Chapter – 6

Two Dimensional Analytical Geometry

Ex 6.1

Question 1.
Find the locus of P, if for all values of a, the co-ordinates of a moving point P is
(i) (9 cos α, 9 sin α)
(ii) (9 cos α, 6 sin α)

Solution:
(i) Let P(h, k) be the moving point.
We are given h = 9 cos α and k = 9 sin α and

$$\Rightarrow \cos\alpha = \frac{h}{9} \text{ and } \sin\alpha = \frac{k}{9}$$

We know $\cos^2\alpha + \sin^2\alpha = 1 \Rightarrow \left(\frac{h}{9}\right)^2 + \left(\frac{k}{9}\right)^2 = 1$

(i.e.) $\frac{h^2}{81} + \frac{k^2}{81} = 1 \Rightarrow h^2 + k^2 = 81$

∴ locus of the point is $x^2 + y^2 = 81$

(ii) Let P(h , k) be a moving point.
We are given h = 9 cos α and k = 6 sin α

$$\Rightarrow \cos\alpha = \frac{h}{9} \text{ and } \sin\alpha = \frac{k}{6}$$

$$\cos^2\alpha + \sin^2\alpha = 1 \Rightarrow \left(\frac{h}{9}\right)^2 + \left(\frac{k}{6}\right)^2 = 1$$

$$\therefore \frac{h^2}{81} + \frac{k^2}{36} = 1$$

Locus of the point is $\frac{x^2}{81} + \frac{y^2}{36} = 1$

Question 2.
Find the locus of a point P that moves at a constant distance of
(i) Two units from the x-axis
(ii) Three units from the y-axis.

Solution:
(i) Let the point (x, y) be the moving point.
The equation of a line at a distance of 2 units from the x-axis is k = 2
So the locus is y = 2 (i.e.) y – 2 = 0

(ii) Equation of a line at a distance of 3 units from y-axis is h = 3
So the locus is x = 3 (i.e.) x – 3 = 0

Question 3.
If θ is a parameter, find the equation of the locus of a moving point, whose coordinates are $x = a\cos^3\theta$, $y = a\sin^3\theta$

Solution:

$$x = a\cos^3\theta \Rightarrow \cos^3\theta = \frac{x}{a} \Rightarrow \cos\theta = \left(\frac{x}{a}\right)^{1/3}$$

$$y = a\sin^3\theta \Rightarrow \sin^3\theta = \frac{y}{a}$$

$$\Rightarrow \sin\theta = \left(\frac{y}{a}\right)^{1/3}$$

$$\text{But } \cos^2\theta + \sin^2\theta = 1 \Rightarrow \left\{\left(\frac{x}{a}\right)^{1/3}\right\}^2 + \left\{\left(\frac{y}{a}\right)^{1/3}\right\}^2 = 1$$

$$\text{(i.e) } \frac{x^{2/3}}{a^{2/3}} + \frac{y^{2/3}}{a^{2/3}} = 1 \Rightarrow x^{2/3} + y^{2/3} = a^{2/3}$$

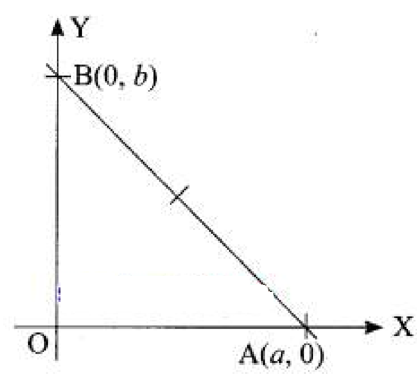
Question 4.
Find the value of k and b, if the points P (-3, 1) and Q (2, b) lie on the locus of $x^2 - 5x + ky = 0$.

Solution:
Given P (-3, 1) lies on the locus of
$x^2 - 5x + ky = 0$
$\therefore (-3)^2 - 5(-3) + k(1) = 0$
$9 + 15 + k = 0$
$\Rightarrow k = -24$
Also given Q(2 , b) lies on the locus of
$x^2 - 5x + ky = 0$

$x^2 - 5x - 24y = 0$
$\therefore (2)^2 - 5(2) - 24(b) = 0$
$4 - 10 - 24b = 0 \Rightarrow -6 - 24b = 0$
$\Rightarrow 24b = -6 \Rightarrow b = -\frac{6}{24} = -\frac{1}{4}$
Thus $k = -24$, $b = -\frac{1}{4}$

Question 5.
A straight rod of length 8 units slides with its ends A and B always on the x and y-axis respectively. Find the locus of the midpoint of the line segment AB.

Solution:
Let P (h, k) be the moving point A (a, 0) and B (0, b) P is the midpoint of AB.
$\Rightarrow P = \left(\frac{a}{2}, \frac{b}{2}\right)$
Given $\frac{a}{2} = h$ *and* $\frac{b}{2} = k \Rightarrow a = 2h$ and $b = 2k$
OAB is a right angled triangle $\Rightarrow OA^2 + OB^2 = AB^2$
(i.e.) $a^2 + b^2 = 8^2$ [$\because AB = 8$]
$\Rightarrow (2h)^2 + (2k)^2 = 8^2$ (i.e) $4h^2 + 4k^2 = 64$
($\div$ by 4) $h^2 + k^2 = 16$
So the locus of P is $x^2 + y^2 = 16$



Question 6.
Find the equation of the locus of a point such that the sum of the squares of the distance from the points (3, 5), (1, -1) is equal to 20.

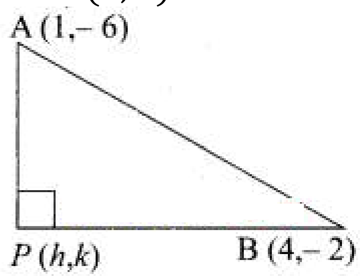
Solution:
Let the given points be A (3, 5) and (1, -1).
Let P (h, k) be the point such that
$PA^2 + PB^2 = 20$ ………….. (1)
$PA^2 = (3 - h)^2 + (5 - k)^2$
$PB^2 = (1 - h)^2 + (-1 - k)^2$
(1) $\Rightarrow (3 - h)^2 + (5 - k)^2 + (1 - h)^2 + (1 + k)^2 = 20$
$9 - 6h + h^2 + 25 - 10k + k^2 + 1 - 2h + h^2 + 1 + 2k + k^2 = 20$
$2h^2 + 2k^2 - 8h - 8k + 36 = 20$
$2h^2 + 2k^2 - 8h - 8k + 16 = 0$

$h^2 + k^2 - 4h - 4k + 8 = 0$
The locus of P (h , k) is obtained by replacing h by x and k by y
∴ The required locus is $x^2 + y^2 - 4x - 4y + 8 = 0$

Question 7.
Find the equation of the locus of the point P such that the line segment AB, joining the points A (1, -6) and B (4, -2), subtends a right angle at P.

Solution:

Let P (h, k) be the moving point



Given A (1, – 6) and B (4, – 2),
Since $\Delta APB = 90°$, $PA^2 + PB^2 = AB^2$
(i.e.) $(h - 1)^2 + (k + 6)^2 + (h - 4)^2 + (k + 2)^2 = (4 - 1)^2 + (-2 + 6)^2$
(i.e) $h^2 + 1 - 2h + k^2 + 36 + 12k + h^2 + 16 - 8h + k^2 + 4 + 4k = 3^2 + 4^2 = 25$
$2h^2 + 2k^2 -10h + 16k + 57 - 25 = 0$
$2h^2 + 2k^2 - 10h + 16k + 32 = 0$
(÷ by 2)$h^2 + k^2 - 5h + 8k + 16 = 0$
So the locus of P is $x^2 + y^2 - 5x + 8y + 16 = 0$

Question 8.
If O is origin and R is a variable point on $y^2 = 4x$, then find the equation of the locus of the mid-point of the line segment OR.

Solution:
Let P(h, k) be the moving point
We are given O (0, 0). Let R = (a, b)

$$\text{Mid point of OR} = \left(\frac{a}{2}, \frac{b}{2}\right) \Rightarrow \left(\frac{a}{2}, \frac{b}{2}\right) = (h, k)$$
$$\Rightarrow a = 2h, b = 2k$$

Substituting a, b values is $y^2 = 4x$
we get $(2k)^2 = 4\ (2h)$

(i.e) $4k^2 = 8h$
($\div$ by 4) $k^2 = 2h$
So the locus of P is $y^2 = 2x$

Question 9.

The coordinates of a moving point P are $\left(\frac{a}{2}(\text{cosec}\,\theta + \sin\theta), \frac{b}{2}(\text{cosec}\,\theta - \sin\theta)\right)$ where θ is a variable parameter. Show that the equation of the locus P is $b^2x^2 - a^2y^2 = a^2b^2$.

Solution:

Let P (h, k) be the moving point

We are given $P = \left[\frac{a}{2}(\text{cosec}\,\theta + \sin\theta), \frac{b}{2}(\text{cosec}\,\theta - \sin\theta)\right]$

$\Rightarrow h = \frac{a}{2}(\text{cosec}\,\theta + \sin\theta)\,(i.e.)\,a[\text{cosec}\,\theta + \sin\theta] = 2h$

$\Rightarrow \text{cosec}\,\theta + \sin\theta = \frac{2h}{a}$ .... (1)

$\frac{b}{2}(\text{cosec}\,\theta - \sin\theta) = k \Rightarrow \text{cosec}\,\theta - \sin\theta = \frac{2k}{b}$ .... (2)

(1) + (2) $\Rightarrow 2\text{cosec}\,\theta = \frac{2h}{a} + \frac{2k}{b}$

($\div$ by 2) $\Rightarrow \text{cosec}\,\theta = \frac{h}{a} + \frac{k}{b}$ .... (3)

(1) – (2) $\Rightarrow 2\sin\theta = \frac{2h}{a} - \frac{2k}{b}\,(\div\, by\ 2) \Rightarrow \sin\theta = \frac{h}{a} - \frac{k}{b}$ .... (4)

But cosec θ sin θ = 1, So from (3) & (4) $\Rightarrow \left(\frac{h}{a} + \frac{k}{b}\right)\left(\frac{h}{a} - \frac{k}{b}\right) = 1$

$\Rightarrow \frac{h^2}{a^2} - \frac{k^2}{b^2} = 1$

So the locus of (h, k) is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

(i.e) $\frac{b^2x^2 - a^2y^2}{a^2b^2} = 1 \Rightarrow b^2x^2 - a^2y^2 = a^2b^2$

Question 10.

If P (2, -7) is a given point and Q is a point on $2x^2 + 9y^2 = 18$, then find the equations of the locus of the mid-point of PQ.

Solution:

P = (2, -7); Let (h, k) be the moving point Q = (a, b)

$$\text{Mid point of } PQ = \left(\frac{2+a}{2}, \frac{-7+b}{2}\right) = (h, k) \text{ say}$$

$$\frac{a+2}{2} = h, \quad \frac{b-7}{2} = k$$

⇒ a = 2h – 2,
b = 2k + l
Q is a point on $2x^2 + 9y^2 = 18$ (i.e) (a, b) is on $2x^2 + 9y^2 = 18$
⇒ $2(2h – 2)^2 + 9 (2k + 7)^2 = 18$
(i.e) $2 [4h^2 + 4 – 8h] + 9 [4k^2 + 49 + 28k] – 18 = 0$
(i.e) $8h^2 + 8 – 16h + 36k^2 + 441 + 252k – 18 = 0$
$8h^2 + 36k^2 – 16h + 252k + 431 = 0$
The locus is $8x^2 + 36y^2 – 16x + 252y + 431 = 0$

Question 11.
If R is any point on the x-axis and Q is any point on the y-axis and Pis a variable point on RQ with RP = b, PQ = a. then find the equation of locus of P.

Solution:

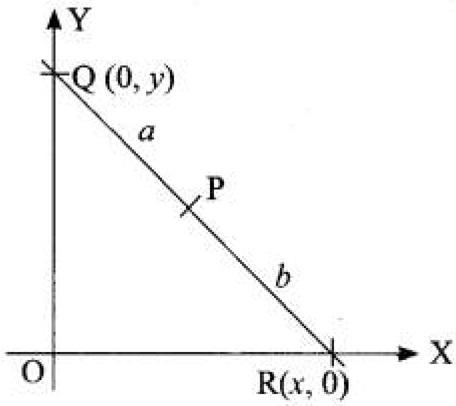
P = (x, 0), Q = (0, y), R (h, k) be a point on RQ such that PR : RQ = b : a

$$\therefore \ P = \left(\frac{ax}{a+b}, \frac{by}{a+b}\right) \Rightarrow \left(\frac{ax}{a+b}, \frac{by}{a+b}\right) = (h, k)$$

$$\Rightarrow ax = (a+b)h \Rightarrow x = \frac{a+b}{a}h$$

$$by = (a+b)\,k \Rightarrow y = \frac{a+b}{b}k$$

From the right-angled triangle OQR, $OR^2 + OQ^2 = QR^2$



(i.e) $X^2 + Y^2 = (a+b)^2$

$$\Rightarrow \left[\frac{a+b}{a}h\right]^2 + \left[\frac{a+b}{b}k\right]^2 = (a+b)^2$$

$$\Rightarrow \frac{(a+b)^2}{a^2}h^2 + \frac{(a+b)^2}{b^2}k^2 = (a+b)^2$$

$$\div by\ (a+b)^2, \frac{h^2}{a^2} + \frac{k^2}{b^2} = 1$$

Locus is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Question 12.

If the points P (6, 2) and Q (-2, 1) and R are the vertices of a ΔPQR and R is the point on the locus $y = x^2 - 3x + 4$, then find the equation of the locus of the centroid of ΔPQR.

Solution:

P (6, 2), Q (-2, 1). Let R = (a, b) be a point on $y = x^2 - 3x + 4$.

