

Number System and its Operations

IIT Foundation Material

SECTION - I

Straight Objective Type

1.
$$\begin{aligned} & \sqrt{14\sqrt{5}-30} \\ &= 5^{1/4} \sqrt{14-6\sqrt{5}} \\ &= 5^{1/4} \sqrt{14-2\sqrt{45}} \\ &= 5^{1/4} (\sqrt{9}-\sqrt{5}) \\ &= 5^{1/4} (3-\sqrt{5}) \end{aligned}$$

Hence, (b) is the correct option.

2.
$$\begin{aligned} & \frac{1}{\sqrt{5-\sqrt{5-\sqrt{24}}}} + \frac{1}{\sqrt{5+\sqrt{5+\sqrt{24}}}} \\ & \frac{1}{\sqrt{5-\sqrt{5-2\sqrt{6}}}} + \frac{1}{\sqrt{5-\sqrt{5+2\sqrt{6}}}} \\ & \frac{1}{\sqrt{5-(\sqrt{3}-\sqrt{2})}} + \frac{1}{\sqrt{5-(\sqrt{3}++\sqrt{2})}} \\ &= \frac{1}{\sqrt{2}} \end{aligned}$$

Hence, (b) is the correct option.

3.
$$\begin{aligned} & \sqrt{a+2\sqrt{a-1}} + \sqrt{a-2\sqrt{a-1}} \\ &= 2\sqrt{a-1} \end{aligned}$$

Hence, (d) is the correct option.

4.
$$\begin{aligned} & (\sqrt{50} + \sqrt{48})^{1/2} = k(\sqrt{3} + \sqrt{2}) \\ & (5\sqrt{2} + 4\sqrt{3})^{1/2} = k(\sqrt{3} + \sqrt{2}) \end{aligned}$$

$$\Rightarrow 2^{1/4}(\sqrt{3} + \sqrt{2}) = k(\sqrt{3} + \sqrt{2})$$

$$2^{1/4}(\sqrt{3} + \sqrt{2}) = k(\sqrt{3} + \sqrt{2})$$

$$\Rightarrow k = 2^{1/4}$$

Hence, (b) is the correct option.

5. Let $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6}}}} \dots \alpha$
 $x = \sqrt{6 + x}$

$$\Rightarrow x^2 = 6 + x$$

$$x^2 - x - 6 = 0$$

$$(x - 3)(x + 2) = 0$$

$$x = 3 \text{ or } -2$$

Hence (d) is the correct option

6. $\sqrt[6]{2\sqrt{2} + 3} - \sqrt[3]{1 - \sqrt{2}} = -1$

Hence, (d) is the correct option.

7. If $(4 + \sqrt{15})^{3/2} - (4 - \sqrt{15})^{3/2} = k\sqrt{6}$

$$(4 + \sqrt{15})^{3/2} - (4 - \sqrt{15})^{3/2} = 9\sqrt{6}$$

$$\Rightarrow k = 9$$

Hence, (c) is the correct option

8. $x = 2 + \sqrt{3}, \quad xy = 1$

$$\Rightarrow y = \frac{1}{x}$$

$$y = 2 - \sqrt{3}$$

$$\frac{x}{\sqrt{2} + \sqrt{x}} + \frac{y}{\sqrt{2} - \sqrt{y}}$$

$$x = 2 + \sqrt{3}$$

$$\begin{aligned}
\sqrt{x} &= \sqrt{\frac{2(2+\sqrt{3})}{2}} \\
&= \sqrt{\frac{4+2\sqrt{3}}{2}} \\
&= \frac{\sqrt{3}+\sqrt{1}}{\sqrt{2}} \\
y &= 2-\sqrt{3} \\
\sqrt{y} &= \sqrt{\frac{4-2\sqrt{3}}{2}} \\
&= \frac{\sqrt{3}-1}{\sqrt{2}} \\
&\frac{2+\sqrt{3}}{\sqrt{2}+\frac{\sqrt{3}+1}{\sqrt{2}}} + \frac{2-\sqrt{3}}{\sqrt{2}-\frac{\sqrt{3}-1}{\sqrt{2}}} \\
&\frac{\sqrt{2}(2+\sqrt{3})}{2+\sqrt{3}+1} + \frac{\sqrt{2}(2-\sqrt{3})}{2+1-\sqrt{3}} \\
&\frac{\sqrt{2}(2+\sqrt{3})}{3+\sqrt{3}} + \frac{\sqrt{2}(2-\sqrt{3})}{3-\sqrt{3}} \\
&= \sqrt{2}
\end{aligned}$$

Hence, (a) is the correct option.

