Chapter 3 Expansions

Exercise – 3.1

1.

Solution

(i) $\mathbf{(2x + 7y)^2}$

It is in the form of $(a + b)^2 = a^2 + 2ab + b^2$

$\therefore a = 2x \; ; \; b = 2y$

$\therefore (2x + 7y)^2 = (2x)^2 + 2.2x.\,7y + (7y)^2$

$= 4x^2 + 28xy + 49y^2$

(ii) $\left(\frac{1}{2}x + \frac{2}{3}y\right)^2$

$\Rightarrow \left(\frac{1}{2}x\right)^2 + 2.\frac{1}{2}\,.x.\frac{2}{3}\,.y + \left(\frac{2}{3}y\right)^2$

$\Rightarrow \frac{x^2}{4} + \frac{2}{3}xy + \frac{4}{9}y^2$

2.

Solution

(i) $\left(3x + \frac{1}{2}x\right)^2$

It is in the form of $(a + b)^2 = a^2 + 2ab + b^2$

$\therefore (3x)^2 + 2.3x \cdot \frac{1}{2x} + \left(\frac{1}{2x}\right)^2$

$\Rightarrow 9x^2 + 3 + \frac{1}{4x^2}$

(ii) $(3x^2y + 5z)^2$

It is in the form of $(a + b)^2 = a^2 + 2ab + b^2$

Here $a = 3x^2 y$; $b = 5z$

$\Rightarrow (3x^2y)^2 + 2.3x^2y.5z + (5z)^2$

$\Rightarrow 9x^4y^2 + 30x^2yz + 25z^2$

3.

Solution

(i) $\left(3x - \frac{1}{2x}\right)^2$

It is in the form of $(a-b)^2 = a^2 - 2ab + b^2$

Here $a = 3x$; $b = \frac{1}{2x}$

$\Rightarrow (3x)^2 - 2.3x \cdot \frac{1}{2x} + \left(\frac{1}{2x}\right)^2$

$\Rightarrow 3^2 - x^2 - 3 + \frac{1}{2^2x^2}$

$\Rightarrow 9x^2 - 3 + \frac{1}{4x^2}$

(ii) $\left(\frac{1}{2}x - \frac{3}{2}y\right)^2$

It is in the form of $(a - b)^2 = a^2 - 2ab + b^2$

Here $a = \frac{1}{2}x$; $b = \frac{3}{2}y$

$\therefore \left(\frac{1}{2}x\right)^2 - 2.\frac{1x}{2}.\frac{3}{2}y + \left(\frac{3}{2}y\right)^2$

$\Rightarrow \frac{x^2}{4} - \frac{3}{2}xy + \frac{9y^2}{4}$

4.

Solution

(i) (x + 3) (x + 5)

= x(x +5) + 3(x +5)

= $x^2 + 5x + 3x + 15$

= $x^2 + 8x + 15$

(ii) (x + 3) (x-5)

= x(x -5) + 3(x -5)

= x . x – x. 5 + 3.x – 3.5

= $x^2 - 5x + 3x - 15$

= $x^2 - 2x - 15$

(iii) (x -7) (x + 9)

$= x(x + 9) – 7(x + 9)$

$= x. x + 9.x - 7.x – 7.9$

$= x^2 + 9x – 7x – 63$

$= x^2 + 2x – 63$

(iv) (x – 2y) (x – 3y)

$= x(x – 3y) - 2y (x – 3y)$

$= x. x – x . 3y – 2y.x + 2y. 3y$

$= x^2 – 3xy – 2xy + 6y^2$

$= x^2 – 5xy + 6y^2$

5.

Solution

(i) $(x – 2y – z)^2$

It is in the form of $(a+ b +c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

There $a = x ; b = -2y ; c = -z$

$\therefore \Rightarrow x^2 + (-2y)^2 + (-z)^2 + 2(x(-2y) +(-2y) (-z) + (-z) .x)$

$\Rightarrow x^2 + 4y^2 + z^2 + 2 (-2xy + 2yz – zx)$

$\Rightarrow x^2 + 4y^2 + z^2 + 4yz - 4xy – 2zx$

(ii) $(2x - 3y + 4z)^2$

It is in the form of $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

Here $a = 2x$; $b = -3y$; $c = 4z$

$\therefore \Rightarrow (2x)^2 + (-3y)^2 + (4z)^2 + 2\ (2x \cdot (-3y) + (3y)(4z) + 4z \cdot 2x$

$\Rightarrow 4x^2 + 9y^2 + 16z^2 + 2(-6xy - 12yz + 8xz)$

$\Rightarrow 4x^2 + 9y^2 + 16z^2 - 12xy - 24yz + 16xz$

6.

Solution

(i) $\left[2x + \frac{3}{x} - 1\right]^2$

It is in the form of $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

Here $a = 2x$; $b =$; $c = -1$

$\therefore \Rightarrow (2x)^2 + \left(\frac{3}{x}\right)^2 + (-1)^2 + 2(2x \cdot \frac{3}{x} + \frac{3}{x}(-1) + (-1) \cdot 2x)$

$\Rightarrow 4x^2 + \frac{9}{x^2} + 1 + 2\ (6 - \frac{3}{x} - 2x)$

$\Rightarrow 4x^2 + \frac{9}{x^2} + 1 + 12 - \frac{6}{x} - 4x$

$\Rightarrow 4x^2 + \frac{9}{x^2} - \frac{6}{x} - 4x + 13$

(ii) $\left(\frac{2}{3}x - \frac{3}{2x} - 1\right)^2$

It is in the form of $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

Here $a = \frac{2}{3}x$; $b = -\frac{3}{2x}$; c= -1

$\therefore \Rightarrow \left(\frac{2}{3}x\right)^2 + \left(-\frac{3}{2x}\right)^2 + (-1)^2 + 2\left[\frac{2}{3}x\left(-\frac{3}{2x}\right) + \left(-\frac{3}{2x}\right).(-1)(-1)\left(\frac{2}{3}x\right)\right]$

$\Rightarrow \frac{4}{9}x^2 - \frac{9}{4x^2} + 1 - 2 + \frac{6}{2x} - \frac{4x}{3}$

$\Rightarrow \frac{4}{9}x^2 - \frac{9}{4x^2} + \frac{3}{x} - \frac{4x}{3} - 1$

7.

Solution

(i) $(x + 3)^3$

It is in the form of $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Here a= x ; b= 2

$\therefore \Rightarrow x^3 + 3.x^2 . 2 + 3.x.2^2 + 2^3$

$\Rightarrow x^3 + 6x^2 + 3x .4 + 8$

$\Rightarrow x^3 + 6x^2 + 12x + 8$

(ii) $(2a + b)^3$

$\Rightarrow (2a)^3 + 3.(2a)^2.b + 3.2a.\ b^2 + b^3$

$\Rightarrow 8a^3 + 3.4a^2\ .b + 6ab^2 + b^3$

$\Rightarrow\ 8a^3 + 12a^2\ b + 6ab^2 + b^3$

8.

Solution

(i) $\left(3x + \frac{1}{x}\right)^3$

It is in the form of $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$a = 3x\ ;\ b = \frac{1}{x}$

$\therefore \Rightarrow (3x)^3 + 3.(3x)^2\ .\frac{1}{x} + 3.3x.\left(\frac{1}{x}\right)^2 + \left(\frac{1}{x}\right)^3$

$\Rightarrow 27x^3 + 3.9x^2\ .\frac{1}{x} + 9x\ .\frac{1}{x^2} + 9x\ .\frac{1}{x^2} + \frac{1}{x^3}$

$\Rightarrow 27x^3 + 27x + \frac{9}{x} + \frac{1}{x^3}$

(ii) $(2x-1)^3$

It is in the form of $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Here $a= 2x\ ;\ b= 1$

$\therefore \Rightarrow (2x)^3 - 3.(2x)^2\ .\ 1 + 3(2x)(1)^2 - (1)^3$

$\Rightarrow 8x^3 - 3.4x^2 + 6x - 1$

$\Rightarrow\ 8x^3 - 12x^2 + 6x - 1$

9.

Solution

(i) $(5x - 3y)^3$

It is in the form of $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$a = 5x$; $b = 3y$

$\therefore \Rightarrow (5x)^3 - 3.(5x)^2 . 3y + 3.5x .(3y)^2 - (3y)^3$

$\Rightarrow 125x^3 - 3.25x^2. 3y + 3.5x . 9y^2 - 27y^3$

$\Rightarrow 125x^3 - 225x^2y + 135y^2. X - 27y^3$

(ii) $\left(2x - \frac{1}{3y}\right)^3$

$\therefore \Rightarrow (2x)^3 - 3.(2x)^2 . \frac{1}{3y} + 3.2x . \left(\frac{1}{3y}\right)^2 - \left(\frac{1}{3y}\right)^3$

$\Rightarrow 8x^3 - 3.4x^2 . \frac{1}{3y} + 3.2x\frac{1}{9y^2} - \frac{1}{27y^3}$

$\Rightarrow 8x^3 - \frac{4x^2}{y} + \frac{2x}{3y^2} - \frac{1}{27y^3}$

10.