Centroid of ΔPQR is $\left(\frac{6-2+a}{3}, \frac{2+1+b}{3}\right)$

(i.e) $\left(\frac{4+a}{3}, \frac{3+b}{3}\right) = (h, k)$

$$\frac{4+a}{3} = h \Rightarrow a = 3h - 4$$

$$\frac{3+b}{3} = k \Rightarrow b = 3k - 3$$

But (a, b) is a point on $y = x^2 - 3x + 4$

$b = a^2 - 3a + 4$
(i.e) $3k - 3 = (3h - 4)^2 - 3(3h - 4) + 4$
(i.e) $3k - 3 = 9h^2 + 16 - 24h - 9h + 12 + 4$
$\Rightarrow 9h^2 - 24h - 9h + 32 - 3k + 3 = 0$
(i.e) $9h^2 - 33h - 3k + 35 = 0$,
Locus of (h, k) is $9x^2 - 33x - 3y + 35 = 0$

Question 13.
If Q is a point on the locus of $x^2 + y^2 + 4x - 3y + 7 = 0$ then find the equation of locus of P which divides segment OQ externally in the ratio 3 : 4, where O is origin.

Solution:
Let (h, k) be the moving point O = (0, 0);
Let PQ = (a, b) on $x^2 + y^2 + 4x - 3y + 7 = 0$

P divides OQ externally in the ratio 3:4 $\Rightarrow P = \left(\frac{3(a)-0}{-1}, \frac{3(b)-0}{-1}\right) = (-3a, -3b) = (h, k)$

$a = -\frac{h}{3}$ and $b = -\frac{k}{3}$

But (a, b) is on $x^2 + y^2 + 4x - 3y + 7 = 0$

$$\Rightarrow \left(-\frac{h}{3}\right)^2 + \left(-\frac{k}{3}\right)^2 + 4\left(-\frac{h}{3}\right) - 3\left(-\frac{k}{3}\right) + 7 = 0$$

(i.e) $\frac{h^2}{9} + \frac{k^2}{9} - \frac{4h}{3} + k + 7 = 0$

$h^2 + k^2 - 12h + 9k + 63 = 0$, Locus of (h, k) is $x^2 + y^2 - 12x + 9y + 63 = 0$

Question 14.
Find the points on the locus of points that are 3 units from the x-axis and 5 units from the point (5, 1).

Solution:
A line parallel to the x-axis is of the form y = k.
Here $k = 3 \Rightarrow y = 3$
A point on this line is taken as P (a, 3).
The distance of P (a, 3) from (5, 1) is given as 5 units
$\Rightarrow (a - 5)^2 + (3 - 1)^2 = 5^2$
$a^2 + 25 - 10a + 9 + 1 - 6 = 25$

$a^2 - 10a + 25 + 4 - 25 = 0$

$a^2 - 10a + 4 = 0$

$$a = \frac{10 \pm \sqrt{100 - 4(1)(4)}}{2(1)} = \frac{10 \pm \sqrt{84}}{2} = \frac{10 \pm 2\sqrt{21}}{2} = 5 \pm \sqrt{21}$$

So the points are $\left(5+\sqrt{21}, 3\right), \left(5-\sqrt{21}, 3\right)$.

Question 15.

The sum of the distance of a moving point from the points (4, 0) and (-4, 0) is always 10 units. Find the equation of the locus of the moving point.

Solution:

Let P (h, k) be a moving point

Here A = (4, 0) and B = (-4, 0)

Given PA + PB = 10

$$\Rightarrow \sqrt{(h-4)^2 + k^2} + \sqrt{(h+4)^2 + k^2} = 10$$

$$\Rightarrow \sqrt{(h-4)^2 + k^2} = 10 - \sqrt{(h+4)^2 + k^2}$$

Squaring on both sides $(h-4)^2 + k^2 = 100 + (h+4)^2 + k^2 - 20\sqrt{(h+4)^2 + k^2}$

(i.e.) $h^2 + 16 - 8h + k^2 = 100 + h^2 + 16 + 8h + k^2 - 20\sqrt{(h+4)^2 + k^2}$

$$\Rightarrow -16h - 100 = -20\sqrt{(h+4)^2 + k^2}$$

$(\div \text{ by } -4)$ $4h + 25 = 5\sqrt{(h+4)^2 + k^2}$

Again Squaring on both sides we get,

$$(4h + 25)^2 = 25\left[(h+4)^2 + k^2\right]$$

(i.e) $16h^2 + 625 + 200h = 25[h^2 + 8h + 16 + k^2]$

$$\Rightarrow 16h^2 + 625 + 200h - 25h^2 - 200h - 400 - 25k^2 = 0$$

$$-9h^2 - 25k^2 + 225 = 0$$

$$\Rightarrow 9h^2 + 25k^2 = 225$$

$$\frac{9h^2}{225}+\frac{25k^2}{225}=1$$

(i.e) $h^2/_{25}+k^2/_9=1$

So the locus is $\frac{x^2}{25}+\frac{y^2}{9}=1$

Ex 6.2

Question 1.
Find the equation of the lines passing through the point (1, 1)
(i) With y-intercept (- 4)
(ii) With slope 3
(iii) And (-2, 3)
(iv) And the perpendicular from the origin makes an angle 60° with x-axis.

Solution:
(i) Given y intercept = – 4,
Let x intercept be a

Now equation of the lines is $\frac{x}{a}+\frac{y}{-4}=1$.

It passes through (1, 1) $\Rightarrow \frac{1}{a}-\frac{1}{4}=1, \frac{4-a}{4a}=1$

$\Rightarrow 4-a=4a \Rightarrow 4=4a+a$

(i.e) $5a=4 \Rightarrow a=4/5$ .... (1)

So the equation of the line is $\frac{x}{4/5}+\frac{y}{-4}=1$ (*i.e.*) $\frac{5x}{4}-\frac{y}{4}=1$

(*i.e*) $\frac{5x-y}{4}=1 \Rightarrow 5x-y=4$ (or) $y=5x-4$

(ii) with slope 3
The equation the line passing through the point (x_1, y_1) and having slope m is
$y - y_1 = m(x - x_1)$
Given $(x_1, y_1) = (1, 1)$, m = 3
∴ The required equation of the line is
y – 1 = 3(x – 1)
y – 1 = 3x – 3

3x – y – 3 + 1 = 0
3x – y – 2 = 0

(iii) Passing through (1, 1) and (-2, 3)

Equation of the line passing through 2 points is $\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$

$(i.e)\quad \frac{y-1}{3-1}=\frac{x-1}{-2-1}, (i.e.)\ \frac{y-1}{2}=\frac{x-1}{-3}$

$-3(y-1)=2(x-1) \Rightarrow -3y+3=2x-2$

$5=2x+3y$

$2x+3y=5$

(iv) P = Distance between (0, 0) and (1, 1) = $\sqrt{(0-1)^2+(0-1)^2}=\sqrt{1+1}=\sqrt{2}$, $\alpha=60°$

Equation of the line is $x\cos\alpha+y\sin\alpha=p$

$(i.e)\ x\cos 60°+y\sin 60°=\sqrt{2}$

$x\left(\frac{1}{2}\right)+y\left(\frac{\sqrt{3}}{2}\right)=\sqrt{2} \Rightarrow \frac{x+\sqrt{3}y}{2}=\sqrt{2}$

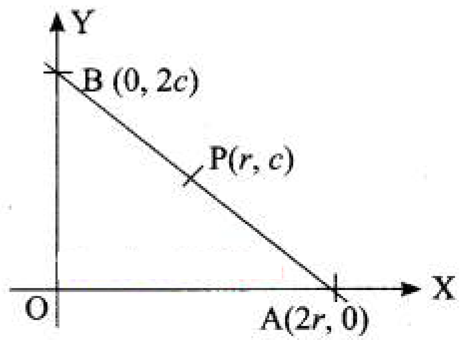
$\Rightarrow x+\sqrt{3}y=2\sqrt{2}$

Question 2.
If P (r, c) is midpoint of a line segment between the axes, then show that
$\frac{x}{r}+\frac{y}{c}=2.$

Solution:
P (r, c) is the midpoint of AB.
⇒ A = (2r, 0) and B = (0, 2c)
(i.e) x intercept = 2r and
y intercept = 2c.

Equation of the line is $\frac{x}{2r}+\frac{y}{2c}=1$

$(i.e)\ \frac{1}{2}\left(\frac{x}{r}+\frac{y}{c}\right)=1 \Rightarrow \frac{x}{r}+\frac{y}{c}=2.$



Question 3.
Find the equation of the line passing through the point (1, 5) and also divides the co-ordinate axes in the ratio 3 : 10.

Solution:
Let x-intercept be 3a and y-intercept be 10a

Equation of the line is $\frac{x}{3a}+\frac{y}{10a}=1$

The line passes through (1, 5)

$\Rightarrow \frac{1}{3a}+\frac{5}{10a}=1 \Rightarrow \frac{1}{3a}+\frac{1}{2a}=1$

$\frac{2+3}{6a}=1 \Rightarrow 6a=5 \rightarrow (1)$

So the equation of the line is $\frac{x}{3a}+\frac{y}{10a}=1 \Rightarrow \frac{10x+3y}{30a}=1$

$\Rightarrow 10x+3y=30a$

$(i.e)$ $10x+3y=5\ (6a)=5\ (5)=25$ .... [from (1)]

$\therefore\ 10x+3y=\ 25$

Question 4.
If p is length of perpendicular from origin to the line whose intercepts on the axes are a and b, then show that $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$

Solution:

$(i.e)\ \frac{bx+ay}{ab}=1 \Rightarrow bx+ay=ab$

$\Rightarrow bx+ay-ab=0 \qquad \ldots\ldots\ldots\ldots\ldots(1)$

p = The length of the perpendicular from the origin to (1)

$\Rightarrow p=\pm\frac{-ab}{\sqrt{a^2+b^2}}$

$(i.e)\ p=\frac{ab}{\sqrt{a^2+b^2}}$

Squaring on both sides $p^2=\frac{a^2b^2}{a^2+b^2}$

$\Rightarrow \frac{1}{p^2}=\frac{a^2+b^2}{a^2b^2}=\frac{1}{b^2}+\frac{1}{a^2} \Rightarrow \frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$

Question 5.

The normal boiling point of water is 100°C or 212°F and the freezing point of water is 0°C or 32°F.

(i) Find the linear relationship between C and F

(ii) Find the value of C for 98.6° F and

(iii) The value of F for 38°C.

Solution:

Given when C = 100, F = 212 and when C = 0, F = 32

$(i.e)\ (x_1, y_1)=(100, 212)$ and $(x_2, y_2)=(0, 32)$ and $(x, y)=(C, F)$

(*i*) The equation of the line passing through (x_1, y_1) and (x_2, y_2) is $\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$

$(i.e)\ \frac{F-212}{32-212}=\frac{C-100}{0-100} \Rightarrow \frac{F-212}{-180}=\frac{C-100}{-100}$

By dividing the denominators by – 20, $\frac{F-212}{9}=\frac{C-100}{5}$

$\Rightarrow \quad 5F-1060=9C-900$

$5F=9C-900+1060$

$(i.e)\ 5F=9C+160$

$$F = \frac{9C+160}{5} = \frac{9}{5}C + 32$$

Again $5F - 1060 = 9C - 900$

(*i.e*) $9C - 900 = 5F - 1060$

$$9C = 5F - 1060 + 900$$

$$9C = 5F - 160$$

$$C = \frac{5F-160}{9} = \frac{5(F-32)}{9}$$

$$C = \frac{5F-160}{9} = \frac{5(F-32)}{9}$$

$$C = \frac{5}{9}(F-32)$$

(*ii*) So $F = \frac{9}{5}C + 32$ (OR) $C = \frac{5}{9}(F-32)$

When $F = 98.6°$ to find C

$$C = \frac{5}{9}(F-32)$$

When $F = 98.6$, $c = \frac{5}{9}(98.6 - 32) = \frac{5}{9}(66.6) = 37°$

(*iii*) When $C = 38°$, To find F

$$F = \frac{9}{5}(38) + 32 = 9 \times 7.6 + 32 = 68.4 + 32$$

$$F = 100.4°$$

Question 6.

An object was launched from a place P in constant speed to hit a target. At the 15th second, it was 1400m away from the target and at the 18th second 800m away. Find

(i) The distance between the place and the target

(ii) The distance covered by it in 15 seconds,

(iii) Time is taken to hit the target.

Solution:

Taking time = x and distance = y

We are given at x = 15, y = 1400 and at x = 18, y = 800

The equation of the line passing through (x_1, y_1) and (x_2, y_2) is $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

(i.e) $\frac{y-1400}{800-1400} = \frac{x-15}{18-15}$

(i.e) $\frac{y-1400}{-600} = \frac{x-15}{3}$

(Denominator divided by 3) we get $\frac{y-1400}{-200} = x-15$

(i.e) $y - 1400 = -200x + 3000$

$\Rightarrow 200x = 1400 - y + 3000 \Rightarrow x = \frac{1400-y}{200} + \frac{3000}{200}$

(i.e) $x = \frac{1400-y}{200} + 15$

Taking x = Time and y = distance (D) we get $T = \frac{1400-\text{D}}{200} + 15$ ... (1)

(i) Let T = 0 in (1)

$0 = \frac{1400-\text{D}}{200} + 15 \Rightarrow 1400 - \text{D} + 3000 = 0 \Rightarrow \text{D} = 4400 \text{ m}$

(ii) Let T = 15 in (1)

$15 = \frac{1400-\text{D}}{200} + 15 \Rightarrow 3000 = 1400 - \text{D} + 3000$

(i.e) $1400 - \text{D} = 0 \Rightarrow d = 1400\text{m}$

So the distance covered in 15 seconds = 4400 – 1400 = 3000 m

(iii) To find T at D = 0

$\text{T} = \frac{1400-0}{200} + 15 \Rightarrow \text{T} = 7 + 15 = 22 \text{ seconds}$

Question 7.
The population of a city in the years 2005 and 2010 are 1,35,000 and 1,45,000 respectively. Find the approximate population in the year 2015. (assuming that the growth of population is constant).

Solution:
Taking the year as x and population as y
We are given when x = 2005,

y = 1,35,000 and
when x = 2010,
y = 1,45,000

(i.e) $(x_1, y_1) = (2005, 135000), (x_2, y_2) = (2010, 145000)$

The equation of the line passing through (x_1, y_1) and (x_2, y_2) is $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

(i.e) $$\frac{y-135000}{145000-135000} = \frac{x-2005}{2010-2005}$$

(i.e) $$\frac{y-135000}{10000} = \frac{x-2005}{5}$$

$$\frac{5}{10000}(y-135000) = x-2005$$

(i.e) $$\frac{1}{2000}(y-135000) = x-2005$$

y – 135000 = 2000 (x – 2005)
y = 2000(x – 2005) + 135000
At x = 2015, y = 2000 (2015 – 2005) + 135000
(i.e) y = 2000 (10) + 135000 = 20000 + 135000 = 1,55,000
The approximate population in the year 2015 is 1,55,000

Question 8.
Find the equation of the line, if the perpendicular drawn from the origin makes an angle 30° with x – axis and its length is 12.