9. $\sqrt{b}+\sqrt{c}, \sqrt{c}+\sqrt{a}, \sqrt{a}+\sqrt{b}$ are in HP

$\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$, are in A.P

$$\begin{aligned}
\Rightarrow & \frac{1}{\sqrt{c}+\sqrt{a}} - \frac{1}{\sqrt{b}+\sqrt{c}} \\
&= \frac{1}{\sqrt{a}+\sqrt{b}} - \frac{1}{\sqrt{c}+\sqrt{a}}
\end{aligned}$$

$$\Rightarrow 2b = a + c$$

$$\Rightarrow a, b, c \text{ are in A.P}$$

Hence, (a) is the correct option

10. If $p = \sqrt{32} - \sqrt{24}$ $q = \sqrt{50} - \sqrt{48}$

$$\frac{1}{p} = \frac{1}{\sqrt{32} - \sqrt{24}} = \frac{\sqrt{32} + \sqrt{24}}{8}$$

$$= \frac{4\sqrt{2} + 2\sqrt{3}}{4} = \frac{2\sqrt{2} + \sqrt{3}}{2}$$

$$\frac{1}{q} = \frac{\sqrt{50} + \sqrt{48}}{2}$$

$$= \frac{5\sqrt{2} + 4\sqrt{3}}{2} = \frac{5}{\sqrt{2}} + 2\sqrt{3}$$

$$\frac{1}{p} = \frac{2}{\sqrt{2}} + \frac{\sqrt{3}}{2}$$

$$\frac{1}{q} = \frac{5}{\sqrt{2}} + 2\sqrt{3}$$

$$\Rightarrow \frac{1}{q} > \frac{1}{p}$$

$$\Rightarrow q < p$$

$$\Rightarrow p > q$$

Hence, (b) is the correct option

11. If $\frac{4+3\sqrt{3}}{\sqrt{7+4\sqrt{3}}} = a + \sqrt{b}$ then (a, b)

$$\frac{4+3\sqrt{3}}{\sqrt{7+2\sqrt{6}}} = \frac{4+3\sqrt{3}}{\sqrt{6}+1} = \frac{4+3\sqrt{3} \times (\sqrt{6}-1)}{6-1}$$

$$= -1 + \sqrt{12}$$

$$a = -1, b = 12$$

Hence, (c) is the correct option

- 12.** Rationalising factor of

$$\left(\sqrt[3]{a^2} + \sqrt[3]{b^2} + \sqrt[3]{ab}\right) \text{ is } \left(\sqrt[3]{a} - \sqrt[3]{b}\right)$$

$$\begin{aligned} \text{Hence, Rationalising factor of } \sqrt[3]{16} + \sqrt[3]{4} + 1 \\ = \sqrt[3]{4} - 1 \end{aligned}$$

Hence, (c) is the correct option.

- 13.** If $x = \sqrt[3]{11}, y = \sqrt[4]{12}, z = \sqrt[6]{13}$

$$x = (11)^{\frac{1}{3} \times \frac{4}{4}}$$

$$\text{L. C. M of } \begin{array}{c} 3, 4, 6 \\ 1, 4, 2 \\ 1, 2, 1 \end{array} = 12$$

