CHAPTER 2

HCF and LCM of Numbers

Factors

A number '*a*' is said to be the measure or factor of an other number '*b*' if it measures or divides the number *b* without leaving a remainder.

e.g., (i) 4 is a factor of 16 as $16 = 4 \times 4$

(ii) The factors of 729 are 9, 9, 9 as

$$729 = 9 \times 9 \times 9$$

Prime Factors

A factor may have other factors, then these can be again factorized such that all factors are prime numbers, then the factors are called prime factors. *e.g.*,

(i) 729 has factors $9 \times 9 \times 9$.

But $3 \times 3 \times 3 \times 3 \times 3 \times 3$ are prime factors of 729.

(ii) 72 has factors 8×9, but $2 \times 2 \times 2 \times 3 \times 3$ are prime factors of 72.

Least Common Multiple (LCM)

Common Multiple A common multiple of two numbers is a number which is exactly divisible by each of the given numbers.

e.g., 30 is a common multiple of 2, 3, 5, 6.

Least Common Multiple The least common multiple of two or more given numbers is the least number which is exactly divisible by each one of them.

e.g., 42 is the common multiple of 2, 3, 4, 7.

84 is the common multiple of 2, 3, 4, 7. But, 42 is the LCM of 2, 3, 4, 7 as it is least among the two.

Methods of Finding LCM

1. **LCM by Factorization** Write down the prime factors of the given numbers. Then, the LCM is the product of the highest powers of all the factors.

Example 1. The LCM of 30, 250, 490 is

(a) 46750 (b) 36750 (c) 26750 (d) None of these

Sol. (*b*) Here, $30 = 2 \times 3 \times 5$

$250 = 5 \times 5 \times 2 \times 5 = 2 \times 5^3$

and $490 = 7 \times 7 \times 2 \times 5 = 2 \times 5 \times 7^2$

$\therefore$ LCM of the number $= 2 \times 5^3 \times 7^2 \times 3$

$= 2 \times 125 \times 49 \times 3$

$= 250 \times 49 \times 3$

$= 36750$

2. **To Find the Least Common Multiple of More Than Two Numbers by Factorization** Divide all number or as many as possible by such a prime common divisors as may be contained in them and then multiply together the divisors and the final quotients.

Example 2. What is the LCM of 30, 250, 490?

(a) 16750 (b) 26750 (c) 36750 (d) 46750

Sol. (*c*)

2	30,	250,	490
3	15,	125,	245
5	5,	125,	245
5	1,	25,	49
5	1,	5,	49
7	1,	1,	49
7	1,	1,	7
	1,	1,	1

LCM $= 2 \times 3 \times 5 \times 5 \times 5 \times 7 \times 7$

$= 2 \times 3 \times 5^3 \times 7^2 = 36750$

Highest Common Factors (HCF)

Common Factor A common factor of two or more numbers is a number which divides each of them exactly.

e.g., 2 is common factor of 2, 10, 20.

Highest Common Factor When two or more number is the greatest number which divides each of them exactly.

e.g., HCF of the numbers 18 and 24 is 6.

Methods of Finding HCF

HCF by Division Method

To find the HCF by division, divide the largest number by the smaller one. Now, you will get the remainder, divide the divisor by the remainder. Repeat this process until no remainder is left, the last divisor used in this process is the desired greatest common divisor.

Example 3. The HCF of 2923 and 3239 is

(a) 69 (b) 71 (c) 75 (d) 79

Sol. (*d*) Here,

```
2923) 3239 (1
      2923
       316) 2923 (9
            2844
              79) 316 (4
                  316
                   ×
```

Hence, HCF = 79

Example 4. The HCF of 204, 1190 and 1445 is

(a) 85 (b) 15 (c) 17 (d) 15

Sol. (*c*) Here,

```
1190)1445 (1
     1190
      255)1190 (4
          1020
           170) 225 (1
                170
                 85)170 (2
                    170
                     ×
```

Here, HCF of 1190 and 1445 is 85.

