# Chapter 3

# Material Requirement Planning and Inventory Control

## CHAPTER HIGHLIGHTS

- *Material Requirement Planning*
- *Objectives of MRP*
- *JIT Philosophy (Just in time)*
- *Push and Pull Systems*
- *Unit Scheduling System*
- *Inventory Control*
- *Economic Batch Production*
- *Expression for Total Cost*
- *Lead Time and Re – Order Level*
- *Periodic Review System*

## Material Requirement Planning

Material requirement planning (MRP) is the technique of determining the quantity and timing to acquire the materials demanded by the master schedule. A master schedule is the expression of production plan into specific products and their requirements and it also specifies the time required for their acquisition.

Similarly, as indicated by the master schedule, it is required to determine the capacity in terms of man power and in terms of machine capacity to meet the production objective. The technique of determining the capacity required is known as the capacity requirement planning.

### Explanation of Terms in MRP

#### *Dependent Demand*

It is the demand of a particular item which stems out from the demand of a complete unit, this item being a part of that unit. In MRP, we plan for the acquisition of dependent demand items as required by the master schedule.

#### *Parent Items and Component Items*

When the basic parts are assembled together, we get a parent item. For the parent item, the parts are the component items. The parent unit can be a component item for a large assembly. For example, a gear box is the assembly of a number of gear wheels. But the gear box itself is a component item when a large equipment assembly is considered.

#### *Lot Size*

It is the quantity of an item procured as a lot as per an order. Lot size is used in a wider sense. Lot sizing is the process of specifying the order size.

#### *Time Phasing*

It is the process of scheduling in such a way that we receive the required quantity of the material exactly at the time we require it. It can also be the process of scheduling such that we produce a certain amount of material required exactly at the time of its requirement – it cannot be before or after.

#### *Explosion of Requirement*

It is the process of breaking down of the parent item to components so that a detailed effective planning of the material requirement can be done.

#### *Bill of Material*

Bill of material (BOM) is the list of all components going into the assembly. The list specifies the part numbers of the components and the number of components required for the assembly. Bill of material totally specifies the item.

## Objectives of MRP

### *Reduction of Inventory*

MRP reduces the excessive build up of an inventory. MRP determines how many components are required, and at what time they are required to meet the master schedule requirements. If it is properly planned, then almost a correct quantity of the material is procured and therefore, the inventory built up is reduced.

### *Reduction of Manufacturing and Delivery Lead Time*

MRP reduces the delay in production, as it identifies the material required and takes appropriate steps to get it at the right time. So, the production is not suffered. It fixes the priorities in production and delivery. It also fixes the dates for the arrival of delivery of materials and products.

### *Realistic Commitments*

With a good system of MRP, the production department can give timely information to the marketing department so that the commitments can be observed, and a better customer relation can be maintained.

### *Increased Efficiency*

MRP maintains good co-ordination among the work centres. Therefore, uninterrupted flow of material occurs and therefore, capacity utilisation and system efficiency improves.

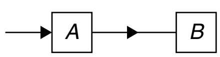
## JIT Philosophy (Just in Time)

It is a philosophy of manufacturing, which aims at having zero – inventory. The philosophy is 'right part at the right time in right quantity'. JIT is a philosophy working in the manufacturing process. The following aspects are the characteristics of JIT:

1. Planned elimination of all wastes.
2. Sustained improvement of productivity.
3. Execution of all production activities (all stages of conversion from raw material to the finished product).

## Push and Pull Systems

Push system is the conventional manufacturing system. It does not permit queuing up in front of a work centre.




Consider two work centres $A$ and $B$. Let '$A$' be faster than $B$. Then, the products and materials queue up, before $B$. To avoid queuing up, $A$ is decelerated to synchronise with $B$. This is the push system. In the pull system, we pull the process from the end point. What is demanded at the end is carried out and accordingly, the process is pulled from the end. In this case, there is no queuing up. The work in a process inventory will be very minimum.

'Kanban' is an integral part of the JIT system. It is an instruction card designed to produce what exactly is needed at the end. This system initiates the pull. It is a demand feeding process. Each preceding process will produce only that many items as have been withdrawn. The withdrawal of material from the preceding process and the production of items to replace it is ordered through a withdrawal and production Kanban (marker card).

## Solved Examples

**Example 1:** The process of fixing the sequence of operation and material flow is

(A) master scheduling (B) routing
(C) scheduling (D) expediting.

**Solution:**

**Example 2:** 'Kanban' is used to

(A) schedule production
(B) determine the optimum path for material movement
(C) request for production or withdrawal of parts
(D) decide the manpower requirement.

**Solution:**
It is an information of production and withdrawal of items.

**Example 3:** The unidirectional flow of work so that all the jobs are subjected to processing in the same order is known as

(A) flow shop (B) job shop
(C) sequencing (D) routing.

**Solution:**
The difference between flow shop and job shop is that in flow shop, the routing of all jobs through the system is unidirectional. But in job shop, the routing may follow any path.

**Example 4:** The average number of jobs in the processing system is known as

(A) mean lateness (B) mean tardiness
(C) in process inventory (D) mean process flow.

**Solution:**

**Example 5:** The most suitable forecasting technique suited for planning in long range is

(A) regression analysis (B) moving average
(C) time series analysis (D) delphi method.

**Solution:** (D)

## Unit Scheduling System

Job shop scheduling is a unit scheduling system. We may have to process varieties of jobs on different machines. The plan for production can be done only after receiving the order. Scheduling is to be done in such a way that the available resources are optimally used. Some of the popular methods of job shop scheduling are as given below:

## Shortest Processing Time (SPT)

According to this method, the job having the shortest processing time is processed first. This is done to avoid queuing up of the jobs.

## Earliest Due Date (EDD)

In this case, processing is done according to their ascending order of the availability of time before the due date. Doing so, we can keep up the commitments of the delivery dates.

## First in First Out (FIFO)

In this case, the processing is done as they come in. The job which comes first, is served first.

## Last Come First Served (LIFO)

In this case, the job which comes last is served first. When a job piles up, the job at the top will be that which came last. It is served first.

**Example 6:** Five jobs have come on a machine. The processing time of each of the job is as given below:

| Job | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Processing time (hrs) | 30 | 7 | 9 | 28 | 15 |

The minimum mean flow time is
(A) 40.4 hrs (B) 43.6 hrs
(B) 46.2 hrs (D) 48.5 hrs.

**NOTE**

Flow time is the sum of the waiting time and the processing time of each job. The average flow time will be least, when the processing is done according to the arrangement with the shortest possible time first and then processing according to the ascending order of the processing time.

**Solution:**
Arranging the jobs according to the ascending order of the processing time.

| Job | 2 | 3 | 5 | 4 | 1 |
|---|---|---|---|---|---|
| Processing time (hrs) | 7 | 9 | 15 | 28 | 30 |
| Time of completion | 7 | 16 | 31 | 59 | 89 |

$$\text{Mean flow time} = \frac{7+16+31+59+89}{5}$$
$$= 40.4 \text{ hrs.}$$

**Example 7:** In a machine shop, eight jobs come for processing. The processing time for each job and the weightages given to them are shown in the table. The minimum flow time shall be
(A) 56.2 (B) 49.2 (C) 42.3 (D) 38.6.

Arranging the job according to the ascending order of the processing time.

| Job | 5 | 4 | 6 | 8 | 1 | 3 | 7 | 2 |
|---|---|---|---|---|---|---|---|---|
| Processing time (hrs) | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 18 |
| Weightage | 1 | 1 | 3 | 2 | 1 | 1 | 2 | 2 |
| Flow time | 8 | 17 | 27 | 38 | 50 | 63 | 77 | 95 |
| Weighted flow time | 8 | 17 | 81 | 76 | 50 | 63 | 154 | 190 |

**Solution:**
Flow time = 8 + 17 + 81 + 76 + 50 + 63 + 154 + 190
= 639.

$$\text{Mean flow time} = \frac{\text{Total flow time}}{\text{Total of weightage}}$$
$$= \frac{639}{13} = 49.15 \text{ min.}$$

**Example 8:** In a work shop, a cost of ₹50 is incurred for each day for a job. The workshop receives six jobs on a day with the following details:

| Job | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Processing time (days) | 6 | 4 | 7 | 2 | 9 | 3 |
| Due dates | 16 | 12 | 20 | 4 | 27 | 7 |

The minimum total cost is (₹)
(A) 4800 (B) 4200 (C) 4000 (D) 3600.

**Solution:**
The cost becomes minimum when the jobs are completed and given off from the work shop as early as possible. Therefore, we have to schedule the jobs according to the shortest processing time first. Accordingly, we have

| Job | Processing time | Total time | Due date |
|---|---|---|---|
| 4 | 2 | 2 | 4 ✓ |
| 6 | 3 | 5 | 7 ✓ |
| 2 | 4 | 9 | 12 ✓ |
| 1 | 6 | 15 | 1 ✓ |
| 3 | 7 | 22 | 20 ✗ |
| 5 | 9 | 31 | 27 ✗ |
| | | 84 | |

Job nos. 3 and 5 are delayed, all the other jobs are completed before time.
Total number of days to be charged = 84
Cost = 84 × 50
= ₹4200.

