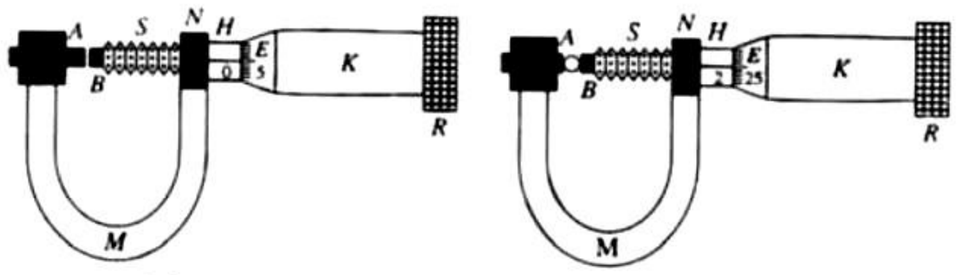
JEE Advanced

Single Correct Answer Type

1. The circular scale of a screw gauge has 50 divisions and pitch of 0.5 mm. Find the diameter of sphere. Main scale reading is 2.



a. 1.2 mm **b.** 1.25 mm
c. 2.20 mm **d.** 2.25 mm

(IIT-JEE 2006)

2. A student performs an experiment for determination of $g\left[=\frac{4\pi^2 L}{T^2}\right]$, $L \approx 1$ m, and he commits an error of ΔL. For T he takes the time of n oscillations with the stop watch of least count ΔT. For which of the following data, the measurement of g will be most accurate?

a. $\Delta L = 0.5, \Delta T = 0.1, n = 20$
b. $\Delta L = 0.5, \Delta T = 0.1, n = 50$
c. $\Delta L = 0.5, \Delta T = 0.01, n = 20$
d. $\Delta L = 0.5, \Delta T = 0.05, n = 50$ **(IIT-JEE 2006)**

3. A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of 0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with an uncertainty of 0.01 mm. Take $g = 9.8$ m s^{-2} (exact). The Young's modulus obtained from the reading is

a. $(2.0 \pm 0.3) \times 10^{11}$ N m^{-2}
b. $(2.0 \pm 0.2) \times 10^{11}$ N m^{-2}
c. $(2.0 \pm 0.1) \times 10^{11}$ N m^{-2}
d. $(2.0 \pm 0.05) \times 10^{11}$ N m^{-2} **(IIT-JEE 2007)**

4. Students I, II, and III perform an experiment for measuring the acceleration due to gravity (g) using a simple pendulum. They use different lengths of the pendulum and/or record time for different number of oscillations. The observations are shown in the following table. Least count for length = 0.1 cm. Least count for time = 0.1 s.

Student	Length of Pendulum (cm)	Number of Oscillations (n)	Total Time for n Oscillations (s)	Time Period (s)
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	4	36.0	9.0

If E_I, E_{II}, and E_{III} are the percentage errors in g, i.e., $\left(\frac{\Delta g}{g} \times 100\right)$ for students I, II, and III, respectively, then

a. $E_I = 0$ **b.** E_I is minimum
c. $E_I = E_{II}$ **d.** E_{II} is maximum

(IIT-JEE 2008)

5. A Vernier callipers has 1 mm marks on the main scale. It has 20 equal divisions on the vernier scale, which match with 16 main scale divisions. For this Vernier callipers, the least count is

a. 0.02 mm **b.** 0.05 mm **c.** 0.1 mm **d.** 0.2 mm

(IIT-JEE 2010)

6. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is

a. 0.9% **b.** 2.4%
c. 3.1% **d.** 4.2% **(IIT-JEE 2011)**

7. In the determination of Young's modulus $\left(Y = \frac{4MLg}{\pi l d^2}\right)$ by using Searle's method, a wire of length $L = 2$ m and diameter $d = 0.5$ mm is used. For a load $M = 2.5$ kg, an extension $l = 0.25$ mm in the length of the wire is observed. Quantities d and l are measured using a screw gauge and a micrometer, respectively. They have the same pitch of 0.5 mm. The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement

a. due to the errors in the measurements of d and l are the same.
b. due to the error in the measurement of d is twice that due to the error in the measurement of l.
c. due to the error in the measurement of l is twice that due to the error in the measurement of d.
d. due to the error in the measurement of d is four times that due to the error in the measurement of l.

