Chapter 2 Linear Equations and Functions

Ex 2.2

Answer 1e.

We know that the vertical change is the rise and the horizontal change is the run. The ratio of rise to run is the slope.

The given statement can be completed as
"The slope of a nonvertical line is the ratio of vertical change to horizontal change."

Answer 1p.

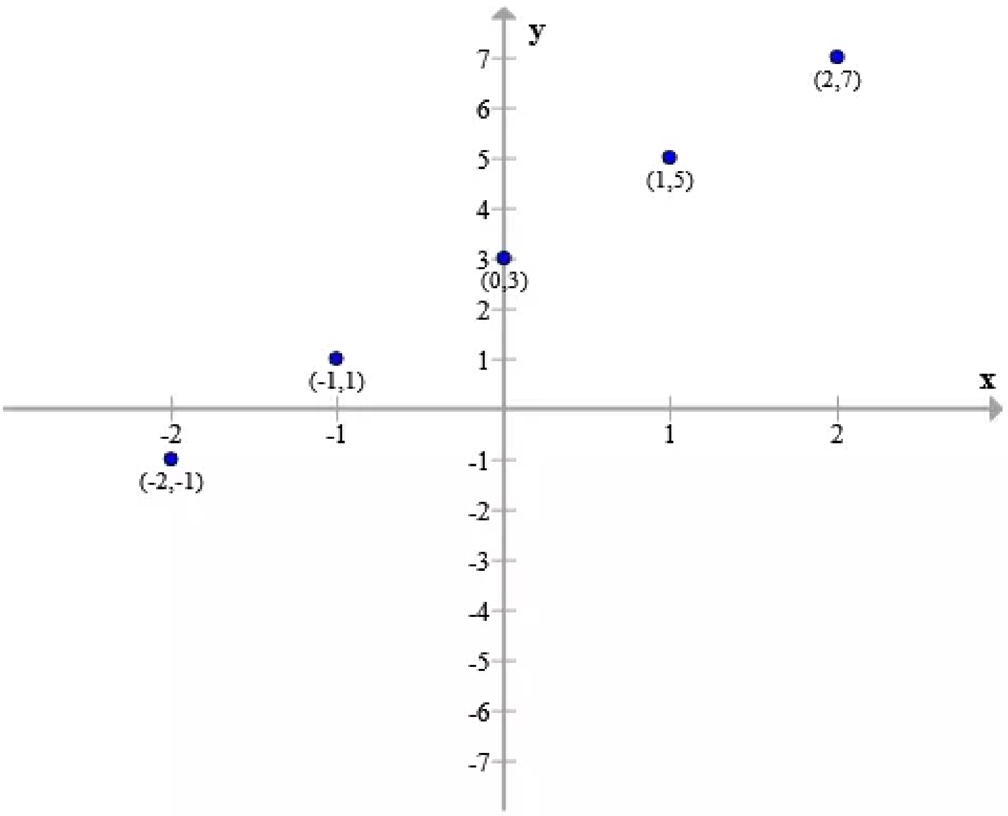
The given function is

$$y = 2x + 3;\ \text{domain } -2, -1, 0, 1, 2$$

We need to classify whether the relation is discrete or continuous and we need to identify the range.
We mark the table using the $x-$values in the domain.

x	-2	-1	0	1	2
y	-1	1	3	5	7

The graph with the corresponding x and y values is shown below:



The graph consists of separate points, so the function is $\boxed{\text{discrete}}$

The range is $\boxed{-1, 1, 3, 5, 7}$

Answer 2e.

Slope can be used to determine whether two different non-vertical lines are parallel or perpendicular.

Two non-vertical lines are parallel if and only if they have the same slope.

$$m_1 = m_2$$

Again, two non-vertical lines are perpendicular if and only if their slopes are negative reciprocals of each other.

$$m_1 = -\frac{1}{m_2};\ \text{or } m_1 m_2 = -1$$

Answer 2gp.

We need to find the slope of the line passing through the points $(-4,9)$ and $(-8,3)$. The given options for the answer are

(A) $-\frac{2}{3}$ (B) $-\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{3}{2}$

The slope *m* of a non vertical line with end points (x_1, y_1) and (x_2, y_2) is the ratio of vertical change (the *rise*) to horizontal change (the *run*). That is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

Now for the given problem let $(x_1, y_1) = (-4,9)$ and $(x_2, y_2) = (-8,3)$. Therefore

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3-9}{-8-(-4)} \quad [\text{By putting } y_1 = 9, y_2 = 3, x_1 = -4, x_2 = -8] \\ &= \frac{-6}{-8+4} \quad [\text{By simplifying}] \\ &= \frac{-6}{-4} \\ &= \frac{3}{2} \end{aligned}$$

Therefore the slope of the given points is $\frac{3}{2}$. Hence the answer is option $\boxed{(D)}$.

Answer 2p.

The given function is

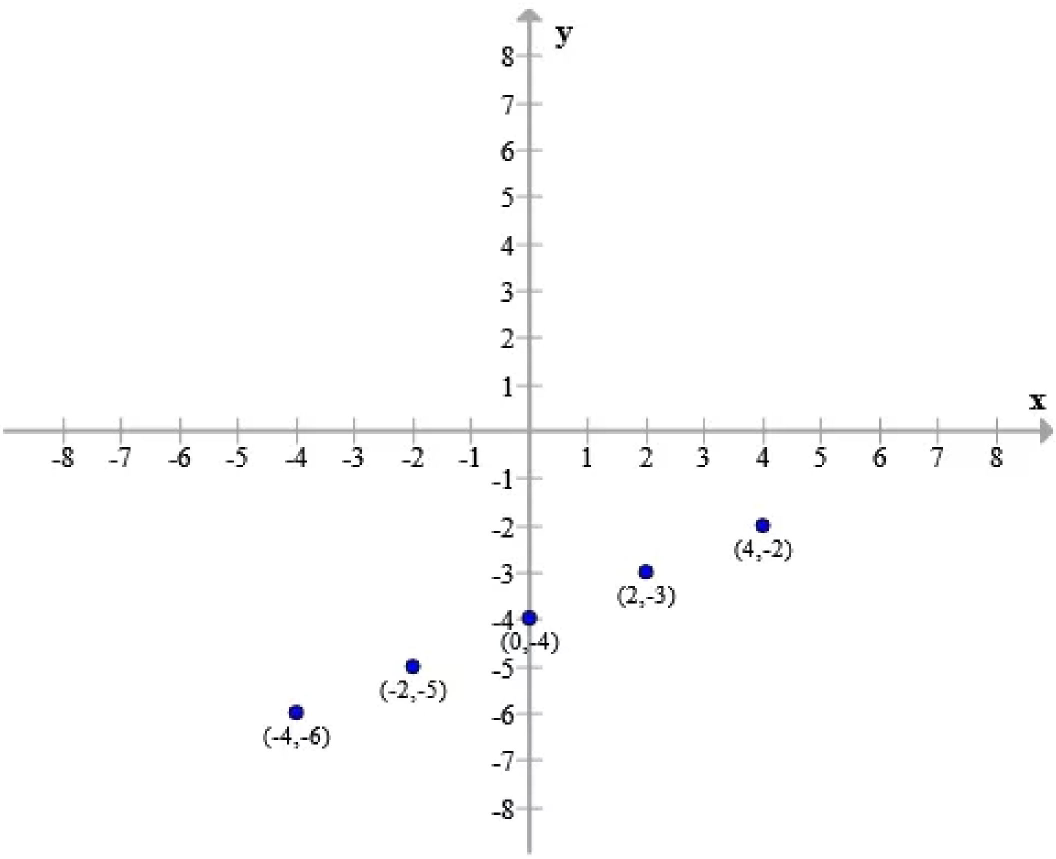
$y = 0.5x - 4$; Domain $-4, -2, 0, 2, 4$

We need to classify whether the relation is discrete or continuous and we need to identify the range.

We mark the table using the $x-$values in the domain.

x	-4	-2	0	2	4
y	-6	-5	-4	-3	-2

The graph with the corresponding x and y values is shown below:



The graph consists of separate points, so the function is $\boxed{\text{discrete}}$

The range is $\boxed{-6, -5, -4, -3, -2}$

Answer 3e.

Find the ratio of vertical change to horizontal change to get the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute (2, −4) for (x_1, y_1), and (4, −1) for (x_2, y_2) and evaluate.

$$\begin{aligned} m &= \frac{-1-(-4)}{4-2} \\ &= \frac{-1+4}{4-2} \\ &= \frac{3}{2} \end{aligned}$$

The slope of the line that passes through (2, −4) and (4, −1) is $\frac{3}{2}$.

The slope of the given line is positive.
Therefore, the line passing through the given points rises from left to right.

Answer 3p.

The given function is

$y = -3x + 9$; Domain $x < 5$

We need to classify whether the relation is discrete or continuous and we need to identify the range.

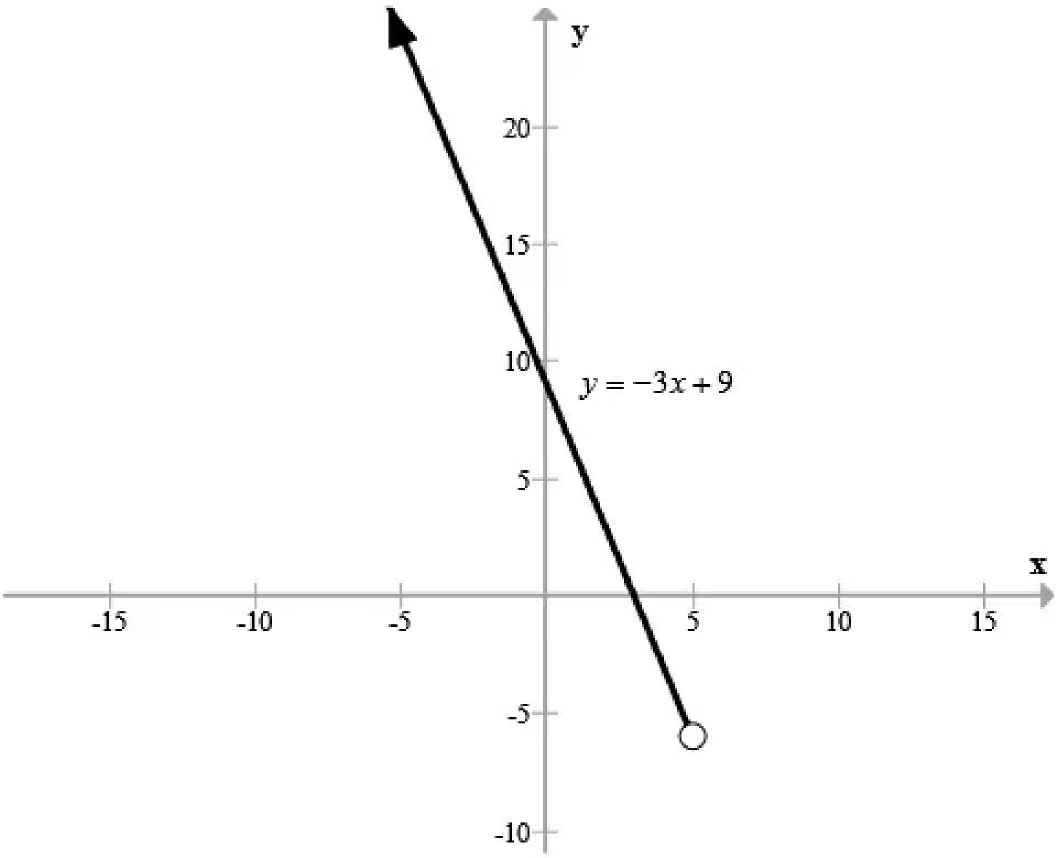
The function y is a linear function defined for $x < 5$.

Substituting $x = 5$ in the equation $y = -3x + 9$

$$\begin{aligned} y(5) &= -3 \times 5 + 9 \\ &= -15 + 9 \\ &= -6 \end{aligned}$$

So the graph is the ray with the end point $(5, -6)$ exclusive.

