DAY TWENTY EIGHT

Ray Optics

Learning & Revision for the Day

- Reflection of Light
- Mirror formula
- Refraction of Light
- Lens
- Total Internal Reflection (TIR)
- Deviation by a Prism
- Dispersion by a Prism
- Human Eye
- Optical Instruments
- Resolving Power of an Optical Instrument

Reflection of Light

The phenomena of bouncing back of light on striking a smooth surface is called reflection of light.

- According to the laws of reflection, (i) the incident ray, reflected ray and the normal drawn on the reflecting surface at the point of incidence lie in the same plane and (ii) the angle of incidence $\angle i$ = angle of reflection $\angle r$.
- Laws of reflection are true for reflection from a polished mirror or from an unpolished surface or for diffused reflection.
- Whenever reflection takes place from a denser medium, the reflected rays undergo a phase change of π.

Reflection from a Plane Mirror

- If a ray is incident on a plane mirror at an angle of incidence i, then it suffers a deviation of $(\pi - 2i)$ and for two inclined plane mirrors deviation is $(360° - 2\theta)$.
- While keeping an object fixed, a plane mirror is rotated in its plane by an angle θ, then the reflected ray rotates in the same direction by an angle 2θ.
- Focal length as well as the radius of curvature of a plane mirror is infinity. Power of a plane mirror is zero.
- If two plane mirrors are inclined to each other at an angle θ, the total number of images formed of an object kept between them, is $n = \frac{2\pi}{\theta}$ or $\left(\frac{2\pi}{\theta} - 1\right)$, when it is odd.
- The minimum size of a plane mirror fixed on a wall of a room, so that a person at the centre of the room may see the full image of the wall behind him, should be 1/3rd the size of the wall.

Reflection from a Spherical Mirror

- A spherical mirror is a part of a hollow sphere whose one surface is polished, so that it becomes reflecting. The other surface of the mirror is made opaque.

- Images formed by a concave mirror may be real or virtual, may be inverted or erect, and may be smaller, larger or equal in size to that of the object. The image is virtual and erect when the object is placed between the pole and the principal focus of concave mirror. In all other cases, the image formed is real and inverted one.
- Image formed by a convex mirror is virtual, erect and diminished in size irrespective of the position of the object. Moreover, image is formed in between the pole and the principal focus of the mirror.
- The focal length of a spherical mirror is half of its radius of curvature, i.e. $f = \frac{R}{2}$.

Mirror Formula

Let an object be placed at a distance u from the pole of a mirror and its image is formed at a distance v from the pole.

Then, according to mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

- The power of a mirror (in dioptre), is given as

$$P = \frac{1}{f(\text{in metre})}$$

- If a thin object of height h is placed perpendicular to the principal axis of a mirror and the height of its image be h', then the transverse or lateral magnification produced is given by

$$m = \frac{h'}{h} = -\frac{v}{u} = \frac{f}{f-u} = \frac{f-v}{f}$$

Negative sign of magnification means the inverted image and positive sign means an erect image.

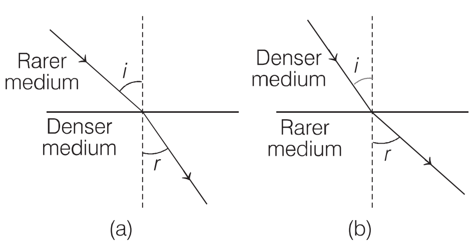
- When a small sized object is placed along the principal axis, then its longitudinal (or axial) magnification is given by

$$\text{Axial magnification} = -\frac{dv}{du} = \left(\frac{v}{u}\right)^2 = \left(\frac{f}{f-u}\right)^2 = \left(\frac{f-v}{f}\right)^2$$

Refraction of Light

When light passes from one medium, say air, to another medium, say glass, a part is reflected back into the first medium and the rest passes into the second medium. When it passes into the second medium, it either bends towards the normal or away from the normal.

This phenomenon is known as refraction.



Refractive Index

For a given pair of media, the ratio of the sine of angle of incidence (i) to the sine of angle of refraction (r) is a constant, which is called the refractive index of second medium, w.r.t. first medium.

Thus, $$\frac{\sin i}{\sin r} = \text{constant} = n_{21} = \frac{n_2}{n_1}$$

This is also called **Snell's law**.

- Refractive index is a unitless, dimensionless and a scalar quantity.
- The refractive index of a medium w.r.t. vacuum (or free space) is known as its absolute refractive index. It is defined as the ratio of the speed of light in vacuum (c) to the speed of light in a given medium (v).

  $\therefore$ $$n = \frac{c}{v}$$

  Value of absolute refractive index of a medium can be 1 or more than 1, but never less than 1.
- When light travels from one material medium to another, the ratio of the speed of light in the first medium to that in the second medium is known as the relative refractive index of second medium, w.r.t. the first medium. Thus,

$$n_{21} = \frac{v_1}{v_2} = \frac{c/v_2}{c/v_1} = \frac{n_2}{n_1}$$

- When light undergoing refraction through several media finally enters the first medium itself, then

$$n_{21} \times n_{32} \times n_{13} = 1 \text{ or } n_{32} = \frac{n_{31}}{n_{21}}$$

- When the object is in denser medium and the observer is in rarer medium, then real and apparent depth have the relationship, $\frac{\text{Real depth}}{\text{Apparent depth}} = n_{21}$

  i.e. real depth > apparent depth shift, $y = h - h' = \left(1 - \frac{1}{n_{21}}\right)h$

  where, h and h' are real and apparent depths.

Refraction from a Spherical Surface

Let an object be placed in a medium of refractive index n_1 at a distance u from the pole of a spherical surface of radius of curvature R and after refraction, its image is formed in a medium of refractive index n_2 at a distance v, then

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

The relation is true for all surfaces, whether the image formed is real or virtual.

Lens

A lens is part of a transparent refracting medium bound by two surfaces, with atleast one of the two surfaces being a curved one. The curved surface may be spherical or cylindrical.

The lens formula is given by $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

For a thin object of height h placed perpendicular to the principal axis at a distance u, if the height of image formed is h', then lateral or transverse magnification m is given by

$$m = \frac{h'}{h} = \frac{v}{u} = \frac{f}{f+u} = \frac{f-v}{f}$$

For a small sized object placed linearly along the principal axis, its **axial** or **longitudinal magnification** is given by

$$\text{Axial magnification} = -\frac{dv}{du} = \left(\frac{v}{u}\right)^2 = \left(\frac{f}{f+u}\right)^2 = \left(\frac{f-v}{f}\right)^2$$

Silvering of Lens

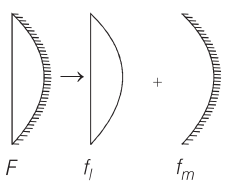
When one surface of a lens is silvered, it behaves as a mirror. The focal length of silvered lens is

$$\frac{1}{F} = \frac{2}{f_l} + \frac{1}{f_m}$$

In case of plano-convex lens

- When **curved surface is silvered**, then focal length of silvered lens is

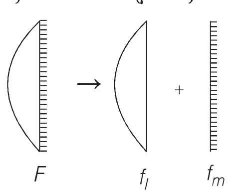
$$F = \frac{R}{2\mu}$$



where, $R = 2f_m$ or $R = f_l(\mu - 1)$

- When **plane surface is silvered**, then

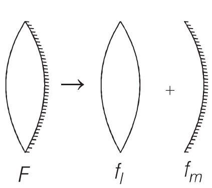
$$F = \frac{R}{2(\mu - 1)} \quad \text{and } f_l = \frac{R}{(\mu - 1)}, f_m = \infty$$



- When **double convex lens is silvered**,

$$F = \frac{R}{2(2\mu - 1)} \quad \text{and} \quad f_l = \frac{R}{2(\mu - 1)}$$

$$\Rightarrow \quad f_m = \frac{R}{2}.$$



Power of a Lens

The power of a lens is mathematically given by the reciprocal of its focal length, i.e. power $P = \frac{1}{f(\text{m})}$

SI unit of power is dioptre (D). Power of a converging lens is positive and that of a diverging lens is negative.

Lens Maker's Formula

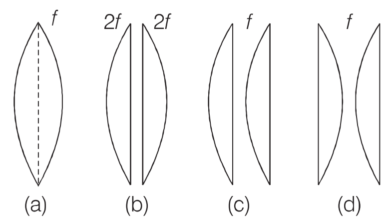
For a lens having surfaces with radii of curvature R_1 and R_2 respectively, its focal length is given by

$$P = \frac{1}{f} = (n_{21} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where, $n_{21} = \frac{n_2}{n_1}$ = refractive index of the lens material w.r.t. the surroundings.

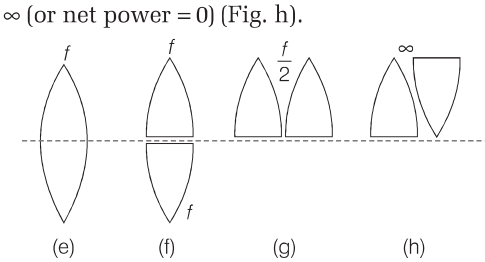
Cutting and Combination of a Lens

If a symmetrical convex lens of focal length f is cut into two parts along its optic axis, then focal length of each part (a plano-convex lens) is $2f$ [fig(b)]. However, if the two parts are joined as shown in the figure (c) and (d), the focal length of the combination is again f.