***Direction for questions 9 and 10:***

| Job | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Processing time | 12 | 18 | 13 | 9 | 8 | 10 | 14 | 11 |
| Weight | 1 | 2 | 1 | 3 | 1 | 3 | 2 | 2 |

A job shop has six orders at hand which are to be completed with one work centre. The processing time and the due dates are as shown.

| Order | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Processing time (days) | 3 | 2 | 9 | 4 | 2 | 4 |
| Due date | 17 | 21 | 5 | 12 | 15 | 24 |

**Example 9:** Minimum average flow time is (days)
(A) 12.5 (B) 11.5 (C) 10.5 (D) 9.5.

**Solution:**

| Order | Processing time | Flow time | Due date | Lateness |
|---|---|---|---|---|
| 5 | 2 | 2 | 15 | - |
| 2 | 2 | 4 | 21 | - |
| 4 | 3 | 7 | 17 | - |
| 6 | 4 | 11 | 24 | - |
| 4 | 4 | 15 | 12 | 3 |
| 3 | 9 | 24 | 5 | 19 |
| | | 63 | | 24 |

(Arranging the orders according to the rule of shortest processing time first)

$$\text{Minimum flow time} = \frac{63}{6} = 10.5.$$

**Example 10:** Total tardiness (days)
(A) 17 days (B) 12 days (C) 5 days (D) 22 days.

**Solution:**
Tardiness means lateness.
Two of the jobs are delayed.
No. of days of delay = 3 + 19 = 22 days.

***Direction for questions 11 and 12:*** Scheduling of jobs on a single machine is to be done. The processing time of each job and their due dates are given below.

| Job | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Processing time | 10 | 8 | 8 | 7 | 12 | 15 | 18 |
| Due date | 15 | 10 | 12 | 11 | 18 | 25 | 30 |

**Example 11:** The optimal sequence which will minimise the maximum lateness is
(A) 2 – 4 – 3 – 1 – 5 – 6 – 7
(B) 2 – 3 – 1 – 4 – 6 – 5 – 7
(C) 2 – 4 – 1 – 5 – 6 – 7 – 3
(D) 3 – 1 – 4 – 5 – 7 – 6 – 2.

**Solution:**
The minimum value of maximum lateness can be obtained by arranging the jobs according to the earliest due date (EDD).

| Job | 2 | 4 | 3 | 1 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Processing time | 8 | 7 | 8 | 10 | 12 | 15 | 18 |
| Due date | 10 | 11 | 12 | 15 | 18 | 25 | 30 |

| Job | Processing time | Flow time | Due date | Lateness |
|---|---|---|---|---|
| 2 | 8 | 8 | 10 | - |
| 4 | 7 | 15 | 11 | 4 |
| 3 | 8 | 23 | 12 | 11 |
| 1 | 10 | 33 | 15 | 18 |
| 5 | 12 | 45 | 18 | 27 |
| 6 | 15 | 60 | 25 | 35 |
| 7 | 18 | 78 | 30 | 48 |

The sequence is 2 – 4 – 3 – 1 – 5 – 6 – 7.

**Example 12:** The maximum lateness in the optimum sequence
(A) 32 (B) 27 (C) 48 (D) 90.

**Solution:**
From the table, it is clear that the maximum delay is 48. It occurred in the case of job no. 7.

***Direction for questions 13 and 14:*** In a work centre, the number of jobs arriving in the system and their due dates are given.

| Job | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Processing time (days) | 9 | 7 | 5 | 11 | 6 |
| Due date (days) | 16 | 20 | 25 | 15 | 40 |

**Example 13:** As per the 'shortest processing time' criterion, the average number of jobs in the system is
(A) 3.4 (B) 3.2 (C) 2.6 (D) 1.8.
Arranging according to SPT,

| Job | Processing time | Flow time | Due date | Job lateness |
|---|---|---|---|---|
| 3 | 5 | 5 | 25 | 0 |
| 5 | 6 | 11 | 40 | 0 |
| 2 | 7 | 18 | 20 | 0 |
| 1 | 9 | 27 | 16 | 11 |
| 4 | 11 | 38 | 15 | 23 |
| | 38 | 99 | | 34 |

**Solution:**
Total flow time = 99
Average no. of jobs in the system is

$$\frac{\text{Total flow time}}{\text{Total completion time}} = \frac{99}{38}$$
$$= 2.6.$$

**Example 14:** Average lateness in the system.
(A) 9.2 days (B) 8.6 days (C) 7.5 days (D) 6.8 days.

**Solution:**
Total lateness = 34 days.

$$\text{Average lateness} = \frac{34}{5} = 6.8 \text{ days.}$$

***Direction for questions 15 and 16:*** In a job centre, there are six jobs at hand. The jobs, their processing time and due dates are as shown in the chart:

| Job | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Processing time | 8 | 6 | 9 | 7 | 4 | 5 |
| Due date | 10 | 12 | 20 | 32 | 36 | 40 |

**Example 15:** As per the 'earliest due date' criterion, the average number of jobs in the system is
(A) 4.2 days (B) 3.8 days (C) 3.2 days (D) 2.9 days.

**Solution:**
Arranging the jobs as per the earliest due date (EDD),

| Job | Processing time | Flow time | Due date | Lateness |
|---|---|---|---|---|
| 1 | 8 | 8 | 10 | - |
| 2 | 6 | 14 | 12 | 2 |
| 3 | 9 | 23 | 20 | 3 |
| 4 | 7 | 30 | 32 | - |
| 5 | 4 | 34 | 36 | - |
| 6 | 5 | 39 | 40 | - |
| | 39 | 148 | | 5 |

Total flow time = 148
Total completion time = 39

Number of jobs at hand = $\dfrac{148}{39} = 3.8$ days.

**Example 16:** Average job lateness is
(A) 0.83 days (B) 1.6 days (C) 3.2 days (D) 4.1 days.

**Solution:**

$$\text{Average lateness} = \frac{\text{Total lateness}}{\text{Number of jobs}}$$

$$\frac{5}{6} = 0.83 \text{ days.}$$

# Inventory Control

Inventory may be defined as the quantity of economic resources (goods) stored at any point of time or remaining idle at any point of time. The variables in inventory control are:

1. Raw materials
2. Semi finished goods
3. Finished products
4. Machinery
5. Fixtures and furniture etc.

Inventories are broadly classified in to:

1. Direct inventory
2. Indirect inventory.

Direct inventories are those materials that become the integral part of the final product. The raw material, semi–finished goods etc. are direct inventories whereas, fuel, cleaning agents, lubricants etc. are indirect inventories.

Inventory control is a planned scientific approach by which we decide when to purchase, how much to purchase and how much to store so that the cost is minimum. At the same time, there should be no interruption in the process of production.

## Purpose of Inventory Control

- During fluctuating demand, the inventory helps in maintaining economy by absorbing the fluctuations.
- It helps in smooth and efficient running of an organisation.
- Service is provided at short notice when an inventory is maintained.
- When there are shop rejections or delay in the raw materials, it acts as a buffer stock.
- Product cost is reduced because of long, uninterrupted production runs and batching.
- Because of the inventory, bulk purchases can be met and the clerical cost be reduced.

In the case of raw material inventory, the basic question is (a) when should the order be placed for the material? (b) how much should be procured through the order?

## Costs Associated with Inventory

There are two costs associated with an inventory. When we are trying to reduce one cost, the other cost increases. Therefore, we have to strike a balance between these costs. Therefore, inventory control becomes an operations research problem.

The costs are:

### *Holding Cost or Inventory Carrying Cost*

This is the cost involved in keeping the goods in the store. When there is an under stock, we have to see that it is maintained properly, (according to its nature), so that it will be available at a required quality at the time of requirement. Moreover, the holding cost depends on its size and the time for which it is to be stored. It indicates the space and time that we should have to maintain it. It involves cost. Generally, holding cost includes the following:

1. Storage cost
2. Handling cost
3. Depreciation cost
4. Rent and capital cost
5. Administrative cost etc.

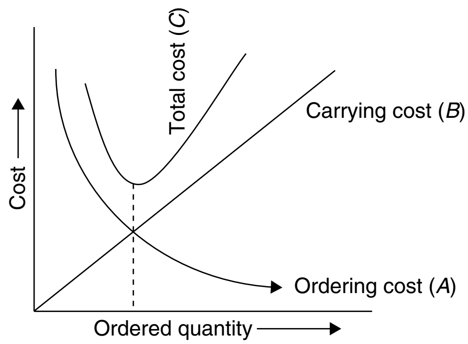
### *Ordering Cost (Set Up Cost)*

This is the sum of the various costs involved in placing the order for a material. All the purchase formalities are cost – involved. It includes

1. The cost of placing an order
2. The cost of transportation
3. The cost of inspection
4. The cost of checking supplies
5. The advertisement cost
6. The cost of communication
7. Cost of stationary etc.

It can be seen that if the quantity purchased per order is more, the number of orders to be placed within a specified time frame will be less. Consequently, ordering cost will decrease. Because ordering cost is directly proportional to the number of orders, more quantity of the material arrives at the stores at a time; the inventory carrying cost will be high.

On the other hand, if the quantity ordered per order is less, more number of orders are to be placed in a specified time. When the number of orders is more, the ordering cost will be high. But, as the number of units arriving in the store per order is less, the inventory carrying cost will be less, in this case.




In the graph shown above, '$A$' indicates the ordering cost which increases as the number of orders increase. At the same time, as the number of orders increase, because the quantity procured per order is less, the inventory carrying cost (curve $B$) decreases.

The other costs are purchase costs and shortage costs or stock-out costs. Purchase cost is the price paid for producing/purchasing the item. Shortage costs or stock out costs are the costs incurred when there is a delay in meeting the demand or an inability to meet it at all.

$\therefore$ Total variable cost = carrying cost + ordering cost + shortage cost.

