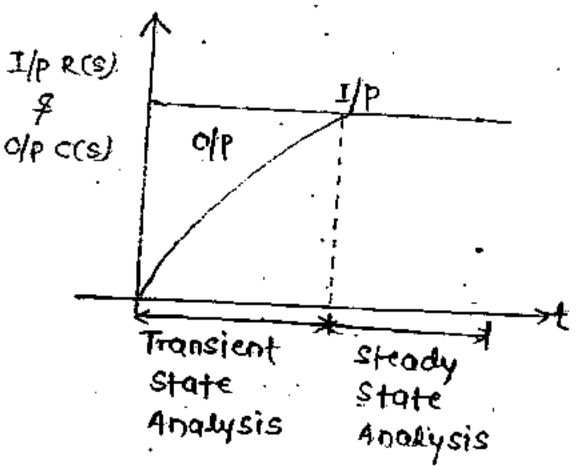
Chapter-02
Time domain analysis

* The time responce analysis is the analysis on time taken by the Responce of the sys. when subjected to an i/p.

* It is devided into 2 parts:-

(i) Transcient state analysis → It deals with the nature of responce of sys. when subjected to an i/p.

(ii) Steady state responce analysis → It deals with the estimation of magnitude of steady state error b/n i/p & o/p.



*** Standard test signals →**

(1) Sudden i/p → Step signal
(2) Velocity type i/p → Ramp signal
(3) Acceleration type i/p → parabolic signal
(4) Sudden shocks → Impulse signal

(1)–(3) → Time domain analysis
(4) → stability

Note:- The transient state analysis & the transient state specification are defined for step signal only because the magnitude of the i/p sig. should not change with time.

* Type & Order →

(1.) Every TF representing the cs has certain type & order.

(2.) Steady state responce analysis depands on type of the cs.

(3.) The type of the sys. obtained from open loop TF $G(s)H(s)$, by observing the no. of open loop poles occuring at origin.

Let $G(s) \cdot H(s) = \frac{k(1+T_a s)}{s^p(1+T_1 s)}$

$p=0$, type-0 system

$p=1$, type-1 system

⋮

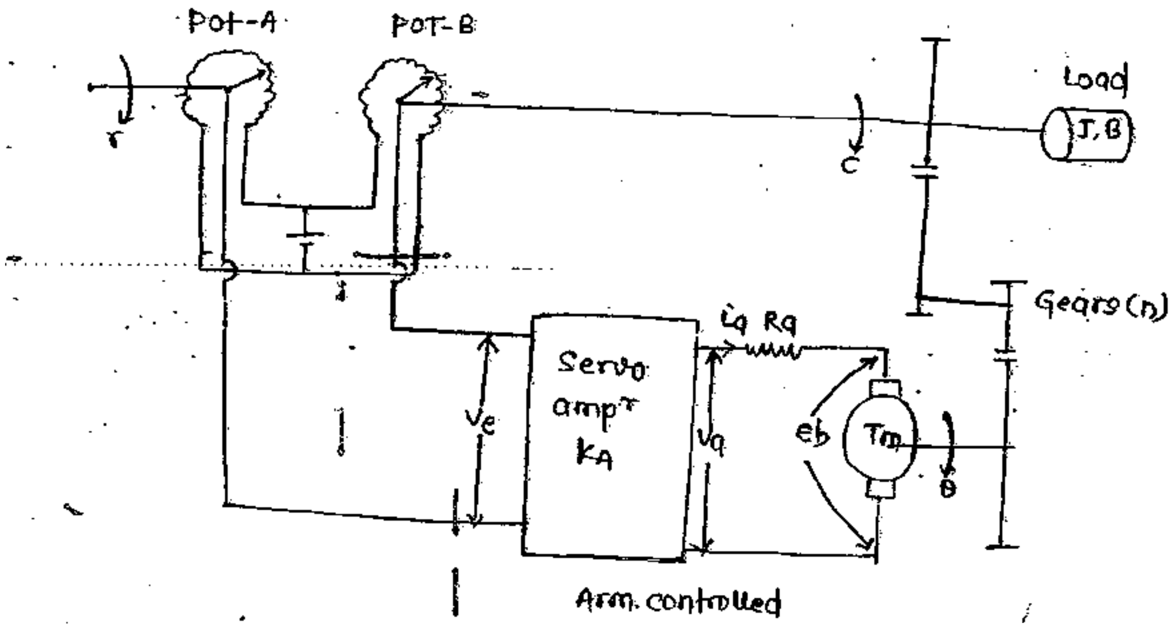
$p=n$, type-n system.

(4.) The transient state analysis depands upon the order of cs.

(5.) The order of sys is obtained from closed loop TF $\frac{G(s)}{1+G(s)H(s)}$ by observing the highest power of a c/s eqn.

$$\boxed{1+G(s)H(s)=0}$$

Eg:- Position Control system



Arm. controlled
dc servomotor.

(1) $I/p = r\,[R(s)]$, $o/p = c\,[C(s)]$

(2) Principle of operation

(i) At potentiometer

$V_e \propto (r-c)$

$V_e = K_p(r-c)$

$V_e(s) = K_p[R(s)-C(s)]$ ----(i)

(ii) At Ampr

$V_a \propto V_e$

$V_a = K_A V_e$

$V_a(s) = K_A V_e(s)$ ------(ii)

(iii) Analysis of arm. controlled dc servomotor

$T_m = K_T I_a(s)$ ------(iii)

$e_b = K_b \frac{d\theta}{dt}$

$E_b(s) = K_b(s)\theta(s)$ -----(iv)

$V_a = i_a R_a + e_b$

$V_a(s) - E_b(s) = I_a(s) R_a$ -----(v)

$T_m = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt}$

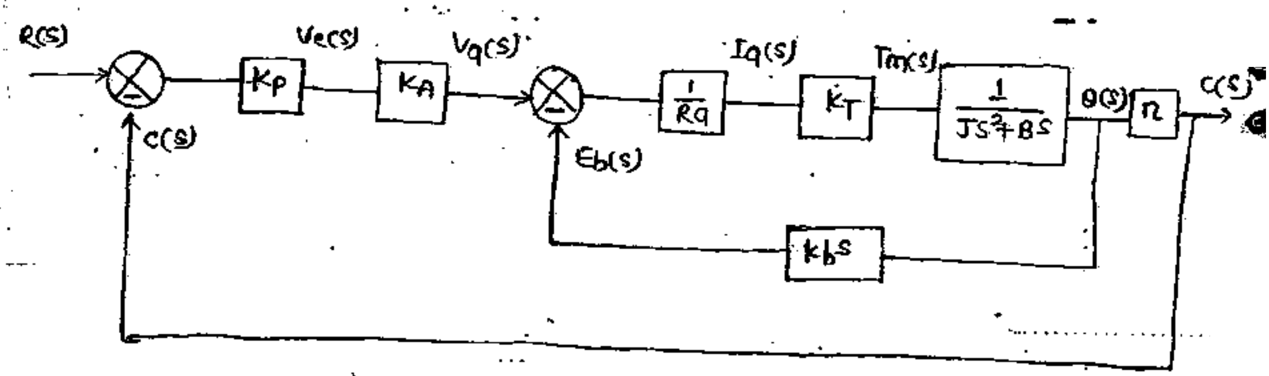
$T_m(s) = (Js^2 + Bs)\theta(s)$ -----(vi)

(iv) At gears:-

$c \propto \theta$

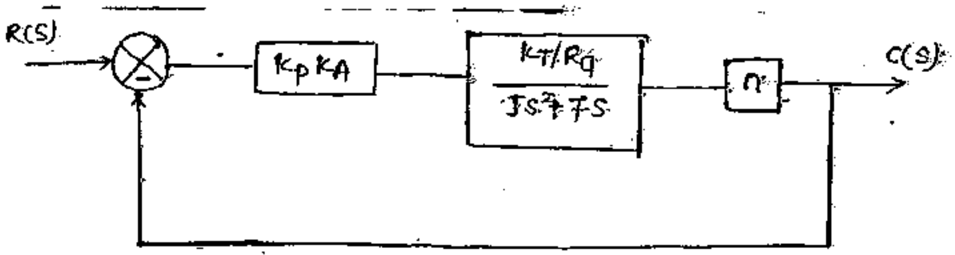
$c = n\theta$

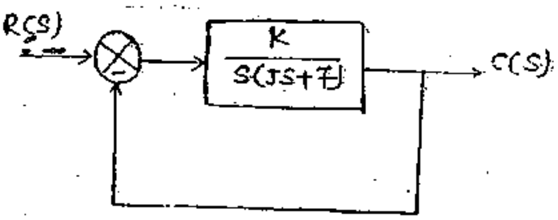
$C(s) = n\theta(s)$ -----(vii)



Inner f/b loop→

$$\frac{\frac{K_T}{R_a(Js^2+Bs)}}{1+\frac{K_T K_b s}{R_a(Js^2+Bs)}} = \frac{K_T}{R_a(Js^2+Bs)+K_T K_b s} = \frac{K_T/R_a}{\underbrace{Js^2+Bs+\frac{K_T K_b s}{R_a}}_{Fs}} = \frac{K_T/R_a}{Js^2+Fs}$$





where; $K = \frac{K_p K_A K_T n}{R_a}$

<u>For type of sys →</u>

$$G(s) = \frac{K}{s(Js+f)}, \quad H(s) = 1$$

$$G(s)\cdot H(s) = \frac{K}{s(Js+f)}$$

$\boxed{\text{Type} = 1}$

<u>Order of the sys →</u>

$$1 + G(s)H(s) = 0$$

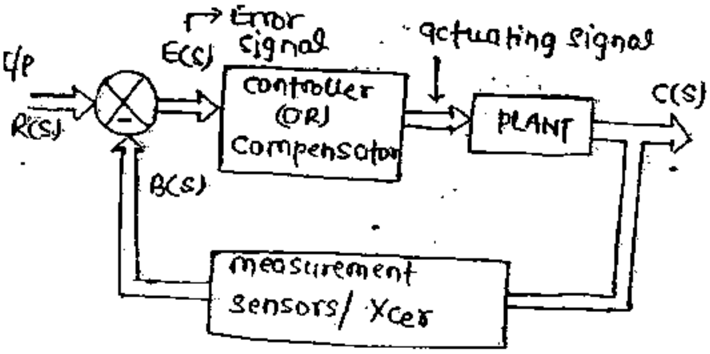
$$1 + \frac{K}{s(Js+f)} = 0$$

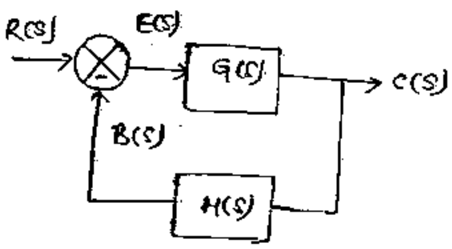
$$Js^2 + fs + K = 0$$

$\boxed{\text{Order} = 2}$

* <u>steady state Responce analysis →</u> It deals with estimation of magnitude of steady state error b/n i/p & o/p & depands on type of cs.

