# 11. QUADRATIC EQUATIONS

## 1. Quadratic Equation : $a x^2 + b x + c = 0$ , $a \ne 0$

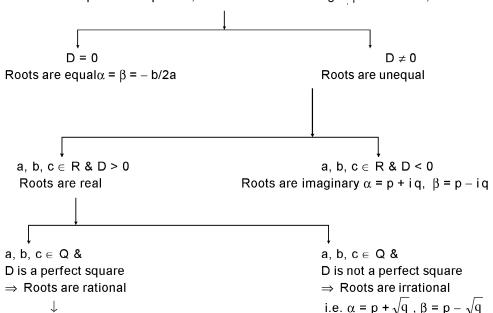
 $x=\frac{-\,b\pm\sqrt{b^2-4\,a\,c}}{2\,a}\,,\, The\,\, expression\,\, b^2-4\,\,a\,\,c\equiv D\,\, is\,\, called\,\, discriminant\,\, of\,\, quadratic\,\, equation.$ 

If  $\alpha$ ,  $\beta$  are the roots, then (a)  $\alpha + \beta = -\frac{b}{a}$  (b)  $\alpha \beta = \frac{c}{a}$ 

A quadratic equation whose roots are  $\alpha \& \beta$ , is  $(x - \alpha)(x - \beta) = 0$  i.e.  $x^2 - (\alpha + \beta)x + \alpha\beta = 0$ 

### 2. Nature of Roots:

Consider the quadratic equation, a  $x^2$  + b x + c = 0 having  $\alpha$   $\beta$  as its roots; D =  $b^2$  - 4 a c



a = 1, b,  $c \in I \& D$  is a perfect square

⇒ Roots are integral.

#### 3. Common Roots:

Consider two quadratic equations  $a_1x^2 + b_1x + c_1 = 0 & a_2x^2 + b_2x + c_2 = 0$ .

- (i) If two quadratic equations have both roots common, then  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ .
- (ii) If only one root  $\alpha$  is common, then  $\alpha = \frac{c_1 a_2 c_2 a_1}{a_1 b_2 a_2 b_1} = \frac{b_1 c_2 b_2 c_1}{c_1 a_2 c_2 a_1}$

# 4. Range of Quadratic Expression f (x) = $ax^2 + bx + c$

Range in restricted domain: Given  $x \in [x_1, x_2]$ 

(a) If 
$$-\frac{b}{2a} \notin [x_1, x_2]$$
 then,  $f(x) \in [\min\{f(x_1), f(x_2)\}, \max\{f(x_1), f(x_2)\}]$ 

$$\text{(b)} \qquad \text{If} - \frac{b}{2a} \in \left[ \mathbf{x}_1, \, \mathbf{x}_2 \right] \text{ then}, \ \ f(\mathbf{x}) \in \left[ \min \left\{ f\left(\mathbf{x}_1\right), \, f\left(\mathbf{x}_2\right), \, -\frac{D}{4a} \right\}, \ \ \max \left\{ f\left(\mathbf{x}_1\right), \, f\left(\mathbf{x}_2\right), \, -\frac{D}{4a} \right\} \right]$$

### 5. Location of Roots:

Let  $f(x) = ax^2 + bx + c$ , where  $a > 0 \& a^{-}b^{-}c \in R$ .

- (i) Conditions for both the roots of f (x) = 0 to be greater than a specified number' $x_0$ ' are  $b^2-4ac \ge 0$ ; f ( $x_0$ ) > 0 & (-b/2a) >  $x_0$ .
- (ii) Conditions for both the roots of f(x) = 0 to be smaller than a specified number ' $x_0$ ' are  $b^2 4ac \ge 0$ ;  $f(x_0) > 0 & (-b/2a) < x_0$ .
- (iii) Conditions for both roots of f(x) = 0 to lie on either side of the number ' $x_0$ ' (in other words the number ' $x_0$ ' lies between the roots of f(x) = 0), is  $f(x_0) < 0$ .
- (iv) Conditions that both roots of f(x) = 0 to be confined between the numbers  $x_1$  and  $x_2$ ,  $(x_1 < x_2)$  are  $b^2 4ac \ge 0$ ;  $f(x_1) > 0$ ;  $f(x_2) > 0$  &  $x_1 < (-b/2a) < x_2$ .
- (v) Conditions for exactly one root of f(x) = 0 to lie in the interval  $(x_1, x_2)$  i.e.  $x_1 < x < x_2$  is  $f(x_1)$ .  $f(x_2) < 0$ .