The graph with the corresponding x and y values is shown below:



The graph is unbroken, so the function is continuous and the range is $\boxed{y > 6}$

Answer 4e.

We need to find the slope of the line passing through the given points. Then tell whether the line rises, falls, is horizontal, or is vertical.

$$(8,9),(-4,3)$$

The slope m of a non-vertical line is the ratio of vertical change to horizontal change. Such that

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{...... (1)}$$

Here $(x_1, y_1) = (8,9)$ and $(x_2, y_2) = (-4,3)$

Therefore using equation (1), we get the slope of the line passing through the given points is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{3-9}{-4-8}$$
$$= \frac{1}{2}$$
$$\boxed{m = \frac{1}{2}}$$

Since $m > 0$, the line rises from left to right.

Answer 4gp.

We need to find the slope of the line passing through the points $(-5,1)$ and $(5,-4)$.

The slope *m* of a non vertical line with end points (x_1, y_1) and (x_2, y_2) is the ratio of vertical change (the *rise*) to horizontal change (the *run*). That is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

Now for the given problem let $(x_1, y_1) = (-5,1)$ and $(x_2, y_2) = (5,-4)$. Therefore

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-4-1}{5-(-5)} \quad [\text{By putting } y_1 = 1, y_2 = -4, x_1 = -5, x_2 = 5]$$
$$= \frac{-5}{5+5} \quad [\text{By simplifying}]$$
$$= \frac{-5}{10}$$
$$= -\frac{1}{2}$$

Therefore the slope of the given points is $\boxed{-\frac{1}{2}}$.

Answer 4p.

The given function is

$$y = \frac{1}{3}x + 6 \text{; Domain } x \geq -6$$

We need to classify whether the relation is discrete or continuous and we need to identify the range.

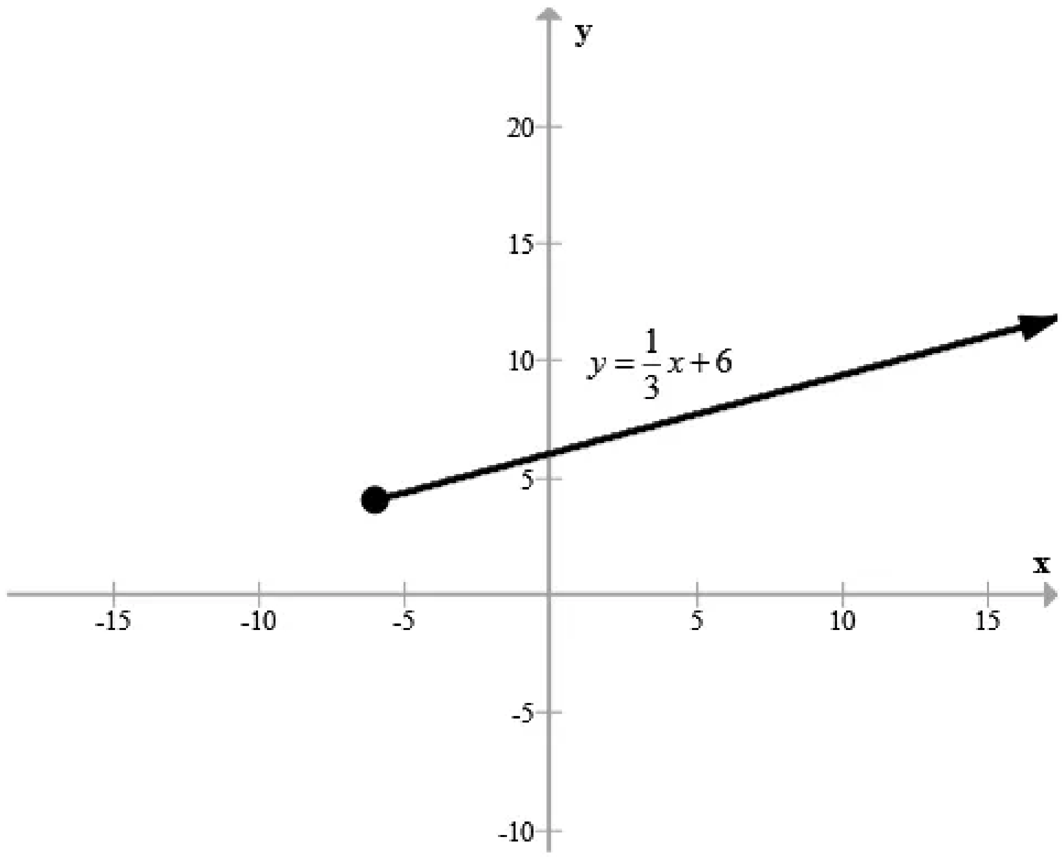
The function y is a linear function defined for $x \geq -6$.

Substituting $x = -6$ in the equation $y = \frac{1}{3}x + 6$

$$y(-6) = \frac{1}{3} \times (-6) + 6$$
$$= -2 + 6$$
$$= 4$$

So the graph is the ray with the end point $(-6, 4)$ inclusive.

The graph is shown below:



The graph is unbroken, so the function is continuous and the range is $\boxed{y \geq 4}$

Answer 5e.

Find the ratio of vertical change to horizontal change to get the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute (5, 1) for (x_1, y_1), and (8, –4) for (x_2, y_2) and evaluate.

$$m = \frac{-4 - 1}{8 - 5}$$

$$= \frac{-5}{3}$$

$$= -\frac{5}{3}$$

The slope of the line that passes through (5, 1) and (8, –4) is $-\frac{5}{3}$.

The slope of the given line is negative.
Therefore, the line passing through the given points falls from left to right.

Answer 5p.

It is given that Amanda walks at an average speed of 3.5miles per hour .We need to find the distance $d(x)$ in miles in x hours.

Given, $v = 3.5$ miles per hour, time $= x$ hours

Using, distance = velocity × time

Therefore the required distance is

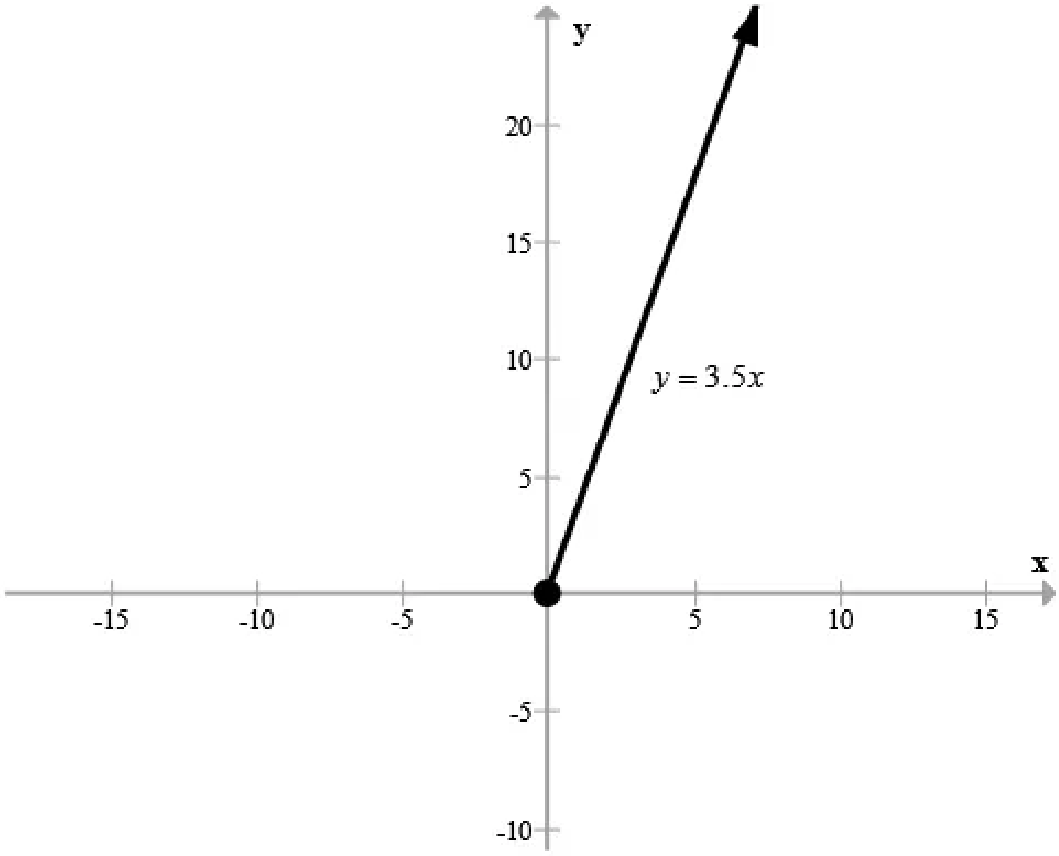
$$\boxed{d(x) = 3.5x}, x \geq 0$$

Substituting $x = 0$ in the equation $y = 3.5x$

$$y(0) = 3.5 \times 0$$
$$= 0$$

So the graph is the ray with the end point $(0,0)$ inclusive.

The graph is shown below:



The graph is unbroken, so the function is $\boxed{\text{continuous}}$ and the range is $\boxed{y \geq 0}$

Answer 6e.

We need to find the slope of the line passing through the given points. Then tell whether the line rises, falls, is horizontal, or is vertical.

$$(-3,-2),(3,-2)$$

The slope m of a non-vertical line is the ratio of vertical change to horizontal change. Such that

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{...... (1)}$$

Here $(x_1, y_1) = (-3, -2)$ and $(x_2, y_2) = (3, -2)$

Therefore using equation (1), we get the slope of the line passing through the given points is

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2 - (-2)}{3 - (-3)} \\ &= 0 \end{aligned}$$

$$\boxed{m = 0}$$

Here, y co-ordinate is same.

Since $m = 0$, the line is horizontal.

Answer 6gp.

We need to find the slope of the line passing through the points $(7,3)$ and $(-1,7)$.

The slope *m* of a non vertical line with end points (x_1, y_1) and (x_2, y_2) is the ratio of vertical change (the *rise*) to horizontal change (the *run*). That is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

Now for the given problem let $(x_1, y_1) = (7,3)$ and $(x_2, y_2) = (-1,7)$. Therefore

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{7-3}{-1-7} \quad [\text{By putting } y_1 = 3, y_2 = 7, x_1 = 7, x_2 = -1] \\ &= \frac{4}{-8} \quad [\text{By simplifying}] \\ &= -\frac{1}{2} \end{aligned}$$

Therefore the slope of the given points is $\boxed{-\frac{1}{2}}$.

Answer 6p.

It is given that a token to ride a subway costs \$1.25.We need to find the function $s(x)$ which gives the cost of riding the subway x times.

