

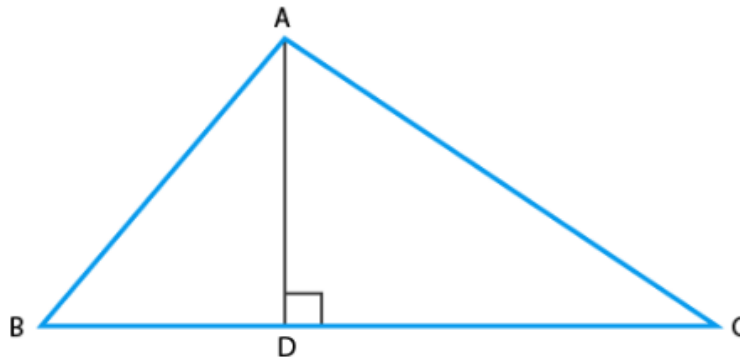
Chapter 6
Triangles

Exercise No. 6.1

Multiple Choice Questions:

Choose the correct answer from the given four options:

1. In figure, if $\angle BAC = 90^\circ$ and $AD \perp BC$. Then,



If the lengths of the diagonals of rhombus are 16 cm and 12 cm. Then, the length of the sides of the rhombus is

- (A) $BD \cdot CD = BC^2$
(B) $AB \cdot AC = BC^2$
(C) $BD \cdot CD = AD^2$
(D) $AB \cdot AC = AD^2$

Solution:

(C) $BD \cdot CD = AD^2$

In $\triangle ADB$ and $\triangle ADC$,

We have,

$$\angle D = \angle D = 90^\circ$$

$$\angle DBA = \angle DAC$$

$$(\because AD \perp BC)$$

$$[\text{each angle} = 90^\circ - \angle C]$$

From AAA similarity rule,

$$\triangle ADB \sim \triangle ADC$$

Therefore,

$$\frac{BD}{AD} = \frac{AD}{CD}$$

$$BD \cdot CD = AD^2$$

2. If the lengths of the diagonals of rhombus are 16 cm and 12 cm. Then, the length of the sides of the rhombus is

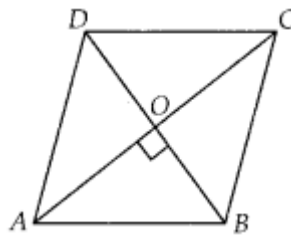
- (A) 9 cm
- (B) 10 cm
- (C) 8 cm
- (D) 20 cm

Solution:

(B) 10 cm

We have,

A rhombus is a simple quadrilateral whose four sides are of same length and diagonals are perpendicular bisector of each other.



Now,

AC = 16 cm and

BD = 12 cm

$\angle AOB = 90^\circ$

AC and BD bisect each other

$$AO = \frac{1}{2} AC$$

$$BO = \frac{1}{2} BD$$

So,

AO = 8 cm

BO = 6 cm

In right angled $\triangle AOB$,

By Pythagoras theorem,

We have,

$$AB^2 = AO^2 + OB^2$$

$$\begin{aligned}
 AB^2 &= 8^2 + 6^2 \\
 &= 64 + 36 \\
 &= 100 \\
 AB &= \sqrt{100} \\
 &= 10 \text{ cm}
 \end{aligned}$$

As the four sides of a rhombus are equal.

So, one side of rhombus = 10 cm.

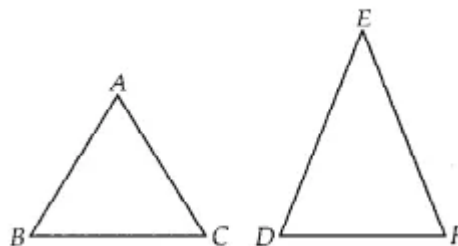
3. If $\triangle ABC \sim \triangle EDF$ and $\triangle ABC$ is not similar to $\triangle DEF$, then which of the following is not true?

- (A) $BC \cdot EF = AC \cdot FD$
- (B) $AB \cdot EF = AC \cdot DE$
- (C) $BC \cdot DE = AB \cdot EF$
- (D) $BC \cdot DE = AB \cdot FD$

Solution:

(C) $BC \cdot DE = AB \cdot EF$

If sides of one triangle are proportional to the side of the other triangle, and the corresponding angles are also equal, then the triangles are similar by SSS similarity.



So, $\triangle ABC \sim \triangle EDF$

By similarity rule,

$$\frac{AB}{ED} = \frac{BC}{DF} = \frac{AC}{EF}$$

At first we take,

$$\frac{AB}{ED} = \frac{BC}{DF}$$

$$\frac{AB}{ED} = \frac{BC}{DF}$$

$$AB \cdot DF = ED \cdot BC$$

Hence, option (D) $BC \cdot DE = AB \cdot FD$ is true

Now taking,

$$\frac{BC}{DF} = \frac{AC}{EF}, \text{ we get}$$

$$BC \cdot EF = AC \cdot DF$$

Hence, option (A) $BC \cdot EF = AC \cdot FD$ is true

Now if,

$$\frac{AB}{ED} = \frac{AC}{EF}, \text{ we get,}$$

$$AB \cdot EF = ED \cdot AC$$

Hence, option (B) $AB \cdot EF = AC \cdot DE$ is true.

4. If in two triangles ABC and PQR, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$, then

(A) $\triangle PQR \sim \triangle CAB$

(B) $\triangle PQR \sim \triangle ABC$

(C) $\triangle CBA \sim \triangle PQR$

(D) $\triangle BCA \sim \triangle PQR$

Solution:

(A) $\triangle PQR \sim \triangle CAB$

We have, from $\triangle ABC$ and $\triangle PQR$,

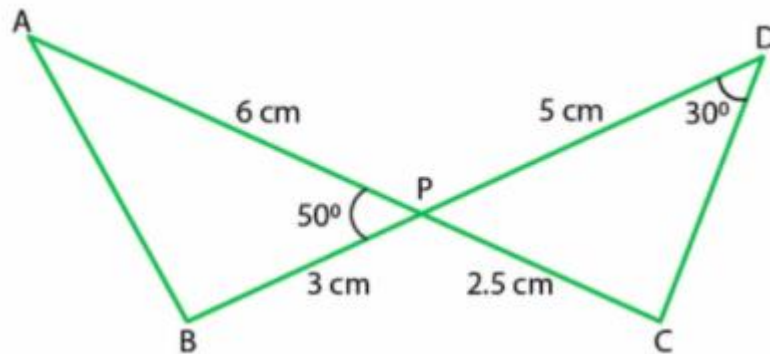
$$\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$$

If sides of one triangle are proportional to the side of the other triangle, and their corresponding angles are also equal, then both the triangles are similar by SSS similarity.

So, we can say that,

$\triangle PQR \sim \triangle CAB$

5. In fig. 6.3, two line segments AC and BD intersect each other at the point P such that PA = 6 cm, PB = 3 cm, PC = 2.5 cm, PD = 5 cm, $\angle APB = 50^\circ$ and $\angle CDP = 30^\circ$. Then, $\angle PBA$ is equal to



- (A) 50°
- (B) 30°
- (C) 60°
- (D) 100°

Solution:

(D) 100°

In $\triangle APB$ and $\triangle CPD$,
 $\angle APB = \angle CPD = 50^\circ$

(vertically opposite angles)

$$\frac{AP}{PD} = \frac{6}{5} \quad \dots (i)$$

And,

$$\frac{BP}{CP} = \frac{3}{2.5}$$

$$\frac{BP}{CP} = \frac{6}{5} \quad \dots (ii)$$

From equations (i) and (ii),

$$\frac{AP}{PD} = \frac{BP}{CP}$$

Therefore,

$\triangle APB \sim \triangle DPC$

[using SAS similarity rule]

$$\angle A = \angle D = 30^\circ \quad [\text{Corresponding angles of similar triangles}]$$

As,

$$\text{Sum of angles of a triangle} = 180^\circ$$

From $\triangle APB$,

$$\angle A + \angle B + \angle APB = 180^\circ$$

$$30^\circ + \angle B + 50^\circ = 180^\circ$$

$$\angle B = 180^\circ - (50^\circ + 30^\circ)$$

$$\angle B = 180 - 80^\circ$$

$$= 100^\circ$$

So,

$$\angle PBA = 100^\circ$$

6. If in two triangles DEF and PQR, $\angle D = \angle Q$ and $\angle R = \angle E$, then which of the following is not true?

(A) $\frac{EF}{PR} = \frac{DF}{PQ}$

(B) $\frac{DE}{PQ} = \frac{EF}{RP}$

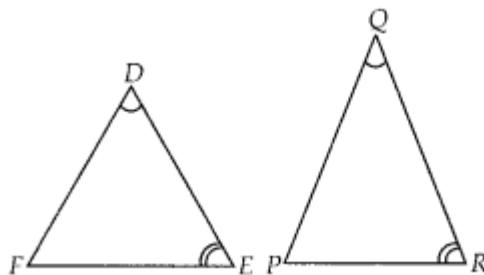
(C) $\frac{DE}{QR} = \frac{DF}{PQ}$

(D) $\frac{EF}{RP} = \frac{DE}{QR}$

Solution:

(B)

We have,



In $\triangle DEF$ and $\triangle PQR$,

$$\angle D = \angle Q,$$

$$\angle R = \angle E$$

$$\triangle DEF \sim \triangle QRP$$

[using AAA similarity criterion]

$$\angle F = \angle P$$

[corresponding angles]

$$\frac{DF}{QP} = \frac{ED}{RQ} = \frac{FE}{PR}$$

7. In triangles ABC and DEF, $\angle B = \angle E, \angle F = \angle C$ and $AB = 3DE$. Then, the two triangles are

- (A) Congruent but not similar
- (B) Similar but not congruent
- (C) Neither congruent nor similar
- (D) Congruent as well as similar

Solution:

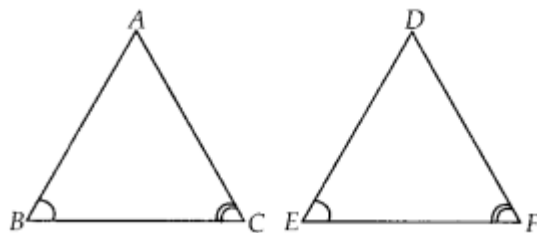
(B)

In $\triangle ABC$ and $\triangle DEF$,

$$\angle B = \angle E,$$

$$\angle F = \angle C \text{ and}$$

$$AB = 3DE$$



We know that, if in two triangles corresponding two angles are same, then they are similar by AA similarity criterion.