Solution:
The equation of the line is x cos α + y sin α = p

Here $\alpha = 30°$, $\cos\alpha = \cos 30° = \frac{\sqrt{3}}{2}$; $\sin\alpha = \sin 30° = 1/2$; P = 12.

So equation of the line is $x\frac{\sqrt{3}}{2} + y\frac{1}{2} = 12$

(i.e) $\sqrt{3}x + y = 12 \times 2 = 24 \Rightarrow \sqrt{3}x + y - 24 = 0$

Question 9.
Find the equation of the straight lines passing through (8, 3) and having intercepts whose sum is 1.

Solution:
Given sum of the intercepts = 1 ⇒ when x-intercept = a then y-intercept = 1 – a

Equation of the line is $\frac{x}{a}+\frac{y}{1-a}=1$

The line passes through (8, 3) ⇒ $\frac{8}{a}+\frac{3}{1-a}=1$

8 (1 – a) + 3a = a (1 – a)
8 – 8a + 3a = a – a^2

$$a^2-6a+8=0$$

$$(a-2)(a-4)=0 \Rightarrow a=2 \text{ or } 4$$

1. When $a=2$ equation of the line is $\frac{x}{2}+\frac{y}{1-2}=1 \Rightarrow \frac{x}{2}-y=1 \Rightarrow x-2y=2$
2. When $a=4$ equation of the line is $\frac{x}{4}+\frac{y}{1-4}=1 \Rightarrow \frac{x}{4}-\frac{y}{3}=1 \Rightarrow 3x-4y=12$

Question 10.

Show that the points (1, 3), (2, 1) and $\left(\frac{1}{2},4\right)$ are collinear, by using (*i*) Concept of slope (*ii*) Using a straight line and (*iii*) Any other method.

Solution:

Let the given points be A (1, 3), B (2, 1), and C $\left(\frac{1}{2},4\right)$

(*i*) Slope of AB = $\frac{1-3}{2-1}=\frac{-2}{1}=-2=m_1$

Slope of BC = $\frac{4-1}{1/2-2}=\frac{3}{-3/2}=-2=m_2$

Slope of AB = Slope of BC ⇒ AB parallel to BC but B is a common point
⇒ The points A, B, C are collinear.

(*ii*) Equation of the line passing through A and B is $\frac{y-1}{3-1}=\frac{x-2}{1-2} \Rightarrow \frac{y-1}{2}=\frac{x-2}{-1}$
$1-y=2x-4,$
$2x+y=5$ .... (1)

Substituting $C\left(\frac{1}{2}, 4\right)$ in (1),

we get LHS = $2\left(\frac{1}{2}\right) + 4 = 1 + 4 = 5 =$ RHS

C is a point on AB

⇒ The points A, B, C lie on a line

⇒ The points A, B, C are collinear

(iii) Area of $\Delta ABC = \frac{1}{2}\left(x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right)$

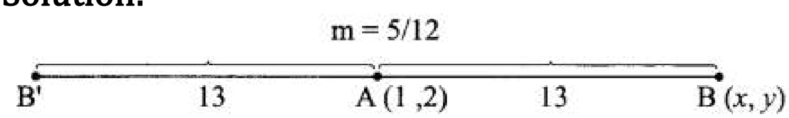
$$= \frac{1}{2}\left\{1(1-4) + 2(4-3) + \frac{1}{2}(3-1)\right\} = \frac{1}{2}(-3+2+1) = 0$$

⇒ The points A, B, C are collinear.

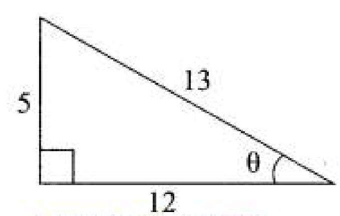
Question 11.

A straight line is passing through the point A (1, 2) with slope 5/12. Find points on the line which are 13 units away from A.

Solution:



Equation of the line in parametric form is $\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$



Here $(x_1, y_1) = (1, 2)$, $r = 13$, $m = \tan\theta = 5/12$

$\sin\theta = 5/13$, $\cos\theta = 12/13$

$\frac{x-1}{12/13} = \frac{y-2}{5/13} = \pm 13$, (i.e) $\frac{13(x-1)}{12} = \frac{13(y-2)}{5} = \pm 13$

(÷ by 13) $\frac{x-1}{12} = \frac{y-2}{5} = \pm 1$

$\frac{x-1}{12} = 1 \Rightarrow x = 12 + 1 = 13$	$\frac{x-1}{12} = -1 \Rightarrow x = -12 + 1 = -11$
$\frac{y-2}{5} = +1 \Rightarrow y = 5 + 2 = 7$	$\frac{y-2}{5} = -1 \Rightarrow y = -5 + 2 = -3$

So the points are (13, 7) or (–11, –3)

Question 12.
A 150m long train is moving with a constant velocity of 12.5 m/s. Find
(i) The equation of the motion of the train,
(ii) Time taken to cross a pole,
(iii) The time taken to cross the bridge of length 850 m is?

Solution:
(i) Now m = y/x = 12.5m / second,

The equation of the line is y = mx + c ....(1)
Put c = -150, m = 12.5 m,
The equation of motion of the train is y = 12.5x – 150

(ii) To find the time taken to cross a pole we take y = 0 in (1)
⇒ 0 = 12.5x – 150 ⇒ 12.5x = 150

$$x = \frac{150}{12.5} = 12 \text{ seconds}$$

(iii) When y = 850 in (1)
850 = 12.5 x – 150 ⇒ 12.5x = 850 + 150 = 1000

$$\Rightarrow x = \frac{1000}{12.5} = 80 \text{ seconds}$$

Question 13.
A spring was hung from a hook in the ceiling. A number of different weights were attached to the spring to make it stretch, and the total length of the spring was measured each time shown in the following table.

Weight (kg)	2	4	5	8
Length (cm)	3	4	4.5	6

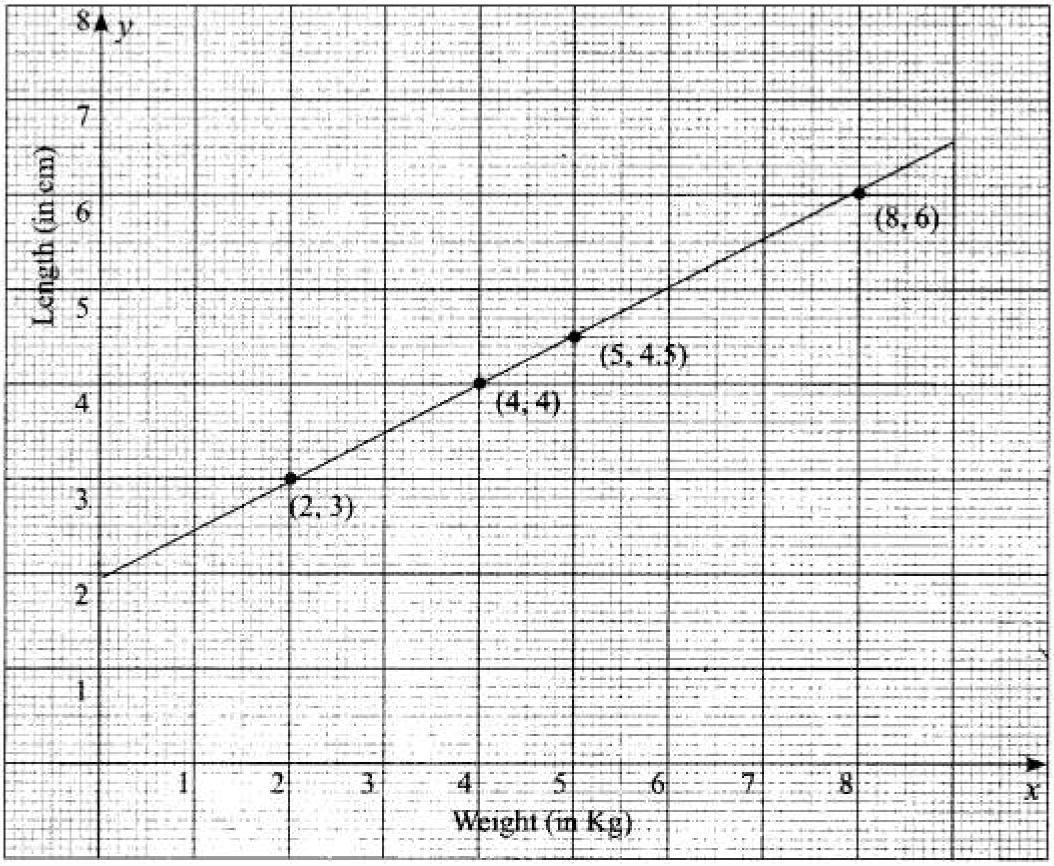
(i) Draw a graph showing the results.
(ii) Find the equation relating the length of the spring to the weight on it.
(iii) What is the actual length of the spring.
(iv) If the spring has to stretch to 9 cm long, how much weight should be added?

(v) How long will the spring be when 6 kilograms of weight on it?

Solution:

Taking weight (kg) as x values and length (cm) as y values we get $(x_1, y_1) = (2, 3)$, $(x_2, y_2) = (4, 4)$

(i)



The equation of the line passing through the above two points is

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1} \quad (i.e) \frac{y-3}{4-3} = \frac{x-2}{4-2}$$

(ii) $\frac{y-3}{1} = \frac{x-2}{2} \Rightarrow 2y - 6 = x - 2$

i.e. $x - 2y = -4 \Rightarrow x - 2y + 4 = 0$

(iii) When $x = 0$, $2y = 4 \Rightarrow y = 2$ cm

(iv) When $y = 9$ cm, $x - 18 = -4$

$x = -4 + 18 = 14$ kg

(v) When x = 6 (kg), 6 – 2y = – 4, -2y = -4 – 6 = -10
⇒ 2y = 10 ⇒ y = 10/2 = 5 cm.

Question 14.
A family is using Liquefied petroleum gas (LPG) of weight 14.2 kg for consumption. (Full weight 29.5 kg includes the empty cylinders tare weight of 15.3 kg.). If it is used at a constant rate then it lasts for 24 days. Then the new cylinder is replaced
(i) Find the equation relating the quantity of gas in the cylinder to the days.
(ii) Draw the graph for the first 96 days.

Solution:
(*i*) Let x represent the number of days.

y represent the weight of gas.

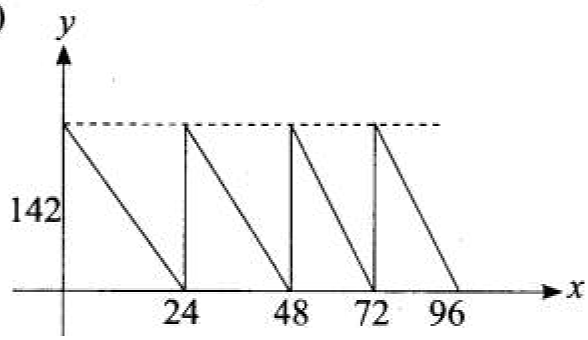
x	0	24
y	14.2	0

$$\therefore \text{ Equation is } \frac{x}{24}+\frac{y}{14.2}=1 \Rightarrow \frac{y}{14.2}=1-\frac{x}{24}$$

$$y=\frac{-14.2x}{24}+14.2$$

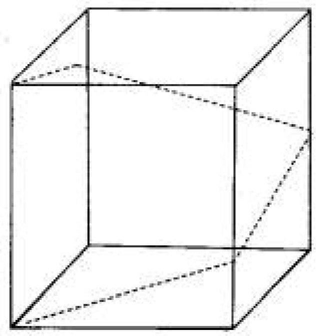
$$y=-\frac{71x}{120}+14.2\,,\ 0\le x\le 24$$

(*ii*)

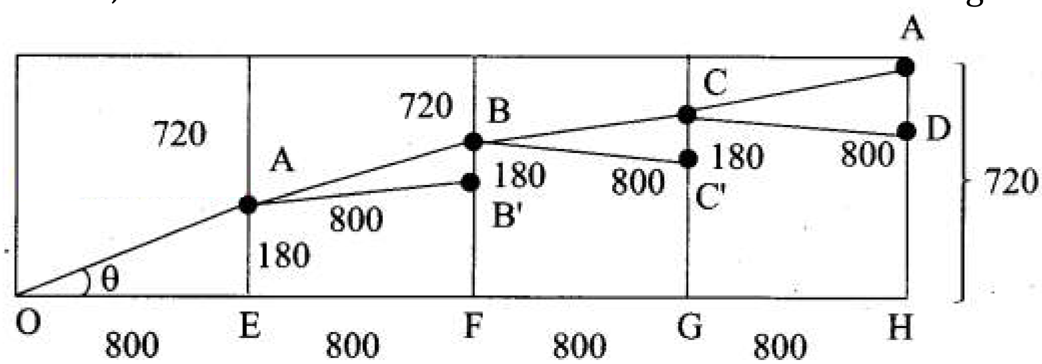


Question 15.
In a shopping mall, there is a hall of cuboid shape with dimension 800 × 800 × 720 units, which needs to be added the facility of an escalator in the path as shown by the dotted line in the figure. Find
(i) The minimum total length of the escalator,

(ii) The heights at which the escalator changes its direction,
(iii) The slopes of the escalator at the turning points.



Solution:
(i) the minimum total length of the escalator.
Shape of the hall in the shopping mall is cuboid. When you open out the cuboid, the not of the cuboid will be as shown in the following diagram.



