Basic Geometrical Ideas

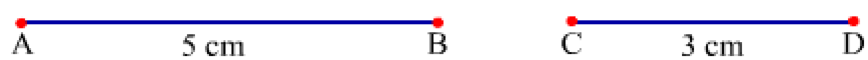
Points, Line Segments, Lines, Rays, Planes and Space

Points, line segments, lines, and rays are the basic ideas on which the world of geometry is based. In order to understand geometry, we should be very clear about these basic ideas.

Let us start with the concept of a point. In order to understand the idea of a point, let us consider the sharp tip of a pencil or the pointed end of a pin.

These sharp tips represent dots. These small dots can be described as points. Let us go through the video to understand the concept of points and other related geometrical concepts such as lines, rays, and line segments.

Now, look at the line segments AB and CD.



Length of segment AB is 5 cm and that of segment CD is 3 cm.

Mathematically, their respective lengths can be written as follows:

***l* (AB) = 5 cm** and ***l* (CD) = 3 cm**

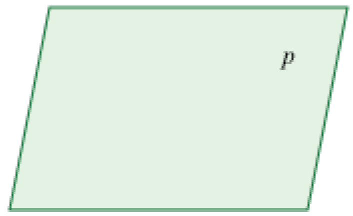
We have seen what lines, rays, and line segments are, so let us now study another concept, which is the concept of planes.

A plane is a flat surface having length and width, but no thickness. We can say that a plane is a flat surface, which extends indefinitely in all directions.

For example, surface of a wall, floor of a ground, etc.

A plane can be denoted by writing small letters inside it such as letters *p, q*, etc.

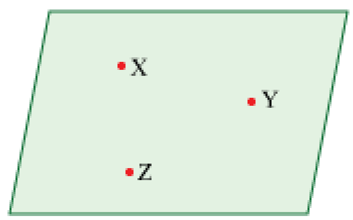
For example:



This plane is read as "plane *p*".

Also, a plane can be denoted by taking 3 different points, say X, Y, Z in the plane, but not on same line.

For example:



This plane is read as "plane XYZ".

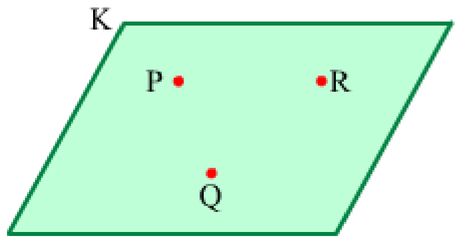
Do you think that there is any relation between points and lines in a plane?

The answer to this question is 'yes' and the relations between them are called incidence properties.

Some axioms:

(1) There is exactly one plane passing through three non-collinear points.

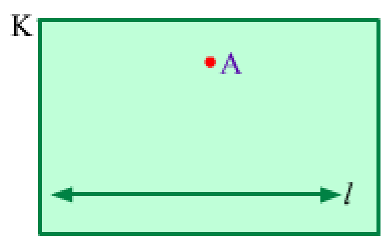
Observe the given figure.



Here, points P, Q and R are three non-collinear points. K is the plane passing through these points.

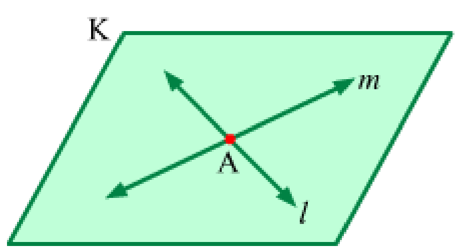
(2) There is exactly one plane which passes through a line and a point not lying on the line.

Look at the following figure.



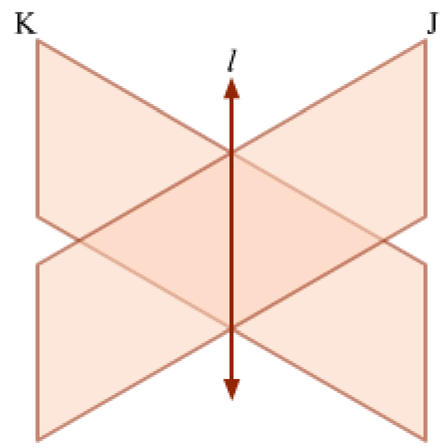
Here, plane K passes through the line *l* and the point A which does not lie on the line *l*.

(3) Only one plane can pass through two distinct intersecting lines.



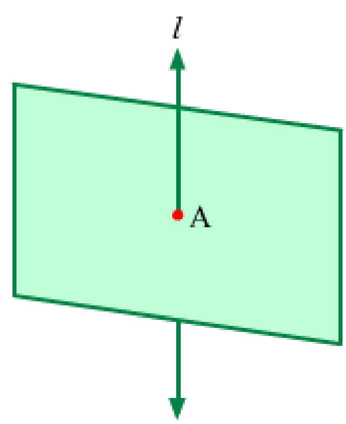
In the figure given above, lines *l* and *m* intersect each other at point A. Plane K passes through these lines.

(4) A line is obtained by intersection of two planes.



In the figure given above, planes J and K intersect each other. Line *l* is obtained by their intersection.

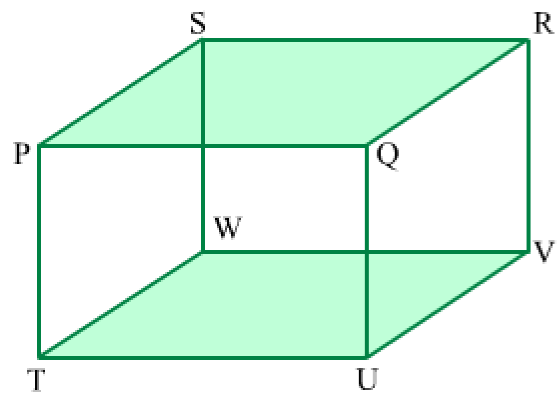
(5) When a plane is intersected by a line, which does not lie in the plane, then a point is obtained.



In the figure given above, line *l* does not lie in the plane K and it intersects the plane at point A.

Parallel planes:

Planes which do not intersect each other are said to be parallel.



In the above given figure, planes PQRS and TUVW do not intersect each other and hence, these are parallel planes.

Similarly, PTWS and QUVR is a pair of parallel planes while PTUQ and SWVR is another pair of parallel planes.

Length of a line segment:

The length of a line segment is the distance between the end points of the line segment.

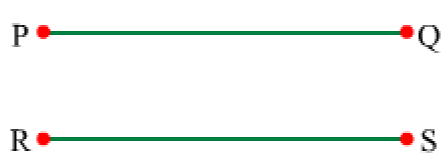
Length of a line segment PQ is denoted as *l*(PQ) and distance between two points is denoted as *d*(P, Q).

Therefore, *l*(PQ) = *d*(P, Q).

Here after *l*(PQ) is denoted as PQ and thus, PQ = *l*(PQ) = *d*(P, Q).

Congruent segment:

If two line segments are of equal length, then they are said to be congruent.



In the above figure, *l*(PQ) = *l*(RS). So, line segments PQ and RS are congruent.

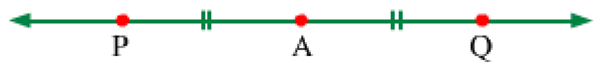
Mathematically, if *l*(PQ) = *l*(RS), then seg PQ $\cong$ seg RS.

Note: While considering the length of segment PQ, we write only PQ but while considering the set of points between P and Q (segment as a whole), we write seg PQ or side PQ.

Mid-point of a segment:

If A is a point such that P – A – Q and d(P, A) = d(A, Q) then A is said to be the mid-point of the segment PQ.

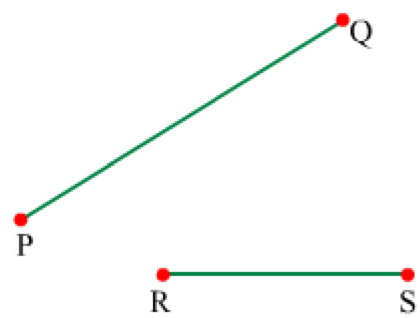
Every line segment has one and only one mid-point.



In the above figure, seg PA ≅ seg AQ.

Comparison of segments:

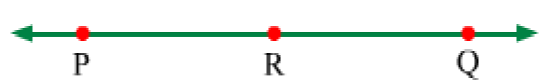
Observe the below given figure.



Here, RS < PQ so, it can be said seg RS is smaller than seg PQ. This information is denoted as seg RS < seg PQ.

Opposite rays:

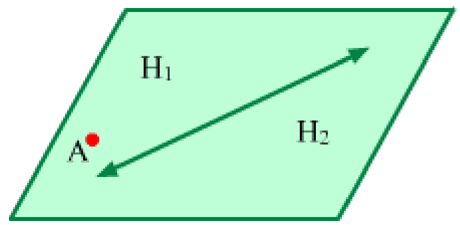
If two rays have same origin and are contained in the same line, then the rays are known as opposite rays.