But,

$$AB \neq DE$$

Therefore $\triangle ABC$ and $\triangle DEF$ are not congruent.

8. It is given that $\triangle ABC \sim \triangle PQR$, with $\frac{BC}{QR} = \frac{1}{3}$. Then, $\frac{\text{ar}(\triangle PQR)}{\text{ar}(\triangle BCA)}$ is equal to

- (A) 9
- (B) 3
- (C) $\frac{1}{3}$
- (D) $\frac{1}{9}$

Solution:

(A)

We have,
 $\triangle ABC \sim \triangle PQR$

$$\frac{BC}{QR} = \frac{1}{3}$$

We know that, the ratio of the areas of two similar triangles is equal to square of the ratio of their corresponding sides.

Therefore,

$$\frac{\text{ar}(\triangle PRQ)}{\text{ar}(\triangle BCA)} = \frac{QR^2}{BC^2}$$

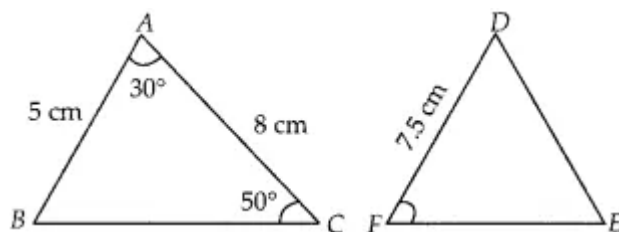
$$\begin{aligned}\frac{QR^2}{BC^2} &= \frac{3^2}{1^2} \\ &= 9\end{aligned}$$

9. It is given that $\triangle ABC \sim \triangle DFE$, $\angle A = 30^\circ$, $\angle C = 50^\circ$, $AB = 5$ cm, $AC = 8$ cm and $DF = 7.5$ cm. Then, the following is true:

- (A) $DE = 12$ cm, $\angle F = 50^\circ$
- (B) $DE = 12$ cm, $\angle F = 100^\circ$
- (C) $EF = 12$ cm, $\angle D = 100^\circ$
- (D) $EF = 12$ cm, $\angle D = 30^\circ$

Solution:

We have,
 $\triangle ABC \sim \triangle DFE$,
 $\angle A = \angle D = 30^\circ$,
 $\angle C = \angle E = 50^\circ$



$$\begin{aligned}\angle B &= \angle F \\ &= 180^\circ - (50^\circ + 30^\circ) \\ &= 100^\circ\end{aligned}$$

Now,

$$\frac{AB}{DF} = \frac{AC}{DE}$$

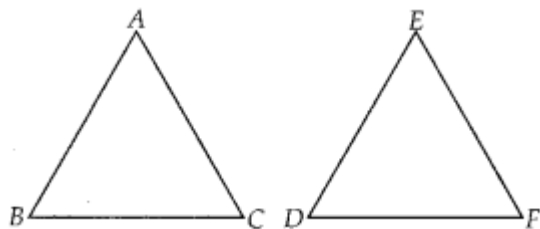
$$\frac{5}{7.5} = \frac{8}{DE}$$

$$DE = 12\text{cm}$$

10. If in triangles ABC and DEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar when

- (A) $\angle B = \angle E$
- (B) $\angle A = \angle D$
- (C) $\angle B = \angle D$
- (D) $\angle A = \angle F$

Solution:



Given, in $\triangle ABC$ and $\triangle EDF$,

$$\frac{AB}{DE} = \frac{BC}{FD}$$

Therefore,

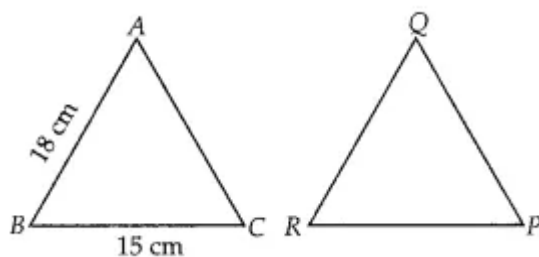
$\triangle ABC \sim \triangle EDF$ if, $\angle B = \angle D$ [By SAS similarity criterion]

11. If $\triangle ABC \sim \triangle QRP$, and $BC = 15$ cm, then PR is equal to

- (A) 10 cm
- (B) 12 cm
- (C) $\frac{20}{3}$ cm
- (D) 8 cm

Solution:

In given question,



We know that the ratio of area of two similar triangles is equal to the ratio of square of their corresponding sides.

$$\frac{ar(ABC)}{ar(QRP)} = \frac{(BC)^2}{(RP)^2} \text{ Also, } \frac{ar(ABC)}{ar(QRP)} = \frac{9}{4}$$

Therefore,

$$\frac{(BC)^2}{(RP)^2} = \frac{9}{4}$$

$$\frac{(15)^2}{(RP)^2} = \frac{9}{4}$$

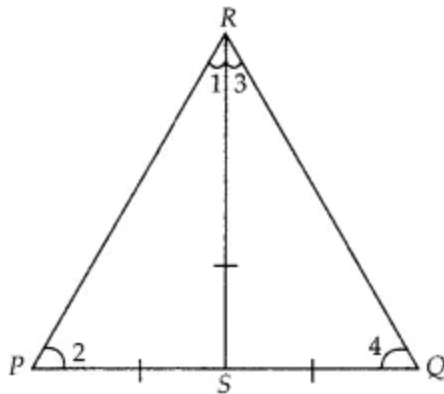
$$RP = 10cm$$

12. If S is a point on side PQ of a $\triangle PQR$ such that $PS = QS = RS$, then

- (A) $PR \cdot QR = RS^2$
- (B) $QS^2 + RS^2 = QR^2$
- (C) $PR^2 + QR^2 = PQ^2$
- (D) $PS^2 + RS^2 = PR^2$

Solution:

In given question,



In $\triangle PQR$,
 $PS = QS = RS$

..... (i)

Now,

In $\triangle PSR$,
 $PS = RS$

(By eqn (i))

$\angle 1 = \angle 2$

..... (ii)

[Angles opposite to equal sides are equal]

Also, in $\triangle RSQ$,

$RS = SQ$

$\angle 3 = \angle 4$

..... (iii)

[angles opposite to equal sides are equal]

We know that, in ΔPQR , sum of angles = 180°

$$\angle P + \angle Q + \angle R = 180^\circ$$

$$\angle 2 + \angle 4 + \angle 1 + \angle 3 = 180^\circ$$

$$\angle 1 + \angle 3 + \angle 1 + \angle 3 = 180^\circ$$

$$2(\angle 1 + \angle 3) = 180^\circ$$

$$= 90^\circ$$

So, $\angle R = 90^\circ$

In ΔPQR , by Pythagoras theorem,

$$PR^2 + QR^2 = PQ^2$$

Exercise No. 6.2

Short Answer Questions with Reasoning:

Question:

1. Is the triangle with sides 25 cm, 5 cm and 24 cm a right triangle? Give reason for your answer.

Solution:

It is not true.

Taking,

a = 25 cm,

b = 5 cm and

c = 24 cm

Now,

$$\begin{aligned}b^2 + c^2 &= (5)^2 + (24)^2 \\&= 25 + 576 \\&= 601 \\&\neq (25)^2\end{aligned}$$

Therefore, given sides do not make a right triangle because it does not satisfy the property of Pythagoras theorem.

2. It is given that $\triangle DEF \sim \triangle RPQ$. Is it true to say that $\angle D = \angle R$ and $\angle F = \angle P$? Why?

Solution:

It is not true

We know that, if two triangles are similar, then their corresponding angles are equal.

$\angle D = \angle R$,

$\angle E = \angle P$ and

$\angle F = \angle Q$

3. A and B are respectively the points on the sides PQ and PR of a $\triangle PQR$ such that PQ = 12.5 cm, PA = 5 cm, BR = 6 cm and PB = 4 cm. Is AB \parallel QR? Give reason for your answer.

Solution:

It is correct.

Given,

PQ = 12.5 cm,

PA = 5 cm,

BR = 6 cm and

$$PB = 4 \text{ cm}$$

Also,

$$\frac{PB}{BR} = \frac{4}{6}$$

$$= \frac{2}{3}$$

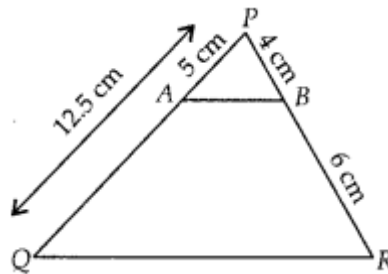
So, $QA = QP - PA$

$$= 12.5 - 5$$

$$= 7.5 \text{ cm}$$

$$\frac{PA}{AQ} = \frac{5}{7.5}$$

$$= \frac{2}{3}$$

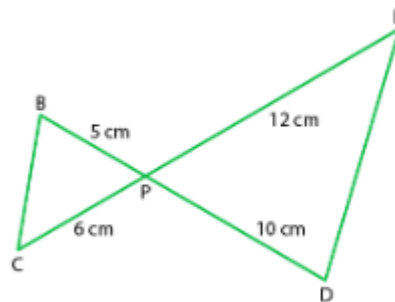


Therefore,

$$\frac{PA}{AQ} = \frac{PB}{BR}$$

So by converse of basic proportionality theorem, $AB \parallel QR$.

4. In figure, BD and CE intersect each other at the point P. Is $\triangle PBC \sim \triangle PDE$? Why?



Solution:

It is correct.

In $\triangle PBC$ and $\triangle PDE$,

$$\angle BPC = \angle EPD$$

[vertically opposite angles]

$$\frac{PB}{PD} = \frac{5}{10}$$
$$= \frac{1}{2}$$

$$\frac{PC}{PE} = \frac{6}{12}$$
$$= \frac{1}{2}$$

So,

$$\frac{PB}{PD} = \frac{PC}{PE}$$

As, one angle of $\triangle PBC$ is equal to one angle of $\triangle PDE$ and the sides including these angles are proportional, so both triangles are similar.

So, $\triangle PBC \sim \triangle PDE$, by SAS similarity criterion.

5. In $\triangle PQR$ and $\triangle MST$, $\angle P = 55^\circ$, $\angle Q = 25^\circ$, $\angle M = 100^\circ$ and $\angle S = 25^\circ$. Is $\triangle PQR \sim \triangle MST$? Why?

Solution:

It is not true.

As, the sum of three angles of a triangle is 180° .