<u>Error Compensation →</u>





To obtain an expression for error

$$E(s) = R(s) - B(s)$$

$R(s) \downarrow$ i/p, $B(s) \downarrow$ o/p

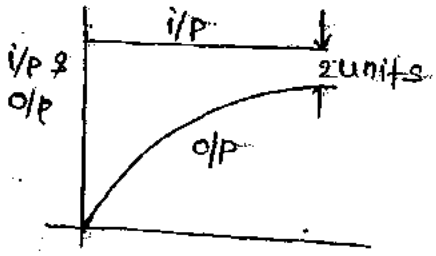
$$E(s) = R(s) - C(s).H(s)$$

$$E(s) = R(s) - E(s).G(s).H(s)$$

$$E(s)\left[1 + G(s).H(s)\right] = R(s)$$

$$\boxed{E(s) = \frac{R(s)}{1+G(s).H(s)}}$$

error Ratio



$$\lim_{t\to\infty} e(t) = \underset{(e_{ss})}{2\,\text{units}}$$

$$\boxed{e_{ss} = \lim_{t\to\infty} e(t)}$$

By FVT

$$\boxed{e_{ss} = \lim_{s\to 0} s \cdot E(s)}$$

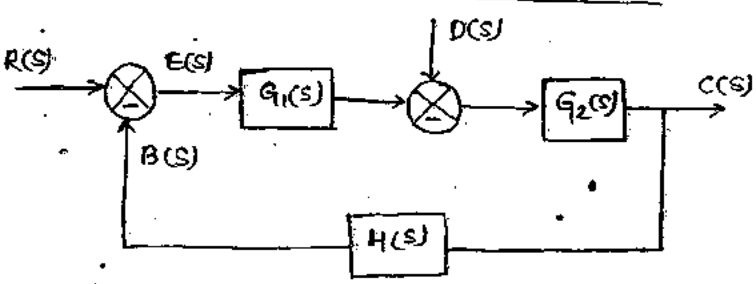
$$e_{ss} = \lim_{s\to 0} \frac{s \cdot R(s)}{1+G(s) \cdot H(s)}$$

$$\boxed{e_{ss} = \frac{\lim_{s\to 0} s \cdot R(s)}{1 + \lim_{s\to 0} G(s)H(s)}}$$

Note:- * Since the steady state error is defined as the diff b/n i/p & o/p, to find the type of the sys. & hence steady state error the F/b gain should be unity [H(s)=1]

* For non-unity F/b elements they should be specified as measuring elemet i.e sensors (OR) Xcer element.

* To obtain an expⁿ for error with disturbance →



$$E(s) = R(s) - B(s)$$

$$E(s) = R(s) - C(s)\cdot H(s)$$

$$\because\ C(s) = [E(s)\cdot G_1(s) + D(s)]\, G_2(s)$$

$$C(s) = E(s)\cdot G_1(s)\cdot G_2(s) + D(s)\cdot G_2(s)$$

$$E(s) = R(s) - E(s)\cdot G_1(s)\cdot G_2(s)\cdot H(s) - D(s)\cdot G_2(s)\cdot H(s)$$

$$E(s)[1 + G_1(s)\cdot G_2(s)\cdot H(s)] = R(s) - D(s)\cdot G_2(s)\cdot H(s)$$

$$E(s) = \frac{R(s)}{1 + G_1(s)G_2(s)H(s)} - \frac{D(s)\cdot G_2(s)H(s)}{1 + G_1(s)G_2(s)H(s)}$$

$$e_{ss} = \lim_{s\to 0} s\cdot E(s)$$

$$\boxed{e_{ss} = \lim_{s\to 0} \frac{s\cdot R(s)}{1 + G_1(s)\cdot G_2(s)H(s)} - \lim_{s\to 0}\frac{s\cdot D(s)\cdot G_2(s)H(s)}{1 + G_1(s)\cdot G_2(s)H(s)}}$$

(4/59)

$$e_{ss} = -\lim_{s\to 0} \frac{s\cdot D(s)\cdot G(s)}{1 + G_1(s)\cdot G_2(s)}$$

$$= -\lim_{s\to 0} \frac{\cancel{s}\times \frac{1}{\cancel{s}}\, G_2(s)}{1 + G_1(s)\cdot G_2(s)}$$

ans. (c)

$$e_{ss} = \frac{-G_2}{G_1G_2 + 1} \Rightarrow |e_{ss}| = \frac{G_2}{1 + G_1G_2}$$

$e_{ss}\downarrow$ by $G_1\uparrow$

* Steady state error for different types of i/p →

(1) Step i/p

$$R(s) = \frac{A}{s}$$

K_p = position error constant

$$= \lim_{s\to 0} G(s)\cdot H(s)$$

$$e_{ss} = \lim_{s\to 0} \frac{\cancel{s}\frac{A}{\cancel{s}}}{1 + G(s)\cdot H(s)}$$

$$= \frac{A}{1 + \lim_{s\to 0} G(s)H(s)} = \frac{A}{1 + K_p}$$

$$\boxed{e_{ss} = \frac{A}{1 + K_p}}$$

(2) Ramp signal

$$R(s) = \frac{A}{s^2}$$

$$e_{ss} = \lim_{s\to 0} \frac{\cancel{s}\cdot\frac{A}{s^{\cancel{2}}}}{1+G(s)\cdot H(s)}$$

$$= \lim_{s\to 0} \frac{A}{s + sG(s)\cdot H(s)}$$

$$= \frac{A}{\lim_{s\to 0} s + \lim_{s\to 0} s\cdot H(s)\cdot G(s)}$$

$$= \frac{A}{K_V}$$

K_V = Velocity error const.

$$= \lim_{s\to 0} s\cdot G(s)\cdot H(s)$$

(3) Parabolic signal

$$R(s) = \frac{A}{s^3}$$

$$e_{ss} = \lim_{s\to 0} \frac{s\cdot\frac{A}{s^3}}{1+G(s)\cdot H(s)}$$

$$\frac{A}{\lim_{s\to 0} s^2 + \lim_{s\to 0} s^2 G(s)\cdot H(s)} = \frac{A}{K_A}$$

K_A = acceleration error const.

$$= \lim_{s\to 0} s^2 G(s)\cdot H(s)$$

Steady state error for different types of system →

Type-0 →

$$G(s)\cdot H(s) = \frac{K(1+T_a s)}{(1+T_1 s)}$$

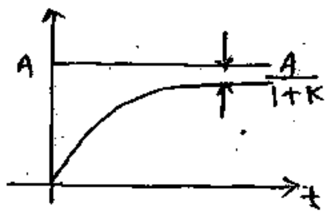
(1) step i/p

$$R(s) = \frac{A}{s}$$

$$e_{ss} = \lim_{s\to 0} \frac{\cancel{s}\cdot\frac{A}{\cancel{s}}}{1+\frac{K(1+T_a s)}{(1+T_1 s)}}$$

$$= \frac{A}{1+\lim_{s\to 0}\frac{K(1+T_a s)}{(1+T_1 s)}}$$

$$= \frac{A}{1+K}$$



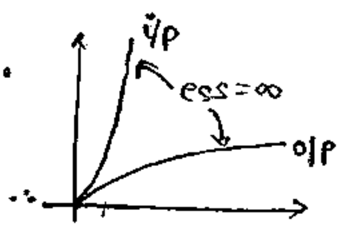
(2) Ramp i/p

$$R(s) = \frac{A}{s^2}$$

$$e_{ss} = \lim_{s\to 0} \frac{\cancel{s}\cdot\frac{A}{s^{\cancel{2}}}}{1+\frac{K(1+T_a s)}{(1+T_1 s)}}$$

$$e_{ss} = \lim_{s\to 0} \frac{A}{s+\frac{Ks(1+T_a s)}{(1+T_1 s)}}$$

$$= \infty$$



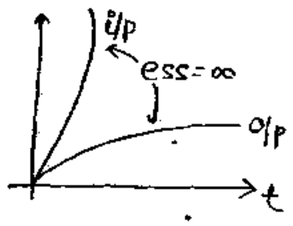
(3) Parabolic i/p

$$R(s) = \frac{A}{s^3}$$

$$e_{ss} = \lim_{s\to 0} \frac{\cancel{s}\cdot\frac{A}{s^{\cancel{3}\,2}}}{1+K\left(\frac{1+T_a s}{1+T_1 s}\right)}$$

$$= \lim_{s\to 0} \frac{A}{s^2 + Ks^2\left(\frac{1+T_a s}{1+T_1 s}\right)}$$

$$= \infty$$



System	Step Input	Ramp input	parabolic Input
TYPE-0	$\frac{A}{1+K}$ $K_p = K$	∞ $K_V = 0$	∞ $K_A = 0$
TYPE-1	0 $K_p = \infty$	$\frac{A}{K}$ $K_V = K$	∞ $K_A = 0$
TYPE-2	0 $K_p = \infty$	0 $K_V = \infty$	A/K $K_A = K$

1	∞	∞
0	1	∞
0	0	1

K	0	0	K_p K_v
∞	K	0	
∞	∞	K	

Observations →

(1) $e_{ss} \propto \frac{1}{K}$

As $K\uparrow$ $e_{ss}\downarrow$

(2) The max^{m} type no. for a linear CS is 2. Beyond type-2 the sys. tends to become unstable & also exibits non-linear c/s more dominatly.

