ICSE Board
Mathematics
Sample Paper – 2

Time: 2 hrs 30 min **Total Marks: 80**

General Instructions:

1. Answers to this paper must be written on the paper provided separately.
2. You will not be allowed to write during the first **15 minutes**.
3. This time is to be spent in reading the question paper.
4. The time given at the head of this paper is the time allowed for writing the answers.
5. Attempt **all** questions from **Section A.** Solve any **four** questions from **Section B.**
6. **All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer.**
7. **Omission of essential working will result in loss of marks.**
8. The intended marks for questions or parts of questions are given in brackets [ ].

Section A (40 marks)

Question 1

(a) Expand the following: [3]

i. $(a + 2b - 3c)^2$

ii. $\left(4-\sqrt{5}x\right)^2$

(b) Find the cube root of 74088. [3]

(c) Let A = {factors of 24} and B = {factors of 30}, find [4]

i. A ∪ B ii. A ∩ B iii. A – B

Also verify that, n(A – B) = n(A) – n(A ∩ B) = n(A ∪ B) – n(B)

Question 2

(a) Solve: $(81)^{-1} \times 3^{-5} \times 3^{9} \times (64)^{5/6} \times \left(\sqrt[3]{3}\right)^{6}$ [3]

(b) If two adjacent sides of a rectangle are $(5x^2 + 25xy + 4y^2)$ and $(2x^2 - 2xy + 3y^2)$, find its area. [3]

(c) A two digit number is three times the sum of its digits. If 45 is added to the number; its digits are reversed. Find the number. [4]

Question 3

(a) Find the square root of 761.9, corrected up to two places of decimal. [3]

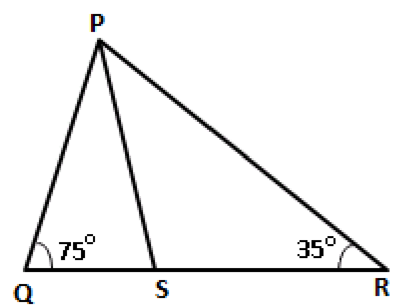
(b) A wire is in the form of a square with each side measuring 27.5 cm. It is straightened and bent into the shape of a circle. Find the area of the circle. [3]

(c) Sumit took a loan of Rs. 16000 from Bank of Baroda for 3 years at the rate of 12.5% p.a. compounded annually. Find the amount and the compound interest he has to pay at the end of 3 years to clear his debt to the nearest rupee. [4]

Question 4

(a) A hot water tap and a cold water tap fill a bath tub in 12 minutes and 15 minutes respectively. An outlet pipe empties it in 10 minutes. If all three are kept open simultaneously, in how much time will the bath tub be full? [3]

(b) In the adjoining diagram, PS bisects $\angle P$. Arrange PQ, QS and SR in ascending order. [3]



(c) Construct ΔABC in which BC = 6 cm, $m\angle B = 120^{\circ}$ and AB = 4.5 cm. Draw its circumcircle. [4]

Section B (40 Marks)

Question 5

(a) Factorise the polynomial $x^4 + 5x^2 - 6$. [3]

(b) Solve to find values of a and b: [3]

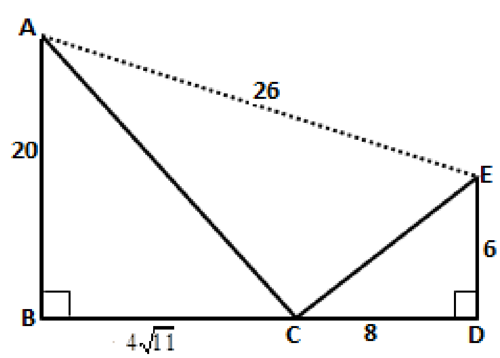
$2(a - 3) + 3(b - 5) = 0$

$5(a - 1) + 4(b - 4) = 0$

(c) In the adjoining figure, all measurements are in centimeters. [4]

Find (i) AC (ii) CE

Hence prove that $AE^2 = AC^2 + CE^2$. Also state the measure of $\angle ACE$.



