6} = 9$

$3x^2 + 6 = 81$

$3x^2 = 81 - 6 = 75$

$x^2 = 25$

$\therefore$ x = ± 5

Hence, Positive root = 5

Section B

21. Total number of letters = 6

$\therefore$ Number of all possible outcomes = 6

i. Let E be the event of getting A. Then, the number of outcomes favourable to E is 2 because there are 2 A's.

   $\therefore$ P(E) = P(A) =

   $Proabibilty \ of \ the \ event = \frac{Number \ of \ favourble \ outcomes}{Total \ number \ of \ possible \ outcomes} = \frac{2}{6} = \frac{1}{3}$

ii. Let E be the $\therefore$ event of getting D. Then, the number of outcomes favourable to E is

1 since there is only one D.

∴ P(E) = P(D)= $Proabibilty\ of\ the\ event = \frac{Number\ of\ favourble\ outcomes}{Total\ number\ of\ possible\ outcomes} = \frac{1}{6}$

22. The given equation is $x^2 + x + 2 = 0$

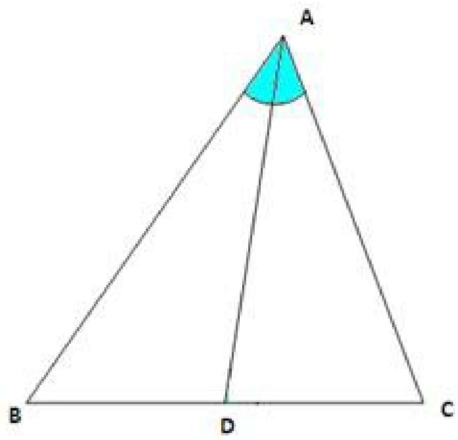
Comparing it with $ax^2 + bx + c = 0$, we get

a = 1, b = 1 and c = 2

∴ D=b^{2-}4ac=$(1)^2$-4(1)(2)=1-8=-7<0

So, the given equation has no real roots.

23.



According to question it is given that ABC is a triangle in which AD is the bisector

Let us suppose that BD = x

Also, DC = 3 cm, AB = 5.6 cm, AC = 4 cm

Therefore, $\frac{BD}{DC} = \frac{AB}{AC}$

$\Rightarrow \frac{x}{3} = \frac{5.6}{4}$

$\Rightarrow x = \frac{5.6 \times 3}{4}$

= 4.2 cm

Hence, BD = 4.2 cm

So BC = BD + AC

= 4.2 + 3

BC = 7.2 cm

OR

Using BPT $\frac{x-2}{x} = \frac{x-1}{x+2}$

$\Rightarrow$ x(x - 1) = (x - 2)(x + 2)

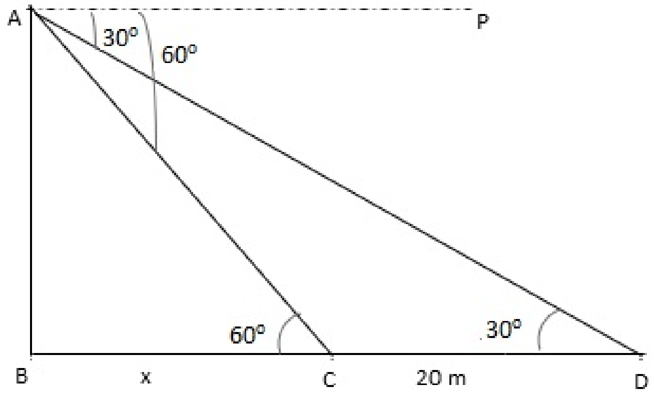
$\Rightarrow$ x^2x = x^2 - 4

$\Rightarrow$ -x = -4

$\Rightarrow$ x = 4 units

Hence, the Value of x is 4 units

24. The above figure can be redrawn as shown below:



i. From the figure,

   let AB = h and BC = x

   In ΔABC,

   $tan\ 60 = \frac{\text{AB}}{\text{BC}} = \frac{\text{h}}{x}$

   $\sqrt{3}$ = $\frac{\text{h}}{x}$

   h = $\sqrt{3}$ x ....(i)

   In ΔABD,

   $tan\ 30 = \frac{\text{AB}}{\text{BD}} = \frac{\text{h}}{x+20}$

   $\frac{1}{\sqrt{3}}$ = $\frac{\sqrt{3}x}{x+20}$ [using (i)]

   x + 20 = 3x

   x = 10m

ii. Height of the building, h = $\sqrt{3}$ x = 10$\sqrt{3}$ = 17.32 m

25. Construction : Draw OC

Proof : Line AB is tangent to smaller circle at point C.