Given, cost, $c = \$1.25$, times $= x$

Therefore the function is

$$\boxed{s(x)=1.25x}$$

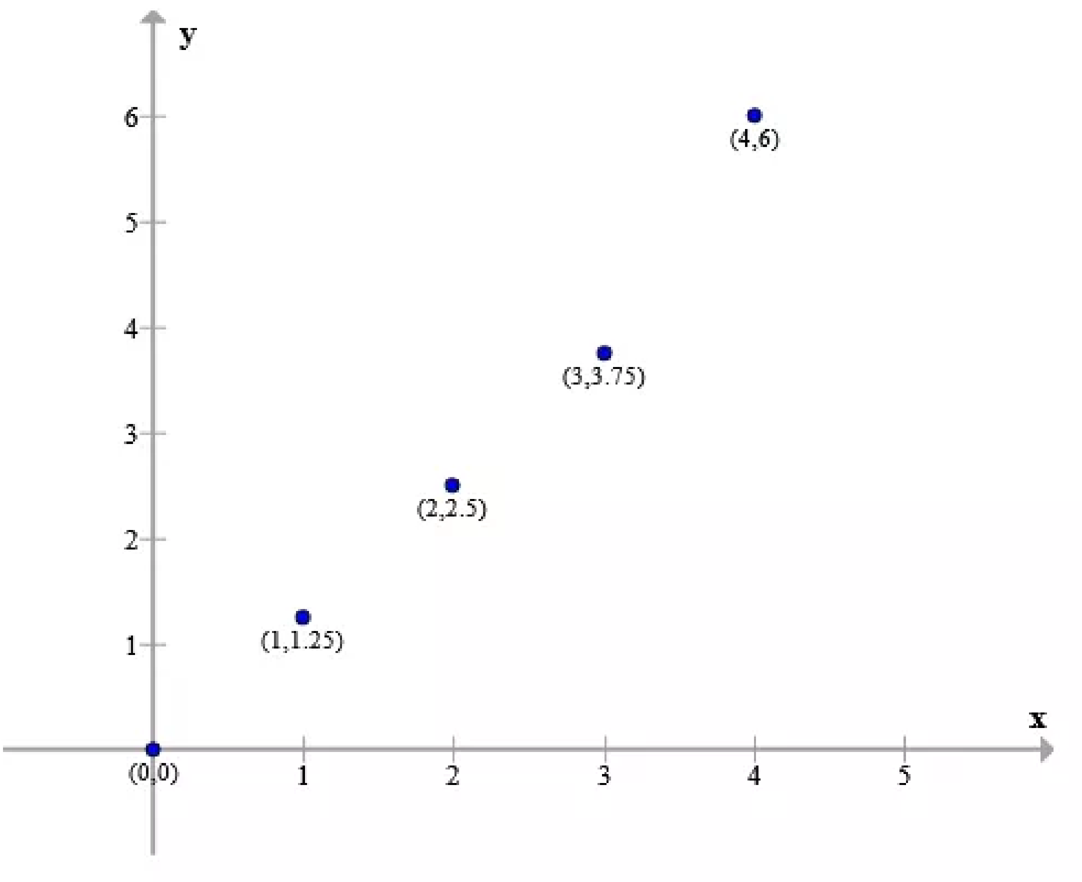
The first 5 points of the function $s(x)=1.25x$ is shown in the table.

x	0	1	2	3	4
y	0	1.25	2.5	3.75	6

The domain is the set of whole numbers $\boxed{0, 1, 2, 3, 4\ldots}$

The range is $\boxed{0,1.25,2.5,3.75,6\ldots}$

The graph is shown below:



The graph consists of separate points, so the function is $\boxed{\text{discrete.}}$

Answer 7e.

Find the ratio of vertical change to horizontal change to get the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute (−1, 4) for (x_1, y_1), and (1, −4) for (x_2, y_2) and evaluate.

$$\begin{aligned} m &= \frac{-4-4}{1-(-1)} \\ &= \frac{-4-4}{1+1} \\ &= \frac{-8}{2} \\ &= -4 \end{aligned}$$

The slope of the line that passes through (−1, 4) and (1, −4) is −4.

The slope of the given line is negative.
Therefore, the line passing through the given points falls from left to right.

Answer 7p.

It is given that a family has 3 gallons of milk delivered every Thursday. We need to find the function $m(x)$ which gives the total amount of milk that is delivered to the family after x weeks.
Given, gallons, $g = 3$, weeks $= x$
Therefore the function is

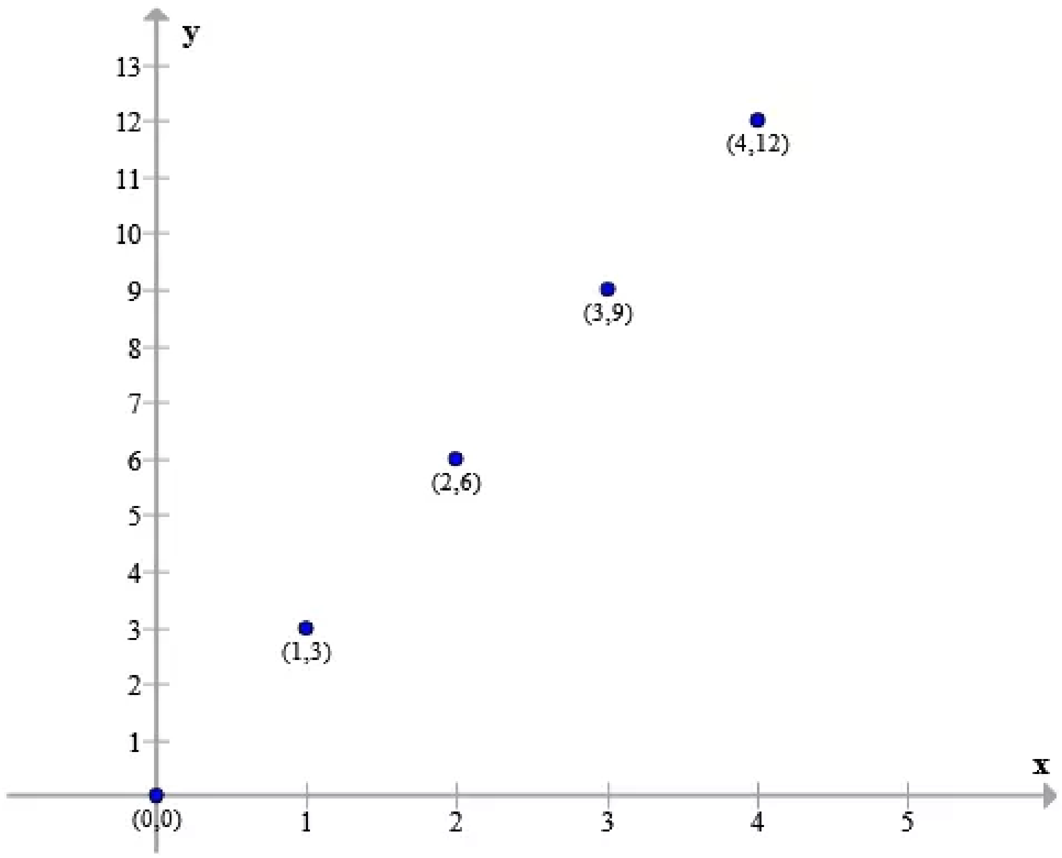
$$\boxed{m(x) = 3x}$$

The first 5 points of the function $m(x) = 3x$ is shown in the table.

x	0	1	2	3	4
y	0	3	6	9	12

The domain is the set of whole numbers $\boxed{0, 1, 2, 3, 4...}$
The range is the multiple of 3 $\boxed{0,3,6,9,12...}$

The graph is shown below:



The graph consists of separate points, so the function is **discrete.**

Answer 8e.

We need to find the slope of the line passing through the given points. Then tell whether the line rises, falls, is horizontal, or is vertical.

$$(-6,5),(-6,-5)$$

The slope m of a non-vertical line is the ratio of vertical change to horizontal change. Such that

$$m=\frac{y_2-y_1}{x_2-x_1} \qquad \ldots\ldots (1)$$

Here $(x_1, y_1) = (-6, 5)$ and $(x_2, y_2) = (-6, -5)$

Therefore using equation (1), we get the slope of the line passing through the given points is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-5 - 5}{-6 - (-6)}$$
$$= \frac{-10}{0}$$
$$\boxed{m = \frac{-10}{0}}$$

Here, x co-ordinate is same.

Since m is undefined, the line is vertical.

Answer 8gp.

We need to find whether the line through the points $(7,1)$ and $(7,-1)$ rises, falls, is horizontal, or is vertical.

The slope of a line indicates whether the line rises from left to right, falls from left to right, is horizontal, or is vertical. Thus

(i) If *m* is positive then the line rises from left to right.

(ii) If *m* is negative then the line falls from left to right.

(iii) If *m* is zero then the line is horizontal.

(iv) If *m* is undefined then the line is vertical.

The slope *m* of a non vertical line with end points (x_1, y_1) and (x_2, y_2) is the ratio of vertical change (the *rise*) to horizontal change (the *run*). That is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

Now for the given problem let $(x_1, y_1) = (7,1)$ and $(x_2, y_2) = (7,-1)$. Therefore

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-1 - 1}{7 - 7} \quad [\text{By putting } y_1 = 1, y_2 = -1, x_1 = 7, x_2 = 7]$$
$$= \frac{-2}{0} \quad [\text{By simplifying}]$$

Thus *m* is undefined.

Since m is undefined, the line is $\boxed{\text{vertical}}$.

Answer 8p.

It is given that steel cable that is $\frac{3}{8}$ inch in diameter weighs 0.24 pound per foot. We need to find the function $w(x)$ that gives the weight of x feet of steel cable.

Given, diameter, $d = \frac{3}{8}$ inch, feet $= x$

Therefore the required distance is

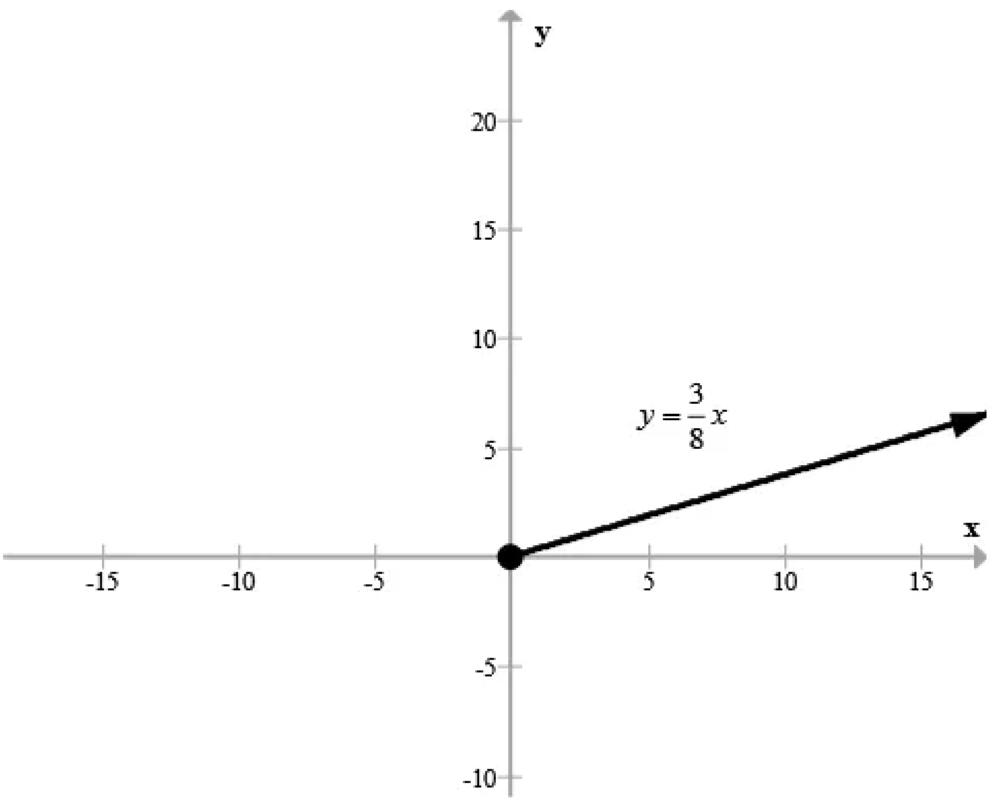
$$\boxed{w(x) = \frac{3}{8}x}, x \geq 0$$

Substituting $x = 0$ in the equation $w(x) = \frac{3}{8}x$

$$\begin{aligned} w(0) &= \frac{3}{8} \times 0 \\ &= 0 \end{aligned}$$

So the graph is the ray with the end point $(0,0)$ inclusive.

The graph is shown below:



The graph is unbroken, so the function is $\boxed{\text{continuous}}$ and the range is $\boxed{y \geq 0}$

Answer 9e.

Find the ratio of vertical change to horizontal change to get the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute (−5, −4) for (x_1, y_1), and (−1, 3) for (x_2, y_2) and evaluate.

$$m = \frac{3-(-4)}{-1-(-5)}$$

$$= \frac{3+4}{-1+5}$$

$$= \frac{7}{4}$$

The slope of the line that passes through (−5, −4) and (−1, 3) is $\frac{7}{4}$.