$$y = (12)^{\frac{1}{4} \times \frac{3}{3}}$$

$$z = (13)^{\frac{1}{6} \times \frac{2}{2}}$$

$$x = 11^{\frac{4}{12}}$$

$$y = 12^{\frac{3}{12}}$$

$$z = 13^{\frac{2}{12}}$$

$$x = \sqrt[12]{11^4}$$

$$y = \sqrt[12]{12^3}$$

$$z = \sqrt[12]{13^2}$$

Hence, (b) is the correct option

- 14.** $4^{\frac{1}{3}}, 5^{\frac{1}{4}}, 6^{\frac{1}{6}}, 8^{\frac{1}{3}}$

$$\text{L.C.M of } 3, 4, 6 = 12$$

$$4^{\frac{1}{3} \times \frac{4}{4}}, 5^{\frac{1}{4} \times \frac{3}{3}}, 6^{\frac{1}{6} \times \frac{2}{2}}, 8^{\frac{1}{3} \times \frac{4}{4}}$$

$$4^{\frac{4}{12}}, 5^{\frac{3}{12}}, 6^{\frac{2}{12}}, 8^{\frac{4}{12}}$$

$$\sqrt[12]{4^4}, \sqrt[12]{5^3}, \sqrt[12]{6^3}, \sqrt[12]{8^4}$$

$$\sqrt[12]{256}, \sqrt[12]{125}, \sqrt[12]{216}, \sqrt[12]{4096}$$

$\Rightarrow 5^{\frac{1}{4}}$ is the smallest number among the Numbers
Hence, (b) is the correct option.

15. $x = \sqrt{5}, y = \sqrt[4]{10}, z = \sqrt[3]{6}$

$$5^{\frac{1}{2}}, 10^{\frac{1}{4}}, 6^{\frac{1}{3}}$$

$$\sqrt[12]{5^6}, \sqrt[12]{10^3}, \sqrt[12]{6^4}$$

$$\sqrt[12]{15625}, \sqrt[12]{1000}, \sqrt[12]{1296}$$

$\Rightarrow x > y > z$

Hence, (c) is the correct option.

16. If $x = \sqrt[2]{9}, y = \sqrt[4]{11}, z = \sqrt[12]{17}$

L. C. M of 3, 4, 6 = 12

$$9^{\frac{1}{3} \times \frac{4}{7}}, 11^{\frac{1}{4} \times \frac{3}{3}}, 17^{\frac{1}{6} \times \frac{2}{2}}$$

$$9^{\frac{4}{12}}, 11^{\frac{3}{12}}, 17^{\frac{2}{12}}$$

$$\sqrt[12]{9^4}, \sqrt[12]{11^3}, \sqrt[12]{17^2}$$

$$\sqrt[12]{6561}, \sqrt[12]{1331}, \sqrt[12]{289}$$

$\Rightarrow x > y > z$

Hence, (a) is the correct option.

17. $a + x + \sqrt{2ax + x^2}$

$$a + x + \sqrt{x(2a + x)}$$

$$\frac{1}{2} [2a + 2x + 2\sqrt{x(2a + x)}]$$

$$\frac{1}{2} [x + 2a + x + 2\sqrt{x(2a + x)}]$$

$$\begin{aligned}
 \Rightarrow & \sqrt{a+x+\sqrt{2ax+x^2}} \\
 &= \sqrt{\frac{x+2a+x+2\sqrt{x(2a+x)}}{2}} \\
 &= \frac{\sqrt{x}+\sqrt{2a+x}}{\sqrt{2}}
 \end{aligned}$$

Hence, (c) is the correct option.

$$\begin{aligned}
 \text{18. } & \sqrt[4]{17+\sqrt{288}} = \sqrt[4]{17+2\sqrt{72}} \\
 &= \sqrt[2]{\sqrt{8}+\sqrt{9}} \\
 &= \sqrt{3+2\sqrt{2}} \\
 &= \sqrt{(\sqrt{2}+1)^2} \\
 &= 1+\sqrt{2}
 \end{aligned}$$

Hence, (d) is the correct option.

$$\begin{aligned}
 \text{19. } & (26+15\sqrt{3})^{\frac{2}{3}} + (26-15\sqrt{3})^{\frac{2}{3}} \\
 &= 14
 \end{aligned}$$

Hence, (d) is the correct option.