(IIT-JEE 2012)

8. The diameter of a cylinder is measured using a Vernier callipers with no zero error. It is found that the zero of the Vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The Vernier scale has 50 divisions equivalent to 2.45 cm. The 24th division of the Vernier scale exactly coincides with one of the main scale divisions. The diameter of the cylinder is

a. 5.112 cm **b.** 5.124 cm
c. 5.136 cm **d.** 5.148 cm

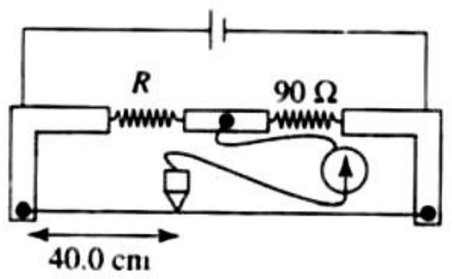
(JEE Advanced 2013)

9. Using the expression $2d \sin\theta = \lambda$, one calculates the values of d by measuring the corresponding angles θ in the range $0°$ to $90°$. The wavelength λ is exactly known and the error in θ is constant for all values of θ. As θ increases from $0°$,

a. the absolute error in d remains constant.
b. the absolute error in d increases.
c. the fractional error in d remains constant.
d. the fractional error in d decreases.

(JEE Advanced 2013)

10. During an experiment with a metre bridge, the galvanometer shows a null point when the jockey is pressed at 40.0 cm using a standard resistance of 90 Ω, as shown in the figure. The least count of the scale used in the metre bridge is 1 mm. The unknown resistance is



a. $60 \pm 0.15\Omega$ **b.** $135 \pm 0.56\Omega$
c. $60 \pm 0.25\Omega$ **d.** $135 \pm 0.23\Omega$

(JEE Advanced 2014)

Multiple Correct Answer Type

1. A student performed the experiment of determination of focal length of a concave mirror by $u - v$ method using an optical bench of length 1.5 meter. The focal length of the mirror used is 24 cm. The maximum error in the location of the image can be 0.2 cm. The 5 sets of (u, v) values recorded by the student (in cm) are: (42, 56), (48,48), (60, 40), (66, 33), (78, 39). The data set(s) that **cannot** come from experiment and is (are) incorrectly recorded is (are)

a. (42, 56) **b.** (48, 48)
c. (66, 33) **d.** (78, 39)

(IIT-JEE 2009)

2. A student uses a simple pendulum of exactly 1 m length to determine g, the acceleration due to gravity. He uses a stop watch with the least count of 1 s for this and records 40 s for 20 oscillations. For this observation, which of the following statement(s) is(are) true?

a. Error ΔT in measuring T, the time period, is 0.05 s
b. Error ΔT in measuring T, the time period, is 1 s
c. Percentage error in the determination of g is 5%
d. Percentage error in the determination of g is 2.5%

(IIT-JEE 2010)

3. Consider a vernier callipers in which each 1 cm on the main scale is divided into 8 equal divisions and a screw gauge with 100 divisions on its circular scale. In the vernier callipers, 5 divisions of the vernier scale coincide with 4 division on the main scale and in the screw gauge, one complete rotation of the circular scale moves it by two divisions on the linear scale. Then,

a. If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm.
b. If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm.
c. If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm.
d. If the least count of the linear scale of the screw gauge is twice the least count of the vernier callipers, the least count of the screw gauge is 0.005 mm.