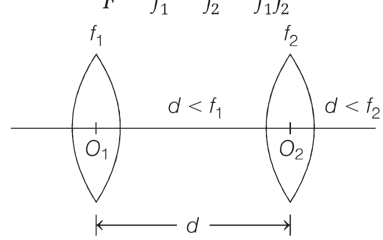


If a symmetrical convex lens of focal length f is cut into two parts along the principal axis, then the focal length of each part remains unchanged, as f[Fig. f].

If these two parts are joined with the curved ends on one side, the focal length of the combination is $\frac{f}{2}$[Fig. g]. But on joining the two parts in opposite sense, the net focal length becomes ∞ (or net power = 0) (Fig. h).



- The equivalent focal length of co-axial combination of two lenses is given by $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$



- If a number of lenses are in contact, then

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \ldots$$

- If two thin lenses of focal lengths f_1 and f_2 are in contact, then their equivalent focal length

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$$

In terms of power, $P_{eq} = P_1 + P_2$

Important Results

- Effective diameter of light transmitting area is called aperture. Intensity of image $\propto$ (Aperture)^2
- Relation between object and image speed, $v_i = \left(\frac{f}{f+u}\right)^2 v_o$
- Newton's formula states, $f^2 = x_1 x_2$ where, x_1 and x_2 = distance of object and image from first and second principlal foci. This formula is also called thin lens formula.
- If lens immersed in a liquid, then

$$\frac{f_l}{f_a} = \frac{({}_a\mu_g - 1)}{({}_l\mu_g - 1)} = \frac{({}_a\mu_g - 1)}{\left(\frac{{}_a\mu_g}{{}_a\mu_l} - 1\right)}$$

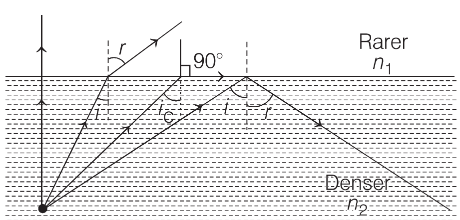
- Displacement method for finding focal length of lens is

$$f = \frac{D^2 - x^2}{4D}$$

Total Internal Reflection (TIR)

When a ray of light goes from a denser to a rarer medium, it bends away from the normal.

For a certain angle of incidence i_C, the angle of refraction in rarer medium becomes 90°. The angle i_C is called the **critical angle**.



$$\sin i_C = \frac{n_1}{n_2} = \frac{1}{n_{21}}$$

For the angle of incidence greater than the critical angle ($i > i_C$) in the denser medium, the light ray is totally internally reflected back into the denser medium itself.

Conditions for Total Internal Reflection

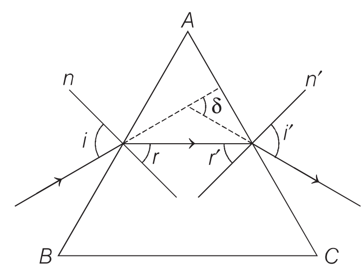
- The light ray should travel from the denser medium towards the rarer medium.
- The angle of incidence should be the greater than the critical angle.

Common Examples of Total Internal Reflection

- **Looming** An optical illusion in cold countries.
- **Mirage** An optical illusion in deserts.
- **Brilliance of diamond** Due to repeated internal reflections diamond sparkles.
- **Optical fibre** Each fibre consists of core and cladding. The refractive index of core material is higher than that of cladding. Light entering at small angle on one end undergoes repeated total internal reflections along the fibre and finally comes out.

Deviation by a Prism

A prism is a homogeneous, transparent medium bounded by two plane surfaces inclined at an angle A with each other. These surfaces are called as refracting surfaces and the angle between them is called angle of prism A.



Deviation produced by a prism is, $\delta = i + i' - A \Rightarrow r + r' = A$

For grazing incidence $i = 90°$

and grazing emergence $i' = 90°$

For minimum deviation

(i) $i = i'$ and $r = r'$ (ii) $\mu = \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin\frac{A}{2}}$

In case of minimum deviation, ray is passing through prism symmetrically.

For maximum deviation (δ_{max}), $i = 90°$ or $i' = 90°$

For thin prism, $\delta = (\mu - 1)A$

Dispersion by a Prism

Dispersion of light is the phenomenon of splitting of white light into its constituent colours on passing light through a prism. This is because different colours have different wavelength and hence different refractive indices.

Angular dispersion $= \delta_v - \delta_r = (n_v - n_r)A$

where, n_v and n_r represent refractive index for violet and red lights.

Dispersive power, $\omega = \frac{n_v - n_r}{n - 1}$, where $n = \frac{n_v + n_r}{2}$ is the mean refractive index.

By combining two prisms with angle A and A' and refractive index n and n' respectively, we can create conditions of

- Dispersion without deviation when, $A' = -\frac{(n-1)A}{(n'-1)}$
- Deviation without dispersion when,

$$A' = -\left[\frac{n_v - n_r}{n'_v - n'_r}\right]A$$

Refraction Through a Prism

A ray of light suffers two refractions at the two surfaces on passing through a prism. Angle of deviation through a prism $\delta = i + e - A$.
where, i is the angle of incidence, e is the angle of emergence and A is the angle of prism.

Scattering of Light

Molecules of a medium after absorbing incoming light radiations, emit them in all directions. This phenomenon is called scattering. According to Rayleigh, intensity of scattered light $\propto \frac{1}{\lambda^4}$.

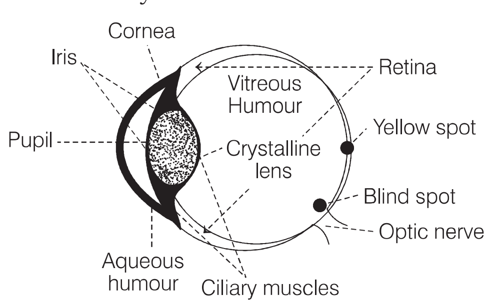
There are some phenomenon based on scattering

- Sky looks blue due to scattering.
- At the time of sunrise and sunset, sun looks reddish.
- Danger signals are made of red colour.

Human Eye

The human eye is one of the most valuable and sensitive sense organs of human being. The cornea, acqueous humour, crystalline lens and vitreous humour together form a lens system which forms an inverted and real image on retina for the objects situated in front of eye.

The optic nerve transmits this image to the brain which makes it erect and analyses it.



The eye lens has power of accommodation to adjust its focal length, so as to focus objects situated at different distances from eye at the retina.

- The least distance of distinct vision (D) or near point of an eye is generally 25 cm. Far point of a normal eye is at infinity. Ciliary muscles play an important role in changing focal length of eye lens.

  Persistence of vision is $\frac{1}{10}$ s, i.e. if time interval between two consecutive light pulses is lesser than 0.1 s, eye cannot distinguish them separately.

  Resolving limit for eye is $1' = \left[\frac{1}{60}\right]^\circ$

- A person suffering from myopia can see near objects clearly but can not see distant objects. For a **myopic** (near sighted) eye near point is at 25 cm, but far point comes closer to eye from infinite. Causes of near sightedness are
  (i) decrease in focal length of eye lens, or
  (ii) elongation of eye ball. The defect can be rectified by use diverging lens.

  If a myopic person cannot see objects situated beyond a distance x, then he should use lens of focal length $f = -x$.

  So, concave lens is used to correct this.

- A person suffering from **hypermetropia** or **long sightedness** can see distant objects clearly, but cannot see nearby objects distinctly. It means that his near point has shifted away from 25 cm to distance x. Possible causes of hypermetropia are
  (i) increase in focal length of eye lens, or
  (ii) shortening of eye ball.

  To rectify hypermetropia, the person should use a converging lens of focal length f, so that it forms virtual image of an object situated at distance ($D = 25\,\text{cm}$) at near point x of defective eye, $f = \frac{xD}{x - D}$.

  So, convex lens is used for its correction.

Some Other Defects of Vision

- **Presbyopia** is due to weakness of ciliary muscles due to advancing age. Generally, defective eye suffers from near sightedness as well as long sightedness. To rectify the defect bifocal lenses are used.
- **Astigmatism** is due to non-uniformity of curvature of the cornea. The defective eye cannot concentrate simultaneously along horizontal as well as vertical. To rectify this defect, cylindrical lenses are used.

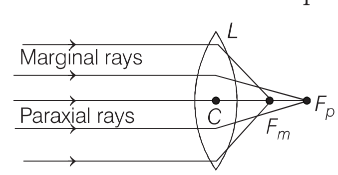
Defects of Images

The two types of defects which are occurred commonly, given below

Spherical Aberration

This defect is present in spherical mirrors as well as in lenses, whose aperture is comparatively large.

In mirrors, the spherical aberration can be almost eliminated by taking a paraboloidal mirror instead of a spherical mirror.