Question 6

(a) Simplify: $\dfrac{\sqrt{15}-2}{\sqrt{15}+2}+\dfrac{\sqrt{15}+2}{\sqrt{15}-2}$ [3]

(b) Find the area of a triangle whose sides are 28 cm, 21 cm and 35 cm. [3]

(c) Draw a histogram for the following data: [4]

Class Interval	Frequency
0 – 5	4
5 – 10	10
10 - 15	18
15 - 20	8
20 - 25	6

Question 7

(a) If $2a - \dfrac{1}{2a} = 3$, find the value of $8a^3 - \dfrac{1}{8a^3}$. [3]

(b) The following table shows the market position of different brands of tea-leaves: [3]

Brand	A	B	C	D	others
% Buyers	35	20	20	15	10

Draw a pie-chart to represent the above information.

(c) Draw the graphs of the equations 2x - y = 3 and 3x + 2y = 1 on the same co-ordinate axes. Also, find the point of intersection of the two lines from the graphs. [4]

Question 8

(a) Simplify: $\dfrac{2x}{x^2-4}+\dfrac{1}{x^2+3x+2}$ [3]

(b) The dimensions of a cube are doubled. Will there be an increase or decrease in its volume and surface area? If yes, by how many times will its volume and surface area change? [3]

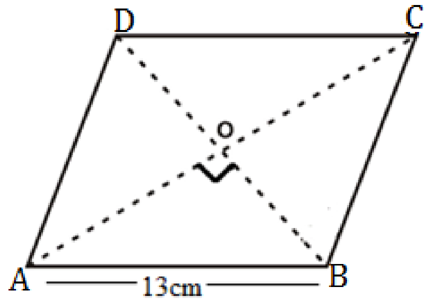
(c) A dealer puts up a sale in his shoe shop. He marks his goods 40% above the cost price and allows a discount of 15%. Find his profit percentage. [4]

Question 9

(a) Prove that a median divides a triangle into two triangles of equal area. [3]

(b) The dimensions of a cuboidal tin box are 30 cm x 40 cm x 50 cm. Find the cost of the tin required for making 20 such tin boxes if the cost of tin sheet is Rs. 25 per square metre. [3]

(c) ABCD is a rhombus having each side measuring 13 cm and one of its diagonal AC of length 24 cm. Find the area of the rhombus. [4]



Solution

Section A (40 marks)

Question 1

(a)

i. $(a + 2b - 3c)^2$
$= (a)^2 + (2b)^2 + (-3c)^2 + 2(a)(2b) + 2(2b)(-3c) + 2(-3c)(a)$
$= a^2 + 4b^2 + 9c^2 + 4ab - 12bc - 6ca$

ii. $\left(4-\sqrt{5}x\right)^2 = (4)^2 - 2(4)\left(\sqrt{5}x\right) + \left(\sqrt{5}x\right)^2 = 16 - 8\sqrt{5}x + 5x^2$

(b)

2	74088
2	37044
2	18522
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

Thus, the cube root of 74088 = 2 × 3 × 7 = 42

(c) A = {factors of 24} = {1, 2, 3, 4, 6, 8, 12, 24}
B = {factors of 30} = {1, 2, 3, 5, 6, 10, 15, 30}

i. A ∪ B = {1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 24, 30}

ii. A ∩ B = {1, 2, 3, 6}

iii. A – B = {4, 8, 12, 24}

Now, n(A) = 8, n(B) = 8, n(A ∪ B) = 12, n(A ∩ B) = 4, n(A – B) = 4
n(A – B) = 4
n(A) – n(A ∩ B) = 8 – 4 = 4
n(A ∪ B) – n(B) = 12 – 8 = 4
Thus, n(A – B) = n(A) – n(A ∩ B) = n(A ∪ B) – n(B)

Question 2

(a) $(81)^{-1} \times 3^{-5} \times 3^{9} \times (64)^{5/6} \times (\sqrt[3]{3})^{6}$