In the above figure, ray RP and ray RQ are opposite rays.

Plane separation axiom:

In a plane, a given line and points which do not lie on the line form two disjoint sets say H_1 and H_2.



Each of the sets H_1 and H_2 is known as the half plane and the given line is called the edge of each half plane.

If A is any point in any of the half planes then then that half plane is known as A side of the half plane.

Let us discuss some examples based on these basic geometrical ideas.

Example 1:

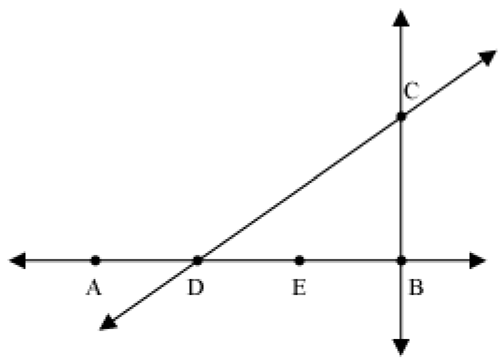
With respect to the given figure, name

(a) any six line segments

(b) a line

(c) three rays

(d) two pairs of intersecting lines



Solution:

(a) Six line segments shown in the figure are $\overline{AD}$, $\overline{AE}$, $\overline{DE}$, $\overline{DB}$, $\overline{BE}$, and $\overline{BC}$.

(b) $\overleftrightarrow{AB}$ is a line in the given figure.

(c) Three rays in the given figure are $\overrightarrow{DA}$, $\overrightarrow{DB}$, and $\overrightarrow{DC}$.

(d) Two pairs of intersecting lines in the given figure are $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$, and $\overleftrightarrow{DC}$ and $\overleftrightarrow{BC}$.

Example 2:

Draw rough figures of the following.

(a) A line $\overleftrightarrow{AB}$

(b) Points P and Q that lie on line $\overleftrightarrow{AB}$

(c) A line $\overleftrightarrow{XY}$ that intersects line $\overleftrightarrow{AB}$ at point Q

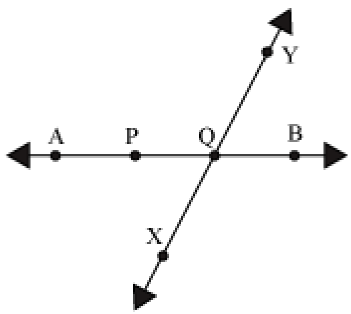
(d) A line $\overleftrightarrow{DE}$ that is parallel to line $\overleftrightarrow{XY}$ and passes through point P

Solution:

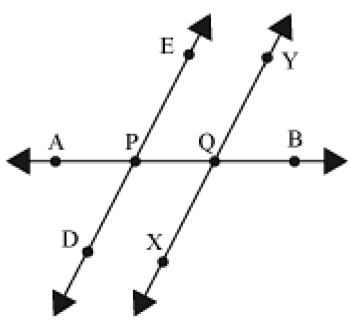
(a)

(b)

(c)

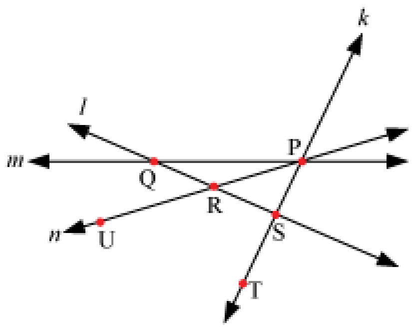


(d)



Example 3:

With respect to the given figure, state whether the following statements are correct or incorrect.



(a) The points P, S, and T are collinear points.

(b) The lines *k*, *l*, and n are concurrent lines.

(c) The point U, R, and S are collinear points.

(d) The point P is the point of concurrence of lines *k*, *m*, and *n*.

Solution:

(a) Correct

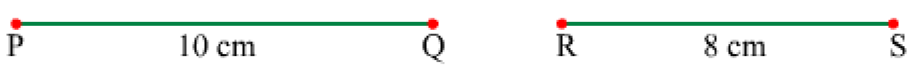
(b) Incorrect; since the lines *k, l*, and *n* do not pass through the same point

(c) Incorrect; since the points U, R, and S do not lie on the same line

(d) Correct

Example 4:

Write the lengths of given line segments.



Solution:

It can be seen that the length of segment PQ is 10 cm and that of segment RS is 8 cm.

Mathematically, their respective lengths can be written as follows:

l (PQ) = 10 cm and *l* (RS) = 8 cm

Classification of Curves as Open and Closed

Look at the following figures.

(i) (ii) (iii)

Each of these shapes is an example of a **curve.** In fact, any shape that we draw is a curve. We can define a curve as follows.

Any figure drawn on a paper is known as a curve. A curve may or may not be straight.

Note: In real life, we do not consider straight lines as curves. However, in mathematics, straight lines are also considered as curves.

Can we find any difference among the three curves that we discussed in the beginning?

Curves (i) and (iii) do not intersect themselves, while curve (ii) does. Also, curves (i) and (iii) are not closed figures, while curve (ii) is a closed figure. On the basis of these observations, we classify curves as follows.

1. Simple curves
2. Closed Curves
3. Open curves

4. Let us discuss some more examples based on classification of curves.

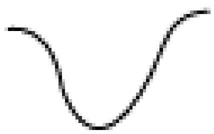
5. **Example 1:**

6. **Classify each of the following curves as open or closed.**

7. **(a)**

8. 

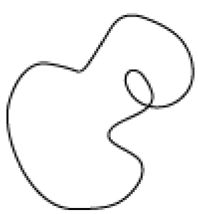
9. **(b)**

10. 

11. **(c)**

12. 

13. **(d)**

14. 

15. **Solution:**

16. **(a)** Since no end points can be seen in the curve, it is an example of a closed curve.

17. **(b)** Since the two end points of the curve can be seen, it is an example of an open curve.

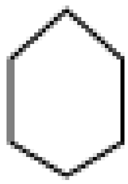
18. **(c)** Since the two end points of the curve can be seen, it is an example of an open curve.

19. **(d)** Since no end points can be seen in the curve, it is an example of a closed curve.

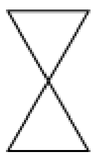
20.**Example 2:**

21.**State whether each of the following curves is simple or not.**

22.**(a)**

23. 

24.**(b)**

25. 

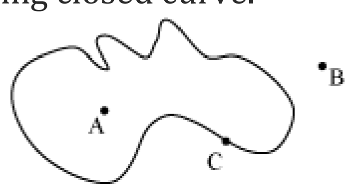
26.**Solution:**

27.**(a)** Since the curve does not cross itself, it is a simple curve.

28.**(b)** Since the curve crosses itself at one point, it is not a simple curve.

29.Identification Of Regions Of A Curve

30.Let us consider the following closed curve.

31. 

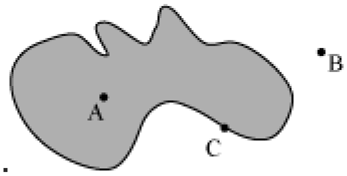
32.Now, how can we classify the parts of the curve according to the position of points A, B, and C?

33.From this figure, we can see that point A lies inside the boundary of the curve. In another way, we can say that point A lies in the **interior** of the curve. Point B lies outside the boundary of the curve. We can also say that point B lies in the **exterior** of the curve. Point C lies **on the curve**, that is, on the boundary of the curve.

34.From this observation, we can say that a closed curve has three parts. They are

35.**(i)** Interior of the curve

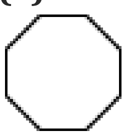
36.**(ii)** Exterior of the curve
37.**(iii)** Boundary of the curve
38.If we shade the interior of this closed curve along with its boundary, then we have the following closed curve.

39. 

40.This shaded portion of the curve is known as **region.** Therefore, the region of a closed curve can be defined as:

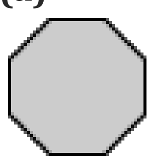
The interior of a closed curve along with its boundary is called its region.

41.Let us discuss some more examples to understand this concept better.
42.**Example 1:**
43.**Shade the interior of the following curves.**
44.**(a)**



45.
46.**(b)**

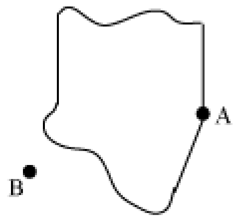
47.
48.**Solution:**
49.The interior region is the region inside the curve. Thus, the interior of the given curves can be shaded as:
50.**(a)**



51.
52.**(b)**

53.
54.**Example 2:**

55.**In which parts of the curve do the points A and B lie?**

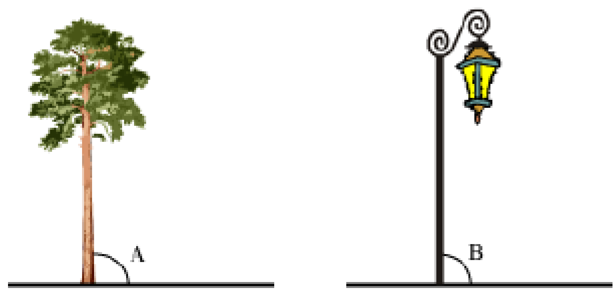
56. 