In $\triangle PQR$,

$$\angle P + \angle Q + \angle R = 180^\circ$$

$$55^\circ + 25^\circ + \angle R = 180^\circ$$

$$\begin{aligned}\angle R &= 180^\circ - (55^\circ + 25^\circ) \\ &= 180^\circ - 80^\circ = 100^\circ\end{aligned}$$

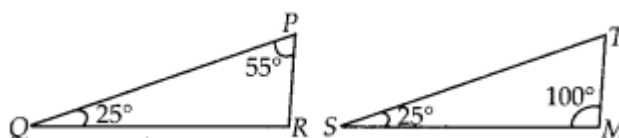
In $\triangle MST$,

$$\angle T + \angle S + \angle M = 180^\circ$$

$$\angle T + 25^\circ + 100^\circ = 180^\circ$$

$$\begin{aligned}\angle T &= 180^\circ - (25^\circ + 100^\circ) \\ &= 180^\circ - 125^\circ\end{aligned}$$

$$= 55^\circ$$



So,

In $\triangle PQR$ and $\triangle MST$,

$$\begin{aligned}\angle P &= \angle T, \\ \angle Q &= \angle S \text{ and} \\ \angle R &= \angle M \\ \angle PQR &= \angle TSM\end{aligned}$$

[As, all corresponding angles are equal]

Therefore,

$\triangle QPR$ is not similar to $\triangle TSM$, because correct correspondence is $P \leftrightarrow T$, $Q \leftrightarrow S$ and $R \leftrightarrow M$.

6. Is the following statement true? Why?

“Two quadrilaterals are similar, if their corresponding angles are equal”.

Solution:

It is not true.

Two quadrilaterals are similar if their corresponding angles are equal and corresponding sides must also be proportional.

7. Two sides and the perimeter of one triangle are respectively three times the corresponding sides and the perimeter of the other triangle. Are the two triangles similar? Why?

Solution:

Yes, It is true.

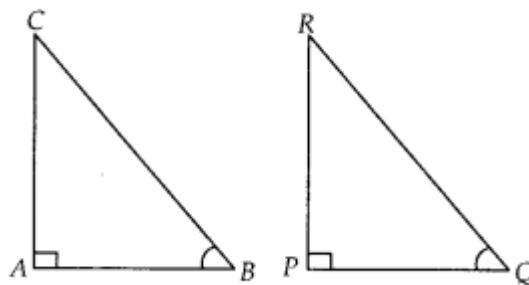
The corresponding two sides and the perimeters of two triangles are proportional, then the third side of both triangles will also in proportion.

8. If in two right triangles, one of the acute angles of one triangle is equal to an acute angle of the other triangle, can you say that the two triangles will be similar? Why?

Solution:

It is false.

Let two right angled triangles be $\triangle ABC$ and $\triangle PQR$



Where,

$$\angle A = \angle P = 90^\circ \text{ and}$$

$$\angle B = \angle Q = \text{acute angle}$$

(Given)

So, by AA similarity criterion, $\triangle ABC \sim \triangle PQR$

9. The ratio of the corresponding altitudes of two similar triangles is $\frac{3}{5}$. Is it correct to say that ratio of their areas is $\frac{6}{5}$? Why?

Solution:

It is false.

Ratio of corresponding altitudes of two triangles having areas A_1 and A_2 respectively is $\frac{3}{5}$.

Using the property of area of two similar triangles,

$$\frac{A_1}{A_2} = \left(\frac{3}{5}\right)^2$$

$$\frac{6}{5} \neq \frac{9}{25}$$

So, the given statement is not correct.

10. D is a point on side QR of $\triangle PQR$ such that $PD \perp QR$. Will it be correct to say that $\triangle PQD \sim \triangle PRD$? Why?

Solution:

No, it is false statement.

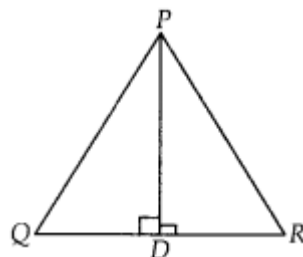
In given $\triangle PQD$ and $\triangle PRD$,

$$PD = PD$$

[common side]

$$\angle PDQ = \angle PDR$$

[each 90°]



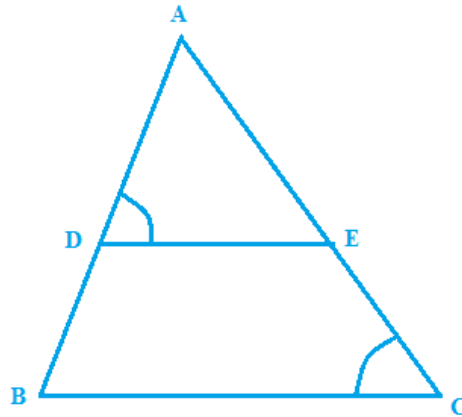
Also, no other sides or angles are equal, so we can say that $\triangle PQD$ is not similar to $\triangle PRD$.

But if $\angle P = 90^\circ$, then

$$\angle DPQ = \angle PRD$$

[each equal to $90^\circ - \angle Q$ and by ASA similarity criterion, $\triangle PQD \sim \triangle PRD$]

11. In Fig. 6.5, if $\angle D = \angle C$, then is it true that $\triangle ADE \sim \triangle ACB$? Why?



Solution:

True

In $\triangle ADE$ and $\triangle ACB$,

$$\angle A = \angle A$$

[common angle]

$$\angle D = \angle C \text{ [given]}$$

$$\triangle ADE \sim \triangle ACB$$

[using AA similarity criterion]

12. Is it true to say that if in two triangles, an angle of one triangle is equal to an angle of another triangle and two sides of one triangle are proportional to the two sides of the other triangle, then the triangles are similar? Give reasons for your answer.

Solution:

False

As, according to SAS similarity criterion, if one angle of a triangle is equal to an angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

In the above question, one angle and two sides of two triangles are equal but these sides does not includes equal angle, so given statement is not true.

Exercise No. 6.3

Short Answer Questions:

Question:

1. In a ΔPQR , $PR^2 - PQ^2 = QR^2$ and M is a point on side PR such that $QM \perp PR$.

Prove that:

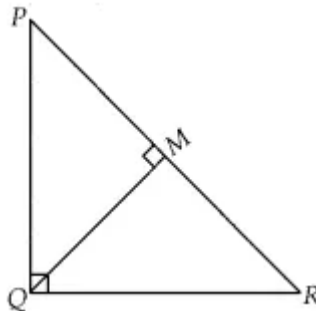
$$QM^2 = PM \times MR.$$

Solution:

In ΔPQR ,

$$PR^2 = PQ^2 + QR^2 \text{ and}$$

$$QM \perp PR$$



Using Pythagoras theorem, we have,

$$PR^2 = PQ^2 + QR^2$$

ΔPQR is right angled triangle at Q.

From ΔQMR and ΔPMQ , we get,

$$\angle M = \angle M$$

$$\angle MQR = \angle QPM$$

[each $90^\circ - \angle R$]

So, using the AAA similarity criteria,

We have,

$$\Delta QMR \sim \Delta PMQ$$

Also,

$$\text{Area of triangles} = \frac{1}{2} \times \text{base} \times \text{height}$$

So, by property of area of similar triangles,

$$\frac{\text{ar}(\Delta QMR)}{\text{ar}(\Delta PMQ)} = \frac{QM^2}{PM^2}$$

$$\frac{\text{ar}(\Delta QMR)}{\text{ar}(\Delta PMQ)} = \frac{\frac{1}{2} RM \times QM}{\frac{1}{2} PM \times QM}$$

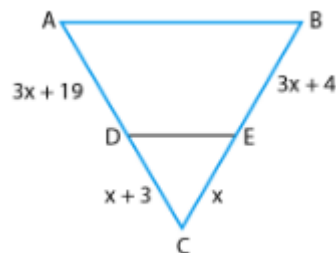
So,

$$\frac{QM^2}{PM^2} = \frac{\frac{1}{2} RM \times QM}{\frac{1}{2} PM \times QM}$$

$$QM^2 = PM \times RM$$

Hence proved.

2. Find the value of x for which $DE \parallel AB$ in given figure.



Solution:

As given in the question,

$$DE \parallel AB$$

Using basic proportionality theorem,

$$\frac{CD}{AD} = \frac{CE}{BE}$$

If a line is drawn parallel to one side of a triangle such that it intersects the other sides at distinct points, then, the other two sides are divided in the same ratio.

Therefore, we can conclude that, the line drawn is equal to the third side of the triangle.

$$\frac{x+3}{3x+19} = \frac{x}{3x+4}$$

$$(x+3)(3x+4) = x(3x+19)$$

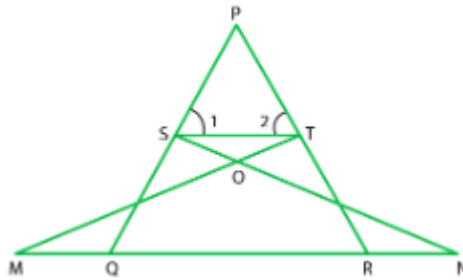
$$3x^2 + 4x + 9x + 12 = 3x^2 + 19x$$

$$19x - 13x = 12$$

$$6x = 12$$

$$x = 2$$

3. In figure, if $\angle 1 = \angle 2$ and $\triangle NSQ \cong \triangle MTR$, then prove that $\triangle PTS \sim \triangle PRQ$.



Solution:

As given in the question,

$$\triangle NSQ \cong \triangle MTR$$

$$\angle 1 = \angle 2$$

As,

$$\triangle NSQ = \triangle MTR$$

So,

$$SQ = TR \quad \dots(i)$$

Also,

$$\angle 1 = \angle 2 \text{ so,}$$

$$PT = PS \quad \dots(ii)$$

[As, sides opposite to equal angles are also equal]

Using Equation (i) and (ii).

$$\frac{PS}{SQ} = \frac{PT}{TR}$$

So, $ST \parallel QR$

By converse of basic proportionality theorem, If a line is drawn parallel to one side of a triangle to intersect the other sides in distinct points, the other two sides are divided in the same ratio.

$$\angle 1 = \angle PQR \text{ and } \angle 2 = \angle PRQ$$

Now, In $\triangle PTS$ and $\triangle PRQ$.

$$\angle P = \angle P$$

$$\angle 1 = \angle PQR$$

$$\angle 2 = \angle PRQ$$

$$\triangle PTS \sim \triangle PRQ$$

[Common angles]

(proved)

(proved)

[By AAA similarity criteria]

Hence proved.

4. Diagonals of a trapezium PQRS intersect each other at the point O, $PQ \parallel RS$ and $PQ = 3RS$. Find the ratio of the areas of $\triangle POQ$ and $\triangle ROS$.

