CHAPTER 12

Permutation and Combination

INTRODUCTION

FACTORIAL

The important mathematical term "Factorial" has extensively used in this chapter.

The product of first n consecutive **natural numbers** is defined as **factorial of n**. It is denoted by n! or $\lfloor n$. Therefore,

$$\boxed{n! = 1 \times 2 \times 3 \times .... \times (n-1) \times n}$$

For example, $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Note that :

$\frac{n!}{r!} \neq \left(\frac{n}{r}\right)!$

$0! = 1$

The factorials of fractions and negative integers are not defined.

EXAMPLE 1. Prove that n! + 1 is not divisible by any natural number between 2 and 'n'.

Sol. Since $n! = 1 . 2 . 3 . 4 ..... (n-1) . n$

Therefore n! is divisible by any number from 2 to 'n'.

Consequently n! + 1, when divided by any number between 2 and 'n' leaves 1 as remainder.

Hence, n! + 1 is not divisible by any number between 2 and 'n'.

Fundamental Principles of Counting

1. **Principle of Addition :** If an event can occur in 'm' ways and another event can occur in 'n' ways independent of the first event, then either of the two events can occur in (m + n) ways.
2. **Principle of Multiplication :** If an operation can be performed in 'm' ways and after it has been performed in any one of these ways, a second operation can be performed in 'n' ways, then the two operations in succession can be performed in (m × n) ways.

EXAMPLE 2. In a class there are 10 boys and 8 girls. The class teacher wants to select a student for monitor of the class. In how many ways the class teacher can make this selection ?

Sol. The teacher can select a student for monitor in two exclusive ways

(i) Select a boy among 10 boys, which can be done in 10 ways OR

(ii) Select a girl among 8 girls, which can be done in 8 ways. Hence, by the fundamental principle of addition, either a boy or a girl can be selected in 10 + 8 = 18 ways.

EXAMPLE 3. In a class there are 10 boys and 8 girls. The teacher wants to select a boy and a girl to represent the class in a function. In how many ways can the teacher make this selection?

Sol. The teacher has to perform two jobs :

(i) To select a boy among 10 boys, which can be done in 10 ways.

(ii) To select a girl, among 8 girls, which can be done in 8 ways.

Hence, the required number of ways = 10 × 8 = 80.

EXAMPLE 4. There are 6 multiple choice questions in an examination. How many sequences of answers are possible, if the first three questions have 4 choices each and the next three have 5 choices each?

Sol. Each of the first three questions can be answered in 4 ways and each of the next three questions can be answered in 5 different ways.

Hence, the required number of different sequences of answers = 4 × 4 × 4 × 5 × 5 × 5 = 8000.

EXAMPLE 5. Five persons entered a lift cabin on the ground floor of an 8-floor house. Suppose that each of them can leave the cabin independently at any floor beginning with the first. What is the total number of ways in which each of the five persons can leave the cabin at any of the 7 floors?

Sol. Any one of the 5 persons can leave the cabin in 7 ways independent of other.

Hence the required number of ways = $7 \times 7 \times 7 \times 7 \times 7 = 7^5$.

Method of Sampling :

Sampling process can be divided into following forms :

1. The order is IMPORTANT and the repetition is ALLOWED, each sample is then a SEQUENCE.
2. The order is IMPORTANT and the repetition is NOT ALLOWED, each sample is then a PERMUTATION.
3. The order is NOT IMPORTANT and repetition is ALLOWED, each sample is then a MULTISET.
4. The order is NOT IMPORTANT and repetition is NOT ALLOWED, each sample is then a COMBINATION.

PERMUTATION

Each of the arrangements, which can be made by taking, some or all of a number of things is called a PERMUTATION.

For Example : Formation of numbers, word formation, sitting arrangement in a row.

The number of permutations of 'n' things taken 'r' at a time is denoted by ${}^nP_r$. It is defind as, ${}^nP_r = \frac{n!}{(n-r)!}$.

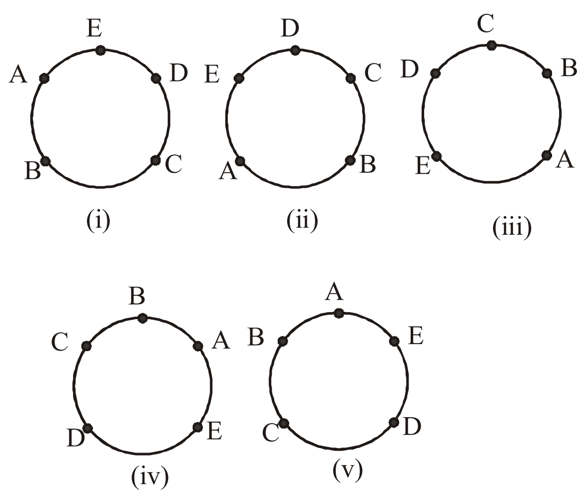
Note that:

${}^nP_n = n!$

Circular permutations:

(i) Arrangements round a circular table :

Consider five persons A, B, C, D and E to be seated on the circumference of a circular table in order (which has no head). Now, shifting A, B, C, D and E one position in anticlockwise direction we will get arrangements as follows:



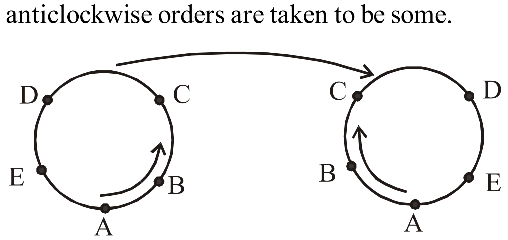
we see that arrangements in all figures are same.