$$\begin{aligned}
 \text{20. } & \text{If } x = 2 + 2^{\frac{1}{3}} + 4^{\frac{1}{3}} \\
 \Rightarrow & x - 2 = 2^{\frac{1}{3}} + 4^{\frac{1}{3}} \\
 & (x-2)^3 = 2 + 4 + 3 \cdot 2^{\frac{1}{3}} \cdot 4^{\frac{1}{3}} \cdot \left(2^{\frac{1}{3}} + 4^{\frac{1}{3}}\right) \\
 &= 6 + 3 \cdot 2(x-2) \\
 &= 6 + (6x-12) \\
 & x^3 - 8 - 6x^2 + 12x = 6x - 6 \\
 \Rightarrow & x^3 - 6x^2 + 6x = 8 - 6
 \end{aligned}$$

$$= 2$$

$$\Rightarrow x^3 - 6x^2 + 6x = 2$$

Hence, (c) is the correct option.

$$\begin{aligned} 21. \quad & \frac{3 + \sqrt{6}}{5\sqrt{3} - 2\sqrt{12} - \sqrt{32 + 50}} \\ &= \frac{3 + \sqrt{6}}{5\sqrt{3} - 2.2\sqrt{3} - \sqrt{32 + 5\sqrt{2}}} \\ &= \frac{3 + \sqrt{6}}{5\sqrt{3} - 4\sqrt{3} - \sqrt{32 + 5\sqrt{2}}} = 2\sqrt{2} \end{aligned}$$

Hence, (c) is the correct option.

$$\begin{aligned} 22. \quad & \text{If } n = \frac{\sqrt{a+2b} + \sqrt{a-2b}}{\sqrt{a+2b} - \sqrt{a-2b}} \\ & \frac{x+1}{x-1} = \frac{\sqrt{a+2b}}{\sqrt{a-2b}} \\ & \frac{(x+1)^2}{(x-1)^2} = \frac{a+2b}{a-2b} \\ & \frac{(x+1)^2 + (x-1)^2}{(x+1)^2 (x-1)^2} = \frac{a+2b + (a-2b)}{(a+2b) - (a-2b)} \\ & \frac{2(x^2+1)}{4x} = \frac{2a}{4b} \end{aligned}$$

$$\Rightarrow bx^2 + b = ax$$

$$\Rightarrow bx^2 - ax + b = 0$$

Hence, (d) is the correct option.

23. The value of

$$(20 + 14\sqrt{2})^{\frac{-1}{3}} + (20 - 14\sqrt{2})^{\frac{-1}{3}} = 2$$

Hence, (a) is the correct option.

24. If $n = 2\sqrt{2} + \sqrt{7}$

$$\frac{1}{x} = \frac{1}{2\sqrt{2} + \sqrt{7}} = 2\sqrt{2} + \sqrt{7}$$

$$\frac{1}{2} \left(x + \frac{1}{x} \right) = \frac{1}{2} \left[\left(2\sqrt{2} + \sqrt{7} \right) + \left(2\sqrt{2} - \sqrt{7} \right) \right]$$

$$= \frac{4\sqrt{2}}{2} = 2\sqrt{2} = \sqrt{8}$$

Hence, (c) is the correct option.

25. $x = 3 + 3^{\frac{1}{3}} + 3^{\frac{2}{3}}$

$$x - 3 = 3^{\frac{1}{3}} + 3^{\frac{2}{3}}$$

$$(x - 3)^3 = \left(3^{\frac{1}{3}} + 3^{\frac{2}{3}} \right)^3$$

$$x^3 - 27 + 27x - 9x^2 = 3 + 9 + 9(x - 3)$$

$$x^3 - 9x^2 + 27x - 27 = 12 + 9x - 27$$

$$\Rightarrow 3x^2 - 9x^2 + 18x = 12$$

Hence, (b) is the correct option.