Solution

(i) $(a + b)^2 + (a - b)^2$

$\Rightarrow a^2 + 2ab + b^2 + a^2 - 2ab + b^2$

$\Rightarrow 2a^2 + 2b^2$

$\Rightarrow 2(a^2 + b^2)$

(ii) $(a+b)^2 - (a-b)^2$

$\Rightarrow (a^2 + 2ab + b^2) - (a^2 - 2ab + b^2)$

$\Rightarrow a^2 + 2ab + b^2 - a^2 + 2ab - b^2$

$\Rightarrow 2ab + 2ab$

$\Rightarrow 4ab$

11.

Solution

(i) $\left(\boldsymbol{a} + \frac{\mathbf{1}}{\boldsymbol{a}}\right)^{\mathbf{2}} + \left(\boldsymbol{a} - \frac{\mathbf{1}}{\boldsymbol{a}}\right)^{\mathbf{2}}$

$\Rightarrow \left(a^2 + 2.a.\frac{1}{a} + \frac{1}{a^2}\right) + \left(a^2 + 2.a.\frac{1}{a} + \frac{1}{a^2}\right)$

$\Rightarrow a^2 + 2 + \frac{1}{a^2} + a^2 - 2 + \frac{1}{a^2}$

$\Rightarrow 2a^2 + \frac{2}{a^2}$

$\Rightarrow 2\left(a^2 + \frac{2}{a^2}\right)$

(ii) $\left(\boldsymbol{a} + \frac{\mathbf{1}}{\boldsymbol{a}}\right)^{\mathbf{2}} - \left(\boldsymbol{a} - \frac{\mathbf{1}}{\boldsymbol{a}}\right)^{\mathbf{2}}$

$\Rightarrow \left(a^2 + 2.a.\frac{1}{a} + \frac{1}{a^2}\right) - \left(a^2 - 2.a.\frac{1}{a} + \frac{1}{a^2}\right)$

$\Rightarrow a^2 + 2 + \frac{1}{a^2} - a^2 + 2 - \frac{1}{a^2}$

$\Rightarrow 2 + 2$

$\Rightarrow 4$

12.

Solution

(i) $(3x - 1)^2 - (3x - 2)(3x + 1)$

$\Rightarrow (3x)^2 - 2.3x.1 + 1^2 - 3x(3x + 1) + 2(3x + 1)$

$\Rightarrow 9x^2 - 6x + 1 - 9x^2 - 3x + 6x + 2$

$\Rightarrow 3x + 3$

$\Rightarrow 3(x+1)$

(ii) $(4x + 3y)^2 - (4x - 3y)^2 - 48$

$\Rightarrow (4x)^2 + 2.3y.\ 4x + (3y)^2 - ((4x)^2 - 2.4x.3y + (3y)^2) - 48$

$\Rightarrow 16x^2 + 24xy + 9y^2 - 16yx^2 + 24xy - 9y^2 - 48$

$\Rightarrow 48xy - 48$

$\Rightarrow 48(xy - 1)$

13.

Solution

(i) $(7p + 9q)(7p - 9q)$

$\Rightarrow 7p(7p - 9q) + 9q(7p - 9q)$

$\Rightarrow 49p^2 - 63pq + 63pq - 81q^2$

$\Rightarrow 49p^2 - 81q^2$

(ii) $\left(2x - \frac{3}{x}\right)\left(2x + \frac{3}{x}\right)$

$\Rightarrow (2x)^2 - \left(\frac{3}{x}\right)^2$

Since it is in the form of (a+b) (a-b) = $a^2 - b^2$

$\Rightarrow 4x^2 - \frac{9}{x^2}$

14.

Solution

(i) (2x – y + 3) (2x –y -3)

((2x –y) + 3) ((2x – y) – 3)

It is in the form of (a +b) (a-b) = $a^2 - b^2$

$\Rightarrow (2x - y)^2 - 3^2$

$\Rightarrow (2x)^2 - 2.2x.y + y^2 - 9$

$\Rightarrow 4x^2 - 4xy + y^2 - 9$

(ii) (3x + y – 5) (3x – y – 5)

⇒ (3x +(y-5)) (3x-(y + 5)

⇒ [(3x- 5) + y] [(3x – 5)-y]

It is in the form of (a + b) (a – b) = $a^2 - b^2$

∵ a = 3x – 5 ; b = y

$\therefore (3x - 5)^2 - y^2$

$\Rightarrow (3x)^2 - 2.3x . 5 + 5^2 - y^2$

$\Rightarrow 9x^2 - 30x + 25 - y^2$

15.

Solution

(i) $\left(\boldsymbol{x} + \frac{\mathbf{2}}{\boldsymbol{x}} - \mathbf{3}\right)\left(\boldsymbol{x} - \frac{\mathbf{2}}{\boldsymbol{x}} - \mathbf{3}\right)$

$\Rightarrow \left((x-3) + \frac{2}{x}\right)\left((x-3) - \frac{2}{x}\right)$

It is in the form of $(a + b)(a - b) = a^2 - b^2$

$\because$ $a = x - 3$; $b = \frac{2}{x}$

$\Rightarrow (x-3)^2 - \left(\frac{2}{x}\right)^2$

$\Rightarrow x^2 - 2.x.3 + 3^3 - \frac{4}{x^2}$

$\Rightarrow x^2 - 6x + 9 - \frac{4}{x^2}$

(ii) $(5 - 2x)(5 + 2x)(25 + 4x^2)$

It is in the form of $(a + b)(a - b) = a^2 - b^2$

$\because$ $(5^2 - (2x)^2)(25 + 4x^2)$

$\Rightarrow (25 - 4x^2)(25 + 4x^2)$

$\Rightarrow (25)^2 - (4x^2)^2$

$\Rightarrow 625 - 16x^4$

16.

Solution

(i) $(x + 2y + 3)(2y + x + 7)$

$\Rightarrow x(2y + x + 7) + 2y(2y + x + 7) + 3(2y + x + 7)$

$\Rightarrow 2xy + x^2 + 7x + 4y^2 + 2xy + 14y + 6y + 3x + 21$

$\Rightarrow x^2 + 4y^2 + 4xy + 10x + 20y + 21$

(ii) $(2x + y + 5)(2x + y - 9)$

$\Rightarrow 2x(2x + y - 9) + y(2x + y - 9) + 5(2x + y - 9)$

$\Rightarrow 4x^2 + 2xy - 18x + 2xy + y^2 - 9y + 10x + 5y - 45$

$\Rightarrow 4x^2 + y^2 + 4xy - 8x - 4y - 45$

(iii) $(x - 2y - 5)(x - 2y + 3)$

$\Rightarrow x(x - 2y + 3) - 2y(x - 2y + 3) - 5(x - 2y + 3)$

$\Rightarrow x^2 - 2xy + 3x - 2xy + 4y^2 - 6y - 5x + 10y - 15$

$\Rightarrow x^2 + 4y^2 - 4xy - 2x - 4y - 15$

(iv) $(3x - 4y - 2)(3x - 4y - 6)$

$\Rightarrow 3x(3x - 4y - 6) - 4y(3x - 4y - 6) - 2(3x - 4y - 6)$

$\Rightarrow 9x^2 - 12xy - 18x - 12xy + 16y^2 + 24y - 6x + 8y + 12$

$\Rightarrow 9x^2 + 16y^2 - 24xy - 24x + 32y + 12$

17.

Solution

(i) $(2p + 3q)(4p^2 – 6pq + 9q^2)$

$(2p + 3q)((2p)^2 – 2p.3q + (3q)^2)$

It is in the form of $(a+b)(a^2 – ab + b^2)$ is $a^3 + b^3$

∴ here $a = 2p$; $b = 3q$

∴ $(2p)^3 + (3q)^3$

⇒ $8p^3 + 27q^3$

(ii) $\left(x + \frac{1}{x}\right)\left(x^2 - 1 + \frac{1}{x^2}\right)$

It is in the form of $(a + b)(a^2 – ab + b^2)$ is $a^3 + b^3$

∴ here $a = x$; $b = \frac{1}{x}$

∴ $x^3 + \frac{1}{x^3}$

18.