(JEE Advanced 2015)

Integer Answer Type

1. During Searle's experiment, zero of the Vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale. The 20^{th} division of the Vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{2} m of the main scale but now the 45^{th} division of Vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is 8×10^{7} m^2. The least count of the Vernier scale is 1.0×10^{-5} m. The maximum percentage error in the Young's modulus of the wire is **(JEE Advanced 2014)**

Subjective Type

1. The nth division of main scale coincides with $(n + 1)^{th}$ divisions of Vernier scale. Given one main scale division is equal to a units. Find the least count of the vernier.

(IIT-JEE 2003)

2. In a Searle's experiment. the diameter of the wire as measured by a screw gauge of least count 0.001 cm is 0.050 cm. The length. measured by a scale of least count 0.1 cm, is 110.0 cm. When a weight of 50 N is suspended from the wire, the extension is measured to be 0.125 cm by a micrometer of least count 0.001 cm. Find the maximum error in the measurement of Young's modulus of the material of the wire from these data.

(IIT-JEE 2004)

3. The side of a cube is measured by vernier callipers (10 divisions of a vernier scale coincide with 9 divisions of main scale, where 1 division of main scale is 1 mm). The main scale reads 10 mm and first division of vernier scale coincides with the main scale. Mass of the cube is 2.736 g. Find the density of the cube in appropriate significant figures.

(IIT-JEE 2005)

ANSWER KEY

JEE Advanced

Single Correct Answer Type

1. a.	**2.** d.	**3.** b.	**4.** b.	**5.** d.
6. c.	**7.** a.	**8.** b.	**9.** d.	**10.** c.

Multiple Correct Answers Type

1. c, d.	**2.** a, c.	**3.** b, c.

Integer Answer Type

1. 4

Subjective Type

1. $\frac{a}{n+1}$

2. 4.89%

3. 2.66 g/cm^3

HINTS AND SOLUTIONS

JEE Advanced

Single Correct Answer Type

1. a. Least count $= \dfrac{\text{Pitch}}{\text{Number of division on circular scale}} = \dfrac{0.5}{50}$

$= 0.01\,\text{mm}$

Now, diameter of ball $= (2 \times 0.5) + (25 - 5)\,(0.01) = 1.2$ mm

2. d. $\dfrac{\Delta g}{g} = \dfrac{\Delta l}{l} + 2\dfrac{\Delta T}{nT}$ (Δl and ΔT are least, and the number of readings are maximum.)

3. b. We know that $Y = \dfrac{mg}{\pi\dfrac{D^2}{4}} \times \dfrac{L}{l}$

$$\frac{\Delta Y}{Y} = \frac{2\Delta D}{D} + \frac{\Delta l}{l}$$

[Since the values of m, g and L are exact]

$$= 2\frac{0.01}{0.4} + \frac{0.05}{0.8}$$

$$= 2 \times 0.025 + 0.0625$$

$$= 0.05 + 0.0625 = 0.1125$$

$$\Delta Y = 2 \times 10^{11} \times 0.1125 = 0.225 \times 10^{11}$$

[Since the value of $Y = 2 \times 10^{11}$ in all the four options]

4. b. $g = 4\pi^2\left(\dfrac{l}{T^2}\right) \Rightarrow \dfrac{\Delta g}{g} = \dfrac{\Delta l}{l} + 2\dfrac{\Delta T}{T}$

$$\Rightarrow \quad E = \left(\frac{\Delta l}{l} + 2\frac{\Delta T}{T}\right) \times 100$$

The greater the value of T, the lesser the error. Hence, fractional error in the first observation is minimum.