(3/61) $G(s) = \frac{10}{s^2(4+s)}$; $r(t) = 2+3t+4t^2$

$R(s) = \frac{2}{s} + \frac{3}{s^2} + \frac{8}{s^3}$

$$e_{ss} = \lim_{s\to 0} \frac{\cancel{s}\left(\frac{2s^2+3s+8}{s^{\cancel{3}2}}\right)}{1+\frac{10}{s^2(4+s)}}$$

$$e_{ss} = \lim_{s\to 0} \frac{2s^2+3s+8}{s^2+s^2\cdot\frac{10}{s^2(4+s)}}$$

$$= \frac{8\times 4}{10}$$

$\boxed{e_{ss} = 3.2}$

Shortcut method

type-2

$R(s) = \frac{2}{s} + \frac{3}{s^2} + \frac{8}{s^3}$

$e_{ss} = 0 \quad 0 \quad \frac{A}{K}$

$A = 8$

$K = K_A = \lim_{s\to 0} \cancel{s^2}\frac{10}{\cancel{s^2}(4+s)} = \frac{10}{4}$

$e_{ss} = \frac{8}{\left(\frac{10}{4}\right)} = 3.2$

(4/61) $r(t) = (1-t^2)\,3u(t)$

$R(s) = \frac{3}{s} - \frac{6}{s^2}$

$$e_{ss} = \lim_{s\to 0} \frac{s\left[\frac{3}{s} - \frac{6}{s^3}\right]}{1+G(s)}$$

$$e_{ss} = \lim_{s\to 0} \frac{\cancel{s}\left(\frac{3}{\cancel{s}}\right)}{1+G(s)} - \lim_{s\to 0} \frac{s\cdot\frac{6}{s^3}}{1+G(s)}$$

$$e_{ss} = \frac{3}{1+\lim\limits_{s\to 0} G(s)} - \frac{6}{\lim\limits_{s\to 0} s^2G(s)}$$

$$\boxed{e_{ss} = \frac{3}{1+K_p} - \frac{6}{K_A}}$$

Shortcut method

$R(s) = \frac{3}{s} - \frac{6}{s^3}$

$= \frac{A}{1+K} - \frac{A}{K}$

$= \frac{3}{1+K_p} - \frac{6}{K_A}$

If I/p is not specified then finite error is taken.

(2/1) Type-0 $\xrightarrow{(1/s)}$ Type-1

$e_{ss} = \frac{1}{1+K}$

$0.2 = \frac{1}{1+K}$

$K = 4$

$e_{ss} = \frac{1}{K}$

$e_{ss} = \frac{1}{4} = 0.25$ units

$\boxed{e_{ss} = 0.25}$

Type-1 system

$e_{ss} = \frac{1}{K}$

$e_{ss} = 5\% \Rightarrow \frac{5}{100} = \frac{1}{20}$

$\frac{1}{20} = \frac{1}{K}$

$\boxed{K = 20 \quad \text{Type-1}}$

(1/61) $G(s) = \frac{10}{s^2+14s+50}$

Type-0

$e_{ss} = \frac{1}{1+K}$

$K = K_p = \lim\limits_{s\to 0} \frac{10}{s^2+14s+50} = \frac{10}{50}$

$e_{ss} = \frac{1}{1+\frac{10}{50}} = \frac{50}{60} = 0.83$ unit

$\boxed{e_{ss} = 0.83}$

3/59 $G(s) = \dfrac{k}{s(s+a)}$

Type-1 system

$e_{ss} = \dfrac{A}{K}$

$A=1,\ K = K_v = \lim_{s\to 0} s \cdot \dfrac{k}{s(s+a)}$

$= \dfrac{k}{a}$

$e_{ss} = \dfrac{a}{k}$

(i) $S_k^{e_{ss}} = \dfrac{k}{e_{ss}} \cdot \dfrac{\partial e_{ss}}{\partial k}$

$e_{ss} = \dfrac{a}{k}$

$\dfrac{e_{ss}}{k} = \dfrac{a}{k^2} \Rightarrow \dfrac{k}{e_{ss}} = \dfrac{k^2}{a}$

$\dfrac{\partial e_{ss}}{\partial k} = \dfrac{\partial}{\partial k}\left(\dfrac{a}{k}\right) = \dfrac{-a}{k^2}$

$S_k^{e_{ss}} = \dfrac{k^2}{a} \times \dfrac{-a}{k^2} = -1$

$\boxed{S_k^{e_{ss}} = -1}$

(ii) $S_a^{e_{ss}} = \dfrac{a}{e_{ss}} \cdot \dfrac{\partial e_{ss}}{\partial a}$

$e_{ss} = \dfrac{a}{k} \Rightarrow \dfrac{a}{e_{ss}} = k$

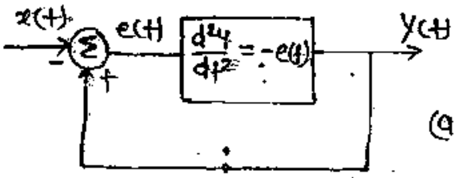
$\dfrac{\partial e_{ss}}{\partial a} = \dfrac{\partial}{\partial a}\left(\dfrac{a}{k}\right) = \dfrac{1}{k}$

$S_a^{e_{ss}} = k \times \dfrac{1}{k} = 1$

$\boxed{S_a^{e_{ss}} = 1}$

Note:- Sensitivity of e_{ss} wrt k & a is same for the above system.

Que.→



For $x(t) = tu(t)$ find $e(t) = ?$

(a) $\sin t$ (b) $\cos t$ (c) $-\sin t$ (d) $-\cos t$

Soln→

$e(t) = -x(t) + y(t)$

$E(s) = -X(s) + Y(s)$

$\dfrac{d^2y}{dt^2} = -e(t)\ ;\ s^2Y(s) = -E(s)$

$Y(s) = \dfrac{-E(s)}{s^2}$

$E(s) = -X(s) - \dfrac{E(s)}{s^2}$

$E(s) + \dfrac{E(s)}{s^2} = -X(s)$

$\left(\dfrac{s^2+1}{s^2}\right) E(s) = -X(s)$

$E(s) = \dfrac{-X(s) \cdot s^2}{s^2+1}$

Given $x(t) = tu(t)$

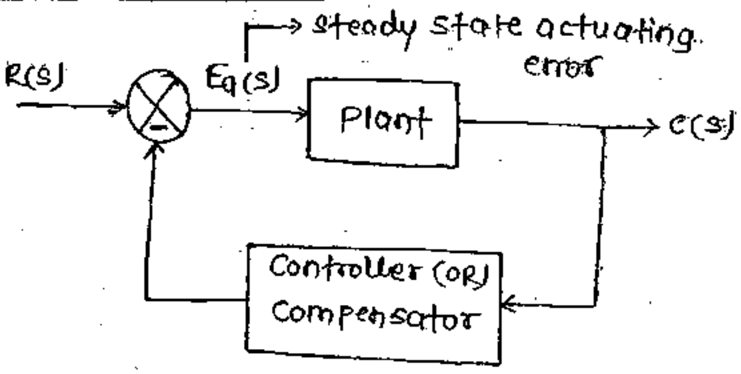
$X(s) = \dfrac{1}{s^2}$

$E(s) = \dfrac{\frac{-1}{s^2} \cdot s^2}{(s^2+1)}$

$= \dfrac{-1}{s^2+1}$

$\boxed{e(t) = -\sin t}$

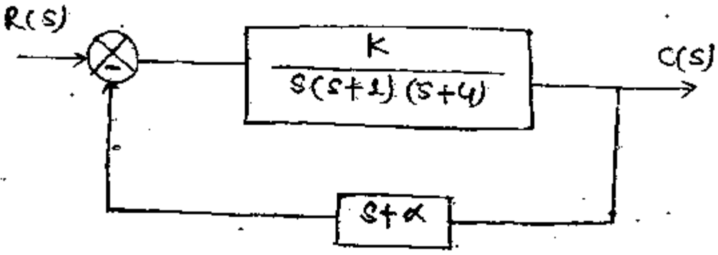
* Output Compensation →



* A controller/compensator placed in the f/b path compensates for changes in o/p & a steady state actuating error signal effects the dynamics of the plant to acheive the control objective.

∴ In such cases to find steady state error which is the diff. b/n i/p & o/p convert the CS into unity f/b system.

Que →



(i) which i/p will yeild constant error?

(a) step i/p (b) ramp i/p (c) parabolic i/p (d) Impulse i/p

(ii) Find steady state error for the above i/p?