$\therefore$ The number of circular permutations of n different things taken all at a time is $\frac{{}^nP_n}{n} = (n-1)!$, if clockwise and anticlockwise orders are taken as different.

(ii) Arrangements of beads or flowers (all different) around a circular necklace or garland:

Consider five beads A, B, C, D and E in a necklace or five flowers A, B, C and D, E in a garland etc. If the necklace or garland on the left is turned over we obtain the arrangement on the right, i.e., anticlockwise and clockwise order of arrangements are not different.

Thus the number of circular permutations of 'n' different things taken. all at a time is $\frac{1}{2}(n-1)!$, if clockwise and anticlockwise orders are taken to be some.



EXAMPLE 6. Prove that ${}^nP_r = {}^{n-1}P_r + r.{}^{n-1}P_{r-1}$

Sol. $${}^{n-1}P_r + r.{}^{n-1}P_{r-1} = \frac{(n-1)!}{(n-1-r)!} + r\frac{(n-1)!}{(n-1-r+1)!}$$

$$= (n-1)!\left\{\frac{1}{(n-1-r)!} + \frac{r}{(n-r)!}\right\}$$

$$= (n-1)!\left\{\frac{n-r+r}{(n-r)!}\right\} = \frac{n!}{(n-r)!} = {}^nP_r$$

EXAMPLE 7. Prove that ${}^nP_r = (n-r+1)\ {}^nP_{r-1}$

Sol. We have

$$(n-r+1)\ {}^nP_{r-1} = (n-r+1)\frac{n!}{(n-r+1)!}$$

$$= (n-r+1)\frac{n!}{(n-r+1)(n-r)!}$$

$$= \frac{n!}{(n-r)!} = {}^nP_r$$

EXAMPLE 8. The number of four digit numbers with distinct digits is :

(a) $9 \times {}^9C_3$ **(b)** $9 \times {}^9P_3$

(c) ${}^{10}C_3$ **(d)** ${}^{10}P_3$

Sol. (b) The thousandth place can be filled up in 9 ways with any one of the digits 1, 2, 3, ..., 9. After that the other three places can be filled up in ${}^9P_3$ ways, with any one of the remaining 9 digits including zero. Hence, the number of four digit numbers with distinct digits = $9 \times {}^9P_3$.

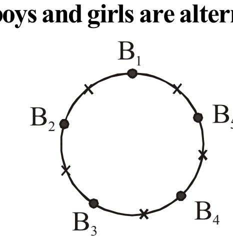
EXAMPLE 9. The number of ways in which 10 persons can sit round a circular table so that none of them has the same neighbours in any two arrangements.

Sol. 10 persons can sit round a circular table in 9! ways. But here clockwise and anticlockwise orders will give the same neighbours. Hence the required number of ways

$= \frac{1}{2}(10–1)! = \frac{1}{2}9!$

EXAMPLE 10. In how many different ways can five boys and five girls form a circle such that the boys and girls are alternate?

Sol. After fixing up one boy on the table the remaining can be arranged in 4! ways.



There will be 5 places, one place each between two boys which can be filled by 5 girls in 5! ways.

Hence by the principle of multiplication, the required number of ways = 4! × 5! = 2880.

EXAMPLE 11. In how many ways can 5 boys and 5 girls be seated at a round table no two girls may be together ?

Sol. Leaving one seat vacant between two boys may be seated in 4! ways. Then at remaining 5 seats, 5 girls any sit in 5! ways. Hence the required number = 4! × 5!

Conditional Permutations

1. **Number of permutations of n things taking r at a time, in which a particular thing always occurs** = $r.\,^{n-1}P_{r-1}$.

Distinguishable Permutations

Suppose a set of n objects has n_1 of one kind of object, n_2 of a second kind, n_3 of a third kind, and so on, with $n = n_1 + n_2 + n_3 + ... + n_k$, Then the number of distinguishable permutations of the n objects is $\dfrac{n!}{n_1!\,n_2!\,n_3!.....\,n_k!}$

EXAMPLE 12. In how many distinguishable ways can the letters in BANANA be written?

Sol. This word has six letters, of which three are A's, two are N's, and one is a B. Thus, the number of distinguishable ways the letters can be written is

$$\frac{6!}{3!\,2!\,1!} = \frac{6\times5\times4\times3!}{3!\,2!} = 60$$

EXAMPLE 13. How many 4 digits number (repetition is not allowed) can be made by using digits 1-7 if 4 will always be there in the number?

Sol. Total digits (n) = 7

Total ways of making the number if 4 is always there $= r \times {}^{n-1}P_{r-1} = 4 \times {}^6P_3 = 480$.

2. **Number of permutations of n things taking r at a time, in which a particular thing never occurs** = $^{n-1}P_r$.

EXAMPLE 14. How many different 3 letter words can be made by 5 vowels, if vowel 'A' will never be included?

Sol. Total letters (n) = 5

So total number of ways $= {}^{n-1}P_r = {}^{5-1}P_3 = {}^4P_3 = 24$.

3. **Number of permutations of n different things taking all at a time, in which m specified things always come together** $= m!(n-m+1)!$.

4. **Number of permutations of n different things taking all at a time, in which m specified things never come together** $= n! - m!(n-m+1)!$

EXAMPLE 15. In how many ways can we arrange the five vowels, a, e, i, o & u if :

(i) two of the vowels e and i are always together.