The path of the escalator is from OA to AB to BC to CD

$$\text{In OAE, } OA^2 = AE^2 + OE^2$$

$$\Rightarrow OA^2 = \left[\frac{1}{4}(720)\right]^2 + OE^2$$

$$\Rightarrow OA^2 = (180)^2 + (800)^2$$

$$= 32400 + 640000 = 672400$$

$$OA = 820 \text{ m}$$

$$\text{Total length of the escalator} = OA + AB + BC + CD$$

$$= 4 \times OA \quad (\text{since } \Delta OAE = \Delta ABB' = \Delta CDD'$$

$$= 4 \times 820$$

The minimum length = 3280 units

(ii) The height at which the escalator changes its direction.

$$AE = \frac{1}{4}(720) = 180 \text{ units}$$

$$BE = \frac{1}{2}(720) = 360 \text{ units and } GE = \frac{3}{4}(720) = 540 \text{ units}$$

(iii) Slope of the escalator at the turning points

$$\text{Let } \angle AOE = \theta$$

$$\text{In } \Delta OAE, \tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{AE}{OE} = \frac{180}{800} = \frac{9}{40}$$

$$\therefore \text{Slope at the point A} = \frac{9}{40}$$

Since ΔOAE = ΔABB' = ΔBCC' = ΔCAD
Slope at the points B, C will be 9/40

Ex 6.3

Question 1.
Show that the lines are 3x + 2y + 9 = 0 and 12x + 8y – 15 = 0 are parallel lines.

Solution:

$$\text{Slope of I line} = m_1 = -\left(\frac{3}{2}\right) = \frac{-3}{2}$$

$$\text{Slope of II line} = m_2 = -\left(\frac{12}{8}\right) = \frac{-3}{2}$$

Here $m_1 = m_2 \Rightarrow$ the two lines are parallel.

Question 2.
Find the equation of the straight line parallel to 5x – 4y + 3 = 0 and having x – intercept 3.

Solution:
Equation of a line parallel to ax + by + c = 0 will be of the form ax + by + k = 0

So equation of a line parallel to 5x – 4y + 3 = 0 will be of the form 5x – 4y = k

$$\Rightarrow \frac{5x}{k} - \frac{4y}{k} = 1 \quad (i.e)\ \frac{x}{k/5} + \frac{y}{k/-4} = 1$$

Here we are given that x intercept $= -3 \Rightarrow \frac{k}{5} = -3 \Rightarrow k = -15$

So equation of the line is $\frac{x}{-3} + \frac{y}{-15/-4} = 1$

$$(i.e)\ \frac{-x}{3} + \frac{4y}{15} = 1 \Rightarrow \frac{-5x+4y}{15} = 1$$

$$-5x + 4y = 15 \Rightarrow 5x - 4y - 15 = 0$$

Question 3.
Find the distance between the line 4x + 3y + 4 = 0 and a point
(i) (-2, 4)
(ii) (7, -3)

Solution:
The distance between the line ax + by + c = 0 and the point(x_1, y_1) is given by

$$\pm \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}$$

(i) Now the distance between the line 4x + 3y + 4 = 0 and (-2, 4) is

$$\pm \frac{4(-2)+3(4)+4}{\sqrt{4^2+3^2}} = \frac{-8+12+4}{5} = 8/5 \text{ units}$$

(ii) The distance between the line 4x + 3y + 4 = 0 and (7, -3) is

$$\pm \frac{4(7)+3(-3)+4}{\sqrt{4^2+3^2}} = \frac{28-9+4}{5} = \frac{23}{5} \text{ units}$$

Question 4.
Write the equation of the lines through the point (1, -1)
(i) Parallel to x + 3y – 4 = 0
(ii) Perpendicular to 3x + 4y = 6

Solution:
(i) Parallel to $x + 3y - 4 = 0$
The equation of any line parallel to the line
$x + 3y - 4 = 0$ is $x + 3y + k = 0$ ………… (1)

This line passes through the point (1, – 1)
$\therefore$ (1) $\Rightarrow 1 + 3(-1) + k = 0$
$1 - 3 + k = 0 \Rightarrow k = 2$
$\therefore$ The equation of the required line is
$x + 3y + 2 = 0$

(ii) Perpendicular to $3x + 4y = 6$
The equation of any line perpendicular to $3x + 4y = 6$ is
$4x - 3y + k = 0$ ………… (2)
This line passes through the point (1,-1)
(2) $\Rightarrow (4) 1 - 3(-1) + k = 0$
$4 + 3 + k = 0 \Rightarrow k = -7$
$\therefore$ The required equation is $4x - 3y - 7 = 0$

Question 5.
If (- 4, 7) is one vertex of a rhombus and if the equation of one diagonal is $5x - y + 7 = 0$, then find the equation of another diagonal.

Solution:
Let the equation of the diagonal AC be $5x - y + 7 = 0$ ……….. (1)
Since (-4, 7) does not satisfy equation (1), (- 4, 7) represents neither A nor C.
Let (-4, 7) represent the vertex D.
The diagonal BD is perpendicular to AC

The equation of any line perpendicular to line (1) is $-x - 5y + k = 0$ ……….. (2)
This line passes through the point D (-4, 7)
$\therefore$ (2) $\Rightarrow -(-4) - 5(7) + k = 0$
$4 - 35 + k = 0 \Rightarrow k = 31$
$\therefore$ The equation of the other diagonal is
$-x - 5y + 31 = 0$
$x + 5y - 31 = 0$

Question 6.
Find the equation of the lines passing through the point of intersection lines $4x - y + 3 = 0$ and $5x + 2y + 7 = 0$, and
(i) Through the point (-1, 2)
(ii) Parallel to $x - y + 5 = 0$
(iii) Perpendicular to $x - 2y + 1 = 0$.

Solution:
To find the point of intersection of the lines we have to solve them

$$\begin{array}{lrl} & 4x - y = -3 & \quad ....(1) \\ & 5x + 2y = -7 & \quad ....(2) \\ (1)\times(2) \Rightarrow & \underline{8x - 2y = -6} & \quad ....(3) \\ (2)+(3) \Rightarrow & 13x = -13 & \Rightarrow x = -1 \end{array}$$

Substituting x = -1 in equation (2) we get
$-5 + 2y = -7$
$\Rightarrow 2y = -7 + 5 = -2$
$\Rightarrow y = -1$
So the point of intersection is (-1, -1)

(i) Now $(x_1, y_1) = (-1, -1)$; $(x_2, y_2) = (-1, 2)$.
Equation of the line passing through 2 points is $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$

(i.e) $\dfrac{y+1}{2+1} = \dfrac{x+1}{-1+1} \Rightarrow \dfrac{y+1}{3} = \dfrac{x+1}{0} \Rightarrow x + 1 = 0 \Rightarrow x = -1$

(ii) Parallel to $x - y + 5 = 0$
Given that the line (1) is parallel to the line
$x - y + 5 = 0$ .... (2)
∴ Slope of line (1) = Slope of line (2)
$(4x - y + 3) + \lambda(5x + 2y + 7) = 0$
$4x - y + 3 + 5\lambda x + 2\lambda y + 7\lambda = 0$
$(4 + 5\lambda)x + (2\lambda - 1)y + 3 + 7\lambda = 0$
Slope of this line $= -\frac{4+5\lambda}{2\lambda-1}$
Slope of line (2) $= -\frac{1}{-1} = 1$
These two slopes are equal

$-\frac{4+5\lambda}{2\lambda-1} = 1$

$-(4 + 5\lambda) = 2\lambda - 1$

$-4 - 5\lambda = 2\lambda - 1$

$2\lambda + 5\lambda - 1 + 4 = 0$

$7\lambda + 3 = 0 \Rightarrow \lambda = -\frac{3}{7}$

Substituting the value of λ in equation (1), we have

$(4x - y + 3) - \frac{3}{7}(5x + 2y + 7) = 0$

$7(4x - y + 3) - 3(5x + 2y + 7) = 0$

$28x - 7y + 21 - 15x - 6y - 21 = 0$

$13x - 13y = 0$

$x - y = 0$

(iii) Equation of a line perpendicular to $x - 2y + 1 = 0$ will be of the form $2x + y + k = 0$. It passes through (-1, -1) $\Rightarrow -2 - 1 + k = 0 \Rightarrow k = 3$.

So the required line is $2x + y + 3 = 0$

Question 7.

Find the equations of two straight lines which are parallel to the line $12x + 5y + 2 = 0$ and at a unit distance from the point (1, -1).

Solution:

Equation of a line parallel to $12x + 5y + 2 = 0$ will be of the form $12x + 5y + k = 0$.

We are given that the perpendicular distance form (1, -1) to the line $12x + 5y + k = 0$ is 1 unit.

$$\Rightarrow \pm\frac{12(1)+5(-1)+k}{\sqrt{12^2+5^2}} = 1 \; (i.e) \; \frac{12-5+k}{13} = \pm 1 \Rightarrow 7 + k = \pm 13$$

$$\begin{array}{c|c} 7+k=13 & 7+k=-13 \\ \Rightarrow k=13-7=6 & \Rightarrow k=-13-7=-20 \end{array}$$

So the required line will be $12x + 5y + 6 = 0$ or $12x + 5y - 20 = 0$

Question 8.
Find the equations of straight lines which are perpendicular to the line 3x + 4y – 6 = 0 and are at a distance of 4 units from (2, 1).

Solution:
Given equation of line is 3x + 4y – 6 = 0.
Any line perpendicular to 3x + 4y – 6 = 0 will be of the form 4x – 3y + k = 0
Given perpendicular distance is 4 units from (2, 1) to line (1)

$$\therefore 4 = \pm\frac{(4(2)-3(1)+k)}{\sqrt{4^2+(-3)^2}}$$

$$\Rightarrow \quad 4 = \pm\frac{(8-3+k)}{\sqrt{16+9}}$$

$$\Rightarrow \quad 4 = \pm\left(\frac{5+k}{5}\right)$$

∴ 20 = + (5 + k) or 20 = – (5 + k)
⇒ k = 20 – 5 or k = -(20 + 5)
k = 15 or k : = -25
∴ Required equation of the lines are 4x – 3y + 15 = 0 and 4x – 3y – 25 = 0

Question 9.
Find the equation of a straight line parallel to 2x + 3y = 10 and which is such that the sum of its intercepts on the axes is 15.

Solution:
The equation of the line parallel to 2x + 3y = 10 will be of the form 2x + 3y = k.

$$(i.e)\ \frac{2x}{k}+\frac{3y}{k}=1 \Rightarrow \frac{x}{k/2}+\frac{y}{k/3}=1.$$

$$\text{Sum of the intercepts} = 15 \Rightarrow \frac{k}{2}+\frac{k}{3}=15 \Rightarrow \frac{3k+2k}{6}=15$$

$5k = 90 \Rightarrow k = 90/5 = 18.$

So the required line is $2x + 3y = 18$ or $2x + 3y - 18 = 0$

Question 10.
Find the length of the perpendicular and the co-ordinates of the foot of the

perpendicular from (-10, -2) to the line x + y – 2 = 0.

Solution:
Length of the perpendicular from (-10, -2) to x + y – 2 = 0 is

$$\pm\left(\frac{-10-2-2}{\sqrt{1^2+1^2}}\right)=\pm\frac{14}{\sqrt{2}}=\frac{14\sqrt{2}}{\sqrt{2}\sqrt{2}}=7\sqrt{2} \text{ units}$$

Let A (a, b) be the foot of the perpendicular the given line is $x+y-2=0 \Rightarrow a+b-2=0$
$\Rightarrow a+b=2$ .... (1)

So of line joining A (a, b) and B $(-10, -2)$ is $\frac{b+2}{a+10}=m_1$.

Slope of $x+y-2=0$ is $\frac{-1}{1}=-1=m_2$

We are given $m_1m_2=-1$

$$\Rightarrow\left(\frac{b+2}{a+10}\right)(-1)=-1 \Rightarrow \frac{b+2}{a+10}=1 \Rightarrow b+2=a+10$$

Solving (1) and (2) $\Rightarrow a-b=-8$ .... (2)

$$(1) \Rightarrow a+b=2$$
$$(2) \Rightarrow \underline{a-b=-8}$$
$$2a=-6$$
$$a=-3$$

Substituting $a=-3$ in (1) we get $-3+b=2 \Rightarrow b=2+3=5$
So the foot of the perpendicular is $(-3, 5)$

Question 11.
If p_1 and p_2 are the lengths of the perpendiculars from the origin to the straight lines. sec θ +y cosec θ = 2a and x cos θ – y sin θ = a cos 2θ, then prove that $p_1^2+p_2^2=a^2$.

Solution:
p_1 = length of perpendicular from (0, 0) to x sec θ + y cosec θ = 2a

$$\Rightarrow p_1 = \pm \frac{2a}{\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}}$$

$$\therefore p_1^2 = \frac{4a^2}{\sec^2\theta + \operatorname{cosec}^2\theta} = \frac{4a^2}{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}}$$

$$= \frac{4a^2\cos^2\theta\sin^2\theta}{\sin^2\theta + \cos^2\theta} = (2a\sin\theta\cos\theta)^2$$

$$= a^2[2\sin\theta\cos\theta]^2 = a^2(\sin 2\theta)^2 = a^2\sin^2 2\theta$$

p_2 = length of perpendicular from (0, 0) to $x\cos\theta - y\sin\theta = a\cos 2\theta$

$$p_2 = \pm\frac{a\cos 2\theta}{\sqrt{\cos^2\theta + \sin^2\theta}} = a\cos 2\theta$$

$$\therefore p_2^2 = (a\cos 2\theta)^2 = a^2\cos^2 2\theta$$

$$\text{LHS} = p_1^2 + p_2^2 = a^2\sin^2 2\theta + a^2\cos^2 2\theta = a^2\left[\sin^2 2\theta + \cos^2 2\theta\right] = a^2(1) = a^2 = \text{RHS}$$

Question 12.
Find the distance between the parallel lines
(i) $12x + 5y = 7$ and $12x + 5y + 7 = 0$
(ii) $3x - 4y + 5 = 0$ and $6x - 8y - 15 = 0$.

Solution:

(i) The distance between the parallel lines $ax + by + c = 0$ and $ax + by + d = 0$ is $\pm\frac{c-d}{\sqrt{a^2+b^2}}$
The distance between $12x + 5y - 7 = 0$ and $12x + 5y + 7 = 0$ is

$\pm 7-(-7)/\sqrt{12^2+5^2} = \frac{14}{13}$ units.

(ii) $3x - 4y + 5 = 0$ (multiplying by 2) $6x - 8y + 10 = 0$, $6x - 8y - 15 = 0$.

The distance between the parallel lines is $\pm\frac{10-(-15)}{\sqrt{6^2+8^2}} = \frac{10+15}{\sqrt{36+64}} = \frac{25}{10} = \frac{5}{2}$ units

Question 13.
Find the family of straight lines
(i) Perpendicular
(ii) Parallel to $3x + 4y - 12 = 0$.