$= (3^4)^{-1} \times 3^{-5} \times 3^9 \times (2^6)^{5/6} \times (3^{1/3})^6$

$= 3^{-4} \times 3^{-5} \times 3^9 \times 2^5 \times 3^{6/3}$

$= 3^{-4-5+9+2} \times 2^5$

$= 3^2 \times 2^5$

$= 9 \times 32$

$= 288$

(b) Area of a rectangle = l × b

= $(5x^2 + 25xy + 4y^2)(2x^2 - 2xy + 3y^2)$

= $10x^4 - 10x^3y + 15x^2y^2 + 50x^3y - 50x^2y^2 + 75xy^3 + 8x^2y^2 - 8xy^3 + 12y^4$

= $10x^4 - 40x^3y - 27x^2y^2 + 67xy^3 + 12y^4$

(c) Let the digit at tens and units place be x and y respectively.

Then, the number formed = 10x + y

Sum of the digits = x + y

According to given condition,

$10x + y = 3(x + y)$

$\Rightarrow 7x - 2y = 0$ ----- (i)

On reversing the digits, we have y at the tens place and x at the units place.

Thus, number formed = 10y + x

By second condition,

$10x + y + 45 = 10y + x$

$\Rightarrow 9x - 9y = -45$

$\Rightarrow x - y = -5$ ----- (ii)

Multiplying (ii) by 2, we get

$2x - 2y = -10$ ---- (iii)

Subtracting (iii) from (i), we get x = 2

Substituting x = 2 in (i) we get y = 7

Tens digit = 2 and units digit = 7

Original number = 27

Question 3

(a) Pair up the digits from right to left before and after the decimals.

	2 7 . 6 0 2
2	$\overline{7}\ \overline{61}.\overline{90}\ \overline{00}\ \overline{00}$
	4
47	361
	329
546	3290
	3276
5520	1400
55202	140000
	110404

(b) Length of the wire = Perimeter of a square = 4(27.5) = 110 cm

Let r be the radius of the circle.

Circumference of the circle = 2πr

As the same wire is bent to form a circle,

$2\pi r = 110 \Rightarrow 2\times\frac{22}{7}\times r = 110 \Rightarrow r = \frac{110\times 7}{22\times 2} \Rightarrow r = 17.5$ cm

Thus, area of the circle = $\pi r^2 = \frac{22}{7}\times 17.5\times 17.5 = 962.5\ cm^2$

(c) Rate of interest = 12.5% = $\frac{25}{2}$% p.a.

Principal for the first year = Rs. 16000

Interest for the first year = Rs. $\frac{16000\times\frac{25}{2}\times 1}{100}$ = Rs. 2000

Amount at the end of 1^{st} year = Rs. 16000 + Rs. 2000 = Rs. 18000

Principal for the second year = Rs. 18000

Interest for the second year = Rs. $\frac{18000\times\frac{25}{2}\times 1}{100}$ = Rs. 2250

Amount at the end of 2^{nd} year = R s. 18000 + Rs. 2250 = Rs. 20250

Principal for third year = Rs. 20250

Interest for the third year = Rs. $\frac{20250\times\frac{25}{2}\times 1}{100}$ = Rs. 2531.5

Amount at the end of 3^{rd} year = Rs. 20250 + Rs. 2531.5 = Rs. 22781.25

∴ Sumit has to pay Rs. 22781.25 i.e. Rs. 22781(rounding to nearest rupee) to clear his debt.

Compound interest paid by Sumit = Final amount – Principal (primary)

= Rs. 22781 – Rs. 16000

= Rs. 6781

Question 4

(a) In 1 minute, the hot water tap fills $\left(\frac{1}{12}\right)^{th}$ part of the bath tub.

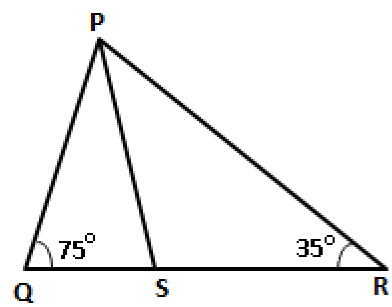
In 1 minute, the cold water tap fills $\left(\frac{1}{15}\right)^{th}$ part of the bath tub.

And, in 1 minute, the outlet pipe empties $\left(\frac{1}{10}\right)^{th}$ part of the bath tub.