If the unit cost is dependent on the quantity purchased, then the total variable inventory cost will be

Total variable inventory cost = purchase cost + carrying cost + ordering cost + shortage cost.

In inventory management, the best policy corresponds to the minimum total cost (curve $C$). Corresponding to the minimum total cost, the quantity purchased per order is known as the Economic Order Quantity ($EOQ$).

## Variables in Inventory Control

There are two types of variables in inventory control:

1. Controllable variables and
2. Uncontrollable variables

### *Controllable Variables*

These are the variables which can be controlled separately or in combination.

1. How much to procure?
   The quantity to be ordered to raise the available stock by a prescribed level and in a particular order can be controlled.
2. When to order?
   The inventory is replenished when the stock is equal to or below the prescribed quantity or at every time interval.
3. Completion of stocked items:
   To meet the demand, having more quantity of the finished goods maintains less delay. But more the stock, higher is the inventory holding cost; less the stock, more is the delay in meeting the demand.

### *Uncontrollable Variables*

1. Various costs involved in inventory control:
   (a) Holding costs
   (b) Shortage costs
   (c) Setup costs.
2. Demand: The demand pattern may be deterministic or probabilistic. In deterministic models, the quantity required for a fixed period of time is known exactly. The known demand maybe fixed or variable.

***Static inventory model*** Models with known demand which is fixed over a period of time are called static inventory models.

***Dynamic inventory model*** Models with known demand which vary with time are called dynamic inventory models.

In the probabilistic model, demand over a certain period of time is known with a certain probability, but the pattern is described by a known probability distribution. It can be stationary or non-stationary over a period of time.

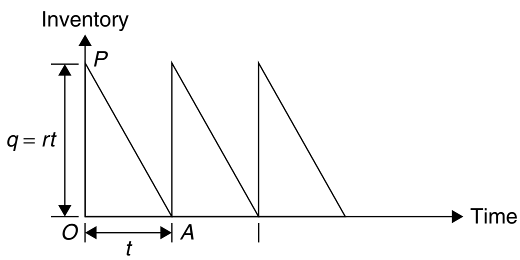
3. Lead time: When the demand is deterministic and the lead time is known, then the order should be placed in advance by an amount equal to the lead time. If the lead time is zero, then the replenishment of stock is instantaneous and there will be no need to order in advance. When the demand or lead time are known with a probability, then the amount and the time of replenishment are found by using expected costs of holding and shortage.
4. Supply of goods: The supply of goods may vary around the quantity of the goods ordered. The amount procured may vary with a known probability density function.

It is difficult to have a single model to evaluate the inventory costs and quantities which take into account all the variations. Therefore, inventory models are classified into two major categories on the basis of demand i.e., deterministic models and probabilistic models. The other categories are models with price breaks and models with restrictions.

## Deterministic Model I(a)

In this model, the demand rate is uniform, lead time is zero i.e., the replenishment time of items is negligible and shortages are not allowed. Let $r$ be the uniform demand rate at which the goods are supplied to the customers. If the orders are placed at time intervals $t$ of quantity $q$, then the quantity $q$ is given by $q = rt$ in each order. In a small time $dt$, the stock is $rt\,dt$, the stock for the time period $t$ is

$$\int_0^t rt\,dt = \frac{1}{2}rt^2 = \frac{1}{2}qt \quad \text{(area of triangle OAP)}$$




$C_2$ = inventory holding cost of one unit per unit time.
$C_1$ = ordering cost per order.

∴ Inventory holding cost during time $t = \frac{1}{2}\ C_2 R t^2$
ordering cost per order = $C_1$

∴ Total cost during time $t = \frac{1}{2}\ C_2\, Rt^2 + C_1$

∴ Average total cost per unit time

$$C(t) = \frac{1}{2}\ C_2\, Rt^2 + C_1$$

∴ The optimum time interval to place an order is

$$t_o = \sqrt{\frac{2C_1}{C_2 r}}$$

By multiplying with $r$, we get the optimum quantity to order during each order.

$$\therefore q_o = r\,t_o = \sqrt{\frac{2C_1 r}{C_2}}$$

where, $q_o$ is called the optimum lot size or economic order quantity.

The minimum average cost per unit time is

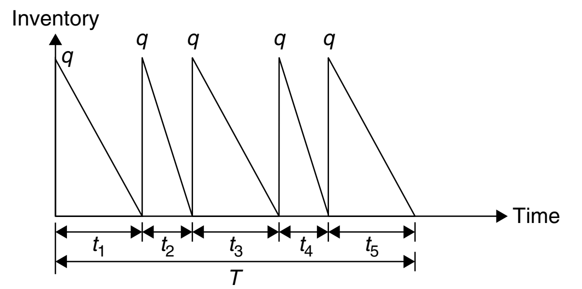
$$C_o = \sqrt{2C_1C_2r}.$$

The total minimum cost per unit time, where $C$ is the cost per unit of the item is

$$= \sqrt{2C_1C_2r + Cr}.$$

## Deterministic Model I(b)

In this model, the demand rate is non-uniform, lead time is zero and shortages are not allowed. As there is a non-uniform demand rate instead of the demand rate, the total demand $D$ for some long time $T$ is given.




The total time period $T$ is given as the sum of the time intervals $t_1$, $t_2$, ….. $t_n$.
Holding cost for time period $T$ is

$$\left(\frac{1}{2}qt_1\right)C_2 + \left(\frac{1}{2}qt_2\right)C_2 + \left(\frac{1}{2}qt_3\right)C_2 + \ldots. + \left(\frac{1}{2}qt_n\right)C_2$$

$$= \frac{1}{2}qC_2\left(t_1 + t_2 + \ldots t_n\right)$$

$$= \frac{1}{2}qTC_2$$

$$\text{Ordereing cost} = N.C_1 = C_1 \cdot \frac{D}{q}$$

$$\therefore \text{Total cost} = \frac{1}{2}qC_2T + C_1\frac{D}{q}$$

$$\text{The optimum lot size} = q_o = \sqrt{\frac{2C_1\left(D/T\right)}{C_2}}$$

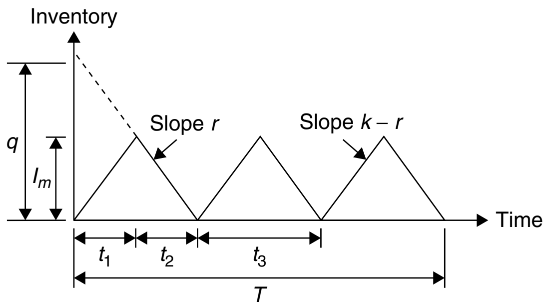
$$\text{Minimum total cost} = \sqrt{2C_1C_2\left(D/T\right)}$$

## Deterministic Model I(c)

In this model, the demand rate is uniform, lead time is not zero i.e., the production rate is finite and shortages are not allowed. The production rate should be higher than the consumption rate to provide an inventory.

$r$ = number of units consumed per unit time
$k$ = number of units produced per unit time
$C_1$ = ordering cost

$C_2$ = inventory holding cost per item per unit time.
$q = rt$ = number of items produced per run
$t$ = interval between runs.




$t_1$ is the time taken to build up the stock with a constant rate $k - r$ and $t_2$ is the time taken to consume the stock during which there is no production rate and the inventory decreases at a constant demand rate $r$.

$I_m$ is the maximum inventory at the end of time $t_1$.

$\therefore I_m = (k - r)t_1$

Total quantity produced during $t_1$ is $q$ and quantity consumed during $t_1$ is $rt_1$.

$$\therefore I_m = q - rt = q - \left(\frac{rI_m}{k - r}\right)$$

$$\therefore I_m = \left(\frac{k - r}{k}\right)q.$$

Total cost per unit time $C(I_m, t) = \frac{1}{2}\ I_m.C_2 + \left(\frac{C_1}{t}\right)$

$$\therefore C(q, t) = \frac{1}{2}\left(\frac{k - r}{k}\cdot q\right)C_2 + \left(\frac{C_1}{t}\right)$$

$$= \frac{1}{2}\left(\frac{k - r}{k}\right)C_2.q + \frac{C_1 r}{t}$$

Optimum lot size $= q_o = \sqrt{\frac{2C_1}{C_2}\cdot\frac{rk}{k - r}}$

Optimum average cost/unit time $= C_o = \sqrt{\frac{k - r}{k}}.\sqrt{2C_1 C_2 r}$

**NOTE**

When $k = r$, $C_o = 0 \Rightarrow$ no holding cost and no set-up cost.
When $k = \infty \Rightarrow$ production rate is infinite i.e., model 1(a)

## Economic Batch Production

Production managers have to decide very often what quantity of the product must be produced, against the demand. If large quantities are produced and they are not sold out immediately, the task of holding the inventory in the store arises. Therefore, a similar situation arises in the case of production of goods as well, which is identical to that of the procurement of raw materials. Often, the quantity produced exceeds the quantity which can be sold. Therefore, the optimum lot size that is to be produced in one batch is to be determined. It is known as economic lot size or economic batch size.

Economic batch size is also given by the formula

$$Q = \sqrt{\frac{2C_1 r}{C_2}}$$

where, $C_1$ represents the set up cost. It is the cost of set up to be made for one batch production.

$C_2$ = inventory carrying cost per unit per year and $r$ = the annual anticipated demand for the quantity produced.

**Example 17:** A company manufactures a product which has a monthly demand of 4000 units. The product requires a component '$A$' priced ₹20. Every product requires one unit of '$A$'. The ordering cost for the component is ₹120/order and the holding cost is 10%/annum/unit.
The economic order quantity is

(A) 2400 units (B) 2600 units
(C) 3200 units (D) 3600 units.