57.**Solution:**

58.Point A lies on the boundary of the curve, i.e., on the curve. Point B lies outside the curve, i.e., in the exterior of the curve.

59.Arms and Vertices of Angles

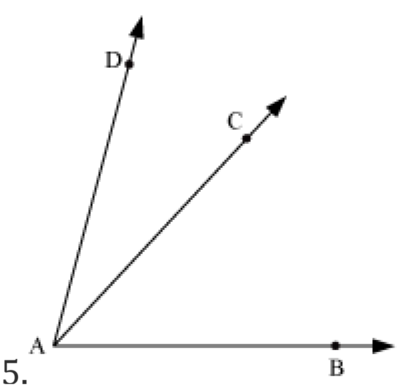
60.We have seen lamp posts and trees on the street side. These lamp posts and trees stand straight on the road as shown in the figure below.

61. 

62.What do we observe in the above figures?

63.We observe that there is a small curve denoted by A between the tree and the ground. Similarly, in the case of lamp post, a curve is denoted by B. These curves are known as **angles.** We say that the tree and the lamp post are making angles A and B respectively with the ground.

64.In some cases, it is difficult to specify an angle by its vertex. For example, in the figure given below, ∠A may denote any of the angles among ∠BAC, ∠CAD, or ∠BAD.

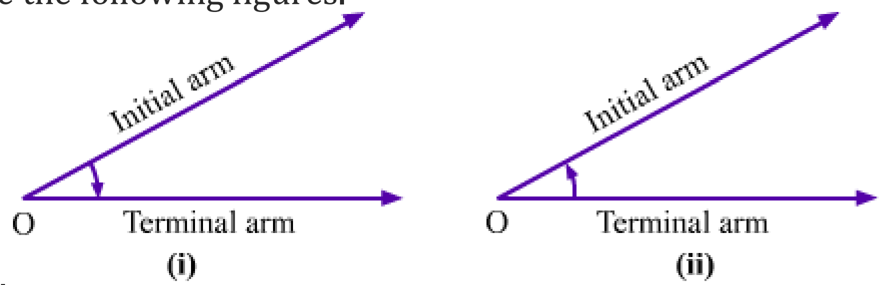
65. 

66.Therefore, it is desirable to represent an angle by three letters (which make the angle) and not just by the vertex letter.

67.

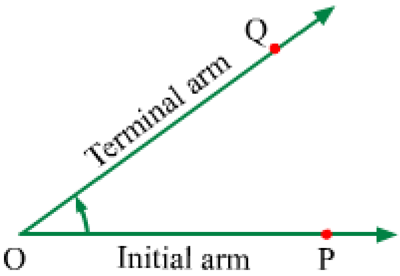
68.**Angles in terms of rotation:**

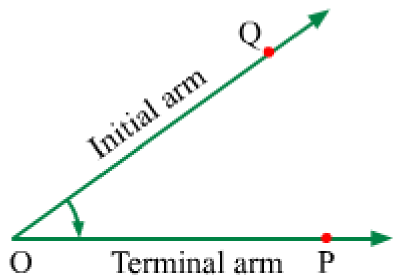
69. An angle is obtained when a ray is rotated about its end point. The ray can be rotated in two ways such as clockwise (direction in which the hands of a clock move) and anticlockwise (direction opposite to clockwise direction).
70. When a ray is rotated clockwise, the obtained angle is regarded as negative while the ray is rotated anticlockwise, the angle obtained is regarded as positive.
71. Observe the following figures.



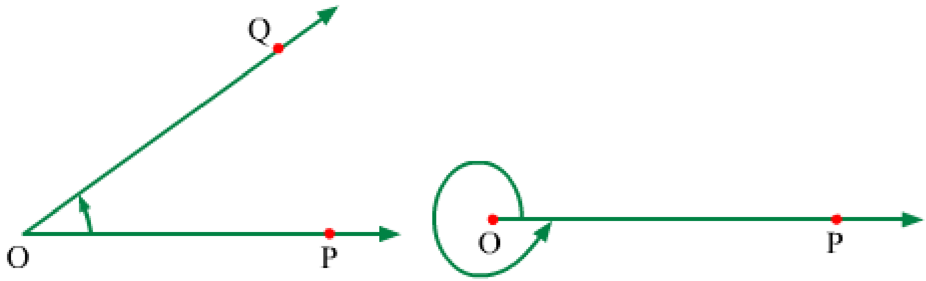
72.

73. In figure (i), the angle is obtained by rotating the initial arm clockwise, thus this angle is negative. On the other hand, the angle obtained in figure (ii) is positive as it is obtained by rotating the initial arm anticlockwise.
74. **The amount of rotation of the ray from its initial position to terminal position is known as the measure of the angle.**
75. In figures (i) and (ii), the angles are not same, even if their measures are equal because one of them is positive and the other is negative.
76.
77. **Directed angle:**
78. When a ray rotates about its origin point to occupy the position of another ray having the same origin point, it forms an angle which is known as the directed angle. Name of the directed angle is written according to the direction of rotation of initial arm.
79. For directed angle, rays can be represented in the form of ordered pair as **(ray acting as initial arm, ray acting as terminal arm).**
80. If we have two rays OP and OQ, then there can be two possibilities for directed angle. These are as follows:
81. **(i) Ray OP is initial arm and ray OQ is terminal arm:**
82. In this case, we get the ordered pair as ray OP, and ray OQ, which represents that the directed angle is obtained by the rotation of ray OP to occupy the position of ray OQ. Thus, obtained angle is called directed angle POQ.

83.


84.Directed angle POQ is denoted as ∡POQ.
85.**(ii) Ray OQ is initial arm and ray OP is terminal arm:**
86.In this case, we get the ordered pair as ray OQ, and ray OP, which represents that the directed angle is obtained by the rotation of ray OQ to occupy the position of ray OP. Thus, obtained angle is called directed angle QOP.
87.


88.Directed angle QOP is denoted as ∡QOP.
89.So, it can easily be concluded that the directed angle POQ and directed angle QOP are different.
90.i.e., ∡POQ ≠ ∡QOP
91.
92.**Positive and negative angles:**
93.Angle obtained on anticlockwise rotation of initial arm is regarded as positive angle.
For example, ∡POQ in the above shown figure is positive angle.
94.Angle obtained on clockwise rotation of initial arm is regarded as negative angle.
For example, ∡QOP in the above shown figure is negative angle.
95.
96.**One complete rotation:**
97.If the initial ray OP is rotated about its end point O in anticlockwise direction such that it comes back to the position OP again for the first time, then it is said that the ray OP has formed one complete rotation.

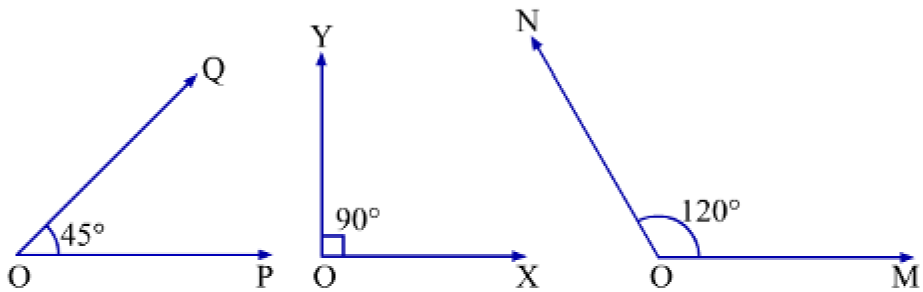
98. 

99.The measure of the angle traced during one complete rotation in anticlockwise direction is 360°. Similarly, the measure of the angle traced during two complete rotations in anticlockwise direction is just double i.e., 720° and so on.

100.

101. **Writing the measure of an angle:**

102. Observe the following angles.

103. 