The slope of the given line is positive.
Therefore, the line passing through the given points rises from left to right.

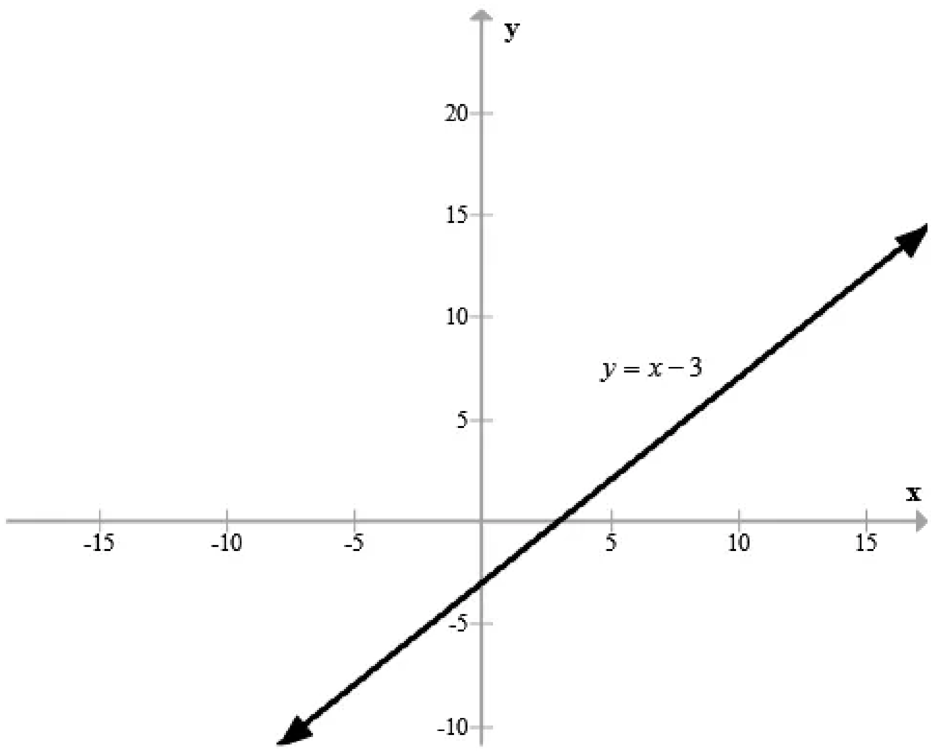
Answer 9p.

It is given that on a number line, the signed distance from a number a to a number b is given by $b-a$. We need to find the function $d(x)$ that gives the signed distance from 3 to any number x

Therefore the function $d(x)$ that gives the signed distance from 3 to any number x

$$\boxed{d(x) = x - 3}$$

The graph is shown below:



The graph is unbroken, so the function is continuous.
The domain and range is the set of all real numbers

Answer 10e.

We need to find the slope of the line passing through the given points. Then tell whether the line rises, falls, is horizontal, or is vertical.

$$(-3,6),(-7,3)$$

The slope m of a non-vertical line is the ratio of vertical change to horizontal change. Such that

$$m=\frac{y_2-y_1}{x_2-x_1} \qquad \text{...... (1)}$$

Here $(x_1,y_1)=(-3,6)$ and $(x_2,y_2)=(-7,3)$

Therefore using equation (1), we get the slope of the line passing through the given points is

$$\begin{aligned} m &= \frac{y_2-y_1}{x_2-x_1} \\ &= \frac{3-6}{-7-(-3)} \\ &= \frac{3}{4} \end{aligned}$$

$$\boxed{m=\frac{3}{4}}$$

Since $m>0$, the line rises from left to right.

Answer 10gp.

We need to find whether the line through the points $(5,6)$ and $(1,-4)$ rises, falls, is horizontal, or is vertical.

The slope of a line indicates whether the line rises from left to right, falls from left to right, is horizontal, or is vertical. Thus

(i) If m is positive then the line rises from left to right.

(ii) If m is negative then the line falls from left to right.

(iii) If m is zero then the line is horizontal.

(iv) If m is undefined then the line is vertical.

The slope *m* of a non vertical line with end points (x_1, y_1) and (x_2, y_2) is the ratio of vertical change (the *rise*) to horizontal change (the *run*). That is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

Now for the given problem let $(x_1, y_1) = (5, 6)$ and $(x_2, y_2) = (1, -4)$. Therefore

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-4-6}{1-5} \quad [\text{By putting } y_1 = 6, y_2 = -4, x_1 = 5, x_2 = 1] \\ &= \frac{-10}{-4} \quad [\text{By simplifying}] \\ &= \frac{5}{2} \\ &= 2.5 \end{aligned}$$

Thus *m* is positive.

Since *m* is positive, the line $\boxed{\text{rises}}$ from left to right.

Answer 11e.

Find the ratio of vertical change to horizontal change to get the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute (4, 4) for (x_1, y_1), and (4, 9) for (x_2, y_2) and evaluate.

$$\begin{aligned} m &= \frac{9-4}{4-4} \\ &= \frac{5}{0} \end{aligned}$$

Since division by 0 is undefined, the slope of the line that passes through (4, 4) and (4, 9) is undefined.

The slope of the given line is undefined.
Therefore, the line passing through the given points is vertical.

Answer 12e.

We need to find the slope of the line passing through the given points. Then tell whether the line rises, falls, is horizontal, or is vertical.

$(5,5),(7,3)$

The slope m of a non-vertical line is the ratio of vertical change to horizontal change. Such that

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{......} (1)$$

Here $(x_1, y_1) = (5,5)$ and $(x_2, y_2) = (7,3)$

Therefore using equation (1), we get the slope of the line passing through the given points is

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3-5}{7-5} \\ &= \frac{-2}{2} \\ &= -1 \end{aligned}$$

$$\boxed{m = -1}$$

Since $m < 0$, the line falls from left to right.

Answer 12gp.

We need to tell whether the given lines are parallel, perpendicular, or neither.

Line 1: through $(-4,-2)$ and $(1,7)$

Line 2: through $(-1,-4)$ and $(3,5)$

Two lines with slopes m_1 and m_2, are parallel if and only if they have the same slope. That is

$$m_1 = m_2$$

Two lines with slopes m_1 and m_2, are perpendicular if and only if their slopes are negative reciprocals of each other. That is

$$m_1 m_2 = -1$$

The slope *m* of a non vertical line with end points (x_1, y_1) and (x_2, y_2) is the ratio of vertical change (the *rise*) to horizontal change (the *run*). That is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

Suppose that m_1 and m_2 are the slopes of line 1 and line 2 respectively. Now we find the slopes of these two lines. Therefore

$$m_1 = \frac{7-(-2)}{1-(-4)}$$
$$= \frac{9}{5}$$
$$m_2 = \frac{5-(-4)}{3-(-1)}$$
$$= \frac{9}{4}$$

Since there is no relation of the two slopes therefore the given lines are $\boxed{\text{neither parallel nor perpendicular}}$.

Answer 13e.

Find the ratio of vertical change to horizontal change to get the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute $(0, -3)$ for (x_1, y_1), and $(4, -3)$ for (x_2, y_2) and evaluate.

$$m = \frac{-3-(-3)}{4-0}$$
$$= \frac{-3+3}{4-0}$$
$$= \frac{0}{4}$$
$$= 0$$

The slope of the given line is 0.
Therefore, the line passing through the given points is horizontal.

Answer 14e.

We need to find the slope of the line passing through the given points. Then tell whether the line rises, falls, is horizontal, or is vertical.

$$(1,-1),(-1,-4)$$

The slope m of a non-vertical line is the ratio of vertical change to horizontal change. Such that

$$m=\frac{y_2-y_1}{x_2-x_1} \qquad \text{...... (1)}$$

Here $(x_1,y_1)=(1,-1)$ and $(x_2,y_2)=(-1,-4)$

Therefore using equation (1), we get the slope of the line passing through the given points is

$$\begin{aligned} m &= \frac{y_2-y_1}{x_2-x_1} \\ &= \frac{-4-(-1)}{-1-1} \\ &= \frac{3}{2} \end{aligned}$$

$$\boxed{m=\frac{3}{2}}$$

Since $m>0$, the line rises from left to right.

Answer 15e.

Though (x_1, y_1) is taken as $(-4, -3)$ and (x_2, y_2) as $(2, -1)$, y_1 is subtracted from y_2, and x_2 is subtracted from y_1.

Therefore, the error is that the x-and y-coordinates were not subtracted in the correct order.

Substitute $(-4, -3)$ for (x_1, y_1), and $(2, -1)$ for (x_2, y_2) in $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ and evaluate.

$$\begin{aligned} m &= \frac{-1-(-3)}{2-(-4)} \\ &= \frac{-1+3}{2+4} \\ &= \frac{2}{6} \\ &= \frac{1}{3} \end{aligned}$$

Therefore, the slope of the line that passes through the given points is $\dfrac{1}{3}$.

Answer 16e.

We need to describe and correct the error in finding the slope of the line passing through the given points.

$$(-1,4),(5,1)$$

$$m = \frac{5-(-1)}{1-4}$$
$$= -2$$

The slope m of a non-vertical line is the ratio of vertical change to horizontal change. Such that

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{......} (1)$$

Student calculated the slope as

$$m = \frac{5-(-1)}{1-4} = -2$$

He used the formula

$$m = \frac{x_2 - x_1}{y_2 - y_1}$$

But it is not the correct formula.

Now, using equation (1), we get the slope of the line passing through the given points is

$$(x_1, y_1) = (-1,4) \text{ and } (x_2, y_2) = (5,1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{1-4}{5+1}$$
$$= \frac{-3}{6}$$
$$= -\frac{1}{2}$$

$$\boxed{m = -\frac{1}{2}}$$

Answer 17e.

Find the slope of the line that passes through (2, –4) and (5, 1).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{1-(-4)}{5-2}$$
$$= \frac{1+4}{5-2}$$
$$= \frac{5}{3}$$

Since the slope of the line is positive, the line rises from left to right.

Therefore, the correct answer is choice **A**.

Answer 18e.

We need to tell whether the lines are parallel, perpendicular, or neither.

Line 1: through $(3,-1)$ and $(6,-4)$

Line 2: through $(-4,5)$ and $(-2,7)$

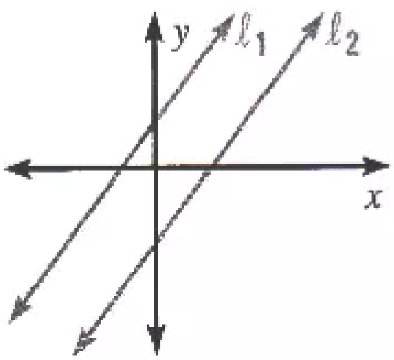
The slope m of a non-vertical line is the ratio of vertical change to horizontal change. Such that

$$m=\frac{y_2-y_1}{x_2-x_1} \qquad \text{...... (1)}$$

Consider two different non-vertical lines l_1 and l_2 with slopes m_1 and m_2.
The lines are parallel if and only if they have the same slope. Such that

$$m_1=m_2$$

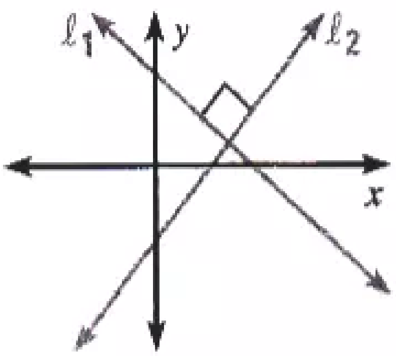
The graph is:



Again, the lines are perpendicular if and only if their slopes are negative reciprocals of each other. Such that

$$m_1=-\frac{1}{m_2} \text{ or } m_1m_2=-1$$

The graph is:



Here, for the line 1:

$$(x_1, y_1) = (3, -1) \text{ and } (x_2, y_2) = (6, -4)$$

Therefore using equation (1), we get the slope of the line passing through the above points is

$$\begin{aligned} m_1 &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-4+1}{6-3} \\ &= \frac{-3}{3} \\ &= -1 \end{aligned}$$