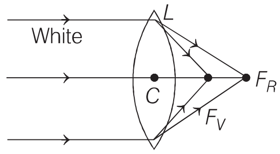
In a lens, we can minimise the aberration by

(a) using plano-convex lens with plane surface on the side of more convergent/divergent light beam.
(b) by using a combination of two lenses separated by a distance $d = f_1 - f_2$.

Chromatic Aberration

This defect is present only in lenses.

Chromatic aberration is the inability of a lens to focus rays of different colours/ wavelengths at a single point.



Longitudinal chromatic aberration = $f_R - f_V = \omega f$, where ω = dispersive power of lens material and f = mean focal length of the lens.

Chromatic aberration can be removed by preparing a combination of two thin lenses of different materials (of dispersive powers ω and ω') and different focal lengths f and f', such that

$$\frac{\omega}{f} + \frac{\omega'}{f'} = 0$$

Optical Instruments

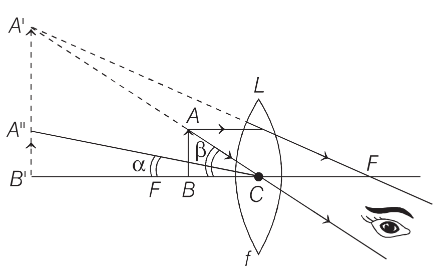
An optical instrument is used to enhance and analyses the light waves. The light waves are in the form of photons, hence optical instruments also determine the characteristics properties of light waves.

Microscope

It is an optical instrument which forms a magnified image of a small nearby object and thus, increases the visual angle subtended by the image at the eye, so that the object is seen to be bigger and distinct.

1. Simple Microscope (Magnifying Glass)

It consists of a single convex lens of small focal length and forms a magnified image of an object placed between the optical centre and the principal focus of the lens.



If the image is formed at the near point of eye, then $m = \left(1 + \frac{D}{f}\right)$

But, if the image is formed at infinity, then

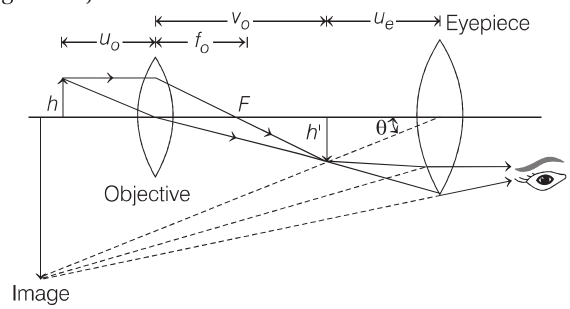
$$m = \frac{D}{f}$$

where, D = normal viewing distance (25 cm),

f = focal length of magnifying lens.

2. Compound Microscope

It consists of two lenses of small focal length and small apertures. Also, the focal length and aperture of objective lens are smaller than that of eyepiece. The image formed by the objective lens is real, inverted and magnified. This image acts as the object for the eyepiece and the final image is highly magnified, virtual and inverted w.r.t. the original object.



If m_o and m_e be the magnifications produced by the objective and the eyepiece respectively, then total magnification of microscope $m = m_o \times m_e$.

If final image is formed at the near point *(D)* of the eye, then

$$m = -\frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right)$$

or $$m = -\frac{L}{f_o}\left(1 + \frac{D}{f_e}\right) \text{(approx)}$$

If final image is formed at infinity, then

$$m = -\frac{v_o}{u_o} \cdot \frac{D}{f_e} = -\frac{L}{f_o} \cdot \frac{D}{f_e} \text{ (approx)}$$

Length of tube of microscope, $L = v_o + u_e$.

NOTE
- Huygens' eyepiece is free from chromatic and spherical aberration, but it cannot be used for measurement purposes.
- Ramsden's eyepiece can be used for precise measurement as cross wires can be fixed in this eyepiece. It slightly suffers from spherical and chromatic aberrations.

Telescope

Telescope is an optical instrument which increases, the visual angle at the eye by forming the image of a distant object at the least distance of distinct vision, so that the object is seen distinct and bigger.

1. Refracting Telescope

It consists of an objective lens of large focal length f_o and large aperture.

The eyepiece consists of a convex lens of small aperture and small focal length f_e. Distance between the two lenses is set as, $L = f_o + f_e$

In normal adjustment, the final image is formed at infinity and magnifying power of the telescope is

$$m = -\frac{f_o}{f_e}$$

In practical adjustment, the final image is formed at the near point of the observer's eye. In this arrangement,

$$m = -\frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$$

2. Reflecting Telescope

It consists of an objective which is a large paraboloid concave mirror of maximum possible focal length f_o and the eyepiece is a convex lens of small focal length and small aperture, then

Magnifying power, $m = -\frac{f_o}{f_e}$

Reflecting type telescope is considered superior as it is free from spherical and chromatic aberrations, is easy to install and maintain, and can produce image of greater intensity.

NOTE
- The large aperture of telescope objective, helps in forming a brighter image.
- If diameter of pupil of human eye is d and that of telescope be D, then image formed by telescope will be $\left(\frac{D}{d}\right)^2$ times brighter than the image of the same object, seen directly by the unaided eye.

Resolving Power of an Optical Instrument

Resolving power of an optical instrument is its ability to produce distinct images of two points of an object (or two nearby objects) very close together. Resolving power of an optical instrument is inverse of its limit of resolution. Smaller the limit of resolution of a device, higher is its resolving power. Limit of resolution of a normal human eye is 1′ .

The minimum distance (or angular distance) between two points of an object whose images can be formed distinctly by the lens of an optical instrument, is called its **limit of resolution**.

Resolving Power of a Telescope

If the aperture (diameter) of the telescope objective be the D, then the minimum angular separation ($d\theta$) between two distant objects, whose images are just resolved by the telescope, is

$$d\theta = \frac{1.22\lambda}{D}$$

and resolving power of the telescope,

$$\text{RP} = \frac{1}{d\theta} = \frac{D}{1.22\lambda}$$

Resolving Power of a Microscope

The least distance (d) between two points, whose images are just seen distinctly by a microscope is given by

$$d = \frac{1.22\lambda}{2n_m \sin\theta}$$

where,

λ = wavelength of light used to illuminate the object,

n_m = refractive index of the medium between the object and the objective lens, and

θ = semi angle of the cone of light from the point object.

The term $n_m \sin\theta$ is generally called the numerical aperture of the microscope.

$\therefore$ Resolving power of the microscope,

$$\text{RP} = \frac{1}{d} = \frac{2\,n_m \sin\theta}{1.22\lambda} = \frac{(\text{NA})}{0.61\lambda}$$

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 A ray of light is successively deflected from two plane mirrors inclined to each other at a certain angle. If the total deviation in the path of the rays reflected from the two mirrors be 300°, then what is the number of images formed?

(a) 30 (b) 15 (c) 11 (d) 5

2 A beam of light from a source L is incident normally on a plane mirror fixed at a certain distance x from the source. The beam is reflected back as a spot on a scale placed just above the source L. When the mirror is rotated through a small angle θ, the spot of the light is found to move through a distance y on the scale. The angle θ is given by ➜ NEET 2017

(a) $\frac{y}{2x}$ (b) $\frac{y}{x}$ (c) $\frac{x}{2y}$ (d) $\frac{x}{y}$

3 A concave mirror forms the real image of an object which is magnified 4 times. The object is moved 3 cm away, the magnification of the image is 3 times. What is the focal length of the mirror?

(a) 3 cm (b) 4 cm (c) 12 cm (d) 36 cm

4 The image formed by a convex mirror of focal length 30 cm is a quarter of the size of the object. The distance of the object from the mirror is

(a) 30 cm (b) 90 cm (c) 120 cm (d) 60 cm

5 The frequency of a light wave in a material is 2×10^{14} Hz and wavelength is 5000 Å. The refractive index of material will be

(a) 1.40 (b) 1.50 (c) 3.00 (d) 1.33

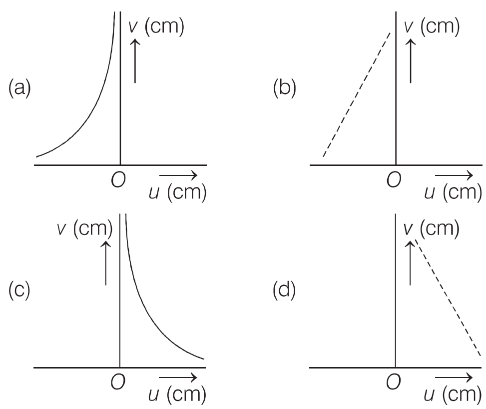
6 A glass slab $(\mu = 1.5)$ of thickness 6 cm is placed over a paper. What is the shift in the letters?

(a) 4 cm (b) 2 cm
(c) 1 cm (d) None of these

7 An air bubble in a glass slab with refractive index 1.5 (near normal incidence) is 5 cm deep when viewed from one surface and 3 cm deep when viewed from the opposite face. The thickness (in cm) of the slab is → NEET 2016

(a) 8 (b) 10 (c) 12 (d) 16

8 A student measures the focal length of a convex lens by putting an object pin at a distance *u* from the lens, and measuring the distance *v* of the image pin. The graph between *u* and *v* plotted by the student should look like.