104. It can be seen that ∠POQ is a 45° angle, or we can say that ∠POQ measures 45°.

105. Mathematically, it is denoted as $m\angle POQ = 45°$.

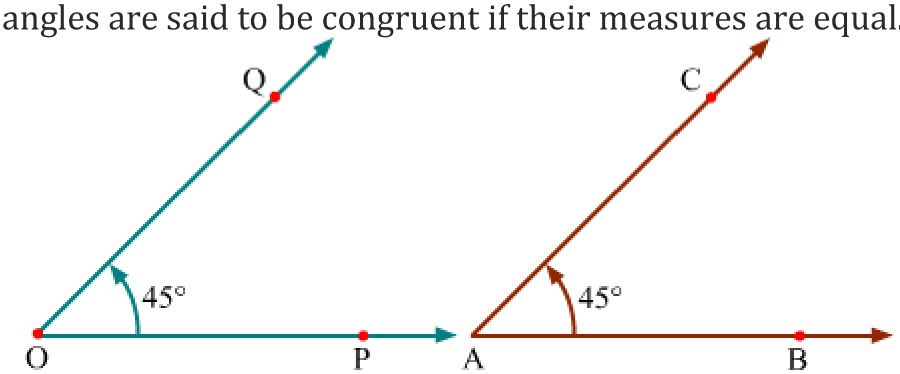
106. Similarly, $m\angle XOY = 90°$, and $m\angle MON = 120°$.

107. This is how we denote the measure of an angle.

108.

109. **Congruent angles:**

110. Two angles are said to be congruent if their measures are equal.

111. 

112. From the above figures, it can be observed that ∠POQ = 45° = ∠BAC.

113. Thus, ∠POQ ≅ ∠BAC.

114.

115. **Properties of congruent angles:**

116. **(i) Reflexivity:** Every angle is congruent to itself i.e., ∠PQR ≅ ∠PQR.

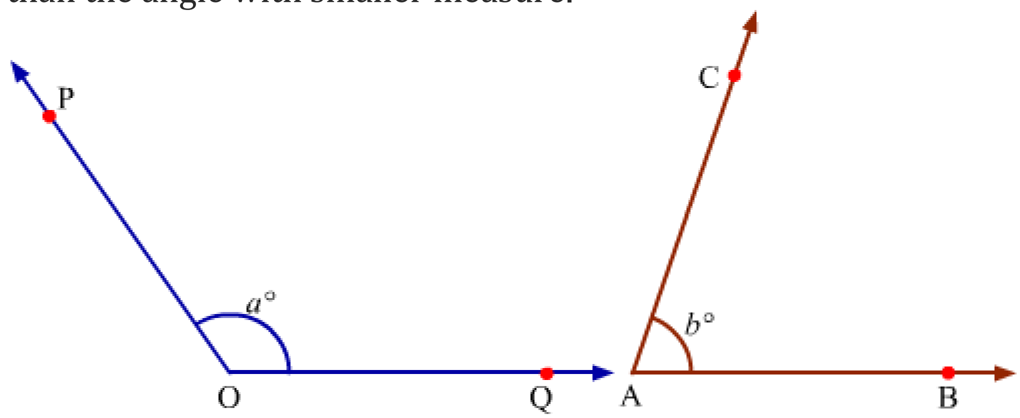
117. **(ii) Symmetry:** If ∠PQR ≅ ∠ABC, then ∠ABC ≅ ∠PQR.

118. **(iii) Transitivity:** If ∠PQR ≅ ∠ABC, and∠ABC ≅ ∠XYZ, then ∠PQR ≅∠XYZ.

119.

120. **Inequality of angles:**

121. Out of two angles, the angle with greater measure is said to be greater than the angle with smaller measure.

122. 

123. In the above figures, we have

124. ∠POQ = a° and ∠CAB = b°

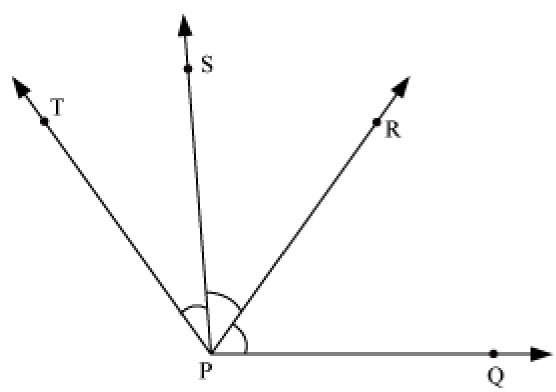
125. If $a > b$, then ∠POQ > ∠CAB

126. Now, let us discuss some examples based on the concept of angle.

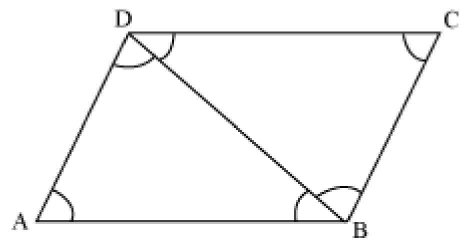
127. **Example 1:**

128. **Name the angles marked in each of the following figures. Name the vertices and arms associated with these angles.**

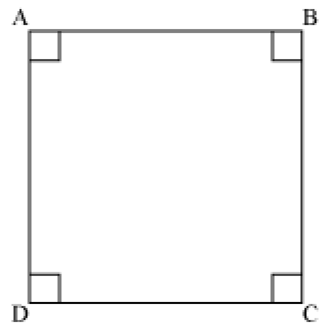
129. **(i)**

130. 

131. **(ii)**

132. 

133. **(iii)**

134. 

135. **Solution:**

136. **(i)** The angles marked in the figure are ∠QPR, ∠RPS, and ∠SPT.

137. The vertices and arms of each of these angles are listed as:

Angle	Vertex	Arms
∠QPR	P	$\overrightarrow{PQ}$ and $\overrightarrow{PR}$
∠RPS	P	$\overrightarrow{PR}$ and $\overrightarrow{PS}$
∠SPT	P	$\overrightarrow{PS}$ and $\overrightarrow{PT}$

138. **(ii)** The angles marked in the figure are ∠BAD, ∠ADB, ∠BDC, ∠BCD, ∠CBD, and ∠ABD.

139. The vertices and arms of each of these angles are listed as:

Angle	Vertex	Arms
∠BAD	A	$\overrightarrow{AB}$ and $\overrightarrow{AD}$
∠ADB	D	$\overrightarrow{DA}$ and $\overrightarrow{DB}$
∠BDC	D	$\overrightarrow{DB}$ and $\overrightarrow{DC}$
∠BCD	C	$\overrightarrow{CB}$ and $\overrightarrow{CD}$
∠CBD	B	$\overrightarrow{BC}$ and $\overrightarrow{BD}$
∠ABD	B	$\overrightarrow{BA}$ and $\overrightarrow{BD}$

140. **(iii)** The angles marked in the figure are ∠ABC, ∠BCD, ∠CDA, and ∠DAB.

141. The vertices and arms of each of these angles are listed as:

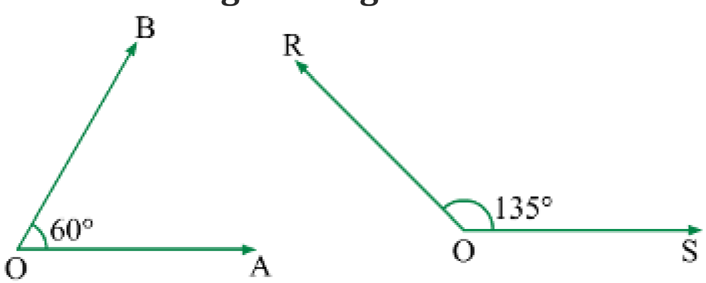
142.

Angle	Vertex	Arms
∠ABC	B	$\overrightarrow{BA}$ and $\overrightarrow{BC}$
∠BCD	C	$\overrightarrow{CB}$ and $\overrightarrow{CD}$
∠CDA	D	$\overrightarrow{DC}$ and $\overrightarrow{DA}$
∠DAB	A	$\overrightarrow{AD}$ and $\overrightarrow{AB}$

143.

144. **Example 2:**

145. **Denote the measures of given angles.**

146. 

147. **Solution:**

148. It can be seen that ∠AOB is a 60° angle, while ∠SOR is a 135° angle.

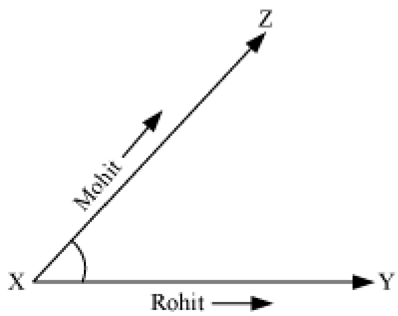
149. Mathematically, we can denote the measures of given angles as follows:

150. $m\angle AOB = 60°$ and $m\angle SOR = 135°$.

LL DOWN FOR THE NEXT TOPIC

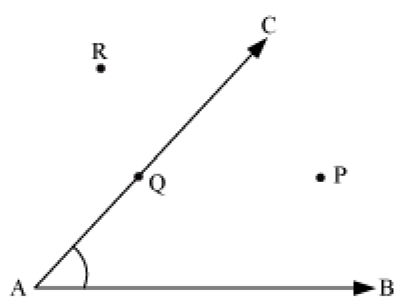
Identification of Regions and Points That Lie Inside, Outside and on the Angles

Rohit and Mohit start walking from the same point in different directions. Let Rohit move towards Y and Mohit towards Z as shown below.