(ii) two of the vowels e and i are never together.

Sol. (i) Using the formula $m!(n-m+1)!$

Here n = 5, m = 2(e & i)

$\Rightarrow$ Required no. of ways = 2!(5 – 2 + 1)! = 2 × 4! = 48

Alternative :

As the two vowels e & i are always together we can consider them as one, which can be arranged among themselves in 2! ways.

Further the 4 vowels (after considering e & i as one) can be arranged in 4! ways.

Total no. of ways = 2! × 4! = 48

(ii) No. of ways when e & i are never together

= total no. of ways of arranging the 5 vowels

– no. of ways when e & i are together = 5! – 48 = 72

Or use $n! - m!(n-m+1)! = 5! - 48 = 72$

5. **The number of permutations of 'n' things taken all at a time, when 'p' are alike of one kind, 'q' are alike of second, 'r' alike of third, and so on** $= \dfrac{n!}{p!\,q!\,r!}$.

EXAMPLE 16. How many different words can be formed with the letters of the world MISSISSIPPI.

Sol. In the word MISSISSIPPI, there are 4 I's, 4S's and 2P's.

Thus required number of words $= \dfrac{(11)!}{4!\,2!\,4!} = 34650$

6. **The number of permutations of 'n' different things, taking 'r' at a time, when each thing can be repeated 'r' times** $= n^r$

EXAMPLE 17. In how many ways can 5 prizes be given away to 4 boys, when each boy is eligible for all the prizes?

Sol. Any one of the prizes can be given in 4 ways; then any one of the remaining 4 prizes can be given again in 4 ways, since it may even be obtained by the boy who has already received a prize. Hence 5 prizes can be given 4 × 4 × 4 × 4 × 4 = 4^5 ways.

EXAMPLE 18. How many numbers of 3 digits can be formed with the digits 1, 2, 3, 4, 5 when digits may be repeated?

Sol. The unit place can be filled in 5 ways and since the repetitions of digits are allowed, therefore, tenth place can be filled in 5 ways.

Furthermore, the hundredth place can be filled in 5 ways also.

Therefore, required number of three digit numbers is 5 × 5 × 5 = 125.

EXAMPLE 19. In how many ways 8 persons can be arranged in a circle?

Sol. The eight persons can be arranged in a circle in (8 – 1)! = 7! = 5040.

EXAMPLE 20. Find the number of ways in which 18 different beads can be arranged to form a necklace.

Sol. 18 different beads can be arranged among themselves in a circular order in (18 – 1)! = 17! ways. Now in the case of necklace there is no distinct between clockwise and anticlockwise arrangements. So, the required number of arrangements $= \dfrac{1}{2}(17!) = \dfrac{17!}{2}$

☞ Shortcut Approach

Number of ways to declare the result where 'n' match are played = 2^n

EXAMPLE 21. In a cricket tournament 5 matches were played, then in how many ways result can be declared?

Sol. Total ways to declare the result = $2^n = 2^5 = 32$ ways

COMBINATION

Each of the different selections that can be made with a given number of objects taken some or all of them at a time is called a COMBINATION.

The number of combinations of 'n' dissimilar things taken 'r' at a time is denoted by ${}^nC_r$ or C(n, r). It is defined as,

$$ {}^nC_r = \frac{n!}{r!(n-r)!} $$

EXAMPLE 22. If ${}^{15}C_{3r} = {}^{15}C_{r+3}$, find r.

Sol. ${}^{15}C_{3r} = {}^{15}C_{r+3}$

$\Rightarrow$ Either $3r = r + 3$ or $3r + r + 3 = 15$

$\Rightarrow$ Either $r = \frac{3}{2}$ or $r = 3$

Since, r cannot be a fraction, so r = 3.

EXAMPLE 23. If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$, then the values of n and r are

(a) 4, 3 **(b)** 3, 2
(c) 4, 2 **(d)** None of these

Sol. (b) We have, ${}^nP_r = {}^nP_{r+1}$

$$ \Rightarrow \frac{n!}{(n-r)!} = \frac{n!}{(n-r-1)!} \Rightarrow \frac{1}{(n-r)} = 1 $$

or $n - r = 1$ ...(1)

Also, ${}^nC_r = {}^nC_{r-1} \Rightarrow r + r - 1 = n \Rightarrow 2r - n = 1$ ...(2)

Solving (1) and (2), we get r = 2 and n = 3

EXAMPLE 24. Prove that

$$ {}^nC_{r-2} + 3.\,{}^nC_{r-1} + 3.\,{}^nC_r + {}^nC_{r+1} = {}^{n+2}C_{r+2} $$

Sol. ${}^nC_{r-2} + 3\,{}^nC_{r-1} + 3\,{}^nC_r + {}^nC_{r+1}$

$= {}^nC_{r-2} + {}^nC_{r-1} + 2({}^nC_{r-1} + {}^nC_r) + ({}^nC_r + {}^nC_{r+1})$

$= {}^{n+1}C_{r-1} + 2.\,{}^{n+1}C_r + {}^{n+1}C_{r+1}$

$= ({}^{n+1}C_{r-1} + {}^{n+1}C_r) + ({}^{n+1}C_r + {}^{n+1}C_{r+1})$

$= {}^{n+2}C_r + {}^{n+2}C_{r+1} = {}^{n+3}C_{r+1}$

EXAMPLE 25. If ${}^nP_r = 720\,{}^nC_r$, then r is equal to

(a) 3 **(b)** 7
(c) 6 **(d)** 4

Sol. (c) ${}^nP_r = 720\,{}^nC_r$

or $\frac{n!}{(n-r)!} = \frac{720(n!)}{(n-r)!r!}$

$\Rightarrow r! = 720 = 1 \times 2 \times 3 \times 4 \times 5 \times 6!$

or r = 6

EXAMPLE 26. In how many ways a hockey team of eleven can be elected from 16 players?