5. d. 16 MSD = 20 VSD

$\Rightarrow$ 1 VSD = 4/5 MSD

LC = 1 MSD − 1 VSD

$$= \left(1 - \frac{4}{5}\right)\text{MSD} = \left(1 - \frac{4}{5}\right)(1\text{ mm}) = 0.2\text{ mm}$$

6. c. Least count $= \dfrac{0.5}{50} = 0.01$ mm

Diameter of ball $D = 2.5$ mm $+ (20)(0.01) = 2.7$ mm

$$\rho = \frac{M}{\text{Vol}} = \frac{M}{\frac{4}{3}\pi\left(\frac{D}{2}\right)^3}$$

$$\left(\frac{\Delta\rho}{\rho}\right) = \frac{\Delta m}{m} + 3\frac{\Delta D}{D} = 2\% + 3\left(\frac{0.01}{2.7}\right) \times 100\% = 3.1\%$$

7. a. $\Delta d = \Delta l = \dfrac{0.5}{100}$ mm

$$y = \frac{4MLg}{\pi l d^2}$$

$$\left(\frac{\Delta y}{y}\right)_{\max} = \frac{\Delta l}{l} + 2\frac{\Delta d}{d}$$

Error due to l measurement, $\dfrac{\Delta l}{l} = \dfrac{0.5/100\,\text{mm}}{0.25\,\text{mm}}$

Error due to d measurement, $2\dfrac{\Delta d}{d} = \dfrac{2 \times \dfrac{0.5}{100}}{0.5\,\text{mm}} = \dfrac{0.5/100}{0.25}$

So error in y due to l measurement = error in y due to d measurement

8. b. Main scale division (s) = .05 cm

Vernier scale division $(v) = \dfrac{49}{1000} = 0.049$

Least count = .05 − .049 = .001 cm

Diameter: $5.10 + 24 \times .001 = 5.124$ cm

9. d. $d = \dfrac{\lambda}{2\sin\theta}$

$$\ln d = \ln\left(\frac{\lambda}{2}\right) - \ln\sin\theta$$

$$\frac{\Delta d}{d} = 0 - \frac{\cos\theta\, d\theta}{\sin\theta}$$

$$\left(\frac{\Delta d}{d}\right)_{\max} = \pm\cot\theta\Delta\theta$$

Also $(\Delta d)_{\max} = d\cot\theta\Delta\theta$

$$= \frac{\lambda}{2\sin\theta}\cot\theta\Delta\theta = \frac{\lambda}{2}\frac{\cos\theta}{\sin^2\theta}\Delta\theta$$

As θ increases, $\cot\theta$ decreases and $\dfrac{\cos\theta}{\sin^2\theta}$ also decreases.

10. c. $\frac{R}{40} = \frac{90}{60}$

$\therefore \quad R = 60\ \Omega$

$$\frac{\Delta R}{R} = \frac{\Delta \ell_1}{\ell_1} + \frac{\Delta \ell_2}{\ell_2} = \frac{0.1}{40} + \frac{0.1}{60} = \frac{5}{1200} = \frac{1}{240}$$

$$\Delta R = \frac{R}{240} = \frac{60}{240} = \frac{1}{4} = 0.25$$

Hence unknown resistance = $60 \pm 0.25\ \Omega$

Multiple Correct Answers Type

1. c., d.

We know $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

Hence $\frac{1}{-|f|} = \frac{1}{-|v|} + \frac{1}{-|u|} \quad \Rightarrow \quad |v| = \frac{|u||f|}{|u| - |f|}$

For $|u| = 42$ cm

We get $|v| = \frac{(42)(24)}{(42-24)} = 56\text{cm}$

So (42, 56) is correct observation.

For $|u| = 66$ cm, we get

$$|v| = \frac{(66)\times(24)}{(66-24)} = 36\text{cm}$$

Hence (66, 33) is not correct observation

For $|u| = 24$ cm, we get

$$|v| = \frac{(48)\times(24)}{(48-24)} = 48\text{cm}$$

Hence (48, 48) is correct observation

For $|u| = 78$ cm, we get

$$|v| = \frac{(78)\times(24)}{(78-24)} = 32\text{cm}$$

Hence (78, 39) is not correct observation

2. a., c. $T = 40/20 = 2$ s

$$\frac{\Delta T}{T} = \frac{\Delta t}{t} = \frac{1}{40} \Rightarrow \Delta T = \frac{T}{40} = \frac{2}{40} = 0.05\text{ s}$$

$$g = \frac{4\pi^2 L n^2}{t^2} \quad \text{where } t = nT$$

$$\frac{\Delta g}{g} = \frac{2\Delta t}{t} \Rightarrow \%\text{error} = \frac{2\Delta t}{t} \times 100 = 5\%$$

3. b., c.