Solution

(i) $(3p – 4q)(9p^2 + 12pq + 16q^2)$

$(3p – 4q)((3p)^2 + 3p.4q + (4q)^2)$

It is in the form of $(a – b)(a^2 + ab + b^2)$ is $a^3 – b^3$

$\therefore$ here 3p = a ; b = 4q

$\therefore (3p)^3 – (4q)^3$

$\Rightarrow 27p^3 – 64q^3$

(ii) $\left(x - \frac{3}{x}\right)\left(x^2 + 3 + \frac{9}{x^2}\right)$

$\therefore \left(x - \frac{3}{x}\right)\left(x^2 + x\frac{3}{x} + \left(\frac{3}{x}\right)^2\right)$

It is in the form of $(a – b)(a^2 + ab + b^2)$ is $a^3 – b^3$

$\therefore$ here a = x ; $b = \frac{3}{x}$

$\therefore x^3 - \left(\frac{3}{x}\right)^3$

$\Rightarrow x^3 - \frac{27}{x^3}$

19.

Solution

Given $(2x + 3y + 4z)(4x^2 + 9y^2 + 16z^2 – 6xy – 12yz – 8zx)$

$\Rightarrow (2x + 3y + 4z)((2x)^2 + (3y)^2 + (4z)^2 – 2x. 3y – 3y . 4z – 4z .2x)$

$\therefore$ it is in the form of

$(a + b+ c)(a^2 + b^2 + c^2 – ab – bc – ca) = a^3 + b^3 + c^3 – 3abc$

$\therefore$ here a = 2x ; b = 3y ; c = 4z

$\therefore (2x)^3 + (3y)^3 + (4z)^3 – 3.2x.3y . 4z$

$\Rightarrow 8x^3 + 27y^3 + 64z^3 – 72xyz$

20.

Solution

(i) (x + 1) (x +2) (x+3)

[x(x+2) +1 (x+2)] (x +3)

$\Rightarrow (x^2 + 2x + x + 2)(x + 3)$

$\Rightarrow (x^2 + 3x + 2)(x + 3)$

$\Rightarrow (x^2 + 3x + 2)(x) + (x^2 + 3x + 2)3$

$\Rightarrow x^3 + 5x^2 + 2x + 3x^2 + 9x + 6$

$\Rightarrow x^3 + 6x^2 + 11x + 6$

(ii) (x- 2) (x-3) (x + 4)

$\Rightarrow [x(x - 3) - 2(x - 3)](x + 4)$

$\Rightarrow (x^2 - 3x - 2x + 6)(x + 4)$

$\Rightarrow (x^2 - 5x + 6)(x + 4)$

$\Rightarrow (x^2 - 5x + 6)x + (x^2 - 5x + 6)4$

$\Rightarrow x^3 - 5x^2 + 6x + 4x^2 - 20x + 24$

$\Rightarrow x^3 - x^2 - 14x + 24$

21.

Solution

Given (x-3) (x+7) (x-4)

$(x(x + 7) - 3(x + 7))(x - 4)$

$(x^2 + 7x - 3x - 21)(x-4)$

$(x^2 + 4x - 21)(x-4)$

$(x^2 + 4x - 21)x - 4(x^2 + 4x - 21)$

$x^3 + 4x^2 - 21x - 4x^2 - 16x + 84$

$x^3 - 37x + 84$

$\therefore$ the coefficient of x^2 is 0

The coefficient of x is – 37

22.

Solution

Given $a^2 + 4a + x = (a + 2)^2$

$\therefore a^2 + 4a + x = a^2 + 2.a.2 + 2^2$

$a^2 + 4a + x = a^2 + 4a + 4$

$\Rightarrow x = a^2 + 4a + 4 - a^2 - 4a$

$X = 4$

23.

Solution

(i) $(101)^2$

$\Rightarrow (100 + 1)^2$

$\Rightarrow (100)^2 + 2.100.1 + 1^2$

$\Rightarrow 10000 + 200 + 1$

$\Rightarrow 10201$

(ii) $(1003)^2$

$\Rightarrow (1000 + 3)^2$

$\Rightarrow (1000)^2 + 2.1000.3 + 3^2$

$\Rightarrow 1000000 + 6000 + 9$

$\Rightarrow 1006009$

(iii) $(10.2)^2$

$(10 + 0.2)^2$

$(10)^2 + 2\times 10 \times 0.2 + (0.2)^2$

$100 + 4 + 0.04$

104.04

24.

Solution

(i) 99^2

$\Rightarrow (100 - 1)^2$

$\Rightarrow (100)^2 - 2.100.1 + 1^2$

$\Rightarrow 10000 - 200 + 1$

$\Rightarrow$ 9801

(ii) $(997)^2$

$\Rightarrow (1000 - 3)^2$

$\Rightarrow 1000^2 - 2.1000.9 + 3^2$

$\Rightarrow$ 1000000 – 6000 + 9

$\Rightarrow$ 1994009

In this we used the $(a - b)^2$ formulae i.e, $a^2 - 2ab + b^2$

(iii) $(9.8)^2$

$\Rightarrow (10 - 0.2)^2$

$\Rightarrow 10^2 - 2 \times 10 \times 0.2 + (0.2)^2$

$\Rightarrow 100 - 4 + 0.4$

$\Rightarrow$ 96.04

25.

Solution

(i) $(103)^2$

$\Rightarrow (100 + 3)^3$

$\therefore$ it is in the form of

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$\therefore$ here a = 100 ; b = 3

$\Rightarrow (100)^3 + 3.(100)^2.3 + 3100 . 3^2 + 3^3$

$\Rightarrow$ 1000000 + 90000 + 2700 + 27

$\Rightarrow$ 1092727

(ii) 99^3

$\Rightarrow (100 -1)^3$

$\Rightarrow 100^3 – 3.100^2 .1 + 3.100.1^2 - 1^3$

$\Rightarrow$ 1000000 – 30000 + 300 -1

$\Rightarrow$ 970299

(iii) $(10.1)^3$

$\Rightarrow (10 + 0.1)^3$

$\Rightarrow 10^3 + 3.10^2.(0.1) + 3.10(0.1)^2 + (0.1)^3$

$\Rightarrow$ 1000 + 30 + 3 + 0.01

$\Rightarrow$ 1030.301

26.

Solution

Given $2a – b + c = 0$

$\Rightarrow (2a + c) = b$

Squaring on both sides

$\Rightarrow (2a + c)^2 = b^2$

$\Rightarrow (2a)^2 + 2.2a.c + c^2 = b^2$

$\Rightarrow 4a^2 + 4ac + c^2 = b^2$

$\Rightarrow 4a^2 - b^2 + c^2 + 4ac = 0$

Hence proved

27.

Solution

Given $a + b + 2c = 0$

$a + b = -2c$ ....(i)

cubing on both sides

$(a+b)^3 = (-2c)^3$

$a^3 + b^3 + 3a^2b + 3ab^2 = -8c^3$

$a^3 + b^3 + 3ab(a + b) = -8c^3$ .... from (i)

$a^3 + b^3 + 3ab(-2c) = -8c^3$

$a^3 + b^3 - 6abc = -8c^3$

$a^3 + b^3 + 8c^3 = 6abc$

hence proved

28

Solution

Given $a + b + c = 0$

$a + b = -c$ .....(i)

cubing on both sides

$(a + b)^3 = (-c)^3$

$\Rightarrow a^3 + b^3 + 3a^2b + 3ab^2 = - c^3$

$\Rightarrow a^3 + b^3 + 3ab (a+ b) = - c^3$

$\Rightarrow a^3 + b^3 + 3ab(-c) = -c^3$

$\Rightarrow a^3 + b^3 – 3abc = - c^3$

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

$\Rightarrow \frac{a^3+b^3+c^3}{abc} = 3$

$\Rightarrow \frac{a^3}{abc} + \frac{b^3}{abc} + \frac{c^3}{abc} = 3$

$\Rightarrow \frac{a^2}{bc} + \frac{b^2}{ac} + \frac{c^2}{ab} = 3$

29

Solution

Given $x+ y = 4$

Cubing on both sides

$(x + y)^3 = 4^3$

$\Rightarrow x^3 + 3x^2y + 3xy^2 + y^3 = 64$

$\Rightarrow x^3 + 3xy (x + y) + y^3 = 64$

$\Rightarrow x^3 + 3xy(4) + y^3 = 64$

$\Rightarrow x^3 + 12xy + y^3 = 64$

$\Rightarrow x^3 + y^3 + 12xy – 64 = 0$

30

Solution

(i) $(27)^3+(-17)^3+(-10)^3$

$\therefore$ if $a + b + c = 0$; then $a^3 + b^3 + c^3 = 3abc$

$\therefore$ here $a = 27$

$b = -17$

$c = -10$

$\therefore 27 - 17 - 10 = 0$

$\therefore a^3 + b^3 + c^3 = 3abc$

$= 3.27.(-17)(-10)$

$= 13770$

(ii) $(-28)^3 + 15^3 + 13^3$

$\therefore$ if $a + b + c = 0$; then $a^3 + b^3 + c^3 = 3abc$

$\Longrightarrow -28 + 15 + 13 = 0$

$\therefore a^3 + b^3 + c^3 = 3abc$

$= 3(-28)(15)(13)$

$= -16380$

31.