**Solution:**
Given, demand ($r$) = 4000 monthly = 4000 × 12 annually
Ordering cost = ₹120.
Inventory carrying cost is 10% of 20 = ₹2.

$$\therefore EOQ = \sqrt{\frac{2 \times 120 \times 4000 \times 12}{2}}$$
$$= \sqrt{144 \times 40000}$$
$$= 12 \times 200 = 2400 \text{ units.}$$

***Direction for questions 18 to 20:*** A firm has a monthly demand for a material equal to 5000 units. The ordering cost is evaluated to be ₹225/order. The inventory carrying cost is 5% of the unit price of the material. Unit price of the material is ₹60.

**Example 18:** The $EOQ$ is

(A) 2600 units (B) 3000 units
(C) 3200 units (D) 3600 units

**Solution:**

$$EOQ = \sqrt{\frac{2C_1 r}{C_2}}$$

$r = 5000 \times 12 = 60000$

$C_1$ = ₹225/order

$C_2 = \frac{5}{100} \times 60 =$ ₹3.

$$\therefore EOQ = \sqrt{\frac{2 \times 225 \times 5000 \times 12}{3}}$$
$$= \sqrt{20000 \times 225 \times 2}$$

$$= \sqrt{40000 \times 225}$$
$$= 200 \times 15$$
$$= 3000 \text{ units.}$$

**Example 19:** If instead, the firm decides to procure 4000 units/order, the additional cost for the company is
(A) ₹675 (B) ₹520 (C) ₹450 (D) ₹375.

**Solution:**
If the purchased quantity is 4000/order.

$$\text{Number of orders per annum} = \frac{60000}{4000} = 15.$$

∴ Ordering cost = 15 × 225 = ₹3375.

$$\text{Inventory carrying cost} = \frac{4000}{2} \times 3$$
$$= \frac{12000}{2} = 6000.$$

Total cost = 6000 + 3375 = 9375.
Total cost in the case of *EOQ*:

$$\text{Number of orders} = \frac{60000}{3000} = 20 \text{ orders.}$$

Ordering cost = 20 × 225 = 4500.

$$\text{Inventory carrying cost} = \frac{3000}{2} \times 3$$
$$= 4500.$$

Total cost = 4500 + 4500 = 9000.
Additional cost = ₹375.

**Example 20:** The minimum carrying cost the company has to incur (₹)
(A) 2450 (B) 2800 (C) 3600 (D) 4500.

**Solution:**
The minimum carrying cost is that which corresponds to *EOQ*
i.e., = ₹4500. It depends on the size of the inventory.

**Example 21:** A company has an annual demand of 400 units of a product. The ordering cost per order is ₹50. The cost per item is ₹10 and the carrying cost is 15% of the unit cost of the item. The total minimum cost is
(A) ₹523 (B) ₹416 (C) ₹380 (D) ₹245.

**Solution:**
The total cost can be directly found from the formula $= \sqrt{2rC_1C_2}$

Ordering cost $C_1 = 50$.
Inventory carrying cost = 15% of ₹10
= ₹1.5.

$$\therefore \text{Total cost} = \sqrt{2 \times 400 \times 50 \times 1.5}$$
$$= \sqrt{800 \times 50 \times 1.5}$$
$$\sqrt{800 \times 75} = ₹245.$$

## Expression for Total Cost

Ordering cost $= \frac{r}{q} C_1$ when $q$ is *EOQ*, $r$ is the annual demand and $C_1$ the ordering cost. Inventory carrying cost is $\frac{q}{2}.C_2$

$$\text{Total cost} = \frac{r}{q} C_1 + \frac{q}{2} C_2$$

$$\text{where } q = \sqrt{\frac{2C_1 r}{C_2}}$$

$$\text{Total cost} = \frac{rC_1}{\sqrt{\frac{2C_1 r}{C_2}}} + \frac{\sqrt{\frac{2C_1 r}{C_2}}}{2} \times C_2$$

$$= \frac{r\sqrt{C_2 C_1}}{\sqrt{2C_1 r}} + \frac{\sqrt{2C_1 r}}{2\sqrt{C_2}} \times C_2$$

$$= \frac{\sqrt{rC_1C_2}}{\sqrt{2}} + \frac{\sqrt{2C_1C_2 r}}{2}$$

$$= \frac{\sqrt{2C_1C_2 r}}{2} + \frac{\sqrt{2C_1C_2 r}}{2}$$

$$\text{Total cost} = \sqrt{2C_1C_2 r}$$

**Example 22:** A manufacturer has to supply at the rate of 1000 units/year. Shortage is not allowed. The storage cost is ₹0.80/unit/year. The set up cost per run of production is ₹80. The minimum average yearly inventory cost is (₹)
(A) 240 (B) 280 (C) 290 (D) 360.

**Solution:**

$$\text{The total cost is } \sqrt{2C_1C_2 r}$$
$$= \sqrt{2 \times 80 \times 0.8 \times 1000}$$
$$\approx 360.$$

***Direction for examples 23 to 25:*** A company requires 1500 items/month at a price of ₹27. The ordering cost is ₹200 and the inventory carrying cost is 20% of the average inventory.

**Example 23:** The economic order quantity is
(A) 1155 (B) 1276 (C) 1315 (D) 1385.

**Solution:**
Annual demand = 1500 × 12.
Ordering cost $C_1 = 200$.
Inventory carrying cost is 20% of 27.
Average inventory of 20% is the same as 20% of the unit cost.
∴ Inventory carrying cost = ₹5.4.

$$\therefore EOQ = \sqrt{\frac{2 \times 200 \times 18000}{5.4}}$$
$$= 1155 \text{ units.}$$

**Example 24:** The number of orders per year is

(A) 12 (B) 14 (C) 16 (D) 18.

**Solution:**
Annual demand is 18000 units.
$EOQ = 1155.$

$\therefore$ Number of orders $= \dfrac{18000}{1155} \approx 16$ orders. $\approx \dfrac{18000}{1155} \approx 16$ orders.

**Example 25:** If 1400 numbers are procured at a time the company can avail a discount of 2% in the price. If the company goes for the option of purchasing 1400 nos./order avoiding *EOQ*, then the saving for the company is
(A) ₹11000 (B) ₹9604 (C) ₹8980 (D) ₹8430.

**Solution:**
The total cost if the purchase is made according to *EOQ* is $\sqrt{2C_1C_2r}$.
$TC = \sqrt{2\times 200\times 5.4\times 18000} = 6235.38.$
Total purchase cost = ₹6235.38 + 27 × 18000 = ₹492235.38.
If procured at the rate of 1400 numbers/order,

$$\text{total purchase cost} = \frac{18000}{1400}\times 200 + \left(\frac{1400}{2}\right)5.4$$
$$+27\times 0.98\times 18000$$
$$= 2571.42 + 3780 + 476280 = 6351.42 + 476280.$$

Total cost = 482, 631.42.
The saving is ₹9604.

## Lead Time and Re – Order Level

Usually when an order is placed, it may take some time to receive the item. The time between the placement of order and the actual receipt of the item is known as the lead time. In most of the cases, lead time cannot be exactly known; it is probabilistic. If it is deterministic, the order can be placed in advance just to take care of the lead time consumption. Then, there will not be any shortage for the item.

At what stock of the item is it essential to initiate the purchase procedures is known as re – order level. Re – order level means the stock that is required to take care of the lead time demand.

$$\therefore ROL = \text{average consumption} \times \text{lead time}.$$

During the lead time, sometimes there can be an unexpected delay. Also in the lead time, there can be an additional demand arising. To take care of this, an additional stock is to maintained. This is known as the safety stock or buffer stock. If the buffer stock is high, naturally the carrying cost will be high. But if the buffer stock is less, there can be a shortage. Therefore, it is desirable to strike a balance between these two.

Buffer stock = (maximum lead time – Minimum lead time) × lead time rate of consumption.

***Direction for questions 26 to 28:*** An inventory problem is as given below. Annual demand for a product = 36000.
Cost/unit = ₹1
Ordering cost = ₹25/order
Lead time = 15 days
Store charge = 5% of unit cost
Cost capital = 15%
Safety stock – one month's consumption.

**Example 26:** The *EOQ* is (units)
(A) 2500 (B) 2800 (C) 3000 (D) 3500.

**Solution:**

$$EOQ = \sqrt{\frac{2C_1r}{C_2}} = \sqrt{\frac{2\times 25\times 36000}{C_2}}$$
$$C_2 = (0.15 + 0.05)1$$
$$= 0.2$$
$$EOQ = \sqrt{\frac{2\times 25\times 36000}{0.2}}$$
$$= 3000 \text{ units}.$$

**Example 27:** The safety stock is
(A) 3000 (B) 3200 (C) 3800 (D) 4200.

**Solution:**
Safety stock is one month's consumption.

Number of orders $= \dfrac{36000}{3000} = 12.$

$\therefore$ *EOQ* is one month's consumption = 3000.