If all are open at the same time, in 1 minute, $\left(\frac{1}{12}+\frac{1}{15}-\frac{1}{10}\right)^{th}$ part of the bath tub is filled

i.e. $\frac{5+4-6}{60} = \frac{3}{60} = \frac{1}{20}$

Thus, the bath tub will be full in 20 minutes.



(b) For ΔPQR,

$m\angle P + 75° + 35° = 180°$ (sum of angles in a triangle = 180°)

$\therefore m\angle P = 180° - 75° - 35° = 70°$

Since PS bisects $\angle P$,

$m\angle QPS = m\angle SPR = \frac{1}{2}(70°) = 35°$

$m\angle PSQ = m\angle SPR + m\angle R = 35° + 35° = 70°$ (Ext. angle = sum of interior angles)

$\therefore$ In ΔPQS, $\angle QPS < \angle PSQ < \angle PQS$

$\Rightarrow QS < PQ < PS$ -------------------- (i) (Triangle Inequality Theorem)

Also in ΔPSR, $m\angle SPR = 35° = m\angle R$

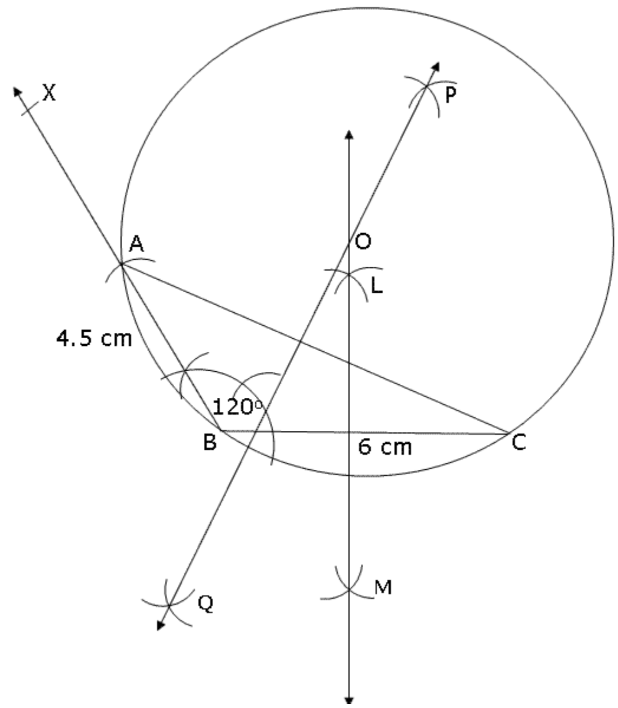
$\Rightarrow PS = SR$ -------------------- (ii) (sides opp. equal angles are equal)

From (i) and (ii), we get

$QS < PQ < SR$

(c) Steps of Construction:

1) Construct ΔABC with given measures.
2) Draw the perpendicular bisectors LM & PQ of sides BC and AC respectively, intersecting each other at O.
3) With O as centre & radius OA = OB = OC, draw a circle which will circumscribe ΔABC.



Section B (40 Marks)

Question 5

(a) For $x^4 + 5x^2 - 6$

Putting $x^2 = y$, the polynomial can be written as $y^2 + 5y - 6$.

Factorising it further,

Here a = 1, c = -6 , ac = -6

b = 5 = 6 - 1

$y^2 + 5y - 6$

$= y^2 + 6y - y - 6$

$= y(y + 6) - 1(y + 6)$

$= (y - 1)(y + 6)$

$= (x^2 - 1)(x^2 + 6)$ [putting back $x^2 = y$]

$= (x - 1)(x + 1)(x^2 + 6)$

(a) For the first equation,

$2(a - 3) + 3(b - 5) = 0$

$\Rightarrow 2a - 6 + 3b - 15 = 0$

$\Rightarrow 2a + 3b = 21$ ....(i)

For second equation,

$5(a - 1) + 4(b - 4) = 0$

$\Rightarrow 5a - 5 + 4b - 16 = 0$

$\Rightarrow 5a + 4b = 21$ ....(ii)

Multiplying (i) by 4 and (ii) by 3, we get

$4(2a + 3b) = 4(21) \Rightarrow 8a + 12b = 84$ ....(iii)

$3(5a + 4b) = 3(21) \Rightarrow 15a + 12b = 63$ ....(iv)

Subtracting (iv) from (iii), we get

$8a + 12b = 84$

$15a + 12b = 63$

$- \quad - \quad -$

$-7a = 21 \Rightarrow a = -3$

Substituting a = –3 in (i), we get b = 5

(b) From the given figure,

i) In ΔABC, $m\angle B = 90°$, so AC is the hypotenuse.