Solution

Given $\frac{86\times86\times86+14\times14\times14}{86\times86-86\times14+14\times14}$

$\therefore$ It is in the form of $\frac{a^3+b^3}{a^2-ab+b^2}$ = (a +b)

$\therefore \frac{86^3+14^3}{80^2-86.14+14^2}$ = 86 + 14

= 100

Exercise 3.2

1.

Solution

Given $x- y = 8$ .... (i)

$xy = 5$ ...... (ii)

squaring on both sides in equal (i)

$\therefore (x - y)^2 = 8^2$

$X^2 - 2.xy + y^2 = 64$

$X^2 - 2(5) + y^2 = 64$ $\therefore$ from eg (ii)

$X^2 - 10 + y^2 = 64$

$X^2 + y^2 = 64 + 10$

$X^2 + y^2 = 74$

2.

Solution

Given $x + y = 10$ ...... (i)

$xy = 21$ ...... (ii)

squaring on both sides in equal(i)

$(x + y)^2 = 10^2$

$x^2 + 2xy + y^2 = 100$

$x^2 + 2(21) + y^2 = 100$

$x^2 + 4^2 + y^2 = 100$

$x^2 + y^2 = 100 – 42$

$x^2 + y^2 = 58$

$2(x^2 + y^2) = 2 \times 58 = 116$

3.

Solution

Given $2a + 3b = 7$ .....(i)

$ab = 2$ ......(ii)

squaring on both sides in equal(i)

$(2a + 3b)^2 = 7^2$

$(2a)^2 + 2.2a.3b + (3b)^2 = 49$

$4a^2 + 12ab + 9b^2 = 49$

$4a^2 + 12(2) + 9b^2 = 49$

$4a^2 + 24 + 9b^2 = 49$

$4a^2 + 9b^2 = 49 – 24$

$4a^2 + 9b^2 = 25$

4.

Solution

Given $3x – 4y = 16$ ...... (i)

$xy = 4$.......(ii)

squaring on both sides in equal(i)

$(3x - 4y)^2 = 16^2$

$(3x)^2 - 2.3x.\ 4y + (4y)^2 = 256$

$9x^2 - 24xy + 16y^2 = 256$

$9x^2 - 24(4) + 16y^2 = 256$

$9x^2 - 96 + 16y^2 = 256$

$9x^2 + 16y^2 = 256 + 96$

$9x^2 + 16y^2 = 352$

5.

Solution

Given $x + y = 8$.....(i)

$x - y = 2$......(ii)

since we know that

$2(x)^2 + 2(y)^2 = (x + y)^2 + (x-y)^2$

From (i) squaring on both sides

$(x + y)^2 = 8^2$

$= 64$

From (ii) squaring on both sides

$(x - y)^2 = 2^2$

$= 4$

$\therefore 2(x^2 + y^2) = (x+ y)^2 + (x - y)^2$

$= 64 + 4$

$= 68$

6.

Solution

Given $a^2 + b^2 = 13$.....(i)

$ab = 6$ ..... (ii)

(i) a + b

$\therefore (a + b)^2 = a^2 + 2ab + b^2$

$= a^2 + b^2 + 2ab$

$= 13 + 2(6)$

$= 13 + 12$

$(a + b)^2 = 25$

$a + b = \sqrt{25}$

$a + b = 5$

(ii) a –b

$\therefore (a - b)^2 = a^2 - 2ab + b^2$

$= a^2 + b^2 - 2ab$

$= 13 - 2(6)$

$= 13 – 12$

$(a – b)^2 = 1$

$(a – b) = \sqrt{1}$

$a – b = 1$

7.

Solution

Given $a + b = 4$ .....(i)

$ab = -12$.....(ii)

(i) a- b

From (i) and (2)

$ab = -12$

$a(4 – a) = -12$

$4a – a^2 = -12$

$a^2 – 12 – 4a = 0$

$a^2 – 4a – 12 = 0$

$a2 – 6a + 2a – 12 = 0$

$a(a-6) + 2(a –b) = 0$

$(a – 6) (a+2) =0$

$a -6 = 0$; $a + 2 = 0$

$a= 6$ $a = -6$

$a + b = 4$ $a + b = 4$

$6 + b = 4$ $-2 + b = 4$

$b = 4 - 6$ $b = 4 + 2$

$b = -2$ $b = 6$

$(a, b) = (6, -2)$

(i) $a - b = 6 - (-2) = 8$

(ii) $a^2 - b^2 = (a + b)(a - b)$

$= (4)(8)$

$= 32$

8.

Solution

Given $p - q = 9$ .....(i)

$pq = 36$.....(ii)

from (i) and (ii)

$p = q + 9$

$\therefore pq = 36$

$(q + 9) q = 36$

$q^2 + 9q = 36$

$q^2 + 9q - 36 = 0$

$q^2 + 19q - 3q - 36 = 0$

$q (q + 12) - 3(q + 12) = 0$

$(q + 12)(q - 3) = 0$

$q + 12 = 0$ $\quad$ $q - 3 = 0$

$q = -12$ $\quad$ $q = 3$

$p - q = 9$ $\quad$ $p - q = 9$

$p - (-12) = 9$ $\quad$ $p - 3 = 9$

$p = 9 - 12$ $\quad$ $p = 9 + 3$

$p = -3$ $\quad$ $p = 12$

$p = 12$; $q = 3$

(i) $p + q$

$12 + 3$

$= 15$

(ii) $p^2 - q^2$

$= (p + q)(p - q)$

$= 15 \times 9$

$= 135$

9

Solution

Given $x + y = 6$ ......(i)

$x - y = 4$ .....(ii)

from (i) squaring on both sides

$(x + y)^2 = 6^2$

$x^2 + y^2 + 2xy = 36$

$x^2 + y^2 = 36 – 2xy$ ......(iii)

from (i) squaring on both sides

$(x – y)^2 = 4^2$

$x^2 – 2xy + y^2 = 16$

$x^2 + y^2 = 16 + 2xy$......(iv)

∴ equate equation (iii) and (iv)

$36 – 2xy = 16 + 2xy$

$36 – 16 = 2xy + 2xy$

$20 = 4xy$

$4xy = 20$

$xy = \frac{20}{9}$

$xy = 5$

(i) $x^2 + y^2$

From equation (iii) $x^2 + y^2 = 36 – 2xy$

$= 36 – 2(5)$

$= 36 - 10 = 26$

(ii) $xy = 5$

10.

Solution

Given $x - 3 = \frac{1}{x}$

$x - \frac{1}{x} = 3$

squaring on both sides

$\left(x - \frac{1}{x}\right)^2 = 3^2$

$x^2 - 2.x.\frac{1}{x} + \frac{1}{x^2} = 9$

$x^2 - 2 + \frac{1}{x^2} = 9$

$x^2 + \frac{1}{x^2} = 9 + 2$

$x^2 + \frac{1}{x^2} = 11$

11.

Solution

Given $x + y = 8$ .....(i)

$xy = 3\frac{3}{4} = \frac{15}{4}$......(ii)

from equation (i) = $y = 8 - x$

from equation (ii) = $xy = \frac{15}{4}$

$x(8 - x) = \frac{15}{4}$

$8x - x^2 = \frac{15}{4}$

$4(8x - x^2) = 15$

$32x - 4x^2 = 15$

$4x^2 - 32x + 15 = 0$

$4x^2 - 30x - 2x + 15 = 0$

$2x(2x - 15) -1 (2x -15) = 0$

$(2x - 15)(2x-1) = 0$

$2x - 15 = 0 \quad ; \quad 2x - 1 = 0$

$2x = 15 \quad\quad 2x = 1$

$x = \frac{15}{2} \quad\quad x = \frac{1}{2}$

$x + y = 8 \quad\quad x + y = 8$

$\frac{15}{2} + y = 8 \quad\quad \frac{1}{2} + y = 8$

$Y = 8 - \frac{15}{2} \quad\quad y = 8 - \frac{1}{2}$

$Y = \frac{16-15}{2} \quad\quad y = \frac{16-1}{2}$

$Y = \frac{1}{2} \quad\quad y = \frac{15}{2}$

$x = \frac{15}{2} \quad ; \quad y = \frac{1}{2}$

(i) x – y

$= \frac{15}{2} - \frac{1}{2}$

$= 15 - \frac{1}{2}$

$= \frac{14}{2} = 7$

(ii) 3(x² + y²)

$= \quad 3\left[\left(\frac{15}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right]$

$= \quad 3\left[\frac{225}{4} + \frac{1}{4}\right]$

$= \quad 3\left[\frac{225+1}{4}\right]$

$= \quad 3\left(\frac{226}{4}\right)$

$= \quad \frac{3\times113}{2}$

$= \quad \frac{339}{2}$

(iii) 5(x² + y²) + 4(x –y)

$5\left(\left(\frac{15}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right) + 4(7)$

$= 5\left(\frac{225}{4} + \frac{1}{4}\right) + 4(7)$

$= 5\left(\frac{113}{2}\right) + 28$

$= \frac{565}{2} + 28$

$= \frac{565+56}{2}$

$= \frac{621}{2}$

12.