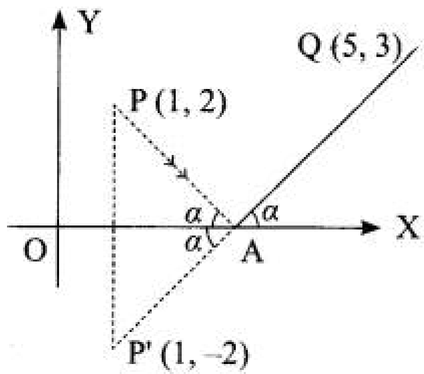
Solution:
(i) The equation of the family of straight lines perpendicular to $3x + 4y - 12 = 0$ is $4x - 3y + k = 0$ where $k \in R$
(ii) The equation of the family of straight lines parallel to the straight line $3x + 4y - 12 = 0$ is $3x + 4y + \lambda = 0, \lambda \in R$

Question 14.
If the line joining two points A (2, 0) and B (3, 1) is rotated about A in an anti-clockwise direction through an angle of 15°, then find the equation of the line in the new position.

Solution:
Slope of A (2, 0) and B (3, 1) is $m = \frac{1-0}{3-2} = \frac{1}{1} = 1$
$(i.e)$ $\tan\theta = 1 \Rightarrow \theta = 45°$.
This line is rotated about 15° in an anti-clockwise direction
⇒ New slope = tan (45° + 15°) = tan 60° = 3–√ (i.e) m = 3–√.
Point A = (2, 0)
Equation of the line is $y - 0 = \sqrt{3}(x-2)$
$y = \sqrt{3}x - 2\sqrt{3} \Rightarrow \sqrt{3}x - y - 2\sqrt{3} = 0$

Question 15.
A ray of light coming from the point (1, 2) is reflected at a point A on the x-axis and it passes through the point (5, 3). Find the coordinates of point A.

Solution:
The image of the point P (1, 2) will be P’ (1, -2).
Since ∠OAP = ∠XAQ (angle of inches = angle of reflection) So ∠OAP’ = ∠XAQ = a (Vertically opposite angles)
⇒ P’, A, Q lie on the same line.



Now equation of the line P', Q is [where P' = (1, -2), Q = (5, 3)]

$$\frac{y+2}{3+2} = \frac{x-1}{5-1} \Rightarrow \frac{y+2}{5} = \frac{x-1}{4}$$

$4y + 8 = 5x - 5$

$5x = 4y + 13 \Rightarrow x = \dfrac{4y+13}{5}.$

Since we find a point of intersection with the x-axis we put y = 0.

So $y = 0 \Rightarrow x = \dfrac{13}{5}$.

$\therefore$ The reflection point is $\left(\dfrac{13}{5}, 0\right)$

Question 16.

A line is drawn perpendicular to 5x = y + 7. Find the equation of the line if the area of the triangle formed by this line with co-ordinate axes is 10 sq. units.

Solution:

Equation of the given lines 5x = y + 7 ⇒ 5x – y = 7.

So its perpendicular will be of the form x + 5y = 7

$$\Rightarrow \frac{x}{k} + \frac{5y}{k} = 1 \quad (i.e) \frac{x}{k} + \frac{y}{k/5} = 1$$

Now x intercept = k and y intercept = $k/5$.

Area of the $\Delta = \dfrac{1}{2}(k)\left(k/5\right) = \dfrac{k^2}{10} = 10$ (given)

$\Rightarrow k^2 = 100$ or $k = 10$.

So equation of the line is $x + 5y = \pm 10$

Question 17.
Find the image of the point (-2, 3) about the line x + 2y – 9 = 0.

Solution:
The coordinates of the image of the point (x_1, y_1) with respect to the line ax + by + c = 0 can be
obtained by the line $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{-2(ax_1+by_1+c)}{a^2+b^2}$
Here $(x_1, y_1) = (-2, 3)$ and the given line is $x + 2y - 9 = 0$.

So the image is $\frac{x+2}{1}=\frac{y-3}{2}=\frac{-2[-2+2(3)-9]}{1+4}$

$(i.e)\ \frac{x+2}{1}=\frac{y-3}{2}=\frac{-2(-2+6-9)}{5}$

$\Rightarrow \frac{x+2}{1}=\frac{y-3}{2}=\frac{-2(-5)}{5}=2$

$\Rightarrow \frac{x+2}{1}=2 \quad \Big| \quad \frac{y-3}{2}=2$

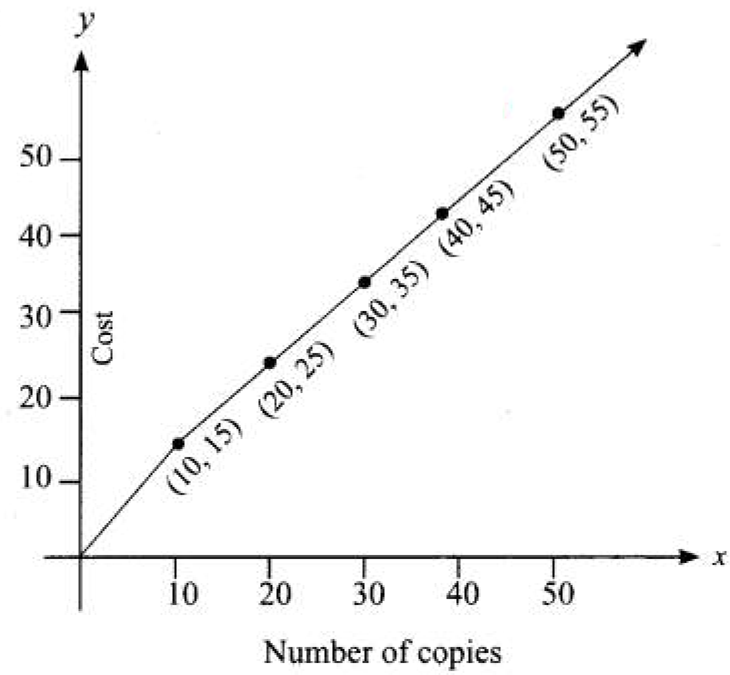
$\Rightarrow x=2-2=0 \quad \Big| \quad \Rightarrow y=4+3=7$

So the image is (0, 7)

Question 18.
A photocopy store charges Rs. 1.50 per copy for the first 10 copies and Rs. 1.00 per copy after the 10th copy. Let x be the number of copies, and let y be the total cost of photocopying.
(i) Draw a graph of the cost as x goes from 0 to 50 copies.
(ii) Find the cost of making 40 copies

Solution:

(i)



$$y = f(x) = \begin{cases} 1.5x & 0 < x \le 10 \\ 15 + 1\,(x - 10) & \text{if } x > 10 \\ (i.e)\ x + 5 & \end{cases}$$

x	10	20	30	40	50
y	15	25	35	45	55

(ii) The cost of making 40 copies is 40 + 5 = Rs. 45.

Question 19.
Find atleast two equations of the straight lines in the family of the lines y = 5x + b, for which b and the x-coordinate of the point of intersection of the lines with 3x – 4y = 6 are integers.

Solution:
y = 5x + b …….. (1)
3x-4y = 6 …….. (2)
Solving (1) and (2)
Substituting y value from (1) in (2) we get

$y = 5x + b$ ....(1)

$3x - 4y = 6$ ....(2)

Solving (1) and (2)

Substituting y value from (1) in (2) we get

$$3x - 4(5x + b) = 6$$

$$3x - 20x - 4b = 6$$

$$-17x = 6 + 4b$$

$$\Rightarrow \quad x = \frac{6+4b}{-17}$$

$$\text{So, } y = 5\left(\frac{6+4b}{-17}\right) + b \Rightarrow y = \frac{30+3b}{-17}$$

$$\text{So, } (x, y) = \left(\frac{6+4b}{-17}, \frac{30+3b}{-17}\right)$$

Since x coordinate and 6 are integers 6 + 46 must be a multiple of 17

$(i.e) \pm 17, \pm 34$

$$\Rightarrow \quad 6 + 4b = +34 \qquad\qquad 6 + 4b = -34$$

$$4b = 28 \qquad\qquad 4b = -34 - 6 = -40$$

$$b = 7 \qquad\qquad b = -10$$

$\therefore$ Equation of lines $y = 5x + b$ will be $y = 5x + 7$, $y = 5x - 10$

Question 20.

Find all the equations of the straight lines in the family of the lines y = mx – 3, for which m and the x-coordinate of the point of intersection of the lines with x – y = 6 are integers.

Solution:

Equation of the given lines are

y= mx – 3 …….. (1)

and x – y = 6 ……. (2)

Solving (1) and (2)

x – (mx – 3) = 6

(i.e) $x - mx + 3 = 6$

$$x(1-m) = 6 - 3 = 3$$

$\Rightarrow$ $$x = \frac{3}{1-m}$$

Now (2) $\Rightarrow$ $x - y = 6$

(i.e) $y = x - 6$

$\Rightarrow$ $$y = \frac{3}{1-m} - 6 = \frac{3-6+6m}{1-m} = \frac{6m-3}{1-m}$$

$$\therefore (x, y) = \left(\frac{3}{1-m}, \frac{6m-3}{1-m}\right)$$

Since m and x coordinates are integers

1 – m is the divisor of 3 (i.e) $\pm 1, \pm 3$

$1-m=+1$ $\Rightarrow m=0$	$1-m=-1$ $\Rightarrow m=2$	$1-m=+3$ $\Rightarrow m=-2$	$1-m=-3$ $\Rightarrow m=4$

So equation of lines are (y = mx – 3) ,y = mx – 3

(i) When m = 0, y = -3

(ii) When m = 2, y = 2x – 3

(iii) When m = -2, y = -2x – 3 or 2x + y + 3 = 0

(iv) When m = 4, y = 4x – 3

Ex 6.4

Question 1.

Find the combined equation of the straight lines whose separate equations are x – 2y – 3 = 0 and x + y + 5 = 0.

Solution:

The given separate equations of the lines are

x – 2y – 3 = 0 and x + y + 5 = 0

∴ The combined equation of the straight lines is

(x – 2y – 3) (x + y + 5) = 0

$x^2 + xy + 5x - 2xy - 2y^2 - 10y - 3x - 3y - 15 = 0$

$x^2 - xy - 2y^2 + 2x - 13y - 15 = 0$

Question 2.
Show that $4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$ represents a pair of parallel lines.

Solution:
Comparing this equation with $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
we get $a = 4$, $h = 4/2 = 2$, $b = 1$, $g = -3$, $f = -3/2$, $c = -4$
The condition for the lines to be parallel is $h^2 - ab = 0$
Now $h^2 - ab = 2^2 - (4)(1) = 4 - 4 = 0$
$h^2 - ab = 0 \Rightarrow$ The given equation represents a pair of parallel lines.

Question 3.
Show that $2x^2 + 3xy - 2y^2 + 3x + y + 1 = 0$ represents a pair of perpendicular lines.

Solution:
The equation of the given pair of straight lines is
$2x^2 + 3xy - 2y^2 + 3x + y + 1 = 0$ ……….. (1)
Compare this equation with the equation
$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ ……… (2)
$a = 2$, $2h = 3$, $b = -2$, $2g = 3$, $2f = 1$, $c = 1$

The condition for pair of straight lines to be perpendicular is $a + b = 0$.
$2 - 2 = 0$
Hence the given pair of lines represents a perpendicular straight lines.

Question 4.
Show that the equation $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$ represents a pair of intersecting lines. Show further that the angle between them is $\tan^{-1}(5)$.

Solution:
Comparing the given equation with general form we get $a = 2$, $b = -3$, $c = -20$, $f = \frac{19}{2}$,
$g = -3$, $h = -1/2$
The condition for the given equation to represent a pair of straight lines is
$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

Now LHS $= abc + 2fgh - af^2 - bg^2 - ch^2$

$$= (2)(-3)(-20) + 2\left(\frac{19}{2}\right)(-3)\left(-\frac{1}{2}\right) - 2\left(\frac{19}{2}\right)^2 + 3(-3)^2 + 20\left(-\tfrac{1}{2}\right)^2$$

$$= 120 + \frac{57}{2} - \frac{361}{2} + 27 + 5 = 180\frac{1}{2} - 180\frac{1}{2} = 0$$

The given equation represents a pair of straight lines.

The angle between the pair of straight lines is given by $\tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a+b}$

$$(i.e)\ \tan\theta = \pm\frac{2\sqrt{\frac{1}{4}+6}}{2+(-3)} = \pm\frac{2\sqrt{\frac{25}{4}}}{-1} = 2\times\frac{5}{2} = 5 \Rightarrow \theta = \tan^{-1}(5)$$

Question 5.