26. $\sqrt{12 - \sqrt{68 + 48\sqrt{2}}}$

$$= \sqrt{12 - \sqrt{68 + 2\sqrt{576 \times 2}}}$$

$$= \sqrt{12 - \sqrt{68 + 2\sqrt{24 \times 48}}}$$

$$= \sqrt{12 - \sqrt{(\sqrt{24} + \sqrt{48})^2}}$$

$$= \sqrt{12 - (2\sqrt{6} + 2\sqrt{12})}$$

$$= 2 + \sqrt{2}$$

Hence, (a) is the correct option.

$$\begin{aligned} 27. \quad & \text{Let } \sqrt[3]{38+17\sqrt{5}} = x + \sqrt{y} \\ \Rightarrow & x^3 + 3xy = 38, 17\sqrt{5} = \sqrt{y}(y + 3x^2) \end{aligned}$$

$$\Rightarrow y = 5$$

$$x^3 + 3.x(5) = 38$$

$$x^3 + 15x = 38$$

$$3x^2 + 5 = 17$$

$$3x^2 = 12$$

$$x^2 = 4$$

$$x = \pm 2$$

$$\text{Hence, } \sqrt[3]{38+17\sqrt{5}} = 2 + \sqrt{5}$$

Hence, (b) is the correct option.

$$28. \quad \left(4 + \sqrt{15}\right)^{\frac{3}{2}} + \left(4 - \sqrt{15}\right)^{\frac{3}{2}} = P\sqrt{10}$$

$$\left(4 + \sqrt{15}\right)^3 + \left(4 - \sqrt{15}\right)^2 +$$

$$2.\left(4 + \sqrt{15}\right)^{\frac{3}{2}}\left(4 - \sqrt{15}\right)^{\frac{3}{2}} = P^2(10)$$

$$2\left(64 + 3.4\left(\sqrt{15}\right)^2\right) = P^2(10)$$

$$\cancel{2}(64 + 12 \times 15) = \cancel{10} P^2$$

$$(64 + 180) = 5P^2$$

$$224 = 5P^2$$

$$\Rightarrow P = 7$$

Hence, (a) is the correct option.

$$29. \quad \text{Let } x = \sqrt{12 + \sqrt{12 + \sqrt{12 + \sqrt{12 + \dots}}}}$$

$$x = \sqrt{12 + \sqrt{12 + \sqrt{12 + \sqrt{12 + \dots}}}}$$

$$x = \sqrt{12 + x}$$

$$x^2 = 12 + x$$

$$x^2 - x - 12 = 0$$

$$(x-4)(x+3) = 0$$

$$\Rightarrow x - 4 = 0 \text{ or } x + 4 = 0$$

$$\Rightarrow x = 4 \quad x = -3$$

Hence, (c) is the correct option.

30. $\sqrt{2}, \sqrt[3]{3}, \sqrt[4]{5}$

$$2^{\frac{1}{2}}, 3^{\frac{1}{3}}, 5^{\frac{1}{4}}$$

$$2^{\frac{1}{2} \times \frac{2}{3}}, 3^{\frac{1}{3} \times \frac{2}{2}}, 5^{\frac{1}{4}}$$

$$2^{\frac{2}{4}}, 3^{\frac{2}{4}}, 5^{\frac{1}{4}}$$

$$\sqrt[4]{4}, \sqrt[4]{9}, \sqrt[4]{5}$$

Hence, (c) is the correct option.

SECTION - II

Assertion - Reason Questions

31. $\sqrt{a^2 - b^2} \neq \sqrt{a - b}$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

Hence (c) is the correct option.

32. $\sqrt{15}, \sqrt{13}, \sqrt{11}$ are pure surds

$$\begin{aligned} \sqrt[5]{320} &= \sqrt[5]{32 \times 10} = \sqrt[5]{32} \times \sqrt[5]{10} \\ &= 2\sqrt[5]{10} \end{aligned}$$

Hence (a) is the correct option.

$$\begin{aligned}
 33. \quad & \sqrt{3} < \sqrt[4]{10} < \sqrt[3]{6} \\
 & 4\sqrt{12} - \sqrt{50} - 7\sqrt{48} \\
 & = 8\sqrt{3} - 5\sqrt{2} - 28\sqrt{3} \\
 & = -20\sqrt{3} - 5\sqrt{2}
 \end{aligned}$$

Hence (a) is the correct option.