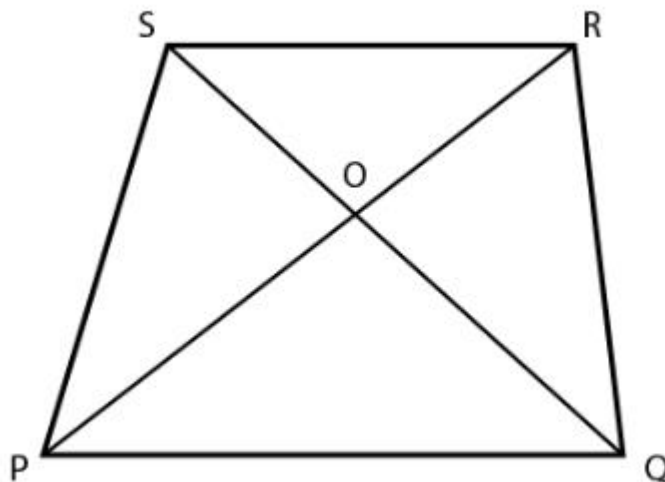
Solution:

As given in the question,

PQRS is a trapezium in which $PQ \parallel RS$ and $PQ = 3RS$

$$\frac{PQ}{RS} = \frac{3}{1}$$

...(i)



In $\triangle POQ$ and $\triangle ROS$,

$$\angle SOR = \angle QOP$$

[vertically opposite angles]

$$\angle SRP = \angle RPQ$$

[alternate angles]

$$\triangle POQ \sim \triangle ROS$$

[by AAA similarity criterion]

Using property of area of similar triangle,

$$\frac{ar(POQ)}{ar(SOR)} = \frac{PQ^2}{RS^2}$$

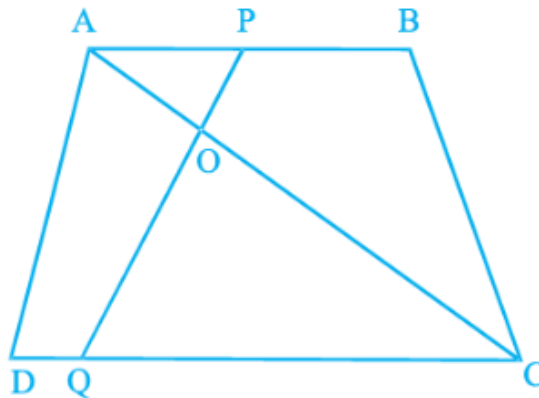
$$\frac{PQ^2}{RS^2} = \left(\frac{PQ}{RS}\right)^2$$

$$= \left(\frac{3}{1}\right)^2$$

$$= 9$$

So, the required ratio = 9:1.

5. In figure, if $AB \parallel DC$ and AC , PQ intersect each other at the point O . Prove that $OA.CQ = OC.AP$.



Solution:

As given in the question,

AC and PQ intersect each other at the point O and $AB \parallel DC$.

Using $\triangle AOP$ and $\triangle COQ$,

$$\angle AOP = \angle COQ$$

[as they are vertically opposite angles]

$$\angle APO = \angle CQO$$

[since, $AB \parallel DC$ and PQ is transversal, Angles are alternate angles]

So,

$$\triangle AOP \sim \triangle COQ$$

[using AAA similarity criterion]

As, corresponding sides are proportional

We have,

$$\frac{OA}{OC} = \frac{AP}{CQ}$$

$$OA \times CQ = OC \times AP$$

Hence Proved!!!

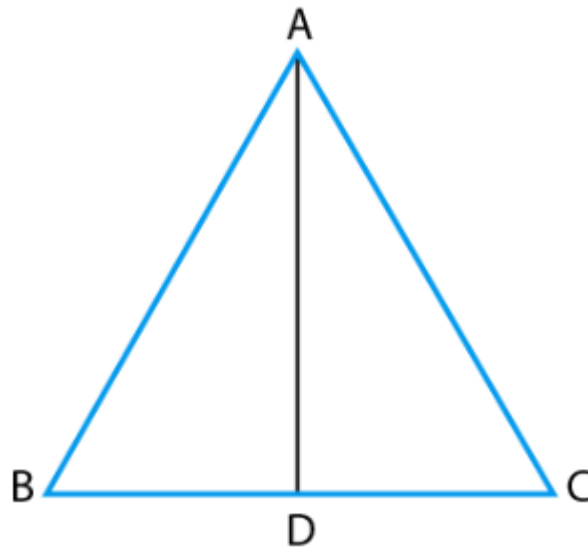
6. Find the altitude of an equilateral triangle of side 8 cm.

Solution:

Taking ABC be an equilateral triangle of side 8 cm.

$$AB = BC = CA = 8 \text{ cm}$$

(sides of an equilateral triangle is equal)



Draw altitude AD which is perpendicular to BC.

Then, D is the mid-point of BC.

$$BD = CD = \frac{1}{2}$$

$$\begin{aligned} BC &= \frac{8}{2} \\ &= 4 \text{ cm} \end{aligned}$$

Now,

Using Pythagoras theorem

$$AB^2 = AD^2 + BD^2$$

$$(8)^2 = AD^2 + (4)^2$$

$$64 = AD^2 + 16$$

$$\begin{aligned} AD &= \sqrt{48} \\ &= 4\sqrt{3} \text{ cm.} \end{aligned}$$

Therefore, altitude of an equilateral triangle is $4\sqrt{3}$ cm.

7. If $\triangle ABC \sim \triangle DEF$, $AB = 4$ cm, $DE = 6$, $EF = 9$ cm and $FD = 12$ cm, then find the perimeter of $\triangle ABC$.

Solution:

As given in the question,

$$AB = 4 \text{ cm,}$$

$$DE = 6 \text{ cm}$$

$$EF = 9 \text{ cm}$$

$$FD = 12 \text{ cm}$$

Also,

$$\triangle ABC \sim \triangle DEF$$

We have,

$$\frac{AB}{ED} = \frac{BC}{EF} = \frac{AC}{DF}$$

$$\frac{4}{6} = \frac{BC}{9} = \frac{AC}{12}$$

Now,

$$\frac{4}{6} = \frac{BC}{9}$$

$$BC = 6 \text{ cm}$$

Similarly,

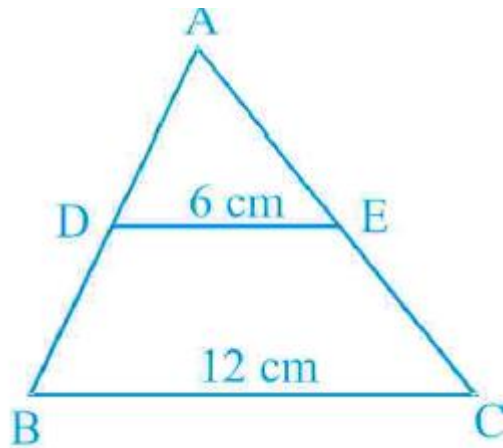
$$\frac{AC}{12} = \frac{4}{6}$$

$$AC = 8 \text{ cm}$$

$$\begin{aligned} \text{Perimeter of } \triangle ABC &= AB + BC + AC \\ &= 4 + 6 + 8 = 18 \text{ cm} \end{aligned}$$

So, the perimeter of the triangle is 18 cm.

8. In Fig. 6.11, if $DE \parallel BC$, find the ratio of $\text{ar}(\triangle ADE)$ and $\text{ar}(\triangle ECB)$.



Solution:

We have,
 $DE \parallel BC$,
 $DE = 6 \text{ cm}$ and
 $BC = 12 \text{ cm}$

In $\triangle ABC$ and $\triangle ADE$,
 $\angle ABC = \angle ADE$
 and
 $\angle A = \angle A$

[corresponding angle]

[common side]

$\triangle ABC \sim \triangle ADE$

[using AA similarity criterion]

$$\begin{aligned}\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle ABC)} &= \frac{DE^2}{BC^2} \\ &= \frac{6^2}{12^2} \\ &= \frac{1}{4}\end{aligned}$$

Taking,
 $\text{ar}(\triangle ADE) = k$, then
 $\text{ar}(\triangle ABC) = 4k$

Now,
 $\text{ar}(\triangle ECB) = \text{ar}(\triangle ABC) - \text{ar}(\triangle ADE)$
 $= 4k - k = 3k$

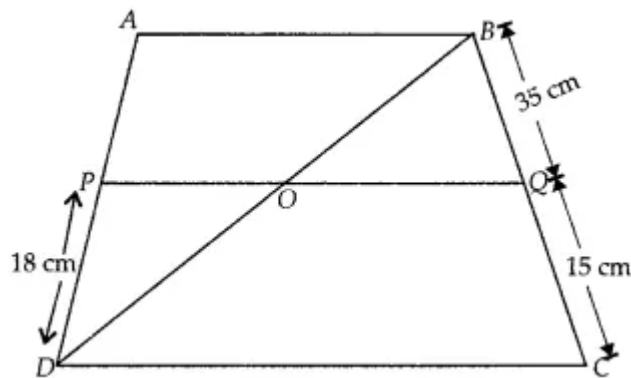
So,
 Required ratio = $\text{ar}(\triangle ADE) : \text{ar}(\triangle ECB)$
 $= k : 3k$
 $= 1 : 3$

9. ABCD is a trapezium in which $AB \parallel DC$ and P and Q are points on AD and BC, respectively such that $PQ \parallel DC$. If $PD = 18$ cm, $BQ = 35$ cm and $QC = 15$ cm, find AD.

Solution:

We have, a trapezium ABCD in which $AB \parallel DC$. P and Q are points on AD and BC, respectively such that $PQ \parallel DC$.

So, $AB \parallel PQ \parallel DC$.



In $\triangle ABD$, $PO \parallel AB$

$$\frac{DP}{AP} = \frac{DO}{OB} \quad \dots(i)$$

In $\triangle BDC$, $OQ \parallel DC$

$$\frac{BQ}{QC} = \frac{OB}{OD}$$

or,

$$\frac{QC}{BQ} = \frac{DO}{OB} \quad \dots(ii)$$

So, from (i) and (ii),

$$\frac{DP}{AP} = \frac{QC}{BQ}$$

$$\frac{18}{AP} = \frac{15}{35}$$

$$AP = 42 \text{ cm.}$$

Also;

$$\begin{aligned} AD &= AP + PD \\ &= 42 + 18 = 60 \end{aligned}$$

So,
 $AD = 60 \text{ cm}$

10. Corresponding sides of two similar triangles are in the ratio of 2:3. If the area of the smaller triangle is 48 cm^2 , find the area of the larger triangle.