9 A lens has focal length 10 cm. An object is placed 15 cm in front of it. Where should a convex mirror be placed, so that image is formed at the object itself, when focal length of convex mirror is 12 cm?

(a) 6 cm from lens (b) 8 cm from lens
(c) 5 cm from lens (d) 4 cm from lens

10 The distance of the image from the focus of a lens is *x* and that of object is *y*. What is the nature of the graph *y versus x*?

(a) Straight line (b) Ellipse
(c) Parabola (d) Hyperbola

11 A lens when placed on a plane mirror, then object needle and its image coincide at 15 cm. The focal length of the lens is

(a) 15 cm (b) 30 cm
(c) 20 cm (d) ∞

12 A concave mirror of focal length f_1 is placed at a distance of *d* from a convex lens of focal length f_2. A beam of light coming from infinity and falling on this convex lens-concave mirror combination returns to infinity. The distance *d* must be equal → CBSE AIPMT 2012

(a) $f_1 + f_2$ (b) $-f_1 + f_2$ (c) $2f_1 + f_2$ (d) $-2f_1 + f_2$

13 A convex lens and a concave lens, each having same focal length of 25 cm, are put in contact to form a combination of lenses. The power in dioptres of the combination is

(a) 25 (b) 50 (c) infinite (d) zero

14 If in a plano-convex lens, radius of curvature of convex surface is 10 cm and the focal length of the lens is 30 cm, the refractive index of the material of the lens will be

(a) 1.5 (b) 1.66
(c) 1.33 (d) 3

15 When a biconvex lens of glass having refractive index 1.47 is dipped in a liquid, it acts as a plane sheet of glass. This implies that the liquid must have refractive index → CBSE AIPMT 2012

(a) equal to that of glass (b) less than one
(c) greater than that of glass (d) less than that of glass

16 Which of the following is not due to total internal reflection? → CBSE AIPMT 2011

(a) Difference between apparent and real depth of a pond
(b) Mirage on hot summer days
(c) Brilliance of diamond
(d) Working of optical fibre

17 If the critical angle for total internal reflection from a medium to vacuum is 30°, the velocity of light in the medium is

(a) 3×10^8 ms^{-1} (b) 1.5×10^8 ms^{-1}
(c) 6×10^8 ms^{-1} (d) $\sqrt{3} \times 10^8$ ms^{-1}

18 A ray of light travelling in a transparent medium of refractive index μ falls on a surface separating the medium from air at an angle of incidence of 45°. For which of the following value of μ, the ray can undergo total internal reflection? → CBSE AIPMT 2010

(a) $\mu = 1.33$ (b) $\mu = 1.40$ (c) $\mu = 1.50$ (d) $\mu = 1.25$

19 A hollow prism is filled with water and placed in air. It will deviate the incident rays

(a) towards the base
(b) away from base
(c) parallel to base
(d) towards or away from base depending on the location

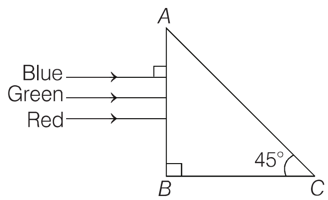
20 A thin prism of angle 7° and refractive index 1.5 is combined with another prism of angle θ and refractive index 1.7. The emergent ray goes undeviated. What is the value of θ?

(a) 3° (b) 5° (c) 9° (d) 1°

21 What is the refractive index of a prism whose angle $A = 60°$ and angle of minimum deviation $d_m = 30°$?

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) $\frac{1}{\sqrt{3}}$

22 A beam of light consisting of red, green and blue colours is incident on a right angled prism. The refractive index of the material of the prism for the above red, green and blue wavelengths are 1.39, 1.44 and 1.47, respectively. ➜ CBSE AIPMT 2015



The prism will

(a) separate the blue colour part from the red and green colours
(b) separate all the three colours from one another
(c) not separate the three colours at all
(d) separate the red colour part from the green and blue colours

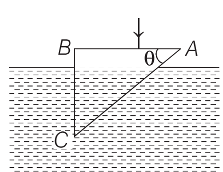
23 The angle of a prism is A. One of its refracting surfaces is silvered. Light rays falling at an angle of incidence $2A$ on the first surface returns back through the same path after suffering reflection at the silvered surface. The refractive index μ of the prism is ➜ CBSE AIPMT 2014

(a) $2 \sin A$ (b) $2 \cos A$ c) $\frac{1}{2} \cos A$ (d) $\tan A$

24 A ray of light is incident at an angle of incidence i on one face of a prism of angle A (assumed to be small) and emerges normally from the opposite face. If the refractive index of the prism is μ, the angle of incidence i, is nearly equal to ➜ CBSE AIPMT 2012

(a) μA (b) $\frac{\mu A}{2}$ (c) A/μ (d) $A/2\mu$

25 A glass prism ABC (refractive index 1.5), immersed in water (refractive index 4/3). A ray of light is incident normally on face AB. If it is totally reflected at face AC, then



(a) $\sin\theta \geq \frac{8}{9}$ (b) $\sin\theta \geq \frac{2}{3}$
(c) $\sin\theta > \frac{\sqrt{3}}{2}$ (d) $\frac{2}{3} < \sin\theta < \frac{8}{9}$

26 To correct myopia, the focal length of the concave lens should be

(a) equal to the distance of far point
(b) less than the distance of far point
(c) less than the distance of near point
(d) equal to the distance of near point

27 For a normal eye, the cornea of eye provides a converging power of 40 D and the least converging power of the eye lens behind the cornea is 20 D. Using this information, the distance between the retina and the cornea-eye lens can be estimated to be ➜ NEET 2013

(a) 5 cm (b) 2.5 m (c) 1.67 cm (d) 1.5 cm

28 A person can see clearly objects only when they lie between 50 cm and 400 cm from his eyes. In order to increase, the maximum distance of distinct vision to infinity, the type and power of the correcting lens, the person has to use, will be ➜ NEET 2016

(a) convex, + 2.25 D (b) concave, – 0.25 D
(c) concave – 0.2 D (d) convex, + 0.15 D

29 A microscope is focussed on a mark on a piece of paper and then a slab of glass of thickness 3 cm and refractive index 1.5 is placed over the mark. How should the microscope be moved to get the mark in focus again?

(a) 1 cm upward (b) 4.5 cm downward
(c) 1 cm downward (d) 2 cm upward

30 In a compound microscope, the intermediate image is

(a) virtual, erect and magnified
(b) real, erect and magnified
(c) real, inverted and magnified
(d) virtual, erect and reduced

31 A simple telescope, consisting of an objective of focal length 60 cm and a single eye lens of focal length 5 cm is focused on a distant object in such a way that parallel rays emerge from the eye lens. If the object subtends an angle of 2° at the objective, the angular width of the image is

(a) 10° (b) 24° (c) 50° (d) (1/6)°

32 The diameter of the moon is 3.5×10^3 km and its distance from the earth is 3.8×10^5 km. Seen by a telescope having focal lengths of the objective and the eyepiece as 40 mm and 1.0 cm respectively, the angular diameter of the image of the moon will be approximately

(a) 2° (b) 10°
(c) 20° (d) None of these

33 In an astronomical telescope in normal adjustment, a straight black line of length L is drawn on inside part of objective lens. The eyepiece forms a real image of this line. The length of this image is l. The magnification of the telescope is ➜ CBSE AIPMT 2015

(a) $\frac{L}{l} + 1$ (b) $\frac{L}{l} - 1$ (c) $\frac{L+1}{L-1}$ (d) $\frac{L}{l}$

34 The magnifying power of a telescope is 9. When it is adjusted for parallel rays, the distance between the objective and eyepiece is 20 cm. The focal length of lenses are ➜ CBSE AIPMT 2012

(a) 10 cm, 10 cm (b) 15 cm, 5 cm
(c) 18 cm, 2 cm (d) 11 cm, 9 cm

35 The focal length of objective lens is increased, then magnifying power of ➜ CBSE AIPMT 2014

(a) microscope will increase, but that of telescope decrease
(b) microscope and telescope both will increase
(c) microscope and telescope both will decrease
(d) microscope will decrease, but that of telescope will increase

36 The ratio of resolving powers of an optical microscope for two wavelengths $\lambda_1 = 4000$ Å and $\lambda_2 = 6000$ Å is ➜ NEET 2017

(a) 8 : 27 (b) 9 : 4
(c) 3 : 2 (d) 16 : 81

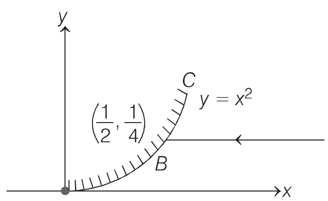
DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 A room (cubical) is made of mirrors. An insect is moving along the diagonal on the floor, such that the velocity of image of insect on two adjacent wall mirrors, is $10\,\text{cms}^{-1}$. The velocity of image of insect in ceiling mirror is