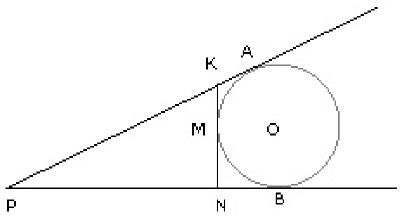
$\therefore$ segment $OC \perp AB$

AB is chord to larger circle and

as perpendicular drawn from centre to chord bisects the chord.

$\therefore AC = CB$

OR



We know that the lengths of the tangents drawn form an external point to a circle are equal.

∴ PA = PB .... (i)

KA = KM .... (ii)

NB = NM ......(iii)

(ii) + (iii)

KA + NB = KM + NM

$\Rightarrow$ AK + BN = KM + MN

$\Rightarrow$ AK + BN = KN

26. For cone, Radius of the base (r)

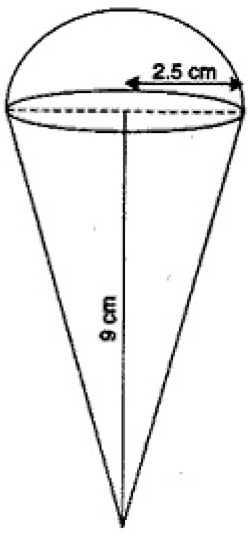
$= 2.5\text{cm} = \frac{5}{2}\text{cm}$

Height (h) = 9 cm

∴ Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9$

$= \frac{825}{14}\text{cm}^3$



For hemisphere,

Radius (r) = $2.5\text{cm} = \frac{5}{2}\text{cm}$

∴ Volume = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{1375}{42}\text{cm}^3$

i. The volume of the ice-cream without hemispherical end = Volume of the cone
$= \frac{825}{14} \mathrm{cm}^3$

ii. Volume of the ice-cream with hemispherical end = Volume of the cone + Volume of the hemisphere
$= \frac{825}{14} + \frac{1375}{42} = \frac{2475+1375}{42}$
$= \frac{3850}{42} = \frac{275}{3} = 91\frac{2}{3} \mathrm{cm}^3$

Section C

27. We have to express the number $0.3\overline{178}$ in the form of rational number $\frac{a}{b}$.

Let x = $0.3\overline{178}$

or, x = .3178178178....

or, 10,000x = 3178.178178.............(i)

or, 10x = 3.178178.............. (ii)

Subtracting (i) and (ii) we get,

9990x = 3175

or, x = $\frac{3175}{9990} = \frac{635}{1998}$

OR

$P(x) = a^8 - b^8 = \left(a^4 + b^4\right)\left(a^4 - b^4\right)$
$= \left(a^4 + b^4\right)\left(a^2 + b^2\right)\left(a^2 - b^2\right)$
$= \left(a^4 + b^4\right)\left(a^2 + b^2\right)(a + b)(a - b)$
$Q(x) = (a + b)\left(a^4 - b^4\right)$
$= (a + b)\left(a^2 + b^2\right)\left(a^2 - b^2\right)$
$= (a + b)\left(a^2 + b^2\right)(a + b)(a - b)$ {Using Identity $a^2 - b^2 = (a + b)(a - b)$}
Common factors: $\left(a^2 + b^2\right), (a - b), (a + b)$
Uncommon factors: $\left(a^4 + b^4\right), (a + b)$
$\therefore LCM$ of $P(x)$ and $Q(x)$
$= \left(a^2 + b^2\right)(a - b)(a + b) \times \left(a^4 + b^4\right)(a + b)$
$= \left(a^4 + b^4\right)\left(a^2 + b^2\right)(a + b)^2(a - b)$

28. Here, d = 5

$S_9 = 75$

We know that

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_9 = \frac{9}{2}[2a + (9-1)d]$

$\Rightarrow S_9 = \frac{9}{2}[2a + 8d]$

$\Rightarrow S_9 = 9\,[a + 4d]$

$\Rightarrow S_9 = 9\,[a + 4 \times 5]$

$\Rightarrow S_9 = 9[a + 20]$

$\Rightarrow$ 75 = 9a + 180

$\Rightarrow$ 9a = 75 - 180

$\Rightarrow$ 9a = -105

$\Rightarrow a = -\frac{105}{9}$

$\Rightarrow a = -\frac{35}{3}$

Again, we know that

$a_n = a + (n - 1)d$

$\Rightarrow a_9 = a + (9 - 1)d$

$\Rightarrow a_9 = a + 8d$

$\Rightarrow a_9 = -\frac{35}{3} + 8(5)$

$\Rightarrow a_9 = -\frac{35}{3} + 40$

$\Rightarrow a_9 = \frac{-35+120}{3}$

$\Rightarrow a_9 = \frac{85}{3}$

29. The given pair of the equation is:

$\frac{1}{(3x+y)} + \frac{1}{(3x-y)} = \frac{3}{4}$ ....(1)