Sol. Total number of ways = ${}^{16}C_{11} = \frac{16!}{11! \times 5!} = 4368.$

$$ = \frac{16 \times 15 \times 14 \times 13 \times 12}{5 \times 4 \times 3 \times 2 \times 1} = 4368. $$

REMEMBER

- ★ ${}^nC_0 = 1,\ {}^nC_n = 1;\ {}^nP_r = r!\,{}^nC_r$
- ★ ${}^nC_r = {}^nC_{n-r}$
- ★ ${}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$
- ★ ${}^nC_x = {}^nC_y \Rightarrow x + y = n$
- ★ ${}^nC_r = {}^nC_{r+1} = {}^{n+1}C_r$
- ★ ${}^nC_r = \frac{n}{r}.\,{}^{n-1}C_{r-1}$
- ★ ${}^nC_r = \frac{1}{r}(n-r+1)\,{}^nC_{r-1}$
- ★ ${}^nC_1 = {}^nC_{n-1} = n$

Conditional Combinations

1. **Number of combinations of n distinct things taking r $(\le n)$ at a time, when k $(0 \le k \le r)$ particular objects always occur** = ${}^{n-k}C_{r-k}$.
2. **Number of combinations of n distinct objects taking $r(\le n)$ at a time, when k $(0 \le k \le r)$ particular objects never occur** $= {}^{n-k}C_r$.
3. **Number of selections of r things from n things when p particular things are not together in any selection** $= {}^nC_r - {}^{n-p}C_{r-p}$
4. **Number of selection of r consecutive things out of n things in a row** = $n - r + 1$
5. **Number of selection of r consecutive things out of n things along a circle**

$$ = \begin{cases} n, & \text{when } r < n \\ 1, & \text{when } r = n \end{cases} $$

6. **The number of Combinations of 'n' different things taking some or all at a time**

$$ = {}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots\ldots\ldots + {}^nC_n = 2^n - 1 $$

EXAMPLE 27. In a class of 25 students, find the total number of ways to select two representative,

(i) if a particular person will never be selected.

(ii) if a particular person is always there.

Sol. (i) Total students (n) = 25

A particular students will not be selected (p) = 1,

So total number of ways = $^{25-1}C_2 = {}^{24}C_2 = 276$.

(ii) Using $^{n-p}C_{r-p}$ no. of ways = $^{25-1}C_{2-1} = {}^{24}C_1 = 24$.

NOTE : *If a person is always there then we have to select only 1 from the remaining 25 – 1 = 24*

Shortcut Approach

Let there are n persons in a hall. If every person shakes his hand with every other person only once, then total number of handshakes

$$= {}^nC_2 = \frac{n(n-1)}{2}$$

Note: *If in place of handshakes each person gives a gift to another person, then formula changes to = n (n – 1)*

EXAMPLE 28. In a party, every person shakes his hand with every other person only once. If total number of handshakes is 210, then find the number of persons.

Sol. Let number of persons be n. Then, according to the question,

$^nC_2 = 210$

$$\Rightarrow \frac{n(n-1)}{2} = 210$$

$\Rightarrow n(n-1) = 420 = 21 \times 20$

$\Rightarrow n = 21$

EXAMPLE 29. There are 10 lamps in a hall. Each of them can be switched on independently. The number of ways in which the hall can be illuminated is

(a) 10^2 **(b)** 1023

(c) 2^{10} **(d)** 10 !

Sol. Since each bulb has two choices, either switched on or off, therefore required number = $2^{10} - 1 = 1023$.

7. **The number of ways of dividing 'm + n' things into two groups containing 'm' and 'n' things respectively**

$$= {}^{m+n}C_m \ {}^nC_n = \frac{(m+n)!}{m!n!}$$

8. **The number of ways of dividing 'm + n + p' things into three groups containing 'm', 'n' and 'p' things respectively**

$$= {}^{m+n+p}C_m \cdot {}^{n+p}C_p = \frac{(m+n+p)!}{m!\,n!\,p!}$$

(i) **If m = n = p i.e. '3m' things are divided into three equal groups then the number of combinations is**

$$\frac{(3m)!}{m!\,m!\,m!\,3!} = \frac{(3m)!}{(m!)^3\,3!}$$

(ii) Buf if '3m' things are to be divided among three persons, then the number of divisions is $\frac{(3m)!}{(m!)^3}$

9. **If mn distinct objects are to be divided into m groups. Then, the number of combination is**

$\frac{(mn)!}{m!\,(n!)^m}$, when the order of groups is not important and

$\frac{(mn)!}{(n!)^m}$, when the order of groups is important

EXAMPLE 30. The number of ways in which 52 cards can be divided into 4 sets, three of them having 17 cards each and the fourth one having just one card

(a) $\frac{52!}{(17!)^3}$ **(b)** $\frac{52!}{(17!)^3\,3!}$

(c) $\frac{51!}{(17!)^3}$ **(d)** $\frac{51!}{(17!)^3\,3!}$

Sol. Here we have to divide 52 cards into 4 sets, three of them having 17 cards each and the fourth one having just one card. First we divide 52 cards into two groups of 1 card and 51 cards. this can be done in $\frac{52!}{1!\,51!}$ ways.