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$= (20)^2 + (4\sqrt{11})^2$

$= 400 + 176$

$= 576$

ii) Now, in ΔEDC, $m\angle D = 90°$, so EC is the hypotenuse.

By Pythagoras theorem,

$EC^2 = DE^2 + CD^2$

$= (6)^2 + (8)^2$

$= 36 + 64$

$= 100$

Next, AE = 26 cm ...(given)

$\Rightarrow AE^2 = (26)^2 = 676$ ....(i)

$AC^2 + EC^2 = 576 + 100 = 676$ ....(ii)

From (i) & (ii), it implies $AE^2 = AC^2 + CE^2$ ....(iii)

For ΔACE, by converse of Pythagoras theorem and as (iii) is proved we can say m∠ACE = 90°

Question 6

(a) $\dfrac{\sqrt{15}-2}{\sqrt{15}+2}+\dfrac{\sqrt{15}+2}{\sqrt{15}-2}$

$$=\frac{\left(\sqrt{15}-2\right)^2+\left(\sqrt{15}+2\right)^2}{\left(\sqrt{15}+2\right)\left(\sqrt{15}-2\right)}$$

$$=\frac{\left(\sqrt{15}\right)^2+(2)^2-2\left(\sqrt{15}\right)(2)+\left(\sqrt{15}\right)^2+(2)^2+2\left(\sqrt{15}\right)(2)}{\left(\sqrt{15}\right)^2-(2)^2}$$

$$=\frac{15+4-4\sqrt{15}+15+4+4\sqrt{15}}{15-4}$$

$$= \frac{38}{11} = 3\frac{5}{11}$$

(b) Let a = 28 cm, b = 21 cm and c = 35cm. Then,

$$s = \frac{a+b+c}{2}=\frac{28+21+35}{2}=\frac{84}{2}=42\text{ cm.}$$

∴ Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$$=\sqrt{42(42-28)(42-21)(42-35)}$$

$$=\sqrt{42\times14\times21\times7}\,cm^2$$

$$=\sqrt{21\times2\times7\times2\times21\times7}\text{ cm}^2$$

$= 21 \times 2 \times 7\text{ cm}^2$

$= 294\text{ cm}^2$

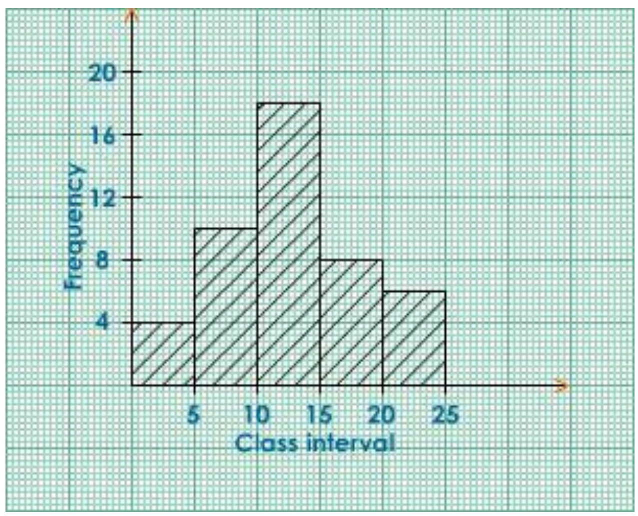
(c) Steps of construction:

i. On a graph paper, draw a horizontal line OX and vertical line OY, representing the x-axis and the y-axis respectively.

ii. Along OX, write the class intervals at points taken at uniform gaps.