$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$ ....(2)

Put, $\frac{1}{3x+y} = u$ ....(3)

and $\frac{1}{3x-y} = v$ ....(4)

Then, the equation (1) and (2) can be rewritten as :

$u + v = \frac{3}{4}$ ....(5)

$\frac{1}{2}u - \frac{1}{2}v = -\frac{1}{8}$ ....(6)

Equation(6) gives

$u - v = -\frac{1}{4}$ ....(7)......Multiplying both sides by 2

Adding equation(5) to equation (7), we get

$2u = \frac{3}{4} - \frac{1}{4} = \frac{1}{2} \Rightarrow u = \frac{1}{4}$............(8)

Subtracting equation(7) from equation (5), we get $2v = \frac{3}{4} + \frac{1}{4} = 1$

$\Rightarrow \quad v = \frac{1}{2}$ ....(9)

From equation(3) and equation(8), we get $\frac{1}{3x+y} = \frac{1}{4}$

$\Rightarrow 3x + y = 4$....................(10)

From equation (4) and equation (9), we get

$\frac{1}{3x-y} = \frac{1}{2} \Rightarrow 3x - y = 2$...............(11)

Adding equation (10) and equation(11), we get 6x = 6

$\Rightarrow x = \frac{6}{6} = 1$

Substitute this value of x in equation (1), we get 3(1) + y =4

$\Rightarrow 3 + y = 4$

$\Rightarrow y = 4 - 3 = 1$

Hence, the solution of the given pair of equation is x = 1, y =1

Verification, Substituting x =1, y =1.

We find that both the equations (1) and (2) are satisfied as shown below:

$\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{1}{3(1)+1} + \frac{1}{3(1)-1} = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} - \frac{1}{2(3.1+1)} - \frac{1}{2(3.1-1)}$

$= \frac{1}{8} - \frac{1}{4} = -\frac{1}{8}.$

This verifies the solution.

OR

Let in the first two digit number,

Unit's digit = x

And, ten's digit = y

Them, the number = 10y + x

On interchanging the digits, in the new number unit's digit = y

And, ten's digit = x

$\therefore$ The new number = 10x + y

According to the question,

(10y + x) + (10x + y) = 110

$\Rightarrow$ 11x + 11y = 110

$\Rightarrow$ x + y = 10 ....Dividing throughout by 11

$\Rightarrow$ x + y - 10 = 0 ....(1)

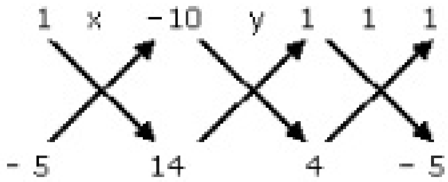
And (10y + x) - 10 = 5(x + y) + 4

$\Rightarrow$ 10y + x - 10 = 5x + 5y + 4

$\Rightarrow$ -4x +5y -14 =0

$\Rightarrow$ 4x - 5y + 14 = 0 ....(2)

To solve the equation (1) and (2) by cross multiplication method, we draw the diagram below;



Then,

$\Rightarrow \frac{x}{(1)(14)-(-5)(-10)} = \frac{y}{(-10)(4)-(14)(1)} = \frac{1}{(1)(-5)-(4)(1)}$

$\Rightarrow \frac{x}{14-50} = \frac{y}{-40-14} = \frac{1}{-5-4}$

$\Rightarrow \frac{x}{-36} = \frac{y}{-54} = \frac{1}{-9}$

$\Rightarrow x = \frac{-36}{-9} = 4$ and $\Rightarrow y = \frac{-54}{-9} = 6$

Hence, the first two digit number = 10 $\times$ 6 + 4 = 60 + 4 = 64

Verification. Substituting x = 4, y = 6, we find that both the equations (1) and (2) are satisfied as shown below:

x + y - 10 = 4 + 6 - 10 = 0

4x - 5y + 14 = 4(4) - 5(6) + 14

= 16 - 30 + 14 = 0

Hence, the solution we have got is correct.

30. The given polynomial is:

f(x) = $2x^3 - x^2 - 4x + 2$.