**Example 28:** Re – order level is
(A) 5200 units (B) 4500 units
(C) 400 units (D) 3500 units.

**Solution:**
Re – order level = lead time consumption + buffer stock.
$$= \frac{1}{2}\times 3000 + 3000$$
$$= 4500 \text{ units}.$$

### Shortage Cost

If there is a delay in meeting the demand or inability to undertake production due to shortage, then there are penalty costs. These costs include:

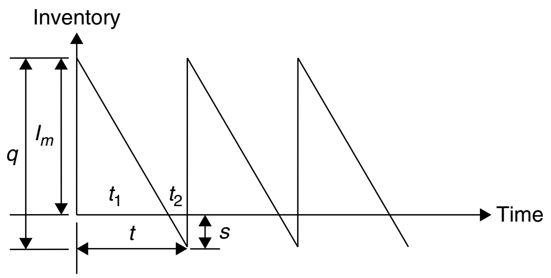
1. loss of future sales (due to loss of customers)
2. extra cost due to emergency purchase
3. loss of good will.

Let $C_p$ be the penalty cost. The inventory carrying cost after accommodating the penalty cost because of shortage is

$$C_2' = C_2\left(\frac{C_p}{C_2 + C_p}\right)$$

## Deterministic Model 2(a)

This model allows shortages with uniform demand rate and zero lead time. Allowing shortages will increase the cycle time which decreases the total ordering cost over the planned period. The inventory stock is also decreased which results in lower inventory carrying cost.




$I_m$ = number of items that are available at the beginning of the inventory.
$q$ = number of items ordered in one order = $rt$
$r$ = demand rate
$C_P$ = shortage cost
The total time $T = Nt$, $N$ is the number of orders.

$$t = t_1 + t_2$$

where, $t_1$ is the time during which inventory items are consumed and $t_2$ is the interval during which the demand is not met.

Using the similar triangles relation,

$$\frac{t_1}{t} = \frac{I_m}{q} \quad \text{and} \quad \frac{t_2}{t} = \frac{q - 1_m}{q}$$

Total cost during time $t$

$$= \frac{1}{2} I_m t_1 C_2 + \frac{1}{2}(q - I_m) t_2 C_p + C_1$$

$$\text{Optimum lot size} = q_o = \sqrt{\frac{2C_1 r}{C_2}\left(\frac{C_p + C_2}{C_p}\right)}$$

## Deterministic Model 2(b)

This model has similar conditions as model 2(a), but the time period $t$ is variable.

The optimum quantity per order is given by,

$$q = \sqrt{\frac{2rC_1}{C_2}\left(\frac{C_p + C_2}{C_p}\right)}$$

$$\text{The re-order time } t = \sqrt{\frac{2C_1(C_p + C_2)}{rC_2C_p}}$$

$$\text{The minimum cost } C_{\min} = \sqrt{2C_1C_2r}\sqrt{\frac{C_2}{C_p + C_2}}$$

**NOTE**

The optimum cost given by model (a) with no shortages gives greater value than the minimum cost provided by model 2(b).

## Quantity Discounts

The quantity to order from the supplier to replenish inventory is an important decision in material/inventory management. Economic order quantity (*EOQ*) model computes the amount to order using the assumptions that the cost per unit of purchased items remains fixed, regardless of the number of units ordered. But, it is common for the suppliers to give discounts when order quantities are high. When discounts are considered, the economic order quantity may change. The optional order quantity model investigates the total annual inventory costs with and without discounts, and the optional order quantity is one that minimises the total annual inventory costs. The result is that larger than usual economic order quantities may be justified.

Steps in quantity discount model:

- Compute *EOQ* for each quantity discount price.
- Find whether the computed *EOQ* is in the discount range.
- If not, use lowest cost quantity in the discount range.
- Compute total cost for *EOQ* or lowest cost quantity in the discounted range.
- Select quantity with the lowest total cost, including the cost of the items purchased.

## Solved Examples

***Direction (29, 30):*** The demand for an item is 25 numbers/month. The set up cost for production is ₹30/batch. The inventory carrying cost/unit/month is ₹1/2. The shortage cost is evaluated to be ₹3/unit/month.

**Example 29:** Then *EOQ* is
(A) 60 units (B) 75 units (C) 85 units (D) 95 units.

**Solution:**

$$ECQ = \sqrt{\frac{2C_1 r}{C_2\left(\frac{C_p}{C_2 + C_p}\right)}}$$

$C_1$ = ₹30, $r = 25 \times 12 = 300$/annum
$C_2 = 0.5 \times 12$ = ₹6, $C_p$ = ₹3

$$EOQ = \sqrt{\frac{2 \times 30 \times 25 \times 12}{6\left[\frac{3}{6+3}\right]}} = 95 \text{ units.}$$

**Example 30:** The time between two consecutive batch productions is
(A) 1.5 months (B) 2 months
(C) 3 months (D) 4 months.

**Solution:**

Number of batches per annum = $\frac{300}{95} \approx 4$ batches.

Time between the batches = $\frac{1}{4} \times 12 = 3$ months.

**Example 31:** A firm has an annual demand for a product at the rate of 1000 units. The cost of ordering is ₹100/order. The carrying cost is also ₹100/unit/year. The stock out cost is evaluated to be ₹300. Everytime, the item is out of stock. The quantity to be stocked (in units) is

(A) 68 (B) 61 (C) 52 (D) 46.

**Solution:**

$r = 1000$

$C_1 = 100$

$C_2 = 100$

Penalty cost $C_p = 300$.

The economic stock level is

$$= \sqrt{\frac{2C_1 r}{C_2\left\{\frac{Cp}{Cp+C_2}\right\}}}$$

$$= \sqrt{\frac{2\times 100\times 1000}{100\left\{\frac{300}{100+300}\right\}}} = \sqrt{\frac{2\times 100\times 1000}{100\times\frac{3}{4}}}$$

$$= \sqrt{\frac{8\times 100\times 1000}{300}} = \sqrt{\frac{8000}{3}}$$

= 52 units.

**Example 32:** In a manufacturing unit, the production rate of a commodity is 36000/annum; the annual demand is only 6000 units/annum. The set up cost for batch production is ₹500. The inventory carrying cost/unit/year ₹8. The penalty cost, if shortage occurs is ₹20. Then, the economic batch size of production is

(A) 1123 (B) 324 (C) 2510 (D) 937.

**Solution:**

As the demand is only 6000 units and the production rate is 36000 units, the carrying cost is applicable to 36000 – 6000 units only. The corresponding batch size is,

$$Q = \sqrt{\frac{2C_1 r}{C_2\left[1-\frac{r}{p}\right]\times\left[\frac{C_p}{C_2+C_p}\right]}}$$

$$= \sqrt{\frac{2\times 500\times 6000}{8\left[1-\frac{6000}{36000}\right]\left[\frac{20}{20+8}\right]}}$$

$$= \sqrt{\frac{1000\times 6000}{8\left[\frac{5}{6}\right]\left[\frac{20}{28}\right]}} = \sqrt{\frac{6000\times 1000}{\frac{40}{6}\times\frac{20}{28}}}$$

$$= \frac{6000}{\sqrt{2857}} = 1123.$$

In real situations, the probability distribution of future demand is usually determined rather than the exact demand itself. The expected costs are obtained and minimised instead of the actual costs.

## Probabilistic Model

In this model, the demand is instantaneous, setup costs are zero, discrete stock levels and lead time are zero. The two types of costs involved are over-stocking cost and under-stocking cost.

Let $r$ = discrete demand rate with probability $P_r$

$I_m$ = discrete stock level for the time interval $t$

$t$ = constant interval between the orders

$C_h$ = unit carrying cost, $S$ = selling price of the unit

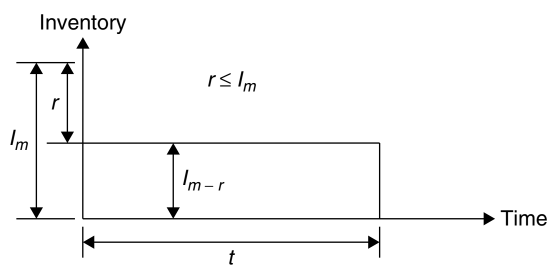
$C$ = cost price of the unit, $V$ = salvage value

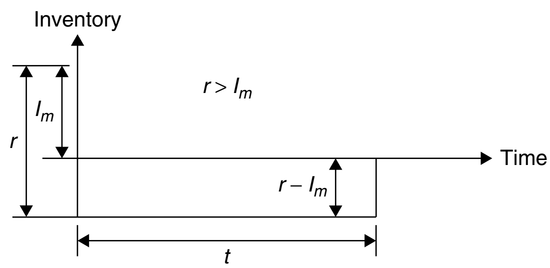
$C_o$ = over-stocking cost = $C + C_h - V$

$C_u$ = under-stocking cost = $S - C - C_h/2 + C_s$

$C_s$ = unit shortage cost.

If any of the values is not given, it is taken as zero.




The two cases where the optimum inventory $I_m$ has to be determined are $r \le I_m$ and $r > I_m$.

There are no shortages when $r \le I_m$ and shortages occur when $r > I_m$.

The cumulative values of the probability distribution are determined and the ratio $C_u/(C_u + C_o)$ is determined. The optimal order quantity $I_{mo}$ is calculated by computing.

$$P_{r\leq I_m-1} < \frac{C_u}{C_u + C_o} < P_{r\leq I_m}$$

## Safety Stock

The excess stock maintained during the lead time demand is called as the safety or buffer or cushion stock.

### Determination of Safety Stock

- Safety stock = (maximum lead time – normal lead time) × average demand.
- According to the standard deviation of the forecast error, safety stock = $Z$ × standard deviation.
- $Z$ is the value given by the service level.
- Mean absolute deviation (MAD) is also used to calculate safety stock.

