Complex Numbers and Quadratic Equations

Case Study Based Questions

Read the following passages and answer the questions that follow:

1. Two complex numbers Z_1 = a + ib and Z_2 = c + id are said to be equal if a = c and b = d.
(A) If (2a + 2b) + i(ba) = -4i, then find the real values of a and b.
(B) If (x + y) + i(x − y) = 4 + 6i, then find xy.
(C) Express the given expression (1 + i)(1 + 2i) in the form a + ib and find the values of a and b.

Ans. (A) We have (2a + 2b) + i(ba) = -4i Here
2a + 2b=0
a+b=0 ..... (i)
and b-a=-4 ...(ii)
On adding eq. (i) and (ii), we get a= 2 and b = -2
(B) Given that (x + y) + i(x − y) = 4 + 6i
Hence,
x+y=4
x-y=6
by solving equations
x= 5,y=-1
then xy=-5
(C) Given expression: (1 + i)(1 + 2))
Hence,
(1+)(1 + 2) = 1(1) +1(2)) + i + 2i()
(1+)(1 +2)=1+2i+i+2i^2
(1+)(1 + 2) = 1 + 2i+i+2(-1) [As, i^2 = -1)
(1+i)(1+2)=1+2i+i-2
(1+)(1 + 2) = -1 + 3/
Hence, the expression (1 + i)(1 + 2) in the
form of a + bi is -1 + 3i.
Thus, the value of a = -1 and b = 3.
2. A complex number z is pure real if and only if z = z and is pure imaginary if and only if

(A) If (1 + i)z = (1-i)z, then iz is:

(a) -z

(b) z

(c) z

(d) z-1

(B)

(B) $\overline{z_1 z_2}$ is:

(a) $\overline{z_1}\,\overline{z_2}$ (b) $\overline{z_1} + \overline{z_1}$

(c) $\dfrac{\overline{z_1}}{\overline{z_2}}$ (d) $\dfrac{1}{\overline{z_1 z_2}}$

(C) If x and y are real numbers and the complex number

$$\frac{(2+i)\,x-i}{4+i}+\frac{(1-i)\,y+2i}{4i}$$

is pure real, the relation between x and y is:

(a) 8x-17y = 16

(b) 8x + 17y = 16

(c) 17x-8y = 16

(d) 17x-8y = -16

(D)

If $z = \dfrac{3+2i\sin\theta}{1-2i\sin\theta}\left(0<\theta\le\dfrac{\pi}{2}\right)$ is pure imaginary, then θ is equal to:

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{6}$

(c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{12}$

(E) Assertion (A): The value of the expression: ¡30 +i40+ ¡60 is 1.

Reason (R): The values of it and i^2 are 1 and -1.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

Ans. (A) (b) z

Explanation: Since, (1 + i)z = (1-i) z

$$\Rightarrow \quad \frac{z}{\overline{z}} = \frac{1-i}{1+i} \times \frac{1-i}{1-i} = \frac{(1-i)^2}{1-i^2}$$

$$= \frac{1+i^2-2i}{1+1} = -i$$

$$\Rightarrow \quad z = -i\overline{z}$$

(B)

$$z = \frac{(2+i)x-i}{4+i} + \frac{(1-i)y+2i}{4i}$$

$$= \frac{2x+(x-1)i}{4+i} + \frac{y+(2-y)i}{4i} \times \frac{i}{i}$$

$$= \frac{(2x+(x-1)i)(4-i)}{(4+i)(4-i)} + \frac{-iy+(2-y)}{4}$$

$$= \frac{8x+x-1+i(4x-4-2x)}{16+1} + \frac{(2-y)-iy}{4}$$

$$= \frac{9x-1+i(2x-4)}{17} + \frac{2-y-iy}{4}$$

Since, z is real

$$\Rightarrow \quad \overline{z} = z$$

$$\Rightarrow \quad Im\, z = 0$$

$$\Rightarrow \quad \frac{2x-4}{17} - \frac{y}{4} = 0$$

$$\Rightarrow \quad 8x - 16 = 17y$$

$$\Rightarrow \quad 8x - 17y = 16$$

(D)

(c) $\frac{\pi}{3}$

Explanation: $z = \frac{3+2i\sin\theta}{1-2i\sin\theta} \times \frac{(1+2i\sin\theta)}{(1+2i\sin\theta)}$

$$= \frac{(3+2i\sin\theta)(1+2i\sin\theta)}{1+4\sin^2\theta}$$

$$= \frac{(3-4\sin^2\theta)+i(8\sin\theta)}{1+4\sin^2\theta}$$

Since, z is pure imaginary.

$\Rightarrow \quad Re(z) = 0$

$\Rightarrow \quad \frac{3-4\sin^2\theta}{1+4\sin^2\theta} = 0$

$\Rightarrow \quad \sin^2\theta = \frac{3}{4}$

$\Rightarrow \quad \sin\theta = \pm\frac{\sqrt{3}}{2}$

$\Rightarrow \quad \theta = \frac{\pi}{3}\left(since, 0 < \theta \leq \frac{\pi}{2}\right)$

(E) (a) Both (A) and (R) are true and (R) is the correct explanation of (A).

Explanation: ¡30 +i40 +i60

The given expression can be simplified as follows:

$i^{30} + i^{40} + i^{60} = (i^4).i^2 + (i^4)^{10} + (i^4)^{15}$

We know that the value of i^4 is 1.

$i^{30} + i^{40} + i^{60} = (1)^7.i^2 + (1)^{10} + (1)^{15}$

$i^{30} + i^{40} + i^{60} = (1)i^2 + 1 + 1$

$i^{30} + i^{40} + i^{60} = -1 + 1 + 1 \qquad$ [Since $i^2 = 1$]

$i^{30} + i^{40} + i^{60} = 1$

Therefore, the simplification of $i^{30} + i^{40} + i^{60}$ is 1.