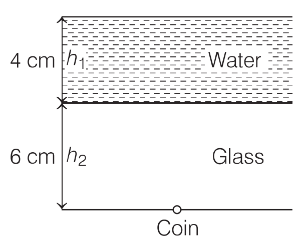
(a) $10\,\text{cms}^{-1}$ (b) $20\,\text{cms}^{-1}$
(c) $\frac{10}{\sqrt{2}}\,\text{cms}^{-1}$ (d) $10\sqrt{2}\,\text{cms}^{-1}$

2 In the given figure, the angle of reflection is



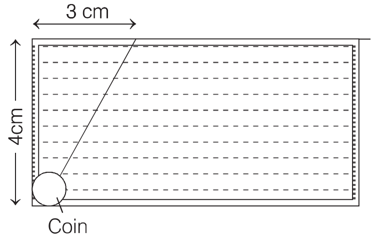
(a) 30° (b) 60°
(c) 45° (d) None of these

3 A 4 cm thick layer of water covers a 6 cm thick glass slab. A coin is placed at the bottom of the slab and is being observed from the air side along the normal to the surface. Find the apparent position of the coin from



(a) 7.0 cm (b) 8.0 cm (c) 10 cm (d) 5 cm

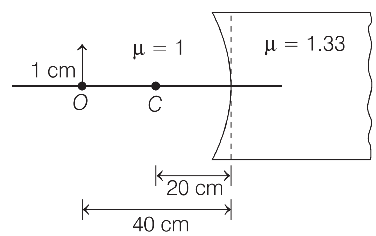
4 A small coin is resting on the bottom of a beaker filled with a liquid. A ray of light from the coin travels upto the surface of the liquid and moves along its surface as shown in figure.



How fast is the light travelling in the liquid?

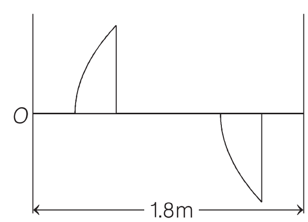
(a) $1.8\times 10^8\ \text{ms}^{-1}$ (b) $2.4\times 10^8\ \text{ms}^{-1}$
(c) $3.0\times 10^8\ \text{ms}^{-1}$ (d) $1.2\times 10^4\ \text{ms}^{-1}$

5 For an optical arrangement as shown in the figure. Find the position and nature of image.



(a) 32 cm (b) 0.6 cm (c) 6 cm (d) 0.5 cm

6 A thin plano-convex lens of focal length f is split into two halves. One of the halves is shifted along the optical axis. The separation between object and image plane is 1.8 m. The magnification of the image formed by one of the half lens is 2. Find the focal length of the lens and separation between the two halves.



(a) 0.1 m (b) 0.4 m (c) 0.9 m (d) 1 m

7 A thin convergent glass lens ($\mu = 1.5$) has a power of + 5.0 D. When this lens is immersed in a liquid of refractive index μ_l, it acts as a divergence lens of focal length 100 cm. The value of μ_l should be

(a) 3/2 (b) 4/3 (c) 5/3 (d) 2

8 A thin glass prism ($\mu = 1.5$) is immersed in water ($\mu = 1.3$). If the angle of deviation in air for a particular ray be D, then that in water will be

(a) $0.2D$ (b) $0.3D$ (c) $0.5D$ (d) $0.6D$

9 A ray of light incident at an angle θ on a refracting face of a prism emerges from the other face normally. If the angle of the prism is 5° and the prism is made of a material of refractive index 1.5, the angle of incidence is

(a) 7.5° (b) 5°
(c) 15° (d) 2.5°

10. The refracting angle of a prism is A and refractive index of the material of the prism is cot(A/2). The angle of minimum deviation is ➜ CBSE AIPMT 2015

(a) $180° - 3A$ (b) $180° - 2A$
(c) $90° - A$ (d) $180° + 2A$

11 Two identical thin plano-convex glass lenses (refractive index 1.5) each having radius of curvature of 20cm are placed with their convex surfaces in contact at the centre. The intervening space is filled with oil of refractive index 1.7. The focal length of the combination is ➜ CBSE AIPMT 2015

(a) –20 cm (b) –25 cm
(c) –50 cm (d) 50 cm

12 The radius of curvature of the curved surface of a plano-convex lens is 20 cm. If the refractive index of the material of the lens be 1.5, it will

(a) act as a convex lens only for the objects that lie on its curved side
(b) act as a concave lens for the objects that lie on its curved side
(c) act as a convex lens irrespective of the side, on which the object lies
(d) act as a concave lens irrespective of side, on which the object lies

13 An thin prism having refracting angle 10° is made of glass of refractive index 1.42. This prism is combined with another thin prism of glass of refractive index 1.7. This combination produces dispersion without deviation. The refracting angle of second prism should be ➜ NEET 2017

(a) 4° (b) 6°
(c) 8° (d) 10°

14 An astronomical telescope has objective and eyepiece of focal lengths 40 cm 4 cm, respectively. To view an object 200 cm away from the objective, the lenses must be separated by a distance ➜ NEET 2016

(a) 46.0 cm (b) 50.0 cm
(c) 54.0 cm (d) 37.3 cm

15 The angle of incidence for a ray of light at a refracting surface of a prism is 45°. The angle of prism is 60°. If the ray suffers minimum deviation through the prism, the angle of minimum deviation and refractive index of the material of the prism respectively, are ➜ NEET 2016

(a) 30°; $\sqrt{2}$ (b) 45°; $\sqrt{2}$
(c) 30°; $\frac{1}{\sqrt{2}}$ (d) 45°; $\frac{1}{\sqrt{2}}$

16 Two identical glass ($\mu_g = 3/2$) equi-convex lenses of focal length f each are kept in contact. The space between the two lenses is filled with water ($\mu_w = 4/3$). The focal length of the combination is ➜ NEET 2016

(a) $f/3$ (b) f (c) $\frac{4f}{3}$ (d) $\frac{3f}{4}$

17 A plano-convex lens fits exactly into a plano-concave lens. Their plane surfaces are parallel to each other. If lenses are made of different materials of refractive indices μ_1 and μ_2; and R is the radius of curvature of the curved surface of the lenses, then the focal length of the combination is ➜ NEET 2013

(a) $\frac{R}{2(\mu_1 + \mu_2)}$ (b) $\frac{R}{2(\mu_1 - \mu_2)}$ (c) $\frac{R}{(\mu_1 - \mu_2)}$ (d) $\frac{2R}{(\mu_2 - \mu_1)}$

18 A biconvex lens has a radius of curvature of magnitude 20 cm. Which one of the following options describe best the image formed of an object of height 2 cm placed 30 cm from the lens? ➜ CBSE AIPMT 2011

(a) Virtual, upright, height = 0.5 cm
(b) Real, inverted, height = 4 cm
(c) Real, inverted, height = 1 cm
(d) Virtual, upright, height = 1 cm

19 A lens having focal length f and aperture of diameter d forms an image of intensity I. Aperture of diameter $\frac{d}{2}$ in central region of lens is covered by a black paper. Focal length of lens and intensity of image now will be respectively ➜ CBSE AIPMT 2010

(a) f and $\frac{I}{4}$ (b) $\frac{3f}{4}$ and $\frac{I}{2}$ (c) f and $\frac{3I}{4}$ (d) $\frac{f}{2}$ and $\frac{I}{2}$

20 An astronomical refracting telescope will have large angular magnification and high angular resolution, when it has an objective lens of ➜ NEET 2018

(a) large focal length and large diameter
(b) large focal length and small diameter
(c) small focal length and large diameter
(d) small focal length and small diameter

21 The refractive index of the material of a prism is $\sqrt{2}$ and the angle of the prism is 30°. One of the two refracting surfaces of the prism is made a mirror inwards, by silver coating. A beam of monochromatic light entering the prism from the other face will retrace its path (after reflection from the silvered surface), if its angle of incidence on the prism is ➜ NEET 2018

(a) 30° (b) 45° (c) 60° (d) zero

22 An object is placed at a distance of 40 cm from a concave mirror of focal length 15 cm. If the object is displaced through a distance of 20 cm towards the mirror, the displacement of the image will be ➜ NEET 2018

(a) 30 cm towards the mirror
(b) 36 cm away from the mirror
(c) 30 cm away from the mirror
(d) 36 cm towards the mirror

Hints and Explanations

1 $\delta = 360° - 2\theta$.

Here, $\delta = 300°$,
Hence, $\theta = 30°$.
So, the number of images
$n = \frac{360}{30} - 1 = 11$.