iii. Then, the heights of the various bars are:
0-5 : 4; 5-10 : 10; 10-15 : 18; 15-20 : 8; 20-25 : 6

iv. On the x-axis, draw bars of equal width and of heights obtained in step (iii) at the points marked in step (ii).
The histogram is as follows:



Question 7

(a) $2a - \frac{1}{2a} = 3$

Cubing both sides,

$$\left(2a - \frac{1}{2a}\right)^3 = 27$$

$$\Rightarrow (2a)^3 - \left(\frac{1}{2a}\right)^3 - 3(2a)\left(\frac{1}{2a}\right)\left(2a - \frac{1}{2a}\right) = 27$$

$$\Rightarrow 8a^3 - \frac{1}{8a^3} - 3(3) = 27$$

$$\Rightarrow 8a^3 - \frac{1}{8a^3} = 27 + 3 = 30$$

(b)

Class	**Frequency**	$\mathbf{x_i}$	$\mathbf{f_i x_i}$
10-16	12	13	156
16-22	8	19	152
22-28	5	25	125
28-34	9	31	279
34-40	6	37	222
Total	40		934

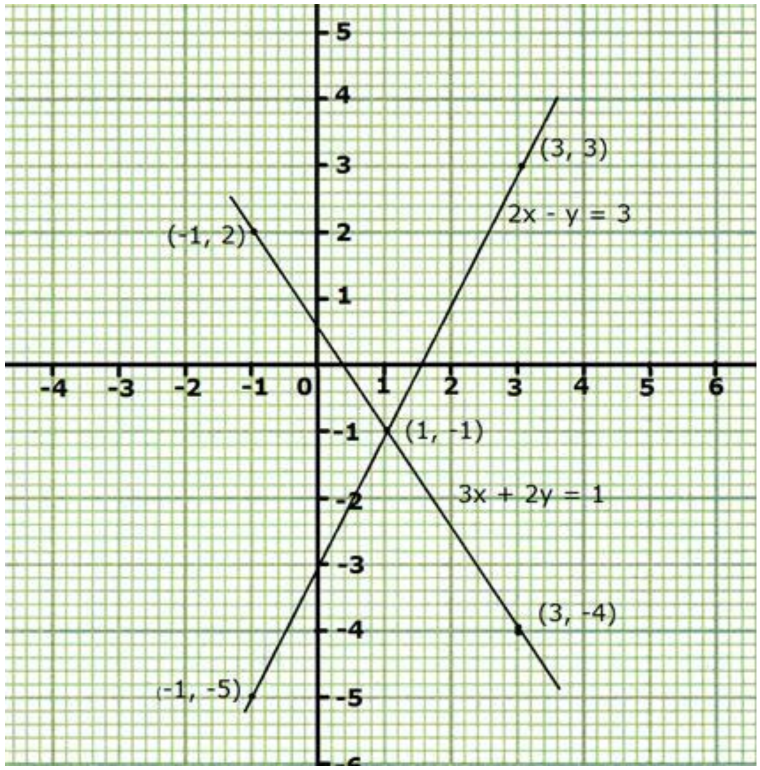
Mean = $\frac{\sum f_i.x_i}{\sum f_i} = \frac{934}{40} = 23.35$

(c) $2x - y = 3$

x	-1	1	3
y	-5	-1	3

$3x + 2y = 1$

x	-1	1	3
y	2	-1	-4



Point of intersection of the two lines is (1,-1).

Question 8

(b) $\frac{2x}{x^2-4}+\frac{1}{x^2+3x+2}$

$= \frac{2x}{(x-2)(x+2)}+\frac{1}{(x+2)(x+1)}$

$= \frac{2x(x+1)+1(x-2)}{(x-2)(x+2)(x+1)}$

$= \frac{2x^2+2x+x-2}{(x-2)(x+2)(x+1)} = \frac{2x^2+3x-2}{(x-2)(x+2)(x+1)}$

$= \frac{(2x-1)(x+2)}{(x-2)(x+2)(x+1)}$

$= \frac{(2x-1)}{(x-2)(x+1)}$ OR $\frac{2x-1}{x^2-x-2}$

(c) Let each side of the original cube measure a**.**

Then measure of the sides of the new cube = 2a

i. Volume of the original cube = a x a x a = a^3

Volume of the new cube = 2a x 2a x 2a = $8a^3$

Volume increases eight times if the side is doubled.

ii. Surface area of original cube = $6a^2$

Surface are of new cube = $6(2a)^2 = 24a^2 = 4(6a^2)$

Hence, surface area increases 4 times and volume of a cube increases 8 times.