Solution:

According to the question,

Ratio of corresponding sides of two similar triangles is $2 : 3$ or $\frac{2}{3}$

Area of smaller triangle $= 48 \text{ cm}^2$

Using the property of area of two similar triangles,

Ratio of area of both triangles $= (\text{Ratio of their corresponding sides})^2$

$$\frac{\text{ar(smaller triangle)}}{\text{ar(larger triangle)}} = \left(\frac{2}{3}\right)^2$$

$$\frac{48}{\text{ar(larger triangle)}} = \left(\frac{2}{3}\right)^2$$

$$\text{ar(larger triangle)} = 108 \text{ cm}^2$$

11. In a triangle PQR, N is a point on PR such that $QN \perp PR$. If $PN \cdot NR = QN^2$, prove that $\angle PQR = 90^\circ$.

Solution:

We have,

In $\triangle PQR$, N is a point on PR, such that $QN \perp PR$ and $PN \cdot NR = QN^2$

To prove: $\angle PQR = 90^\circ$

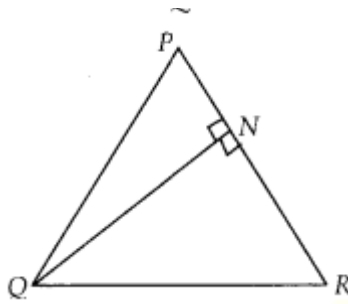
Proof:

We have, $PN \cdot NR = QN^2$

$$PN \cdot NR = QN \cdot QN$$

So,

$$\frac{PN}{QN} = \frac{QN}{NR}$$



Also,

$\angle PNQ = \angle RNQ$ [each equal to 90°]

$\triangle QNP \sim \triangle RNQ$

[by SAS similarity criterion]

So we can say, $\triangle QNP$ and $\triangle RNQ$ are equiangular.

$\angle PQN = \angle QRN$

$\angle RQN = \angle QPN$

On adding both sides,

$\angle PQN + \angle RQN = \angle QRN + \angle QPN$

$\angle PQR = \angle QRN + \angle QPN$ (ii)

We have, sum of angles of a triangle is 180°

In $\triangle PQR$,

$\angle PQR + \angle QPR + \angle QRP = 180^\circ$

$\angle PQR + \angle QPN + \angle QRN = 180^\circ$

[$\because \angle QPR = \angle QPN$ and $\angle QRP = \angle QRN$]

$\angle PQR + \angle PQR = 180^\circ$ [using Eq. (ii)]

$2\angle PQR = 180^\circ$

$\angle PQR = 90^\circ$

Hence proved.

12. Areas of two similar triangles are 36 cm^2 and 100 cm^2 . If the length of a side of the larger triangle is 20 cm, find the length of the corresponding side of the smaller triangle.

Solution:

We have,

Area of smaller triangle = 36 cm^2

Area of larger triangle = 100 cm^2

And, length of a side of the larger triangle = 20 cm

Let length of the corresponding side of the smaller triangle = x cm

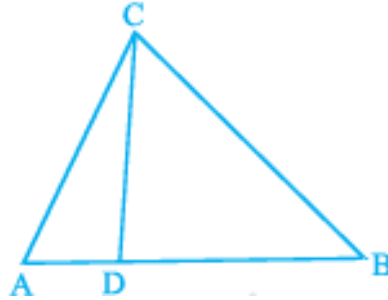
By property of area of similar triangles,

$$\frac{\text{ar(larger triangle)}}{\text{ar(smaller triangle)}} = \frac{(\text{side of larger triangle})^2}{(\text{side of smaller triangle})^2}$$

$$\frac{100}{36} = \frac{20^2}{x^2}$$

$$x = 12 \text{ cm}$$

13. In the given fig., if $\angle ACB = \angle CDA$, $AC = 8 \text{ cm}$ and $AD = 3 \text{ cm}$, find BD .



Solution:

We have,

$$AC = 8 \text{ cm},$$

$$AD = 3 \text{ cm}$$

$$\angle ACB = \angle CDA$$

In $\triangle ACD$ and $\triangle ABC$,

$$\angle A = \angle A$$

$$\angle ADC = \angle ACB$$

So,

[Common angle]

[Given]

$$\triangle ADC \sim \triangle ACB$$

[By AA similarity criterion]

$$\frac{AC}{AD} = \frac{AB}{AC}$$

$$\frac{8}{3} = \frac{AB}{8}$$

$$AB = \frac{64}{3} \text{ cm}$$

Also,

$$AB = BD + AD$$

$$\frac{64}{3} = BD + 3$$

$$BD = \frac{55}{3} \text{ cm}$$

14. A 15 meters high tower casts a shadow 24 meters long at a certain time and at the same time, a telephone pole casts a shadow 16 meters long. Find the height of the telephone pole.

Solution:

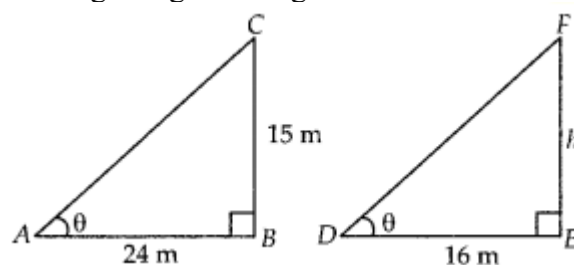
Taking $BC = 15$ m be the tower and its shadow AB is 24 m.

Let $\angle CAB = \theta$.

Again, let $EF = h$ be a telephone pole and its shadow $DE = 16$ m.

At the same time $\angle EDF = \theta$.

$\triangle ABC$ and $\triangle DEF$ both are right angled triangles.



In $\triangle ABC$ and $\triangle DEF$,

$$\angle CAB = \angle EDF$$

$$\angle B = \angle E$$

So, by AA rule,

$$\triangle ABC \sim \triangle DEF$$

$$\frac{AB}{DE} = \frac{BC}{EF}$$

$$\frac{24}{16} = \frac{15}{h}$$

$$h = 10$$

Hence, the height of the point on the wall where the top of the ladder reaches is 8 m.

15. Foot of a 10 m long ladder leaning against a vertical wall is 6 m away from the base of the wall. Find the height of the point on the wall where the top of the ladder reaches.

Solution:

Let AB be a vertical wall and $AC = 10$ m is a ladder.

The top of the ladder reached to A and distance of ladder from the base of the wall BC is 6 m.

In right angled $\triangle ABC$

$$AC^2 = AB^2 + BC^2$$

[by Pythagoras theorem]

$$(10)^2 = AB^2 + (6)^2$$

$$100 = AB^2 + 36$$

$$AB^2 = 100 - 36 = 64$$

$$AB = 8 \text{ m}$$

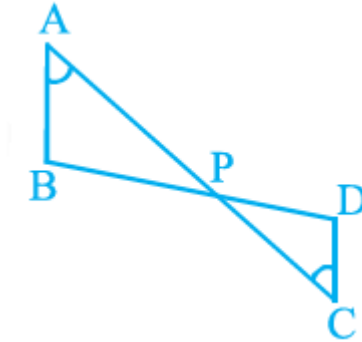
Therefore, the height of the point on the wall where the top of the ladder reaches is 8 m.

Exercise No. 6.4

Long Answer Questions:

Question:

1. In Fig., if $\angle A = \angle C$, $AB = 6$ cm, $BP = 15$ cm, $AP = 12$ cm and $CP = 4$ cm, then find the lengths of PD and CD .



Solution:

We have,
 $\angle A = \angle C$,
 $AB = 6$ cm,
 $BP = 15$ cm,
 $AP = 12$ cm and
 $CP = 4$ cm

In $\triangle APB$ and $\triangle CPD$,

$$\angle A = \angle C$$

$$\angle APB = \angle CPD$$

$$\triangle APB \sim \triangle CPD$$

[given]

[vertically opposite angles]

[by AA similarity criterion]

$$\frac{AP}{CP} = \frac{PB}{PD} = \frac{AB}{CD}$$

$$\frac{12}{4} = \frac{15}{PD} = \frac{6}{CD}$$

So,

$$\frac{12}{4} = \frac{15}{PD}$$

$$PD = 5 \text{ cm}$$

Also,

$$\frac{12}{4} = \frac{6}{CD}$$

$$CD = 2 \text{ cm}$$

Therefore, length of PD is 5 cm and length of CD is 2 cm.

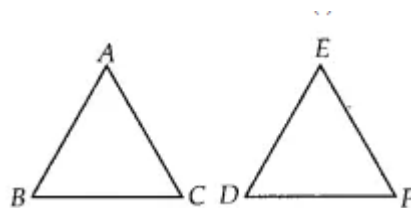
2. It is given that $\triangle ABC \sim \triangle EDF$ such that $AB = 5$ cm, $AC = 7$ cm, $DF = 15$ cm and $DE = 12$ cm. Find the lengths of the remaining sides of the triangles.

Solution:

We have,

$\triangle ABC \sim \triangle EDF$, so the corresponding sides of $\triangle ABC$ and $\triangle EDF$ are in the same ratio

$$\frac{AB}{ED} = \frac{AC}{EF} = \frac{BC}{DF} \dots\dots\dots (i)$$



Also, we have,

$AB = 5$ cm,

$AC = 7$ cm,

$DF = 15$ cm and

$DE = 12$ cm

Putting value in $\frac{AB}{ED} = \frac{AC}{EF} = \frac{BC}{DF}$,

$$\frac{5}{12} = \frac{7}{EF} = \frac{BC}{15}$$

So,

$$\frac{5}{12} = \frac{7}{EF}$$

$$EF = 16.8 \text{ cm}$$

Also,

$$\frac{5}{12} = \frac{BC}{15}$$

$$BC = 6.25 \text{ cm}$$

So, lengths of the remaining sides of the triangles are $EF = 16.8$ cm and $BC = 6.25$ cm.

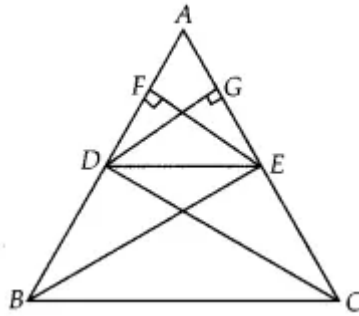
3. Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides, then the two sides are divided in the same ratio.

Solution:

Let us take $\triangle ABC$ in which a line DE parallel to BC intersects AB at D and AC at E .