Solution

Given $x^2 + y^2 = 34$

$xy = 10\frac{1}{2} = \frac{21}{2}$

$(x + y)^2 = x^2 + y^2 + 2xy$

$= 34 + 2.\frac{21}{2}$

$= 34 + 21$

$= 55$

$(x - y)^2 = x^2 + y^2 - 2xy$

$= 34 - 2.\frac{21}{2}$

$= 34 - 21$

$= 13$

$2(x + y)^2 + (x - y)^2$

$= 2(55) + 13$

$= 110 + 13$

$= 123$

13.

Solution

Given $a - b = 3$ .....(i)

$ab = 4$ ......(ii)

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

from (i) squaring on both sides

$(a - b)^2 = 3^2$

$a^2 + b^2 - 2ab = 9$

$a^2 + b^2 - 2(4) = 9$

$a^2 + b^2 - 8 = 9$

$a^2 + b^2 = 9 + 8$

$a^2 + b^2 = 17$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$= 3 . (a^2 + b^2 + ab)$

$= 3 (17 + 4)$

$= 3(21)$

$= 63$

14.

Solution

Given $2a - 3b = 3$......(i)

$ab = 2$....(2)

from (i) equation on both sides

$(2a - 3b)^2 = 3^2$

$(2a)^2 - 2.2a.\ 3b + (3b)^2 = 9$

$4a^2 - 12ab + 9b^2 = 9$

$4a^2 + 9b^2 - 12(2) = 9$

$4a^2 + 9b^2 - 24 = 9$

$4a^2 + 9b^2 - 24 = 9$

$4a^2 + 9b^2 = 9 + 24$

$= 33$

$(2a)^3 - (3b)^3 = 8a^3 - 27b^3$

$= (2a - 3b)((2a)^2 + 2a.\ 3b + (3b)^2)$

$= 3(4a^2 + 6ab + 9b^2)$

$= 3(33 + 6(2))$

$= 3(33 + 12)$

$= 3(45)$

$= 135$

15.

Solution

given $x + \frac{1}{x} = 4$

(i) squaring on both sides

$$\left(x + \frac{1}{x}\right)^2 = 4^2$$

$$x^2 + 2.x . \frac{1}{x} + \frac{1}{x^2} = 16$$

$$x^2 + 2 + \frac{1}{x^2} = 16$$

$$x^2 + \frac{1}{x^2} = 16 - 2$$

$$x^2 + \frac{1}{x^2} = 14$$

(ii) $x^4 + \frac{1}{x^4}$

We know that $x^4 + \frac{1}{x^4} = 14$

Squaring on both sides

$$\left(x^2 + \frac{1}{x^2}\right)^2 = 14^2$$

$$(x^2)^2 + 2. x^2. \frac{1}{x^2} + \left(\frac{1}{x^2}\right)^2 = 196$$

$$x^4 + 2 + \frac{1}{x^4} = 196$$

$$x^4 + \frac{1}{x^4} = 196 - 2$$

$$x^4 + \frac{1}{x^4} = 194$$

(iii) $x^3 + \frac{1}{x^3}$

We know $x + \frac{1}{x} = 4$

Cubing on both sides

$\left(x + \frac{1}{x}\right)^3 = 4^3$

$x^3 + 3.\ x.\ \frac{1}{x}\left(x + \frac{1}{x}\right) + \frac{1}{x^3} = 64$

$x^3 + 3\left(x + \frac{1}{x}\right) + \frac{1}{x^3} = 64$

$x^3 + \frac{1}{x^3} + 3(4) = 64$

$x^3 + \frac{1}{x^3} = 64 - 12$

$= 52$

(iv) $x - \frac{1}{x}$

$\left(x - \frac{1}{x}\right)^2 = x^2 - 2.x.\ \frac{1}{x} + \frac{1}{x^2}$

$= x^2 + \frac{1}{x^2} - 2$

from (i) $x^2 + \frac{1}{x^2} = 14$

$= 14 - 2$

$\left(x - \frac{1}{x}\right)^2 = 12$

$x - \frac{1}{x} = \sqrt{12}$

$= \sqrt{4 \times 3}$

$= 2\sqrt{3}$

16.

Solution

Given $x - \frac{1}{x} = 5$

Squaring on both sides

$\left(x - \frac{1}{x}\right)^2 = 5^2$

$x^2 - 2.x \cdot \frac{1}{x} + \frac{1}{x^2} = 25$

$x^2 + \frac{1}{x^2} = 25 + 2$

$x^2 + \frac{1}{x^2} = 23$

squaring on both sides

$\left(x^2 - \frac{1}{x^2}\right)^2 = 23^2$

$(x^2)^2 + 2. x^2 . \frac{1}{7^2} + \left(\frac{1}{x^2}\right)^2 = 529$

$x^4 + 2 + \frac{1}{x^4} = 529$

$x^4 + \frac{1}{x^4} = 529 - 2$

$x^4 + \frac{1}{x^4} = 527$

17.

Solution

Given

$\mathbf{x - \frac{1}{x} = \sqrt{5}}$

(i) squaring on both sides

$\left(x - \frac{1}{x}\right)^2 = \left(\sqrt{5}\right)^2$

$x^2 - 2. x. \frac{1}{x} + \frac{1}{x^2} = 5$

$x^2 - 2 + \frac{1}{x^2} = 5$

$x^2 + \frac{1}{x^2} = 5 + 2$

$x^2 + \frac{1}{x^2} = 7$

(ii) $\mathbf{x + \frac{1}{x}}$

$= \left(x + \frac{1}{x}\right)^2 = x^2 + 2.\ x.\ \frac{1}{x} + \frac{1}{x^2}$

$= x^2 + \frac{1}{x^2} + 2 \quad [\therefore \text{from (i)}]$

$= 7 + 2$

$= 9$

$\left(x + \frac{1}{x}\right)^2 = 9$

$x + \frac{1}{x} = \sqrt{9}$

$x + \frac{1}{x} = 3$

(iii) $\mathbf{x^3 + \frac{1}{x^3}}$

$= \left(x + \frac{1}{x}\right) \left(x^2 - x.\frac{1}{x} + \frac{1}{x^2}\right)$

$= \left(x + \frac{1}{x}\right) \left(x^2 + \frac{1}{x^2} - 1\right)$

$= 3.\ (7-1)$

$= 3 \times 6$

$= 18$

18.

Solution

Given $x + \frac{1}{x} = 6$

Squaring on both sides

$\left(x + \frac{1}{x}\right)^2 = 6^2$

$x^2 + 2. x . \frac{1}{x} + \frac{1}{x^2} = 36$

$x^2 + 2 + \frac{1}{x^2} = 36$

$x^2 + \frac{1}{x^2} = 36 - 2$

$= 34$

(i) $\mathbf{x - \frac{1}{x}}$

$= \left(x - \frac{1}{x}\right)^2 = x^2 - 2. x . \frac{1}{x} + \frac{1}{x^2}$

$= x^2 + \frac{1}{x^2} - 2$

$= 34 - 2$

$\left(x - \frac{1}{x}\right)^2 = 32$

$x - \frac{1}{x} = \sqrt{32}$

$x - \frac{1}{x} = \sqrt{16 \times 2}$

$x - \frac{1}{x} = 4\sqrt{2}$

(ii) $\mathbf{x^2 - \frac{1}{x^2}}$

$= x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

$= 6 \cdot 4\sqrt{2}$

$= 24\sqrt{2}$

19.

Solution

Given $x + \frac{1}{x} = 2$

Squaring on both sides

$\left(x + \frac{1}{x}\right)^2 = 2^2$

$x^2 + 2 . x . \frac{1}{x} + \frac{1}{x^2} = 4$

$x^2 + 2 + \frac{1}{x^2} = 4$

$x^2 + \frac{1}{x^2} = 4 - 2$

$x^2 + \frac{1}{x^2} = 2$ ......(i)

$\therefore x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)\left(x^2 - x . \frac{1}{x} + \frac{1}{x^2}\right)$

$= 2 . \left(x^2 + \frac{1}{x^2} - 1\right)$

$= 2 (2 -1)$

$= 2(1)$

$= 2$.....(ii)

From (1) $= x^2 + \frac{1}{x^2} = 2$

Squaring on both sides

$= \left(x^2 + \frac{1}{x^2}\right)^2 = 2^2$

$= (x^2)^2 + 2.\ x^2 \cdot \frac{1}{x^2} + \left(\frac{1}{x^2}\right)^2 = 4$

$= x^4 + 2 + \frac{1}{x^4} = 4$

$= x^4 + \frac{1}{x^4} = 4 - 2$

$= x^4 + \frac{1}{x^4} = 4 - 2$

$= 2$ ......(iii)

From equation (i) , (ii) and (iii)

$x^2 + \frac{1}{x^2} = 2$

$x^3 + \frac{1}{x^3} = 2$

$x^4 + \frac{1}{x^4} = 2$

$\therefore x^2 + \frac{1}{x^2} = x^3 + \frac{1}{x^3} = x^4 + \frac{1}{x^4}$

Hence proved

20.