Again, for the line 2:

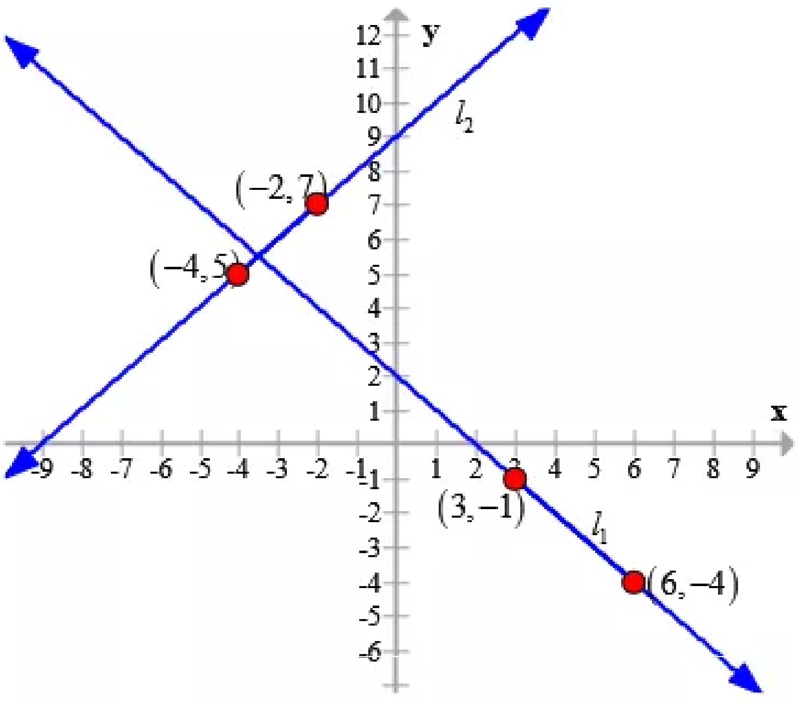
$$(x_1, y_1) = (-4, 5) \text{ and } (x_2, y_2) = (-2, 7)$$

Therefore using equation (1), we get the slope of the line passing through the above points is

$$\begin{aligned} m_2 &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{7-5}{-2-(-4)} \\ &= \frac{2}{2} \\ &= 1 \end{aligned}$$

Since $m_1 m_2 = -1$, m_1 and m_2 are negative reciprocals of each other. So, the lines are perpendicular.

The graph of the lines l_1 and l_2 is shown below:



Answer 19e.

Find the slope of line 1 that passes through (1, 5) and (3, −2).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-2 - 5}{3 - 1}$$
$$= -\frac{7}{2}$$

Find the slope of line 2 that passes through (−3, 2) and (4, 0).

$$m = \frac{0 - 2}{4 - (-3)}$$
$$= -\frac{2}{7}$$

We know the lines are perpendicular if and only if their slopes are negative reciprocals of each other.

$$-\frac{7}{2}\left(-\frac{2}{7}\right) = 1$$

Since the product of the slopes is not −1, the lines are not perpendicular.

The lines are parallel if and only if they have the same slope.
Since the given lines have different slope, they are not parallel.

Therefore, the lines are neither perpendicular nor parallel.

Answer 20e.

We need to tell whether the lines are parallel, perpendicular, or neither.

Line 1: through $(-1,4)$ and $(2,5)$

Line 2: through $(-6,2)$ and $(0,4)$

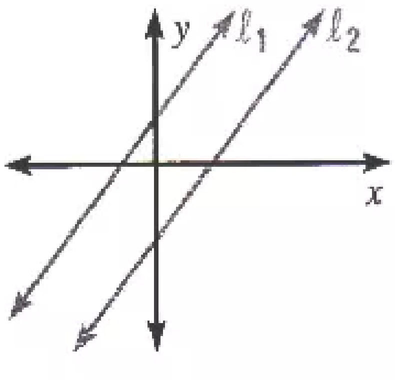
The slope m of a non-vertical line is the ratio of vertical change to horizontal change.
Such that

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{...... (1)}$$

Consider two different non-vertical lines l_1 and l_2 with slopes m_1 and m_2.
The lines are parallel if and only if they have the same slope. Such that

$$m_1 = m_2$$

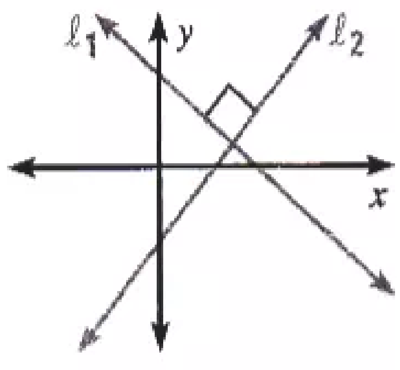
The graph is:



Again, the lines are perpendicular if and only if their slopes are negative reciprocals of each other. Such that

$$m_1 = -\frac{1}{m_2} \text{ or } m_1 m_2 = -1$$

The graph is:



Here, for the line 1:

$$(x_1, y_1) = (-1, 4) \text{ and } (x_2, y_2) = (2, 5)$$

Therefore using equation (1), we get the slope of the line passing through the above points is

$$\begin{aligned} m_1 &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5-4}{2+1} \\ &= \frac{1}{3} \end{aligned}$$

Again, for the line 2:

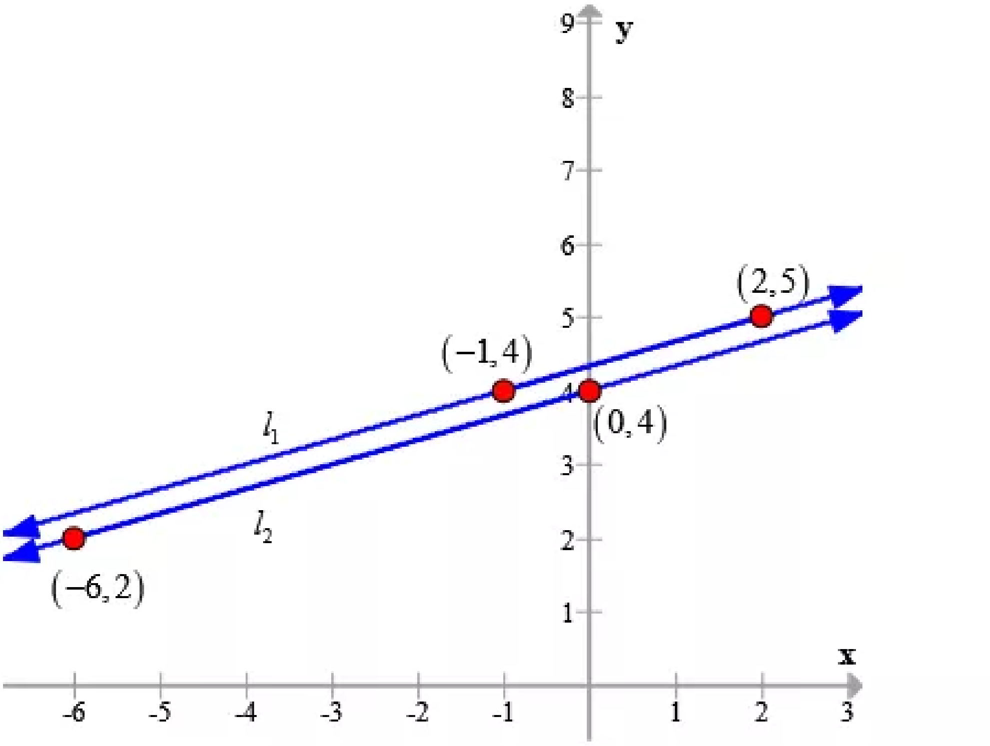
$(x_1, y_1) = (-6, 2)$ and $(x_2, y_2) = (0, 4)$

Therefore using equation (1), we get the slope of the line passing through the above points is

$$\begin{aligned} m_2 &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4-2}{-(-6)} \\ &= \frac{2}{6} \\ &= \frac{1}{3} \end{aligned}$$

Since $m_1 = m_2$ (and the lines are different). So, the lines are parallel.

The graph of the lines l_1 and l_2 is shown below:



Answer 21e.

Find the slope of line 1 that passes through (5, 8) and (7, 2).

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2-8}{7-5} \\ &= \frac{-6}{2} \\ &= -3 \end{aligned}$$

Find the slope of line 2 that passes through (−7, −2) and (−4, −1).

$$m = \frac{-1-(-2)}{-4-(-7)}$$
$$= \frac{-1+2}{-4+7}$$
$$= \frac{1}{3}$$

We know the lines are perpendicular if and only if their slopes are negative reciprocals of each other.

$$-3\left(\frac{1}{3}\right) = -1$$

Since the product of the slopes is−1, the given lines are perpendicular.

Answer 22e.

We need to tell whether the lines are parallel, perpendicular, or neither.

Line 1: through $(-3,2)$ and $(5,0)$

Line 2: through $(-1,-4)$ and $(3,-3)$

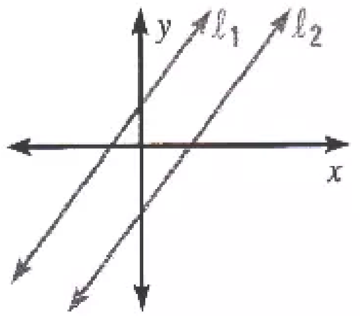
The slope m of a non-vertical line is the ratio of vertical change to horizontal change . Such that

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{......} (1)$$

Consider two different non-vertical lines l_1 and l_2 with slopes m_1 and m_2.
The lines are parallel if and only if they have the same slope. Such that

$$m_1 = m_2$$

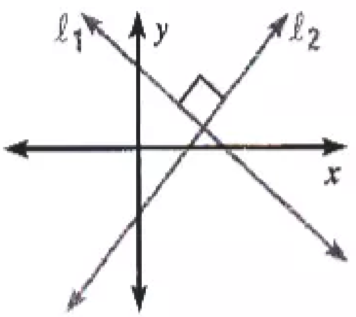
The graph is:



Again, the lines are perpendicular if and only if their slopes are negative reciprocals of each other. Such that

$$m_1 = -\frac{1}{m_2} \text{ or } m_1 m_2 = -1$$

The graph is:



Here, for the line 1:

$$(x_1, y_1) = (-3, 2) \text{ and } (x_2, y_2) = (5, 0)$$

Therefore using equation (1), we get the slope of the line passing through the above points is

$$\begin{aligned} m_1 &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2}{5+3} \\ &= -\frac{2}{8} \\ &= -\frac{1}{4} \end{aligned}$$

Again, for the line 2:

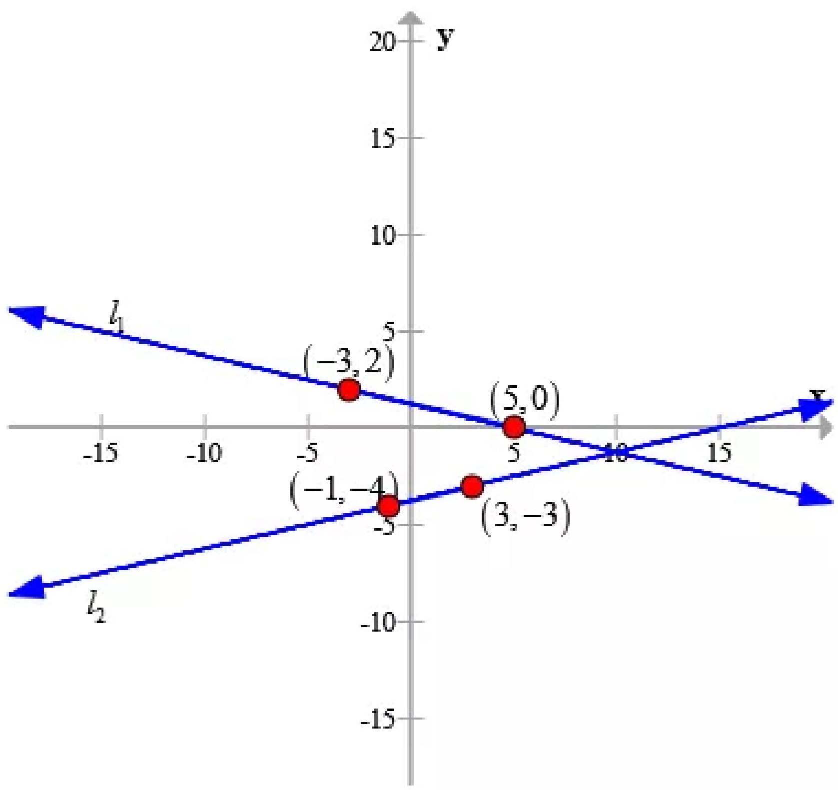
$$(x_1, y_1) = (-1, -4) \text{ and } (x_2, y_2) = (3, -3)$$

Therefore using equation (1), we get the slope of the line passing through the above points is

$$\begin{aligned} m_2 &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-3+4}{3+1} \\ &= \frac{1}{4} \end{aligned}$$

Since $m_1 \neq m_2$ and $m_1 m_2 \neq -1$, therefore the lines are neither parallel nor perpendicular.