SS = safety stock = k.MAD

where, $k$ = service factor = $\sqrt{\frac{\Pi}{2}} \cdot Z$

$$\therefore \text{SS} = \sqrt{\frac{\Pi}{2}} \cdot Z \cdot \text{MAD}.$$

### Implications of Safety Stock

1. Safety stocks are used when the lead time of manufacturing is too long and is difficult to meet the customer demand.
2. It is used as a buffer to protect the organisation when there is a rise in the demand during the lead time.
3. Increase in safety stock increases the inventory holding cost. Therefore, it is necessary to have a minimum safety stock.
4. Lean manufacturing, just in time methods, have the least amount of safety stocks. Enterprise resource planning also reduces the safety stock of an organisation.

## Service Level

The amount of safety stock required is dependent on the service level required by the organisation. A 98% service level means that the demand has a 2% of chance to exceed the re-order level during the lead time. Safety stock can be calculated as

$$\text{safety stock} = z.\ \sigma_x = 2.055\ \sigma_x$$

$z$ = 2.055 for 98% service level from the normal distribution tables.

$\sigma_x$ = Standard deviation of random demand during lead time ($x$).

$\overline{X}$ = Expected demand during the lead time

$\therefore$ Re-order point = $\overline{X} + 2.055\ \sigma_x$.

## Types of Inventory Control

### Selective Inventory Control

In inventory management, some items are very important and given special attention on the basis of usage, lead time, technical problems, costs involved etc. All items of the inventory need not and cannot be controlled with equal attention. For this reason, the Management gives different items, different priorities for their control and management.

1. **ABC Analysis (always better control):** It is based on Pareto's Law – A few high usage items constitute a major part of the capital whereas, bulk of the items in an inventory having low usage value constitute insignificant part of the capital. The inventory items are categorised into $A$, $B$ and $C$ categorises based on the usage value in monetary terms.

| | Usage value | Item no. |
|---|---|---|
| $A$ | 60 – 70% | 10 – 20% |
| $B$ | 30 – 40% | 40 – 50% |
| $C$ | 10 – 20% | 60 – 70% |

2. **VED (vital essential desirable):** Items are categorised on the basis of the criticality of the inventory for the production system.
3. **HML analysis (high medium low):** The classification of items is on the basis of the unit costs involved. It is similar to $ABC$ control.
4. **SDE analysis (scare difficult easy):** Classification of items is done according to the ease or difficulty of the availability of the item.
5. **S-OS analysis (seasonal-*S*, off seasonal-*OS*):** Items are classified according to the nature of availability of the inventory items depending on seasonal demand/availability.
6. **XYZ analysis:** Classification is done on the basis of closing inventory values. $X$ items have high closing inventory values, $Z$ items have a low value while $Y$ items have in-between value. This analysis helps to reduce the capital investment in $X$ items.
7. **FNSD analysis:** Items are categorised on the basis of their consumption rate:
   $F$ – fast moving items (high consumption)
   $N$ – normal moving items
   $S$ – slow moving items
   $D$ – dead stock

## Periodic Review System

### Two-bin System or Q-system or Perpetual Inventory System

In this type of inventory control, a fixed size of the order is placed whenever the depleting stock level of the item reaches the re-order level. The time interval between the orders is not fixed, but the order quantity is fixed. This is named two-bin because it is divided into two bins, the first bin is from the receiving of goods to the placement of order whereas, the second bin contains the stock required to meet the demand during the lead time. This system is based on three parameters: re-order level, economic order quantity and safety stock.

The two types of periodic review systems commonly used are – Fixed order cycle system and sS system.

### *Fixed Order Quantity System*

These are the situations in which the demand is assumed to be fixed and known exactly. These are also called economic lot size models or economic order quantity models.

## Exercises

### Practice Problems I

1. The process of determining the sequence of operation and allocation of facilities is known as
   (A) aggregate planning (B) scheduling
   (C) routing (D) forecasting.
2. A work centre receives six job orders. The processing time and weightage to be given are given in the table below.

| Job | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Processing time (hrs) | 12 | 18 | 7 | 9 | 5 | 8 |
| Weightage | 1 | 1 | 2 | 1 | 3 | 1 |

   The minimum mean flow time is
   (A) 18.8 hrs (B) 21.2 hrs (C) 23.4 hrs (D) 25.3 hrs.
3. In the VED analysis of inventory control, the classification of the items is based on
   (A) cost of the item
   (B) criticality of the item
   (C) availability of the item
   (D) size of the item.
4. In an assembly line wherein four persons are working in series $(A, B, C, D)$, the time on an average taken by each is 12 min, 10 min, 13 min, 9 min respectively. The balance delay in the assembly line expressed in percentage is
   (A) 16.8% (B) 15.4%
   (C) 12.3% (D) 8.6%.
5. A system consists of four components, each of which having an efficiency of performance 96%, 97%, 93% and 80%. The inefficiency of the system is
   (A) 26.8% (B) 28.2%
   (C) 30.7% (D) 32.4%.
6. In a workshop, seven jobs arrive as job orders. They are to be completed. The processing time of the jobs is as given in table.
   The details are shown below:

| Job | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Processing time (days) | 9 | 7 | 11 | 4 | 2 | 6 | 5 |

   The minimum average flow time is
   (A) 20 days (B) 24 days
   (C) 26 days (D) 28 days.

***Direction for questions 7 to 9:*** A job shop has to schedule six jobs, the details of which are as follows.

| Job | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Processing time (days) | 4 | 7 | 3 | 2 | 6 | 9 |
| Due date | 12 | 20 | 4 | 5 | 18 | 25 |

7. According to the shortest processing schedule, the last flow time is
   (A) 12 days (B) 14 days
   (C) 16 days (D) 18 days.
8. In the above case, the average lateness is
   (A) 1.3 days (B) 2 days
   (C) 2.5 days (D) 3 days.
9. The average number of jobs in the system at any point of time is
   (A) 4.2 jobs (B) 3.8 jobs
   (C) 3.2 jobs (D) 2.7 jobs.
10. For every job remaining in the work shop, the workshop incures a cost of ₹150/day. At a point of time, the workshop receives orders for 7 jobs whose details are given below.

| Job | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Processing time (days) | 11 | 4 | 5 | 12 | 9 | 3 | 6 |
| Due date (days) | 35 | 11 | 15 | 40 | 30 | 12 | 20 |

   The minimum cost the workshop incurs is
   (A) ₹25450 (B) ₹23250
   (C) ₹22150 (D) ₹19750.
11. A work shop has to schedule eight jobs received from an order. The jobs, their processing time and due dates are as given below.
   When the jobs are scheduled as per the shortest processing time, the tardiness is $t_1$. When it is scheduled as per early due date basis, the tardiness is $t_2$.

| Job | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Processing time (days) | 12 | 6 | 7 | 3 | 11 | 8 | 5 | 4 |
| Due date (days) | 60 | 20 | 21 | 2 | 40 | 37 | 10 | 7 |

   Then the ratio $\frac{t_1}{t_2}$ is
   (A) 1 (B) 15 (C) 2 (D) 1.6.

***Direction for questions 12 to 15:*** Following are the details of six jobs for a workshop which are to be scheduled.

| Jobs | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Processing time (days) | 10 | 8 | 7 | 11 | 12 | 9 |
| Due date (days) | 12 | 24 | 32 | 8 | 15 | 40 |

**12.** When it is scheduled as per the shortest processing time, the average tardiness is
(A) 20.32 days (B) 18.62 days
(C) 16.83 days (D) 15.65 days.

**13.** The number of jobs, on an average, with the workshop at any point of time is
(A) 5.23 (B) 4.87 (C) 4.23 (D) 3.19.

**14.** When the jobs are arranged according to earliest due date and schedule, the average tardiness is
(A) 12.86 (B) 13.67 (C) 14.12 (D) 15.22.

**15.** The average number of jobs in the system when scheduled as per the earliest due date criterion is
(A) 5.6 jobs (B) 4.8 jobs
(C) 4.2 jobs (D) 3.7 jobs.

**16.** A company requires raw material at the rate of 85000 tonnes/annum. The cost of placing an order is on an average ₹1500/order. When the item reaches the stores the holding cost/ton/year is ₹60. Then the economic order quantity is
(A) 3410 $^{t}/_{\text{order}}$ (B) 3075 $^{t}/_{\text{order}}$
(C) 2061 $^{t}/_{\text{order}}$ (D) 1098 $^{t}/_{\text{order}}$

***Direction for questions 17 and 18:*** The annual demand for a product is 2,400 units. The cost of placing the order is ₹100/order. The storage cost is 24% of the unit cost. The price break as proposed by the supplier is

| Qty | Price/unit |
|---|---|
| $0 < 500$, | ₹10 |
| $500 < Q$ | ₹9 |

**17.** The economic order quantity is
(A) 500 (B) 447
(C) 471 (D) 480

**18.** The number of orders per year is
(A) 6 (B) 5
(C) 3 (D) 2.

***Direction for questions 19 and 20:*** For a product, the annual consumption rate is 5000 units. Unit cost is ₹1. The set up ₹ cost for a batch is ₹12/run. Inventory carrying cost is 0.25/units/year.

From the records, the past lead times were founds to be 16 days, 14 days, 18 days, 13 days, 14 days.

**19.** The economic batch size is
(A) 842 units (B) 741 units
(C) 693 units (D) 524 units

**20.** The Re–order level shall be
(A) 312 units (B) 279 units
(C) 242 units (D) 212 units

## Practice Problems 2

1. There are three components in the stores which are parts of an assembly. Each of the components have 95% service level in the stock. The proportion of time during which there will be a shortage for assembling is
(A) 5% of the time (B) 15% of the time
(C) 20% of the time (D) 25% of the time.