Solution

Given $x- \frac{2}{x} = 3$

Cubing on both sides

$\left(x - \frac{2}{x}\right)^3 = 3^3$

$x^3 - 3 . x . \frac{2}{x}\left(x - \frac{2}{x}\right) - \left(\frac{2}{x}\right)^3 = 27$

$x^3 - 6\left(x - \frac{2}{x}\right) - \frac{8}{x^3} = 27$

$x^3 - \frac{8}{x^3} - 6(3) = 27$

$x^3 - \frac{8}{x^3} = 27 + 18$

$= 45$

21.

Solution

Given $a+ 2b = 5$

Cubing on both sides

$(a + 2b)^2 = 5^3$

$a^3 + 3a . 2b(a + 2b) + (2b)^3 = 125$

$a^3 + 6ab(5) + 8b^3 = 125$

$a^3 + 30ab + 8b^3 = 125$

$\therefore a^3 + 8b^3 + 30ab = 125$

22.

Solution

Given $a + \frac{1}{a} = p$

Cubing on both sides

$\left(a + \frac{1}{a}\right)^3 = p^3$

$a^3 + 3a \cdot \frac{1}{a} \left(a + \frac{1}{a}\right) + \frac{1}{a^3} = p^3$

$a^3 + 3(p) + \frac{1}{a^3} = p^3$

$a^3 + \frac{1}{a^3} = p^3 - 3p$

$a^3 + \frac{1}{a^3} = p(p^2 - 3)$

23.

Solution

Given $x^2 + \frac{1}{x^2} = 27$

$\therefore \left(x - \frac{1}{x}\right)^2 = x^2 - 2x.\frac{1}{x} + \frac{1}{x^2}$

$= x^2 - 2 + \frac{1}{x^2}$

$= x^2 + \frac{1}{x^2} - 2$

$= 27 - 2$

$\left(x - \frac{1}{x}\right)^2 = 25$

$x - \frac{1}{x} = \sqrt{25}$

$x - \frac{1}{x} = 5$

24.

Solution

Given $x^2 + \frac{1}{x^2} = 27$

Take $\left(x - \frac{1}{x}\right)^2 = x^2 - 2.x.\frac{1}{x} + \frac{1}{x^2}$

$= x^2 - 2 + \frac{1}{x^2}$

$= x^2 + \frac{1}{x^2} - 2$

$= 27 - 2$

$\left(x - \frac{1}{x}\right)^2 = 25$

$x - \frac{1}{x} = \sqrt{25}$

$x - \frac{1}{x} = 5$

$\therefore 3x^3 + 5x - \frac{3}{x^3} - \frac{5}{x}$

$= 3x^3 - \frac{3}{x^3} + 5x - \frac{5}{x}$

$= 3\left(x^3 - \frac{1}{x^3}\right) + 5\left(x - \frac{1}{x}\right)$

$= 3\left(x - \frac{1}{x}\right)\left(x^2 + x \cdot \frac{1}{x} + \frac{1}{x^2}\right) + 5\left(x - \frac{1}{x}\right)$

$= 3\left(x - \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + 1\right) + 5\left(x - \frac{1}{x}\right)$

$= 3(5)(27 + 1) + 5(5)$

$= 15(28) + 25$

$= 420 + 25 = 445$

25.

Solution

Given $x^2 + \frac{1}{25x^2} = 8\frac{3}{5}$

$x^2 + \left(\frac{1}{5x}\right)^2 = \frac{43}{5}$

$\therefore$ let us consider

$\left(x + \frac{1}{5x}\right)^2 = x^2 + 2 \cdot x \cdot \frac{1}{5x} + \left(\frac{1}{5x}\right)^2$

$= x^2 + \frac{2}{5} + \frac{1}{25x^2}$

$= x^2 + \frac{1}{25x^2} + \frac{2}{5}$

$= \frac{43}{5} + \frac{2}{5}$

$$\left(x + \frac{1}{5x}\right)^2 = \frac{43+2}{5}$$

$$\left(x + \frac{1}{5x}\right)^2 = \frac{47}{5}$$

$$x + \frac{1}{5x} = \sqrt{\frac{47}{5}}$$

26.

Solution

Given $x^2 + \frac{1}{4x^2} = 8$

$$x^2 + \left(\frac{1}{2x}\right)^2 = 8$$

let us consider

$$\left(x + \frac{1}{2x}\right)^2 = x^2 + 2.x.\frac{1}{2x} + \left(\frac{1}{2x}\right)^2$$

$$= x^2 + 1 + \frac{1}{4x^2}$$

$$= x^2 + \frac{1}{4x^2} + 1$$

$$= 8 + 1$$

$$\left(x + \frac{1}{2x}\right)^2 = 9$$

$$x + \frac{1}{2x} = \sqrt{9}$$

$$\therefore x^3 + \left(\frac{1}{2x}\right)^3$$

$$= x^3 + \frac{1}{8x^3} = \left(x + \frac{1}{2x}\right)\left(x^2 - x.\frac{1}{2x} + \left(\frac{1}{2x}\right)^2\right)$$

$= x^3 + \frac{1}{8x^3} = \left(x + \frac{1}{2x}\right)\left(x^2 + \frac{1}{4x^2} - 1\right)$

$= 3(8-1)$

$= 3(7)$

$= 21$

27.

Solution

Given $a^2 - 3a + 1 = 0$

Dividing each term by a , we get

$\frac{a^2}{a} - \frac{3a}{a} + \frac{1}{a} = 0$

$a - 3 + \frac{1}{a} = 0$

$a + \frac{1}{a} = 3$

now

(i) $\left(\boldsymbol{a} + \frac{\mathbf{1}}{\boldsymbol{a}}\right)^{\mathbf{2}} = a^2 + 2 \,.\, a \,.\, \frac{1}{a} + \frac{1}{a^2}$

$\left(a + \frac{1}{a}\right)^2 = a^2 + 2 + \frac{1}{a^2}$

$a^2 \frac{1}{a^2} = \left(a + \frac{1}{a}\right)^2 - 2$

$= 3^2 - 2$

$= 9 - 2$

$= 7$

(ii) $a^3 + \frac{1}{a^3}$

$= \left(a + \frac{1}{a}\right)\left(a^2 - a\,.\frac{1}{a} + \frac{1}{a^2}\right)$

$= \left(a + \frac{1}{a}\right)\left(a^2 + \frac{1}{a^2} - 1\right)$

= (3)(7-1)

= 3× 6

= 18

28.

Solution

Given $a = \frac{1}{a-5}$

$a(a - 5) = 1$

$a^2 - 5a = 1$

$a^2 - 5a - 1 = 0$

(i)

Dividing each term by a , we get

$\frac{a^2}{a} - \frac{5a}{a} - \frac{1}{a} = 0$

$a - 5 - \frac{1}{a} = 0$

$\therefore\ a - \frac{1}{a} = 5$

(ii) now $\left(\boldsymbol{a}+\frac{\mathbf{1}}{\boldsymbol{a}}\right)$

$\therefore$ a - $\frac{1}{a}$ = 5

Squaring on both sides

$\left(a-\frac{1}{a}\right)^2 = 5^2$

$a^2 - 2 \cdot a \cdot \frac{1}{a} + \frac{1}{a^2} = 25$

$a^2 + \frac{1}{a^2} = 25 - 2$

$a^2 + \frac{1}{a^2} = 23$

$\left(a+\frac{1}{a}\right)^2 = a^2 + 2 \cdot a \cdot \frac{1}{a} + \frac{1}{a^2}$

$= a^2 + \frac{1}{a^2} + 2$

$= 23 + 2$

$= 25$

$\left(a+\frac{1}{a}\right)^2 = 25$

$a + \frac{1}{a} = \sqrt{25}$

$= 5$

(iii) $\mathbf{a^2} - \frac{\mathbf{1}}{\boldsymbol{a^2}} = \left(a + \frac{1}{a}\right)\left(a - \frac{1}{a}\right)$

$= 5 \times 5 = 25$

29.

Solution

Given $\left(x + \frac{1}{x}\right)^2 = 3$

$x^2 + 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} = 3$

$x^2 + \frac{1}{x^2} = 3 - 2$

$x^2 + \frac{1}{x^2} = 1$

$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)\left(x^2 - x.\frac{1}{x} + \frac{1}{x^2}\right)$

$= \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - 1\right)$

$= \sqrt{3}(1 - 1)$

$= \sqrt{3}(0)$

$= 0$

30.