Now every group of 51 cards can be divided into 3 groups of 17 each in $\frac{51!}{(17!)^3\,3!}$.

Hence the required number of ways

$$= \frac{52!}{1!\,51!} \cdot \frac{51!}{(17!)^3\,3!} = \frac{52!}{(17!)^3\,3!}$$

NUMBER OF RECTANGLES AND SQUARES

(a) Number of rectangles of any size in a square of size n × n is $\sum_{r=1}^{n} r^3$ and number of squares of any size is $\sum_{r=1}^{n} r^2$.

(b) Number of rectangles of any size in a rectangle size n × p (n < p) is $\frac{np}{4}(n+1)(p+1)$ and number of squares of any size is $\sum_{r=1}^{n}(n+1-r)(p+1-r)$.

EXAMPLE 31. The number of squares that can be formed on a chessboard is

(a) 64 **(b)** 160

(c) 224 **(d)** 204

Sol. (d) A chessboard is made up of 9 equispaced horizontal and vertical line. To make a 1 × 1 square, we must choose two consecutive horizontal and vertical lines from among these. This can be done in $8 \times 8 = 8^2$ ways. A 2×2 square needs three consecutive horizontal and vertical lines, and we can do this in $7 \times 7 = 7^2$ ways. Continuing in this manner, the total number of square is

$$8^2 + 7^2 + 6^2 + ... + 2^2 + 1^2 = \frac{8(8+1)[(2 \times 8)+1]}{6} = 204.$$

Shortcut Approach

If there are n non-collinear points in a plane, then
(i) Number of straight lines formed = $^nC_2$
(ii) Number of triangles formed = $^nC_3$
(iii) Number of quadrilaterals formed = $^nC_4$

EXAMPLE 32. In a plane, there are 16 non-collinear points. Find the number of straight lines formed.

Sol. Here, n = 16

$\therefore$ Required number of straight lines formed = $^nC_2$

$$= {}^{16}C_2 = \frac{16!}{2!(16-2)!} = \frac{16 \times 15 \times 14!}{2 \times 14!}$$

$= 8 \times 15 = 120$

Shortcut Approach

If there are n points in a plane out of which m are collinear, then
(i) Number of straight lines formed = $^nC_2 - {}^mC_2 + 1$
(ii) Number of triangles formed = $^nC_3 - {}^mC_3$

EXAMPLE 33. In a plane, there are 11 points, out of which 5 are collinear. Find the number of triangles made by these points.

Sol. Here, n = 11, m = 5

Then, required number of triangles = $^nC_3 - {}^mC_3 = {}^{11}C_3 - {}^5C_3$

$$= \frac{11 \times 10 \times 9}{3 \times 2 \times 1} - \frac{5 \times 4 \times 3}{3 \times 2 \times 1}$$

$= 165 - 10 = 155$

Shortcut Approach

Number of diagonals in a polygen of n sides = $^nC_2 - n$

EXAMPLE 34. How many diagonals will be there in an 5 - sided regular polygon?

Sol. The number of diagonals = $^5C_2 - 5 = \frac{5 \times 4}{2 \times 1} - 5 = 5$

EXERCISE

1. In how many different ways can be letters of the word SOFTWARE be arranged in such a way that the vowels always come together?
(a) 13440 (b) 1440
(c) 360 (d) 120
(e) None of these

2. In how many different ways can a group of 4 men and 4 women be formed out of 7 men and 8 women?
(a) 2450 (b) 105
(c) 1170 (d) Cannot be determined
(e) None of these

3. A bag contains 2 red, 3 green and 2 blue balls. 2 balls are to be drawn randomly. What is the probability that the balls drawn contain no blue ball?
(a) $\frac{5}{7}$ (b) $\frac{10}{21}$
(c) $\frac{2}{7}$ (d) $\frac{11}{21}$
(e) None of these

4. In how many different ways can the letters of the word BOOKLET be arranged such that B and T always come together?
(a) 360 (b) 720
(c) 480 (d) 5040
(e) None of these

5. In a box there are 8 red, 7 blue and 6 green balls. One ball is picked up randomly. What is the probability that it is neither red nor green?
(a) $\frac{7}{19}$ (b) $\frac{2}{3}$
(c) $\frac{3}{4}$ (d) $\frac{9}{21}$
(e) None of these

6. In how many different ways can the letters of the word RUMOUR be arranged?
(a) 180 (b) 720
(c) 30 (d) 90
(e) None of these

7. 765 chairs are to be arranged in a column in such a way that the number of chairs in each column should be equal to the columns. How many chairs will be excluded to make this arrangement possible?
(a) 6 (b) 36
(c) 19 (d) 27
(e) None of these

8. In how many different ways can the letters of the word JUDGE be arranged so that the vowels always come together?
(a) 48 (b) 24
(c) 120 (d) 60
(e) None of these

9. How many words can be formed from the letters of the word SIGNATURE so that the vowels always come together?
(a) 720 (b) 1440
(c) 3600 (d) 2880
(e) None of these

10. In how many ways a committee consisting of 5 men and 6 women can be formed from 8 men and 10 women?
(a) 266 (b) 86400
(c) 11760 (d) 5040
(e) None of these