2 When the plane mirror is rotated by an angle θ, the reflected ray or beam of light must rotate by angle 2θ, from refraction at plane surface theory.
From the figure

$$\tan 2\theta = \frac{BS}{SO} = \frac{y}{x}$$

If the angle is small $\tan 2\theta \approx 2\theta$

So, $2\theta = \frac{y}{x}$

$\Rightarrow \theta = \frac{y}{2x}$

3 In the first case, $m = \frac{f}{u - f} = 4$

In the second case, $\frac{f}{(u + 3) - f} = 3$

On solving, we find $f = 36\,\text{cm}$

4 $\therefore m = \frac{f}{f - u} \Rightarrow \frac{1}{4} = \frac{30}{30 - u} \Rightarrow u = -\,90\,\text{cm}$

5 Velocity of light waves in material is

$$v = \nu\lambda \quad \ldots\text{(i)}$$

Refractive index of material is

$$\mu = \frac{c}{v} \quad \ldots\text{(ii)}$$

where, c is speed of light in vacuum or air.

or $\mu = \frac{c}{\nu\lambda}$

Given, $\nu = 2 \times 10^{14}\,\text{Hz}$

$\lambda = 5000\,\text{Å} = 5000 \times 10^{-10}\,\text{m}$,

$c = 3 \times 10^{8}\,\text{ms}^{-1}$

This gives $\mu = 3$

6 Shift $= x\left(1 - \frac{1}{\mu}\right) = 6\left(1 - \frac{1}{3/2}\right)$

$= 6 \times \frac{1}{3} = 2\,\text{cm}$

7 Let thickness of the given slab is t. According to the question, when viewed from both the surfaces

$$\frac{x}{\mu} + \frac{t - x}{\mu} = 3 + 5 \Rightarrow \frac{t}{\mu} = 8\,\text{cm}$$

$\Rightarrow t = 8 \times \mu \Rightarrow t = 8 \times \frac{3}{2} = 12\,\text{cm}$

8 From the lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} = \text{constant}$$

u is always negative, v is positive.

9 $\frac{1}{v} = \frac{1}{10} - \frac{1}{15} = \frac{1}{30}$

$\Rightarrow v = 30\,\text{cm}$.

We have to place the convex mirror in such a way that it receives normal radiation. Hence, we shall place at a distance of radius of curvature (24 cm) from I (position of image) or 6 cm from lens.

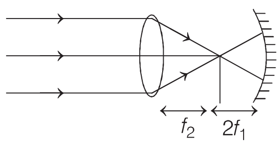
10 According to Newton's formula, $xy = f^2$.

Hence, y *versus* x graph is a rectangular hyperbola.

11 When the object is placed at the focus, the rays are parallel.

The mirror placed normal sends them back. Hence, image is formed at the object itself.

12 According to question, the ray diagram will be



So, distance d between convex lens and concave mirror is given by

$$d = 2f_1 + f_2$$

13 Focal length of combination of lenses placed in contact, is

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

For convex lens, $f_1 = 25\,\text{cm}$

For concave lens, $f_2 = -25\,\text{cm}$

Hence, $\frac{1}{F} = \frac{1}{25} + \frac{1}{(-25)}$

$$= \frac{1}{25} - \frac{1}{25} = 0$$

$\therefore \quad F = \frac{1}{0} = \infty$

Hence, power of combination,

$$P = \frac{1}{F} = 0$$

14 For plano-convex lens, we have

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R}\right)$$

Here, $f = 30\,\text{cm}$, $R = 10$ cm,

This gives $\mu = 1.33$.

15 If biconvex lens behaves like a plane sheet of glass, ray will pass undeviated through it only when medium has same refractive index as that of biconvex lens.

16 Real and apparent depth are explained on the basis of refraction only. The concept of TIR is not involve here.

17 $\therefore \mu = \frac{1}{\sin i_C} = \frac{1}{\sin 30°} = 2$

Hence, $\quad v = \frac{c}{\mu}$

$$= \frac{3 \times 10^8\,\text{ms}^{-1}}{2}$$

$$= 1.5 \times 10^8\,\text{ms}^{-1}$$

18 For total internal reflection, $i > C$

$\Rightarrow \quad \sin i > \sin C$

$\Rightarrow \quad \sin 45° > \frac{1}{\mu}$

$\Rightarrow \quad \mu > \sqrt{2} \Rightarrow \mu > 1.4$

19 The prism deviates the light rays towards its base.

20 Here, $(\mu_1 - 1) A_1 = (\mu_2 - 1)\theta$

i.e. $(1.5 - 1)7 = (1.7 - 1)\,\theta$

Hence, $\theta = 5°$.

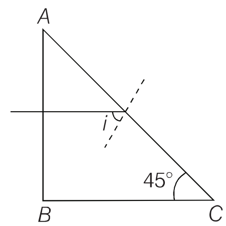
21 Refractive index of prism,

$$\mu = \frac{\sin \frac{A + \delta_m}{2}}{\sin \frac{A}{2}} = \frac{\sin \frac{60° + 30°}{2}}{\sin \frac{60°}{2}}$$

$$= \frac{\sin 45°}{\sin 30°} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}} = \frac{1}{\sqrt{2}} \times \frac{2}{1} = \sqrt{2}$$

22 For refractive index of a prism,

$$\mu = \frac{1}{\sin i_c} = \frac{1}{\sin 45°} = \sqrt{2} = 1.414$$



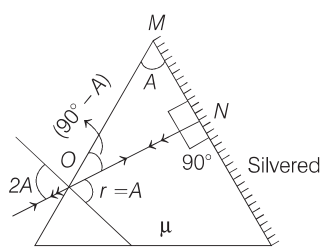
As, $\mu_{\text{red}} = 1.39$, $\mu_{\text{green}} = 1.44$ and

$\mu_{\text{blue}} = 1.47$

$\therefore (\mu_{\text{red}} = 1.39) < \mu, \mu_{\text{green}} > \mu, \mu_{\text{blue}} > \mu$

Thus, only red colour do not suffer total internal reflection.

23 According to question, diagram is shown below.



So, $\angle MON = 90° - A$

and $\angle r = 90° - (90° - A)$

$\Rightarrow \quad \angle r = A$

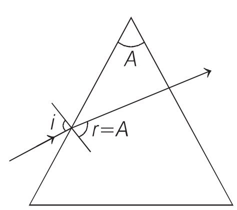
By Snell's law, $\frac{\sin i}{\sin r} = \mu \Rightarrow \frac{\sin(2A)}{\sin(A)} = \mu$

$$\frac{2\sin A \cos A}{\sin A} = \mu \Rightarrow \mu = 2\cos A$$

24 Refractive index,

$$\mu = \frac{\sin i}{\sin r}, \ \mu = \frac{\sin i}{\sin A}$$

In case of small angle, $\sin i \approx i$ and $\sin A \approx A$



$$\mu = \frac{i}{A}$$

$$i = \mu A$$

25 For total internal reflection at AC face,

$$\sin i \geq \frac{\mu_w}{\mu_g} \quad \Rightarrow \quad \sin\theta \geq \frac{4}{3 \times 1.5}$$

$$\sin\theta \geq \frac{8}{9}$$

26 To correct myopia focal length should be equal to the distance of far point.

27 Given, $P_1 = 40\,\text{D}$ and $P_2 = 20\,\text{D}$

We have, $P_{\text{eff}} = 40\,\text{D} + 20\,\text{D} = 60\,\text{D}$

$\therefore \quad f = \frac{100}{P_{\text{eff}}} = \frac{100}{60} = 1.67$ cm

28 Given, image distance, $v = 400\,\text{cm} = 4$ m

$\Rightarrow u = \infty$

Using lens equation,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$\Rightarrow \quad \frac{1}{-4} - \frac{1}{\infty} = \frac{1}{f}$

$\Rightarrow \quad f = -4$ m

Now, power of the required lens is,

$$P = \frac{1}{f} = \frac{1}{-4} = -0.25 \text{ D}$$

Thus, the person require a concave lens of power -0.25 D.

29 Apparent depth of mark as seen through a glass slab of thickness x and refractive index μ is

Apparent depth

$$= \frac{\text{real depth}}{\text{refractive index}} = \frac{3}{1.5} = 2 \text{ cm}$$

As image appears to be raised by 1 cm, therefore microscope must be moved upward by 1 cm.

30 In compound microscope, intermediate image (image formed by objective lens) is real, inverted and magnified.

31 It is a case of normal adjustment.

Hence, $M = f_o/f_e$.

Also, $\quad M = \beta/\alpha$.

Therefore, $\frac{\beta}{\alpha} = \frac{f_o}{f_e}$

Here, $f_o = 60$ cm, $f_e = 5$ cm, $\alpha = 2°$

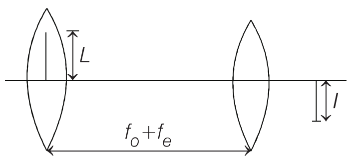
Hence, $\beta = 24°$.

32 $\because \frac{\beta}{\alpha} = \frac{f_o}{f_e}$

Here, $f_o = 40$ mm, $f_e = 1.0$ cm $= 10$ mm

$\alpha = 3.5 \times 10^3 / 3.8 \times 10^5$ rad. This gives $\beta \cong 2°$.

33



We know, magnification of telescope,

we have $M = \dfrac{f_o}{f_e}$,

Here, $\dfrac{f_e}{f_e + u} = \dfrac{-I}{L}$

$\Rightarrow \dfrac{f_e}{f_e - (f_o + f_e)} = \dfrac{-I}{L} \Rightarrow \dfrac{f_e}{f_o} = \dfrac{I}{L}$

i.e. $M = \dfrac{L}{I}$

34 Given, magnification $= \dfrac{f_o}{f_e} = 9$

and $f_o + f_e = 20 \Rightarrow f_o = 9f_e$

So, $9f_e + f_e = 20 \Rightarrow f_e = 2\text{ cm}$

$\therefore \quad f_o = 9 \times 2 \Rightarrow f_o = 18\text{ cm}$

35 Magnifying power of microscope

$$= \frac{L}{f_o}\left[1 + \frac{P}{f_e}\right]$$

Magnifying power of telescope

$$= \frac{f_o}{f_e}\left[1 + \frac{f_e}{D}\right]$$

So, the focal length of objective lens is increased, then magnifying power of microscope will decrease but that of telescope will increase.