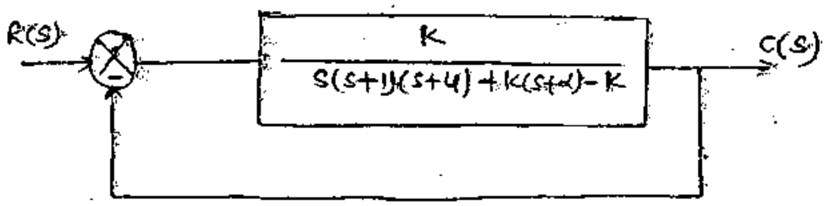
(a) $\frac{\alpha}{\alpha-1}$ (b) $\frac{\alpha-1}{\alpha}$ (c) α (d) $\alpha-1$

Sol^n →

$$\frac{C(s)}{R(s)} = \frac{\frac{K}{s(s+1)(s+4)}}{1+\frac{K(s+\alpha)}{s(s+1)(s+4)}}$$

$$= \frac{K}{s(s+1)(s+4)+K(s+\alpha)}$$

$$G(s) = \frac{K}{s^0\{[(s+1)(s+4)s+K(s+\alpha)]-K\}}$$



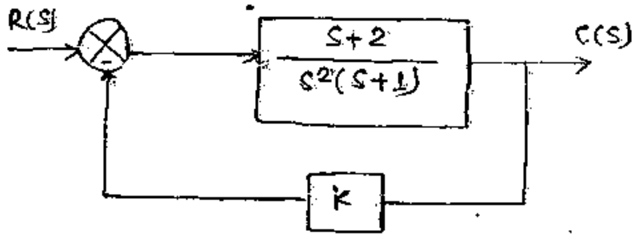
$$e_{ss} = \lim_{s\to 0} \frac{\cancel{s}\cdot\frac{1}{\cancel{s}}}{1+\dfrac{K}{s(s+1)(s+4)+K(s+\alpha)-K}}$$

$$= \frac{1}{1+\dfrac{K}{K\alpha-K}} = \frac{K\alpha-K}{K\alpha-K+K}$$

$$e_{ss} = \frac{K(\alpha-1)}{K\alpha}$$ ans (a) & (a)

$$\boxed{e_{ss} = \frac{\alpha-1}{\alpha}}$$

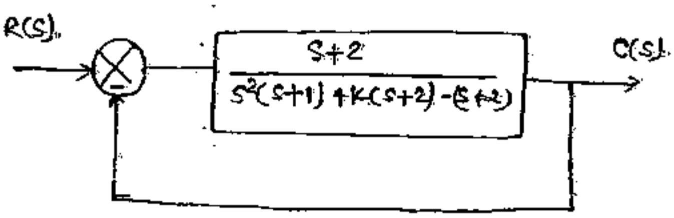
Que.→



Solⁿ→

$$\frac{C(s)}{R(s)} = \frac{\dfrac{s+2}{s^2(s+1)}}{1+\dfrac{(s+2)K}{s^2(s+1)}} = \frac{s+2}{s^2(s+1)+(s+2)K}$$

$$G(s) = \frac{(s+2)}{s^0[s^2(s+2)+K(s+2)-(s+2)]}$$



$$e_{ss} = \lim_{s\to 0} \frac{s\cdot\frac{1}{s}}{1+\dfrac{(s+2)}{s^2(s+1)+K(s+2)-(s+2)}}$$

$$e_{ss} = \frac{1}{1+\dfrac{2}{2K-2}}$$

$$\boxed{e_{ss} = \frac{K-1}{K}}$$

* Error Series →

$$E(s) = \frac{R(s)}{1+G(s)\cdot H(s)}$$

Let; $F(s) = \frac{1}{1+G(s)\cdot H(s)}$

$$\mathcal{L}^{-1} E(s) = R(s)\cdot F(s)$$

$$\mathcal{L}^{-1} E(s) = \mathcal{L}^{-1} R(s)\cdot F(s)$$

$$e(t) = \int_0^\infty f(T)\cdot r(t-T)\, dT$$

Expanding $r(t-T)$ using Taylor series

$$r(t-T) = r(t) - T\dot{r}(t) + \frac{T^2}{2!}\ddot{r}(t) - \frac{T^3}{3!}\dddot{r}(t) + \cdots\cdots$$

$$e(t) = r(t)\int_0^\infty f(T)\, dT - \dot{r}(t)\int_0^\infty T\, f(T)\, dT + \frac{\ddot{r}(t)}{2!}\int_0^\infty T^2 f(T)\, dT - \frac{\dddot{r}(t)}{3!}\int_0^\infty T^3 f(T)\, dT \cdots$$

Defining "Dynamic error const's"

$$k_0 = \int_0^\infty f(T)\, dT;\quad k_1 = -\int_0^\infty T f(T)\, dT,\quad k_2 = \int_0^\infty T^2 f(T)\, dT,\quad k_3 = -\int_0^\infty T^3 f(T)\, dT.$$

$$\boxed{e(t) = k_0 r(t) + k_1\dot{r}(t) + \frac{k_2}{2!}\ddot{r}(t) + \frac{k_3}{3!}\dddot{r}(t) + \cdots}$$

$$\boxed{e_{ss} = \lim_{t\to\infty} e(t)}$$

To find dynamic error constants →

$$\mathcal{L}\, f(T) = F(s) = \int_0^\infty f(T)\, e^{-sT}\, dT$$

$$\lim_{s\to 0} F(s) = \lim_{s\to 0}\int_0^\infty f(T)\cdot e^{-sT}\, dT = \int_0^\infty f(T)\, dT \Rightarrow k_0$$

$$\frac{d}{ds}F(s) = \frac{d}{ds}\int_0^\infty f(T)\, e^{-sT}\, dT = -\int_0^\infty T f(T)\, e^{-sT}\, dT$$

$$\lim_{s\to 0}\frac{d}{ds}F(s) = \lim_{s\to 0} -\int_0^\infty T f(T)\, e^{-sT}\, dT \Rightarrow -\int_0^\infty T\cdot f(T)\, dT \Rightarrow k_1$$

$$K_0 = \lim_{s \to 0} F(s)$$

$$K_1 = \lim_{s \to 0} \frac{d}{ds} F(s)$$

$$K_2 = \lim_{s \to 0} \frac{d^2 F(s)}{ds^2}$$

$$\vdots \qquad \vdots$$

where; $F(s) = \dfrac{1}{1 + G(s) \cdot H(s)}$

* Relationship b/n static & dynamic error constants →

$$G(s)H(s) = \frac{100}{s(s+2)}$$

(I) Static error constants.

$$K_p = \lim_{s \to 0} \frac{100}{s(s+2)} = \infty$$

$$K_v = \lim_{s \to 0} \frac{\cancel{s} \cdot 100}{\cancel{s}(s+2)} = 50$$

$$K_A = \lim_{s \to 0} \frac{s^2 \cdot 100}{s(s+2)} = 0$$

(II) Dynamic error constants.

$$F(s) = \frac{1}{1 + G(s) \cdot H(s)} = \frac{1}{1 + \frac{100}{s(s+2)}}$$

$$K_0 = \lim_{s \to 0} F(s)$$

$$K_0 = \lim_{s \to 0} \frac{1}{1 + \frac{100}{s(s+2)}}$$

$$K_0 = \frac{1}{1 + \lim_{s \to 0} \frac{100}{s(s+2)}} = \frac{1}{1+\infty} = 0$$

$$\boxed{K_0 = \frac{1}{1 + K_p}}$$

$$K_1 = \lim_{s \to 0} \frac{d}{ds} F(s)$$

$$\frac{d}{ds} F(s) = \frac{d}{ds} \frac{1}{1 + \frac{100}{s(s+2)}} = \frac{d}{ds} \frac{s(s+2)}{s^2 + 2s + 100}$$

$$= \frac{(s^2+2s+100)(2s+2) - s(s+2)(2s+2)}{(s^2+2s+100)^2}$$

$$K_1 = \lim_{s \to 0} \frac{d}{ds} F(s) = \frac{(0+0+100)(0+2) - 0}{(0+0+100)^2} = \frac{100 \times 2}{(100)^2} = \frac{1}{50}$$

$$\boxed{K_1 = \frac{1}{K_V}} \quad \boxed{K_2 = \frac{1}{K_A}}$$

<u>Notes:-</u> Static & dynamic error constant are inversely related to each other however they need not be direct reciprocal value because the dynamic error constant are defined for error series.

<u>Que→</u> $G(s)\cdot H(s) = \frac{100}{s(s+2)}$

Find e_{ss} for $r(t) = 5+2t$

<u>Soln→</u> <u>(i) Error ratio</u>

$$R(s) = \frac{5}{s} + \frac{2}{s^2} = \frac{5s+2}{s^2}$$

$$e_{ss} = \lim_{s\to 0} \frac{s\cdot\left(\frac{5s+2}{s^2}\right)}{1+\frac{100}{s(s+2)}}$$

$$e_{ss} = \frac{2\times 2}{100} = \frac{2}{50} \text{ units}$$

<u>(ii) Error series</u>

$$e_{ss} = \lim_{t\to 0}\left[k_0 r(t) + k_1 \dot{r}(t) + \frac{k_2}{2!}\ddot{r}(t) + \cdots\right]$$

$$r(t) = 5+2t \Rightarrow k_0 = 0$$

$$\dot{r}(t) = 0+2 = 2 \Rightarrow k_1 = \frac{1}{50}$$

$$\ddot{r}(t) = 0$$

$$e_{ss} = \lim_{t\to\infty}\left[0\times(5+2t) + \frac{1}{50}\times 2\right] = \frac{2}{50}$$

<u>(iii) Short cut methods</u>

Type-(1)

$$R(s) = \frac{5}{s} + \frac{2}{s^2}$$

$$e_{ss} = 0 + \frac{A}{k}$$

$A = 2,\ k = K_V = 50$

$$e_{ss} = \frac{2}{50}$$

*** Transient state analysis →** * It deals with the nature of responce of sys. when subjected to an i/p & depends on order of CS.