**2.** ‘Dependent demand’ item means
(A) items which are very costly
(B) items which cannot be directly traced
(C) items which have derived demand
(D) items which cannot be singly obtained.

**3.** In a job shop, at the beginning of a month, there are five jobs. Their processing time and weightage are shown in the chart below.

| Job | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Processing time (hrs) | 8 | 5 | 9 | 6 | 4 |
| Weightage | 1 | 2 | 2 | 1 | 3 |

The minimum mean flow time is
(A) 17.32 (B) 16.81 (C) 15.26 (D) 14.67.

***Direction for questions 4 and 5:*** The population in five towns and the demand of a household product in the towns are as follows.

**4.** By interpolation (regression method), the demand for a product in a town where the population is 10 lakhs is

| Population in lakhs | 5 | 7 | 8 | 11 | 14 |
|---|---|---|---|---|---|
| Series in thousands | 9 | 12 | 11 | 14 | 18 |

(A) 13.74 thousands (B) 14.08 thousands
(C) 14.82 thousands (D) 15.32 thousands.

**5.** The demand for the product in thousands where the population is 20 lakhs is
(A) 20.56 (B) 23.14 (C) 24.86 (D) 25.10.

***Direction for questions 6 and 7:*** Seven jobs are to be scheduled against a single machine in a work shop. The jobs and their processing time together with the due dates of the job order are as given below.

| Job | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Processing time | 10 | 8 | 6 | 7 | 12 | 14 | 18 |
| Due day (days) | 15 | 10 | 11 | 12 | 17 | 24 | 28 |

**6.** The sequence of working which will minimise the maximum lateness is
(A) 2 – 4 – 1 – 3 – 6 – 5 – 7
(B) 2 – 1 – 4 – 1 – 5 – 6 – 7
(C) 2 – 3 – 4 –1 – 5 – 6 – 7
(D) 3 – 2 – 5 – 1 – 4 –3 – 7.

7. Maximum Lateness is (days)
(A) 16 (B) 26 (C) 33 (D) 45.

***Direction for questions 8 to 11:*** The processing time and due dates for six jobs in a work shop are given below.

| Job | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Processing time (days) | 11 | 8 | 7 | 9 | 6 | 4 |
| Due date (days) | 32 | 27 | 19 | 30 | 12 | 6 |

8. According to the consideration of processing priority as shortest processing time, the average job lateness is
(A) 3.24 days (B) 2.86 days
(C) 2.43 days (D) 1.16 days.

9. The average number of jobs in the system is
(A) 4.31 jobs (B) 3.57 jobs
(C) 3.23 jobs (D) 2.84 jobs.

10. For the same data above, from the consideration of processing priority as earliest due date, the average job lateness is
(A) 1.16 days (B) 2.32 days
(C) 2.84 days (D) 3.12 days

11. The average number of jobs at hand is
(A) 4.12 (B) 3.92 (C) 3.57 (D) 2.82

***Direction for questions 12 to 15:*** Consider the case of a job shop which receives five job orders. The details of the jobs regarding the processing time in days and due date in days are as given below.

| Job | Processing time | Due date |
|---|---|---|
| 1 | 9 | 16 |
| 2 | 7 | 20 |
| 3 | 5 | 25 |
| 4 | 11 | 15 |
| 5 | 6 | 40 |

12. With shortest processing time scheduling consideration, the average lateness is
(A) 6.8 days (B) 7.2 days
(C) 7.8 days (D) 8.3 days.

13. With the shortest processing time scheduling, the average number of jobs in the system is
(A) 3.2 jobs (B) 2.6 jobs
(C) 2.2 jobs (D) 1.8 jobs.

14. For the same data furnished above, if the scheduling is as per 'earlier due date' scheduling, the average lateness is
(A) 5.2 (B) 4.8 (C) 3.6 (D) 3.1.

15. Referring to the same data according to earlier due date scheduling, the average number of jobs in the system is
(A) 5.41 (B) 5.2 (C) 4.61 (D) 3.37

***Direction for questions 16 to 18:*** Six orders arrive at a job shop which are to be processed by a single machine. The processing time and due dates of the job are as follows.

| Job | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Processing time (days) | 4 | 2 | 9 | 5 | 3 | 5 |
| Due date (days) | 17 | 21 | 5 | 12 | 15 | 24 |

16. The production schedule that minimises the average flow time is
(A) 2 – 5 – 1 – 4 – 6 – 3 (B) 2 – 4 – 1 – 5 – 6 – 3
(C) 2 – 6 – 1 – 3 – 4 – 5 (D) 3 – 2 – 4 – 6 – 1 – 5

17. The average lateness in the schedule is (in days)
(A) 3.33 (B) 4.33 (C) 5.33 (D) 6.33.

18. In the above scheduling, the average number of jobs in the system is
(A) 4.32 jobs (B) 3.41 jobs
(C) 2.72 jobs (D) 1.76 jobs

***Direction for questions 19 to 22:*** In a job shop, six jobs arrive at a time. The set up of the shop is such that everyday the job is with the shop; the shop incurs a cost of ₹100/day/job. The jobs with their processing time and due dates are given below.

| Job | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Processing time (days) | 5 | 3 | 8 | 2 | 6 | 9 |
| Due date | 10 | 12 | 18 | 9 | 7 | 9 |

19. The schedule which minimises the total cost is
(A) 4 – 2 – 1 – 5 – 3 – 6 (B) 4 – 1 – 5 – 2 – 3 – 6
(C) 4 – 5 – 1 – 3 – 2 – 6 (D) 2 – 4 – 6 – 3 – 1.

20. In the case above, the total cost incurred which is minimum is
(A) ₹8600 (B) ₹8800
(C) ₹9000 (D) ₹9500

21. The average tardiness in the process is
(A) 6.5 days (B) 5.3 days
(C) 4.8 days (D) 4.2 days.

22. The average number of jobs at hand is
(A) 3.42 jobs (B) 2.72 jobs
(C) 1.84 jobs (D) 1.12 jobs.

23. In ABC Analysis of inventory control, the major consideration is given for
(A) criticality of the item
(B) demand for the item
(C) which consumes for money
(D) which are not easily available.

24. A company purchases an item from the market and sells it. It purchases the item according to the demand for the item in the selling market. The demand is evaluated to be 800000 pieces annually. The cost of storing the procured item in the stores is ₹120/piece/year. The cost of placing the order is ₹1200. The optimum quantity to be procured in a batch is
(A) 4000 prices (B) 36000 prices
(C) 3200 prices (D) 3000 prices.

**25.** In an assembly line, there are seven workmen doing the work in series. They take time as 20 min, 25 min, 17 min, 18 min, 22 min, 19 min, 21 min. The balance delay for the assembly line is
(A) 18.8% (B) 17.6%
(C) 16.8% (D) 15.2%.

***Direction for questions 26 to 28:*** A company proposes to purchase 2 items '*x*' and '*y*' from two vendors. The ordering cost for both are ₹400 each. The unit price of item *x* is ₹60 and that of item *y* is ₹80. The inventory carrying cost is 15% of the unit price/unit/year. The annual demands for *x* and *y* are 10000 and 20000, respectively.

**26.** *EOQ* of an item *x* is
(A) 927 nos. (B) 943 nos.
(C) 1012 nos. (D) 1087 nos.

**27.** *EOQ* of an item *y* is
(A) 1155 nos. (B) 1218 nos.
(C) 1326 nos. (D) 1372 nos.

**28.** The minimum total cost for item *x* is
(A) ₹8618 (B) ₹8112
(C) ₹7956 (D) ₹7350.

**29.** The demand for an item is in a uniform rate of 300 units/year. The cost price of each item is ₹50. The inventory carrying cost/unit/year is ₹6. The set up cost is evaluated to be ₹30/set up. The number of times in an year the batch production must be made, for least cost of production is
(A) 4 (B) 5
(C) 6 (D) 7.

**30.** The demand for an item annually is 10000 units. The ordering cost is ₹200 and the inventory carrying cost is 20% of the cost price. The unit price of the commodity is ₹100. If there is a penalty cost of ₹150 for the shortage, then the economic order quantity is
(A) 523 (B) 476
(C) 412 (D) 365.

## Previous Years' Questions

**1.** There are two products *P* and *Q* with the following characteristics: **[2004]**

| Product | Demand (units) | Order cost (₹/order) | Holding cost (₹ unit/year) |
|---|---|---|---|
| *P* | 100 | 50 | 4 |
| *Q* | 400 | 50 | 1 |

The economic order quantity (*EOQ*) of products *P* and *Q* will be in the ratio
(A) 1: 1 (B) 1: 2
(C) 1: 4 (D) 1: 8.

**2.** A company has an annual demand of 1000 units, ordering cost of ₹100/order and carrying cost of ₹100/unit/year. If the stock-out costs are estimated to be nearly ₹400 each time the company runs out-of-stock, the safety stock justified by the carrying cost will be **[2004]**
(A) 4 (B) 20 (C) 40 (D) 100.

**3.** The distribution of lead-time demand for an item is as follows: **[2005]**

| Lead time demand | Probability |
|---|---|
| 80 | 0.20 |
| 100 | 0.25 |
| 120 | 0.30 |
| 140 | 0.25 |

The recorder level is 1.25 times the expected value of the lead-time demand. The service level is
(A) 25% (B) 50%
(C) 75% (D) 100%.