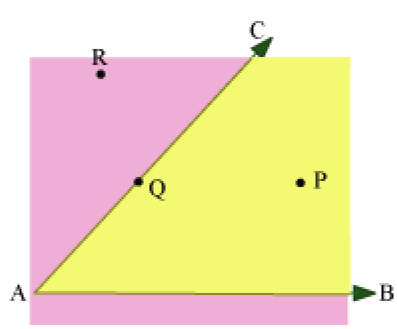


Here, we can say that the rays $\overrightarrow{XY}$ and $\overrightarrow{XZ}$ form an angle ZXY. These rays, i.e. $\overrightarrow{XY}$ and $\overrightarrow{XZ}$, are known as the arms of ∠ZXY. The point, i.e. X, that is common to these arms is called the vertex of ∠ZXY.

Let us consider the following ∠CAB with some points P, Q, and R.



Let us shade the different parts of the angle as shown below.



Here, we can see that the region of the angle shaded by yellow colour lies between the two arms of the angle. This region is called the **interior region** of the angle. It extends indefinitely, since the two arms also extends indefinitely. Every point in this region is said to lie in the **interior** of the angle**.** In this figure, point P lies in the interior of ∠CAB.

The region of the angle shaded pink lies outside the two arms of the angle. This region is called the **exterior region** of the angle. Like the interior region of an angle, the exterior region also extends indefinitely. Every point in this region is said to lie in the **exterior of the angle.** In this figure, point R lies in the exterior of ∠CAB.

The boundary of ∠CAB is formed by its arms $\overrightarrow{AB}$ and $\overrightarrow{AC}$. These arms are called the **boundary** of the angle. Every point lying on the arms is said to lie on the boundary of the angle, or simply, **on the angle.** In this figure, points A, B, C, and Q lie on the angle.

Using this concept, we can say that an angle has three regions. They are interior region, exterior region, and boundary region. Using this idea, we can easily identify whether a point lies inside, outside, or on the given angle. Let us discuss one more example to understand the concept better.

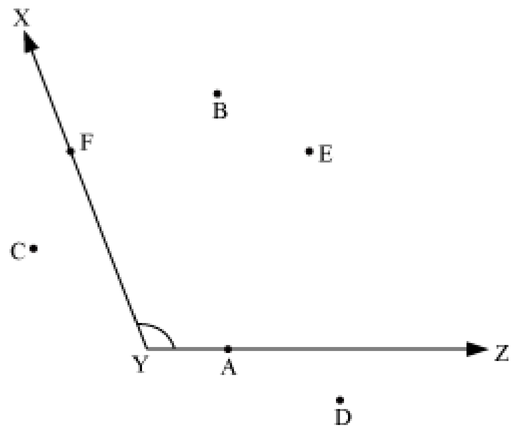
Example 1:

In the figure given below, name the point or points that lie

(i) in the interior of the angle

(ii) in the exterior of the angle

(iii) on the angle



Solution:

(i) Points B and E lie in the interior of the angle.

(ii) Points C and D lie in the exterior of the angle.

(iii) Points A, F, X, Y, and Z lie on the angle.

Polygons and Their Attributes

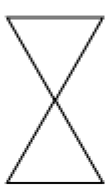
Polygons can be classified on the basis of number of sides. But before we learn about this, we should know the basic properties of polygons such as vertices, adjacent sides, diagonals, etc. This is the foundation of higher polygon concepts and the given video will help us get familiar with these properties.

Let us discuss some examples based on polygon.

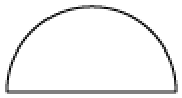
Example 1:

State whether each of the following curves is a polygon or not.

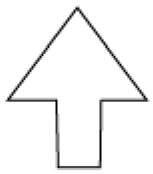
(a)



(b)



(c)



Solution:

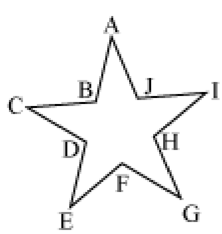
(a) A polygon is always a simple and closed curve, entirely made up of line segments. Since the given curve crosses itself, it is not a simple curve and thus, not a polygon.

(b) The given curve is not entirely made up of line segments. Therefore, it is not a polygon.

(c) The given curve is a simple and closed curve and is entirely made up of line segments. It is, thus, a polygon.

Example 2:

Answer the questions below with respect to the given figure.



(a) Name the vertices of the polygon.

(b) Name the adjacent sides of AJ, GF, and BC.

(c) Name the adjacent vertices of A, G, and E.

Solution:

(a) The point where two sides of a polygon meet is called its vertex. The vertices of the polygon are A, B, C, D, E, F, G, H, I, and J.

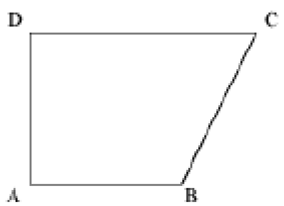
(b) Any two sides of a polygon with a common vertex are called adjacent sides. Thus, the adjacent sides of AJ are AB and IJ; that of GF are GH and EF; and that of BC are AB and CD.

(c) The vertices of a polygon that lie on the same side are called adjacent vertices. Thus, the adjacent vertices of A are B and J; that of G are H and F; and that of E are D and F.

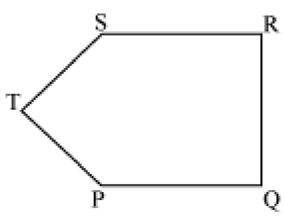
Example 3:

Name all the sides of the following polygons. Also, draw and count all the possible number of diagonals.

(a)

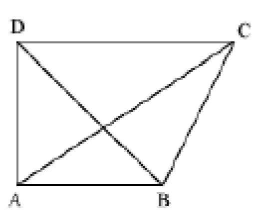


(b)

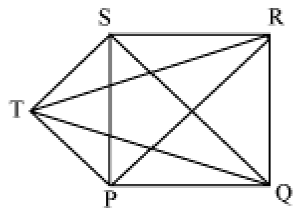


Solution:

(a) The sides of the given polygon are AB, BC, CD, and DA. It has two diagonals, AC and BD.

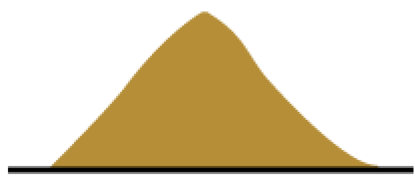


(b) The sides of the given polygon are PQ, QR, RS, ST, and TP. It has five diagonals, PR, PS, QS, QT, and RT.

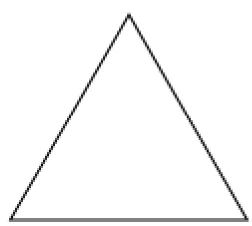


Triangles and Their Attributes

Let us look at the figure of a heap of sand given below.



If we draw the above figure in a simple way, then it will look similar to the following figure.



We must have seen these types of shapes before. This is a three-sided polygon. It is called a **triangle.** We define a triangle as follows.

A polygon formed by three line segments is called a triangle i.e., a triangle is a polygon having three sides.

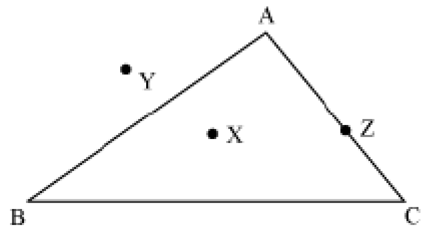
Let us discuss some characteristics of a triangle through various observations.

We know that a polygon has three regions.

1. Interior
2. Exterior

3. Boundary

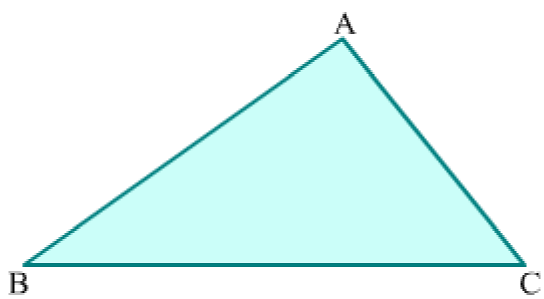
A triangle is a polygon, therefore, it also has the above three regions. It can be clearly understood with the help of the following figure.



In the above figure, point X lies in the interior of ΔABC; point Y lies in the exterior of ΔABC, while points A, B, C, and Z lie on the boundary of ΔABC.

The triangular area:

Look at the following triangle.