To prove: DE divides the two sides in the same ratio.

$$\frac{AD}{DB} = \frac{AE}{EC}$$



Construction:

Join BE and CD

$EF \perp AB$

$DG \perp AC$

Proof:

$$\begin{aligned} \frac{ar(\triangle ADE)}{ar(\triangle BDE)} &= \frac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times DB \times EF} \\ &= \frac{AD}{DB} \quad \dots(i) \end{aligned}$$

Also,

$$\begin{aligned} \frac{ar(\triangle ADE)}{ar(\triangle DEC)} &= \frac{\frac{1}{2} \times AE \times DG}{\frac{1}{2} \times EC \times DG} \\ &= \frac{AE}{EC} \quad \dots(ii) \end{aligned}$$

As,

$\triangle BDE$ and $\triangle DEC$ lie between the same parallel lines DE and BC and on the same base DE

So,

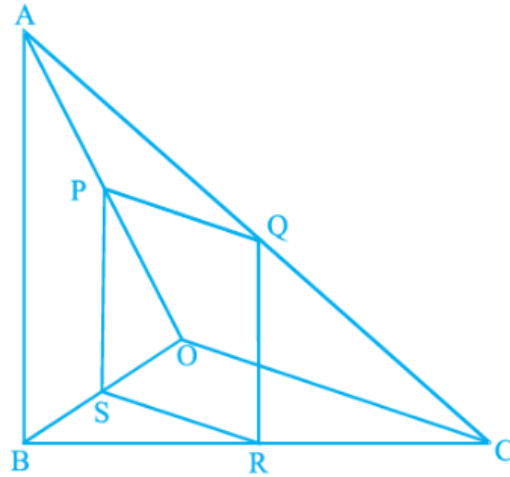
$$ar(\triangle BDE) = ar(\triangle DEC) \quad \dots (iii)$$

From Eqs. (i), (ii) and (iii),

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence proved!!!

4. In Fig., if PQRS is a parallelogram and $AB \parallel PS$, then prove that $OC \parallel SR$.



Solution:

We have,

PQRS is a parallelogram, so $PQ \parallel SR$ and $PS \parallel QR$.

Also

$AB \parallel PS$.

To prove: $OC \parallel SR$

Proof: In $\triangle OPS$ and $\triangle OAB$, $PS \parallel AB$

$$\angle POS = \angle AOB$$

[common angle]

$$\angle OSP = \angle OBA$$

[corresponding angles]

$$\triangle OPS \sim \triangle OAB$$

[by AA similarity criterion]

$$\text{Then } \frac{PS}{AB} = \frac{OS}{OB}$$

..... (i)

In $\triangle CQR$ and $\triangle CAB$, $QR \parallel PS \parallel AB$

$$\angle QCR = \angle ACB$$

[common angle]

$$\angle CRQ = \angle CBA$$

[corresponding angles]

So,

$$\triangle CQR \sim \triangle CAB$$

$$\frac{QR}{AB} = \frac{CR}{OB}$$

$$\frac{PS}{AB} = \frac{CR}{OB}$$

.....(ii)

($PS = QR$, opposite sides of parallelogram)

From (i) and (ii),

$$\frac{OS}{OB} = \frac{CR}{CB}$$

or,

$$\frac{OB}{OS} = \frac{CB}{CR}$$

Subtracting 1 from both sides, we get,

$$\frac{OB}{OS} - 1 = \frac{CB}{CR} - 1$$

$$\frac{OB - OS}{OS} = \frac{CB - CR}{CR}$$

$$\frac{BS}{OS} = \frac{BR}{CR}$$

By using converse of basic proportionality theorem, $SR \parallel OC$.

Hence proved

5. A 5 m long ladder is placed leaning towards a vertical wall such that it reaches the wall at a point 4 m high. If the foot of the ladder is moved 1.6 m towards the wall, then find the distance by which the top of the ladder would slide upwards on the wall.

Solution:

Taking AC be the ladder of length 5 m and BC = 4 m be the height of the wall, which ladder is placed.

If the foot of the ladder is moved 1.6 m towards the wall so, AD = 1.6 m, then the ladder is slide upward i.e., CE = x m.

In right angled $\triangle ABC$,

$$AC^2 = AB^2 + BC^2 \quad \text{[using Pythagoras theorem]}$$

$$(5)^2 = (AB)^2 + (4)^2$$

$$\begin{aligned} AB^2 &= 25 - 16 \\ &= 9 \end{aligned}$$

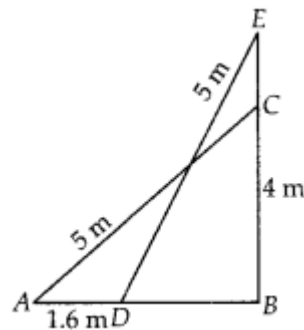
$$AB = 3\text{m}$$

Now,

$$DB = AB - AD$$

$$= 3 - 1.6$$

$$= 1.4 \text{ m}$$



In right angled $\triangle EBD$,

$$ED^2 = EB^2 + BD^2$$

[using Pythagoras theorem]

$$(5)^2 = (EB)^2 + (1.4)^2$$

[$\because BD = 1.4 \text{ m}$]

$$25 = (EB)^2 + 1.96$$

$$(EB)^2 = 25 - 1.96$$

$$= 23.04$$

$$EB = 4.8$$

Now,

$$EC = EB - BC$$

$$= 4.8 - 4$$

$$= 0.8$$

Therefore, the top of the ladder would slide upwards on the wall at distance is 0.8 m.

6. For going to a city B from city A, there is a route via city C such that $AC \perp CB$, $AC = 2x \text{ km}$ and $CB = 2(x + 7) \text{ km}$. It is proposed to construct a 26 km highway which directly connects the two cities A and B. Find how much distance will be saved in reaching city B from city A after the construction of the highway.

Solution:

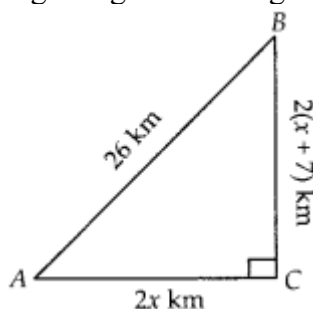
We have,

$AC \perp CB$,

$AC = 2x \text{ km}$,

$CB = 2(x + 7) \text{ km}$ and $AB = 26 \text{ km}$

On drawing the figure, we get the right angle $\triangle ACB$ right angled at C.



Now,

In $\triangle ACB$, by Pythagoras theorem,

$$AB^2 = AC^2 + BC^2$$

$$(26)^2 = (2x)^2 + \{2(x + 7)\}^2$$

$$676 = 4x^2 + 4(x^2 + 49 + 11x)$$

$$676 = 4x^2 + 4x^2 + 196 + 56x$$

$$676 = 8x^2 + 56x + 196$$

$$8x^2 + 56x - 480 = 0$$

On dividing by 8, we get,

$$x^2 + 7x - 60 = 0$$

$$x^2 + 12x - 5x - 60 = 0$$

$$x(x + 12) - 5(x + 12) = 0$$

$$(x + 12)(x - 5) = 0$$

$$x = -12,$$

$$x = 5$$

As, distance cannot be negative.

$$x = 5$$

$$[\because x \neq -12]$$

Now,

$$AC = 2x$$

$$= 10 \text{ km and}$$

$$BC = 2(x + 7)$$

$$= 2(5 + 7)$$

$$= 24 \text{ km}$$

The distance covered to reach city B from city A via city C = AC + BC

$$= 10 + 24$$

$$= 34 \text{ km}$$

Distance covered to reach city B from city A after the construction of the highway is

$$BA = 26 \text{ km}$$

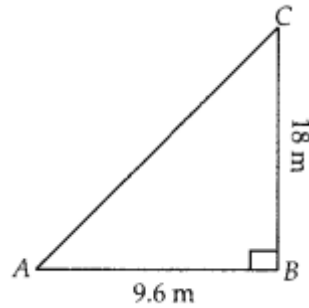
So, the required saved distance is $34 - 26 = 8 \text{ km}$.

7. A flag pole 18 m high casts a shadow 9.6 m long. Find the distance of the top of the pole from the far end of the shadow.

Solution:

Let BC = 18 m be the flag pole and its shadow be AB = 9.6 m.

The distance of the top of the pole, C from the far end which is A of the shadow is AC



In right angled $\triangle ABC$

$$AC^2 = AB^2 + BC^2$$

[using Pythagoras theorem]

$$AC^2 = (9.6)^2 + (18)^2$$

$$AC^2 = 92.16 + 324$$

$$AC^2 = 416.16$$

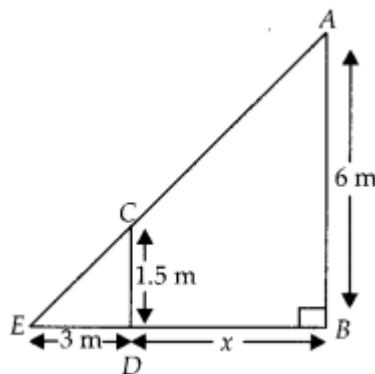
$$AC = 20.4 \text{ m}$$

So, the required distance is 20.4 m.

8. A street light bulb is fixed on a pole 6 m above the level of the street. If a woman of height 1.5 m casts a shadow of 3m, find how far she is away from the base of the pole.

Solution:

Taking A be the position of the street bulb fixed on a pole $AB = 6 \text{ m}$ and $CD = 1.5 \text{ m}$ be the height of a woman and her shadow be $ED = 3 \text{ m}$. And distance between pole and woman be $x \text{ m}$.



In this question, woman and pole both are standing vertically

So,

$$CD \parallel AB$$

In $\triangle CDE$ and $\triangle ABE$,

$$\angle E = \angle E$$

$$\angle ABE = \angle CDE$$

$$\triangle CDE \sim \triangle ABE$$

[common angle]

[each equal to 90°]

[by AA similarity criterion]

Then,

$$\frac{ED}{EB} = \frac{CD}{AB}$$
$$\frac{3}{3+x} = \frac{1.5}{6}$$

$$3 \times 6 = 1.5(3 + x)$$

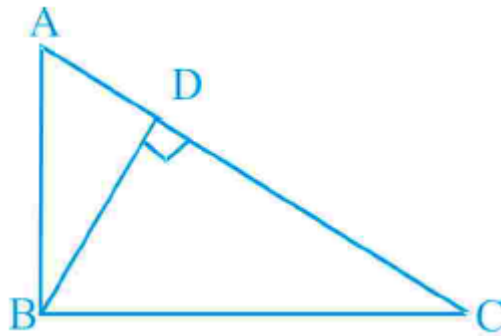
$$18 = 1.5 \times 3 + 1.5x$$

$$1.5x = 18 - 4.5$$

$$x = 9 \text{ m}$$

So, she is at the distance of 9 m from the base of the pole.