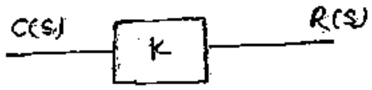
$$\frac{C(S)}{R(S)} = \frac{b_0}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}$$

(1.) zero order system →

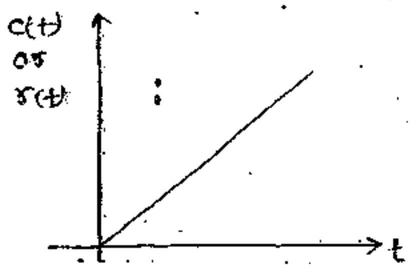
$$\frac{C(S)}{R(S)} = \frac{b_0}{a_0}$$

let $k = \text{gain} = \frac{b_0}{a_0}$

$$\frac{C(S)}{R(S)} = K$$



Ex:- Sensors / Xcers



(2.) 1st order sys →

$$\frac{C(S)}{R(S)} = \frac{b_0}{a_1 s + a_0}$$

$$= \frac{b_0/a_0}{\frac{a_1}{a_0} s + 1}$$

Let $k = \text{gain} = \frac{b_0}{a_0}$

T = time constant element $= \frac{a_1}{a_0}$

$$\frac{C(S)}{R(S)} = \frac{K}{1+TS}$$

RC n/w

$$\frac{V_o(S)}{V_i(S)} = \frac{1}{RCS+1} \quad \therefore T = RC$$

Transient analysis →

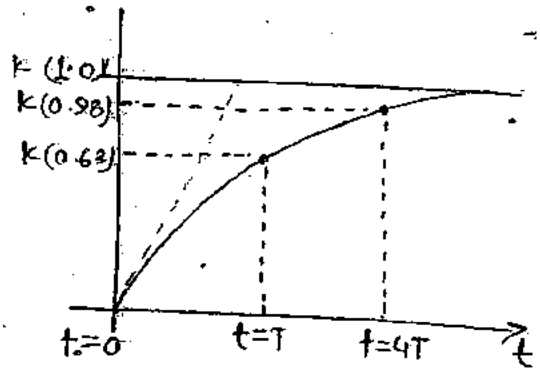
Let $R(s) = 1/s$

$$C(s) = \frac{k}{s(1+Ts)}$$

$$= \frac{k}{s(1+Ts)}$$

$$= k\left[\frac{1}{s} - \frac{T}{1+Ts}\right]$$

$$= k\left[\frac{1}{s} - \frac{1}{s+\frac{1}{T}}\right]$$



$$c(t) = k(1-e^{-t/T})$$

$$\lim_{t\to\infty} c(t) = k$$

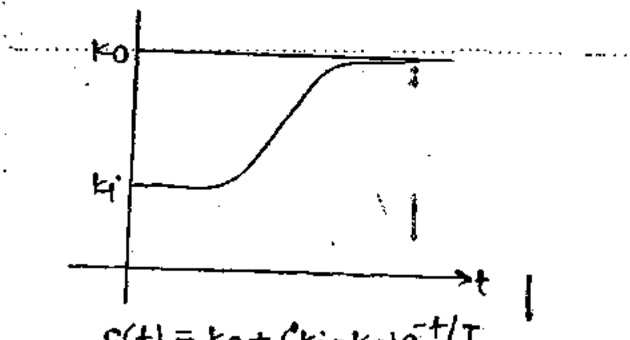
at $t=0$, $c(t) = k(1-e^{0}) = 0$

$t=T$, $c(t) = k(1-e^{-1}) = (0.63)k$

$t=4T$, $c(t) = k(1-e^{-4}) = (0.98)k$

The time const is defined as time taken by the responce of the sys to reach 63% of the final value.

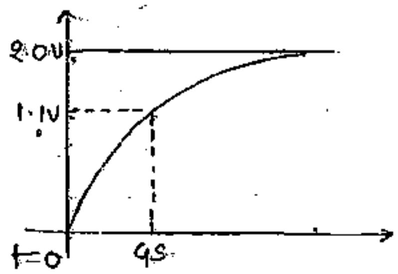
Liquid level sys., pneumatic sys., thermometers, RC (or) RL n/w are eg. of 1^{st} order system.



$$c(t) = k_o + (k_i - k_o)e^{-t/T}$$

Que→ Certain 1st order sys. is initialy at rest & subjected to sudden i/p at t=0, its responce reaches 1.1V is 4s & evantualy reaches a steady state of 2V. find the time const.

Sol^n→



$c(t) = k(1-e^{-t/T})$

$\lim_{t\to\infty} c(t) = k = 2$

at $t = 4s$

$1.1 = 2(1-e^{-4/T})$

$2e^{-4/T} = 0.9$

$\boxed{T = 5s}$

(17)/(63)

$H(s) = \frac{C(s)}{R(s)} = \frac{1}{s+2}$

$r(t) = 10u(t)$

$R(s) = \frac{10}{s}$

$C(s) = \frac{10}{s(s+2)} = \frac{5}{s} - \frac{5}{s+2}$

$c(t) = 5(1-e^{-2t})$

$\lim_{t\to\infty} c(t) = 5$

$\frac{99}{100} \times 5 = 4.95$

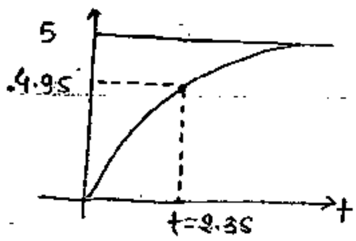
$4.95 = 5(1-e^{-2t})$

$5e^{-2t} = 0.05$

$e^{-2t} = 0.01$

$-2t = \ln(0.01)$

$t = 2.3s$



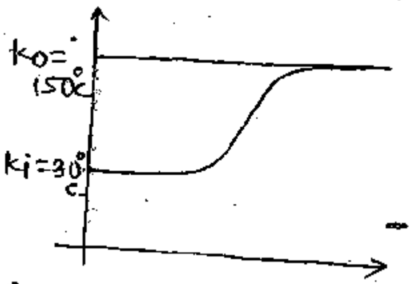
Que→ A thermometer having 1st order dynamics is subjected to sudden temp. change of 30°c – 150°c. If it has a time constant of 4s. what temp. it will indicate after 4s.

<u>Sol^n</u> $\rightarrow$ $c(t) = k_o + (k_i - k_o)e^{-t/T}$

at $\Rightarrow t = 4s$

$c(t) = 150 + (30-150)e^{-4/4}$

$c(t) = 105.6°C$



<u>Que.</u> $\rightarrow$ The TF of the 1st order sys is

$$\frac{C(s)}{R(s)} = \frac{1}{1+Ts}$$

Its type & ess| unit ramp i/p are

(a) 0, ∞ (b) 0, T (c) 1, ∞ (d) 1, T

<u>Sol^n</u> $\rightarrow$ $G(s) = \frac{1}{1+Ts-1} = \frac{1}{Ts}$

$\boxed{\text{Type-1 system}}$

$e_{ss} = \frac{1}{K}$ where $K = K_v = \lim_{s\to 0} \cancel{s} \cdot \frac{1}{T\cdot\cancel{s}} = \frac{1}{T}$

$\boxed{e_{ss} = T}$

<u>2nd order system</u> $\rightarrow$

The responce of 2nd order (or) higher order sys. exibits contineous (or) & sustained oscillation about the steady state value of i/p with a freq. known as undamped natural freq. ω_n.

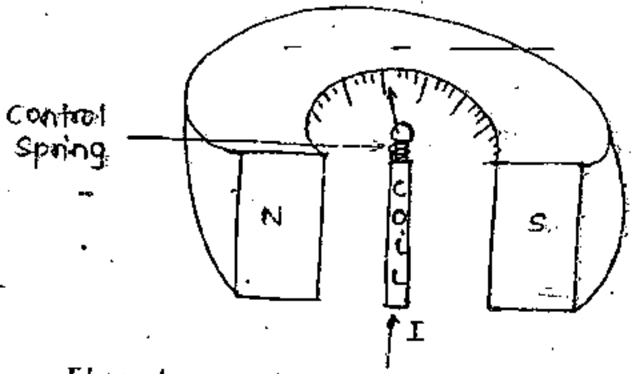
This oscillation in the responce are damped to the steady state value of i/p using appropriate damping methods, the damping is mathemoticaly expressed as damping ratio ξ (ζ).

<u>eg:-</u> PMMC.

Undamped Natural freq. ~~ω_m~~ ω_n r/s

damping Ratio "ξ (Zeta)"

$$\boxed{\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n + \omega_n^2}}$$



I/P = deflecting torque (T_d)

O/P = angular deflection of pointer (θ)

J = Inertia of moving system , B = Inherent friction

k = Spring constant.

$$T_d = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + k\theta \quad ; \quad T_d(s) = (Js^2 + Bs + k)\,\theta(s)$$

$$\frac{\theta(s)}{T_d(s)} = \frac{1}{\cancel{Js^2+Bs}+k} = \frac{1/J}{s^2 + \frac{B}{J}s + \frac{K}{J}}$$

$$s^2 + \frac{B}{J}s + \frac{K}{J} = s^2 + 2\xi\omega_n s + \omega_n^2$$

$$\boxed{\omega_n = \sqrt{\frac{K}{J}}\ r/s} \quad ; \quad 2\xi\sqrt{\frac{K}{J}} = \frac{B}{J} \quad ; \quad \boxed{\xi = \frac{B}{2\sqrt{KJ}}} \quad \boxed{\xi \propto B}$$

"TYPE" of std. 2nd order system →

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n + \omega_n^2}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \cancel{\omega_n^2} - \cancel{\omega_n^2}}$$

$$G(s) = \frac{\omega_n^2}{s(s + 2\xi\omega_n)} \qquad \text{Type-1 sys.}$$

Effect of damping on the nature of responce →

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$s^2 + 2\xi\omega_n s + 1 = 0$$

$$= \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2}$$

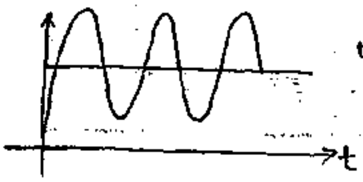
$$= -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$$

$D = \xi^2 - 1 = 0 \Rightarrow \xi = 1$

$D = \xi^2 - 1 < 0 \Rightarrow \xi < 1$

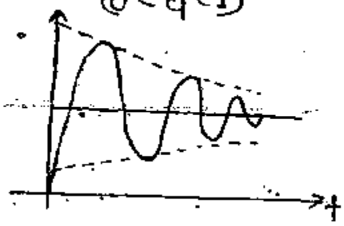
$D = \xi^2 - 1 > 0 \Rightarrow \xi > 1$

Case(1) → Un-damped case ($\xi = 0$)