**4.** In an MRP system, component demand is: **[2006]**
(A) forecasted
(B) established by the master production schedule
(C) calculated by the MRP system from the master production schedule
(D) ignored.

**5.** A manufacturing shop processes sheet metal jobs, wherein each job must pass through two machines (*M*1 and *M*2, in that order). The processing time (in hours) for these jobs is: **[2006]**

| Machine | Jobs | | | | | |
|---|---|---|---|---|---|---|
| | P | Q | R | S | T | U |
| M1 | 15 | 32 | 8 | 27 | 11 | 16 |
| M2 | 6 | 19 | 13 | 20 | 14 | 7 |

The optimal make–span (in hours) of the shop is
(A) 120 (B) 115 (C) 109 (D) 79.

**6.** Consider the following data for an item. Price quoted by a supplier **[2006]**

| Order quantity (units) | Unit price (₹) |
|---|---|
| < 500 | 10 |
| ≥ 500 | 9 |

Annual demand: 2500 units/year.
Ordering cost: ₹100/order.
Inventory holding rate: 25% of the unit price.
The optimum order quantity (in units) is:

(A) 447 (B) 471
(C) 500 (D) ≥ 600.

**7.** A stockist wishes to optimise the number of perishable items he needs to stock in any month in his store. The demand distribution for this perishable item is: **[2006]**

| Demand (in units) | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| Probability | 0.10 | 0.35 | 0.35 | 0.20 |

The stockist pays ₹70 for each item and sells each at ₹90. If the stock is left unsold in any month, he can sell the item at ₹50 each. There is no penalty for the unfulfilled demand. To maximise the expected profit, the optimal stock level is:
(A) 5 units (B) 4 units
(C) 3 units (D) 2 units.

**8.** Capacities of production of an item over three consecutive months in regular time are 100, 100 and 80 and in overtime are 20, 20 and 40. The demands over those 3 months are 90, 130 and 110. The cost of production in regular time and overtime are ₹20/item and ₹24/item, respectively. Inventory carrying cost is ₹2/item per month. The levels of starting and final inventory are nil; backorder is not permitted. For minimum cost of the plan, the level of planned production in overtime in the third month is **[2007]**
(A) 40 (B) 30 (C) 20 (D) 0.

**9.** The maximum level of inventory of an item is 100 and it is achieved with infinite replenishment rate. The inventory becomes zero over one and half month because of the consumption at a uniform rate. This cycle continues throughout the year. The ordering cost is ₹100/order and the inventory carrying cost is ₹10/item/month. Annual cost (in ₹) of the plan, neglecting the material cost, is **[2007]**
(A) 800 (B) 2800
(C) 4800 (D) 6800.

**10.** In a machine shop, pins of 15 mm diameter are produced at a rate of 1000/month and the same is consumed at a rate of 500/month. The production and consumption continue simultaneously till a maximum inventory is reached. Then, the inventory is allowed to reduce to zero becasue of consumption. The lot size of the production is 1000. If backlog is not allowed, the maximum inventory level is **[2007]**
(A) 400 (B) 500 (C) 600 (D) 700.

**11.** The net requirements of an item over five consecutive weeks are 50-0-15-20-20. The inventory carrying cost and ordering cost are ₹1/item/week and ₹100/order, respectively. The starting inventory is zero. Use 'least unit cost technique' for developing the plan. The cost of the plan (in ₹) is **[2007]**
(A) 200 (B) 250 (C) 255 (D) 260.

**12.** A set of 5 jobs is to be processed on a single machine. The processing time (in days) is given in the table below. The holding cost for each job is ₹*K*/day. **[2008]**

| Job | Processing time |
|---|---|
| P | 5 |
| Q | 2 |
| R | 3 |
| S | 2 |
| T | 1 |

A schedule that minimises the total inventory cost is
(A) T-S-Q-R-P (B) P-R-S-Q-T
(C) T-R-S-Q-P (D) P-Q-R-S-T.

**13.** Six jobs arrived in a sequence as given below: **[2009]**

| Jobs | Processing time (days) |
|---|---|
| I | 4 |
| II | 9 |
| III | 5 |
| IV | 10 |
| V | 6 |
| VI | 8 |

Average flow time (in days) for the above jobs using shortest processing time rule is
(A) 20.83 (B) 23.16
(C) 125.00 (D) 139.00.

**14.** The Annual demand for certain window frames is 10000. Each frame costs ₹200 and the ordering cost is ₹300/order. The inventory holding cost is ₹40/frame/year. The supplier is willing to offer 2% discount if the order quantity is 1000 or more, and 4% if the order quantity is 2000 or more. If the total cost is to be minimised, the retailer should **[2010]**
(A) order 200 frames every time
(B) accept 2% discount
(C) accept 4% discount
(D) order Economic Order Quantity.

***Direction for questions 15 and 26:*** Four jobs are to be processed on a machine as per the data listed in the table.

| Job | Processing time (days) | Due date |
|---|---|---|
| 1 | 4 | 6 |
| 2 | 7 | 9 |
| 3 | 2 | 19 |
| 4 | 8 | 17 |

**15.** If the earliest due date (EDD) rule is used to sequence the jobs, the number of jobs delayed is **[2010]**
(A) 1 (B) 2 (C) 3 (D) 4.

**16.** Using the shortest processing time (SPT) rule, total tardiness is **[2010]**
(A) 0 (B) 2 (C) 6 (D) 8.

**17.** The word **kanban** is most appropriately associated with **[2011]**
(A) economic order quantity
(B) just-in-time production
(C) capacity planning
(D) product design.

**18.** A component can be produced by any of the four processes I, II, III and IV. The most economical process for producing a batch of 100 pieces is **[2014]**

| Process | Fixed cost (in ₹) | Variable cost per piece (in ₹) |
|---|---|---|
| I | 20 | 3 |
| II | 50 | 1 |
| III | 40 | 2 |
| IV | 10 | 4 |

(A) I (B) II (C) III (D) IV.

**19.** Consider the following data with reference to elementary deterministic economic order quantity model: **[2014]**

| Annual demand of an item | 100000 |
|---|---|
| Unit price of the item (in ₹) | 10 |
| Inventory carrying cost per unit per year (in ₹) | 1.5 |
| Unit order cost (in ₹) | 30 |

The total number of economic orders per year to meet the annual demand is _____.

**20.** A manufacturer can produce 12000 bearings per day. The manufacturer received an order of 8000 bearings/day from a customer. The cost of holding a bearing in stock is ₹0.20/month. The set-up cost per production run is ₹500. Assuming that there are 300 working days in a year, the frequency of production run should be **[2014]**
(A) 4.5 days (B) 4.5 months
(C) 6.8 days (D) 6.8 months.

**21.** Demand during the lead time with associated probabilities is shown below: **[2014]**

| Demand | 50 | 70 | 75 | 80 | 85 |
|---|---|---|---|---|---|
| Probability | 0.15 | 0.14 | 0.21 | 0.20 | 0.30 |

Expected demand during the lead time is _____.

**22.** Annual demand of a product is 50,000 units and the ordering cost is ₹7000 per order. Considering the basic economic order quantity model, the economic order quantity is 10,000 units. When the annual inventory cost is minimized, the annual inventory holding cost (in ₹) is ______. **[2015]**

**23.** The annual requirement of rivets at a ship manufacturing company is 2000 kg. The rivets are supplied in units of 1 kg costing ₹25 each. If it costs ₹100 to place an order and the annual cost of carrying one unit is 9% of its purchase cost, the cycle length of the order (in days) will be ______. **[2015]**

**24.** The annual demand for an item is 10,000 units. The unit cost is ₹100 and inventory carrying charges are 14.4% of the unit cost per annum. The cost of one procurement is ₹2000. The time between two consecutive orders to meet the above demand is ______ month(s). **[2016]**

**25.** A food processing company uses 25,000 kg of corn flour every year. The quantity-discount price of corn flour is provided in the table below:

| Quantity (kg) | Unit price (₹/kg) |
|---|---|
| 1–749 | 70 |
| 750–1499 | 65 |
| 1500 and above | 60 |

The order processing charges are ₹500/order. The handling plus carry-over charge on an annual basis is 20% of the purchase price of the corn flour per kg. The optimal order quantity (in kg) is _______.**[2016]**

## ANSWER KEYS

# EXERCISES

## Practice Problems 1

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** C | **2.** A | **3.** B | **4.** B | **5.** C | **6.** A | **7.** B | **8.** A | **9.** D | **10.** B |
| **11.** A | **12.** C | **13.** D | **14.** B | **15.** D | **16.** C | **17.** A | **18.** B | **19.** C | **20.** B |

## Practice Problems 2

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** B | **2.** C | **3.** D | **4.** A | **5.** B | **6.** C | **7.** D | **8.** D | **9.** B | **10.** A |
| **11.** C | **12.** A | **13.** B | **14.** C | **15.** D | **16.** A | **17.** B | **18.** C | **19.** A | **20.** C |
| **21.** A | **22.** B | **23.** C | **24.** A | **25.** A | **26.** B | **27.** A | **28.** D | **29.** C | **30.** B |

## Previous Years' Questions

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** C | **2.** C | **3.** D | **4.** C | **5.** B | **6.** C | **7.** B | **8.** B | **9.** D | **10.** B |
| **11.** D | **12.** A | **13.** A | **14.** C | **15.** C | **16.** D | **17.** B | **18.** B | **19.** 49 to 51 | |
| **20.** C | **21.** 74 to 75 | | **22.** ₹34,000 to 36,000 | | | **23.** 76 to 78 | | **24.** 2 | **25.** 1500 |