11. Out of 15 students studying in a class, 7 are from Maharashtra, 5 are from Karnataka and 3 are from Goa. Four students are to be selected at random. What are the chances that at least one is from Karnataka?
(a) $\frac{12}{13}$ (b) $\frac{11}{13}$
(c) $\frac{10}{15}$ (d) $\frac{1}{15}$
(e) None of these

12. 4 boys and 2 girls are to be seated in a row in such a way that the two girls are always together. In how many different ways can they be seated?
(a) 120 (b) 720
(c) 148 (d) 240
(e) None of these

13. In how many different ways can the letters of the word DETAIL be arranged in such a way that the vowels occupy only the odd positions?
(a) 120 (b) 60
(c) 48 (d) 32
(e) None of these

14. In a box carrying one dozen of oranges, one-third have become bad. If 3 oranges are taken out from the box at random, what is the probability that at least one orange out of the three oranges picked up is good?
(a) $\frac{1}{55}$ (b) $\frac{54}{55}$
(c) $\frac{45}{55}$ (d) $\frac{3}{55}$
(e) None of these

15. Letters of the word DIRECTOR are arranged in such a way that all the vowels come together. Find out the total number of ways for making such arrangement.
(a) 4320 (b) 2720
(c) 2160 (d) 1120
(e) None of these

16. A box contains 5 green, 4 yellow and 3 white marbles, 3 marbles are drawn at random. What is the probability that they are not of the same colour?
(a) $\frac{13}{44}$ (b) $\frac{41}{44}$
(c) $\frac{13}{55}$ (d) $\frac{52}{55}$
(e) None of these

17. How many different letter arrangements can be made from the letters of the word RECOVER?
(a) 1210 (b) 5040
(c) 1260 (d) 1200
(e) None of these

18. How many three digit numbers can having only two consecutive digits identical is
(a) 153 (b) 162
(c) 168 (d) 163
(e) None of these

19. How many total numbers of seven-digit numbers can be formed having sum of whose digits is even is
(a) 9000000 (b) 4500000
(c) 8100000 (d) 4400000
(e) None of these

20. How many total numbers of not more than 20 digits that can be formed by using the digits 0, 1, 2, 3, and 4 is
(a) 5^{20} (b) $5^{20} - 1$
(c) $5^{20} + 1$ (d) 6^{20}
(e) None of these

21. The number of six digit numbers that can be formed from the digits 1, 2, 3, 4, 5, 6 and 7 so that digits do not repeat and the terminal digits are even is
(a) 144 (b) 72
(c) 288 (d) 720
(e) None of these

22. Three dice are rolled. The number of possible outcomes in which at least one dice shows 5 is
(a) 215 (b) 36
(c) 125 (d) 91
(e) None of these

23. The number of ways in which ten candidates $A_1, A_2, ..., A_{10}$ can be ranked so that A_1 is always above A_2 is
(a) $\frac{10!}{2}$ (b) $10!$
(c) $9!$ (d) $\frac{8!}{2}$
(e) None of these

24. How many total number of ways in which n distinct objects can be put into two different boxes is
(a) n^2 (b) 2^n
(c) $2n$ (d) 3^n
(e) None of these

25. In how many ways can the letters of the word 'PRAISE' be arranged. So that vowels do not come together?
(a) 720 (b) 576
(c) 440 (d) 144
(e) None of these

26. There are 6 tasks and 6 persons. Task 1 cannot be assigned either to person 1 or to person 2; task 2 must be assigned to either person 3 or person 4. Every person is to be assigned one task. In how many ways can the assignment be done?
(a) 144 (b) 180
(c) 192 (d) 360
(e) None of these

27. The number of ways in which one or more balls can be selected out of 10 white, 9 green and 7 blue balls is
(a) 892 (b) 881
(c) 891 (d) 879
(e) None of these

28. If 12 persons are seated in a row, the number of ways of selecting 3 persons from them, so that no two of them are seated next to each other is
(a) 85 (b) 100
(c) 120 (d) 240
(e) None of these

29. The number of all possible selections of one or more questions from 10 given questions, each question having one alternative is
(a) 3^{10} (b) $2^{10} - 1$
(c) $3^{10} - 1$ (d) 2^{10}
(e) None of these

30. A lady gives a dinner party to 5 guests to be selected from nine friends. The number of ways of forming the party of 5, given that two of the friends will not attend the party together is
(a) 56 (b) 126
(c) 91 (d) 94
(e) None of these

31. All possible two factors products are formed from the numbers 1, 2, 3, 4, ..., 200. The number of factors out of total obtained which are multiples of 5 is
(a) 5040 (b) 7180
(c) 8150 (d) 7280
(e) None of these

Directions (Qs. 32-33): Answer these questions on the basis of the information given below:

From a group of 6 men and 4 women a committee of 4 persons is to be formed.

32. In how many different ways can it be done so that the committee has at least one woman?
(a) 210 (b) 225
(c) 195 (d) 185
(e) None of these

33. In how many different ways can it be done so that the committee has at least 2 men?
(a) 210 (b) 225
(c) 195 (d) 185
(e) None of these

34. In how many different ways can the letters of the word ORGANISE be arranged in such a way that all the vowels always come together and all the consonants always come together?
(a) 576 (b) 1152
(c) 2880 (d) 1440
(e) None of these

ANSWER KEY

1	(e)	5	(e)	9	(e)	13	(e)	17	(c)	21	(d)	25	(b)	29	(c)	33	(d)
2	(a)	6	(a)	10	(c)	14	(b)	18	(b)	22	(d)	26	(a)	30	(c)	34	(b)
3	(b)	7	(b)	11	(b)	15	(c)	19	(b)	23	(a)	27	(d)	31	(b)		
4	(b)	8	(a)	12	(d)	16	(b)	20	(a)	24	(b)	28	(c)	32	(c)		

Hints & Explanations

1. (e)

O, A, E	S	F	T	W	R

When the vowels are always together, then treat all the vowels as a single letter and then all the letters can be arranged in 6! ways and also all three vowels can be arranged in 3! ways. Hence, required no. of arrangements $= 6! \times 3! = 4320$.