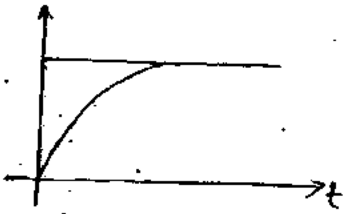


ω_n r/s

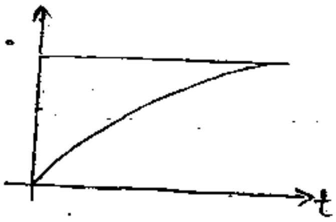
Case(2) → Under-damped case ($0 < \xi < 1$)



Case(3) → Criticaly damped case ($\xi = 1$)



Case(4) → Overdamped case ($\xi > 1$)



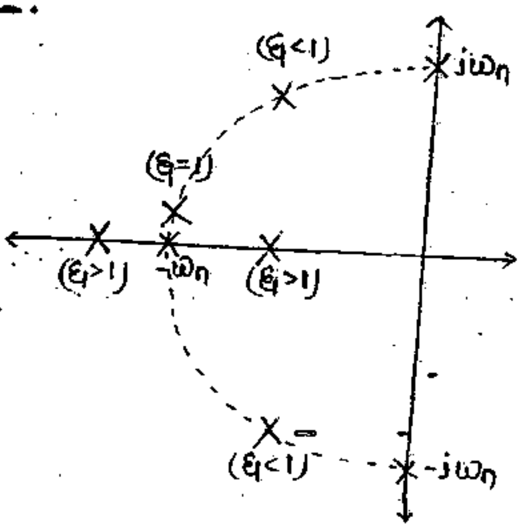


Fig :- Root locus

* Most of the CS are designed $\xi<1$ because the responce can be analysised using more no. of performance specification (optimum values of ξ in CS design are b/n 0.3–0.8).

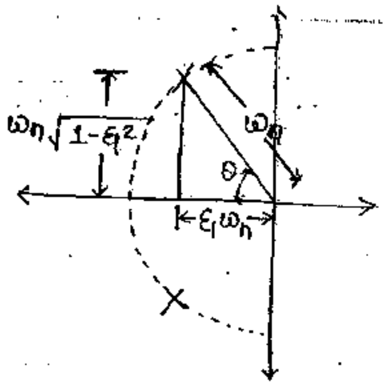
* The root locus of 2nd order sys obtained by varying the damping ratio ξ is a semicircular path with a radius of ω_n & breakaway point at $-\omega_n$ on the -ve real axis.

c/s of underdamped sys. →

$$s^2+2\xi\omega_n s+\omega_n^2=0$$

$$s=-\xi\omega_n \pm \sqrt{(\xi^2-1)}\ \omega_n$$

For $\xi<1$



1) $\cos\theta = \frac{\xi\omega_n}{\omega_n} = \xi$

$$\boxed{\begin{array}{l} \theta=\cos^{-1}\xi \\ \sin\theta=\sqrt{1-\xi^2} \\ \tan\theta=\dfrac{\sqrt{1-\xi^2}}{\xi} \end{array}}$$

2) Damping cofficient (or) actual damping (or) damping factor

$$\boxed{\alpha=\xi\omega_n}$$

3) Timeconstant of underdamped responce

$$T=\frac{1}{\alpha}=\frac{1}{\xi\omega_n}$$

4) Damped natural Freq: $\omega_d=\omega_n\sqrt{1-\xi^2}$ r/s

) For $\xi<1$

$$s^2+2\xi\omega_n s+\omega_n^2=(s+\xi\omega_n)^2+\omega_d^2$$

) Damping ratio $= \dfrac{\text{Actual damping}}{\text{critical damping}} = \dfrac{\xi\cancel{\omega_n}}{\cancel{\omega_n}} = \xi$

actual damping $=\xi\omega_n$, at → $\xi=1$

Actual damping becomes critical damping

critical damping $=1$

* Transient analysis → (Underdamped Responce)

Let $R(s) = \frac{1}{s}$, $C(s) = \frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)}$

$$C(s) = \frac{1}{s} - \frac{(s+2\xi\omega_n)}{s^2+2\xi\omega_n s+\omega_n^2}$$

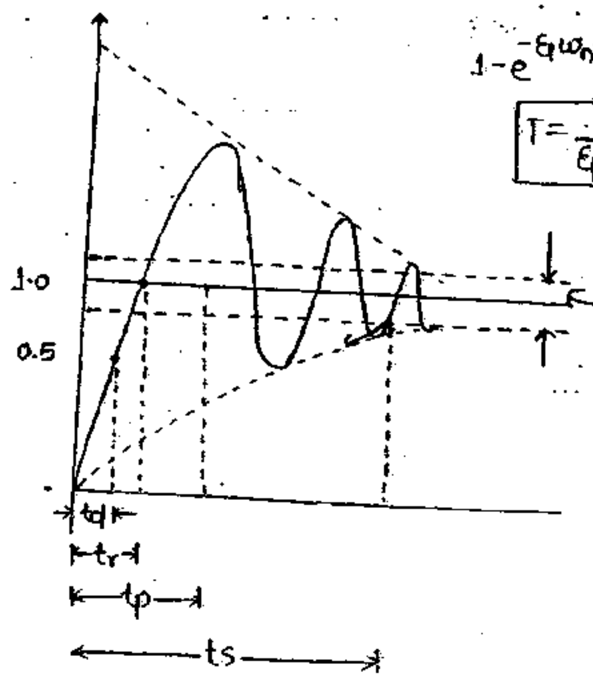
$$= \frac{1}{s} - \frac{(s+\xi\omega_n)}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\omega_d}{(s+\xi\omega_n)^2+\omega_d^2} \times \frac{\xi\cancel{\omega_n}}{\cancel{\omega_n}\sqrt{1-\xi^2}}$$

$$c(t) = 1 - e^{-\xi\omega_n t}\cdot\cos\omega_d t - e^{-\xi\omega_n t}\sin\omega_d t \cdot \frac{\xi}{\sqrt{1-\xi^2}}$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sqrt{1-\xi^2}\cos\omega_d t + \xi\cdot\sin\omega_d t\right]$$

$\because A\sin\omega t + B\cos\omega t = \sqrt{A^2+B^2}\,\sin\left[\omega t + \tan^{-1}\left(\frac{B}{A}\right)\right]$

$$\boxed{c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\left[\omega_d t + \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right]}$$



$1 - e^{-\xi\omega_n t} = 1 - e^{-t/T}$

$\boxed{T = \frac{1}{\xi\omega_n}}$

Tolerance Band 2-5%.

(1) Delay time (t_d) →

$$t_d = \frac{1+0.7\xi}{\omega_n} \text{ secs.}$$

(2) Rise time (t_r) →

At $t = t_r$; $c(t) = 1$

$$\therefore c(t)\Big|_{t=t_r} = 1 - \frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta) = 1$$

$$\frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta) = 0$$

Since $\sin(\omega_d t_r + \theta) = 0$

$\omega_d t_r + \theta = \pi$

$$\boxed{t_r = \frac{\pi - \theta}{\omega_d}}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right) \text{ rad.}$$

(3) Settling time (t_s) →

For 2% of Tolerance band ; $t_s = 4T \rightarrow \frac{4}{\xi\omega_n}$ secs.

For 5% of tolerance band ; $t_s = 3T \rightarrow \frac{3}{\xi\omega_n}$ secs.

(4) No. of cycles →

$\omega_d = 2\pi f_d$; $f_d = \frac{\omega_d}{2\pi} \left(\frac{\text{cycles}}{\text{sec}}\right)$

2% of (TB) → $t_s \times f_d \Rightarrow \frac{4 f_d}{\xi\omega_n}$ (cycles)

5% of TB → $t_s \times f_d \Rightarrow \frac{3 f_d}{\xi\omega_n}$ ~~(secs)~~(cycles)

(5) Time period / Time interval of damped sinusoid →

$$\boxed{T = \frac{1}{f_d}} \text{ secs.}$$

(6) Peak time (t_p) →

At $t = t_p$; $c(t) = \max^m$ value

$$\frac{d}{dt} c(t) = 0$$

$$\Rightarrow \frac{d}{dt}\left[1-\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\cdot\sin(\omega_d t+\theta)\right]=0$$

$$\Rightarrow \quad 0-\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\cdot\cos(\omega_d t+\theta)\,\omega_d+\sin(\omega_d t+\theta)\,\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}(-\xi\omega_n)=0$$

$$\Rightarrow \quad \cancel{\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}}\cdot\cos(\omega_d t+\theta)\cdot\omega_d=\sin(\omega_d t+\theta)\cdot\cancel{\frac{e^{-\omega_n\xi t}}{\sqrt{1-\xi^2}}}\cdot\xi\omega_n$$

$$\Rightarrow \quad \frac{\sin(\omega_d t+\theta)}{\cos(\omega_d t+\theta)}=\frac{\omega_d}{\xi\omega_n}=\frac{\cancel{\omega_n}\sqrt{1-\xi^2}}{\xi\cancel{\omega_n}}$$

$$\tan(\omega_d t+\theta)=\tan\theta$$

$$\tan(n\pi+\theta)=\tan\theta$$

$$\omega_d t=n\pi$$

$$\therefore \quad t=t_p$$

$$\omega_d t_p=n\pi$$

$$\boxed{t_p=\frac{n\pi}{\omega_d}}\ \text{sec.}$$

Maximum peak overshoot (m_p) →

$$c(t)\Big|_{t=t_p=\frac{\pi}{\omega_d}}$$

$$c(t)=1-\frac{e^{-\xi\omega_n\cdot\frac{\pi}{\omega_d}}}{\sqrt{1-\xi^2}}\cdot\sin\left(\cancel{\omega_d}\cdot\frac{\pi}{\cancel{\omega_d}}+\theta\right)$$

$$c(t)=1+e^{-\xi\cancel{\omega_n}\cdot\frac{\pi}{\cancel{\omega_n}\sqrt{1-\xi^2}}}$$