For vernier Calliper.

$$1\text{MSD} = \frac{1}{8}\text{ cm}$$

$$1\text{ VSD} = \frac{4}{5} \times \frac{1}{8} = \frac{1}{10}\text{ cm}$$

Least Count = 1 M.S.D – 1 V.S.D.

$$= \frac{1}{8} - \frac{1}{10} = \frac{1}{40}\text{ cm}$$

$$= 0.25\text{ mm}$$

For screw gauge

Let Least Count. of linear scale be l

Pitch = $2l$

$$\text{Least Count} = \frac{\text{pitch}}{100} = \frac{2l}{100}$$

If pitch = 2 × L.C. of V.C. = 0.5 mm.

$$\text{Least Count of screw gauge} = \frac{0.5}{100} = 0.005\text{ mm}$$

If l = 2 × L.C. of V.C. = 0.5 mm

$$\text{Least Count of screw gauge} = \frac{2\times 0.5}{100} = 0.01\text{ mm}$$

Integer Answer Type

1. (4)

Value of mass scale division = $3.25 \times 10^{-2} - 3.20 \times 10^{-2}$ m = 0.05×10^{-2} m = 0.05 cm = 0.5 mm

Least count (L.C.) = 1.0×10^{-5}m

From 1st observation L_1 = main scale reading (MSR) + 20 (LC)

$L_i = (3.2 \times 10^{-2} + 20 \times 1.0 \times 10^{-5})$ m

$= (3.2 + 0.02) \times 10^{-2} = 3.22 \times 10^{-2}$ m

From 2nd observation L_2 = MSR + 45 (LC)

$L_f = (3.2 + 45 \times 10^{-3}) \times 10^{-2}$ m

$= 3.245 \times 10^{-2}$ m

Change in length

$x = L_f - L_i = 0.025 \times 10^{-2}\text{ m} = 25 \times 10^{-5}$ m

$$\frac{\Delta Y}{Y}\% = \frac{\Delta X}{X}\%$$

$\Delta x = LC = 1 \times 10^{-5}$m

Hence $\frac{\Delta Y}{Y}\% = \frac{1\times 10^{-5}}{25\times 10^{-5}} = 4\%$

We are assuming l, F, A to be known with proper accuracy.

Subjective Type

1. $(n + 1)$ divisions of vernier scale = n divisions of main scale.

$\therefore$ One vernier division = $\frac{n}{n+1}$ main scale division

$$\text{Least count} = 1\text{ MSD} - 1\text{ VSD} = \left(1 - \frac{n}{n+1}\right)\text{MSD}$$

$$= \frac{1}{n+1}\text{MSD} = \frac{a}{a+1}$$

2. Maximum percentage error in Y is given by $Y = \frac{W}{\frac{\pi D^2}{4}} \times \frac{L}{X}$

$$\left(\frac{\Delta Y}{Y}\right)_{\max} = 2\left(\frac{\Delta D}{D}\right) + \frac{\Delta x}{x} + \frac{\Delta L}{L}$$

$$= 2\left(\frac{0.001}{0.05}\right) + \left(\frac{0.001}{0.125}\right) + \left(\frac{0.1}{110}\right) = 0.0489\ .$$

So maximum percentage error = 4.89 %

3. Least count of vernier calliper

$$= \frac{1\text{ division of main scale}}{\text{number of division in vernier scale}} = \frac{1}{10} = 0.1\text{ mm}$$

The side of cube= 10 mm + 1 × 0.1 mm = 1.01 cm

Now, density = $\frac{\text{mass}}{\text{volume}} = \frac{2.736\,g}{(1.01)^3\text{cm}^3} = 2.66\text{ g/cm}^3$ (to correct significant figures)