2. (a) Reqd no. of ways $= {}^7C_4 \times {}^8C_4$

$$= \frac{7\times6\times5\times4}{1\times2\times3\times4}\times\frac{8\times7\times6\times5}{1\times2\times3\times4}$$

$$= 35\times70 = 2450$$

3. (b) Reqd probability $= \frac{{}^5C_2}{{}^7C_2} = \frac{5\times4}{7\times6} = \frac{10}{21}$

4. (b) Treat B and T as a single letter. Then the remaining letters $(5 + 1 = 6)$ can be arranged in 6! ways. Since, O is repeated twice, we have to divide by 2 and the B and T letters can be arranged in 2! ways.

Total no. of ways $= \frac{6!\times2!}{2} = 720$

5. (e) If the drawn ball is neither red nor green, then it must be blue, which can be picked in ${}^7C_1 = 7$ ways. One ball can be picked from the total $(8 + 7 + 6 = 21)$ in ${}^{21}C_1 = 21$ ways.

$\therefore$ Reqd probability $= \frac{7}{21} = \frac{1}{3}$

6. (a) Reqd. number of ways

$$\frac{6!}{2!\times2!} = \frac{6\times5\times4\times3}{1\times2} = 180$$

7. (b) $27^2 < 765 < 28^2$

$\therefore$ required no. of chairs to be excluded $= 765 - 729 = 36$

8. (a) Reqd. number $= 4! \times 2! = 24\times 2 = 48$

9. (e) The word SIGNATURE consists of nine letters comprising four vowels (A, E, I and U) and five consonants (G, N, R, T and S). When the four vowels are considered as one letter, we have six letters which can be arranged in ${}^6P_6$ ways ie 6! ways. Note that the four vowels can be arranged in 4! ways.

Hence required number of words
$= 6! \times 4!$
$= 720\times24 = 17280$

10. (c) Here, 5 men out of 8 men and 6 women out of 10 women can be chosen in ${}^8C_5 \times {}^{10}C_6$ ways, i.e., 11760 ways.

11. (b) Total possible ways of selecting 4 students out of 15 students $= {}^{15}C_4 = \frac{15\times14\times13\times12}{1\times2\times3\times4} = 1365$

The no. of ways of selecting 4 students in which no student belongs to Karnataka$= {}^{10}C_4$

$\therefore$ Hence no. of ways of selecting at least one student from Karnataka $= {}^{15}C_4 - {}^{10}C_4 = 1155$

$\therefore$ Probability $= \frac{1155}{1365} = \frac{77}{91} = \frac{11}{13}$

12. (d) Assume the 2 given students to be together (i.e one]. Now there are five students.
Possible ways of arranging them are $= 5! = 120$
Now, they (two girls) can arrange themselves in 2! ways.
Hence total ways $= 120 \times 2 = 240$

13. (e) 3 vowels can be arranged in three odd places in 3!ways. Similarly, 3 consonants can be arranged in three even places in 3! ways. Hence, the total number of words in which vowels occupy odd positions $= 3! \times 3! = 6 \times 6 = 36$ ways.

14. (b) $n(S) =$ No. of selection of 3 oranges out of the total 12 oranges

$= {}^{12}C_3 = 2\times11\times10 = 220$.

No. of selection of 3 bad oranges out of the total 4 bad oranges $= {}^4C_3 = 4$

$\therefore$ $n(E) =$ no. of desired selection of oranges $= 220 - 4 = 216$

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{216}{220} = \frac{54}{55}$

15. (c) Taking all vowels (IEO) as a single letter (since they come together) there are six letters

Hence no. of arrangements $= \frac{6!}{2!}\times3! = 2160$

[Three vowels can be arranged 3! ways among themselves, hence multiplied with 3!.]

16. (b) Total no. of ways of drawing 3 marbles

$$= {}^{12}C_3 = \frac{12\times11\times10}{1\times2\times3} = 220$$

Total no. of ways of drawing marbles, which are of same colour = ${}^5C_3 + {}^4C_3 + {}^3C_3 = 10 + 4 + 1 = 15$

$\therefore$ Probability of same colour = $\frac{15}{220} = \frac{3}{44}$

$\therefore$ Probality of not same colour = $1 - \frac{3}{44} = \frac{41}{44}$

17. (c) Possible arrangements are :

$$\frac{7!}{2!2!} = 1260$$

[division by 2 times 2! is because of the repetition of E and R]

18. (b) When 0 is the repeated digit like
100, 200, ..., 9 in number
When 0 occurs only once like
110, 220, ..., 9 in number
When 0 does not occur like
112, 211, ..., $2 \times (8 \times 9) = 144$ in number.
Hence, total = 9 + 9 + 144 = 162.

19. (b) Suppose $x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7$ represents a seven digit number. Then x_1 takes the value 1, 2, 3, ..., 9 and x_2, x_3, ..., x_7 all take values 0, 1, 2, 3, ..., 9.
If we keep $x_1, x_2, ..., x_6$ fixed, then the sum $x_1 + x_2 + ... + x_6$ is either even or odd. Since x_7 takes 10 values 0, 1, 2, ..., 9, five of the numbers so formed will have sum of digits even and 5 have sum odd.
Hence the required number of numbers
= 9 . 10 . 10 . 10 . 10 . 10 . 5 = 4500000.