Now,

```
85) 204 (2
    170
     34) 85 (2
         68
         17) 34 (2
             34
              ×
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Here, HCF of 85 and 204 is 17.

So, HCF of 204,1190 and 1145 is 17.

HCF by Prime Factorization

Write the given number into prime factors and then find the product of all the prime factors common to all the numbers. This product is the required HCF of numbers.

Example 5. The HCF of 63 and 81 is

(a) 7 (b) 8
(c) 9 (d) 11

Sol. (*c*) Here, $63 = 7 \times 3 \times 3$ and $81 = 3 \times 3 \times 3 \times 3$

$\therefore$ HCF $= 3 \times 3 = 9$

Example 6. The HCF of 65, 75 and 105 is

(a) 4 (b) 5
(c) 6 (d) 8

Sol. (*b*) Here, $65 = 13 \times 5$

$75 = 5 \times 5 \times 3$ and $105 = 21 \times 5 = 7 \times 3 \times 5$

$\therefore$ HCF $= 5$

Formulae to be Remember

- Product of two numbers = (Their HCF) × (Their LCM)
- LCM of the number $= \dfrac{\text{Product of numbers}}{\text{HCF of numbers}}$
- HCF of the numbers $= \dfrac{\text{Product of numbers}}{\text{LCM of numbers}}$
- One number $= \dfrac{\text{LCM of the numbers} \times \text{HCF of the numbers}}{\text{Second number}}$
- HCF of fractions $= \dfrac{\text{HCF of numerators}}{\text{LCM of denominators}}$
- LCM of fractions $= \dfrac{\text{LCM of numerators}}{\text{HCF of denominators}}$

Example 7. The HCF of $\frac{14}{5}, \frac{21}{5}, \frac{7}{5}$ is

(a) $1\frac{2}{5}$ (b) $2\frac{2}{5}$
(c) $\frac{2}{5}$ (d) None of these

Sol. (*a*) HCF of numerators *i.e.*, 14, 21 and 7 is 7 and LCM of denominators *i.e.*, 5, 5 and 5 is 5. So, HCF of given fractions $= \frac{7}{5}$ or $1\frac{2}{5}$

Points to be Remember

- The greatest number that will divide x, y and z leaving remainders a, b and c, respectively is given by HCF of $(x-a), (y-b), (z-c)$.
- The least number which, when divided by x, y and z leaves the remainders a, b and c respectively is given by [LCM of $(x, y, z) - p$], where $p = (x-a) = (y-b) = (z-c)$.
- The least number which when divided by x, y and z leaves the same remainder R in each case is given by [LCM of $(x, y, z) + R$].
- The greatest number that will divide x, y and z leaving the same remainder in each case is given by [HCF of $|x-y|, |y-z|, |z-x|$]

Exercise

1. The HCF of 185, 333 and 481 is
   (a) 74 (b) 37 (c) 38 (d) 78

2. If $x = 2^3 \times 3^2 \times 5^4$ and $y = 2^2 \times 3^2 \times 5 \times 7$, then HCF of x and y is
   (a) 180 (b) 360 (c) 540 (d) 35

3. The product of two numbers is 960. If HCF is 8, then the numbers are
   (a) 24, 40 (b) 8, 120 or 24, 40
   (c) 8, 140 (d) None of these

4. The product of HCF and LCM of 14 and 16 is
   (a) 2 (b) 12 (c) 224 (d) 112

5. The HCF of 5/6, 10/18, 25/36 is
   (a) $\frac{5}{36}$ (b) $\frac{25}{6}$ (c) $\frac{25}{36}$ (d) $\frac{5}{18}$

6. The LCM of $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$ is
   (a) 42 (b) 24 (c) 12 (d) 4/5

7. LCM of $\frac{4}{5}, \frac{3}{10}$ and $\frac{7}{15}$ is
   (a) $8\frac{2}{3}$ (b) $\frac{8}{15}$ (c) 20 (d) $16\frac{4}{5}$

8. LCM of $2^3 \times 3 \times 5$ and $2^4 \times 5 \times 7$ is
   (a) $2^{12} \times 3 \times 5^2 \times 7$ (b) $2^4 \times 5 \times 7 \times 3$
   (c) $2^4 \times 3 \times 5 \times 7$ (d) $2^3 \times 3 \times 5 \times 7$