$$\begin{aligned}
 34. \quad & \sqrt{14} \times \sqrt{21} = \sqrt{294} \\
 & \left(\text{since } \sqrt{a} \times \sqrt{b} = \sqrt{ab} \right) \\
 & \sqrt[n]{x} \times \sqrt[n]{y} = \sqrt[n]{xy}
 \end{aligned}$$

Hence (a) is the correct option.

$$\begin{aligned}
 35. \quad & (5 - \sqrt{3}) + (5 + \sqrt{3}) = 10 \text{ is rational} \\
 & (5 - \sqrt{3})(5 + \sqrt{3}) = 25 - 3 = 22 \text{ is also rational} \\
 & \text{Hence } (5 - \sqrt{3}) \text{ and } (5 + \sqrt{3}) \text{ are conjugate surds} \\
 & \text{Hence (a) is the correct option.}
 \end{aligned}$$

$$\begin{aligned}
 36. \quad & \frac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} \\
 & = \frac{35+14\sqrt{3}-20\sqrt{3}-8 \times 3}{49-48} \\
 & = 11-6\sqrt{3} = a+b\sqrt{3} \\
 & a=11 \\
 & \frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}} \\
 & = \frac{\sqrt{5}+\sqrt{10}-\sqrt{3}-\sqrt{6}+\sqrt{5}-\sqrt{10}+\sqrt{3}-\sqrt{6}}{5-3}
 \end{aligned}$$

$$= \frac{\cancel{2}(\sqrt{5} - \sqrt{6})}{\cancel{2}}$$

$$= \sqrt{5} - \sqrt{6} = -0.213$$

Hence (a) is the correct option.

37. $x = 5 + 2\sqrt{6}$

$$x + \frac{1}{x} = (5 + 2\sqrt{6}) + (5 - 2\sqrt{6}) = 10$$

$$(a+b)(a-b) = a^2 - b^2$$

Hence (b) is the correct option.

38. $\sqrt{75} = 5\sqrt{3}$

$$5\sqrt{3} \times \sqrt{3} = 5 \times 3 = 15 \text{ which is rational}$$

$$\sqrt{3} \text{ is R.F.}$$

Hence (b) is the correct option.

39. $\sqrt{10} = 3.162$

$$\frac{1}{\sqrt{10}} = \frac{1}{3.162} = 0.316$$

$$\sqrt[3]{36} = \sqrt[3]{3 \times 3 \times 2 \times 2}$$

Hence (b) is the correct option.

40. $4\sqrt{3} - 3\sqrt{12} + 2\sqrt{75}$

$$= 4\sqrt{3} - 6\sqrt{3} + 10\sqrt{3} = 8\sqrt{3}$$

$$\sqrt[4]{81} - 8.\sqrt[3]{216} + 15.\sqrt[5]{32} + \sqrt{225}$$

$$= 3 - 8.6 + 15.2 + 15$$

$$= 3 - 48 + 30 + 15 = 0$$

Hence (b) is the correct option.

SECTION - III

Linked Comprehension Type

$$\begin{aligned}
 41. \quad & \text{If } \frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a+b\sqrt{3} \\
 & \frac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} \\
 & = 35+14\sqrt{3}-20\sqrt{3}-24 \\
 & = 11-6\sqrt{3}
 \end{aligned}$$

$$\Rightarrow a=11, \quad b=-6$$

Hence (a) is the correct option.

$$\begin{aligned}
 42. \quad & x=5-2\sqrt{6} \\
 & \frac{1}{x} = 5-2\sqrt{6} \\
 & x+\frac{1}{x} = (5+2\sqrt{6})+(5-2\sqrt{6})=10
 \end{aligned}$$

Hence (b) is the correct option.

$$\begin{aligned}
 43. \quad & 2\sqrt{3}-\sqrt{5} \text{ is a rationalising factor of } 2\sqrt{3}+\sqrt{5} \\
 & (2\sqrt{3}-\sqrt{5})(2\sqrt{3}+\sqrt{5}) = (2\sqrt{3})^2 - (\sqrt{5})^2 \\
 & = 12-5=7 \text{ which is rational} \\
 & \text{Hence (c) is the correct option.}
 \end{aligned}$$

$$\begin{aligned}
 44. \quad & \sqrt{11-4\sqrt{7}} \\
 & = \sqrt{11-2\sqrt{7} \times 4} \\
 & = \sqrt{7}-2
 \end{aligned}$$