Prove that the equation to the straight lines through the origin, each of which makes an angle α with the straight line y = x is $x^2 - 2xy \sec 2\alpha + y^2 = 0$

Solution:

Slope of y = x is m = tan θ = 1

⇒ θ = 45°

The new lines slopes will be

m = tan(45 + α) and m = tan (45 – α)

∴ The equations of the lines passing through the origin is given by

y = tan(45 + α)x and y = tan(45 – α)x

(i.e) y = tan(45 + α)x = 0 and y = tan(45 – α)x = 0

The combined equation is [y – tan (45 + α)x] [y – tan (45 – α)x] = 0

$y^2 + \tan(45 + \alpha)\tan(45 - \alpha)x^2 - xy[\tan(45 - \alpha) + \tan(45 + \alpha)] = 0$

$$(i.e)\ y^2 + \frac{1+\tan\alpha}{1-\tan\alpha}\times\frac{1-\tan\alpha}{1+\tan\alpha}x^2 - xy\left[\frac{\sin(45-\alpha)}{\cos(45-\alpha)} + \frac{\sin(45+\alpha)}{\cos(45+\alpha)}\right] = 0$$

$$(i.e)\ x^2 + y^2 - xy\frac{[\sin(45-\alpha)\cos(45+\alpha) + \cos(45-\alpha)\sin(45+\alpha)]}{\cos(45-\alpha)\cos(45+\alpha)} = 0$$

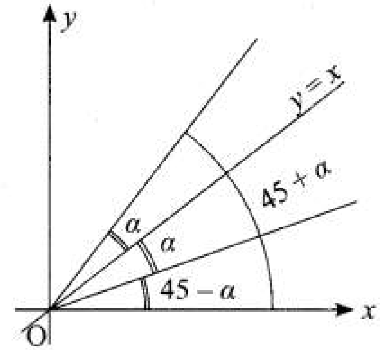
$$(i.e)\ x^2 + y^2 - xy\frac{[\sin(45+45)]}{\frac{1}{2}[2\cos(45-\alpha)\cos(45+\alpha)]} = 0$$

$$x^2 + y^2 - \frac{2xy}{\cos 90 + \cos 2\alpha} = 0 \Rightarrow x^2 - \frac{2xy}{\cos 2\alpha} + y^2 = 0$$

$(i.e)\ x^2 - 2xy \sec 2\alpha + y^2 = 0$

Aliter:

Let the slopes of the lines be m_1 and m_2 where



$m_1 = \tan(45 - \alpha) = \dfrac{1 - \tan\alpha}{1 + \tan\alpha}$ and $m_2 = \tan(45 + \alpha) = \dfrac{1 + \tan\alpha}{1 - \tan\alpha}$

$$m_1 + m_2 = \frac{1 - \tan\alpha}{1 + \tan\alpha} + \frac{1 + \tan\alpha}{1 - \tan\alpha}$$

$$= 2\left(\frac{1}{\cos 2\alpha}\right) = 2\sec 2\alpha$$

$$m_1 m_2 = \frac{1 - \tan\alpha}{1 + \tan\alpha} \times \frac{1 - \tan\alpha}{1 - \tan\alpha} = 1$$

Let the equation of lines passes through the origin
So the equations are $y = m_1x = 0$ and $y = m_2x = 0$
So the combined equations is $(y - m_1x)(y - m_2x) = 0$
$(i.e) y^2 - xy(m_1 + m_2) + m_1m_2x = 0$
$(i.e)\ y^2 - xy(2\sec\alpha) + x^2(1) = 0$
$(i.e)\ y^2 - 2xy \sec 2\alpha + x^2 = 0$

Question 6.
Find the equation of the pair of straight lines passing through the point (1, 3) and perpendicular to the lines $2x - 3y + 1 = 0$ and $5x + y - 3 = 0$

Solution:
The equation of the given lines are
$2x - 3y + 1 = 0$ ……….. (1)
$5x + y - 3 = 0$ ……….. (2)
Equation of any line perpendicular to $2x - 3y + 1 = 0$ is
$-3x - 2y + k = 0$
$3x + 2y - k = 0$
This line passes through the point (1, 3)
$\therefore 3(1) + 2(3) - k = 0$
$3 + 6 - k = 0 \Rightarrow k = 9$
Substituting the value of k in the above equation we have
$3x + 2y - 9 = 0$ ………. (3)
Equation of any line perpendicular to $5x + y - 3 = 0$ is
$x - 5y + k_1 = 0$

This line passes though the point (1 , 3)
$\therefore 1 - 5(3) + k_1 = 0$
$1 - 15 + k_1 \Rightarrow k_1 = 14$
Substituting the value of k_1 in the above equation we have
$x - 5y + 14 = 0$ ……….. (4)
The combined equation of (3) and (4) is
$(3x + 2y - 9)(x - 5y + 14) = 0$
$3x^2 - 15xy + 42x + 2xy - 10y^2 + 28y - 9x + 45y - 126 = 0$
$3x^2 - 13xy - 10y^2 + 33x + 73y - 126 = 0$ Question 7.

Find the separate equation of the following pair of straight lines
(i) $3x^2 + 2xy - y^2 = 0$
(ii) $6(x - 1)^2 + 5(x - 1)(y - 2) - 4(y - 2)^2 = 0$
(iii) $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$

Solution:
(i) $3x^2 + 2xy - y^2 = 0$
The given equation is
$3x^2 + 2xy - y^2 = 0$ ……. (1)
$3x^2 + 3xy - xy - y^2 = 0$
$3x(x + y) - y(x + y) = 0$
$(3x - y)(x + y) = 0$
$3x - y = 0$ and $x + y = 0$
$\therefore$ The separate equations are
$3x - y = 0$ and $x + y = 0$

(ii) $6(x - 1)^2 + 5(x - 1)(y - 2) - 4(y - 2)^2 = 0$
$\Rightarrow 6(x^2 - 2x + 1) + 5(xy - 2x - y + 2) - 4(y^2 - 4y + 4) = 0$
(i.e) $6x^2 - 12x + 6 + 5xy - 10x - 5y + 10 - 4y^2 + 16y - 16 = 0$
(i.e) $6x^2 + 5xy - 4y^2 - 22x + 11y = 0$
Factorising $6x^2 + 5xy - 4y^2$ we get
$6x^2 - 3xy + 8xy - 4y^2 = 3x(2x - y) + 4y(2x - y)$
$= (3x + 4y)(2x - y)$
So, $6x^2 + 5xy - 4y^2 - 22x + 11y = (3x + 4y + l)(2x - y + m)$

Equating coefficient of x $\Rightarrow 3m + 21 = -22$ …….. (1)
Equating coefficient of y $\Rightarrow 4m - l = 11$ ……. (2)

Solving (1) and (2) we get $l = -11$, $m = 0$
So the separate equations are $3x + 4y - 11 = 0$ and $2x - y = 0$

(iii) $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$
Factorising $2x^2 - xy - 3y^2$ we get
$2x^2 - xy - 3y^2 = 2x^2 + 2xy - 3xy - 3y^2$
$= 2x(x + y) - 3y(x + y) = (2x - 3y)(x + y)$
$\therefore 2x^2 - xy - 3y^2 - 6x + 19y - 20 = (2x - 3y + l)(x + y + m)$

Equating coefficient of x $2m + l = -6$ ……. (1)
Equating coefficient of y $-3m + l = 19$ …….. (2)

Constant term $-20 = lm$
Solving (1) and (2) we get $l = 4$ and $m = -5$ where $lm = -20$.
So the separate equations are $2x - 3y + 4 = 0$ and $x + y - 5 = 0$

Question 8.
The slope of one of the straight lines $ax^2 + 2hxy + by^2 = 0$ is twice that of the other, show that $8h^2 = 9ab$.

Solution:
$ax^2 + 2hxy + by^2 = 0$
We are given that one slope is twice that of the other.
So let the slopes be m and 2m.
Now sum of the slopes $= m + 2m$

$$= 3m = -\frac{2h}{b} \Rightarrow m = -\frac{2h}{3b} \quad \text{..............(A)}$$

Product of the slopes $= m \times 2m = 2m^2 = a/b$

$$\Rightarrow m^2 = \frac{a}{2b} \quad \text{..............(B)}$$

Eliminating m from (A) and (B) we get $\left(\frac{-2h}{3b}\right)^2 = \frac{a}{2b}$

$$(i.e)\ \frac{4h^2}{9b^{\cancel{2}}} = \frac{a}{2\cancel{b}}$$

$$8h^2 = 9ab$$

Question 9.
The slope of one of the straight lines $ax^2 + 2hxy + by^2 = 0$ is three times the other, show that $3h^2 = 4ab$.

Solution:
Let the slopes be m and 3m.

$$\text{Now } m+3m=4m=-\frac{2h}{b}$$

$$\Rightarrow m=-\frac{2h}{4b}=-\frac{h}{2b} \qquad \text{...............(1)}$$

$$m\times 3m=\frac{a}{b}\Rightarrow 3m^2=\frac{a}{b}\Rightarrow m^2=\frac{a}{3b} \qquad \text{...............(2)}$$

Eliminating m from (1) and (2)

$$\text{we get} \quad \left(-\frac{h}{2b}\right)^2=\frac{a}{3b}\Rightarrow\frac{h^2}{4b^2}=\frac{a}{3b}\Rightarrow 3h^2=4ab$$

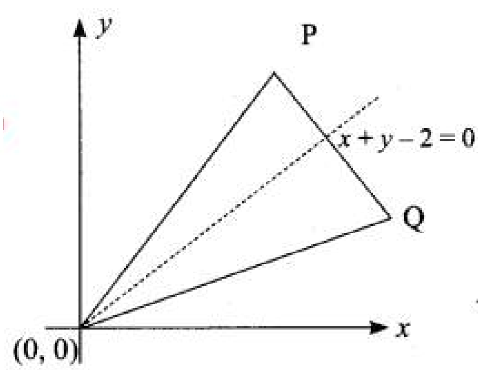
Question 10.
A ΔOPQ is formed by the pair of straight lines $x^2 - 4xy + y^2 = 0$ and the line PQ. The equation of PQ is $x + y - 2 = 0$. Find the equation of the median of the triangle ΔOPQ drawn from the origin O.

Solution:
Equation of pair of straight lines is $x^2 - 4xy + y^2 = 0$ ..... (1)
Equation of the given line is $x + y - 2 = 0 \Rightarrow y = 2 - x$ .... (2)
On solving (1) and (2) we get $x^2 - 4x (2 - x) + (2 - x)^2 = 0$
(i.e) $x^2 - 8x + 4x^2 + 4 + x^2 - 4x = 0$
(i.e) $6x^2 - 12x + 4 = 0$
(÷ by 2) $3x^2 - 6x + 2 = 0$

$$x=\frac{6\pm\sqrt{36-24}}{6}=\frac{6\pm\sqrt{12}}{6}$$

$$x=\frac{6\pm 2\sqrt{3}}{6}==\frac{3\pm\sqrt{3}}{3}=1\pm\frac{1}{\sqrt{3}}$$

$$x=1+\frac{1}{\sqrt{3}}\ ,\ 1-\frac{1}{\sqrt{3}}$$



$$\text{When } x=1+\frac{1}{\sqrt{3}}, y=2-1-\frac{1}{\sqrt{3}}=1-{}^{1}\!/\!\sqrt{3}$$

$$\text{When } x=1-{}^{1}\!/\!\sqrt{3}, y=2-1+\frac{1}{\sqrt{3}}=1+\frac{1}{\sqrt{3}}$$

$$\therefore\ P=\left(1+\frac{1}{\sqrt{3}},1-\frac{1}{\sqrt{3}}\right);Q=\left(1-\frac{1}{\sqrt{3}},1+\frac{1}{\sqrt{3}}\right)$$

The midpoint of PQ is

$$\left(\frac{1+\frac{1}{\sqrt{3}}+1-\frac{1}{\sqrt{3}}}{2}, \frac{1-\frac{1}{\sqrt{3}}+1+\frac{1}{\sqrt{3}}}{2}\right) = (1, 1)$$

Now the equation of the median through (0, 0) and (1, 1) is $\frac{y-1}{0-1} = \frac{x-1}{0-1}$

$(i.e)\ \frac{y-1}{-1} = \frac{x-1}{-1} \Rightarrow y-1 = x-1$

$(i.e)\ x-y=0 \Rightarrow y=x$

Question 11.
Find p and q, ¡f the following equation represents a pair of perpendicular lines
$6x^2 + 5xy - py^2 + 7x + qy - 50$

Solution:
$6x^2 + 5xy - py^2 + 7x + qy - 50$
The given equation represents a pair of perpendicular lines
⇒ coefficient of x^2 + coefficient of $y^2 = 0$
(i.e) $6 - p = 0 \Rightarrow p = 6$

Now comparing the given equation with the general form
$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
we get a = 6, b = -6 and c = -5, f = q/2, g = 7/2 and h = 5/2
The condition for the general form to represent a pair of straight lines is $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$$(i.e)\ (6)(-6)(-5)+2\left(\frac{q}{2}\right)\left(\frac{7}{2}\right)\left(\frac{5}{2}\right)-6\left(\frac{q^2}{4}\right)+6\left(\frac{49}{4}\right)+5\left(\frac{25}{4}\right)=0$$

$$(i.e)\ 180+\frac{35q}{4}-\frac{6q^2}{4}+\frac{294}{4}+\frac{125}{4}=0$$

$$\frac{-6q^2}{4}+\frac{35q}{4}+\frac{720+294+125}{4}=0$$

$$-6q^2+35q+1139=0$$

Changing the sign throughout we get $6q^2 - 35q - 1139 = 0$

$$q = \frac{35 \pm \sqrt{1225 + 4(6)(1139)}}{2(6)}$$

$$q = \frac{35 \pm \sqrt{28561}}{12}$$

$$q = \frac{35 \pm 169}{12}$$

$$q = \frac{35+169}{12}, \frac{35-169}{12}$$

$$q = 17 \text{ or } -\frac{67}{6} \text{ and } p = 6$$

Question 12.
Find the value of k, if the following equation represents a pair of straight lines. Further, find whether these lines are parallel or intersecting, $12x^2 + 7xy - 12y^2 - x + 7y + k = 0$.

Solution:
Comparing the given equation with the general form $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
we get $a = 12$, $b = -12$, $c = k$, $f = 7/2$, $g = -1/2$, $h = 7/2$

Here $a + b = 0 \Rightarrow$ the given equation represents a pair of perpendicular lines
To find k: The condition for the given equation to represent a pair of straight lines is $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

(i.e) $$(12)(-12)(k) + \cancel{2}\left(\frac{7}{\cancel{2}}\right)\left(-\frac{1}{2}\right)\left(\frac{7}{2}\right) - 12\left(\frac{49}{4}\right) + 12\left(\frac{1}{4}\right) - k\left(\frac{49}{4}\right) = 0$$

$$-144k - \frac{49}{4} - 147 + 3 - \frac{49k}{4} = 0$$

$$\frac{-576k - 49k}{4} = \frac{49 + 588 - 12}{4}$$

$$-\frac{625k}{4} = \frac{625}{4} \Rightarrow k = -1$$

Question 13.
For what value of k does the equation $12x^2 + 2kxy + 2y^2, + 11x - 5y + 2 = 0$ represent two straight lines.

Solution:
$12x^2 + 2\ kxy + 2y^2 + 11x - 5y + 2 = 0$
Comparing this equation with the general form we get

$a = 12,\ b = 2,\ c = 2,\ f = -5/2, g = 11/2, h = \frac{2k}{2} = k$

The condition for the given equation to represent a pair of straight lines is

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$(i.e)\ (12)\ (2)\ (2) + 2\left(-\frac{5}{2}\right)\left(\frac{11}{2}\right)(k) - 12\left(\frac{25}{4}\right) - 2\left(\frac{121}{4}\right) - 2\left(k^2\right) = 0$

$$48\frac{-55k}{2} - 75 - \frac{121}{2} - 2k^2 = 0$$

$$-27 - \frac{55k}{2} - \frac{121}{2} - 2k^2 = 0$$

$(\times \text{ by } 2)\qquad -4k^2 - 121 - 55k - 54 = 0$

$4k^2 + 55k + 175 = 0$
$4k^2 + 20k + 35k + 175 = 0$
$4k(k + 5) + 35(k + 5) = 0$
$(4k + 35)\ (k + 5) = 0$
$k = -5$ or $-35/4$

Question 14.
Show that the equation $9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = 0$ represents a pair of parallel lines. Find the distance between them.