The graph of the lines l_1 and l_2 is shown below:



Answer 23e.

Find the slope of line 1 that passes through (1, −4) and (4, −2).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-2 - (-4)}{4 - 1}$$
$$= \frac{-2 + 4}{4 - 1}$$
$$= \frac{2}{3}$$

Find the slope of line 2 that passes through (8, 1) and (14, 5).

$$m = \frac{5 - 1}{14 - 8}$$
$$= \frac{4}{6}$$
$$= \frac{2}{3}$$

We know the lines are perpendicular if and only if their slopes are negative reciprocals of each other.

$$\frac{2}{3}\left(\frac{2}{3}\right) = \frac{4}{9}$$

Since the product of the slopes is not −1, the given lines are not perpendicular.

The lines are parallel if and only if they have the same slope.

The given lines have the same slope. Therefore, they are parallel.

Answer 24e.

We need to find the average rate of change in y relative to x for the ordered pairs. $(2,12)$ and $(5,30)$, x is measured in hours and y is measured in dollars.

The average rate of change is how much one quantity changes, on average, relative to the change in another quantity. It is the ratio of two quantities that have different units.

Here, $(x_1, y_1) = (2,12)$ and $(x_2, y_2) = (5,30)$

The average rate of change in y relative to x is

$$\text{Average rate of change} = \frac{\text{change in dollars}}{\text{change in hours}(\text{time})}$$
$$= \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{(30-12)\text{ dollars}}{(5-2)\text{ hours}}$$
$$= \frac{18\text{ dollars}}{3\text{ hours}}$$
$$= 6\text{ dollars per hour}$$

$$\boxed{\text{Average rate of change} = 6\text{ dollars per hour}}$$

Answer 25e.

The average rate of change in y relative to x can be represented as $\frac{\text{change in } y}{\text{change in } x}$.

$$\frac{\text{Change in } y}{\text{Change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute (0, 11) for (x_1, y_1), and (3, 50) for (x_2, y_2).

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{50\text{ mi} - 11\text{ mi}}{3\text{ gal} - 0\text{ gal}}$$

Evaluate.

$$\frac{50\text{ mi} - 11\text{ mi}}{3\text{ gal} - 0\text{ gal}} = \frac{39\text{ mi}}{3\text{ gal}}$$
$$= 13\text{ mi/gal}$$

Therefore, the average rate of change is 13 mi/gal.

Answer 26e.

We need to find the average rate of change in y relative to x for the ordered pairs. $(3,10)$ and $(5,18)$, x is measured in seconds and y is measured in feet.

The average rate of change is how much one quantity changes, on average, relative to the change in another quantity. It is the ratio of two quantities that have different units.

Here, $(x_1, y_1) = (3, 10)$ and $(x_2, y_2) = (5, 18)$

The average rate of change in y relative to x is

$$\begin{aligned}\text{Average rate of change} &= \frac{\text{change in feet}}{\text{change in seconds (time)}} \\ &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{(18-10)\text{ feet}}{(5-3)\text{ seconds}} \\ &= \frac{8\text{ feet}}{2\text{ seconds}} \\ &= 4 \text{ feet per second}\end{aligned}$$

$$\boxed{\text{Average rate of change} = 4 \text{ feet per second}}$$

Answer 27e.

The average rate of change in y relative to x can be represented as $\frac{\text{change in } y}{\text{change in } x}$.

$$\frac{\text{Change in } y}{\text{Change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute (1, 8) for (x_1, y_1), and (7, 20) for (x_2, y_2).

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{20\text{ m} - 8\text{ m}}{7\text{ sec} - 1\text{ sec}}$$

Evaluate.

$$\begin{aligned}\frac{20\text{ m} - 8\text{ m}}{7\text{ sec} - 1\text{ sec}} &= \frac{12\text{ m}}{6\text{ sec}} \\ &= 2\text{ m/sec}\end{aligned}$$

Therefore, the average rate of change is 2 m/sec.

Answer 28e.

We need to explain why the two lines l_1 and l_2 must be non-vertical in case of parallel and perpendicular conditions.

In case of parallel and perpendicular conditions, two lines must be non-vertical because if they are vertical then slope will be undefined. But we have the conditions that:
Two lines are parallel if and only if they have the same slope.

$$m_1 = m_2$$

Again, two lines are perpendicular if and only if their slopes are negative reciprocals of each other.

$$m_1 = -\frac{1}{m_2}; \text{ or } m_1 m_2 = -1$$

Above conditions are satisfied if and only if the two lines are non-vertical.

Answer 29e.

Find the ratio of vertical change to horizontal change to get the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute $\left(-1, \frac{3}{2}\right)$ for (x_1, y_1), and $\left(0, \frac{7}{2}\right)$ for (x_2, y_2).

$$m = \frac{\frac{7}{2} - \frac{3}{2}}{0 - (-1)}$$

Evaluate.

$$\frac{\frac{7}{2} - \frac{3}{2}}{0 - (-1)} = \frac{\frac{4}{2}}{0 + 1}$$

$$= \frac{2}{1}$$

$$= 2$$

The slope of the line that passes through $\left(-1, \frac{3}{2}\right)$ and $\left(0, \frac{7}{2}\right)$ is 2.

Answer 30e.

We need to find the slope of the line passing through the given points.

$$\left(-\frac{3}{4}, -2\right), \left(\frac{5}{4}, -3\right)$$

The slope m of a non-vertical line is the ratio of vertical change to horizontal change.
Such that

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \ldots\ldots (1)$$

Here $(x_1, y_1) = \left(-\frac{3}{4}, -2\right)$ and $(x_2, y_2) = \left(\frac{5}{4}, -3\right)$.

Therefore using equation (1), we get the slope of the line passing through the given points is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-3 - (-2)}{\frac{5}{4} - \left(-\frac{3}{4}\right)}$$

$$= -\frac{1}{2}$$

$$\boxed{m = -\frac{1}{2}}$$

Answer 31e.

Find the ratio of vertical change to horizontal change to get the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute $\left(-\frac{1}{2}, \frac{5}{2}\right)$ for (x_1, y_1), and $\left(\frac{5}{2}, 3\right)$ for (x_2, y_2).

$$m = \frac{3 - \frac{5}{2}}{\frac{5}{2} - \left(-\frac{1}{2}\right)}$$

Evaluate.

$$\frac{3 - \frac{5}{2}}{\frac{5}{2} - \left(-\frac{1}{2}\right)} = \frac{3 - \frac{5}{2}}{\frac{5}{2} + \frac{1}{2}}$$

$$= \frac{\frac{1}{2}}{3}$$

$$= \frac{1}{6}$$

The slope of the line that passes through $\left(-\frac{1}{2}, \frac{5}{2}\right)$ and $\left(\frac{5}{2}, 3\right)$ is $\frac{1}{6}$.

Answer 32e.

We need to find the slope of the line passing through the given points.

$$(-4.2, 0.1), (-3.2, 0.1)$$

The slope m of a non-vertical line is the ratio of vertical change to horizontal change. Such that

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{...... (1)}$$

Here $(x_1, y_1) = (-4.2, 0.1)$ and $(x_2, y_2) = (-3.2, 0.1)$.

Therefore using equation (1), we get the slope of the line passing through the given points is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0.1 - 0.1}{-3.2 - (-4.2)}$$

$$= 0$$

$$\boxed{m = 0}$$

Answer 33e.

Find the ratio of vertical change to horizontal change to get the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute (–0.3, 2.2) for (x_1, y_1), and (1.7, –0.8) for (x_2, y_2) and evaluate.

$$\begin{aligned} m &= \frac{-0.8 - 2.2}{1.7 - (-0.3)} \\ &= \frac{-0.8 - 2.2}{1.7 + 0.3} \\ &= -\frac{3}{2} \end{aligned}$$

The slope of the line that passes through (–0.3, 2.2) and (1.7, –0.8) is $-\frac{3}{2}$.

Answer 34e.

We need to find the slope of the line passing through the given points.

$(3.5,-2),(4.5,0.5)$

The slope m of a non-vertical line is the ratio of vertical change to horizontal change. Such that

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \ldots\ldots (1)$$

Here $(x_1, y_1) = (3.5, -2)$ and $(x_2, y_2) = (4.5, 0.5)$

Therefore using equation (1), we get the slope of the line passing through the given points is

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{0.5 + 2}{4.5 - 3.5} \\ &= \frac{2.5}{1.5} \\ &= 1.67 \end{aligned}$$

$$\boxed{m = 1.67}$$

Answer 35e.

The slope of a nonvertical line is the ratio of vertical change (the rise) to horizontal change (the run). The ratio of vertical change to horizontal change remains the same for any two points on a line. Therefore, it does not make sense which two points on a line we choose to find the slope.

The formula for the slope of a line is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Factor out -1 from both the numerator and the denominator and simplify.

$$\begin{aligned} m &= \frac{-1(y_1 - y_2)}{-1(x_1 - x_2)} \\ &= \frac{y_1 - y_2}{x_1 - x_2} \end{aligned}$$

Therefore, it does not make a difference which point is (x_1, y_1) and which point is (x_2, y_2) in the formula for slope.

Find the slope of PQ.
Substitute (−1, 1) for (x_1, y_1), and (−3, 2) for (x_2, y_2) in the slope formula and evaluate.

$$m = \frac{2-1}{-3-(-1)}$$
$$= \frac{2-1}{-3+1}$$
$$= -\frac{1}{2}$$

Find the slope of QR.
Let (x_1, y_1) be (1, 0), and (x_2, y_2) be (−1, 1).

$$m = \frac{1-0}{-1-1}$$
$$= -\frac{1}{2}$$

Find the slope of RS.
Let (x_1, y_1) be (3, −1) and(x_2, y_2) be (1, 0).

$$m = \frac{0-(-1)}{1-3}$$
$$= \frac{0+1}{1-3}$$
$$= -\frac{1}{2}$$

Therefore, we can conclude that the slopes are the same for three different pairs of points on the line.

Answer 36e.

We need to find two additional points on the line that passes through the given point and has a slope of -4.

$$(0,3)$$

The slope m of a non-vertical line is the ratio of vertical change to horizontal change. Such that

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{......} (1)$$

Here, given that $(x_1, y_1) = (0,3)$

Therefore using equation (1), we get the two additional points are

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-4 = \frac{y_2 - 3}{x_2}$$

$$\frac{-4}{1} = \frac{y_2 - 3}{x_2}$$

Comparing both sides, we have

$$y_2 = -1$$

$$x_2 = 1$$

If we put the above values in equation (1), we have

$$(x_1, y_1) = (0,3) \text{ and } (x_2, y_2) = (1,-1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-1-3}{1-0}$$

$$= -4$$

Therefore, we observed that by putting the additional two points, we got the same slope as given.

Answer 37e.