9. In Fig., ABC is a triangle right angled at B and $BD \perp AC$. If $AD = 4 \text{ cm}$, and $CD = 5 \text{ cm}$, find BD and AB .



Solution:

Given,

$\triangle ABC$ in which $\angle B = 90^\circ$ and

$BD \perp AC$

Also, $AD = 4 \text{ cm}$ and

$CD = 5 \text{ cm}$

In $\triangle DBA$ and $\triangle DCB$,

$$\angle ADB = \angle CDB$$

and

$$\angle BAD = \angle DBC$$

$$\triangle DBA \sim \triangle DCB$$

[each equal to 90°]

[each equal to $90^\circ - \angle C$];
[by AA similarity criterion]

So,

$$\frac{DB}{DA} = \frac{DC}{DB}$$

$$DB^2 = DA \times DC$$

$$= 4 \times 5$$

$$DB = 2\sqrt{5} \text{ cm}$$

In $\triangle BDC$,

$$BC^2 = BD^2 + CD^2 \text{ (Using pythagoras theorem)}$$

$$= (2\sqrt{5})^2 + 5^2$$

$$= 3\sqrt{5}$$

Also,

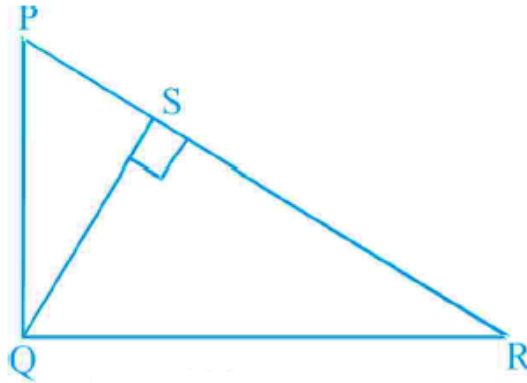
$$\triangle DBA \sim \triangle DBC$$

$$\frac{DB}{DC} = \frac{BA}{BC}$$

$$\frac{(2\sqrt{5})}{5} = \frac{BA}{(3\sqrt{5})}$$

$$AB = 6 \text{ cm}$$

10. In Fig., PQR is a right triangle right angled at Q and $QS \perp PR$. If $PQ = 6$ cm and $PS = 4$ cm, find QS, RS and QR.



Solution:

We have,

In $\triangle PQR$,

$\angle Q = 90^\circ$,

$QS \perp PR$ and

$PQ = 6$ cm,

$PS = 4$ cm

In $\triangle SQP$ and $\triangle SRQ$,

$$\angle PSQ = \angle RSQ$$

[each equal to 90°]

$$\angle SPQ = \angle SQR$$

[each equal to $90^\circ - \angle R$]

$\triangle SQP \sim \triangle SRQ$ [By AA similarity criterion]

$$\text{Then, } \frac{SQ}{PS} = \frac{SR}{SQ}$$

$$SQ^2 = PS \times SR$$

..... (i)

In right angled $\triangle PSQ$,

$$PQ^2 = PS^2 + QS^2$$

[using Pythagoras theorem]

$$(6)^2 = (4)^2 + QS^2$$

$$36 = 16 + QS^2$$

$$QS^2 = 36 - 16$$

$$= 20$$

$$QS = 2\sqrt{5} \text{ cm}$$

From eqn (i),

Putting value of PS and QS we get,

$$RS = 5 \text{ cm}$$

Now, In QSR,

$$QR^2 = QS^2 + SR^2$$

So, putting value of QS and SR we get,

$$QR = 3\sqrt{5} \text{ cm}$$

11. In $\triangle PQR$, $PD \perp QR$ such that D lies on QR. If $PQ = a$, $PR = b$, $QD = c$ and $DR = d$, prove that $(a+b)(a-b) = (c+d)(c-d)$.

Solution:

Given:

In $\triangle PQR$, $PD \perp QR$,

$$PQ = a,$$

$$PR = b,$$

$$QD = c \text{ and}$$

$$DR = d$$

To prove: $(a+b)(a-b) = (c+d)(c-d)$

Proof:

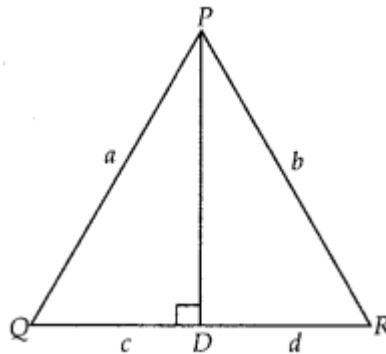
In right angled $\triangle PDQ$,

$$PQ^2 = PD^2 + QD^2$$

[using Pythagoras theorem]

$$a^2 = PD^2 + c^2$$

$$PD^2 = a^2 - c^2 \quad \dots\dots\dots (i)$$



In right angled $\triangle PDR$,

$$\begin{aligned} PR^2 &= PD^2 + DR^2 && \text{[using Pythagoras theorem]} \\ b^2 &= PD^2 + d^2 \\ PD^2 &= b^2 - d^2 && \dots\dots\dots (ii) \end{aligned}$$

$$\begin{aligned} \text{From Eqs. (i) and (ii)} \\ a^2 - c^2 &= b^2 - d^2 \\ a^2 - b^2 &= c^2 - d^2 \\ (a - b)(a + b) &= (c - d)(c + d) \end{aligned}$$

Hence proved.

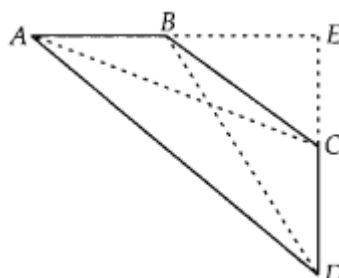
12. In a quadrilateral ABCD, $\angle A + \angle D = 90^\circ$. Prove that $AC^2 + BD^2 = AD^2 + BC^2$
[Hint: Produce AB and DC to meet at E.]

Solution:

Given:
 Quadrilateral ABCD,
 $\angle A + \angle D = 90^\circ$

To prove: $AC^2 + BD^2 = AD^2 + BC^2$

Construct: Produce AB and CD to meet at E
 Also join AC and BD



Proof:

In $\triangle AED$,

$$\angle A + \angle D = 90^\circ$$

[given]

$$\begin{aligned}\angle E &= 180^\circ - (\angle A + \angle D) \\ &= 90^\circ \quad [\text{sum of angles of a triangle} = 180^\circ]\end{aligned}$$

So, by Pythagoras theorem,

$$AD^2 = AE^2 + DE^2$$

In $\triangle BEC$, by Pythagoras theorem,

$$BC^2 = BE^2 + EC^2$$

Adding both equations, we get

$$AD^2 + BC^2 = AE^2 + DE^2 + BE^2 + EC^2 \quad \dots\dots\dots (i)$$

In $\triangle AEC$, by Pythagoras theorem,

$$AC^2 = AE^2 + CE^2$$

In $\triangle BED$, by Pythagoras theorem,

$$BD^2 = BE^2 + DE^2$$

Adding both equations, we get

$$AC^2 + BD^2 = AE^2 + CE^2 + BE^2 + DE^2 \quad \dots\dots\dots (ii)$$

From Eqs. (i) and (ii)

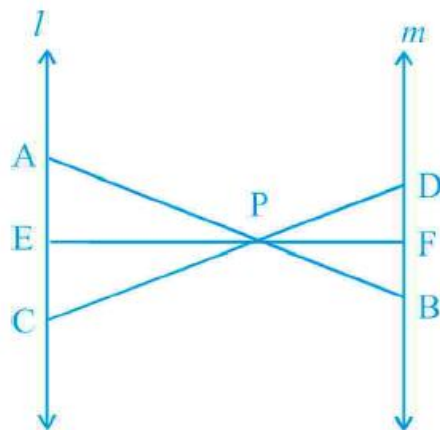
$$AC^2 + BD^2 = AD^2 + BC^2$$

Hence proved.

13. In Fig., $l \parallel m$ and line segments AB, CD and EF are concurrent at point P.

Prove that

$$\frac{AE}{BF} = \frac{AC}{BD} = \frac{CE}{FD}$$



Solution:

We have, $l \parallel m$ and line segments AB, CD and EF are concurrent at point P

To Prove,

$$\frac{AE}{BF} = \frac{AC}{BD} = \frac{CE}{FD}$$

In $\triangle APC$ and $\triangle BPD$,

$\angle APC = \angle BPD$ (vertically opposite angles)

$\angle PAC = \angle PBD$ (Alternate angles)

so,

$\triangle APC : \triangle BPD$ (By AA Similarity)

$$\frac{AP}{PB} = \frac{AC}{BD} = \frac{PC}{PD}$$

Now,

In $\triangle APE$ and $\triangle BPF$,

$\angle APE = \angle BPF$ (vertically opposite angles)

$\angle PAE = \angle PBF$ (Alternate angles)

so,

$\triangle APE : \triangle BPF$ (By AA Similarity)

$$\frac{AP}{PB} = \frac{AE}{BF} = \frac{PE}{PF}$$

Now,

In $\triangle PEC$ and $\triangle PFD$,

$\angle PEC = \angle PFD$ (vertically opposite angles)

$\angle PAC = \angle PBD$ (Alternate angles)

so,

$\triangle PEC : \triangle PFD$ (By AA Similarity)

$$\frac{PC}{PD} = \frac{PE}{PF} = \frac{EC}{FD}$$

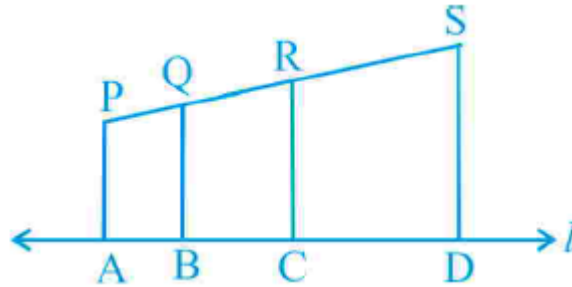
So, from above equations,

$$\frac{AP}{PB} = \frac{AC}{BD} = \frac{PE}{PF} = \frac{EC}{FD} = \frac{AE}{BF}$$

$$\frac{AE}{BF} = \frac{AC}{BD} = \frac{EC}{FD}$$

Hence proved!!

14. In Fig., PA, QB, RC and SD are all perpendiculars to a line l , $AB = 6$ cm, $BC = 9$ cm, $CD = 12$ cm and $SP = 36$ cm. Find PQ, QR and RS.