Solution:
Comparing the given equation with $ax^2 + 2kxy + by^2 = 0$ we get $a = 9$, $h = -12$, $b = 16$.
Now $h^2 = (-12)^2 = 144$, $ab = (9)\ (16) = 144$
$h^2 = ab \Rightarrow$ The given equation represents a pair of parallel lines.

To find their separate equations:
$9x^2 - 24xy + 16y^2 = (3x - 4y)^2$
So, $9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = (3x - 4y + l)(3x - 4y + m)$

Here coefficient of x ⇒ $3m + 3l = -12 \Rightarrow m + l = -4$
coefficient of y ⇒ $-4m - 4l = 16 \Rightarrow m + l = -4$
Constant term $l\,m = -12$
Now $l + m = -4$ and $lm = -12 \Rightarrow l = -6$ and $m = 2$
So the separate equations are $3x - 4y - 6 = 0$ and $3x - 4y + 2 = 0$

The distance between the parallel lines is $\pm\frac{2+6}{\sqrt{3^2+4^2}} = \frac{8}{5}$ units

Question 15.
Show that the equation $4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$ represents a pair of parallel lines. Find the distance between them.

Solution:
$4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$
$a = 4$,
$b = 1$,
$h = 4/2 = 2$
$h^2 - ab = 2^2 - (4)(1) = 4 - 4 = 0$

⇒ The given equation represents a pair of parallel lines.
To find the separate equations $4x^2 + 4xy + y^2 = (2x + y)^2$
So, $4x^2 + 4xy + y^2 - 6x - 3y - 4 = (2x + y + l)(2x + y + m)$
Coefficient of x ⇒ $2m + 2l = -6 \Rightarrow l + m = -3$ ……. (1)

Coefficient of y ⇒ $l + m = -3$ ……… (2)
Constant term ⇒ $l\,m = -4$ ……… (3)
Now $l + m = -3$ and $lm = -4 \Rightarrow l = -4, m = 1$
So the separate equations are $2x + y + 1 = 0$ and $2x + y - 4 = 0$

The distance between the parallel lines is $\pm\frac{1+4}{\sqrt{4+1}} = \frac{5}{\sqrt{5}} = \sqrt{5}$ units

Question 16.
Prove that one of the straight lines given by $ax^2 + 2hxy + by^2 = 0$ will bisect the angle between the co-ordinate axes if $(a + b)^2 = 4h^2$.

Solution:
Let the slopes be l and m
∵ One line bisects the angle between the coordinate axes ⇒ $\theta = 45°$
So $\tan\theta = 1$

The slopes are l and m

Sum of the slopes $= -\frac{2h}{b}$

$$\Rightarrow 1+m=-\frac{2h}{b}\Rightarrow m=-\frac{2h}{b}-1=-\frac{2h-b}{b}$$

Product of the slopes $= \frac{a}{b}\Rightarrow(1)(m)=\frac{a}{b}\Rightarrow m=\frac{a}{b}$

$$\Rightarrow\frac{-2h-b}{b}=\frac{a}{b}\Rightarrow a=-2h-b\Rightarrow a+b=-2h$$

So $(a+b)^2=(-2h)^2=4h^2$

$(i.e)$ $(a+b)^2=4h^2$

Question 17.

If the pair of straight lines $x^2 - 2kxy - y^2 = 0$ bisect the angle between the pair of straight lines $x^2 - 2lxy - y^2 = 0$, show that the latter pair also bisects the angle between the former.

Solution:

Given that $x^2 - 2kxy - y^2 = 0$ …….. (1)

Bisect the angle between the lines $x^2 - 11xy - y^2 = 0$ …… (2)

Equation of the angle bisector of line (2) is $\left(\frac{x^2-y^2}{a+b}=\frac{xy}{h}\right)$

$(i.e)$ $\frac{x^2-y^2}{2}=\frac{xy}{-l}$

$$\Rightarrow x^2-y^2=\frac{-2xy}{l}\Rightarrow x^2-\frac{2}{l}xy-y^2=0 \qquad \text{...............(3)}$$

Equations (2) and (3) represents the same

$$\Rightarrow\frac{1}{1}=\frac{-2k}{2/l}=\frac{-1}{-1}\Rightarrow 1=\frac{-2lk}{2}=1\Rightarrow lk=-1$$

To prove that $x^2-2lxy-y^2=0$ is the angle bisector of $x^2-2kxy-y^2=0$

Equation of the bisector of the lines $x^2-2kxy-y^2=0$ is

$$\left(\frac{x^2-y^2}{a+b}=\frac{xy}{h}\right)\Rightarrow\frac{x^2-y^2}{2}=\frac{xy}{-2k}\Rightarrow x^2-y^2=\frac{2xy}{-k}$$

$(i.e)\ x^2 - y^2 = -\frac{2xy}{k}$

$(i.e)\ x^2 + \frac{2xy}{k} - y^2 = 0$

Using $lk = -1$ we get $k = -1/l$.

So, $x^2 + \frac{2xy}{k} - y^2 = 0$ becomes $x^2 - 2lxy - y^2 = 0$ is the bisector of the pair of straight lines $x^2 - 2kxy - y^2 = 0$

Question 18.
Prove that the straight lines joining the origin to the points of intersection of $3x^2 + 5xy - 3y^2 + 2x + 3y = 0$ and $3x - 2y - 1 = 0$ are at right angles.

Solution:
The equation of the pair of straight lines is
$3x^2 + 5xy - 3y^2 + 2x + 3y = 0$ ……… (1)
The given line is $3x - 2y - 1 = 0$
$3x - 2y = 1$ ……….. (2)

The equation of the straight lines joining the origin to the points of intersection of the pair of lines (1) and the line (2) is obtained by homogeneous using equation (1) by using equation (2)

(1) $\Rightarrow (3x^2 + 5xy - 3y^2) + (2x + 3y)(1) = 0$
$(3x^2 + 5xy - 3y^2) + (2x + 3y)(3x - y) = 0$
$3x^2 + 5xy - 3y^2 + 6x^2 - 4xy + 9xy - 6y^2 = 0$
$9x^2 + 10xy - 9y^2 = 0$ ………. (3)

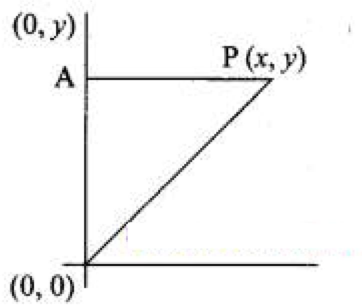
Coefficient of x^2 + coefficient of $y^2 = 9 - 9 = 0$
∴ The pair of straight line (3) represents a perpendicular straight lines.

Ex 6.5

Choose the correct or more suitable answer

Question 1.
The equation of the locus of the point whose distance from y-axis is half the distance from origin is ……..
(a) $x^2 + 3y^2 = 0$
(b) $x^2 - 3y^2 = 0$
(c) $3x^2 + y^2 = 0$
(d) $3x^2 - y^2 = 0$

Solution:
(c) $3x^2 + y^2 = 0$
Hint:
Given that PA = \([\frac{1}{2}/latex]OP
2PA = OP



$4PA^2 = OP^2$
$4(x)^2 = x^2 + y^2 \Rightarrow 3x^2 - y^2 = 0$

Question 2.
Which of the following equation is the locus of $(at^2, 2at)$ ……
(a) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (b) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (c) $x^2 + y^2 = a^2$ (d) $y^2 = 4ax$

Solution:
(d) $y^2 = 4ax$
Hint:
Given $x = at^2$, $y = 2at$

y = 2at ⇒ t = [latex]\frac{y}{2 a}\)

x = at2 ⇒ x = a × $\left(\frac{y}{2a}\right)^2$

x = a × $\frac{y^2}{4a^2}$ = $\frac{y^2}{4a^2}$

$y^2 = 4ax$

Question 3.
Which of the following point lie on the locus of $3x^2 + 3y^2 - 8x - 12y + 17 = 0$?
(a) (0, 0)
(b) (-2, 3)
(c) (1, 2)
(d) (0, -1)

Solution:
(c) (1, 2)
Hint:
The equation of the given locus is
$3x^2 + 3y^2 - 8x - 12y + 17 = 0$
(0, 0) does not lie on the locus since the locus contains constant term.

Substituting (-2, 3) in the locus
$3(-2)^2 + 3(3)^2 - 8 \times -2 - 12 \times 3 + 17$
$= 3 \times 4 + 3 \times 9 + 16 - 36 + 17$
$= 12 + 27 + 16 - 36 + 17 \neq 0$
∴ (- 2, 3) does not lie on the locus

Substituting (1, 2) on the locus
$3(1)^2 + 3(2)^2 - 8 \times 1 - 12 \times 2 + 17$
$= 3 + 12 - 8 - 24 + 17$
$= 32 - 32$
$= 0$
∴ (1, 2) lies on the locus

Question 4.
If the point (8, –5) lies on the locus $\frac{x^2}{16} - \frac{y^2}{25} = k$ then the value of k is ....

(a) 0
(b) 1

(c) 2
(d) 3

Solution:
(d) 3

$$\text{Sub } (x, y) = (8, -5) \Rightarrow \frac{\cancel{64}^{4}}{\cancel{16}} - \frac{25}{25} = k$$

$$4 - 1 = k \Rightarrow k = 3$$

Question 5.
Straight line joining the points (2, 3) and (-1, 4) passes through the point (α, β) if
(a) α + 2β = 7
(b) 3α + β = 9
(c) α + 3β = 11
(d) 3α + 3β = 11

Solution:
(c) α + 3β = 11
Hint:
Equation joining (2, 3), (-1, 4)

$$\frac{y-4}{3-4} = \frac{x+1}{2+1} \Rightarrow \frac{y-4}{-1} = \frac{x+1}{3}$$

3y – 12 = – x -1 ⇒ x + 3y – 11 = 0, (α, β) lies on it ⇒ α + 3β – 11 = 0.

Question 6.
The slope of the line which makes an angle 45° with the line 3x – y = – 5 are
(a) 1, -1
(b) $\frac{1}{2}, -2$
(c) $1, \frac{1}{2}$
(d) $2, -\frac{1}{2}$

Solution:
(c) $1, \frac{1}{2}$

Hint:
Equation of line $3x - y = -5$, $y = 3x + 5$, $m_1 = 3$
Angle between two lines $\theta = 45°$

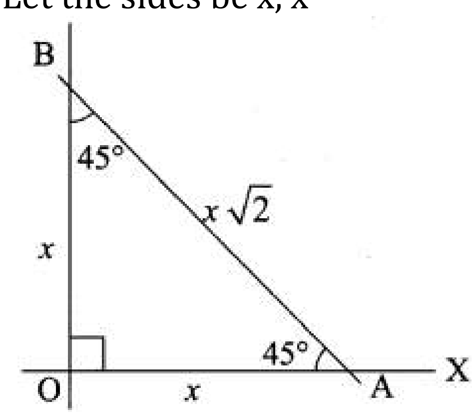
$$\tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = 45° \Rightarrow \frac{m_1 - m_2}{1 + m_1 m_2} = \pm \tan 45°$$

$\frac{3 - m_2}{1 + 3m_2} = 1$	$\frac{3 - m_2}{1 + 3m_2} = -1$
$3 - m_2 = 1 + 3m_2$	$3 - m_2 = -1 - 3m_2$
$2 = 4m_2$	$2m_2 = -4$
$m_2 = 2/4$	$m_2 = -\frac{4}{2}$
$m_2 = 1/2$	$m_2 = -2$

The slopes are $-2, \frac{1}{2}$

Question 7.
Equation of the straight line that forms an isosceles triangle with coordinate axes in the I-quadrant with perimeter $4+2\sqrt{2}$ is
(a) $x + y + 2 = 0$
(b) $x + y - 2 = 0$
(c) $x + y - \sqrt{2} = 0$
(d) $x + y + \sqrt{2} = 0$

Solution:
(b) $x + y - 2 = 0$
Hint.
Let the sides be x, x



$\therefore AB = \sqrt{x^2 + x^2} = x\sqrt{2}$, Perimeter $= 4 + 2\sqrt{2}$

$2x + x\sqrt{2} = 4 + 2\sqrt{2}$

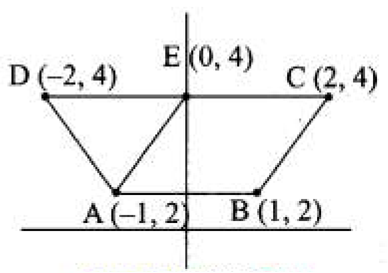
$x(2 + \sqrt{2}) = 2(2 + \sqrt{2})$

x, y intercept $= 2$, Equation of line is $\frac{x}{2} + \frac{y}{2} = 1$

$x + y = 2 \Rightarrow x + y - 2 = 0$

Question 8.
The coordinates of the four vertices of a quadrilateral are (-2, 4), (-1, 2), (1, 2) and (2, 4) taken in order. The equation of the line passing through the vertex (-1, 2) and dividing the quadrilateral into the equal areas is ....
(a) x + 1 = 0
(b) x + y = 1
(c) x + y + 3 = 0
(d) x – y + 3 = 0

Solution:
(b) x + y = 1
Hint:
This equation passes through (-1, 2)
-1 + 2 = 1 ⇒ 1 = 1



Question 9.
The intercepts of the perpendicular bisector of the line segment joining (1, 2) and (3, 4) with coordinate axes are ....
(a) 5, -5
(b) 5, 5
(c) 5, 3
(d) 5, -4

Solution:
(b) 5, 5

$(1, 2), (3, 4) \Rightarrow \text{Mid Point} = \left(\frac{1+3}{2}, \frac{2+4}{2}\right) = (2, 3)$

$\text{Slope} = \frac{4-2}{3-1} = \frac{2}{2} = 1$; $\text{Perpendicular Slope} = \frac{-1}{m} = -1$

Equation of the perpendicular bisector is $y - y_1 = m(x - x_1)$

$y - 3 = -1\,(x - 2) \Rightarrow y - 3 = -x + 2 \Rightarrow x + y = 5 \Rightarrow \frac{x}{5} + \frac{y}{5} = 1$

x, y intercepts are (5, 5)

Question 10.
The equation of the line with slope 2 and the length of the perpendicular from the origin equal to √5 is ……
(a) x + 2y = √5
(b) 2x + y = √5
(c) 2x + y = 5
(d) x + 2y – 5 = 0

Solution:
(c) 2x + y = 5

Hint: $y = 2x + c$

$$\text{Perpendicular distance from origin} = \frac{c}{\sqrt{a^2 + b^2}} = \sqrt{5}$$

$$\frac{c}{\sqrt{1+4}} = \sqrt{5} \Rightarrow c = \sqrt{5} \cdot \sqrt{5} = 5$$

The required line is y = 2x + 5 ⇒ 2x – y + 5 = 0

Question 11.
A line perpendicular to the line 5x – y = 0 forms a triangle with the coordinate axes. If the area of the triangle is 5 sq. units, then its equation is …….