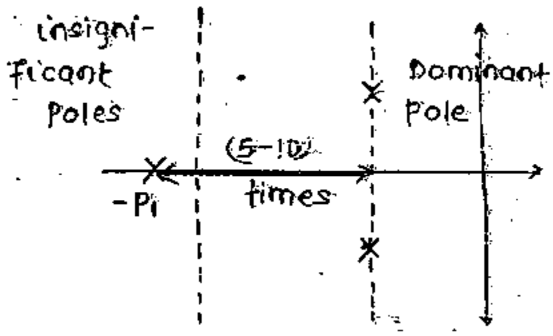
$$c(t)=1+e^{-\xi\pi/\sqrt{1-\xi^2}}$$

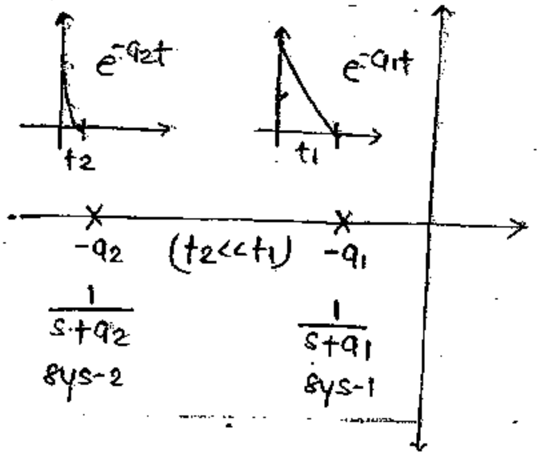
$$m_p=c(t)\Big|_{t=t_p}-1=e^{-\xi\pi/\sqrt{1-\xi^2}}\cong e^{-\xi n\pi/\sqrt{1-\xi^2}}$$

*** Time Responce analysis for higher order sys →** consider a 3rd order c/s eqn;

$$s^3 + ps^2 + qs + K = 0$$

$$(s+p_1)(s^2 + q_1 s + K_1) = 0$$





* The time responce analysis of higher order sys is obtained by approximating to 2nd order system wrt dominant poles.

* The time domain specification obtained for approximated lower order sys are valid for original higher order sys also because poles lie in the insignificant region have insignificant affect on the time responce c/s.

Que → The 2nd order approximation using dominant pole concept is

$$T(s) = \frac{10}{(s+5)(s^2+s+1)}$$

(a.) $\frac{10}{s^2+s+1}$ (b.) $\frac{2}{s^2+s+1}$ (c.) $\frac{10}{(s+5)(s+1)}$ (d.) $\frac{2}{(s+5)(s+1)}$

Soln → **Note:-** When approximating higher order TF to a lower order TF convert in time constant form before eliminating the insignificant pole.

$$T(s) = \frac{10}{(s+5)(s^2+s+1)}$$

$$= \frac{\cancel{10}\,2}{\cancel{5}(\frac{s}{5}+1)(s^2+s+1)}$$

$$T(s) = \frac{2}{s^2+s+1}$$

Que→ The c/s eqⁿ of the sys. is

$$s(s^2+6s+13)+k=0$$

Find the value of k such that the c/s eqⁿ has a pair of complex roots with real part -1.

(a) 10 (b) 20 (c) 30 (d) 40

Solⁿ→

$$s^3+6s^2+13s+k=0 \quad \text{----(i)}$$

$$(s+a)(s^2+bs+c)=0$$

Roots are $s=\dfrac{-b\pm\sqrt{b^2-4ac}}{2}$

Given $\dfrac{-b}{2}=-1$, $b=2$

$$(s+a)(s^2+2s+c)=0$$

$$s^3+as^2+2s^2+2as+sc+ac=0 \quad \text{----(ii)}$$

From eqⁿ (i) & (ii)

$$s^3+s^2(2+a)+s(2a+c)+ac=0 \quad \text{----(iii)}$$

$(2+a)=6$, $a=4$, $2a+c=13$, $c=5$ $\quad$ $ac=k$, $6\times5=k$, $k=30$

Que→ The open loop TF of a unity f/b sys. is $G(s)=\dfrac{k(s+b)}{s^2(s+20)}$

For what value of b does all the 3 roots of the c/s eqⁿ converge at the same point on the real axis.

Solⁿ→

$$1+\frac{k(s+b)}{s^2(s+20)}=0$$

$$s^2(s+20)+k(s+b)=0$$

$$s^3+20s^2+ks+kb=0 \quad \text{----(i)}$$

& $(s+a)(s+a)(s+a)=0$

$$s^3+s^2(3a)+s(3a^2)+a^3=0 \quad \text{----(ii)}$$

$3a=20$ $\quad$ $a^3=3a^2b$

$3a^2=k$ $\quad$ $b=\dfrac{a}{3}$ $\quad$ $\because a=\dfrac{20}{3}$

$a^3=kb$

$$\boxed{b=\frac{20}{9}}$$

(5/64) $15\% < m_p < 30\%$, $t_s < 0.75s$.

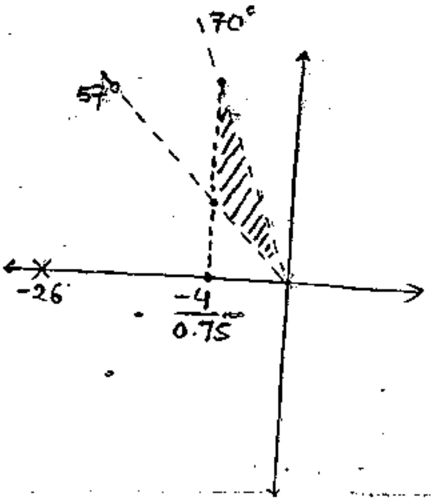
* (a.) $\% m_p = 15\%$ $\quad\quad$ $\% m_p = 30\%$

$m_p = 0.15$ $\quad\quad$ $m_p = 0.3$

$e^{-\xi\pi/\sqrt{1-\xi^2}} = 0.15$ $\quad\quad$ $e^{-\xi\pi/\sqrt{1-\xi^2}} = 0.3$

$\xi = 0.55$ $\quad\quad$ $\xi = 0.35$

$\theta = \cos^{-1}\xi = 57^\circ$ $\quad\quad$ $\theta = \cos^{-1}(0.35) = 70^\circ$



$$t_s = \frac{4}{\xi\omega_n} = 0.75$$

$$\xi\omega_n = \frac{4}{0.75}$$

* (b.) 3rd roots ; $s = 5 \times \frac{-4}{0.75} = -26$

* (c.) $\% m_p = 30\%$; $\xi = 0.35$

$$t_s = 0.75s = \frac{4}{0.35 \times \omega_n} = 0.75$$

$$\omega_n = 15\ r/s.$$

$$(s+26)(s^2 + 2 \times 0.35 \times 15s + 225) = 0$$

$$s^3 + 36.5s^2 + 498s + 5850 = 0$$

$$1 + \frac{5850}{s^3 + 36.5s^2 + 498s} = 0$$

$$1 + G(s) = 0$$

$$\boxed{G(s) = \frac{5850}{s(s^2 + 36.5s + 498)}}$$

(3/63)

$$1 + \frac{K(s+2)}{s^3 + \alpha s^2 + 4s + 1} = 0$$

$$s^3 + \alpha s^2 + 4s + 1 + Ks + 2K = 0$$

$$s^3 + \alpha s^2 + s(4+K) + 2K + 1 = 0 \text{ ---- (i)}$$

$$\because (s+a)(s^2 + bs + c) = 0$$

given $\xi = 0.2$, $\omega_n^2 = 9$

$c = \omega_n^2 = 9$, $b = 2\xi\omega_n = 2 \times 0.2 \times 3 = 1.2$

$$(s+a)(s^2 + 1.2s + 9) = 0$$

$$s^3 + 1.2s^2 + 9s + as^2 + 1.2as + 9a = 0$$

(6/GT) (a)

$$s^3+s^2(1.2+a)+s(9+1.2a)+9a=0 \quad ----(ii)$$

$$\alpha=1.2+a$$

$$4+k=9+1.2a$$

$$2k+1=9a$$

$$\boxed{\alpha=2.8,\ k=7}$$

(2/3)

$$1+G(s)=0$$

$$1+\frac{K}{s(1+Ts)}=0$$

$$s+Ts^2+k=0$$

$$s^2+\frac{s}{T}+\frac{k}{T}=0$$

$$\omega_n^2=\frac{k}{T},\quad \omega_n=\sqrt{\frac{K}{T}}\ r/s$$

$$2\xi\sqrt{\frac{K}{T}}=\frac{1}{T},\quad \boxed{\xi=\frac{1}{2\sqrt{KT}}}$$

<u>Case(1)</u> % mp = 40%

$$mp=0.4$$

$$e^{-\xi\pi/\sqrt{1-\xi^2}}=0.4$$

$$\xi=\xi_1=0.28$$

let $K=K_1$

<u>Case(2)</u> % mp = 60%

$$mp=0.6$$

$$e^{-\xi\pi/\sqrt{1-\xi^2}}=0.6$$

$$\xi=\xi_2=0.16$$

Let $k=k_2$

$$\frac{\xi_1}{\xi_2}=\frac{\frac{1}{2\sqrt{K_1T}}}{\frac{1}{2\sqrt{K_2T}}}$$

$$\left(\frac{0.28}{0.16}\right)^2=\left(\frac{K_2}{K_1}\right)$$

$$\boxed{K_2=3K_1}$$

Note:- The general relations b/n damping ratio(ξ) & sys. gain(K)

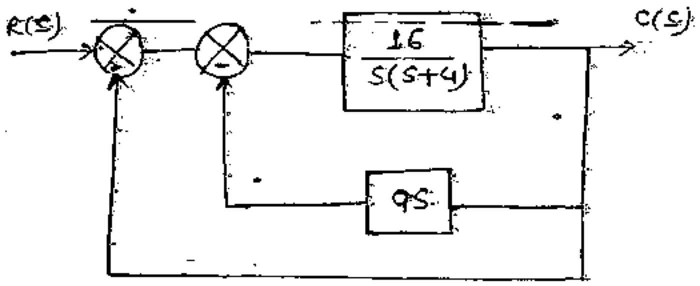
$$\boxed{\xi\propto\frac{1}{\sqrt{K}}}\quad \boxed{\text{As } K\uparrow\ \xi\downarrow,\ mp\uparrow}$$

→ The sys. is originaly criticaly damped. if its gain is doubled then it will become :-

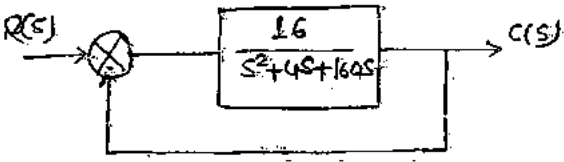
-(a) Undamped (b) Overdamped (C) Underdamped (d) None.