9. Consider those numbers between 300 and 400 such that when each number is divided by 6, 9 and 12, it leaves 4 as remainder in each case. What is the sum of the numbers? **(CDS 2010 I)**
   (a) 692 (b) 764 (c) 1080 (d) 1092

10. The least number which when divided by 5, 6, 7 and 8 leaves a remainder 3, but when divided by 9 leaves no remainder is
    (a) 3246 (b) 1863 (c) 1368 (d) 1683

11. The least number divisible by 12, 15, 20 and is a perfect square is
    (a) 900 (b) 400 (c) 36 (d) 256

12. The largest number which divides 133 and 245 leaving a remainder 5 is
    (a) 17 (b) 15 (c) 8 (d) 16

13. The HCF of two numbers is 1/5th of their LCM. If the product of the two numbers is 720, then the HCF of the numbers is
    (a) 13 (b) 12 (c) 14 (d) 18

14. Let p, q, r be natural numbers. If m is their LCM and n is their HCF, consider the following
    I. $mn = pqr$ iff each p, q, r is prime.
    II. $mn = pqr$ iff p, q, r are ralatively prime in pairs.
    Which of the above is/are correct? **(CDS 2008 I)**
    (a) I only (b) II only
    (c) Both I and II (d) Neither I nor II

15. The LCM of two numbers is 39780 and their ratio is 13 : 15. Then, the numbers are
    (a) 273, 315 (b) 2652, 3060
    (c) 516, 685 (d) None of these

16. The HCF of 11, 1.1, 0.11, 0.011 is
    (a) 0.011 (b) 1.1 (c) 0.11 (d) 0.111

17. The LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one of the number is 80, then the other is
    (a) 65 (b) 280 (c) 25 (d) 45

18. Four bells begin to toll together and toll, respectively at intervals of 5, 6, 8 and 12 s. How many times will they toll together in an hour excluding the one at the start?
    (a) 10 (b) 19 (c) 13 (d) 9

19. Two ropes of length 28 m and 36 m are to be cut into bits of same length. The greatest possible length of each is
    (a) 7 (b) 3 (c) 4 (d) 5

20. The largest size of handkerchief's which can be cut from a cloth having length 275 m and breadth 75 cm is
    (a) 25 cm (b) 30 cm (c) 24 cm (d) 29 cm

21. If 'a' is a prime number, then HCF of 'a' and $(a+1)$ is
    (a) 1 (b) a
    (c) $a(a+1)$ (d) None of these

22. If 'b' is a prime number, then LCM of 'b' and $(b+1)$ is
    (a) $b(b+1)$ (b) b^2 (c) b (d) $(b+1)$

23. There are 28 mango trees, 42 apple trees and 21 orange trees have to be planted in rows such that each row contains the same number of trees of one variety only, then minimum number of rows in which the above trees may be planted is
    (a) 13 (b) 12 (c) 11 (d) 10

24. A heap is to be formed with lots of 8, 10 and 15 pebbles of different colours. The smallest number of pebbles in the heap is
    (a) 121 (b) 120 (c) 110 (d) 8

25. What is the greatest number which divides 392, 486 and 627 so as to leave the same remainder in each case? **(CDS 2009 II)**
    (a) 47 (b) 43 (c) 37 (d) 34

26. What is the least number which when divided by 42, 72 and 84 leaves the remainders 25, 55 and 67, respectively? **(CDS 2009 II)**
    (a) 521 (b) 512 (c) 504 (d) 487

27. 21 mango trees, 42 apple trees and 56 orange trees have to be planted in rows such that each row contains the same number of trees of one variety only. What is the minimum number of rows in which the above trees may be planted? **(CDS 2008 II)**