(a) $x + 5y \pm 5\sqrt{2} = 0$
(b) $x - 5y \pm 5\sqrt{2} = 0$
(c) $5x + y \pm 5\sqrt{2} = 0$
(d) $5x - y \pm 5\sqrt{2} = 0$

Solution:
(a) $x + 5y \pm 5\sqrt{2} = 0$

Hint:
Equation of a line perpendicular to 5x – y = 0 is

$$x + 5y = k, \ \frac{x}{k} + \frac{y}{\frac{k}{5}} = 1$$

x, y intercepts are $k, \ \frac{k}{5}$

Area = 5

$$\frac{1}{2} k \times \frac{k}{5} = 5, k^2 = 50, k^2 = \pm 5\sqrt{2}$$

Equation of line is $x + 5y \pm 5\sqrt{2} = 0$

Question 12.
Equation of the straight line perpendicular to the line x – y + 5 = o, through the point of intersection the y-axis and the given line …….
(a) x – y – 5 = 0
(b) x + y – 5 = 0
(c) x + y + 5 = 0
(d) x + y + 10 = 0

Solution:
(b) x + y – 5 = 0
Hint:
x – y + 5 = 0 ⇒ put x = 0, y = 5
The point is (0, 5)
Equation of a line perpendicular to x – y + 5 = 0 is x + y + k = 0
This passes through (0, 5)
k = -5
x + 7 – 5 = 0

Question 13.
If the equation of the base opposite to the vertex (2, 3) of an equilateral triangle is x + y = 2, then the length of a side is ………

(a) $\sqrt{\frac{3}{2}}$ (b) 6 (c) $\sqrt{6}$ (d) $3\sqrt{2}$

Solution: (b) √6
Hint:

In an equilateral, Δ the perpendicular wall bisects the base into two equal parts. Length of the perpendicular drawn from (2, 3) to the line x + 7 – 2 = 0

$$= \left|\frac{2+3-2}{\sqrt{1+1}}\right| = \left|3/\sqrt{2}\right|$$

This is equal to $\frac{3}{\sqrt{2}} = \frac{\sqrt{3}a}{2}$

$$a = 6/\sqrt{6} = \sqrt{6}$$

Question 14.
The line (p + 2q) x + (p – 3q)y = p – q for different values of p and q passes through the point ……

(a) $\left(\frac{3}{2}, \frac{5}{2}\right)$ (b) $\left(\frac{2}{5}, \frac{2}{5}\right)$ (c) $\left(\frac{3}{5}, \frac{3}{5}\right)$ (d) $\left(\frac{2}{5}, \frac{3}{5}\right)$

Solution:
(d) $\left(\frac{2}{5}, \frac{3}{5}\right)$

Hint:
(p + 2 q)x + (p – 3q)y = p – q
px + 2qx + py – 3qy = p – q
P(x + y) + q (2x – 3y) = p – q
The fourth option x = 2/5, y = 3/5

$$p\left(\frac{2}{5}+3/5\right)+q\left(\frac{4}{5}-\frac{9}{5}\right)=p(5/5)+q\left(\frac{-5}{5}\right)$$

= p – q = RHS

Question 15.
The point on the line 2x – 3y = 5 is equidistance from (1, 2) and (3, 4) is …
(a) (7, 3)
(b) (4, 1)
(c) (1, -1)
(d) (-2, 3)

Solution:
(b) (4, 1)
Hint:
Let (a, b) be on 2x – 3y = 5 ⇒ 2a – 3b = 5
It is equidistance from (1, 2) and (3, 4)

$$\sqrt{(a-1)^2+(b-2)^2}=\sqrt{(a-3)^2+(b-4)^2}$$

$(a - 1)^2 + (b - 2)^2 = (a - 3)^2 + (6 - 4)^2$
$a^2 - 2a + 1 + b^2 - 4b + 4 = a^2 - 6a + 9 + b^2 - 8b + 16$
4a + 4b = 20
2a+ 2b = 10
2a – 3b = 5
5b = 5
b = 1 ∴ a = 4
∴ The point is (4, 1)

Question 16.
The image of the point (2, 3) in the line y = – x is ………
(a) (-3, -2)
(b) (-3, 2)
(c) (-2, -3)
(d) (3, 2)

Solution:
(a) (-3, -2)

Hint: $$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{-2(ax_1+by_1+c)}{\sqrt{a^2+b^2}}$$

$$\frac{x-2}{1}=\frac{y-3}{1}=\frac{-2(2+3)}{1+1}$$

$$\frac{x-2}{1}=\frac{y-3}{1}=\frac{-\cancel{10}\,5}{\cancel{2}}$$

x – 2 = -5, y – 3 = -5
x = -3, y = -2
(-3, -2)

Question 17.
The length of ⊥ from the origin to the line $\frac{x}{3}-\frac{y}{4}=1$ is ……

(a) $\frac{11}{5}$ (b) $\frac{5}{12}$ (c) $\frac{12}{5}$ (d) $-\frac{5}{12}$

Solution:

(c) 12/5
Hint:
4x – 3y = 12 ⇒ 4x – 3y – 12 = 0

$$d=\left|\frac{-12}{\sqrt{16+9}}\right|=\frac{12}{5}$$

Question 18.
The y-intercept of the straight line passing through (1, 3) and perpendicular to 2x – 3y + 1 = 0 is ……..

(a) $\frac{3}{2}$ (b) $\frac{9}{2}$ (c) $\frac{2}{3}$ (d) $\frac{2}{9}$

Solution:
(b) 9/2
Hint:
Equation of a line perpendicular to 2x – 3y + 1 = 0 is 3x + 2y = k. It passes through (1, 3).
3 + 6 = k ⇒ k = 9, 3x + 2y = 9
To find y-intercept x = 0, 2y = 9, y = 9/2

Question 19.
If the two straight lines x + (2k – 7)y + 3 = 0 and 3kx + 9y – 5 = 0 are perpendicular then the value of k is ……

(a) $k=3$ (b) $k=\frac{1}{3}$ (c) $k=\frac{2}{3}$ (d) $k=\frac{3}{2}$

Solution:
(a) k = 3
Hint.

$$x + (2k - 7)y + 3 = 0 \Rightarrow m_1 = \frac{-a}{b} = \frac{-1}{2k-7}$$

$$3kx + 9y - 5 = 0,\ m_2 = -\frac{3k}{9} = -\frac{k}{3}$$

Since the lines are perpendicular $m_1 m_2 = -1$

$$\left(\frac{-1}{2k-7}\right)\left(-\frac{k}{3}\right) = -1 \Rightarrow k = -3(2k-7) \Rightarrow k = -6k + 21 \Rightarrow 7k = 21 \Rightarrow k = 21/7 = 3$$

Question 20.
If a vertex of a square is at the origin and it's one side lies along the line 4x + 3y – 20 = 0, then the area of the square is ……..
(a) 20 sq. units
(b) 16 sq. units
(c) 25 sq. units
(d) 4 sq. units

Solution:
(b) 16 sq. units
Hint:
One side of a square = Length of the perpendicular from (0, 0) to the line.

$$= \left|\frac{20}{\sqrt{16+9}}\right| = \left|\frac{20}{5}\right|,\ a = 4.\ \text{Area of square} = a^2 = 16 \text{ sq. units}$$

Question 21.
If the lines represented by the equation $6x^2 + 41xy - 7y^2 = 0$ make angles α and β with the x-axis, then tan α tan β =

(a) $-\frac{6}{7}$ (b) $-\frac{6}{7}$ (c) $-\frac{7}{6}$ (d) $\frac{7}{6}$

Solution:
(a) $-\frac{6}{7}$
Hint.
$6x^2 + 41xy - 7y^2 = 0$
$\Rightarrow 6x^2 - xy + 42xy - 7y^2 = 0$
$\Rightarrow x(6x - y) + 7y(6x - y) = 0$
$\Rightarrow (x + 7y)(6x - y) = 0$

$\Rightarrow x + 7y = 0, 6x - y = 0$

$$m_1 = \frac{-1}{7}, m_2 = \frac{-6}{-1} = 6$$

$$\tan\alpha = -\frac{1}{7}, \tan\beta = 6$$

$$\tan\alpha \tan\beta = (-1/7)(6)$$

$$\tan\alpha \tan\beta = {}^{-6}\!/_{7}$$

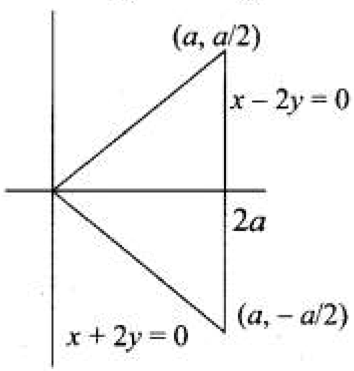
Question 22.
The area of the triangle formed by the lines $x^2 - 4y^2 = 0$ and $x = a$ is ....

(a) $2a^2$ (b) $\frac{\sqrt{3}}{2}a^2$ (c) $\frac{1}{2}a^2$ (d) $\frac{2}{\sqrt{3}}a^2$

Solution:
(c) $\frac{1}{2}a^2$

Hint:
$x^2 - 4y^2 = 0$, $(x - 2y)(x + 2y) = 0 \Rightarrow x - 2y = 0, x + 2y = 0$

$$\text{Area} = \frac{1}{2}bh = \frac{1}{2} \times a \times a = {}^{a^2}\!/_{2}$$



Question 23.
If one of the lines given by $6x^2 - x + 4x^2 = 0$ is $3x + 4y = 0$, then c equals to ……

(a) -3
(b) -1
(c) 3
(d) 1

Solution:
(a) -3
Hint.
$6x^2 - xy + 4cy^2 = 0$, $3x + 4y = 0$
The other line may be $(2x + by)$
$(3x + 4y)(2x + by) = 6x^2 - xy + 4cy^2$
$6x^2 + 3xby + 8xy + 4by^2 = 6x^2 - xy + 4cy^2$
$6x^2 + xy(3b + 8) + 4by^2 = 6x^2 - xy + 4cy^2$
paring, $3b + 8 = -1$
$3b = -9 \Rightarrow b = -3$
$4b = 4c \Rightarrow 4(-3) = 4c$
$-12 = 4c \Rightarrow c = -3$

Question 24.
θ is acute angle between the lines $x^2 - xy - 6y^2 = 0$ then $\dfrac{2\cos\theta + 3\sin\theta}{4\sin\theta + 5\cos\theta}$ is ....

(a) 1 (b) $-\dfrac{1}{9}$ (c) $\dfrac{5}{9}$ (d) $\dfrac{1}{9}$

Solution:
(c) 5/9

Hint:
Hint: $x^2 - xy - 6y^2 = 0$, $a = 1, b = -6, h = -1/2$

$$Q = \tan^{-1}\left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$$

$$Q = \tan^{-1}\left|\frac{2\sqrt{\frac{1}{4} + 6}}{-5}\right| \Rightarrow Q = \tan^{-1}\left|\frac{2 \times 5/2}{5}\right|$$

$$Q = \tan^{-1}(1)\left(\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}\right)$$
$$Q = 45^\circ$$

$$\frac{2\cos\theta + 3\sin\theta}{4\sin\theta + 5\cos\theta} = \frac{\frac{2}{\sqrt{2}} + \frac{3}{\sqrt{2}}}{\frac{4}{\sqrt{2}} + \frac{5}{\sqrt{2}}} = \frac{\frac{5}{\cancel{\sqrt{2}}}}{\frac{9}{\cancel{\sqrt{2}}}} = \frac{5}{9}$$

Question 25.
The equation of one the line represented by the equation $x^2 + 2xy \cot \theta - y^2 = 0$ is ....
(a) $x - y \cot\theta = 0$
(b) $x + y \tan \theta = 0$
(e) $x \cos \theta + y(\sin \theta + 1) = 0$
(d) $x \sin \theta + y(\cos \theta + 1) = 0$

Solution:
(d) $x \sin \theta + y(\cos \theta + 1)=0$

Hint: $x^2 + 2xy\cot\theta - y^2 = 0 \Rightarrow x^2 + 2xy\cot\theta = y^2 \Rightarrow \frac{x^2}{y^2} + \frac{2x}{y}\cot\theta = 1$

By completing the squares

$$\left(\frac{x}{y} + \cot\theta\right)^2 = 1 + \cot^2\theta$$

$$\left(\frac{x}{y} + \cot\theta\right)^2 = \operatorname{cosec}^2\theta$$

$$\frac{x}{y} + \cot\theta = \pm \operatorname{cosec}\theta$$

$x + y\cot\theta = y\operatorname{cosec}\theta$	$x + y\cot\theta = -y\operatorname{cosec}\theta$
$x + y\frac{\cos\theta}{\sin\theta} = \frac{y}{\sin\theta}$	$x + y\frac{\cos\theta}{\sin\theta} = -\frac{y}{\sin\theta}$
$x\sin\theta + y\cos\theta = y$	$x\sin\theta + y\cos\theta = -y$
	$x\sin\theta + y\cos\theta + y = 0$
	$x\sin\theta + y(\cos\theta + 1) = 0$