Hence (a) is the correct option.

$$\begin{aligned}
 45. \quad & \sqrt{15-x\sqrt{14}} = \sqrt{8}-\sqrt{7} \\
 & 15-x\sqrt{14} = 15-2\sqrt{56}
 \end{aligned}$$

$$= 15 - 4\sqrt{14}$$

$$\Rightarrow x = 4$$

Hence, (c) is the correct option.

$$\begin{aligned} 46. \quad & \sqrt{(2x-3)+2\sqrt{x^2-3x+2}} \\ &= \sqrt{x-1+x-2+2\sqrt{(x-1)(x-2)}} \\ &= \sqrt{[\sqrt{(x-1)}+\sqrt{(x-2)}]^2} \\ &= \sqrt{x-1}+\sqrt{x-2} \end{aligned}$$

Hence (a) is the correct option.

$$\begin{aligned} 47. \quad & \text{Let } \sqrt[3]{37-30\sqrt{3}} = x - \sqrt{y} \\ & x^3 + 3x^2 = 37 \text{ and } \sqrt{y}(y+3x^2) = 30\sqrt{3} \end{aligned}$$

$$\Rightarrow y=3, \quad 3x^2+y=30$$

$$3x^2 = 27$$

$$x=3; y=3$$

Hence (c) is the correct option.

$$48. \quad (4+\sqrt{15})^{\frac{3}{2}} + (4-\sqrt{15})^{\frac{3}{2}} = p\sqrt{10}$$

$$\Rightarrow P = 7$$

Hence (c) is the correct option.

$$49. \quad \sqrt[3]{9\sqrt{3}} = 11\sqrt{2} = \sqrt{3} - \sqrt{2}$$

Hence (a) is the correct option.

$$\begin{aligned} 50. \quad & 10^{\frac{1}{4}}, 6^{\frac{1}{3}}, 3^{\frac{1}{2}} \\ & 10^{\frac{1}{4} \times \frac{3}{5}}, 6^{\frac{1}{3} \times \frac{4}{4}}, 3^{\frac{1}{2} \times \frac{1}{6}} \end{aligned}$$

L. C. M of 2, 3, 4 = 12

$$\sqrt[12]{10^3}, \sqrt[12]{6^4}, \sqrt[12]{3^6}$$

$$\sqrt[12]{1000}, \sqrt[12]{1296}, \sqrt[12]{729}$$

Hence, $\sqrt{3}$ is the smallest number

Hence, (c) is the correct option.

51. $\sqrt[5]{2}, \sqrt[6]{5}, \sqrt[9]{6}$

$$\sqrt[3]{3, 6, 9}$$

1,2,3

$$2^{\frac{1}{3}}, 5^{\frac{1}{6}}, 6^{\frac{1}{9}}$$

L.C.M. of 3, 6, 9 = 18

$$2^{\frac{1}{3} \times \frac{6}{6}}, 5^{\frac{1}{6} \times \frac{3}{3}}, 6^{\frac{1}{9} \times \frac{2}{2}}$$

$$\sqrt[18]{2^6}, \sqrt[18]{5^3}, \sqrt[18]{6^2}$$

$$\sqrt[18]{64}, \sqrt[18]{125}, \sqrt[18]{36}$$

$\Rightarrow \sqrt[6]{5}$ is the smaller surd

Hence, (b) is the correct option.

52. $3^{\frac{1}{3}}, 8^{\frac{1}{6}}, 25^{\frac{1}{9}}$

$$3^{\frac{1}{3} \times \frac{6}{6}}, 8^{\frac{1}{6} \times \frac{3}{3}}, 25^{\frac{1}{9} \times \frac{2}{2}}$$

$$\sqrt[18]{3^6}, \sqrt[18]{8^3}, \sqrt[18]{25^2}$$

$$\sqrt[18]{729}, \sqrt[18]{512}, \sqrt[18]{625}$$

$\Rightarrow \sqrt[6]{8}$ is the smaller surd

Hence, (b) is the correct option.