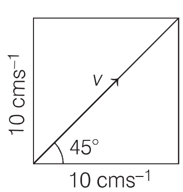
36 As, resolving power of a microscope

$$(\text{RP}) \propto \frac{1}{\lambda_{(\text{wavelength})}}$$

$$\therefore \quad \frac{\text{RP}_1}{\text{RP}_2} = \frac{\lambda_2}{\lambda_1} = \frac{6000}{4000} = \frac{3}{2}$$

SESSION 2

1 $\because v\cos 45° = 10 \Rightarrow v = 10\sqrt{2}\text{ cms}^{-1}$



In the ceiling mirror, the original velocity will be seen.

2 Given, $y = x^2$

$\therefore \quad \dfrac{dy}{dx} = 2x = 2 \times \dfrac{1}{2} = 1$

$\Rightarrow \quad \tan\theta = 1 \Rightarrow \theta = 45°$

The slope of normal at B,

$m_2 = -\dfrac{1}{m} = -\dfrac{1}{1} = -1 \Rightarrow \theta = -45°$

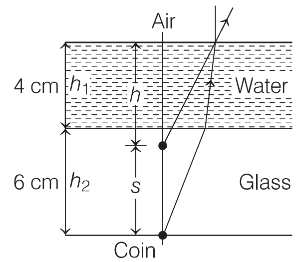
From the figure, angle of incidence is 45°. Hence, angle of reflection is 45°.

3 Using equation, the total apparent shift is

$$s = h_1\left(1 - \frac{1}{\mu_1}\right) + h_2\left(1 - \frac{1}{\mu_2}\right)$$

or $\quad s = 4\left(1 - \dfrac{1}{4/3}\right) + 6\left(1 - \dfrac{1}{3/2}\right)$

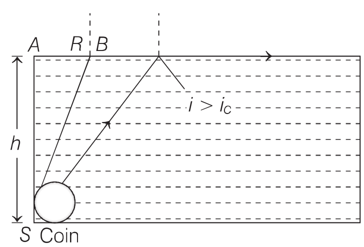
$= 3.0\text{ cm}$



Thus, $h = h_1 + h_2 - s = 4 + 6 - 3$

$= 7.0\text{ cm}$

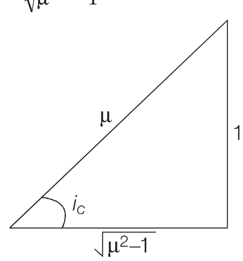
4 As shown in figure, a light ray from the coin will not emerge out of liquid, if $i > i_c$.



Therefore, minimum radius R corresponds to $i = i_c$.

In $\dfrac{R}{h} = \tan i_c$ or $R = h\tan i_c$

or $\quad R = \dfrac{h}{\sqrt{\mu^2 - 1}}$



Given, $R = 3\text{ cm}, h = 4\text{ cm} \Rightarrow \mu = \dfrac{5}{3}$

But $v = \dfrac{c}{\mu} = \dfrac{3 \times 10^8}{5/3} = 1.8 \times 10^8\text{ ms}^{-1}$

5 According to Cartesian sign convention,

$u = -40\text{ cm}, R = -20\text{ cm},$

$\mu_1 = 1, \mu_2 = 1.33$

Applying equation for refraction through spherical surface, we get

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.33}{v} - \frac{1}{(-40)} = \frac{1.33}{-20}$$

After solving, $v = -32\text{ cm}$

The magnification is $m = \dfrac{h_2}{h_1} = \dfrac{\mu_1 v}{\mu_2 u}$

$\therefore \quad \dfrac{h_2}{h_1} = \dfrac{1(32)}{1.33\,(-40)}$

or $\quad h_2 = 0.6\text{ cm}$

The positive sign shows that the image is erect.

6 This is a modified displacement method problem.

Here, $a = 1.8\text{ m}$ and $\dfrac{a + d}{a - d} = \dfrac{2}{1}$

Solving, we get $d = 0.6\text{ m}$

$\therefore \quad f = \dfrac{a^2 - d^2}{4a} = 0.4\text{ m}$

7 When the lens is in air, we have

$$P_a = \frac{1}{f} = \frac{\mu_g - \mu_a}{\mu_a}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

When the lens is in liquid, we have

$$P_l = \frac{1}{f_l} = \frac{\mu_g - \mu_l}{\mu_l}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Here, $P_a = 5, P_l = -1, \mu_a = 1, \mu_g = 1.5$

This gives, $\mu_l = 5/3$.

8 $\quad \delta \cong (\mu - 1)A$

For air, $\quad D = (1.5 - 1)A$

For water,

$$\delta = ({}^g\mu_w - 1)A$$

$$= \left(\frac{1.5}{1.3} - 1\right)A$$

Hence, $\quad \delta = \dfrac{0.2}{1.3} \times \dfrac{D}{0.5} \cong 0.3\,D$

9 Here, $A = 5°, \mu = 1.5, i_1 = ?$

As, the ray emerges from the other face of prism normally,

$i_2 = 0 \quad\quad (\therefore\ r_2 = 0°)$

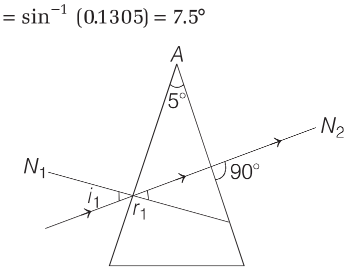
As, $\quad r_1 + r_2 = A$

$r_1 = A - r_2 = 5 - 0 = 5°$

From $\quad \mu = \dfrac{\sin i_1}{\sin r_1}, \sin i_1 = \mu \sin r_1$

$= 1.5 \times \sin 5° = 1.5 \times 0.087 = 0.1305$

$i_1 = \sin^{-1}(0.1305) = 7.5°$



10 $$\mu = \frac{\sin\left(\frac{A+\delta}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

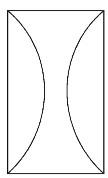
$$\Rightarrow \cot\frac{A}{2} = \frac{\sin\left(\frac{A+\delta}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\frac{\cos\left(\frac{A}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin\left(\frac{A+\delta}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\Rightarrow \sin\left(\frac{\pi}{2} - \frac{A}{2}\right) = \sin\left(\frac{A}{2} + \frac{\delta}{2}\right)$$

$$\delta = \pi - 2A = 180° - 2A$$

11 $$\therefore \frac{1}{f} = (1.5 - 1)\left[\frac{1}{R}\right]$$



$$\frac{1}{f_{\text{lens}}} = \frac{0.5}{R}$$

$$\frac{1}{f_{\text{concave}}} = (1.7 - 1)\left(-\frac{1}{R} - \frac{1}{R}\right)$$

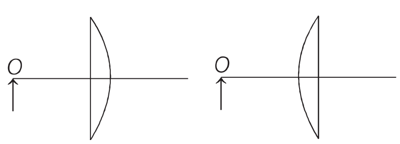
$$\frac{1}{f_{\text{concave}}} = \frac{-0.7 \times 2}{R} = \frac{-1.4}{R}$$

$$\frac{1}{f_{\text{eq}}} = \frac{0.5}{R} - \frac{1.4}{R} + \frac{0.5}{R}$$

$$= \frac{-0.4}{R} = \frac{-0.4}{20}$$

$$\Rightarrow f_{\text{eq}} = -50 \text{ cm}$$

12 Here, $\mu = 1.5$



(a) (b)

If object lies on plane side,
$R_1 = \infty, R_2 = -20$ cm

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.5 - 1)\left(\frac{1}{\infty} + \frac{1}{20}\right)$$

$$= \frac{1}{40}$$

$f = +40$ cm. The lens behaves as convex.

If object lies on its curved side,

$R_1 = 20$ cm, $R_2 = \infty$

$$\frac{1}{f'} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.5 - 1)\left(\frac{1}{20} + \frac{1}{\infty}\right) = \frac{1}{40}$$

$f' = 40$ cm. The lens behaves as convex.

13 Let prism angle of the first and second prisms are A_1 and A_2, respectively. Similarly, their refractive indices are μ_1 and μ_2.