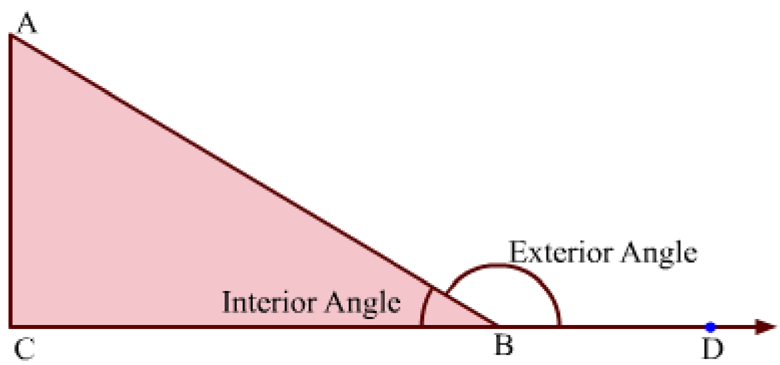


The interior of ΔABC is shaded. The whole shaded part of ΔABC along with its boundary is its area.

Thus, **interior and boundary of a triangle together form the area of triangle or triangular area.**

Exterior angle of a triangle:

Look at the triangle shown below.



It can be seen that in **Δ**ABC, side CB is extended up to point D. This extended side forms an angle with side AB, i.e., ∠ABD. This angle lies exterior to the triangle. Hence, ∠**ABD is an exterior angle of ΔABC.**

An exterior angle of a triangle can be defined as follows:

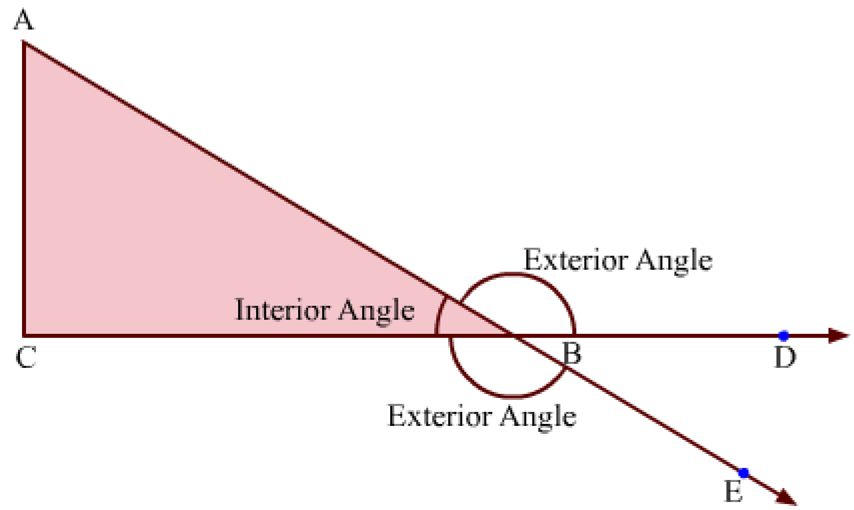
The angle formed by a side of a triangle with an extended adjacent side is called an exterior angle of the triangle.

Also, it can be seen that CBD is a line and hence, ∠ABC and ∠ABD form a linear pair at vertex B.

So, an exterior angle of a triangle can be defined in another way as follows:

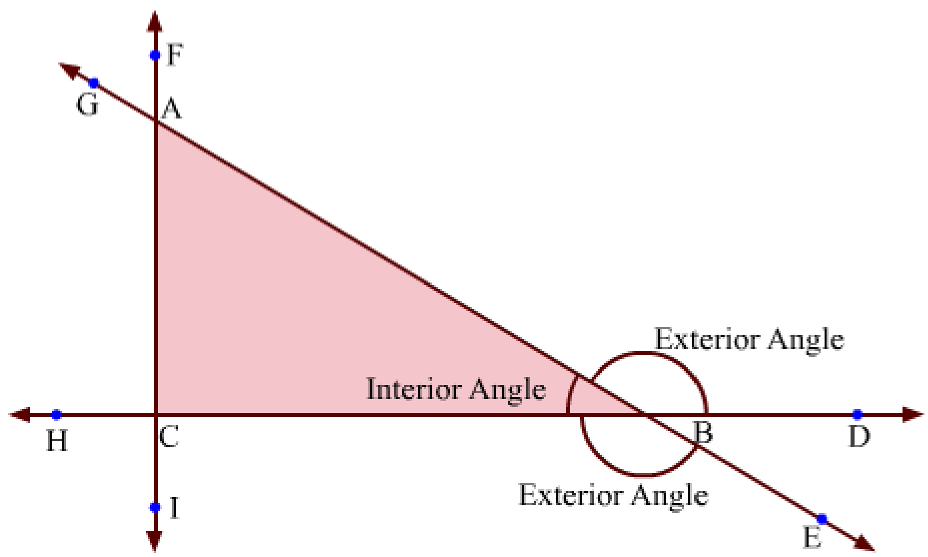
An angle forming linear pair with an interior angle of a triangle is known as exterior angle of that triangle.

Now, look at the following figure.



Here, two exterior angles such as ∠ABD and ∠CBE are formed at the vertex B.

Similarly, two exterior angles can be formed at each vertex of a triangle.



It can be seen that ∠BAF and ∠CAG are exterior angles formed at vertex A whereas, ∠ACH and ∠BCI are exterior angles formed at vertex C.

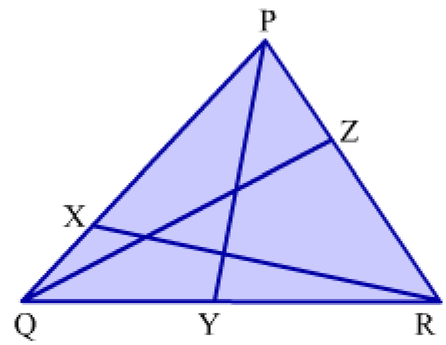
Thus, a triangle has six exterior angles.

In the above figure, it can be seen that there are three more angles such as ∠FAG, ∠DBE and ∠ICH which are vertically opposite angles of interior angles ∠BAC, ∠ABC and ∠ACB respectively. These angles are neither interior nor exterior angles of **Δ**ABC.

Cevians of a triangle:

A line segment joining a vertex of the triangle to any point on the opposite side (or its extension) is known as a cevian of that triangle.

Observe the give figure.



Here, line segments PY, QZ and RX all are cevians.

Note: Infinitely many cevians can be drawn from each vertex of a triangle. In other words, a triangle can have infinitely many cevians.

In a triangle there are few cevians such as **medians, altitudes** and **angle bisectors** are very special as these exhibit interesting properties.

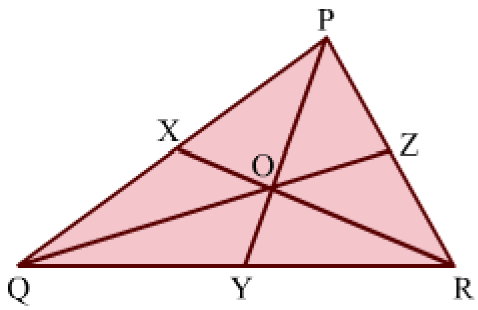
We know that all three medians are concurrent and the point of concurrence is known as centroid which divides each median in the ratio 2 : 1. Thus, centroid trisect each median. Also, centroid is the centre of gravity of any triangular lamina (cut out).

Also, all three altitudes are also concurrent and point of concurrence is known as orthocentre.

Similarly, all three angle bisectors are concurrent and the point of concurrence is known as incentre.

Ceva's Theorem:

A great Italian mathematician **Giovanni Ceva (Dec. 7, 1647 - June 15, 1734)** derived a very interesting property of cevians which is known as Ceva's theorem.



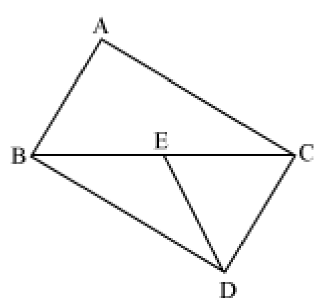
According to the theorem,
In ΔPQR, any three points X, Y and Z on the sides PQ, QR and RP respectively, the line segments PY, QZ and RX will be concurrent, if and only if

$$\frac{PX}{XQ} \cdot \frac{QY}{YR} \cdot \frac{RZ}{ZP} = 1$$

Let us discuss the examples based on these concepts of a triangle.

Example 1:

With respect to the given figure, answer the following questions.



(a) How many triangles are there in the figure?

(b) Write the names of any nine angles.

(c) Write the names of all the line segments.

(d) Which two triangles have DBE common?

(e) Which two triangles have line segment ED as the common side?

(f) Which two triangles have line segment BC as the common side?

Solution:

(a) There are four triangles, namely, ΔABC, ΔBCD, ΔBED, and ΔCED in the given figure.

(b) ∠BAC, ∠ABC, ∠ACB, ∠DBE, ∠BED, ∠BDE, ∠CED, ∠EDC, and ∠DCE are 9 angles of the given figure.

(c) The line segments are $\overline{AB}$, $\overline{BC}$, $\overline{CA}$, $\overline{BD}$, $\overline{DC}$, $\overline{BE}$, $\overline{ED}$, and $\overline{EC}$.