It is given that the two zeroes of the above polynomial are $\sqrt{2}$ and -$\sqrt{2}$

Therefore, (x - $\sqrt{2}$)(x + $\sqrt{2}$) = (x^2 - 2) is a factor of f(x).

Now we divide (x) = $2x^3 - x^2 - 4x + 2$ by (x^2- 2), we obtain

$$x^2 - 2\overline{)2x^3 - x^2 - 4x + 2}(2x - 1$$
$$\underline{2x^3 \quad - 4x}$$
$$-x^2 \quad + 2$$
$$\underline{-x^2 \quad + 2}$$
$$\underline{\times}$$

Where quotient = (2x - 1)

$\therefore$ f(x) = 0 $\Rightarrow$ (x^2 - 2)(2x - 1) = 0

$\Rightarrow$ (x - $\sqrt{2}$) (x + $\sqrt{2}$)(2x - 1) = 0

$\Rightarrow$ (x - $\sqrt{2}$) = 0 or (x + $\sqrt{2}$) = 0 or (2 x -1) = 0

$\Rightarrow$ x = $\sqrt{2}$ or x = - $\sqrt{2}$ or x = $\frac{1}{2}$.

Hence, all zeros of f(x) are $\sqrt{2}$, -$\sqrt{2}$ and $\frac{1}{2}$.

31. i. A(1, 7), B(4, 2), C(-4, 4)

    Distance travelled by Seema, AC = $\sqrt{[-4-1]^2 + [4-7]^2} = \sqrt{34}$ units

    Distance travelled by Aditya, BC = $\sqrt{[-4-4]^2 + [4-2]^2} = \sqrt{68}$ units

    $\therefore$ Aditya travels more distance

    ii. By using mid-point formula,

    Coordinates of D are $\left(\frac{1+4}{2}, \frac{7+2}{2}\right) = \left(\frac{5}{2}, \frac{9}{2}\right)$

    iii. ar$(\Delta ABC) = \frac{1}{2}$ [1(2 - 4) + 4(4 - 7) - 4(7 - 2)]

    = 17 sq. units

32. Given,

tan A = n tan B

$\Rightarrow$ tanB = $\frac{1}{n}$ tan A

$\Rightarrow$ cotB = $\frac{n}{\tan A}$ ....(1)

Also given,

sin A = m sin B

$\Rightarrow$ sin B = $\frac{1}{m}$ sin A

$\Rightarrow$ cosec B = $\frac{m}{\sin A}$ .....(2)

We know that, $\text{cosec}^2B - \cot^2B = 1$, hence from (1) & (2) :-

$$\frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A} = 1$$

$$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2 \cos^2 A}{\sin^2 A} = 1$$

$$\Rightarrow \frac{m^2 - n^2 \cos^2 A}{\sin^2 A} = 1$$

$\Rightarrow m^2 - n^2\cos^2A = \sin^2A$

$\Rightarrow m^2 - n^2\cos^2A = 1 - \cos^2A$

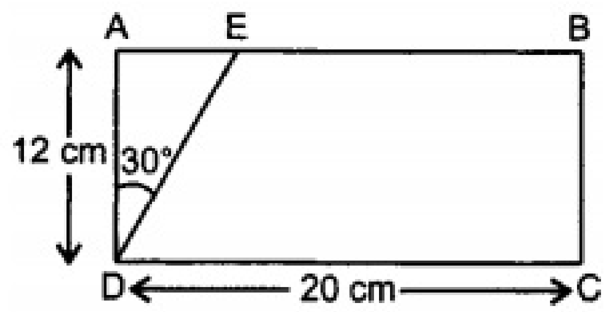
$\Rightarrow m^2 - 1 = n^2\cos^2A - \cos^2A$

$\Rightarrow m^2 - 1 = (n^2 - 1)\cos^2A$

$$\Rightarrow \frac{m^2-1}{n^2-1} = \cos^2A$$

OR

In right $\triangle$DAE,



$$\frac{\text{AE}}{\text{AD}} = \tan 30^\circ$$

$$\Rightarrow \frac{\text{AE}}{12} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \text{AE} = \frac{12}{\sqrt{3}}$$

$= 4\sqrt{3}$ cm

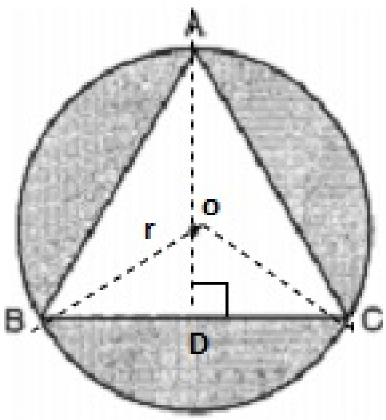
Also $\frac{\text{DE}}{\text{AD}} = \sec 30^\circ$

$$\Rightarrow \frac{\text{DE}}{12} = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \text{DE} = \frac{2}{\sqrt{3}} \times 12$$

$$= \frac{24}{\sqrt{3}}$$

$= 8\sqrt{3}$ cm

33. 