20. (a) Number of single digit numbers = 5
Number of two digits numbers = 4 × 5
[$\because$ 0 cannot occur at first place and repetition is allowed]
Number of three digits numbers
$= 4 \times 5 \times 5 = \quad 4 \times 5^2$
....
....
Number of 20 digits numbers = 4×5^{19}
$\therefore$ Total number of numbers

$$= 5 + 4.5 + 4.5^2 + 4.5^3 \text{ ....} 4.5^{19}$$

$$= 5 + 4.\frac{5(5^{19}-1)}{5-1} = 5 + 5^{20} - 5 = 5^{20}$$

21. (d) The first and the last (terminal) digits are even and there are three even digits. This arrangement can be done in ${}^3P_2$ ways. For any one of these arrangements, two even digits are used; and the remaining digits are 5 (4 odd and 1 even) and the four digits in the six digits (leaving out the terminal digits) may be arranged using these 5 digits in ${}^5P_4$ ways. The required number of numbers is ${}^3P_2 \times {}^5P_4 = 6 \times 120 = 720$.

22. (d) Required number of possible outcomes
= Total number of possible outcomes –
Number of possible outcomes in which 5 does not appear on any dice. (hence 5 possibilities in each throw)
$= 6^3 - 5^3 = 216 - 125 = 91$

23. (a) Ten candidates can be ranked in 10! ways. In half of these ways A_1 is above A_2 and in another half A_2 is above A_1. So, required number of ways is $\frac{10!}{2}$.

24. (b) Let the two boxes be B_1 and B_2. There are two choices for each of the n objects. So, the total number of ways is $\underbrace{2\times2\times....\times2}_{n-\text{times}} = 2^n$.

25. (b) Required number of possible outcomes
= Total number of possible outcomes –
Number of possible outcomes in which all vowels are together
$= 6! - 3! \times 4! = 720 - 144 = 576$

26. (a) Task 1 can not be assigned to either person 1 or 2 i.e. there are 4 options.
Task 2 can be assigned to 3 or 4
So, there are only 2 options for task 2.
So required no. of ways = 2 options for task 2 × 3 options for task 1 × 4 options for task 3 × 3 options for task 4 × 2 options for task 5 × 1 option for task 6.
$= 2 \times 3 \times 4 \times 3 \times 2 \times 1 = 144$

27. (d) The required number of ways
$= (10+1)(9+1)(7+1) - 1 = 879$.

28. (c) The number of ways of selecting 3 persons from 12 people under the given conditon :
Number of ways of arranging 3 people among 9 people seated in a row, so that no two of them are consecutive
= Number of ways of choosing 3 places out of the 10 [8 in between and 2 extremes]

$$= {}^{10}C_3 = \frac{10\times9\times8}{3\times2\times1} = 5\times3\times8 = 120$$

29. (c) Since each question can be selected in 3 ways, by selecting it or by selecting its alternative or by rejecting it. Thus, the total number of ways of dealing with 10 given questions is 3^{10} including a way in which we reject all the questions.
Hence, the number of all possible selections is $3^{10} - 1$.

30. (c) Number of ways of selecting 5 guests from nine friends $= {}^9C_5$
Out of these, ${}^7C_3$ ways are those in which two of the friends occur together [3 more persons to be selected out of remaining 7]
$\therefore$ Number of ways, in which two of the friends will not attend the party together $= {}^9C_5 - {}^7C_3 = 91$.

31. (b) The total number of two factor products = ${}^{200}C_2$. The number of numbers from 1 to 200 which are not multiples of 5 is 160. Therefore the total number of two factor products which are not multiple of 5 is ${}^{160}C_2$. Hence, the required number of factors which are multiples of 5 $= {}^{200}C_2 - {}^{160}C_2 = 7180$.

32. (c) Reqd. no. of ways

$$= {}^4C_1 \times {}^6C_3 + {}^4C_2 \times {}^6C_2 + {}^4C_3 \times {}^6C_1 + {}^4C_4$$

$$= 4 \times \frac{6 \times 5 \times 4}{1 \times 2 \times 3} + \frac{4 \times 3}{1 \times 2} \times \frac{6 \times 5}{1 \times 2} + \frac{4 \times 3 \times 2}{1 \times 2 \times 3} \times 6 + 1$$

$$= 80 + 90 + 24 + 1 = 195$$

33. (d) Reqd. no. of ways

$$= {}^6C_2 \times {}^4C_2 + {}^6C_3 \times {}^4C_1 + {}^6C_4$$

$$= \frac{6 \times 5}{1 \times 2} \times \frac{4 \times 3}{1 \times 2} + \frac{6 \times 5 \times 4}{1 \times 2 \times 3} \times 4 + \frac{6 \times 5 \times 4 \times 3}{1 \times 2 \times 3 \times 4}$$

$$= 90 + 80 + 15 = 185.$$

34. (b) The word ORGANISE has 4 vowels and 4 consonants. Now, both groups (vowels and consonants) can be treated as two letters.This can be arranged in 2! ways. Now, the 4 letters of each group can be arranged.in 4! ways.

So, total possible ways of arrangement

$= 2! \times 4! \times 4!$

$= 2 \times 24 \times 24 = 1152.$