Solution

Given $x = 5 - 2\sqrt{6}$

Squaring on both sides

$x^2 = 5$

31.

Solution

Given $a + b + c = 12$

Squaring on both sides

$(a + b + c)^2 = 12^2$

$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 144$

$\therefore$ from given $ab + bc + ca = 22$

$a^2 + b^2 + c^2 + 2(22) = 144$

$a^2 + b^2 + c^2 + 44 = 144$

$a^2 + b^2 + c^2 = 144 - 44$

$a^2 + b^2 + c^2 = 100$

32.

Solution

Given $a + b + c = 12$

Squaring on both sides

$(a + b + c)^2 = 12^2$

$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 144$

$\therefore a^2 + b^2 + c^2 = 100$

$\therefore 100 + 2(ab + bc + ca) = 144$

$2(ab + bc + ca) = 144 - 100$

$2(ab + bc + ca) = 44$

ab + bc + ca = $\frac{44}{2}$ = 22

33.

Solution

Given $a^2 + b^2 + c^2 = 125$

∴ ab + bc + ca = 50

∵ $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$= 125 + 2(50)$

$= 125 + 100$

$= 225$

$(a+b+c)^2 = 225$

$a + b + c = \sqrt{225}$

a+ b+ c = 15

34.

Solution

Given a+b –c = 5

$a^2 + b^2 + c^2 = 29$

$(a+b-c)^2 = a^2 + b^2 + c^2 + 2(ab - bc - ca)$

$5^2 = 29 + 2(ab - bc - ca)$

$25 = 29 + 2(ab - bc - ca)$

$25 - 29 = 2(ab - bc - ca)$

$-4 = 2(ab - bc - ca)$

$ab - bc - ca = -\frac{4}{2} = -2$

35.

Solution

Given $a - b = 7$

$a^2 + b^2 = 85$

$(a - b)^2 = a^2 + b^2 - 2ab$

$7^2 = 85 - 2ab$

$49 = 85 - 2ab$

$2ab = 85 - 49$

$2ab = 36$

$ab = \frac{36}{2}$

$ab = 18$

$\therefore a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$= 7(a^2 + b^2 + ab)$

$= 7(85 + 18)$

$= 7(103)$

$= 721$

36.

Solution

Given the number is x

$\therefore\ x = y - 3$

$x - y = -3$

$y - x = 3$

and $x^2 + y^2 = 29$

$\therefore y - x = 3$

Squaring on both sides

$(y - x)^2 = 3^2$

$y^2 + x^2 - 2xy = 9$

$29 - 2xy = 9$

$29 - 9 = 2xy$

$2xy = 20$

$xy = \frac{20}{2}$

$xy = 10$

37

Solution

Given sum of two numbers = 8

Product of two numbers = 15

Let number be x and y

$\therefore x + y = 8$

$xy = 15$

$x + y = 8$

squaring on both sides

$(x+y)^2 = 8^2$

$x^2 + y^2 + 2xy = 64$

$x^2 + y^2 + 2xy = 64$

$x^2 + y^2 + 2(15) = 64$

$x^2 + y^2 = 64 - 30$

$x^2 + y^2 = 34$

$\therefore x^3 + y^3$

$= (x + y)(x^2 - xy + y^2)$

$= 8(x^2 + y^2 - xy)$

$= 8(34 - 8)$

$= 8(26)$

$= 208$

Chapter Test

1. Find the expansions of the following :

(i) $(2x + 3y + 5)(2x + 3y - 5)$

(ii) $(6 - 4a - 7b)^2$

(iii) $(7 - 3xy)^3$

(iv) $(x + y + 2)^3$

Solution:

(i) $(2x + 3y + 5)(2x + 3y - 5)$

Let us simplify the expression, we get

$(2x + 3y + 5)(2x + 3y - 5) = [(2x + 3y) + 5][(2x - 3y) - 5]$

By using the formula, $(a)^2 - (b)^2 = [(a + b)(a - b)]$

$= (2x + 3y)^2 - (5)^2$

$= (2x)^2 + (3y)^2 + 2 \times 2x \times 3y - 5 \times 5$

$= 4x^2 + 9y^2 + 12xy - 25$

(ii) $(6 - 4a - 7\,b)^2$

Let us simplify the expression, we get

$(6 - 4a - 7\,b)^2 = [\,6 + (-\,4a) + (-7b)]^2$

$= (6)^2 + (-\,4a)^2 + (-\,7b)^2 + 2\,(6)\,(-\,4a) + 2\,(-\,4a)\,(-7b) + 2\,(-7b)\,(6)$

$= 36 + 16a^2 + 49b^2 - 48a + 56ab - 84b$

(iii) $(7 - 3xy)^3$

Let us simplify the expression

By using the formula, we get

$(7 - 3xy)^3 = (7)^3 - (3xy)^3 - 3\,(7)\,(3xy)\,(7 - 3xy)$

$= 343 - 27x^3y^3 - 63xy\,(7 - 3xy)$

$= 343 - 27x^3y^3 - 441xy + 189x^2y^2$

(iv) $(x + y + 2)^3$

Let us simplify the expression

By using the formula, we get

$(x + y + 2\,)^3 = [(x + y) + 2]^3$

$= (x + y)^3 + (2)^3 + 3\,(x + y)\,(2)\,(x + y + 2)$

$= x^3 + y^3 + 3x^2y + 3xy^2 + 8 + 6\,(x + y)\,[(x + y) + 2]$

$= x^3 + y^3 + 3x^2y + 3xy^2 + 8 + 6\,(x + y)^2 + 12(x + y)$

$= x^3 + y^3 + 3x^2y + 3xy^2 + 8 + 6\,(x^2 + y^2 + 2xy) + 12x + 12y = x^3 +$

$y^3 + 3x^2y + 3xy^2 + 8 + 6x^2 + 6y^2 + 12xy + 12x + 12y$

$= x^3 + y^3 + 3x^2y + 3xy^2 + 8 + 6x^2 + 6y^2 + 12x + 12y + 12xy$

2. Simplify: $(x – 2) (x + 2) (x^2 + 4) (x^4 + 16)$

Solution:

Let us simplify the expression, we get

$(x – 2) (x + 2) (x^4 + 4) (x^4 +16) = (x^2 – 4) (x^4 + 4) (x^4 + 16)$

$= [(x^2)^2 – (4)^2] (x^4 + 16)$

$= (x^4 – 16) (x^4 + 16)$

$= (x^4)^2 – (16)^2$

$= x^8 – 256$

3. Evaluate 1002 × 998 by using a special product.

Solution:

Let us simplify the expression, we get

$1002 \times 998 = (1000 + 2) (1000 – 2)$

$= (1000)^2 – (2)^2$

$= 1000000 – 4$

$= 999996$

4. If $a + 2b + 3c = 0$, Prove that $a^3 + 8b^3 + 27c^3 = 18abc$

Solution:

Given:

$a + 2b + 3c = 0$, $a + 2b = -3c$

Let us cube on both the sides, we get

$(a + 2b)^3 = (-3c)^3$

$a^3 + (2b)^3 + 3(a)(2b)(a + 2b) = -27c^3$

$a^3 + 8b^3 + 6ab(-3c) = -27c^3$

$a^3 + 8b^3 - 18abc = -27c^3$

$a^3 + 8b^3 + 27c^3 = 18abc$

Hence proved.

5. If $2x = 3y - 5$, then find the value of $8x^3 - 27y^3 + 90xy + 125$.

Solution:

Given:

$2x = 3y - 5$

$2x - 3y = -5$

Now, let us cube on both sides, we get

$(2x - 3y)^3 = (-5)^3$

$(2x)^3 - (3y)^3 - 3 \times 2x \times 3y\,(2x - 3y) = -125$

$8x^3 - 27y^3 - 18xy\,(2x - 3y) = -125$

Now, substitute the value of $2x - 3y = -5$

$8x^3 - 27y^3 - 18xy\,(-5) = -125$

$8x^3 - 27y^3 + 90xy = -125$

$8x^3 - 27y^3 + 90xy + 125 = 0$

6. If $a^2 - \left(\frac{1}{a^2}\right) = 5$, evaluate $a^4 + \left(\frac{1}{a^4}\right)$

Solution:

It is given that,

$a^2 - \left(\frac{1}{a^2}\right) = 5$

So,

By using the formula, $(a + b)^2$

$\left[a^2 - \frac{1}{a^2}\right]^2 = a^4 + \left(\frac{1}{a^4}\right) - 2$

$\left[a^2 - \frac{1}{a^2}\right]^2 + 2 = a^4 + \frac{1}{a^4}$

Substitute the value of $a^2 - \frac{1}{a^2} = 5$, we get

$5^2 + 2 = a^4 + \frac{1}{a^4}$

$a^4 + \frac{1}{a^4} = 25 + 2$

= 27

7. If $a + \frac{1}{a} = p$ and $a - \frac{1}{a} = q$, Find the relation between p and q.

Solution:

It is given that,

$a + \frac{1}{a} = p$ and $a - \frac{1}{a} = q$

so,

$\left(a + \frac{1}{a}\right)^2 - \left(a - \frac{1}{a}\right)^2 = 4(a)\left(\frac{1}{a}\right)$

= 4

By substituting the values, we get

$p^2 - q^2 = 4$

Hence the relation between p and q is that $p^2 - q^2 = 4$.