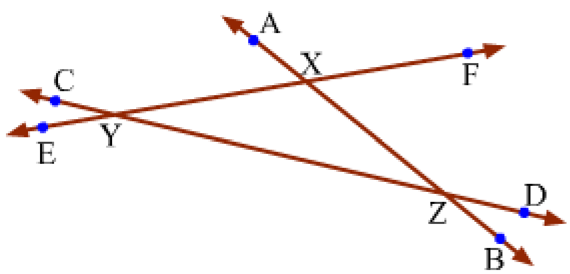
(d) ΔBCD and ΔBED have ∠DBE as common angle.

(e) ΔBED and ΔCED have line segment ED as the common side.

(f) ΔABC and ΔBCD have line segment BC as the common side.

Example 2:

With respect to the given figure, answer the following questions.



(a) Name all the exterior angles of ΔXYZ along with their respective vertices.

(b) Name all the angles which are neither exterior nor interior angles of ΔXYZ.

(c) Name the angles forming linear pair with ∠YXZ.

(d) Name the angle of ΔXYZ forming linear pair with ∠YZB.

Solution:

(a) The exterior angles of ΔXYZ along with their respective vertices are as follows:

∠YXA and ∠ZXF are formed at vertex X.

∠XYC and ∠ZYE are formed at vertex Y.

∠YZB and ∠XZD are formed at vertex Z.

(b) ∠AXF, ∠DZB and ∠EYC are neither exterior nor interior angles of ΔXYZ.

(c) The angles forming linear pair with ∠YXZ are ∠YXA and ∠ZXF.

(d) ∠YZX forms linear pair with ∠YZB.

Quadrilaterals and Their Attributes

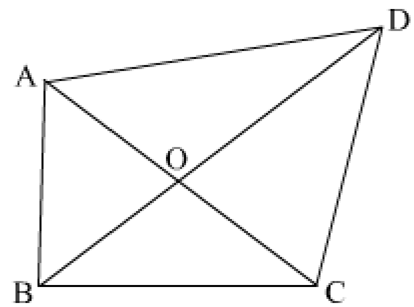
Let us look at the following figures.

These are the figures of a kite, window, door, and a painting. If we notice these figures carefully, then we will observe that each of these figures is a polygon of four sides. Such polygons are known as **quadrilaterals.** We can define it as follows.

A polygon formed by four line segments is called a quadrilateral i.e., a quadrilateral is a four-sided polygon.

Note: Adjacent sides and adjacent angles are also known as consecutive sides and consecutive angles respectively.

Now, look at the following quadrilateral.



Let us name this quadrilateral as ABCD. Symbolically, it can be represented as □ ABCD.

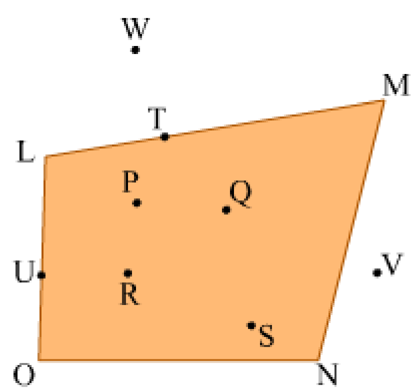
Now, it can be seen that vertices A and C are opposite to each other, so these make a pair of **opposite vertices**. Similarly, B and D also make a pair of opposite vertices.

If we join opposite vertices A and C as well as B and D, we get two line segments such as AC and BD. These are the **diagonals** of □ ABCD. Also, O is the point of intersection of the diagonals.

We can define the diagonals of a quadrilateral as follows:

Line segments obtained after joining the opposite vertices of a quadrilateral are its diagonals.

Now, observe □ LMNO.



The whole coloured part is the **interior** of □ LMNO which means that points P, Q, R and S lie in the interior of the quadrilateral.

Points, T and U lie on the **boundary** of □ LMNO.

The interior and the boundary together form the **region** of the quadrilateral.

The part outside the region of the quadrilateral is **exterior** of the quadrilateral.

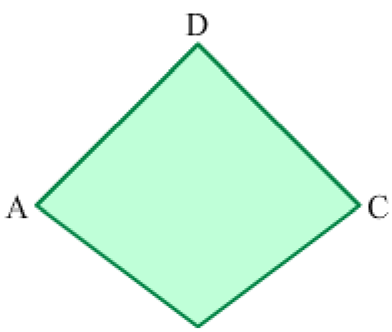
From the figure, it can be seen that the points V and W lie in the exterior of □ LMNO.

On the basis of the internal angles of a quadrilateral, it can be classified into two types: convex and concave quadrilaterals.

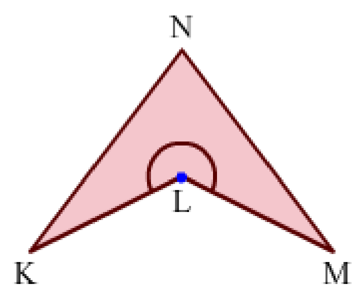
A quadrilateral is said to be **convex**, if each of its internal angles are of the measure less than 180º.

Otherwise, it is a **concave** quadrilateral.

Examples of convex and concave quadrilaterals are shown below:



Convex Quadrilateral



Concave Quadrilateral

It can be seen that each internal angle of quad. ABCD is less than 180º, whereas the marked angle of quad. KLMN is more than 180º.

Thus, quad. ABCD is a convex quadrilateral and quad. KLMN is a concave quadrilateral.

Let us discuss some examples based on these concepts of a quadrilateral.

Example 1:

Draw a rough sketch of a quadrilateral and name it. Also, in the quadrilateral, identify the following.

(i) Sides

(ii) Angles

(iii) Opposite sides

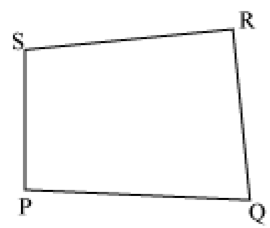
(iv) Adjacent sides

(v) Opposite angles

(vi) Adjacent angles

Solution:

Let us draw a quadrilateral PQRS as



(i) The four sides are $\overline{PQ}$, $\overline{QR}$, $\overline{RS}$, and $\overline{SP}$.

(ii) The four angles are ∠SPQ, ∠PQR, ∠QRS, and ∠RSP. We can also simply say that the four angles are ∠P, ∠Q, ∠R, and ∠S.

(iii) The opposite sides are $\overline{PQ}$ and $\overline{RS}$, $\overline{QR}$ and $\overline{SP}$.

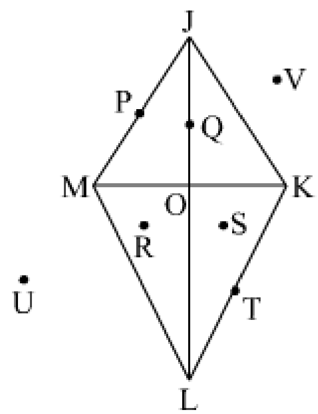
(iv) The adjacent sides are $\overline{PQ}$ and $\overline{QR}$, $\overline{QR}$ and $\overline{RS}$, $\overline{RS}$ and $\overline{SP}$, $\overline{SP}$ and $\overline{PQ}$.

(v) The opposite angles are ∠PQR and ∠RSP, ∠SPQ and ∠QRS.

(vi) The adjacent angles are ∠PQR and ∠QRS, ∠QRS and ∠RSP, ∠RSP and ∠SPQ, ∠SPQ and ∠PQR.

Example 2:

Observe the following figure.



Identify the following from the figure.

(a) Pair of opposite vertices

(b) Diagonals of the quadrilateral

(c) Points lying in the interior of the quadrilateral

(d) Points lying in the exterior of the quadrilateral

(e) Points lying on the boundary of the quadrilateral

Solution:

(a) Pair of opposite vertices are J, L and K, M.

(b) Diagonals of the quadrilateral are JL and KM

(s) Points lying in the interior of the quadrilateral are Q, R, S and O.

(d) Points lying in the exterior of the quadrilateral are V and U.

(e) Points lying on the boundary of the quadrilateral are P and T.

Circle and Its Attributes

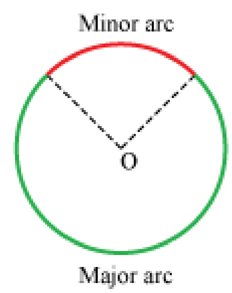
Circle is a simple closed curve. Is it a polygon?? If not, then does it exhibit any special properties?

A circle exhibits various interesting properties which make it a special geometric figure.

Let us discuss the same.