53. $n = \frac{\sqrt{2}+1}{\sqrt{2}-1} \quad y = \frac{\sqrt{2}-1}{\sqrt{2}+1}$

Then $x + y = \frac{\sqrt{2}+1}{\sqrt{2}-1} + \frac{\sqrt{2}-1}{\sqrt{2}+1}$

$$= \frac{(\sqrt{2}+1)+(\sqrt{2}-1)}{2-1}$$

$$= 2(2+1) = 6$$

Hence, (c) is the correct option.

54.

$$\begin{aligned} & \frac{2}{\sqrt{10+2\sqrt{21}}} - \frac{1}{\sqrt{12-2\sqrt{35}}} + \frac{1}{\sqrt{8-2\sqrt{15}}} \\ &= \frac{2}{\sqrt{(\sqrt{7}+\sqrt{3})^2}} - \frac{1}{\sqrt{(\sqrt{7}+\sqrt{5})^2}} + \frac{1}{\sqrt{(\sqrt{5}+\sqrt{3})^2}} \\ &= \frac{2}{\sqrt{7}+\sqrt{3}} - \frac{1}{\sqrt{7}-\sqrt{5}} + \frac{1}{\sqrt{5}-\sqrt{3}} \\ &= \frac{\sqrt{7}-\sqrt{3}}{2} - \frac{\sqrt{7}+\sqrt{5}}{2} + \frac{\sqrt{5}+\sqrt{3}}{2} \\ &= \frac{\sqrt{7}-\sqrt{3}-\sqrt{7}-\sqrt{5}+\sqrt{5}+\sqrt{3}}{2} \\ &= \frac{0}{2} = 0 \end{aligned}$$

Hence (b) is the correct option.

55. If $x = \frac{\sqrt{3}}{2}$ then

$$\begin{aligned} & \frac{1+x}{1+\sqrt{1+x}} + \frac{1+\frac{\sqrt{3}}{2}}{1+\sqrt{1+\frac{\sqrt{3}}{2}}} \\ 1+x &= 1 + \frac{\sqrt{3}}{2} = \frac{2+\sqrt{3}}{2} = \frac{4+2\sqrt{3}}{4} \\ \sqrt{1+x} &= \sqrt{\frac{4+2\sqrt{3}}{4}} = \frac{\sqrt{3+1}}{2} \end{aligned}$$

Hence,

$$\begin{aligned}\frac{1+x}{1+\sqrt{1+x}} &= \frac{\frac{2+\sqrt{3}}{2}}{1+\frac{\sqrt{3}+1}{2}} = \frac{2+\sqrt{3}}{3+\sqrt{3}} \\ &= \frac{(2+\sqrt{3}) \times (3-\sqrt{3})}{9-3} \\ &= \frac{6+3\sqrt{3}-2\sqrt{3}-3}{6} = \frac{3+\sqrt{3}}{6}\end{aligned}$$

Hence (a) is the correct option.

SECTION - IV

Matrix - Match Type

56.

	p	q	r	s
A	☐	☒	☐	☐
B	☒	☐	☐	☐
C	☐	☐	☒	☐
D	☐	☐	☐	☒

57.

	p	q	r	s
A	☐	☐	☒	☐
B	☐	☒	☐	☐
C	☒	☐	☐	☐
D	☐	☐	☐	☒

58.

	p	q	r	s
A	☐	☐	☐	☒
B	☐	☐	☒	☐
C	☐	☒	☐	☐
D	☒	☐	☐	☐

59.

	p	q	r	s
A	☒	☐	☐	☐
B	☐	☒	☐	☐
C	☐	☐	☒	☐
D	☐	☐	☐	☒

60.

	p	q	r	s
A	☐	☒	☐	☐
B	☐	☐	☒	☐
C	☒	☐	☐	☐
D	☐	☐	☐	☒