Substitute $(2, -3)$ for (x_1, y_1), $(k, 7)$ for (x_2, y_2), and -2 for m in the formula for the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-2 = \frac{7 - (-3)}{k - 2}$$

Simplify.

$$-2 = \frac{10}{k - 2}$$

Multiply each side by $k - 2$.

$$-2(k - 2) = \frac{10}{k - 2}(k - 2)$$

$$-2k + 4 = 10$$

Subtract 4 from both the sides.

$$-2k + 4 - 4 = 10 - 4$$

$$-2k = 6$$

Divide both the sides by -2.

$$\frac{-2k}{-2} = \frac{6}{-2}$$

$$k = -3$$

Check the value of k by substituting in $-2 = \dfrac{10}{k - 2}$.

$$-2 \stackrel{?}{=} \frac{10}{-3 - 2}$$

$$-2 \stackrel{?}{=} \frac{10}{-5}$$

$$-2 \stackrel{?}{=} -2 \qquad \text{TRUE}$$

Thus, the solution checks.

Answer 38e.

We need to find the value of k so that the line through the given points has the given slope and check the solution.

$$(0, k), (3, 4); m = 1$$

The slope m of a non-vertical line is the ratio of vertical change to horizontal change. Such that

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{...... (1)}$$

Here $(x_1, y_1) = (0, k)$ and $(x_2, y_2) = (3, 4)$.

Therefore using equation (1), we get the value of k so that the line through the above points has the given slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$1 = \frac{4 - k}{3}$$

$$k = 1$$

$$\boxed{k = 1}$$

Now, putting the value of k in equation (1), we get

$$(x_1, y_1) = (0,1) \text{ and } (x_2, y_2) = (3,4).$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{4-1}{3-0}$$
$$= \frac{3}{3}$$
$$= 1$$

Therefore, we observed that by putting the value of k we got the same slope (given).

Answer 39e.

Substitute $(-4, 2k)$ for (x_1, y_1), $(k, -5)$ for (x_2, y_2), and -1 for m in the formula for the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$-1 = \frac{-5 - 2k}{k - (-4)}$$

Simplify.

$$-1 = \frac{-5 - 2k}{k + 4}$$

Multiply each side by $k + 4$.

$$-1(k + 4) = \frac{-5 - 2k}{k + 4}(k + 4)$$
$$-k - 4 = -5 - 2k$$

Add 4 to both the sides.

$$-k - 4 + 4 = -5 - 2k + 4$$
$$-k = -1 - 2k$$

Add $2k$ to both the sides.

$$-k + 2k = -1 - 2k + 2k$$
$$k = -1$$

Check the value of k by substituting in $-1 = \frac{-5 - 2k}{k + 4}$.

$$-1 \stackrel{?}{=} \frac{-5 - 2(-1)}{-1 + 4}$$
$$-1 \stackrel{?}{=} \frac{-5 + 2}{-1 + 4}$$
$$-1 \stackrel{?}{=} \frac{-3}{3}$$
$$-1 \stackrel{?}{=} -1 \qquad \text{TRUE}$$

Thus, the solution checks.

Answer 40e.

We need to find the value of k so that the line through the given points has the given slope and check the solution.

$$(-2,k),(2k,2);m=-0.25$$

The slope m of a non-vertical line is the ratio of vertical change to horizontal change. Such that

$$m=\frac{y_2-y_1}{x_2-x_1} \qquad \text{...... (1)}$$

Here $(x_1,y_1)=(-2,k)$ and $(x_2,y_2)=(2k,2)$.

Therefore using equation (1), we get the value of k so that the line through the above points has the given slope.

$$m=\frac{y_2-y_1}{x_2-x_1}$$
$$-0.25=\frac{2-k}{2k+2}$$
$$k=5$$
$$\boxed{k=5}$$

Now, putting the value of k in equation (1), we get

$$(x_1,y_1)=(-2,5)\text{and}(x_2,y_2)=(10,2)$$
$$m=\frac{y_2-y_1}{x_2-x_1}$$
$$=\frac{2-5}{10+2}$$
$$=\frac{-3}{12}$$
$$=-0.25$$

Therefore, we observed that by putting the value of k we got the same slope (given).

Answer 41e.

Find the ratio of vertical change (rise) to horizontal change (run) to get the slope.

$$m = \frac{\text{rise}}{\text{run}}$$

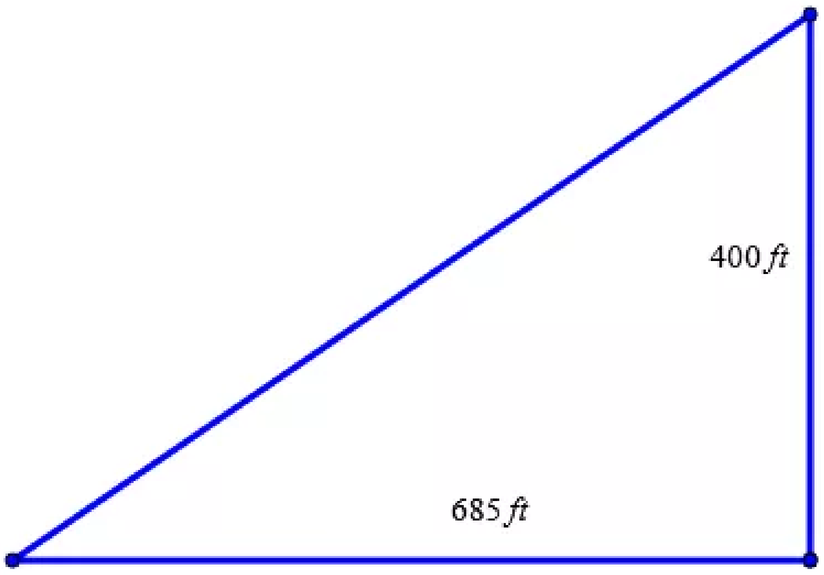
From the given information, we note that the vertical change is 28 ft, and the horizontal change is 48 ft.

$$m = \frac{28}{48}$$
$$= \frac{7}{12}$$

Therefore, the slope of the escalator is $\frac{7}{12}$.

Answer 42e.

It is given that a cable car railway rises 400 ft over a horizontal distance of 685 ft on its ascent to a n overlook of Pittsburgh, Pennsylvania, We need to slope of the incline
The diagram is shown below:



The slope m of a non vertical line is the ratio of vertical change to the horizontal change .

$$m = \frac{\text{verticalchange}}{\text{horizontalchange}}$$

$$m = \frac{400}{685}$$

$$\boxed{m = 0.584}$$

Answer 43e.

The grade of the road is equal to the slope of the road. The slope is the ratio of vertical change (rise) to horizontal change (run).

$$m = \frac{\text{rise}}{\text{run}}$$

From the given information, we note that the rise is 195 ft and run is 3000 ft.

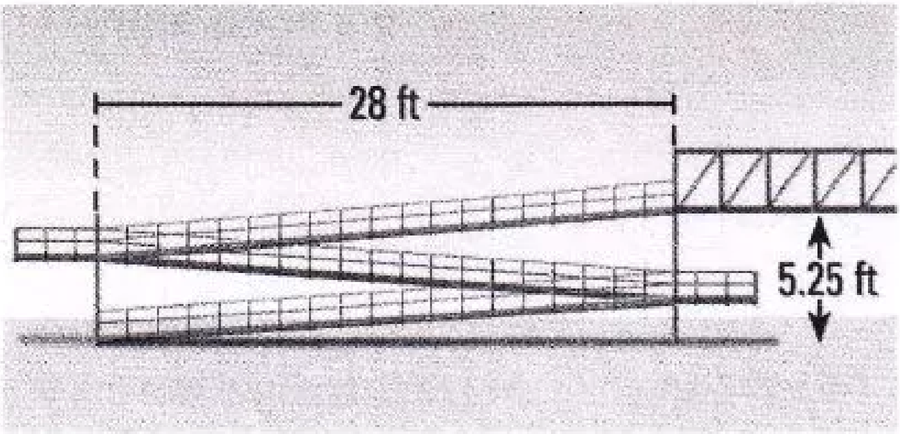
$$m = \frac{195}{3000}$$

Simplify.
$m = 0.065$ or 6.5%

Therefore, the grade of the road is 6.5%.

Answer 44e.

The diagram is shown below:



It is given that the diagram shows a three section ramp to a bridge and each section of the ramp has the same slope.
The benefits of the three section ramp is that the person can walk comfortably.

Answer 45e.

The change in the amount of propane is from 400 gallons to 214 gallons in 30 days.

$$\text{Average rate of change} = \frac{\text{Change in gallons}}{\text{Change in days}}$$

$$= \frac{214 \text{ gallons} - 400 \text{ gallons}}{30 - 0}$$

Simplify.

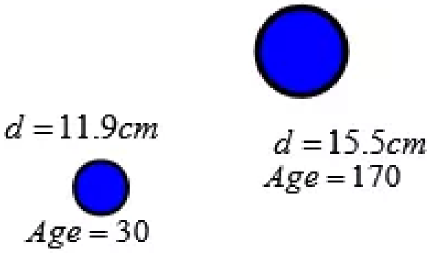
$$\text{Average rate of change} = \frac{-186 \text{ gallons}}{30 \text{ days}}$$

$$= -6.2 \text{ gallons per day}$$

The correct answer is choice **A**.

Answer 46e.

It is given that a red urchin grows its entire life which can last 200 years. The diagram gives information about the growth in the diameter d of one of the red urchin. We need to find the average growth rate of this urchin over the given period.



The average rate of change

$$= \frac{\text{change in diameter}}{\text{change in time}}$$

$$= \frac{15.5 - 11.9}{170 - 30}$$

$$= \frac{3.6}{140}$$

$$= 0.0257$$

The average growth rate of the red urchin over the given period is

$$\boxed{0.0257}$$

Answer 47e.

a. The rise of the roof is 15 ft and its run is 40 ft.
The ratio of rise to run is the slope.

$$m = \frac{\text{rise}}{\text{run}}$$

$$= \frac{15}{40}$$

$$= \frac{3}{8}$$

The slope of the roof is $\frac{3}{8}$.

b. Find the minimum slope required for the building code.

The rise is 3 ft, and the run is 12 ft.

$$m = \frac{3}{12} = \frac{1}{4}$$

The building code requires a minimum slope of $\frac{1}{4}$.

From part (a), we have the slope of the roof as $\frac{3}{8}$.

Since $\frac{3}{8} > \frac{1}{4}$, the slope of the roof exceeds the minimum slope required.

Thus, the roof satisfies the building code.

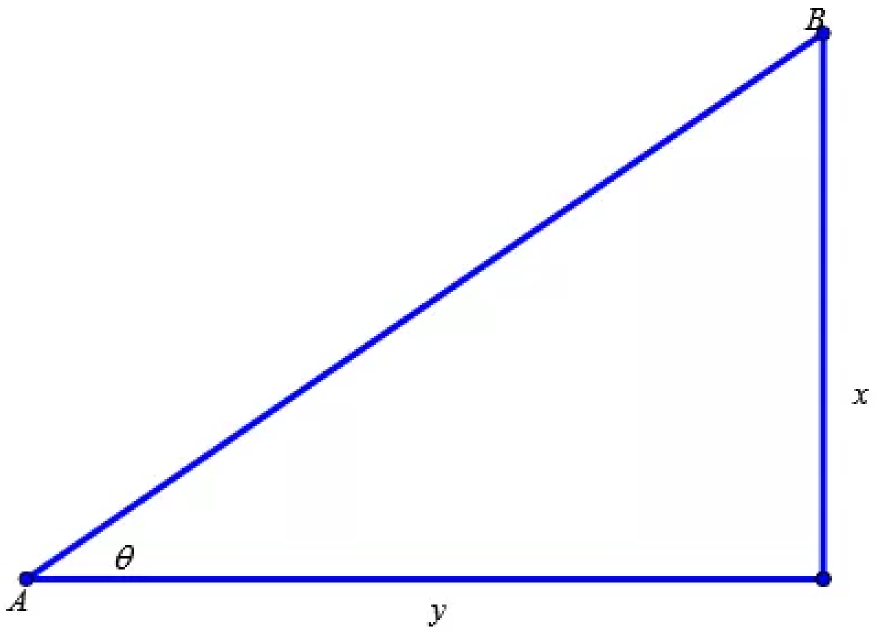
c. Subtract $\frac{1}{4}$ from $\frac{3}{8}$.

$$\frac{3}{8} - \frac{1}{4} = \frac{1}{8}$$

Therefore, the rise exceeds the code minimum by $\frac{1}{8}$.