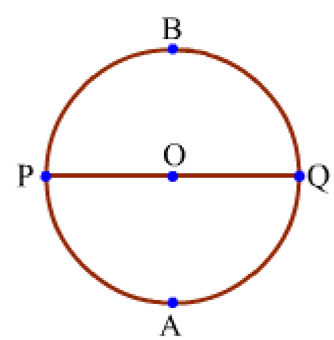
Minor and major arc:

An arc less than one-half of the entire arc of a circle is called the minor arc of the circle, while an arc greater than one-half of the entire arc of a circle is called the major arc of the circle.



Semicircular arc:

Diameter of a circle divides it into two congruent arcs. Each of these arcs is known as semicircular arc.



In the above figure, PQ is diameter which formed semicircular arcs PBQ and PAQ.

Finding radius of a circle when its diameter is given:
We know that the radius of a circle is half of its diameter.

Let r be the radius and d be the diameter of a circle, then we have $r = \frac{d}{2}$.
Using this formula, we can find the radius of the circle if its diameter is given.

Let us take a look at some examples.

We have to find the radius of the circle when diameter is given.

(i) d = 12 cm

$r = \frac{d}{2}$

$\Rightarrow r = \frac{12}{2}$

$\Rightarrow r = 6$ cm

(ii) d = 25 cm

$r = \frac{d}{2}$

$\Rightarrow r = \frac{25}{2}$

$\Rightarrow r = 12.5$ cm

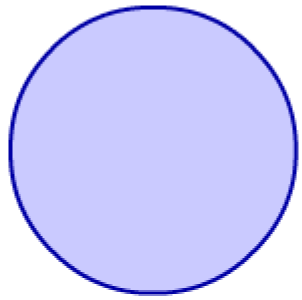
Finding diameter of a circle when its radius is given:
We know that the diameter of a circle is twice its radius.
∴ $d = 2r$
Using this formula, we can find the diameter of the circle when its radius is given.

Let us take a look at some examples.
We have to find the radius of the circle when diameter is given.
(i) r = 15.5 cm
$d = 2r$
$d = 2 \times 15.5$
d = 31 cm

(ii) r = 13 cm
$d = 2r$
$d = 2 \times 13$
d = 26 cm

Let us discuss some more concepts related to circles.

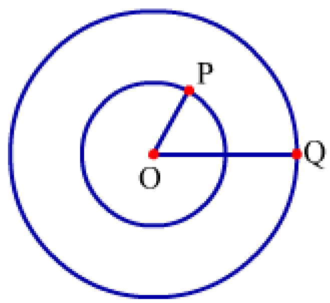
Circular region: Look at the following circle.



The whole shaded part is the region of this circle.

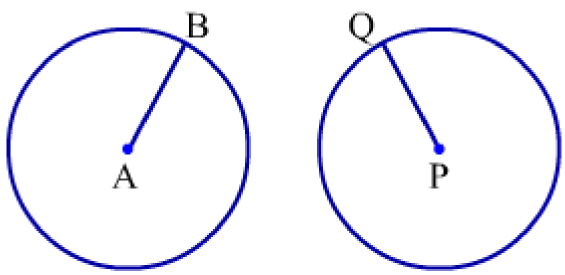
Thus, the interior and boundary together make the region of the circle.

Concentric circles: Circles of different radii but having the same centre are known as concentric circles.



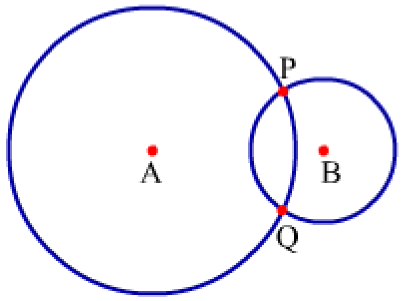
In the above figure, two circles have the same centre O but the different radii OP and OQ such that OQ > OP. These circles are concentric circles.

Congruent circles: If the radii of two or more circles are equal, then the circles are said to be congruent to each other.



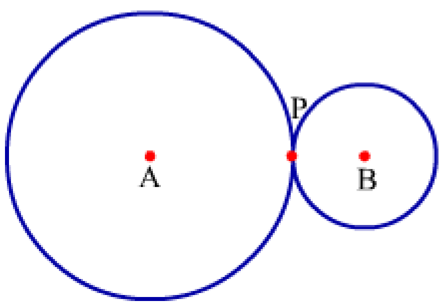
In the above figure, AB and PQ are the radii of the circles such that AB = PQ. Thus, these circles are congruent to each other.

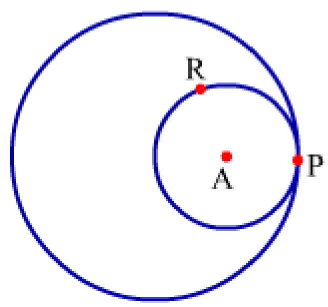
Intersecting circles: Two coplanar circles (circles in the same plane) which intersect each other at two distinct points are known as intersecting circles.



In the above figure, circles with centres A and B intersect each other at two distinct points P and Q. Thus, these are intersecting circles.

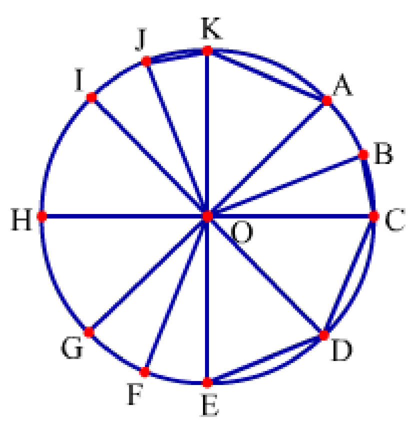
If two coplanar circles intersect each other at only one point, then the circles are known as touching circles.





In each of both the above figure, circles touch each other at only one point P. Thus, circles in each figure are touching circles.

Now, observe the following figure.



Here, OA, OB, OC, ..., OK are all radii of the circle. Similarly, we can draw many more radii of this circle.

So, it can be said that **a circle has innumerable radii.**

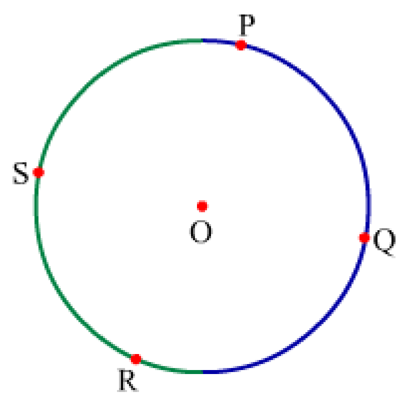
It can be seen that AG, CH, DI and EK all are diameters of the circle. Similarly, we can draw many more diameters of this circle.

So, it can be said that **a circle has innumerable diameters.**

Also, BC, CD, DE, JK and KA are the chords of the circle. Similarly, many more chords of this circle can be drawn.

Thus, it can be said that **a circle has innumerable chords.**

Now, observe the following circle.



It can be seen that points P and R divide this circle into two parts or arcs which are coloured differently. The name "arc PR" does not explain that which of two arcs we are talking about. So, we marked a point on each arc to clarify this. It can be seen that point S is marked on the green arc and point Q is marked on the blue arc. Now, we can give a three letters name to each arc. Thus, green arc can be named as arc PSR or arc RSP whereas blue arc can be named as arc PQR or arc RQP.

Similarly, we can denote any arc by three letters.

Let us discuss some examples to understand this concept better.

Example 1:

With respect to the figure drawn below, name

(a) the centre

(b) the diameter

(c) any two radii

(d) a chord

(e) a point lying in the interior of the circle

(f) a point lying in the exterior of the circle

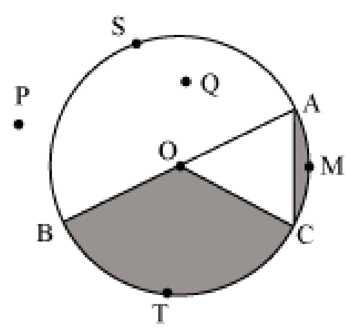
(g) a sector

(h) a segment

(i) a point lying on the circle

(j) two semi-circles

(k) any two arcs



Solution:

(a) O is the centre of the circle.

(b) $\overline{AB}$ is the diameter of the circle.

(c) Two radii of the circle are $\overline{OB}$ and $\overline{OC}$.

(d) $\overline{AC}$ is a chord of the circle.

(e) Q is a point that lies in the interior of the circle.

(f) P is a point that lies in the exterior of the circle.

(g) BOC is a sector of the circle.

(h) AMC is a segment of the circle.

(i) S is a point that lies on the boundary of the circle (or simply, on the circle).

(j) The semi-circles in the given figure are ASB and ATB.

(k) BTC and AMC are two arcs of the circle

Example 2:

Using ruler and compass, draw circle of radius 5 cm. Mark its centre and draw the radius.

Solution:

On using a ruler, first we draw the radius 5 cm of the circle and then assuming O as a centre we draw a circle of radius 5 cm by using a compass. Thus, we get a circle of radius 5 cm as shown below.