Since $\triangle$ABC is an equilateral triangle.

Therefore, AB = BC = AC

We know that equal chords subtend equal angles at the center. Here, AB, BC, CA are the chord of the circle.

$\therefore \angle AOB = \angle BOC = \angle AOC = \frac{360}{3} = 120^{\circ}$

Given: AB = BC = AC = 6 cm

Draw $OD \perp BC$

In $\triangle OBD \& \triangle OCD$

$\angle ODB = \angle ODC \quad (\text{ each } 90^{\circ})$

OD = OD (common)

OB = OC (radii of circle)

Then $\triangle OBD \cong \triangle OCD$ (by RHS theorem)

$\therefore \angle BOD = \angle COD = 60^{\circ}$ (C.P.C.T.)

And $\mathrm{BD} = \mathrm{CD} = \frac{6}{2} = 3\mathrm{cm}$ (C.P.C.T.)

In $\triangle BOD$,

$\sin 60^{\circ} = \frac{BD}{OB}$

$\frac{\sqrt{3}}{2} = \frac{3}{OB}$

$\Rightarrow OB = \frac{6}{\sqrt{3}} = 2\sqrt{3}\mathrm{cm}$, i.e. The radius of circle = $2\sqrt{3}$ cm

$\therefore$ Area of shaded region = Area of circle - Area of equilateral $\triangle$

$= \pi r^2 - \frac{\sqrt{3}}{4}(\text{ side })^2$square units

$= 3.14 \times (2\sqrt{3})^2 - \frac{\sqrt{3}}{4} \times 6 \times 6\mathrm{cm}^2$ [Given , $\pi = 3.14$]

$= 3.14 \times 12 - 9 \times 1.732\mathrm{cm}^2$

= 37.68 - 15.588 cm^2

= 22.092 cm^2

34.

C.I.	f_i	x_i	$x_i f_i$
$0-20$	15	10	150
$20-40$	$37-15=22$	30	660
$40-60$	$74-37=37$	50	1850
$60-80$	$99-74=25$	70	1750
$80-100$	$120-99=21$	90	1890
Total	$\Sigma f_i = 120$		$\Sigma x_i f_i = 6,300$

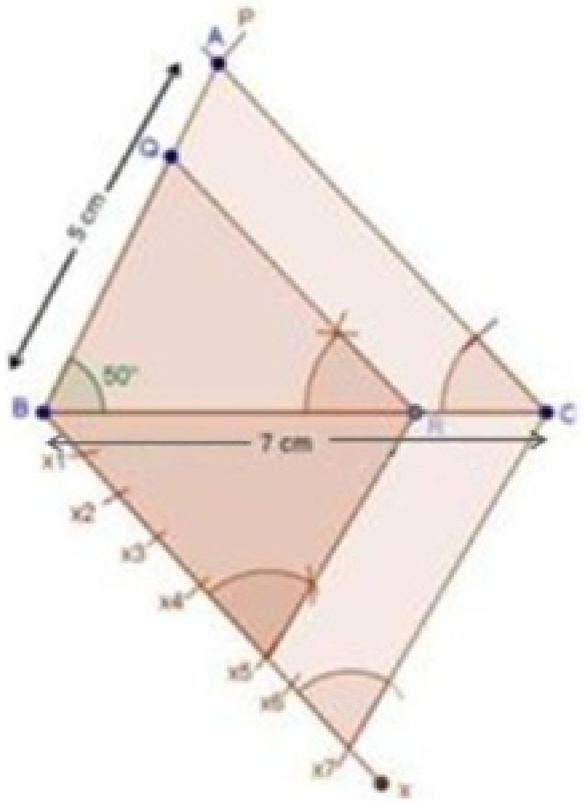
Mean = $\overline{x} = \frac{\Sigma f_1 x_1}{\Sigma f_i}$

$= \frac{6300}{120}$

= 52.5

Section D

35. Steps of construction

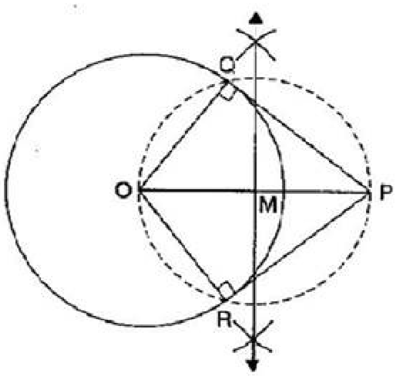


i. Draw a line segment BC of 7 cm.