8. If $\frac{a^2 + 1}{a} = 4$, find the value of $\frac{2a^3 + 2}{a^3}$

Solution:

It is given that,

$\frac{a^2 + 1}{a} = 4$

$\frac{a^2}{a} + \frac{1}{a} = 4$

$a + \frac{1}{a} = 4$

So by multiplying the expression by 2a, we get

$2a^3 + \frac{2}{a^3} = 2\left[a^3 + \frac{1}{a^3}\right]$

$= 2\left[\left(a + \frac{1}{a}\right)^3 - 3(a)\left(\frac{1}{a}\right)\left(a + \frac{1}{a}\right)\right]$

$= 2\ [(4)^3 - 3(4)]$

$= 2\ [64 - 12]$

$= 2\ (52)$

$= 104$

9. If $x = \frac{1}{4 - x}$, find the value of

(i) $x + \frac{1}{x}$

(ii) $x^3 + \frac{1}{x^3}$

(iii) $x^6 + \frac{1}{x^6}$

Solution:

It is given that,

$x = \frac{1}{4 - x}$

So,

(i) x(4 – x) = 1

$4x - x^2 = 1$

Now let us divide both sides by x, we get

$4 - x = \frac{1}{x}$

$4 = \frac{1}{x} + x$

$\frac{1}{x} + x = 4$

$\frac{1}{x} + x = 4$

(ii) $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^2 - 3\left(x + \frac{1}{x}\right)$

By substituting the values, we get

$= (4)^3 - 3(4)$

$= 64 - 12$

$= 52$

(iii) $x^6 + \frac{1}{x^6} = \left(x^3 + \frac{1}{x^3}\right)^2 - 2$

$= (52)^2 - 2$

$= 2704 - 2$

$= 2702$

10. If $\mathbf{x - \frac{1}{x} = 3 + 2\sqrt{2}}$, find the value of $\mathbf{\frac{1}{4}\left(x^3 - \frac{1}{x^3}\right)}$

Solution:

It is given that,

$x - \frac{1}{x} = 3 + 2\sqrt{2}$

So,

$x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right)$

$= (3 + 2\sqrt{2})^3 + 3(3 + 2\sqrt{2})$

By using the formula, $(a+b)^3 = a^3 + b^3 + 3ab\,(a + b)$

$= (3)^3 + (2\sqrt{2})^3 + 3\,(3)\,(2\sqrt{2})\,(3 + 2\sqrt{2}) + 3(3 + 2\sqrt{2})$

$= 27 + 16\sqrt{2} + 54\sqrt{2} + 72 + 9 + 6\sqrt{2}$

$= 108 + 76\sqrt{2}$

Hence,

$\frac{1}{4}\left(x^3 - \frac{1}{x^3}\right) = \frac{1}{4}(108 + 76\sqrt{2})$

$= 27 + 19\sqrt{2}$

11. If $x + \frac{1}{x} = 3\frac{1}{3}$, find the value of $x^3 - \frac{1}{x^3}$

Solution:

It is given that,

$x + \frac{1}{x} = 3\frac{1}{3}$

we know that,

$\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$

$= x^2 + \frac{1}{x^2} + 2 - 4$

$= \left(x + \frac{1}{x}\right)^2 - 4$

But $x + \frac{1}{x} = 3\frac{1}{3} = \frac{10}{3}$

So,

$\left(x - \frac{1}{x}\right)^2 = \left(\frac{10}{3}\right)^2 - 4$

$= \frac{100}{9} - 4$

$= \frac{100 - 36}{9}$

$= \frac{64}{9}$

$x - \frac{1}{x} = \sqrt{\frac{64}{9}}$

$= \frac{8}{3}$

Now,

$x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 + 3\,(x)\left(\frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

$= \left(\frac{8}{3}\right)^3 + 3\left(\frac{8}{3}\right)$

$= \left(\frac{512}{27} + 8\right)$

$= \frac{728}{27}$

$= 26\,\frac{26}{27}$

12. If x = 2 – $\sqrt{3}$, then find the value of $x^3 - \frac{1}{x^3}$

Solution:

It is given that,

$x = 2 - \sqrt{3}$

so,

$\frac{1}{x} = \frac{1}{2 - \sqrt{3}}$

By rationalizing the denominator, we get

$= \left[\frac{1(2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})}\right]$

$= \left[\frac{(2 + \sqrt{3})}{2^2 - \sqrt{3}^2}\right]$

$= \left[\frac{2 + \sqrt{3}}{4 - 3}\right]$

$= 2 + \sqrt{3}$

Now,

$x - \frac{1}{x} = 2 - \sqrt{3} - 2 - \sqrt{3}$

$= - 2\sqrt{3}$

Let us cube on both sides, we get

$\left(x - \frac{1}{x}\right)^3 = \left(-2\sqrt{3}\right)^3$

$x^3 - \frac{1}{x^3} - 3\ (x)\ \left(\frac{1}{x}\right)\left(x - \frac{1}{x}\right) = 24\sqrt{3}$

$x^3 - \frac{1}{x^3} - 3(-2\sqrt{3}) = -24\sqrt{3}$

$x^3 - \frac{1}{x^3} + 6\sqrt{3} = -24\sqrt{3}$

$x^3 - \frac{1}{x^3} = -24\sqrt{3} - 6\sqrt{3}$

$= -30\sqrt{3}$

Hence,

$x^3 - \frac{1}{x^3} = -30\sqrt{3}$

13. If the sum of two numbers is 11 and sum of their cubes is 737, find the sum of their squares.

Solution:

Let us consider x and y be two numbers

Then,

$x + y = 11$

$x^3 + y^3 = 735$ and $x^2 + y^2 = ?$

Now,

$x + y = 11$

Let us cube on both the sides,

$(x + y)^3 = (11)^3$

$x^3 + y^3 + 3xy\ (x + y) = 1331$

$737 + 3x \times 11 = 1331$

$33xy = 1331 – 737$

$= 594$

$xy = \frac{594}{33}$

$xy = 8$

We know that, $x + y = 11$

By squaring on both sides, we get

$(x + y)^2 = (11)^2$

$x^2 + y^2 + 2xy = 121$ ^{2} $x^2 + y^2 + 2 \times 18 = 121$

$x^2 + y^2 + 36 = 121$

$x^2 + y^2 = 121 – 36$
$= 85$

Hence sum of the squares = 85

14. If $a – b = 7$ and $a^3 – b^3 = 133$, find:

(i) ab

(ii) $a^2 + b^2$

Solution:

It is given that,

$a - b = 7$

let us cube on both sides, we get

(i) $(a - b)^3 = (7)^3$

$a^3 + b^3 - 3ab\ (a - b) = 343$

$133 - 3ab \times 7 = 343$

$133 - 21ab = 343$

$- 21ab = 343 - 133\ 21ab$

$= 210$

$ab = - \frac{210}{21}$

$ab = -10$

(ii) $a^2 + b^2$

Again $a - b = 7$

Let us square on both sides, we get

$(a - b)^2 = (7)^2$

$a^2 + b^2 - 2ab = 49$

$a^2 + b^2 - 2 \times (-10) = 49$

$a^2 + b^2 + 20 = 49$

$a^2 + b^2 = 49 - 20$
$= 29$

Hence, $a^2 + b^2 = 29$

15. Find the coefficient of x^2 expansion of $(x^2 + x + 1)^2 + (x^2 - x + 1)^2$

Solution:
Given:
The expression, $(x^2 + x + 1)^2 + (x^2 - x + 1)^2$

$(x^2 + x + 1)^2 + (x^2 - x + 1)^2 = [((x^2 + 1) + x)^2 + [(x^2 + 1) - x)^2]$

$= (x^2 + 1)^2 + x^2 + 2\ (x^2 + 1)\ (x) + (x^2 + 1)^2 + x^2 - 2\ (x^2 + 1)\ (x)$

$= (x^2)^2 + (1)^2 + 2 \times x^2 \times 1 + x^2 + (x^2)^2 + 1 + 2 \times x^2 + 1 + x^2$

$= x^4 + 1 + 2x^2 + x^2 + x^4 + 1 + 2x^2 + x^2$

$= 2x^4 + 6x^2 + 2$

$\therefore$ Co-efficient of x^2 is 6.