<u>S:-</u> Underdamped.

1/63



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{16}{s(s+4)}}{1+\frac{16}{s(s+4)}(as)} = \frac{16}{s^2+4s+16as}$$



(a) $\% m_p = 15\%$

$\cancel{0.15} = 0.015$

$e^{-\xi\pi/\sqrt{1-\xi^2}} = 0.015$

$\xi = 0.8$

$1 + \frac{16}{s^2+4s+16as} = 0$

$s^2 + s(4+16a) + 16 = 0$

$\omega_n = 4$ rad/s

$2\xi \times 4 = 4 + 16a$

$2 \times 0.8 \times 4 = 4 + 16a$

$\boxed{a = 0.15}$

(b) $e_{ss}\big|_{\text{unit ramp}}$

(i) without 'a' $(a=0)$

$G(s) = \frac{16}{s(s+4)}$

$$e_{ss} = \lim_{s\to 0} \frac{s\cdot\frac{1}{s^2}}{1+\frac{16}{s(s+4)}}$$

$e_{ss} = \frac{4}{16} = 0.25$ units

(ii) with 'a' $(a = 0.15)$

$$G(s) = \frac{16}{s(s+4)+16\times 0.15 s} = \frac{16}{s(s+6.4)}$$

$$e_{ss} = \lim_{s\to 0} \frac{s\cdot\frac{1}{s^2}}{1+\frac{16}{s(s+6.4)}} = \frac{6.4}{16} = 0.4 \text{ units}$$

(c) $G(s) = \frac{K}{s(s+4)+K\times 0.15s}$

$$e_{ss} = \lim_{s\to 0} \frac{\cancel{s}\cdot\frac{1}{s^{\cancel{2}}}}{1+\frac{K}{s(s+4)+0.15Ks}}$$

$0.25 = \frac{0.15K+4}{K}$ $\boxed{K=40}$

$$G(s) = \frac{40}{s(s+4)+40\times 0.15s} = \frac{40}{s(s+10)}$$

$s^2+10s+40=0$

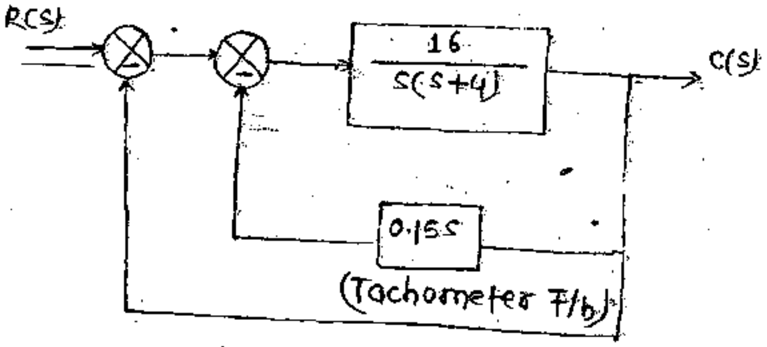
$\omega_n = \sqrt{40} = 6.32$

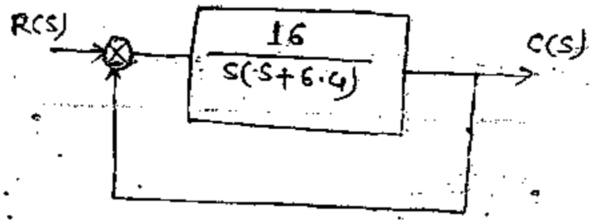
$2\xi\times 6.32 = 0$

$\boxed{\xi = 0.8}$

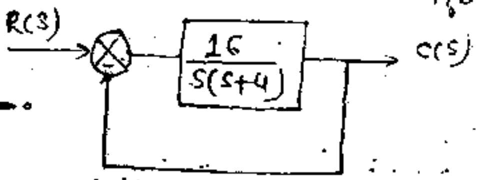
If the pre

*** Effect of tachometer f/b on the performance c/s →**





<u>case(1) →</u> without tachometer f/b.



$$1+\frac{16}{s(s+4)}=0$$

$$s^2+4s+16=0$$

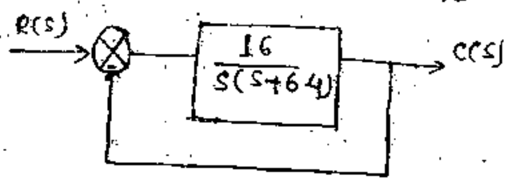
$$\omega_n=4,\quad 2\xi\times 4=4$$

$$\omega_n=4;\quad \xi=0.5$$

$$T=\frac{1}{\xi\omega_n}=\frac{1}{0.5\times 4}=0.5s$$

$$\boxed{\xi=0.5\ ;\ T=0.5s \quad \omega_n=4}$$

<u>case(2) →</u> with tachometer f/b



$$1+\frac{16}{s(s+6.4)}=0$$

$$\omega_n=4,\ \xi=0.8$$

$$T=\frac{1}{\xi\omega_n}=0.32s$$

$$\boxed{\xi=0.8,\ \omega_n=4 \quad T=0.32s}$$

ω_n = Remains fixed
$\xi = \xi\uparrow$
$T=0.32s;\ T\downarrow$
Responce is faster

(19/63) (d)

(13/62) $G(s) = \frac{10K}{s(1+0.1s)}$

$I/p = \frac{1}{2}$ rps

$e_{ss} < 0.2^\circ$

(a.) $I/p = \frac{1}{2}$ rps $\Rightarrow \pi$ r/s

$e_{ss} = \frac{0.2\pi}{180}$ radians

$$\frac{0.2\pi}{180} = \lim_{s\to 0} \frac{s \cdot \frac{\pi}{s^2}}{1 + \frac{10k}{s(1+0.1s)}}$$

$$\frac{0.2\pi}{180} = \frac{\pi}{10k}$$

$\boxed{K=90}$

(b.) $1 + \frac{1000}{s(1+0.1s)} = 0$

$0.1s^2 + s + 900 = 0$

$s^2 + 10s + 9000 = 0$

$\omega_n = \sqrt{9000} = 95$ r/s

$2\xi \times 95 = 10$

$\boxed{\xi = 0.05}$

(12/62) $$\frac{4d^2c(t)}{dt^2} + \frac{8dc(t)}{dt} + 16c(t) = 16u(t)$$

$(4s^2 + 8s + 16)\, c(s) = 16\, u(s)$

$$\frac{c(s)}{u(s)} = \frac{16}{4s^2 + 8s + 16}$$

$$= \frac{4}{s^2 + 2s + 4}$$

$\omega_n = 2$ rad/s

$2\xi \times 2 = 2$

$\boxed{\xi = 0.5}$

(11/62) $\frac{d^2y}{dt^2} + \frac{3dy}{dt} + 2y = x(t)$

at $t = 0$

$y(t) = 0$

ans (a)

(10/62) $\omega_n = 4$ rad/s

$2\xi \times 4 = 4$

$\xi = 0.5$

$$t_p = \frac{3\pi}{4\sqrt{1-(0.5)^2}}$$

$\boxed{t_p = \frac{1.5\pi}{\sqrt{3}}}$

(9/62) $\frac{d}{dt}(1 - e^{-5t} - 5te^{-5t})$

Imp. res. $= 25te^{-5t}$

$$TF = \frac{25}{(s+5)^2} = \frac{25}{s^2+10s+25}$$

$\boxed{\omega_n = 5 \text{ r/s};\ \xi = 1}$ ans (d)

8/62 $\quad \frac{H(s+c)}{(s+a)(s+b)}$ $\quad$ (1) $u(t) = 2 + De^{-t} + Ee^{-3t}$

(2) $e^{-2t}u(t) = Fe^{-t} + Ge^{-3t}$

(i) $\frac{H(s+c)}{s(s+a)(s+b)} = \frac{k_1}{s} + \frac{k_2}{(s+a)} + \frac{k_3}{(s+b)}$ $\qquad$ (ii) $\frac{H(s+c)}{(s+2)(s+a)(s+b)}$

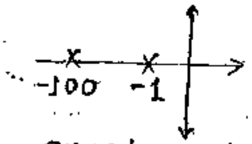
$= 2 + De^{-t} + Ee^{-3t}$ $\qquad \boxed{\begin{matrix} C=2 \\ H=3 \end{matrix}}$

$a=1,\ b=3$

$$\lim_{s\to 0} \frac{s\, H(s+c)}{s(s+a)(s+b)} = \lim_{t\to\infty}\left[2+De^{-t}+Ee^{-3t}\right]$$

$\frac{HC}{ab} = 2$; $\frac{HC}{3} = 2$, $HC = 6$

7/61 $\quad TF = G(s) = \frac{100}{(s+1)(s+100)}$



Overdamped sys.

$$\frac{1}{(1+s)(\cancel{1+\frac{s}{100}})} = \frac{1}{(1+Ts)} \qquad T = 1\,\text{sec.}$$

$2\%\ T_B = 4T = 4\,\text{sec.}$

2nd pole is ~~domin~~ insignificant effect.

7/5 (A) $\quad$ 6/65 (C)