ii. At B, draw $\angle PBC = 50^\circ$
iii. With centre B and radius 5 cm drawn an arc which intersect PB at A
iv. Join AC
v. Starting from B, cut 7 equal parts on BX such that $BX_1 = X_1X_2 = X_2X_3 = X_3X_4 = X_4X_5 = X_5X_6 = X_6X_7$
vi. Join X_7C.
vii. Through X_5, draw $X_5R \| X_7C$
viii. Through R, draw $RQ \| CA$
$\therefore \triangle QBR \sim \triangle ABC$

OR

Given: A circle whose centre is O and radius is 6 cm and a point P is 10 cm away from its centre.
To construct: To construct the pair of tangents to the circle and measure their lengths.

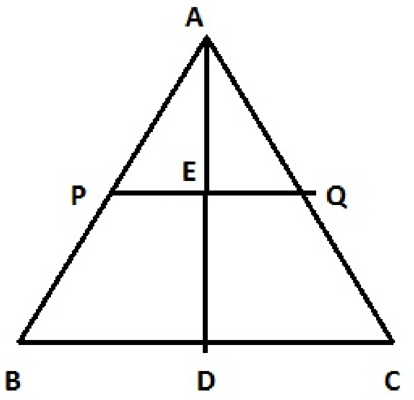


Steps of Construction:

i. Join PO and bisect it. Let M be the mid-point of PO.
ii. Taking M as centre and MO as radius, draw a circle. Let it intersects the given circle at the points Q and R.
iii. Join PQ and PR.
Then PQ and PR are the required two tangents.
By measurement, PQ = PR = 8 cm
Justification: Join OQ and OR.
Since $\angle QPO$ and $\angle ORP$ are the angles in semicircles.
$\therefore \angle OQP = 90^\circ = \angle ORP$
Also, since OQ, OR are radii of the circle, PQ and PR will be the tangents to the

circle at Q and R respectively.

$\therefore$ We may see that the circle with OP as diameter increases the given circle in two points. Therefore, only two tangents can be draw.

36. 

To Prove PE = EQ

Proof In $\triangle APE$ and $\triangle APD$, we have

$\angle PAE = \angle BAD$ [common]

$\angle APE = \angle ABD$ [corresponding angles]

$\therefore \quad \triangle APE \sim \triangle ABD$ [by AA-similarity].

$\therefore \quad \frac{AE}{AD} = \frac{PE}{BD}$ ....(i)

In $\triangle AEQ$ and $\triangle ADC$, we have

$\angle QAE = \angle CAD$ [common]

$\angle AQE = \angle ACD$ [corresponding angles]

$\therefore \quad \triangle AEQ \sim \Delta ADC$ [by AA-similarity].

But, in similar triangles, the corresponding sides are proportional.

$\therefore \quad \frac{AE}{AD} = \frac{EQ}{DC}$ ...(ii)

From (i) and (ii), we get $\frac{PE}{BD} = \frac{EQ}{DC}$

But, BD = DC [$\because$ AD is the median]

$\therefore$ PE = EQ

37. $3x - 4y - 1 = 0$

$= x = \frac{4y+1}{3}$

When $y = 2 \Rightarrow x = 3$

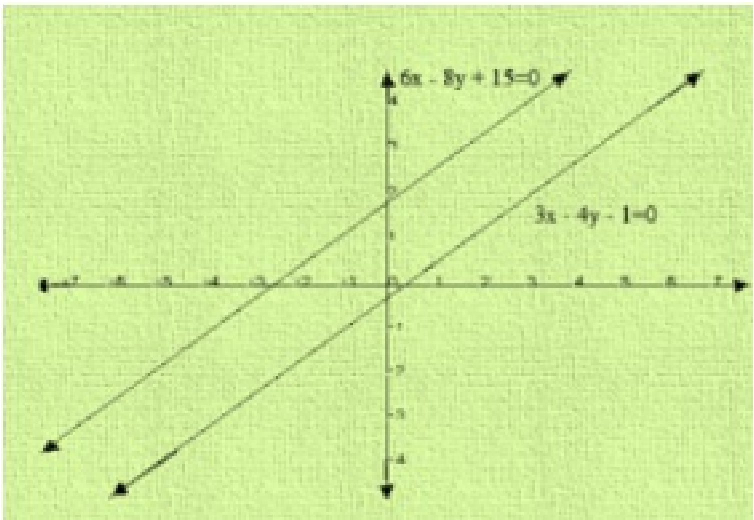
When $y = -1 \Rightarrow x = -1$

Thus, we have the following table giving points on the line $3x - 4y - 1 = 0$

x	-1	3
y	-1	2

Now, $2x + \frac{8}{3}y + 5 = 0$
$= x = \frac{8y-15}{6}$
When $y = 0 \Rightarrow x = -2.5$
When $y = 3 \Rightarrow x = 1.5$
Thus, we have the following table giving points on the line $2x - \frac{8}{3}y + 5 = 0$