Condition for dispersion without deviation is $\delta_1 - \delta_2 = 0$

$$\Rightarrow (\mu_1 - 1) A_1 - (\mu_2 - 1) A_2 = 0$$

$$\Rightarrow A_2 = \left(\frac{\mu_1 - 1}{\mu_2 - 1}\right) \cdot A_1 = \left(\frac{1.42 - 1}{1.7 - 1}\right)(10°)$$

$$\Rightarrow A_2 = 6°$$

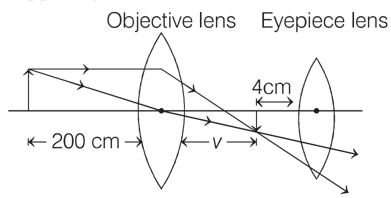
14 According to question,

Focal length of objective lens (f_o)
$= +40$ cm

Focal length of eyepiece lens (f_e)
$= 4$ cm

Object distance for objective lens
$(u_o) = -200$ cm

Applying lens formula for objective lens,



$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f} \Rightarrow \frac{1}{v_o} - \frac{1}{(-200)} = \frac{1}{40}$$

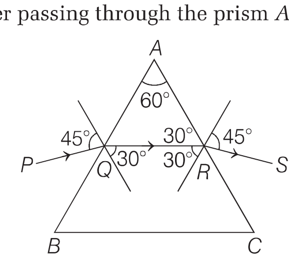
$$\Rightarrow \frac{1}{v_o} = \frac{1}{40} - \frac{1}{200} = \frac{5 - 1}{200} = \frac{4}{200}$$

$$\Rightarrow v_o = 50 \text{ cm}$$

Image will be form at first focus of eyepiece lens.
So, for normal adjustment distance between objectives and eyepiece lense (length of tube) will be

$$v + f_e \Rightarrow 50 + 4 = 54 \text{ cm}$$

15 Consider a ray of light PQ incident on the surface AB and moves along RS, after passing through the prism ABC.



It is given that the incident ray suffers minimum deviation. Therefore, the ray inside the prism must be parallel to the base BC of the prism.

From the geometry of the prism and the ray diagram, it is clear that
angle of incidence, $i = 45°$
angle of refraction $r = r' = 30°$
angle of emergence, $e = 45°$
Therefore, minimum deviation suffered by the ray is

$$\delta_{\min} = i + e - (r + r')$$
$$= 90° - 60° = 30°$$

Also, we know that,

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}}$$

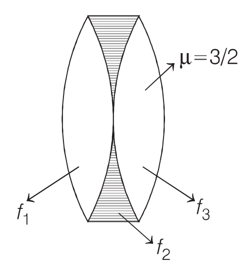
where, μ = refractive index of the material of the prism.

A = angle of prism = 60°

$$\therefore \mu = \frac{\sin\left(\frac{60° + 30°}{2}\right)}{\sin\frac{60°}{2}} = \frac{\sin 45°}{\sin 30°}$$

$$= \frac{1/\sqrt{2}}{1/2} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

16 Consider the situation shown in figure. Let radius of curvature of lens surface is R. The combination is equivalent to three lenses in contact.



$$\therefore \frac{1}{f_{\text{eq}}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} = \frac{2}{f_1} + \frac{1}{f_2}$$

$(\because f_1 = f_3)$

$$\text{Now, } \frac{1}{f_1} = \frac{1}{f_3} = (\mu - 1)\left(\frac{2}{R}\right) = \frac{1}{f}$$

$$\frac{1}{f_2} = (\mu_w - 1)\left(-\frac{2}{R}\right) = \left(\frac{4}{3} - 1\right)\left(\frac{-2}{R}\right)$$

$$= \left(\frac{1}{3}\right)\left(-\frac{2}{R}\right) = \left(\frac{-2}{3}\right)\left(\frac{1}{2(\mu - 1)}\right)$$

$$\Rightarrow \frac{1}{f_2} = \left(-\frac{1}{3}\right)\left(\frac{1}{\frac{3}{2} - 1}\right)\left(\frac{1}{f}\right) = -\frac{2}{3} \times \frac{1}{f}$$

$$\therefore \frac{1}{f_{\text{eq}}} = \frac{2}{f} - \frac{2}{3} \times \frac{1}{f}$$

$$= \frac{6 - 2}{3f} = \frac{4}{3f}$$

$$\Rightarrow f_{\text{eq}} = \frac{3f}{4}$$

17 Focal length of the combination,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \qquad \ldots\text{(i)}$$

We have,

$$\frac{1}{f_1} = (\mu_1 - 1)\left[\frac{1}{\infty} - \left(\frac{1}{-R}\right)\right] = \frac{\mu_1 - 1}{R}$$

and $\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right)$

$$= -\frac{(\mu_2 - 1)}{R}$$

Putting these values of $\frac{1}{f_1}$ and $\frac{1}{f_2}$ in Eq. (i), we get

$$\frac{1}{f} = \frac{(\mu_1 - 1)}{R} - \frac{(\mu_2 - 1)}{R}$$

$$= \frac{[\mu_1 - 1 - \mu_2 + 1]}{R} = \frac{\mu_1 - \mu_2}{R}$$

$$\therefore \quad f = \frac{R}{\mu_1 - \mu_2}$$

18 Given, $R = 20$ cm,

$h_o = 2$ cm and $u = -30$ cm

In general we have assumed $\mu = 1.5$

As, $\frac{1}{f} = (1.5 - 1) \times \frac{2}{20} = \frac{1}{20}$

So, $\quad f = 20$ cm

Now, as $\quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$$\frac{1}{20} = \frac{1}{v} + \frac{1}{30}$$

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{30} = \frac{10}{600}$$

$\therefore \quad v = 60$ cm

As, $m = \frac{v}{u} = \frac{60}{-30} = -2$

$\Rightarrow \quad m = \frac{h_i}{h_o} = -2$

$\therefore \quad h_i = -2 \times 2 = -4$ cm

Here, image is real, inverted, magnified and height of image is 4 cm.

19 As we know that,

(Intensity) $I \propto A$ (Area exposed)

$$\Rightarrow \frac{I_2}{I_1} = \left[\frac{A_2}{A_1}\right] = \frac{\frac{\pi d^2}{4} - \frac{\pi d^2/4}{4}}{\frac{\pi d^2}{4}} = \frac{3}{4}$$

$$\Rightarrow I_2 = \frac{3}{4} I_1$$

and focal length remains unchanged.

20 Angular magnification of an astronomical refracting telescope is given as $\quad M = \frac{f_o}{f_e}$

where, f_o and f_e are the focal length of objective and eyepiece, respectively.

From the given relation, it is clear that for large magnification either f_o has to be large or f_e has to be small.

Angular resolution of an astronomical refracting telescope is given as

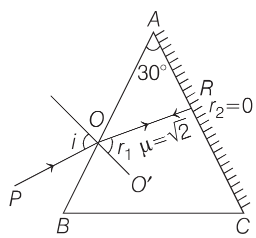
$$R = \frac{a}{1.22\lambda}$$

where, a is the diameter of the objective.

Thus, to have large resolution, the diameter of the objective should be large.

Hence, from the above objective lens should have large focal length (f_o) and large diameter (a).

21 According to the question, the figure of mentioned prism is given as



Here, $\quad r_1 + r_2 = A = 30°$

But $\quad r_2 = 0 \qquad$ [reflected normally]

$\therefore \quad r_1 = 30°$

From Snell's law,

$$\frac{\sin i_1}{\sin r_1} = \frac{\mu_2}{\mu_1} = \frac{\sqrt{2}}{1}$$

$$\Rightarrow \quad \sin i_1 = \sqrt{2}\sin r_1$$

$$= \sqrt{2}\sin 30°$$

$$= \sqrt{2} \times \frac{1}{2}$$

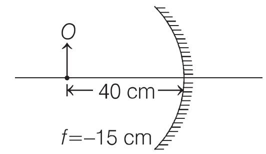
$$= \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \sin i_1 = \frac{1}{\sqrt{2}} = \sin 45° \text{ or } i_1 = 45°$$

22 **Case I** When the object distance,

$u_1 = -40$ cm

Focal length of mirror, $f = -15$ cm



Using the mirror formula, we get

$$\frac{1}{f} = \frac{1}{v_1} + \frac{1}{u_1}$$

Substituting the given values, we get

$$-\frac{1}{15} = \frac{1}{v_1} + \left(\frac{-1}{40}\right)$$

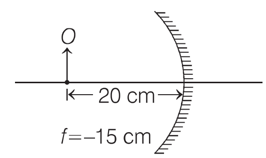
$$\Rightarrow \quad \frac{1}{v_1} = \frac{1}{40} - \frac{1}{15}$$

$$= \frac{3 - 8}{120} = \frac{-5}{120}$$

$$\Rightarrow \quad v_1 = \frac{-120}{5} = -24 \text{ cm}$$

Case II When the object distance,

$u_2 = 20$ cm



Using the mirror formula, we get

$$\frac{1}{f} = \frac{1}{v_2} + \frac{1}{u_2}$$

Substituting the given values, we get

$$-\frac{1}{15} = \frac{1}{v_2} + \left(-\frac{1}{20}\right)$$

$$\Rightarrow \quad \frac{1}{v_2} = \frac{1}{20} - \frac{1}{15}$$

$$= \frac{3 - 4}{60} = \frac{-1}{60}$$

$$\Rightarrow \quad v_2 = -60 \text{ cm}$$

$\therefore$ The displacement of the image is

$= v_2 - v_1$

$= -60 - (-24)$

$= -60 + 24$

$= -36$ cm

$= 36$ cm away from the mirror