(d) Let the cost price of the shoes be Rs. *x*.

Since the dealer marks his shoes 40% above the cost price,

M.P. = C.P. + 40% of C.P.

$= \text{Rs. } x + \dfrac{40}{100} \text{ of Rs. } x$

$= \text{Rs. } x + \text{Rs. } \dfrac{40}{100}x$

$= \text{Rs. } \left(x + \dfrac{2}{5}x\right) \quad = \quad \text{Rs. } \dfrac{7}{5}x$

Now, as per formula, $\quad \text{S.P.} = \left(1 - \dfrac{d}{100}\right) \text{ of M.P.}$

i.e. $\text{S.P.} = \left(1 - \dfrac{15}{100}\right) \text{ of Rs. } \dfrac{7}{5}x$

$= \quad \text{Rs. } \left(\dfrac{85}{100} \times \dfrac{7}{5}x\right)$

$= \quad \text{Rs. } \dfrac{119}{100}x$

Now, Profit = S.P. – C.P.

$= \text{Rs. } \dfrac{119}{100}x \; - \text{Rs. } x$

$= \text{Rs.} \left(\dfrac{119}{100}x - x\right)$

$= \text{Rs. } \dfrac{19}{100}x$

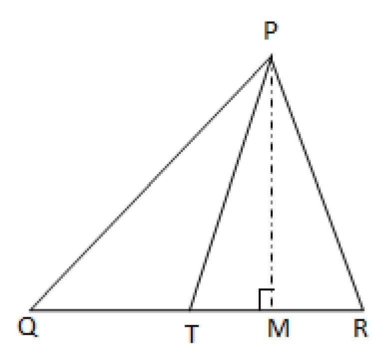
Next, Profit Percentage $= \left(\dfrac{\text{profit}}{\text{C.P.}} \times 100\right)\% \; = \; \left(\dfrac{\dfrac{19}{100}x}{x} \times 100\right)\% \; = 19\ \%$

Question 9

(a) Let ΔPQR be any triangle with PT as the median i.e. T is the midpoint of QR.

⇒ QT = TR.

To get the area of a triangle we need to construct an altitude from P to QR. Let PM $\perp$ QR.



Area of ΔPRT $= \dfrac{1}{2} \times \text{PM} \times \text{TR}$

$= \dfrac{1}{2} \times \text{PM} \times \text{QT}$

Area of ΔPRT = Area of ΔPQT

Hence a median of a triangle divides it into two triangles of equal area.

(b) Surface area of one tin box = 2(lb + bh + hl)

$= 2(30 \times 40 + 40 \times 50 + 50 \times 30)$

$= 2(1200 + 2000 + 1500)$

$= 2 \times 4700$

$= 9400 cm^2$

Therefore, surface area of 20 such tins = 20 x 9400 = 188000 cm^2

$= \frac{188000}{100 \times 100} m^2 = 18.8 m^2$

Hence, cost of 18.8 m^2 tin sheet = Rs. (18.8 x 25) = Rs. 470.

(c) For rhombus ABCD, AB = 13 cm and AC = 24 cm.

The diagonals of a rhombus bisect each other at right angles at O.

AO = OB = ½(24) = 12 cm.

In ΔAOB, by Pythagoras theorem.,

$AB^2 = OA^2 + OB^2 \Rightarrow 13^2 = 12^2 + OB^2$

$\Rightarrow OB^2 = 25 \Rightarrow OB = 5$ cm

O is the midpoint of BD ⇒ OB = OD = 5cm ⇒BD = 10 cm

Area of rhombus = ½ × product of diagonals = ½ × 24 × 10 = 120 cm^2