x	-2.5	1.5
y	0	3



We find the lines represented by equations $3x - 4y - 1 = 0$ and $2x - \frac{8}{3}y + 5 = 0$ are parallel. So, the two lines have no common point.
Hence, the given system of equations is inconsistent.

OR

Given equations, $x + 2y - 7 = 0 \; and \; 2x - y - 4 = 0$
Now, $x + 2y - 7 = 0$
$x = 7 - 2y$
When $y = 1 \Rightarrow x = 5$
When $y = 2 \Rightarrow x = 3$
Thus, we have the following table giving points on the line $x + 2y - 7 = 0$

x	5	3
y	1	2

Now, $2x - y - 4 = 0$

$\Rightarrow y = 2x - 4$

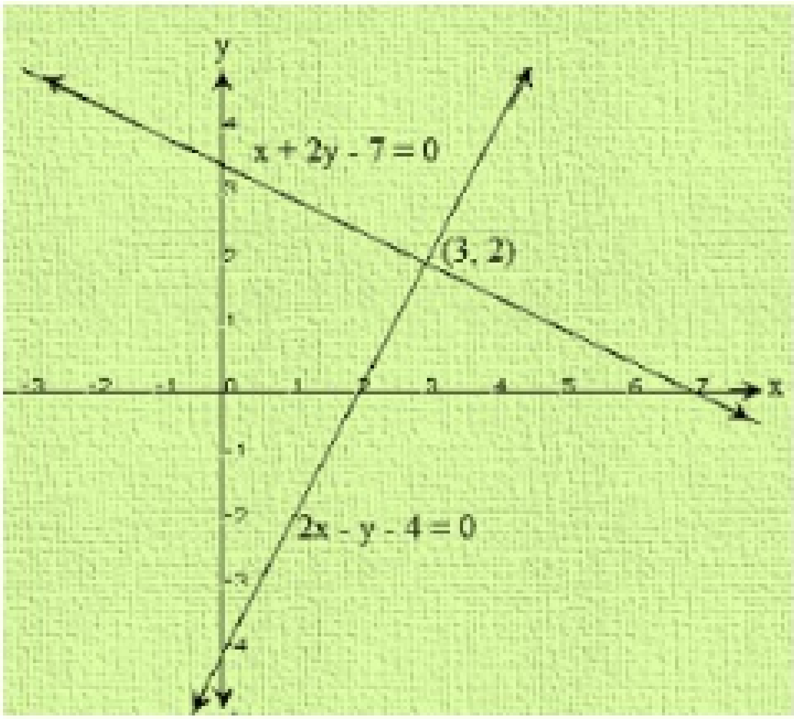
When x = 2, then y = 0

When x = 0, then y = -4

Thus, we have the following table giving points on the line $2x - y - 4 = 0$

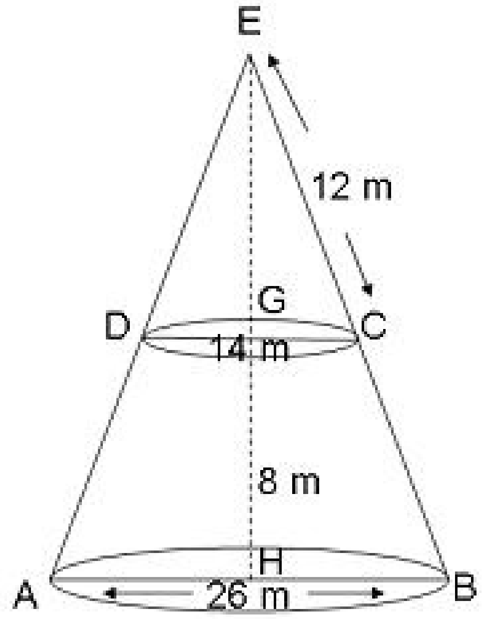
x	2	0
y	0	-4

Graph:



Clearly, two intersect at P (3, 2)

Hence, x = 3 and y = 2 is the solution of the given system of equations.