    (a) 3 (b) 15
    (c) 17 (d) 20

28. A person has four iron bars whose lengths are 24 m, 36 m, 48 m and 72 m, respectively. This person wants to cut pieces of same length from each of four bars. What is the least number of total pieces if he is to cut without any wastage? **(CDS 2007 II)**

(a) 10 (b) 15 (c) 20 (d) 25

29. A bell rings every 5 s. A second bell rings every 6 s and a third one rings every 8 s. If all the three ring at the same time at 8:00 am, at what time will they all ring together next? **(CDS 2007 I)**

(a) 1 min past 8:00 am (b) 2 min past 8:00 am
(c) 3 min past 8:00 am (d) 4 min past 8:00 am

30. Two numbers have 16 as their HCF and 146 as their LCM. How many such pairs of numbers are there? **(CDS 2007 I)**

(a) Zero (b) Only 1 (c) Only 2 (d) Many

Answers

1. (b)	**2.** (a)	**3.** (b)	**4.** (c)	**5.** (a)	**6.** (c)	**7.** (d)	**8.** (c)	**9.** (a)	**10.** (d)
11. (a)	**12.** (d)	**13.** (b)	**14.** (c)	**15.** (b)	**16.** (a)	**17.** (b)	**18.** (a)	**19.** (c)	**20.** (a)
21. (a)	**22.** (a)	**23.** (a)	**24.** (b)	**25.** (a)	**26.** (d)	**27.** (c)	**28.** (b)	**29.** (b)	**30.** (a)

Hints and Solutions

1. Here, $185 = 5 \times 37 : 333 = 3 \times 3 \times 37 : 481 = 13 \times 37$

$\therefore$ The HCF of 185, 333 and 481 = 37

2. $x = 2^3 \times 3^2 \times 5^4$ and $y = 2^2 \times 3^2 \times 5 \times 7$

$\therefore$ HCF = Common power of prime $= 2^2 \times 3^2 \times 5 = 180$

3. As HCF of numbers is 8.

So, it as factor of both.

Here, 64 is factor of product.

Now, as $960 \div 64 = 15$

Now, prime factor of $15 = 3 \times 5$ or 1×15

Hence, numbers are $8 \times 1 = 8$, $8 \times 15 = 120$

or $8 \times 3 = 24$, $8 \times 5 = 40$

$\therefore$ Numbers are 8, 120 or 24, 40.

4. Product of HCF $\times$ Product of LCM = Product of numbers

$= 14 \times 16 = 224$

5. Here, HCF $= \dfrac{\text{HCF of 5, 10, 25}}{\text{LCM of 6, 18, 36}} = \dfrac{5}{36}$

6. Here, LCM $= \dfrac{\text{LCM of 1, 2, 3, 4}}{\text{HCF of 2, 3, 4, 5}} = \dfrac{12}{1} = 12$

7. Here, LCM $= \dfrac{\text{LCM of 4, 3, 7}}{\text{HCF of 5, 10, 15}} = \dfrac{84}{5} = 16\dfrac{4}{5}$

8. Here, say $a = 2^3 \times 3 \times 5$ and $b = 2^4 \times 5 \times 7$, then

LCM = prime factor with highest power $= 2^4 \times 3 \times 5 \times 7$

9. LCM of (6, 9, 12) = 36

$\therefore$ Number is the form of $36p + 4$.

Since, the required number between 300 and 400.

$\therefore$ $p = 9$ and 10

$\therefore$ Required sum $= 328 + 364 = 692$

10. Here, the number which is completely divisible by 5, 6, 7 and 8 is LCM of 5, 6, 7, 8, *i.e.*, 1680 and add 3 as remainder.

So, number $= 1680 + 3 = 1683$

11. LCM $= 2 \times 2 \times 3 \times 5$

2	12,	15,	20
2	6,	15,	10
3	3,	15,	5
5	1,	5,	5
	1,	1,	1

$\therefore$ Required perfect square $= 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 900$

12. When divide 133 and 245 by the number the remainder is 5, so find HCF of $133 - 5$ and $245 - 5$,

i.e., 128 and 240.

Here,

128)240(1
128
112)128(1
112
16)112(7
112
×

The number is 16.

13. Let LCM $= 5x$, then HCF $= x$

Now, product of numbers = 720

($\because$ HCF $\times$ LCM = Product of the numbers)