Solution:

We have,

$$AB = 6 \text{ cm,}$$

$$BC = 9 \text{ cm,}$$

$$CD = 12 \text{ cm and}$$

$$SP = 36 \text{ cm}$$

Also, PA, QB, RC and SD are all perpendiculars to line l ,

$$PA \parallel QB \parallel RC \parallel SD$$

Using Basic proportionality theorem,

$$\begin{aligned} PQ : QR : RS &= AB : BC : CD \\ &= 6 : 9 : 12 \end{aligned}$$

Taking,

$$PQ = 6x,$$

$$QR = 9x \text{ and}$$

$$RS = 12x$$

As,

$$\text{Length of } PS = 36 \text{ cm}$$

$$PQ + QR + RS = 36$$

$$6x + 9x + 12x = 36$$

$$27x = 36$$

$$x = \frac{4}{3}$$

Now,

$$PQ = 6x$$

$$= 6 \times \frac{4}{3}$$

$$= 8 \text{ cm}$$

$$QR = 9x$$

$$= 9 \times \frac{4}{3}$$

$$= 12 \text{ cm}$$

$$RS = 12x$$

$$= 12 \times \frac{4}{3}$$

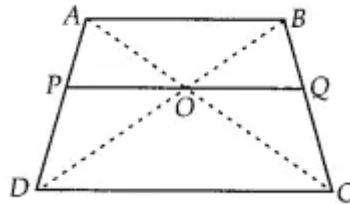
$$= 16 \text{ cm}$$

15. O is the point of intersection of the diagonals AC and BD of a trapezium ABCD with $AB \parallel DC$. Through O, a line segment PQ is drawn parallel to AB meeting AD in P and BC in Q. Prove that $PO = QO$.

Solution:

Given ABCD is a trapezium. Diagonals AC and BD are intersect at O.
 $PQ \parallel AB \parallel DC$

To prove: $PO = QO$



Proof:

In $\triangle ABD$ and $\triangle POD$,
 $PO \parallel AB$

[as, $PQ \parallel AB$]

$\angle D = \angle D$
 $\angle ABD = \angle POD$
 $\triangle ABD \sim \triangle POD$

[common angle]
 [corresponding angles]
 [by AA similarity criterion]

Then,

$$\frac{OP}{AD} = \frac{PD}{AD} \quad \dots\dots\dots (i)$$

In $\triangle ABC$ and $\triangle OQC$, $OQ \parallel AB$

$\angle C = \angle C$
 $\angle BAC = \angle QOC$
 $\therefore \triangle ABC \sim \triangle OQC$

[common angle]
 [corresponding angles]
 [by AA similarity criterion]

$$\frac{OQ}{AB} = \frac{QC}{BC}$$

Also, In $\triangle ADC$, $OP \parallel DC$

$$\frac{AP}{PD} = \frac{OA}{OC}$$

In $\triangle ABC$, $OQ \parallel AB$

$$\frac{BQ}{QC} = \frac{OA}{OC}$$

Therefore,

$$\frac{AP}{PD} = \frac{BQ}{QC}$$

Adding 1 on both sides,

$$\frac{AP}{PD} + 1 = \frac{BQ}{QC} + 1$$

$$\frac{AP + PD}{PD} = \frac{BQ + QC}{QC}$$

$$\frac{AD}{PD} = \frac{BC}{QC}$$

or,

$$\frac{PD}{AD} = \frac{QC}{BC}$$

Also,

$$\frac{OP}{AB} = \frac{QC}{BC} \text{ and } \frac{OP}{AB} = \frac{OQ}{AB}$$

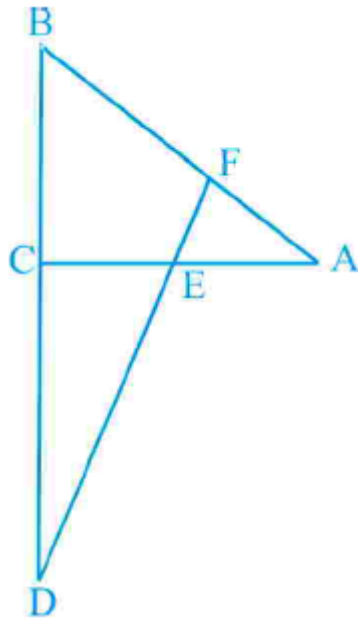
Therefore,

$$OP = OQ$$

16. In Fig., line segment DF intersect the side AC of a triangle ABC at the point E such that E is the mid-point of CA and $\angle AEF = \angle AFE$. Prove that

$$\frac{BD}{CD} = \frac{BF}{CE}$$

[Hint: Take point G on AB such that $CG \parallel DF$.]



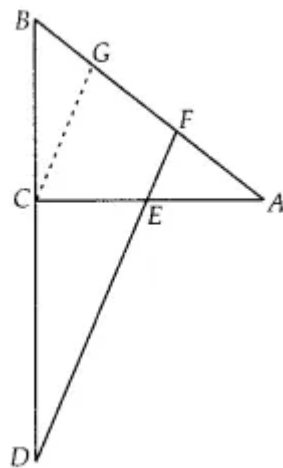
Solution:

Given $\triangle ABC$, E is the mid-point of CA and $\angle AEF = \angle AFE$

To prove: $\frac{BD}{CD} = \frac{BF}{CE}$

Construction: Take a point G on AB such that $CG \parallel DF$

Proof: As, E is the mid-point of CA



$$CE = AE \quad \dots(i)$$

In $\triangle ACG$, $CG \parallel EF$ and E is mid-point of CA

$$\text{So, } CE = GF \quad \dots (ii) \text{ [by mid-point theorem]}$$

Now, in $\triangle BCG$ and $\triangle BDF$, $CG \parallel DF$

$$\frac{BC}{CD} = \frac{BG}{GF}$$

$$\frac{BC}{CD} = \frac{BF - GF}{GF}$$

$$\frac{BC}{CD} = \frac{BF}{GF} - 1$$

$$\frac{BC}{CD} + 1 = \frac{BF}{GF} \quad (\text{from (ii)})$$

$$\frac{BC + CD}{CD} = \frac{BF}{GF}$$

$$\frac{BD}{CD} = \frac{BF}{GF}$$

17. Prove that the area of the semicircle drawn on the hypotenuse of a right angled triangle is equal to the sum of the areas of the semicircles drawn on the other two sides of the triangle.

Solution:

ABC is a right triangle, right angled at B in which

$$AB = y,$$

$$BC = x$$

We will draw three semi-circles are drawn on the sides AB, BC and AC, respectively with diameters AB, BC and AC, respectively.

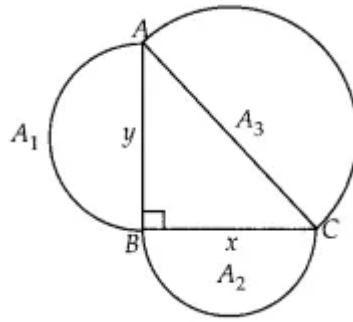
Again,

Taking area of circles with diameters AB, BC and AC are respectively A_1 , A_2 and A_3

To prove : $A_3 = A_1 + A_2$

Proof :

In $\triangle ABC$, by Pythagoras theorem,



$$AC^2 = AB^2 + BC^2$$

$$AC^2 = y^2 + x^2$$

$$AC = \sqrt{y^2 + x^2}$$

$$\begin{aligned} \text{Also, area of semicircle drawn on } AC &= \frac{\pi r^2}{2} \\ &= \frac{\pi}{2} \left(\frac{AC}{2} \right)^2 \end{aligned}$$

$$A_3 = \frac{\pi(y^2 + x^2)}{8}$$

Now,

$$\begin{aligned} \text{area of semicircle drawn on } AB &= \frac{\pi r^2}{2} \\ &= \frac{\pi}{2} \left(\frac{AB}{2} \right)^2 \end{aligned}$$

$$A_1 = \frac{\pi(y^2)}{8}$$

Now,

$$\begin{aligned} \text{area of semicircle drawn on } BC &= \frac{\pi r^2}{2} \\ &= \frac{\pi}{2} \left(\frac{BC}{2} \right)^2 \end{aligned}$$

$$A_2 = \frac{\pi(x^2)}{8}$$

From above equations, we see that,

$$A_3 = A_1 + A_2$$

Hence Proved!!!

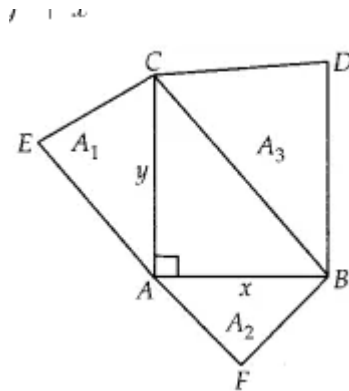
18. Prove that the area of the equilateral triangle drawn on the hypotenuse of a right angled triangle is equal to the sum of the areas of the equilateral triangles drawn on the other two sides of the triangle.

Solution:

BAC is a right triangle in which $\angle A$ is right angle and
 $AC = y$,
 $AB = x$

Now we draw three equilateral triangles on the three sides of $\triangle ABC$,
 $\triangle AEC$,
 $\triangle AFB$ and
 $\triangle CBD$

Let us assume area of triangles made on AC, AB and BC are A_1 , A_2 and A_3 respectively.
We need to prove that,
 $A_3 = A_1 + A_2$



Proof :

In $\triangle CAB$, using Pythagoras theorem,

$$BC^2 = AC^2 + AB^2$$

$$BC^2 = y^2 + x^2$$

$$BC = \sqrt{y^2 + x^2}$$

$$\text{Also Area of equilateral triangle} = \frac{\sqrt{3}}{4} a^2$$

Now we calculate the area A_1 , A_2 , and A_3 respectively

$$ar(\triangle AEC) = A_1$$

$$A_1 = \frac{\sqrt{3}}{4} AC^2$$

$$A_1 = \frac{\sqrt{3}}{4} y^2$$

Now,

$$ar(\triangle AFB) = A_2$$

$$A_2 = \frac{\sqrt{3}}{4} AB^2$$

$$A_2 = \frac{\sqrt{3}}{4} x^2$$

$$ar(\triangle CBD) = A_3$$

$$A_3 = \frac{\sqrt{3}}{4} CB^2$$

$$A_3 = \frac{\sqrt{3}}{4} (y^2 + x^2)$$

$$A_3 = \frac{\sqrt{3}}{4} x^2 + \frac{\sqrt{3}}{4} y^2$$

$$A_3 = A_1 + A_2$$

Hence Proved!!