38. 

ABCD is the frustum in which upper and lower radii are r = 7 m and R = 13 m

Height of frustum, h = 8 m

Slant height CB, l_1 of frustum

$= \sqrt{h^2 + (R - r)^2}$

$= \sqrt{8^2 + (13 - 7)^2}$

$= \sqrt{64 + 36}$

$= \sqrt{100} = 10\text{m}$

Radius of the cone = 7 m

Slant height l_2 of cone = 12 m

$\therefore$ Surface area of canvas required = CSA of frustrum + CSA of cone

$= \pi(R + r)l_1 + \pi r l_2$

= π[(13 + 7)$\times$10 + 7 $\times$12]

= $\frac{22}{7} \times [200 + 84] = \frac{22}{7} \times 284m^2$

= 892.6 m^2

OR

Height of the cylinder (h) = 14 cm,

Base diameter = 7 cm

$\Rightarrow$ Radius of the base of the cylinder (r) = 3.5 cm

Volume of the cylinder = $\pi r^2 h$

= $\frac{22}{7} \times 3.5 \times 3.5 \times 14$

= 22 $\times$3.5 $\times$3.5$\times$14

= 539 cm^3

Radius of the conical holes (r1) = 2.1 cm,

Height of the conical holes (h1) = 4 cm,

volume of the conical hole $= \frac{1}{3}\pi r_1^2 h_1$

$= \frac{1}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 4$

= 18.48 cm^3

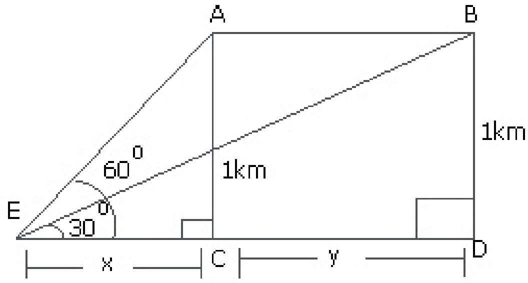
Volume of the two conical hole = 2 $\times$18.48

= 36.96 cm^3

Volume of the remaining solid = Volume of the cylinder - Volume of two conical hole

= 539 - 36.96

= 502.04 cm^3

39. 

In right $\triangle ACE$

$\frac{AC}{x} = \tan 60^\circ$

$\Rightarrow \frac{1}{x} = \frac{\sqrt{3}}{1}$

$\Rightarrow x = \frac{1}{\sqrt{3}} m$

In right $\triangle BDE$

$\frac{1}{x+y} = \tan 30^\circ$

$\Rightarrow \frac{1}{\frac{1}{\sqrt{3}}+y} = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{\sqrt{3}}{1+\sqrt{3}y} = \frac{1}{\sqrt{3}}$

$\Rightarrow 1 + \sqrt{3y} = 3$

$\Rightarrow y = \frac{2}{\sqrt{3}} km$

Speed = $\frac{distance}{time}$

$= \frac{\frac{2}{\sqrt{3}}}{\frac{10}{3600}}$

$= 240\sqrt{3}\text{km/hr}$

40. Table:

Class	Frequency	Mid value x_i	$u_i = \left(\frac{x_i - A}{h}\right)$	$f_i u_i$	Cumulative Frequency
120 - 130	2	125	-2	-4	2
130 - 140	8	135	-1	-8	10
140 - 150	12	145 = A	0	0	22
150 - 160	20	155	1	20	42
160 - 170	8	165	2	16	50
	N = 50				$\Sigma f_i u_i = 24$

i. Let the assumed mean A be 145. Class interval h = 10

Mean$(\overline{x}) = A + h\left(\frac{\sum f_i u_i}{N}\right)$

$= 145 + 10 \times \left(\frac{24}{50}\right)$

$= 145 + 4.8 = 149.8$

ii. N = 50; $\frac{N}{2} = \frac{50}{2} = 25$

Cumulative Frequency just after 25 is 42.

Therefore, median class is 150 - 160.

$l = 150, h = 10, f = 20, c.f. = 22$

Median (M)$= l + h\left(\frac{\frac{N}{2} - c.f.}{f}\right)$

$= 150 + 10 \times \left(\frac{25 - 22}{20}\right)$

$= 150 + \frac{10 \times 3}{20}$

$= 150 + 1.5 = 151.5$

iii. we know that, Mode = 3 median - 2 mean

$= 3(151.5) - 2(149.8)$

$= 454.5 - 299.6$

$= 154.9$

Thus, Mean = 149.8, Median = 151.5, Mode = 154.9