Answer 48e.

It is given that a new water slide in an amusement park call for the slide to descend from a platform 80 *ft* tall and the slide will drop 1 *foot* for every 3 *ft* of horizontal distance.
We need to find the horizontal distance when descending the slide, the length of the slide.
The diagram is shown below:



From the given conditions, we can form a statement

$$y = 3x \qquad \ldots\ldots (1)$$

When $x = 80\,ft$

Substituting the value of x in the equation (1) we have

$$y = 3 \times 80$$
$$= 240$$

Therefore the horizontal distance is $\boxed{240\,ft}$

Using the Pythagoras theorem, we have

$$(\text{Hypotenuse distance})^2 = (\text{Horizontal distance})^2 + (\text{Perpendicular distance})^2$$

From the diagram,

$$AB^2 = x^2 + y^2$$
$$= 80^2 + 240^2$$
$$= 64000$$
$$= 80\sqrt{10}$$

Therefore the length of the slide is $\boxed{80\sqrt{10}\,ft}$

If the length of the horizontal distance y is shorten by 5 ft, then the angle θ will increase.

Answer 49e.

The amount of gasoline used in driving 1 mile on the highway is $\frac{1}{36}$ gallons, and that on the city is $\frac{1}{24}$ gallons.

Let x be the equal distance traveled on the highway and in the city.

The amount of gasoline used in driving x miles on the highway is $\frac{x}{36}$ gallons, and that on the city is $\frac{x}{24}$ gallons.

Add $\frac{x}{36}$ and $\frac{x}{24}$ to find the total amount of gasoline used.

$$\frac{x}{24}+\frac{x}{36}=\frac{5x}{72}$$

Divide the total distance traveled by the total amount of gasoline used to find the average fuel efficiency for all the driving.

The total distance traveled is $2x$ miles.

$$\frac{2x}{\frac{5x}{72}}=\frac{144}{5}$$

$$=28.8$$

Therefore, the average fuel efficiency for all the driving is 28.8 miles per gallon.

Answer 50e.

The given statement is

$$5(8+12)=5(8)+5(12) \qquad \ldots\ldots (1)$$

By distributive law, we have

$$a(b+c)=a\cdot b+a\cdot c$$

From the equation (1), we have

$$a=5, b=8, c=12$$

Therefore the given statement is $5(8+12)=5(8)+5(12)$ by distributive law.

Answer 51e.

Let us consider 7 as a, 9 as b, and 13 as c. Then, the given statement is of the form $(a+b)+c = a+(b+c)$. This is the associative property of addition. According to this property, the way in which we group the numbers does not affect the sum.

Therefore, the associative property of addition justifies the given statement.

Answer 52e.

The given statement is

$$4+(-4)=0 \qquad \ldots\ldots (1)$$

By additive inverse law, we have

$$a+(-a)=0$$

From the equation (1), we have

$$a=4$$

Therefore the given statement is $4+(-4)=0$ $\boxed{\text{by additive inverse law.}}$

Answer 53e.

Let us consider 5 as a, and 10 as b. Then, the given statement is of the form $a \cdot b = b \cdot c$. This is the commutative property of multiplication. According to this property, the way in which we order the numbers does not affect the product.

Therefore, the commutative property of multiplication justifies the given statement.

Answer 54e.

The given statement is

$$15\cdot\left(\frac{1}{15}\right)=1 \qquad \ldots\ldots (1)$$

By multiplicative inverse law, we have

$$a\cdot\frac{1}{a}=1$$

From the equation (1), we have

$$a=15$$

Therefore the given statement is $15\cdot\left(\frac{1}{15}\right)=1$ $\boxed{\text{by multiplicative inverse law.}}$

Answer 55e.

By the identity property of multiplication, any number multiplied by 1 gives that number itself.

$a \cdot 1 = a$

Therefore, the identity property of multiplication justifies the given statement.

Answer 56e.

To solve the equation for y

$$3x + y = 7 \qquad \ldots\ldots (1)$$

To isolate the terms containing y we subtract $-3x$ to both the sides of the equation (1), we have

$$3x + y - 3x = 7 - 3x$$

$$y = 7 - 3x$$

The required solution of $\boxed{y = 7 - 3x}$

Answer 57e.

Subtract $2x$ from both the sides.

$$2x - y - 2x = 3 - 2x$$

$$-y = 3 - 2x$$

Divide both the sides by -1.

$$\frac{-y}{-1} = \frac{3 - 2x}{-1}$$

$$y = -3 + 2x$$

Therefore, the solution for y is $-3 + 2x$.

Answer 58e.

To solve the equation for y

$$y - 4x = -6 \qquad \ldots\ldots (1)$$

To isolate the terms containing y we add $4x$ to both the sides of the equation (1), we have

$$y - 4x + 4x = -6 + 4x$$

$$y = -6 + 4x$$

The required solution of $\boxed{y = -6 + 4x}$

Answer 59e.

Subtract $2x$ from both the sides.

$$2x + 3y - 2x = -12 - 2x$$

$$3y = -12 - 2x$$

Divide both the sides by 3.

$$\frac{3y}{3} = \frac{-12 - 2x}{3}$$

$$y = -4 - \frac{2}{3}x$$

Therefore, the solution for y is $-4 - \frac{2}{3}x$.

Answer 60e.

To solve the equation for y

$$7x-4y=10 \qquad \ldots\ldots (1)$$

To isolate the terms containing y we add $-7x$ to both the sides of the equation (1), we have

$$7x-4y-7x=10-7x$$

$$-4y=10-7x$$

$$y=\frac{10}{-4}-\frac{7x}{(-4)} \qquad \text{[Dividing both sides by} -4\text{]}$$

$$y=-\frac{5}{2}+\frac{7}{4}x \qquad \text{[Dividing the first term of the right hand side by 2]}$$

The required solution of $\boxed{y=-\frac{5}{2}+\frac{7}{4}x}$

Answer 61e.

Add x to both the sides.

$$-x+2y+x=9+x$$

$$2y=9+x$$

Divide both the sides by 2.

$$\frac{2y}{2}=\frac{9+x}{2}$$

$$y=\frac{9}{2}+\frac{1}{2}x$$

Therefore, the solution for y is $\frac{9}{2}+\frac{1}{2}x$.

Answer 62e.

To solve the equation

$$|5+2x|=7$$

The equations are

$$5+2x=7 \quad \ldots\ldots (1)$$

$$5+2x=-7 \quad \ldots\ldots (2)$$

From the equation (1), we have

$$5+2x=7$$

Adding -5 to both sides of the equation $5+2x=7$, we have

$$5+2x-5=7-5$$

$$2x=2$$

$$x=\frac{2}{2}$$

$$x=1$$

From the equation (2), we have

$$5+2x=-7$$

Adding -5 to both sides of the equation $5+2x=-7$, we have

$$5+2x-5=-7-5$$

$$2x=-12$$

$$x=\frac{-12}{2}$$

$$x=-6$$

Therefore the solution of $\boxed{x=1,-6}$

Answer 63e.

The equation $|ax+b|=c$ where $c>0$ is equivalent to $ax+b=c$ or $ax+b=-c$.
Thus, $|4x-9|=5$ is equivalent to $4x-9=5$ or $4x-9=-5$.

Solve each equation for x.
Add 9 to both sides of both the equations.

$$4x-9+9=5+9 \quad \text{or} \quad 4x-9+9=-5+9$$

$$4x=14 \qquad\qquad 4x=4$$

Divide both the sides by 4.

$$\frac{4x}{4}=\frac{14}{4} \quad \text{or} \quad \frac{4x}{4}=\frac{4}{4}$$

$$x=\frac{7}{2} \qquad\qquad x=1$$

Check the solution by substituting in the original equation and simplify.

$$|4x-9| = 5$$
$$\left|4\left(\frac{7}{2}\right)-9\right| \stackrel{?}{=} 5$$
$$|14-9| \stackrel{?}{=} 5$$
$$|5| \stackrel{?}{=} 5$$
$$5 \stackrel{?}{=} 5 \checkmark$$

$$|4x-9| = 5$$
$$|4(1)-9| \stackrel{?}{=} 5$$
$$|4-9| \stackrel{?}{=} 5$$
$$|-5| \stackrel{?}{=} 5$$
$$5 \stackrel{?}{=} 5 \checkmark$$

Therefore, the solutions are 1 and $\frac{7}{2}$.

Answer 64e.

To solve the equation

$$|6-5x| = 9$$

The equations are

$$6-5x=9 \quad \text{...... (1)}$$
$$6-5x=-9 \quad \text{...... (2)}$$

From the equation (1), we have

$$6-5x=9$$

Adding -6 to both sides of the equation $6-5x=9$, we have

$$6-5x-6=9-6$$
$$-5x=3$$
$$x=\frac{3}{-5}$$
$$x=-0.6$$

From the equation (2), we have

$$6-5x=-9$$

Adding -6 to both sides of the equation $6-5x=-9$, we have

$$6-5x-6=-9-6$$
$$-5x=-15$$
$$x=\frac{-15}{-5}$$
$$x=3$$

Therefore the solution of $\boxed{x=3,-0.6}$

Answer 65e.

The inequality $|ax+b| < c$ is equivalent to $-c < ax+b < c$.

Thus, $|3-7x| < 10$ is equivalent to $-10 < 3-7x < 10$.

Solve the equation for x.

Subtract 3 from each part of the inequality.

$$-10-3 < 3-7x-3 < 10-3$$

$$-13 < -7x < 7$$

Divide each part by -7.

$$\frac{-13}{-7} > \frac{-7x}{-7} > \frac{7}{-7}$$

$$\frac{13}{7} > x > -1$$

The solutions are all the numbers between -1 and $\frac{13}{7}$.

Answer 66e.

We need to solve the inequality

$$|3x+1| > 25$$

Using $|x| > a$, where $a > 0$

$$x > a \text{ or } x < -a$$

The equations are

$$3x+1 > 25 \quad \ldots\ldots (1)$$

$$3x+1 < -25 \quad \ldots\ldots (2)$$

From the equation (1), we have

$$3x+1 > 25$$

Adding -1 to both sides of the equation $3x+1>25$, we have

$$3x+1-1 > 25-1$$

$$3x > 24$$

$$x > \frac{24}{3}$$

$$x > 8$$

From the equation (2), we have

$$3x+1 < -25$$

Adding -1 to both sides of the equation $3x+1<-25$, we have

$$3x+1-1 < -25-1$$

$$3x < -26$$

$$x < \frac{-26}{3}$$

Therefore the solution of $\boxed{x = \left(-\infty, -\frac{26}{3}\right) \cup (8, \infty)}$

Answer 67e.

The inequality $|ax+b| \geq c$ is equivalent to $ax+b \leq -c$ or $ax+b \geq c$.

Thus, $|3-4x| \geq 7$ is equivalent to $3-4x \leq -7$ or $3-4x \geq 7$.

Solve each equation for x.

Subtract 3 from each part of the inequalities.

$$3-4x-3 \leq -7-3 \qquad \text{or} \qquad 3-4x-3 \leq 7-3$$

$$-4x \leq -10 \qquad\qquad -4x \geq 4$$

Divide each part by −4.

$$\frac{-4x}{-4} \geq \frac{-10}{-4} \qquad \text{or} \qquad \frac{-4x}{-4} \leq \frac{4}{-4}$$

$$x \geq \frac{5}{2} \qquad\qquad x \leq -1$$

The solutions are all the numbers greater than or equal to $\frac{5}{2}$ and all the numbers less than or equal to -1.