So, here $(5x)x = 720$

$5x^2 = 720$

$x^2 = 144$

$x = 12$

14. I. Since, p, q and r are prime, then $mn = pqr$

II. Let $p = 6, q = 5, r = 7$

LCM (6, 5, 7) = 210, HCF (6, 5, 7) = 1

$\therefore$ $mn = 210 \times 1 = 210$ and $pqr = 6 \times 5 \times 7 = 210$

$\therefore$ $mn = pqr$

15. The numbers are $13x$ and $15x$.

So, x is HCF. Now, HCF $\times$ LCM = Product of numbers

$x \times 39780 = 13x \times 15x$

$$x \times 39780 = 13 \times 15 \times x^2$$

$$x = \frac{39780}{13 \times 15} = 204$$

$\therefore$ Numbers are $13 \times 204 = 2652$ and $15 \times 204 = 3060$

16. HCF of (11, 1.1, 0.11, 0.011) = HCF of $\left(\frac{11}{1}, \frac{11}{10}, \frac{11}{100}, \frac{11}{1000}\right)$

$$= \frac{\text{HCF of } (11, 11, 11, 11)}{\text{LCM of } (1, 10, 100, 1000)} = \frac{11}{1000} = 0.011$$

17. LCM = 14 HCF and LCM + HCF = 600

$$14 \text{ HCF} + \text{HCF} = 600$$
$$15 \text{ HCF} = 600$$
$$\text{HCF} = 40$$

$\therefore$ $\text{LCM} = 14 \times 40 = 560$

$\therefore$ Other number $= \frac{\text{LCM} \times \text{HCF}}{\text{Given number}} = \frac{560 \times 40}{80} = 280$

18. Here, LCM of 5, 6, 8 and 12 is 360 so the bells will toll after 360 s.

So, in an hour they will toll together $= \frac{60 \times 60}{360} = 10$ times

19. Here, the greatest possible length of piece is equal to HCF of 28 and 36. *i.e.,* $28 = 2 \times 2 \times 7$

$$36 = 2 \times 2 \times 9$$

$\therefore$ HCF = 4 and length of each piece = 4 cm

20. Largest size of handkerchief = HCF of 275 cm and 75 cm

$$= 25 \text{ cm}$$

21. As prime number has only two factor 1 and the number. So, HCF of a and $(a+1)$ is 1.

22. As LCM of prime number and a number is product of numbers.

i.e., $b(b+1)$

23. Here, HCF of 28, 42 and 21 is 7.

$\therefore$ Number of rows of mango $= \frac{28}{7} = 4$

Number of rows of apple $= \frac{42}{7} = 6$

Number of rows of orange $= \frac{21}{7} = 3$

$\therefore$ Number of rows $= 3 + 6 + 4 = 13$

24. The smallest number of pebbles in the heap is LCM of 8, 1[illegible] and 15. So, the number of pebbles in heap = 120.

25. Let the remainder be x.

Then, the given number will be $(392 - x)$, $(486 - x)$ and $(627 - x)$.

Now, $(486 - x) - (392 - x) = 94$

$(627 - x) - (486 - x) = 141$

and $(627 - x) - (392 - x) = 235$

$\therefore$ HCF (94, 141, 235) = 47

26. Here, difference $= (42 - 25) = (72 - 55) = (84 - 67) = 17$

Now, LCM (42, 72, 84) = 504

$\therefore$ The required number $= 504 - 17 = 487$

27. The HCF of (21, 42, 56) is 7.

$\therefore$ The minimum number of rows $= 3 + 6 + 8 = 17$

28.

$$24 = 12 \times 2$$
$$36 = 12 \times 3$$
$$48 = 12 \times 4$$
$$72 = 12 \times 6$$

$\therefore$ HCF (24, 36, 48, 72) = 12

Total piece $= 2 + 3 + 4 + 6 = 15$

29. Required time = LCM of (5, 6, 8) = 120 s = 2 min

Hence, all bells will ring 2 min past 8:00 am.

30. HCF of two numbers = 16

and LCM of two numbers = 146

Let the first number, $x = 16a$

and the second number, $y = 16b$

We know that,

HCF × LCM = Product of two numbers

$\Rightarrow$ $16 \times 146 = x \times y$

$\Rightarrow$ $16 \times 146 = 16a \times 16b$

$\Rightarrow$ $a \times b = \frac{146}{16} = 9.125$

The number of such pairs is zero, since, $a \times b$ is not a integer.